DESIGN, FABRICATION, TEST, AND EVALUATION OF SMALL-SCALE
TILTROTOR WHIRL FLUTTER WIND TUNNEL MODELS

A Thesis in
Aerospace Engineering
by
Guillermo J. Costa

© 2015 Guillermo J. Costa

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

August 2015
The thesis of Guillermo J. Costa was reviewed and approved* by the following:

Edward C. Smith  
Professor of Aerospace Engineering  
Thesis Advisor

Jose L. Palacios  
Assistant Professor of Aerospace Engineering

George Lesieutre  
Professor of Aerospace Engineering  
Head of the Department of Aerospace Engineering

*Signatures are on file in the Graduate School.
Abstract

The modern tiltrotor is an aircraft capable of efficient hover as well as high-speed forward flight. The large, flexible rotors needed for good hover performance are susceptible to an aeroelastic phenomenon known as whirl flutter in high-speed flight: high rotor inflow and the flexible nature of the rotors will result in negative aerodynamic damping when the tiltrotor is in airplane mode, causing the coupled wing/rotor/pylon system to become unstable. Traditionally, the tiltrotor whirl flutter margin has been increased through the use of thick wings of low aspect ratio, which maximize the stiffness of the wings at the expense of increased structural weight and reduced aerodynamic efficiency. To increase the top speed of tiltrotors, new methods of analyzing and mitigating the whirl flutter phenomenon must be developed.

This study focuses on the design and development of a semispan wind tunnel model that permits the testing of whirl flutter stability in a controlled, low-risk environment. The wind tunnel model was dynamically tested in the Penn State Hammond Low-Speed Wind Tunnel. Five configurations were tested for modal damping variations with respect to tunnel speed. Two of these configurations (“Gen-2”) were modified versions of the first-generation (“Gen-1”) wind tunnel model designed and tested by S. C. Johnson in the 2012-2013 timeframe. Three configurations of an all-new model (“Gen-3”) were developed to increase the realism of the test specimen, and included features such as tuned wing modal frequencies, higher-Lock number rotor blades, and blade twist. Most importantly, the Gen-3 models were designed to flutter without the use of an added mass or unconventional center of gravity placement.

Gen-3 was developed through the use of an in-house computer code, which generated damping predictions for the wing modes at various tunnel speeds. Model components and support equipment were fabricated in-house by the author, with some components (e.g. those requiring computer-controlled machining) outsourced to local manufacturers. Wind tunnel tests were performed in the Hammond tunnel from October 2014 through January 2015; these tests showed that the experimentally-measured flutter speeds of the Gen-3 models were within 5-7% of the predicted values, with the Gen-3a (untwisted composite blades) fluttering at approximately 101 ft./sec., and the Gen-3b model (twisted composite blades) fluttering at approximately 97 ft./sec.

The present work has further validated the feasibility of whirl flutter testing at a small scale. The Gen-3 model was able to exhibit whirl flutter without the use of an additional mass, and will permit the testing of devices designed to enhance the whirl flutter margin within the current facility. The techniques used to fabricate the Gen-3 model show potential for introducing features such as blade flexibility and an modular wing root flexure for a wider test regime.
Table of Contents

List of Tables vii
List of Figures viii
List of Symbols xii
Acknowledgments xvi

Chapter 1
Introduction 1
1.1 Historical Background .................................................. 1
1.2 Previous Wind Tunnel Experiments at Penn State: The Generation-1 Whirl Flutter Model .................................................. 13
1.3 Scope of Present Work and Project Objectives ......................... 18
  1.3.1 Rotor .............................................................. 18
  1.3.2 Wing .............................................................. 19
  1.3.3 Control system ..................................................... 19
  1.3.4 Experimental validation ............................................ 19

Chapter 2
Model Design and Fabrication 21
2.1 Model Design .................................................................. 21
  2.1.1 Design methodology ................................................ 22
  2.1.2 Rotor design .......................................................... 23
    2.1.2.1 Gimbaled hub ............................................... 23
    2.1.2.2 Rotor blade design ......................................... 29
    2.1.2.3 Composite rotor blade fabrication ....................... 37
  2.1.3 Wing design and fabrication ....................................... 49
2.2 Control System .................................................................. 60
  2.2.1 Rotor electromechanical control ................................... 60
  2.2.2 Operating range ....................................................... 64
2.3 Inputs to Analytical Model .................................................. 68
  2.3.1 Operational inputs ................................................... 68
  2.3.2 Rotor inputs .......................................................... 68
## Chapter 3
**Experimental Setup and Stability Predictions**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>71</td>
</tr>
<tr>
<td>3.2</td>
<td>76</td>
</tr>
<tr>
<td>3.3</td>
<td>79</td>
</tr>
<tr>
<td>3.4</td>
<td>84</td>
</tr>
<tr>
<td>3.5</td>
<td>88</td>
</tr>
<tr>
<td>3.6</td>
<td>95</td>
</tr>
<tr>
<td>3.7</td>
<td>98</td>
</tr>
<tr>
<td>3.8</td>
<td>102</td>
</tr>
<tr>
<td>3.8.1</td>
<td></td>
</tr>
<tr>
<td>3.8.2</td>
<td>105</td>
</tr>
</tbody>
</table>

## Chapter 4
**Results and Discussion**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>109</td>
</tr>
<tr>
<td>4.2</td>
<td>111</td>
</tr>
<tr>
<td>4.3</td>
<td>117</td>
</tr>
<tr>
<td>4.4</td>
<td>124</td>
</tr>
<tr>
<td>4.5</td>
<td>130</td>
</tr>
<tr>
<td>4.5.1</td>
<td></td>
</tr>
<tr>
<td>4.5.2</td>
<td>133</td>
</tr>
<tr>
<td>4.6</td>
<td>137</td>
</tr>
<tr>
<td>4.6.1</td>
<td></td>
</tr>
<tr>
<td>4.6.2</td>
<td>140</td>
</tr>
<tr>
<td>4.6.3</td>
<td>152</td>
</tr>
<tr>
<td>4.6.4</td>
<td>154</td>
</tr>
<tr>
<td>4.6.5</td>
<td>158</td>
</tr>
</tbody>
</table>

## Chapter 5
**Conclusions and Recommendations for Future Work**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>162</td>
</tr>
<tr>
<td>5.2</td>
<td>165</td>
</tr>
<tr>
<td>5.3</td>
<td>168</td>
</tr>
</tbody>
</table>

## Appendix A
**System Dynamics**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>171</td>
</tr>
<tr>
<td>A.2</td>
<td>175</td>
</tr>
<tr>
<td>A.3</td>
<td>179</td>
</tr>
<tr>
<td>A.4</td>
<td>185</td>
</tr>
</tbody>
</table>

## Appendix B
**Mechanics of Composite Structures**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1</td>
<td>189</td>
</tr>
<tr>
<td>B.2</td>
<td>194</td>
</tr>
<tr>
<td>B.2.1</td>
<td>194</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>B.2.2</td>
<td>Maximum strain criterion</td>
</tr>
<tr>
<td>B.2.3</td>
<td>Quadratic interaction criteria</td>
</tr>
<tr>
<td>B.3</td>
<td>Classical Laminated Plate Theory</td>
</tr>
<tr>
<td>B.4</td>
<td>Sandwich Panel Structures</td>
</tr>
<tr>
<td>B.5</td>
<td>Equivalent Properties of Composite Structures</td>
</tr>
<tr>
<td>B.5.1</td>
<td>Equivalent properties of symmetric laminates</td>
</tr>
<tr>
<td>B.5.2</td>
<td>Equivalent properties of nonsymmetric laminates</td>
</tr>
</tbody>
</table>

**Appendix C**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.1</td>
<td>Rotor Dynamics</td>
<td>210</td>
</tr>
<tr>
<td>C.2</td>
<td>Rotor Aerodynamics and Blade Element Theory</td>
<td>215</td>
</tr>
<tr>
<td>C.3</td>
<td>Wing Dynamics</td>
<td>220</td>
</tr>
<tr>
<td>C.4</td>
<td>Wing/Rotor Coupling</td>
<td>225</td>
</tr>
</tbody>
</table>

**Bibliography** | 228
List of Tables

1.1 Gen-2 and Gen-3 test configurations. ........................................ 20
2.1 Composite material properties. ................................................ 32
2.2 Gen-1 measured modal frequencies and Gen-3 targeted modal frequencies. ..... 49
2.3 Geometric and material properties of Gen-3 spar. .......................... 52
3.1 OIML Class M2 maximum allowable mass variations, from [70]. .............. 73
3.2 Applied and recorded masses for UniShow gram scale. ........................ 74
3.3 Operating ranges and resolutions of ATI Delta load cell, from [74]. ........... 77
3.4 Pressure transducer characterization results. ................................... 81
3.5 Pitot-static transducer voltages by tunnel operating percentage, empty tunnel. . 85
3.6 Pitot-static transducer voltages by tunnel operating percentage, Gen-2a model tunnel. .......................................................... 85
4.1 Measured masses of composite blades. ......................................... 110
4.2 Lock number results of composite blades. ..................................... 110
4.3 Masses used for static stiffness testing. ....................................... 112
4.4 Blade deflections versus applied loads. ....................................... 112
4.5 Vibration test results for untwisted composite blades. ......................... 119
4.6 One-way ANOVA of untwisted composite blades, 95% confidence level. .......... 119
4.7 Vibration test results for twisted composite blades. ........................ 120
4.8 One-way ANOVA of twisted composite blades, 95% confidence level. .......... 120
4.9 Gen-3 rap test frequency and damping results. ............................... 125
4.10 Comparison of Gen-1 and Gen-2 model properties. Aft-mass c.g. located 5.5 in. aft of wing elastic axis. .............................. 131
4.11 Gen-2 trim pitch versus tunnel speed. ....................................... 131
4.12 Gen-2 damping versus tunnel speed. Flutter regime in red. .................. 135
4.13 Gen-3 model properties by configuration. .................................... 138
4.14 Gen-3a trim pitch versus tunnel speed. .................................... 138
4.15 Comparison of Gen-1 and Gen-3 measured wing stiffness values. ............ 143
4.16 Gen-3a wing beamwise damping versus tunnel speed. ......................... 143
4.17 Gen-3a wing chordwise damping versus tunnel speed. Flutter regime in red. .... 144
4.18 Gen-3a wing torsional damping versus tunnel speed. ........................ 144
4.19 Gen-3b trim pitch versus tunnel speed. .................................... 152
4.20 Gen-3b modal damping versus tunnel speed. Flutter regime in red. .......... 155
4.21 Gen-3c wing beamwise damping versus tunnel speed. ......................... 159
5.1 Wind tunnel model features by generation. .................................. 164
List of Figures

1.1 Hover efficiency versus disk loading of VTOL aircraft, from [2] ......................... 8
1.2 Transcendental 1-G in hover ................................................................. 8
1.3 Bell XV-3 in hover ................................................................................. 9
1.4 Bell 25-ft. diameter proprotor on semispan wing in the NASA Ames 40x80-foot wind tunnel ................................................................. 10
1.5 Bell XV-15 N702NA ........................................................................ 10
1.6 Bell XV-15 wing modes ..................................................................... 11
1.7 Bell-Boeing V-22 Osprey (top) and WRATS model (bottom) ............ 12
1.8 Nacelle and excitation jet .................................................................. 15
1.9 Gen-1 root roll moment with 1/rev at 26.79 Hz, from [32] ............... 15
1.10 Gen-1 local angle of attack by blade station, from [32] ................... 16
1.11 Gen-1 configuration 4, from [32] ..................................................... 16
1.12 Gen-1 damping vs. tunnel speed by configuration, from [32] .......... 17
2.1 Front view of rotor gimbal, from [32] .................................................. 25
2.2 Exploded views of gimbal assembly, from [32] .................................... 25
2.3 Gimbal housing dimensioned and isometric views, from [32] .......... 26
2.4 Blade yoke root peg, from [32] ............................................................. 26
2.5 Exploded view of rotor hub assembly, from [32] ................................. 27
2.6 Rotor kinematic pitch-flap coupling; negative sense of $\delta_3$ shown ...... 27
2.7 Alignment of pitch links for positive (a) and negative (b) sense of $\delta_3$, from [32] .................. 28
2.8 Wind tunnel blockage survey ............................................................. 33
2.9 Local angle of attack by blade station for untwisted (a) and twisted (b) blades (Re: 100,000 – 120,000 at 0.75 R) .................................................. 34
2.10 Rapid prototype blade design with trellising ........................................ 34
2.11 SolidWorks stress simulation for centrifugal loading ......................... 35
2.12 Simplified model of rotor blade as cantilevered beam subjected to line load................................................................. 35
2.13 SolidWorks stress simulation for flapwise bending ............................. 35
2.14 Calculated maximum blade stresses at 2,000 RPM and 150 ft./sec. tunnel speed with 1.5 factor of safety. Black: maximum stress criterion. Blue: maximum strain criterion. Red: Tsai-Hill criterion .................................................. 36
2.15 NACA 0011 section sketch on base plane ............................................ 40
2.16 NACA 0011 section sketch on base plane ............................................ 40
2.17 Blade female tool half and tool assembly for twisted blade .................. 41
2.18 Divinycell H80 blade core ................................................................ 41
2.19 Core sanding to approximate airfoil section profile .............................. 42
2.20 End-grain balsa blade root attachment ............................................... 42
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.21 Tool release film preparation.</td>
<td>43</td>
</tr>
<tr>
<td>2.22 Unidirectional fabric layup.</td>
<td>44</td>
</tr>
<tr>
<td>2.23 Twill layup and trim.</td>
<td>44</td>
</tr>
<tr>
<td>2.24 Blade wet-out.</td>
<td>45</td>
</tr>
<tr>
<td>2.25 Blade placement in tool.</td>
<td>45</td>
</tr>
<tr>
<td>2.26 Blade secured in tool (a) and clamped to table (b) for sixteen-hour cure at room temperature.</td>
<td>46</td>
</tr>
<tr>
<td>2.27 Raw blade (a) and template mounting (b).</td>
<td>46</td>
</tr>
<tr>
<td>2.28 Blade trimming, sanding, and drilling according to template.</td>
<td>47</td>
</tr>
<tr>
<td>2.29 Finished blade.</td>
<td>47</td>
</tr>
<tr>
<td>2.30 Rotor blade chordwise c.g. balancing assembly.</td>
<td>48</td>
</tr>
<tr>
<td>2.31 Rotor blade radial c.g. balancing assembly.</td>
<td>48</td>
</tr>
<tr>
<td>2.32 Still frame of gimbal bucket striking rotor shaft.</td>
<td>53</td>
</tr>
<tr>
<td>2.33 Shaft damage at the conclusion of Gen-1 testing.</td>
<td>54</td>
</tr>
<tr>
<td>2.34 First mode shape of cantilevered beam.</td>
<td>55</td>
</tr>
<tr>
<td>2.35 Gen-3 wing assembly.</td>
<td>55</td>
</tr>
<tr>
<td>2.36 Dimensioned drawing of Gen-3 wing.</td>
<td>56</td>
</tr>
<tr>
<td>2.37 Gen-3 wing core.</td>
<td>56</td>
</tr>
<tr>
<td>2.38 Gen-3 wing core and spar mounted in foam bed.</td>
<td>57</td>
</tr>
<tr>
<td>2.39 Gen-3 nacelle.</td>
<td>57</td>
</tr>
<tr>
<td>2.40 Gen-3 nacelle secured to wing using machine screws.</td>
<td>58</td>
</tr>
<tr>
<td>2.41 Gen-3 wing mounted to wind tunnel wall.</td>
<td>58</td>
</tr>
<tr>
<td>2.42 Gen-3 wing frequencies versus rotor RPM.</td>
<td>59</td>
</tr>
<tr>
<td>2.43 Rotor actuation assembly.</td>
<td>62</td>
</tr>
<tr>
<td>2.44 Swashplate bottom.</td>
<td>62</td>
</tr>
<tr>
<td>2.45 Swashplate assembly and exploded view, from [32].</td>
<td>63</td>
</tr>
<tr>
<td>2.46 HiTec X1 multifunction controller, used to power and command pitch control servo.</td>
<td>63</td>
</tr>
<tr>
<td>2.47 Servo connector diagram.</td>
<td>66</td>
</tr>
<tr>
<td>2.48 Diagram of pulse-width modulated signal.</td>
<td>66</td>
</tr>
<tr>
<td>2.49 Collective pitch measurement diagram, from [32].</td>
<td>67</td>
</tr>
<tr>
<td>2.50 Rotor root collective pitch versus servo PWM setting.</td>
<td>67</td>
</tr>
<tr>
<td>3.1 UniShow mass readings versus applied mass.</td>
<td>75</td>
</tr>
<tr>
<td>3.2 UniShow measurement error versus applied mass.</td>
<td>75</td>
</tr>
<tr>
<td>3.3 Hammond wind tunnel diagram.</td>
<td>77</td>
</tr>
<tr>
<td>3.4 Wind tunnel test section top view.</td>
<td>78</td>
</tr>
<tr>
<td>3.5 Hammond wind tunnel diagram.</td>
<td>78</td>
</tr>
<tr>
<td>3.6 Wind tunnel contraction section.</td>
<td>82</td>
</tr>
<tr>
<td>3.7 Pressure transducer characterization setup.</td>
<td>82</td>
</tr>
<tr>
<td>3.8 Pressure transducer voltages per load.</td>
<td>83</td>
</tr>
<tr>
<td>3.9 Pitot-static transducer voltage versus tunnel speed, empty tunnel.</td>
<td>86</td>
</tr>
<tr>
<td>3.10 Measured tunnel speed versus percent tunnel speed commanded, empty tunnel.</td>
<td>86</td>
</tr>
<tr>
<td>3.11 Measured tunnel speed versus percent tunnel speed commanded, wind tunnel model installed.</td>
<td>87</td>
</tr>
<tr>
<td>3.12 Load cell characterization test setup.</td>
<td>90</td>
</tr>
<tr>
<td>3.13 Mass measurement for applied roll moment calculation.</td>
<td>91</td>
</tr>
<tr>
<td>3.14 Moment arm distance 1.</td>
<td>91</td>
</tr>
<tr>
<td>3.15 Moment arm distance 2.</td>
<td>92</td>
</tr>
</tbody>
</table>
3.16 Sample histogram of measured roll moment values; applied roll moment: 4.431 Nm.
3.17 Sample boxplot of measured roll moment values; applied roll moment: 4.431 Nm.
3.18 Observed mean roll moment versus applied roll moment.
3.19 Errors of measured roll moments.
3.20 Time histories of applied roll moments.
3.21 Equipment setup for whirl flutter testing.
3.22 Servo and controller wiring.
3.23 Predicted trim collective pitch versus tunnel speed, Gen-3a.
3.24 Predicted wing modal damping versus tunnel speed, Gen-3a.
3.25 Predicted trim collective pitch versus tunnel speed, Gen-3a.
3.26 Predicted wing modal damping versus tunnel speed, Gen-3b.
3.27 Predicted wing modal damping versus tunnel speed, Gen-3c.
3.28 Schematic of moving-block method, from [22].
3.29 Moving-block data curve fit, linear portion only.
3.30 Diagram of passband, from [79].
3.31 Sample unfiltered time history.
3.32 Sample unfiltered frequency spectrum.
3.33 Sample filtered time history.
3.34 Sample filtered frequency spectrum.
4.1 T-plate used for blade stiffness testing.
4.2 Dimensioned drawing of end-mass hanger.
4.3 End-mass hanger installed onto composite blade.
4.4 Dimensioned drawing of base plate.
4.5 Composite blade stiffness testing setup.
4.6 Composite blade deflection versus applied load.
4.7 Closeup of composite blade mounted to shaker.
4.8 Shaker data acquisition wiring.
4.9 Sample frequency response function magnitude (top) and phase (bottom) of un-twisted composite blade.
4.10 Boxplot of coning natural frequencies, untwisted composite blades.
4.11 Sample frequency response function magnitude (top) and phase (bottom), twisted composite blade.
4.12 Boxplot of coning natural frequencies, twisted composite blades.
4.13 Gen-3a model mounted for rap testing.
4.14 Beam mode rap test time history.
4.15 Beam mode rap test frequency spectrum.
4.16 Chord mode rap test time history.
4.17 Chord mode rap test frequency spectrum.
4.18 Torsion mode rap test time history.
4.19 Torsion mode rap test frequency spectrum.
4.20 Gen-2 calculated and experimental trim pitch values.
4.21 Gen-2a and Gen-1 Config. 1 damping versus tunnel speed.
4.22 Gen-2b and Gen-1 Config. 4 damping versus tunnel speed.
4.23 Calculated and experimental trim pitch values, Gen-3a.
4.24 Sample time history of stable Gen-3a test point, tunnel speed = 80 ft./sec.
4.25 Sample frequency spectra of stable Gen-3a test point, tunnel speed = 80 ft./sec.
4.26 Sample frequency spectra of unstable Gen-3a test point, tunnel speed = 115 ft./sec.
List of Symbols

\( A \) area, \( ft.^2 \)

\( a \) airfoil lift-curve slope (\( C_{\text{l}_{\alpha}} \)), \( /\text{rad} \); local speed of sound, \( ft./\text{sec.} \)

\( b \) wing half-span, \( ft. \)

\( c \) chord, \( ft. \)

\( C_d \) airfoil drag coefficient, dimensionless

\( C_l \) airfoil lift coefficient, dimensionless

\( C_{M_x} \) roll moment coefficient, \( \frac{M_x}{\rho A(\Omega R)^2 R} \), dimensionless

\( C_{M_y} \) roll moment coefficient, \( \frac{M_y}{\rho A(\Omega R)^2 R} \), dimensionless

\( c_w \) wing chord, \( ft. \)

\( c_x \) pitch damping of pylon about pivot

\( c_y \) yaw damping of pylon about pivot

\( D \) drag, \( lb_f \)

\( d_{\text{mast}} \) distance from the wing elastic axis to the c.g. of the rotor hub, \( ft. \)

\( e \) chordwise distance from the aerodynamic center to the elastic axis (positive aft), \( ft. \)

\( EI \) bending stiffness, \( lb_f ft.^2 \)

\( EI_b \) wing beam mode stiffness, \( lb_f ft.^2 \)

\( EI_c \) wing chord mode stiffness, \( lb_f ft.^2 \)

\( GJ \) torsional stiffness, \( lb_f ft.^2 \)

\( H \) rotor axial force, \( lb_f \)

\( H_{\beta} \) blade aerodynamic in-plane shear coefficient, dimensionless

\( h \) mast height of rotor, \( ft. \)
\( \bar{h} \) mast height normalized by rotor radius, \( \bar{h} = \frac{h_{\text{mast}}}{R} \).

\( I_b \) rotor blade flap inertia, \( \text{slug} - \text{ft}^2 \).

\( I_n \) torsional inertia of nacelle, \( \text{slug} - \text{ft} \).

\( I_n^* \) torsional inertia of nacelle normalized by element length, \( \text{slug} - \text{ft} \).

\( I_o \) rotor speed perturbation inertia, \( \text{slug} - \text{ft}^2 \).

\( I_x \) pitching moment of pylon about pivot, \( \text{lb}_{f \text{ft}} \).

\( I_y \) yawing moment of pylon about pivot, \( \text{lb}_{f \text{ft}} \).

\( I_\beta \) blade cyclic flap inertia, \( \text{slug} - \text{ft}^2 \).

\( I_0 \) wing element pitch inertia, \( \text{slug} - \text{ft}^2 \).

\( J_0 \) rotational inertia, \( \text{slug} - \text{ft}^2 \).

\( K_p \) rotor kinematic pitch-flap coupling, dimensionless

\( K_t \) transducer correction factor, dimensionless

\( K_v \) wind tunnel boundary layer correction factor, dimensionless

\( k_x \) pitch stiffness of pylon about pivot

\( k_y \) yaw stiffness of pylon about pivot

\( L \) lift, \( \text{lb}_{f \text{ft}} \); wing element length, \( \text{ft} \).

\( L_n \) pylon element length, \( \text{ft} \).

\( L_{\text{span}} \) wing semispan length, from wind tunnel wall to outside plane of nacelle, \( \text{ft} \).

\( m \) elemental mass, \( \text{slug} \)

\( M \) Mach number, dimensionless

\( M_b \) blade mass, \( \text{slug} \)

\( M_b^* \) blade mass normalized by radius and flap inertia, \( M_b^* = \frac{M_b R^2}{I_b} \)

\( M_{F_0} \) rotor collective moment in nonrotating frame, \( \text{lb}_{f \text{ft}} \).

\( M_{F_1} \) rotor pitch moment in nonrotating frame, \( \text{lb}_{f \text{ft}} \).

\( M_{F_{1s}} \) rotor yaw moment in nonrotating frame, \( \text{lb}_{f \text{ft}} \).

\( m^{(n)} \) wing element mass, \( \text{slug} - \text{ft} \).

\( m_{\text{tip}} \) nacelle tip mass, \( \text{slug} \)

\( M(x) \) bending moment at \( x \), \( \text{lb}_{f \text{ft}} \).

\( M_x \) rotor yaw moment, \( \text{lb}_{f \text{ft}} \).
My rotor pitch moment, \( lb\ ft. \)

\( M_\beta \) rotor blade aerodynamic flap moment, \( lb\ ft.^2 \)

\( N \) number of rotor blades

\( P \) power, \( lb\ ft./sec.; \) pressure, \( lb/ft.^2 \)

\( P(x) \) load distribution per unit length at \( x \).

\( Q \) torque, \( lb\ ft. \)

\( R \) rotor radius, \( ft. \)

\( R_\beta \) blade aerodynamic radial shear force

\( r \) local blade station

\( S(x) \) shear at \( x \), \( lb \)

\( S_\alpha \) wing element static imbalance, \( slug - ft. \)

\( S_\alpha^{(n)} \) normalized static imbalance of wing element, \( slug \)

\( S_{\alpha_n} \) normalized static imbalance of nacelle, \( slug \)

\( S_{\beta_0} \) inertia of blade collective flap, \( slug - ft.^2 \)

\( S_{\zeta_0} \) inertia of blade collective lag, \( slug - ft.^2 \)

\( T \) rotor thrust, \( lb \)

\( U \) resultant velocity, \( ft./sec. \)

\( u_g \) gust velocity, \( ft./sec. \)

\( U_p \) perpendicular (axial) velocity, \( ft./sec. \)

\( U_r \) radial velocity, \( ft./sec. \)

\( U_t \) tangential velocity, \( ft./sec. \)

\( v_i \) induced velocity, \( ft./sec. \)

\( V \) freestream speed (also as \( V_\infty \)), \( ft./sec. \)

\( Y \) rotor side force, \( lb \)

\( y \) displacement normal to a beam at distance \( x \) along the beam

\( \alpha \) angle of attack, \( rad \)

\( \alpha_g \) vertical gust angle, \( rad \)

\( \alpha_x \) pylon yaw degree of freedom, positive for outward rotation of rotor hub

\( \dot{\alpha}_x \) hub roll perturbation velocity
\( \alpha_y \) pylon pitch degree of freedom, positive for upward rotation of rotor hub
\( \alpha_y \) hub yaw perturbation velocity
\( \beta_g \) lateral gust angle, \( rad \)
\( \beta^{(m)} \) flap angle of \( m^{th} \) blade, \( rad \)
\( \beta_0 \) rotor coning angle, \( rad \)
\( \beta_{nc} \) longitudinal tilt of tip-path plane, \( rad \)
\( \beta_{ns} \) lateral tilt of tip-path plane, \( rad \)
\( \phi \) inflow angle, \( rad \)
\( \gamma \) shear strain, \( in./in.; \) rotor Lock number, dimensionless
\( \Lambda \) wing sweep angle (positive aft), \( rad \)
\( \lambda \) inflow ratio
\( \mu \) beam mass per unit length, \( slug/ft. \); rotor advance ratio, \( \mu = \frac{V}{\Omega R} \), dimensionless
\( \nu_\beta \) rotor blade rotating natural frequency, dimensionless
\( \theta \) pitch angle, \( rad \)
\( \theta_0 \) rotor blade root collective pitch angle, \( rad \)
\( \rho \) freestream density, \( slug/ft. \)
\( \sigma \) stress, \( psi \); blade geometric solidity, \( (A_{blade}/A_{rotor}) \), dimensionless
\( \Omega \) rotor rotational speed, \( rad/sec. \)
\( \omega_n \) undamped natural frequency, \( rad/sec. \)
\( \omega_d \) damped natural frequency, \( rad/sec. \)
\( \psi \) azimuth angle, rad
\( \zeta \) damping ratio, dimensionless
\( \zeta_{wb} \) wing beam mode structural damping at zero tunnel speed, dimensionless
\( \zeta_{wc} \) wing chord mode structural damping at zero tunnel speed, dimensionless
\( \zeta_{wt} \) wing torsional mode structural damping at zero tunnel speed, dimensionless
\( \dot{} \) first derivative with respect to time
\( \ddot{} \) second derivative with respect to time
Acknowledgments

My parents deserve thanks first and foremost, for putting up with me and trying their best to raise me right... although they might have sometimes wondered where they went wrong. Maybe I didn’t turn out to be quiet, conventional, a doctor, or a lawyer, but I managed to stay out of trouble long enough to become an engineer. Hopefully, that’s not too bad of a consolation prize.

Dr. Edward C. Smith, my thesis advisor, is to be acknowledged for his deep and pervasive knowledge of aeromechanics, and for demonstrating a level of patience that occasionally bordered on the superhuman. Thank you for indulging me. The research took some usually interesting (and occasionally frustrating) turns throughout my time here, which I hope will one day open up entire paradigms of flight that we, as of right now, probably can’t fathom.

Thank you to Dr. Jose L. Palacios for always being as quick with a joke as with a helpful suggestion, and for generously providing the LabVIEW DAQ modules and corresponding data acquisition codes necessary for this work. The sheer number of data channels this project needed far eclipsed what my little myDAQ could handle, and your toys are simply much cooler than mine. Thanks for being the voice of reason.

Thank you to Mr. Rick Auhl for making sure I was well-versed in using the wind tunnel properly, and for trusting me to not destroy it; I appreciate that you were always willing to help me with whatever I needed, and I am happy that I fulfilled my promise to not burn the building down from incompetence. Thank you to Bill Genet and Rob McAllister of the Penn State Learning Factory, for always being willing to help me fabricate parts, being willing to offer suggestions on how to make said fabrication easier and more logical, and for being patient with my constant questions. Thank you also for reminding me why a knee mill needs to be in gear before I turn it on... You two truly are artists, and I appreciate you tremendously.

Thanks to Sandilya Kambampati for being an awesome research partner and for always helping me figure out what was going on with my codes no matter how simple the mistake might be – and thanks for getting me to finally start using \LaTeX like a proper, civilized human being. Thank you to all of my colleagues at the Penn State VLRCOE who inspired me to grow intellectually and helped me maintain my sanity during the past two years, including Matthew Bailey, Adam Thorsen (and the immortal turkey call), and Jared Soltis. Thank you to Ben Pipenberg for showing me how to greatly reduce my fabrication efforts without sacrificing part quality.

Thank you, Hovig, for reminding me why it’s important to occasionally go outside and have fun. Thank you, Dr. Edberg, for reminding me of why it’s important to occasionally go outside
and laugh at Hovig. Thank you, Ray Hudson, for always offering me the perspective I needed, even if it wasn’t always the one I wanted. Thanks, Professor Dobbs, for giving me my first taste of flutter testing. Thanks to the Babas for always encouraging me to keep going — ruck hard, fight hard, and win hard.

I wish to extend a very heartfelt note of thanks to Drs. Walter Silva and Elizabeth Ward of NASA Langley Research Center, and Dr. William Warmbrodt of NASA Ames Research Center. Walt and Liz, you continued to believe in me when even I wouldn’t, for which I am and will always be immensely grateful. Lord knows how you managed; you both have the patience of a saint, and are some of the finest people I’ve ever met. “Uncle Willie,” you’re the one who made me think any of this was even possible, and you’re the one who got me started down the whole rotorcraft road in the first place. I would not have made it this far without your help. A special thank you is also due to Mr. Wally Acree of NASA Ames Research Center, for serving as the technical monitor for this project.

And because not a single engineering document can be printed without some manner of regulatory acknowledgement:

The research presented herein was partially funded by the Government under Agreement No. W911W6-11-2-0011. The U.S. Government is authorized to reproduce and distribute reprints notwithstanding any copyright notation thereon. The views and conclusions contained in this document are those of the author and should not be interpreted as representing the official policies, either expressed or implied, of the U.S. Government.
We are not now that strength which in old days
Moved earth and heaven;
That which we are, we are;
One equal temper of heroic hearts
Made weak by time and fate, but strong in will
To strive, to seek, to find, and not to yield.

- Tennyson
Chapter 1

Introduction

1.1 Historical Background

The desire to conquer all three dimensions of movement through flight - and specifically, through the capability of vertical takeoff and landing (VTOL) - has been one of humanity’s ultimate dreams for centuries. The first step toward the realization of this dream was taken by Igor Sikorsky and his VS-300, which was the first practical manifestation of what would today be considered a helicopter: a single main rotor providing both lift and propulsive thrust, and a single tail rotor providing anti-torque and yaw control. Vertical lift was - and is still - achieved through the combination of high power-to-weight ratio powerplants and low disk loading, both of which are required to achieve efficient hover and controllable low-speed maneuverability; however, the forward flight capability of the helicopter is limited: edgewise flow across the rotor creates advancing and retreating portions of the disk, which experience high oscillatory loads and high drag. Higher cruise speeds can cause the advancing blade to encounter sharp increases in drag due to compressibility effects, while simultaneously causing the retreating blade to stall near the tip and experience reverse flow near the root. Newer rotorcraft designs attempt to address these problems with developments such as slowed rotors, lift- and thrust-compounding configurations, etc. (cf. [1], for instance), but generally speaking, the adverse effects of edgewise flow cannot be eliminated.

Higher forward flight speeds may be obtained with machines of higher disk loading, but this capability comes at the expense of hover efficiency and low-speed maneuverability; therefore, a compromise must be made based upon the trade space created by (likely contradictory) mission requirements. This compromise is illustrated by Fig. 1.1 (from [2]): for all aircraft capable of VTOL flight, there is an inverse relationship between hover efficiency (measured as gross weight divided by installed power) and disk loading. The tiltrotor, with a disk loading in the 7-30 lb/ft\(^2\) range, occupies a unique position in this chart: it is capable of achieving efficient hover with reasonable downwash velocities, but can avoid the problems encountered by rotors in high-
speed edgewise flight by rotating its nacelles forward. This capability has made tiltrotors an attractive option for both military and civil applications which require both efficient hover as well as high-speed cruise; however, tiltrotor aircraft present their own unique technical challenges, one of the most important of which is known as whirl flutter.

Whirl flutter is a type of wing-rotor-pylon instability that occurs during high-speed flight, and which is caused by the interaction of rotor aerodynamic, inertial, and gyroscopic forces with the aerodynamic, structural, and inertial properties of the wing and support structure (see [3] and [4]); these interactions may result in highly-coupled, dynamically unstable vibration modes. The whirl flutter phenomenon relies heavily on the concepts of gyroscopic motion and precession.

A spinning body has rotational inertia that resists any change to the angular momentum of the body; when a force is applied to the spinning body, the rotating mass will react 90° past the point of perturbation. This is referred to as cyclic lag, and the resulting motion is often coupled with a change in the rotational axis of the body, called precession. These coupled motions form the basis of the gyroscopic modes of the rotor. On a tiltrotor, these gyroscopic modes can impart a vertical shear force to the wing, which - due to the rotor hub and rotor center of gravity (c.g.) being offset from the wing c.g. and aerodynamic center - can impart a combination of beamwise, chordwise, and torsional perturbations to the wing. These wing perturbations further disturb the rotor, which can then lead to continuous feedback, initiating the whirl mode. Traditionally, this phenomenon has been mitigated through very thick wings of low aspect ratio, which house very large spars to resist the aeroelastic loads: for example, the V-22 Osprey features wings on the order of 23% thickness with an aspect ratio of 6. This use of thick, low-aspect ratio wings limits the flight speed of tiltrotors in airplane mode, and represents a major obstacle to achieving cruise flight performance on par with that of fixed-wing aircraft.

Several convertiplane concepts had been developed in the 1930s and 1940s, but the Transcendental 1-G (Fig. 1.2, from [5]) is widely considered to have been the first true attempt at building a tiltrotor. The Transcendental Aircraft Company was founded in 1947 in New Castle, Delaware by two employees formerly of the Kellett Autogiro Corporation [6]. The open-cockpit 1-G was built in 1951, and a single 160 hp Lycoming O-290-A engine was used to power a three-bladed rotor at each wingtip via manual two-speed gearbox that drove shafts down each wing. During conversion, three concentric shafts supplied inputs for rotor tilt angle, cyclic pitch, and collective pitch; the rotors required three minutes to rotate through 82° during conversion, including the gear change [7]. The 1,750-lb craft was so small that the pilot’s head actually rose above the windscreen. The 1-G made its maiden flight (in helicopter mode) on 6 July 1954. Although an attempt was made at in-flight conversion, the aircraft crashed on 20 July 1955, when the rotors were tilted 10° from vertical, due to an unintentional reduction in collective pitch. The 1-G had performed over 100 test flights, accumulating twenty-three flight hours. Although the 1-G never achieved complete transition, it demonstrated an ability to rotate the rotors up to 75° from vertical, and was able to generate over 90% of its required lift from its wings. A new development contract was received by Transcendental from the Air Force in 1956 to build a heavier, more aerodynamically efficient, and more powerful Model 2; this aircraft flew in late 1956, but funding
was redirected and Transcendental was sold to Republic Aviation [6].

The 1-G was succeeded by the Bell 200 (later designated XV-3), which was the result of the joint U.S. Army - U.S. Air Force Convertiplane Program. The XV-3 (Fig. 1.3) was first flown on 11 August 1955 (sn 54-147) by Bell Chief Test Pilot Floyd Carlson, and experienced significant vibrations; the aircraft experienced a hard landing on 18 August 1955 after a rotor instability was encountered. Flight tests were resumed on 29 March 1956 and continued until 25 October 1956, when extremely high cockpit vibrations caused the test pilot to lose consciousness; these vibrations resulted when the rotors were tilted 17° from vertical. The test pilot was seriously injured, and the aircraft was damaged beyond repair [8].

The second XV-3 (sn 54-148) was modified by replacing the three-bladed articulated rotor with a two-bladed stiff-in-plane rotor, and began testing at the NACA Ames Aeronautical Laboratory 40x80-foot wind tunnel facility on 18 July 1957; a full-scale vehicle was tested in the wind tunnel, in several flight configurations and operating conditions. The aircraft was perturbed at a given test condition, and the vibrations were allowed to dampen; the subcritical damping of the aircraft modes were determined from the time histories of the decaying vibration levels. Flight testing for XV-3 Ship 2 began on 21 January 1958 at Bell’s facilities [5]. By April of that year, Ship 2 had reached 110 knots, had demonstrated full-autorotation landings, and had transitioned the rotors thirty degrees; however, on 6 May 1958, another instance of rotor instability occurred when the rotors were rotated to forty degrees, and the XV-3 was again grounded [8]. The aircraft was returned to the Ames wind tunnel in October 1958; as a result of this second round of wind tunnel testing, the rotor diameter was reduced, the wing structure was strengthened, and the control system was stiffened. Ship 2 was returned to Bell’s facility on 12 December 1958 to resume flight testing; on 18 December 1958, Bell test pilot Bill Quinlan accomplished the first dynamically stable conversion to airplane mode [5]; this feat was repeated by Air Force Captain Robert Ferry, who on 6 January 1959 became the first military pilot to complete a tiltrotor conversion to airplane mode [9]. The XV-3 program came to a close when both nacelles separated from the aircraft during wind tunnel tests, although subsequent investigation revealed that the nacelles separated due to fatigue rather than a dynamic instability. The XV-3 did not demonstrate that a tiltrotor could exceed the performance of helicopters and fixed-wing aircraft, but rather proved that the concept of converting from helicopter to airplane mode was feasible, and provided critical groundwork for further design and testing of tiltrotors.

The 1960s and 1970s saw an increase in both industry and government tiltrotor research programs. Reference 10 provides an excellent compilation of the experimental and analytical tiltrotor aeroelasticity studies conducted at the NASA Langley Research Center’s Aeroelasticity Branch and the Transonic Dynamics Tunnel (TDT) during this period; a thorough overview of the TDT’s research contributions to rotorcraft technology - including whirl flutter stability testing - is given by [11]. Small-scale wind tunnel tests were conducted to create databases for validating the analytical techniques being developed; however, the small-scale effects of structural and aerodynamic characteristics required empirical corrections to represent the aeroelastic behavior of full-scale aircraft. The use of a heavy gas test medium (currently R-134a, and previously
R-12) enabled Mach scaling and nearly-matched Reynolds scaling, full-scale test models were desired to correctly capture dynamic instabilities \cite{12}. During this time, Bell began developing the Model 300, and Boeing began developing the 222. One of the primary differences between the Bell 300 and Boeing 222 was in the treatment of the nacelle rotation during conversion: while both prototypes featured wingtip-mounted engines (in contrast to the fuselage-mounted engines of earlier tiltrotors), the Bell 300 would rotate the engines and rotors together (similar to the V-22 Osprey), whereas the Boeing 222 would maintain the engines horizontal and instead rotate a “pod” containing the rotor assembly (an arrangement similar to the forthcoming Bell V-280 Valor). A very detailed description of the Model 300’s history and development is given in \cite{5}, which is summarized here.

The knowledge gained from the wind tunnel and flight tests of the XV-3 allowed Bell to design a new rotor system with the goal of increasing the stability boundary of the aircraft. The result was a gimbaled, stiff-inplane rotor, which used kinematic pitch-flap coupling (known as delta-3, or $\delta_3$) to reduce blade flapping. Wind tunnel tests were conducted in 1970 of a full-scale, semispan aeroelastic model (Fig. 1.4) featuring a 25-foot diameter rotor; these tests were conducted in the NASA Ames 40x80-foot wind tunnel and were jointly funded by NASA, the Army, and the Air Force. Although wind tunnel speed limitations precluded the testing of the semispan model at the maximum flight speed of the aircraft, the high-speed operating condition was simulated by reducing the stiffness of the semispan wing as well as the RPM of its rotor \cite{5}. These test results - along with the subcritical damping tests performed on a Boeing soft-inplane, hingeless, 26-foot diameter rotor in 1972 – validated a comprehensive analytical model developed at NASA Ames, which would become the standard for aeromechanical analysis and serve as an important tool in the prediction of aeroelastic stability margins. The theory and derivation used in this analytical model, as well as a complete description of the Bell and Boeing models and test results, are given in \cite{13} & \cite{14}. These methods form the foundation of the majority of the tiltrotor whirl flutter analysis currently performed.

The Bell Model 300 was developed into the Model 301, and NASA selected the Bell 301 for further development, edging out the Boeing 222; a contract for further research work on the Bell 301 (now XV-15) was issued on 31 July 1973. Engineering and testing work was performed for four years on the XV-15, and the first of two Bell XV-15s (N702NA, Fig. 1.5) achieved maiden flight on 3 May 1977; this first flight demonstrated satisfactory handling qualities and safe structural loading. Subsequent early tests validated the longitudinal and directional controllability of the aircraft, as well as hover performance over a fixed point. Nacelles were tilted to 85° from vertical, and assessments were made of the aircraft handling qualities in this “near-airplane” mode. N702NA was then reconfigured with a remote control system for wind tunnel testing, and was disassembled for transport to Moffett Field on 23 March 1978.

The XV-15 wind tunnel tests were conducted prior to the expansion of the flight envelope, not only to check the aircraft systems (mechanical, hydraulic, and electrical) under flight loads, but also to obtain an accurate picture of the XV-15’s loads, performance, and aerodynamic characteristics. Testing took place in the Ames 40x80-foot wind tunnel; over the course of the
two-month test period, a total of fifty-four hours of wind-on time were logged, of which nineteen hours were with the rotors operating. Aerodynamic forces and moments were obtained from the wind tunnel balances, and structural loads were obtained from the instrumentation on board the aircraft; critical test parameters such as static and dynamic loads were monitored in real time in the wind tunnel control room. The tests were performed over a range of flight configurations and airspeeds; modifications to the aircraft (e.g. pylon strakes, vortex generators, etc.) were tested as well. A report [15] summarizing these wind tunnel tests, including data and photographs, was released in April 1980.

Once the wind tunnel tests had been completed, XV-15 Ship 2 (N703NA) began envelope expansion flight tests on 23 April 1979 at Bell’s flight test facility at Arlington Municipal Airport. By mid-June of 1979, XV-15 Ship 2 had been tested from helicopter mode to 60° of nacelle tilt. After fifteen hours of flight testing over three months of envelope expansion, XV-15 Ship 2 reached full conversion on 24 July 1979 - the first time a tiltrotor had attained full in-flight conversion from helicopter to airplane mode. The initial flight in airplane mode lasted approximately forty minutes, during which time climbs, descents, turns, accelerations, and decelerations were evaluated; the aircraft also reached an airspeed of 160 knots during this flight test. The success of the envelope-expansion tests paved the way for a more careful and detailed analysis of the XV-15’s performance, including its aeroelastic stability.

The aeroelastic stability of the XV-15 in high-speed forward flight can be considered the most important technical area to be explored by the test program. Predictions of in-flight structural stability were produced by both Bell and the Army/NASA offices, Bell using an in-house method and the Army/NASA office using the aforementioned methods presented in [13] and [14]. Both methods showed that the XV-15 would be stable throughout the flight envelope except for one condition: high-speed forward flight in airplane mode with the rotors operating at the RPM used for hover and low-speed maneuvering; this was resolved by limiting the rotor RPM in airplane mode below the point at which 1/rev excitation of the wing modes could occur. As this reduction in operating RPM after conversion was accounted for during the design of the XV-15 (in order to improve the performance of the rotor in airplane mode), it has since become standard procedure among subsequent tiltrotors to reduce the rotor RPM after conversion to airplane mode.

Evaluating the aeroelastic stability of the XV-15 required the perturbation of the wing-pylon-rotor system at the various natural frequencies that corresponded to the six different primary modes of the wing (symmetric and antisymmetric beamwise, chordwise, and torsional vibration; Fig. 1.6). The response of the aircraft to these perturbations was then measured, with oscillations whose amplitudes diminished after excitation indicating a stable system (positive damping); potential danger could occur in situations where the amplitude of the oscillations increased after perturbation (negative damping). The initial approach taken by the XV-15 researchers was to excite the structural modes of the wing through the use of limited-authority electrohydraulic actuators in the flaperons and collective linkages on the starboard side of the aircraft. The amplitudes and excitation frequencies of these perturbation actuators were controlled from the cockpit; the perturbations would be initiated by the actuators (one at a time), at a fixed frequency,
and then turned off. The resulting rates of decay of the vibrations were measured to determine
the structural damping at that flight speed. Excitations were initiated in airplane mode, and
only at lower flight speeds (in order to reduce the chance of encountering an instability); after
the data from a given flight test were analyzed, the airspeed for the subsequent flight test was
increased by a small increment (provided that the prediction for the higher flight speed also
indicated that the aircraft would be stable), and the test cycle was repeated.

The aeroelastic stability tests were conducted in March and April 1987. Early tests revealed
that many of the modal frequencies were closely spaced, and certain modes could not be easily
excited; furthermore, the scattering of the damping estimates was too great for meaningful cor-
relation with (and therefore validation of) the stability predictions. Tests were repeated after
installing another set of actuators onto the port side of the aircraft (identical to the actuators
already installed on the starboard side); other excitation schemes (such as frequency sweeps) were
attempted, but the damping scatter still remained too large. The application of a frequency do-
main methodology by the Army Aeroflightdynamics Directorate (AFDD) at Ames improved the
quality of the flight test results, and reduced the total flight time required for stability evaluation;
the details and results of the frequency domain method are described in detail in [16] and [17].
The aeroelastic stability program at Bell focused on the evaluation of two configurations of pro-
protor yokes and a steel hub; these tests showed that the hub configuration did not significantly
affect the stability of the rotor. Most importantly, the tests performed at Ames and Bell verified
positive damping at all elastic modes and flight conditions examined.

The success of the XV-15 program allowed for the Department of Defense to initiate yet an-
other sizable tiltrotor program in 1981: the Joint-Service Vertical Takeoff/Landing Experimental
Aircraft Program (JVX). The Bell-Boeing team was awarded a contract in 1983 to develop a large
tiltrotor for military applications, the end result of which would be the V-22 Osprey (Fig. 1.7).
The first phase of development focused on the construction of full- and semispan 1/5-scale models
to determine the stability of the wing/pylon/rotor design, as well as being used for parametric
studies, evaluation of the impact on whirl flutter stability of various design improvements, and
experimental validation of the whirl flutter analytical methods. These tests were conducted from
1984 to 1988 over a range of simulated flight conditions, from hover at zero forward speed in
helicopter mode to high-speed flight in airplane mode. A detailed summary of the JVX tests, as
well as a detailed description of the models, is presented in [18], along with a discussion of the
wind tunnel tests of a semispan, scaled V-22 model tested in the Langley TDT. In particular, the
tests described in [18] focus on the effects of wing, control system, blade stiffness, and kinematic
pitch-flap coupling on the whirl flutter stability of the scale V-22 model; the methods described
in [18] - specifically the modal excitation methods and moving block analysis - set the precedent
for a great deal of subsequent aeroelastic research, including the present work.

The semispan model described in [18] consists of a gimbaled, stiff-inplane rotor, a dynamically
scaled pylon, and a composite wing. Rather than exciting the model across a range of frequencies,
as had been done with the XV-15 flight tests, the semispan model was excited at a specific
frequency - the measured modal frequency of the model - for each test. Excitation was provided
by small pneumatic nozzles that were aligned with the wing beamwise and chordwise directions; servo-actuated valves controlled the flow of compressed gas to each of these nozzles. These servo valves were then used to excite the model at its beamwise or chordwise natural frequency. The transient response of the model was measured using strain gages and accelerometers.

The time domain data were transformed into frequency domain data using discrete Fourier transforms (DFT), with the modal frequencies being determined from the frequency response magnitude and phase; this allowed the excitation frequency of the excitation jets to be calculated, as well as calculating modal damping for a given configuration as a function of airspeed. The latter calculation was performed through the use of a moving block method, described by [21] and [22]. In the moving block method, the DFT of a “block” of time domain data is calculated at a specific frequency; the block is then stepped forward in time and the process is repeated until the block reaches the end of the recorded data. The calculated magnitudes of the DFTs are plotted, and the slope of the least-squares line gives the modal damping of the model for a given excitation frequency.

The semispan model used during the JVX program was revived in 1994 for a joint NASA–Army–Bell research program, and became known as the Wing and Rotor Aeroelastic Test Stand (WRATS; Fig.1.7) [23], [24]. WRATS was used to investigate the effects of tailored wing stiffness, higher harmonic control, and alternative rotor designs on whirl flutter stability. WRATS continues to be used for wind tunnel research, including whirl flutter stability testing of a stiff-in-plane, four-bladed tiltrotor model with a unique “stepover” control system and an active stability control algorithm to increase the subcritical damping of the four-bladed rotor. These details of this updated WRATS model are described in [25]. Research has also been conducted into examining the whirl flutter stability of tiltrotors using tuned wing extensions and winglets, as shown in [26], [27], and [28].

Research conducted using the WRATS model has provided extensive opportunities to investigate and expand the flight envelope of tiltrotor aircraft. The V-22 has proven to be an effective tool for both the U.S. Marine Corps and U.S. Air Force, and has paved the way for follow-on projects – such as the Bell HV-911 Eagle Eye, part of the U.S. Coast Guard’s Integrated Deepwater System Program [29]. Much work has been performed - and much more work continues to be performed - on the development of a civilian tiltrotor aircraft. Perhaps most famous of these near-horizon configurations is the Agusta-Westland AW609 (formerly Bell-Boeing 609), a small tiltrotor designed to carry six to nine passengers; the AW609 has, at the time of this writing, recently completed autorotation tests and is well on its way toward becoming the first non-military tiltrotor to hold a type certificate [30]. Another civilian application currently in development is that of tiltrotors supplementing or replacing conventional fixed-wing airline service, such as envisioned by the NASA Large Civil Tiltrotor project [31]. These applications certainly challenge the conventional paradigm of what rotorcraft can attain, but it is undeniable is that the whirl flutter boundary will always exist, and dynamic testing of aeroelastic stability will be an integral part of the research and development process of future tiltrotors.
Figure 1.1: Hover efficiency versus disk loading of VTOL aircraft, from [2].

Figure 1.2: Transcendental 1-G in hover.
Figure 1.3: Bell XV-3 in hover.
Figure 1.4: Bell 25-ft. diameter proprotor on semispan wing in the NASA Ames 40x80-foot wind tunnel.

Figure 1.5: Bell XV-15 N702NA.
Figure 1.6: Bell XV-15 wing modes.
Figure 1.7: Bell-Boeing V-22 Osprey (top) and WRATS model (bottom)
1.2 Previous Wind Tunnel Experiments at Penn State: The Generation-1 Whirl Flutter Model

As the current work is a follow-on to the research performed in the 2011 - 2013 timeframe [32], a brief overview of this work is appropriate. The previous wind tunnel model (“Generation 1”, or “Gen-1”) was sized primarily by using geometric relationships from the XV-15 (e.g. chord length normalized by rotor radius); no Reynolds, Mach, or Froude scaling of the XV-15 was used for the Gen-1 model, nor were these scaling parameters incorporated into the models designed for the present work. The objective of the previous research effort was to design and fabricate a low-cost, small-scale that would exhibit whirl flutter within the operating limits of the Penn State Hammond Low-Speed Wind Tunnel. Gen-1 consisted of a hollow wing with integrated root plate and nacelle; a nacelle “cap” allowed the nacelle to be closed, and provided mounting points for the excitation jets used to perturb the model during testing. The monocoque structure of the wing created a pass-through for the model’s pneumatic lines and servo wires. The root plate allowed the model to be mounted to the load cell in the Hammond tunnel, and the nacelle provided mounting points for the pitch control servo and rotor bearing. These components were made of an acrylonitrile butadiene styrene (ABS) thermoplastic polymer and manufactured in a Dimension 1200sst printer at the Penn State Learning Factory. Two #3-56 to 1/16 in. NPT barb fittings were mounted to the nacelle cap at the elastic axis of the wing; these were mounted to the horizontally flat portion of the nacelle cap with the barbs facing in (Fig. 1.8).

Several potential candidates for the Gen-1 rotor were examined, including commercially available hobby-scale propellers manufactured by Master Airscrew. Most of the off-the-shelf propellers caused the model to experience severe 1/rev vibratory loads, and were not used for flutter testing. The final iteration of the Gen-1 rotor consisted of balsa/spruce blades from the tail rotor of a 600-size radio-controlled helicopter; these blades were of a constant NACA 0011 cross-section. The blades were manually trimmed and mounted to the blade yokes with M3x8 machine screws and M3 nylon locking nuts. The manual trimming of the blade length resulted in blades that were non-uniform: there existed length variations of approximately 0.010 in. and mass variations of approximately 0.15 grams between the three blades; while these differences may seem slight, the noticeable 1/rev loads experienced during testing (Fig. 1.9) may be explained in part by differences in balance between the blades; the rotor imbalance served as a point of improvement for subsequent iterations of the wind tunnel model. The pitch link assembly of the model permitted both positive and negative kinematic pitch-flap coupling to be tested; the magnitude of the $\delta_3$ angle was fixed at $\pm 47^\circ$ due to constraints imposed by the gimbal geometry. The fact that the Gen-1 blades were untwisted meant that large portions of the blade were above the static stall angle of the airfoil (Fig. 1.10), which was another point of focus for improving the model.

An external air compressor was connected to an SMC Pneumatics 24-volt solenoid valve, which was used to excite the model near its fundamental beamwise frequency. Root forces and moments were measured using an ATI Delta SI-660-60 load cell built into the wind tunnel wall, and were recorded using a National Instruments cDAQ module and custom LabVIEW data
acquisition software developed in-house. A moving block MATLAB code, based on the methods outlined in [21] & [22], was used to determine the beamwise damping of the model at each wind tunnel speed; the model was tested at subsequently higher wind tunnel speeds until it experienced flutter at a tunnel speed of approximately 115 ft./sec. The Gen-1 model did not exhibit whirl flutter below a tunnel speed of 140 ft./sec. without the use of a one-pound steel mass bolted aft of the wing’s trailing edge (Fig. 1.11). The addition of this aft-mass reduced the modal frequency of the wing, but the torsional motion of the wing also caused the gimbal retaining ring to strike the rotor shaft after the wing had been excited. The wing damping of the various Gen-1 configurations is shown as a function of tunnel speed in Fig. 1.12.
Figure 1.8: Nacelle and excitation jet.

Figure 1.9: Gen-1 root roll moment with 1/rev at 26.79 Hz, from [32].
Figure 1.10: Gen-1 local angle of attack by blade station, from [32].

Figure 1.11: Gen-1 configuration 4, from [32].
Figure 1.12: Gen-1 damping vs. tunnel speed by configuration, from [32].
1.3 Scope of Present Work and Project Objectives

The primary objective of the present work is to perform experimental validation of a new analytical model developed in-house to predict the whirl flutter stability of a semispan tiltrotor wind tunnel model. The design, development, and testing of several iterations of new wind tunnel models, which are the focus of the present work, address the shortcomings of the previous wind tunnel model (termed “Gen-1”). The new wind tunnel models are designed to be more representative of full-scale tiltrotors. As with Gen-1 testing, the operating capabilities of the test facility precluded the dynamic scaling of a full-scale aircraft; thus, the present work is concerned primarily with experimental validation, rather than accurately representing a particular full-scale aircraft. The improvements to Gen-1 were focused on causing the model to flutter at a wind tunnel speed below 140 ft./sec. The improvements to the Gen-1 model are detailed below.

1.3.1 Rotor

New rotors were designed, with the objectives being:

- reduce the total blade mass and balance variations between each independent blade
- increase the rotor Lock number while maintaining functionally equivalent geometric solidity
- incorporate twist to reduce the percentage of the blade span above the static stall angle of attack within the tunnel speed range of interest

The Gen-1 blades exhibited a mass variation of approximately 0.15 grams, which caused an undesirable 1/rev excitation during testing. Additionally, the untwisted blades experienced large regions that were above the static stall angle of attack for the airfoil sections of the rotor (NACA 0011). The Lock number of the Gen-1 rotor (approximately 2.36) is low for a tiltrotor: for example, the XV-15 had a Lock number of 3.83 [33]. Because the perturbation forces imparted to the wing by the rotor during flapping are Lock number-driven, it stands to reason that a higher Lock number will increase these perturbation forces and cause the model to flutter at a lower wind tunnel speed; however, the commercially available blade options examined were all far too heavy (approximately 20-250 grams per blade) to have a desirable impact on rotor Lock number. Therefore, new blades would need to be designed and fabricated in order to attain reasonable Lock numbers.

The rotor blade geometry was also modified to better represent full-scale tiltrotors. The blade chord was linearly tapered (taper ratio 0.70) to more closely match other tiltrotor wind tunnel models, such as TRAM (descriptions of TRAM physical details may be found in [34] and [35]). Linear taper was selected purely for ease of fabrication. As with the Gen-1 blades, a NACA 0011 section was used throughout all blade stations.
1.3.2 Wing

The Gen-1 model was only able to exhibit whirl flutter through the use of a steel mass aft of the wing’s trailing edge; the c.g. of this mass was located 5.5 in. aft of the elastic axis of the wing. High-speed video recorded during Gen-1 wind tunnel testing showed that contact between the gimbal and the rotor shaft was likely due to the magnitude of the coupled bending/torsion motion of the wing. This was only noticed during tests of the unstable configuration, and the aft-mass is likely the cause. The entire purpose of the aft-mass - viz. lowering the modal frequencies of the wing and forcing the c.g. aft of the wing trailing edge - came about as a result of the wing stiffness being too high for the model to exhibit whirl flutter within the operating limits of the tunnel; this was likely due to the monocoque construction of the root/wing/pylon structure in ABS plastic. The monocoque design of the Gen-1 wing also created a high frequency spread between the beam, chord, and torsion modes of the wing (i.e. chord and torsion modes were, respectively, approximately two to three times as stiff as the beam mode); similar whirl flutter models, such as WRATS, have chord and torsion modes that are less than twice as stiff as the beam mode [23].

1.3.3 Control system

The Gen-1 model maintained rotor RPM by varying collective pitch of the rotor blades through the use of a servo control board that varied the pulse width modulation (PWM) signal sent to the pitch servo. Although this is a valid method for commanding a PWM-controlled servo, the Gen-1 setup required a separate power supply and secondary test computer to be used during wind tunnel testing. Additionally, the servo control board was only able to command the servo over a PWM range of approximately 150 milliseconds in one-millisecond increments; this not only limited the resolution of the PWM signal sent to the servo, but also resulted in the servo being unable to maintain the shaft speed of the rotor to within less than $\pm 50$ RPM. The servo control board was replaced with a multifunction controller that eliminated the need for a second power supply, and was able to command the pitch control servo to a PWM resolution of four microseconds. This allowed for finer pitch control at all tunnel speeds of interest.

1.3.4 Experimental validation

Stability tests of the wind tunnel models created for the present work were conducted using a similar approach as that used by the WRATS team. The Gen-1 model demonstrated that low-cost flutter testing was indeed feasible within the Penn State facilities; the development of this capability is critical, as most flutter models are very complex and require expensive test facilities (e.g. Langley TDT), or are merely small demonstrators of limited functionality. The development of the current wind tunnel models continued the trend of fast, simple, low-cost flutter testing that was begun by the Gen-1 model. The wind tunnel models presented herein are representative of a tiltrotor in cruise flight; each model consists of a cantilevered, semispan wing with an unpowered, gimbaled rotor housed in a nacelle at the wingtip. The unpowered rotor maintains constant RPM
through collective pitch control, which is actuated by an internally-mounted servo. The wind tunnel models created for the present work – referred to as Gen-2 and Gen-3 – were designed, fabricated, and tested over the course of August 2013 – January 2015. Several configurations were tested, the details of which are summarized in Table 1.1.

Chapter 2 discusses the details of the design, fabrication, and physical properties of the Gen-2 and Gen-3 models. The wing spar of the Gen-3 model was designed so that all three wing modes (beam, chord, and torsion) were near enough to each other to be excited by simple beamwise perturbation of the model; bandpass filters were written in MATLAB in order to isolate the model’s known modal frequencies for damping analysis. The methods used to obtain the modal frequencies of the wind tunnel models are discussed in Chapter 3. Measured physical properties of the models are presented in Chapter 4, as well as a discussion of the wind tunnel test results. Finally, the conclusions and recommendations for future work are given in Chapter 5.

Table 1.1: Gen-2 and Gen-3 test configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Aft-mass</th>
<th>( \delta_3 )</th>
<th>Wing</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen-2a</td>
<td>None</td>
<td>+47°</td>
<td>ABS monocoque</td>
<td>Composite, untwisted, tapered</td>
</tr>
<tr>
<td>Gen-2b</td>
<td>Extended</td>
<td>−47°</td>
<td>ABS monocoque</td>
<td>Composite, untwisted, tapered</td>
</tr>
<tr>
<td>Gen-3a</td>
<td>None</td>
<td>−47°</td>
<td>Composite</td>
<td>Composite, untwisted, tapered</td>
</tr>
<tr>
<td>Gen-3b</td>
<td>None</td>
<td>−47°</td>
<td>Composite</td>
<td>Composite, twisted, tapered</td>
</tr>
<tr>
<td>Gen-3c</td>
<td>None</td>
<td>N/A</td>
<td>Composite</td>
<td>Blades-off</td>
</tr>
</tbody>
</table>
Model Design and Fabrication

An overview of the design and fabrication used for the Gen-2 and Gen-3 wind tunnel models is presented in this chapter. The choices made during the design of the wind tunnel models are explained in this chapter, and justifications for these design choices are also provided. The physical descriptions and operating ranges of the subsystems of the wind tunnel models are also given. Estimates of geometric and mass properties that were difficult or impossible to physically measure are provided, using the measurement tools in SolidWorks 2014. Finally, the input variables necessary for the analytical model are discussed, and the resulting performance and stability predictions are given.

2.1 Model Design

The goal of this section is to provide insight into the model features and its physical parameters. The objective of the Gen-2 wind tunnel model was to perform comparative tests of wind-on damping to determine the effects of increased Lock number on the damping of the monocoque ABS wing. The objective of the Gen-3 wind tunnel model was to exhibit whirl flutter at a tunnel speed of less than 140 ft./sec. without the use of an added mass. All of the wind tunnel models used for the present work are composed of three subsystems: the rotor, the wing, and the control system. Components of these subsystems were carried over from the Gen-1 model when feasible and appropriate; other components were fabricated at the Penn State Learning Factory or the VLRCOE benchtop laboratory. Unless explicitly stated otherwise, features of size in dimensioned drawings are given in inches per ANSI Y-14.5M standards, with tolerances given by ANSI B4.1 Standard Fit designations. Three decimal places are used for features of size, in accordance with standard machining practice.
2.1.1 Design methodology

Aeroelastic testing is a critical part of any aircraft certification procedure, and the advantages of small-scale testing (primarily cost and safety) are immediately apparent. Typically, a wind tunnel model is dynamically scaled from the full-sized aircraft, and great effort is made to ensure that the interactions of the full-scale aircrafts aerodynamic, elastic, and inertial forces are adequately represented by the wind tunnel model. As detailed in [63], in the ideal case the following rotor parameters are scaled:

- Nondimensional wing frequency
  \[ f = \frac{\omega_{wing}}{\Omega} \]  
  \[ (2.1) \]

- Lock number
  \[ \gamma = \frac{\rho ac R^4}{I_b} \]  
  \[ (2.2) \]

- Advance ratio
  \[ \mu = \frac{V_\infty}{\Omega R} \]  
  \[ (2.3) \]

- Froude number
  \[ Fr = \frac{\Omega^2 R}{g} \]  
  \[ (2.4) \]

- Mach number
  \[ M = \frac{\Omega R}{a_\infty} \]  
  \[ (2.5) \]

- Nondimensional wing and rotor modal frequencies
  \[ \nu_i = \frac{\omega_i}{\Omega} \]  
  \[ (2.6) \]

Modeling a tiltrotor requires that similar considerations be used for scaling of the wing. It is very difficult - and sometimes impossible - to match all of the desired parameters, and compromises are usually made based on the nature of the test being conducted. Oftentimes, fluids other than air must be used: for example, it is difficult to simultaneously match Reynolds number and Mach number using air, but the use of R-134a (such as is normally done at the Langley TDT) makes such simultaneous scaling possible. A detailed discussion of the similitude requirements as applied to scale-model testing is given in [64]. Accurate dynamic scaling of an existing tiltrotor is beyond the scope of the present work - and, given the operating capabilities of the test facility used, is not likely feasible. The Gen-2 and Gen-3 model development efforts, therefore, focused on obtaining physical parameters that were sufficiently similar to full-scale aircraft to be deemed valid subscale test articles.

As mentioned in Chapter 1, the purpose of this work was to experimentally assess the validity of using a computational model to predict the flutter behavior of a small-scale wind tunnel model. Because modeling a specific aircraft was not the goal of the present work, most of the
scaling parameters were relaxed, in order to simplify the design and fabrication process. The one exception to this was the rotor Lock number: the rotor used on the Gen-1 model possessed a Lock number that was uncharacteristically low for a tiltrotor (on the order of 2.36). Therefore, increasing the Lock number of the Gen-2 and Gen-3 models was of interest, and indeed was the impetus for the design and fabrication of composite rotor blades. The Lock number of the XV-15 (3.85, from [33]) was used as a target value for the Gen-3 rotor. Since dynamic scaling was not a predominant concern, the majority of the design effort went into ensuring that the wind tunnel model would be geometrically representative of a full-scale tiltrotor. This necessitated that many of the features of the Gen-1 model - three-bladed, gimbaled rotor; forward-swept wing; collective pitch control for maintaining a constant RPM; and compressed air jets for vibratory excitation - be carried over into the Gen-2 and Gen-3 models. The nondimensional rotor flapping frequency for a gimbaled rotor \( (\nu_\beta = 1) \) was also maintained.

2.1.2 Rotor design

2.1.2.1 Gimbaled hub

A gimbaled hub is required to differentiate between propeller and tiltrotor whirl flutter; the use of a gimbal introduces the rotor dynamics discussed in Appendix C. A front view of the rotor gimbal is shown in Fig. 2.1, and an exploded view of the gimbal is shown in Fig. 2.2. The rotor root cutout is approximately 19\% for the Gen-2 and Gen-3a models, and approximately 18\% for the Gen-3b model. The hub used for all wind tunnel tests was the same hub used for the Gen-1 model, as detailed in [32]. The tolerances of the gimbal - as well as the tolerances of all of the rotor components - were kept very tight to minimize 1/rev vibrations during testing.

The gimbal teeter axis is composed of an alloy steel pin, 0.472 in. long by 0.078 in. diameter, which is pressed into the steel rotor shaft. This steel pin was manufactured to a lengthwise tolerance of \( \pm 0.002 \) in. and a diametral tolerance of 0.0003 in. by the manufacturer [65]. This pin was press-fit into flanged ball bearings, which were themselves pressed into the hub. Retaining clips were then attached to the teeter pin, and this inner assembly was attached to the gimbal housing. The gimbal housing was machined from 6061 billet, and serves as a mounting for both the inner gimbal assembly as well as the blade yokes. Three holes for the blade yokes were drilled and counterbored at 120° intervals around the circumference of the gimbal housing; these mounting holes were tapped to fit the root pegs, which would in turn secure the blade yokes. Two additional smooth-bore holes were drilled through the gimbal housing in the same plane as the blade yoke holes (Fig. 2.3); these two additional holes allowed a second teeter pin to be passed through the gimbal housing and secured with retaining clips. Three root pegs (Fig. 2.4) were screwed into the tapped blade yoke mounting holes and secured using permanent thread locker. To ensure smooth pitch motion and prevent the pitch bearings from binding, the highlighted dimension in Fig. 2.4 needed to be accurate to a tolerance of \( \pm 0.001 \) in. The complete hub assembly is shown in Fig. 2.5; the blade yokes connect to the root pegs through the use of concentric bearings, a washer, and a socket head cap screw. The pitch control assembly attached
to the blade yoke was composed of commercially-available parts for a 600-size hobby helicopter; the ball links used in the wind tunnel model were JMT F-h60071 links from a T-Rex 600 Nitro, and the ball joints were from a Team Associated 6271 short set.

The blades are mounted in the blade yokes via friction fit, and are secured using an M3x0.50 socket head cap screw with a Nylock M3x0.50 locking nut. A minimum screw engagement length of 8.4 mm was used as a guide to select the screw shank length. Commercially-available blades are easily mounted into the blade yokes, due to the factory-installed doublers on the blades (the black, triangular shape at the top of Fig. 2.6); however, the composite blades used for the Gen-2 and Gen-3 models were slightly thinner than the commercial blades by approximately 0.060 in. Several potential items were explored for use as doublers, including nylon and steel washers and shims, which would be bonded to the blade via cyanoacrylate or epoxy adhesives. These solutions proved intractable, as the shear strength of the adhesives was too low to endure even routine mounting and removal of the blades. The problem was finally resolved by using pliers to crimp the ends of the blade yokes and wrapping the root of each blade with electrical tape. The mounting screw was then tightened to ensure that lead-lag motion of the blades was minimized; this permitted the assumption that all lagwise blade motion was due solely to the inherent flexibility of the rotor blades themselves.

Kinematic pitch-flap coupling of the blades was built in to the rotor gimbal. The pitch-flap coupling parameter, $K_p$, is a function of the $\delta_3$ angle of the blades ([2],[25]):

$$K_p = \tan \delta_3$$  \hspace{1cm} (2.7)

Two hub configurations with different pitch-flap coupling are shown in Fig. 2.7. The sense of $K_p$ is considered positive if the sense of $\delta_3$ is positive; this corresponds to the blade pitching down as it flaps up (also known as negative pitch-flap coupling). Conversely, the sense of $K_p$ is negative if the sense of $\delta_3$ is negative; this corresponds to the blade pitching up as it flaps up (also known as positive pitch-flap coupling). These definitions follow the standard Bell convention for $\delta_3$, as detailed in [23] and [25]. Positive $\delta_3$ (negative pitch-flap coupling) exists for the case of the pitch horn extending from the leading edge of the blade, as is shown in Fig. 2.7a; conversely, $\delta_3$ is negative (positive pitch-flap coupling) for the case of the pitch horn extending from the trailing edge of the blade, as shown in Fig. 2.7b. The pitch-flap coupling has a strong effect on the $\beta - 1$ and $\beta + 1$ modal frequencies of the rotor, and can have a profound effect on rotor stability; these effects, and studies of $\delta_3$ effects on stability, are discussed further in [23]. Since the hub for the Gen-2 and Gen-3 models was carried over from the Gen-1 model, the pitch-flap coupling angles $\delta_3 = \pm 47.3^\circ$ were retained. The sense of $\delta_3$ could be changed by rotating the blade yokes 180$^\circ$ so that the tapped hole for the pitch horn faced aft relative to the direction of rotation; the pitch links were relocated on the rotating swashplate (Fig. 2.7). Relocation of the pitch links was not found to affect the root collective pitch angle needed to maintain rotor RPM during testing.
Figure 2.1: Front view of rotor gimbal, from [32].

Figure 2.2: Exploded views of gimbal assembly, from [32].
Figure 2.3: Gimbal housing dimensioned and isometric views, from [32].

Figure 2.4: Blade yoke root peg, from [32].
Figure 2.5: Exploded view of rotor hub assembly, from [32].

Figure 2.6: Rotor kinematic pitch-flap coupling; negative sense of $\delta_3$ shown.
Figure 2.7: Alignment of pitch links for positive (a) and negative (b) sense of $\delta_3$, from [32].
2.1.2.2 Rotor blade design

The Gen-1 model focused exclusively on the use of commercially-available rotor blades, which resulted in high 1/rev vibrations (Fig. 1.9) and an unrealistically low rotor Lock number. The design and fabrication of new rotor blades was therefore a key focus of the present work. Given the relatively light weight of the balsa/spruce Gen-1 blades (on the order of 9.1 grams each) and the constraints of the wind tunnel test section (2 x 3 ft.), it was readily apparent that increasing the rotor Lock number would necessitate reducing the blade mass (which would subsequently reduce the blade flap inertia) and increasing the rotor radius, while still maintaining adequate clearance between the blade tips and the wind tunnel wall.

The blade radius issue was addressed first. Ideally, the radius of the rotor would be extended as much as possible, since Lock number is proportional to \( R^4 \). Initial estimates of rotor radius for the Gen-2 and Gen-3 models were constrained only by the clearance required when the rotor was mounted to the wing (which itself was constrained in semispan to 60% of the test section width); however, minimizing blockage is critical in wind tunnel testing. Therefore a survey of unpowered rotor tests was conducted in order to establish a design space for the Gen-2 and Gen-3 rotor radius. Twenty-one university and industry research studies, comprising both tiltrotors and wind turbines, were examined; the rotor disk area was normalized by the cross-sectional area of the wind tunnel test section to obtain the total area blockage in nondimensional form. The results of this survey, as well as the blockage for the final selection of the Gen-2 and Gen-3 rotor radii, are shown in Fig. 2.8. The final rotor radius for the Gen-2 and Gen-3 wind tunnel models was, respectively, 8.05 and 8.55 inches. It may be seen from Fig. 2.8 that although the Gen-2 and Gen-3 blockages are not ideal, they are reasonably within the range of several other rotor tests.

The Gen-1 blades were symmetrical and approximated a NACA 0011 section, as given by measurements of the blade thickness-to-chord ratio; this airfoil was maintained for the Gen-2 and Gen-3 models. One of the primary issues with the Gen-1 blades was that the untwisted blades produced large regions of the blade with a local angle of attack above the static stall angle of attack for the given airfoil; this may be seen in Fig. 2.9a, which graphs local angle of attack by blade station as calculated from blade element theory. It is evident that the inner 45% of the Gen-1 blades were above the static stall angle of the airfoil. The stall issue was addressed through the use of blade twist; the twist rate (deg. /R) was selected specifically for the operating conditions of the test facility, and was iterated until less than 30% of the blade was above the static stall angle within the operating range of interest (approximately 75 - 120 ft./sec.). Given the low rotor Reynolds numbers expected during testing (approximately 100,000 - 120,000 at 0.75R), a static stall angle of attack of 12° was assumed [83]. The final twist rate selected was \(-25°/R\), which would result in less than 25% of the blade being stalled (Fig. 2.9b).

Initially, rapid prototyping was examined as a possible method of blade fabrication, which would permit almost any amount of twist to be built in to the blade; the blade mass could then be reduced through the use of trellising throughout the blade span and an aerodynamic shape
maintained with a thin MonoKote overwrap (Fig. 2.10a). This design approach presented some rather serious drawbacks, not the least of which would be the dubious structural integrity along the trailing edge of the blade due to the trellising (Fig. 2.10b). Although the trellising could be adjusted to leave more material on the trailing edge of the blade, the sheer amount of trellis cutouts needed to meet the blade mass target (the blade shown in Fig. 2.10a, for instance, had over 2,500 cutout elements) precluded this from being done in a reasonable fashion.

The use of composites allowed the fabrication of blades that were both strong and lightweight. The composite sandwich panel method presented in Appendix B was used to analyze potential candidates for the blade core and skin; these candidates included standard hobby-grade foams (such as expanded polystyrene and expanded polypropylene), surfboard foam (e.g. Spyderfoam), and construction foam (e.g. partially cross-linked polyvinylchloride). Divinycell H80 foam was eventually selected for the blade core, primarily for its strength-to-weight ratio, low open cell content, and high glass transition temperature; a complete list of Divinycell H properties may be found in [67]. A survey of potential reinforcing fabrics showed that Toray T300-1k 3x1 twill weave fabric (part number CF101) would be an ideal candidate for the blade skin, due to the reliability of its room-temperature cure properties as well as its drapability. A single layer of HexTow AS4-12k (part number CF31x) was used beneath the Toray skin. All carbon fiber and core material was secured from CST Composites in Tehachapi, CA. Material properties for the composite components were obtained from manufacturer data sheets, and are tabulated in Table 2.1.

Prior to fabrication, the maximum operating stresses of the blades were analyzed. The blades of the Gen-3b model were considered the worst-case scenario, as they were the largest and heaviest of the blades tested for this project. It was assumed that the primary stresses that would be experienced by the blades during testing would be from centrifugal loading (caused by the rotation of the rotor about the shaft) and a flapwise bending moment (caused by the force exerted on the blades by the freestream air). Lagwise bending was not examined. A factor of safety of 1.5 was applied to all of the expected loads. The centrifugal loading on the blade is a function of its mass, radius, and the square of its rotational speed:

\[ F_{CF} = \int_0^R m g \Omega^2 dy \]  

(2.8)

This integration was performed numerically, as the mass distribution of the composite blades was not uniform due to chordwise taper along the blade span. This calculation was then compared to a SolidWorks analysis, and in both cases the stresses caused by centrifugal loading of the blade was found to be less than 13 MPa (Fig. 2.11).

To calculate the flapwise bending force experienced by the blades, each blade was treated as a cantilevered beam subjected to a distributed line load (Fig. 2.12) that was taken to represent the load exerted on the blade by the oncoming air at maximum tunnel speed. Note that the orientation of the simplified blade shown in Fig. 2.12 represents a full “face-on” flow and neglects
centrifugal stiffening; this would not be the case during wind tunnel testing, but was an assumption made to simplify the stress analysis of the blade. Thus, the flapwise bending analysis is overly conservative. The blade was discretized, and a constant $C_d$ of 1.28 was assumed for all blade stations. The differential flapwise bending moment on each element was calculated as

$$M_\beta = \int_0^R \frac{1}{2} C_d c \rho V_\infty^2 y dy$$  \hspace{1cm} (2.9)

The flapwise bending moments were integrated and used to find the axial stress at the outermost face sheet, located a distance $z$ from the blade section centerline:

$$\sigma_{x,\beta} = \frac{M_\beta z}{I_y}$$  \hspace{1cm} (2.10)

For this calculation, the blade cross-section was approximated as a rectangle, and thus the second moment of area was calculated as

$$I_y = \frac{1}{12} b^3 h$$  \hspace{1cm} (2.11)

As seen in Fig. 2.13, the maximum expected flapwise bending stress for the rotor blade was on the order of 400 MPa. The resultant stresses of the blade were transferred from the structural axes of the blade to the fiber axes of the composite, and were then compared to the composite strength criteria presented in Appendix B (maximum stress criterion, maximum strain criterion, Tsai-Hill criterion). The maximum stresses predicted for the rotor blades are shown in Fig. 2.14 relative to these strength criteria. Even with a factor of safety of 1.5 at the maximum operating speed of the test facility, the composite blades are not in danger of exceeding any of the strength criteria bounds.
Table 2.1: Composite material properties.

<table>
<thead>
<tr>
<th>Divinycell H80</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal density</td>
<td>80 kg/m$^3$</td>
</tr>
<tr>
<td>Compressive strength (ASTM D1621)</td>
<td>1.2 MPa</td>
</tr>
<tr>
<td>Compressive modulus (ASTM D1621)</td>
<td>85 MPa</td>
</tr>
<tr>
<td>Tensile strength (ASTM D1623)</td>
<td>2.2 MPa</td>
</tr>
<tr>
<td>Shear strength (ASTM C273)</td>
<td>1.0 MPa</td>
</tr>
<tr>
<td>Shear modulus (ASTM C273)</td>
<td>31 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spyderfoam</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal density</td>
<td>36.85 kg/m$^3$</td>
</tr>
<tr>
<td>Compressive strength (ASTM D1621)</td>
<td>0.6895 MPa</td>
</tr>
<tr>
<td>Compressive modulus (ASTM D1621)</td>
<td>25.51 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HexTow AS4 Unidirectional</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal density</td>
<td>1790 kg/m$^3$</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>4,620 MPa</td>
</tr>
<tr>
<td>Tensile modulus</td>
<td>231 GPa</td>
</tr>
<tr>
<td>Compressive modulus</td>
<td>128 GPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Toray T300 3x1 0/90 Fabric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal density</td>
<td>1760 kg/m$^3$</td>
</tr>
<tr>
<td>Tensile strength (ASTM D3039)</td>
<td>1,860 MPa</td>
</tr>
<tr>
<td>Tensile modulus (ASTM D3039)</td>
<td>135 GPa</td>
</tr>
<tr>
<td>Compressive modulus (ASTM D695)</td>
<td>1,470 GPa</td>
</tr>
</tbody>
</table>
Figure 2.8: Wind tunnel blockage survey.
Figure 2.9: Local angle of attack by blade station for untwisted (a) and twisted (b) blades (Re: 100,000 – 120,000 at 0.75 R).

Figure 2.10: Rapid prototype blade design with trellising.
Figure 2.11: SolidWorks stress simulation for centrifugal loading.

Figure 2.12: Simplified model of rotor blade as cantilevered beam subjected to line load.

Figure 2.13: SolidWorks stress simulation for flapwise bending.
Figure 2.14: Calculated maximum blade stresses at 2,000 RPM and 150 ft./sec. tunnel speed with 1.5 factor of safety. Black: maximum stress criterion. Blue: maximum strain criterion. Red: Tsai-Hill criterion.
2.1.2.3 Composite rotor blade fabrication

An overview of a fabrication process used for the rotor blades of the Gen-2 and Gen-3 wind tunnel models is detailed here. Note that while only one set of blades incorporated twist, the fabrication process did not change between the two sets of blades. The process for fabricating the Gen-3 composite wing is virtually identical to the method presented here, with differences noted in the wing fabrication section as necessary.

The first step was to model the outer mold line of the blade in SolidWorks, which required the coordinates of the blade section from a public airfoil database [68]. The coordinates were loaded into SolidWorks as a two-dimensional sketch on a plane (Fig. 2.15); this plane is referred to as the base plane. Loading the coordinates onto the Right Plane in SolidWorks is standard, but not required. An offset plane was created, using the base plane as a reference and with the offset distance set to the blade span. The Convert Entities function was used to project the original sketch onto the offset plane; at this point, the projected sketch may be modified in scale (to create taper), location (to create blade sweep), and rotation (to create blade twist). The blade outer mold line is created by using the Lofted Boss/Base feature to loft from one sketch to another. Note that this procedure is not limited to lofting between two planes: rotor blades may be designed with multiple twist rates, airfoils, taper, and sweep angles (all of which can vary along the blade span) simply by lofting between multiple planes. A very simple example (Fig. 2.16) would be bi-linear twist distribution.

Once the blade shape was finalized, the Trim Entities function was used to cut the blade root and tip sketches in half; these halves were lofted together to create a thin surface. This lofted surface formed the inner surface of the female tool; the Sketch and Extruded Boss/Base functions were then used to create a rectangular support structure for the tool (Fig. 2.17a). The process was repeated with the top half of the blade. This created two female tool halves, which when combined (Fig. 2.17b) created a cavity for forming the blade. The CAD files were saved in STL format, and were loaded into an Objet 260 Connex rapid prototyping machine at the Penn State Learning Factory; the blade tools were fabricated in green Digital ABS from RGD515 and RGD535 materials, and thoroughly cleaned with water and air-dried for twelve hours before use.

To make the blade core, rectangular strips of Divinycell foam (0.125 in. thick) were hand-cut to the approximate length and width of the tool (Fig. 2.18); the foam strips were cut to the width of the blade root chord. Each foam strip was clamped to the lab benchtop, and the long sides of the foam strip were sanded (Fig. 2.19) with 300-grit sandpaper to approximate an airfoil shape. A small piece of end-grain balsa (approximately 1.250 x 1.280 x 0.125 in.) was sanded in a fashion similar to the foam strips, and bonded to one end of the foam strip (Fig. 2.20) using Kwik Bond five-minute epoxy. The balsa/foam assembly was allowed to set for ten minutes; a piece of masking tape was used to balsa/foam butt joint during this set time. The end-grain balsa root not only provides additional structural support at the blade root, but also provides a secure core for drilling and tightening the screws that hold the blades within the blade yoke. Note that the sanding only approximates an airfoil shape within the core – the actual airfoil
shape is imparted by the compaction pressure applied to the tool during cure.

While the blade root joint was curing, the blade tool was prepared for layup. Each tool half was wrapped with cellophane, which was secured to the tool with masking tape (Fig. 2.21). The cellophane was then sprayed with Polytek PolEase 2300 release spray and the tool halves were set aside. Strips of the AS4 unidirectional fiber were sprayed with 3M Super 77 adhesive, and set atop the joint between the balsa and the foam strip (Fig. 2.22) with the binder threads facing outward from the blade core; these strips of unidirectional fiber were cut to uniform length, and span approximately 75% of the blade. Rectangular pieces of the Toray twill fabric were cut and biased to 45° before being wrapped around the blade core (Fig. 2.23); the wrap will create a seam along the blade, so the twill fabric was wrapped around the blade in such a fashion as to place this seam on the pressure side of the blade. Excess fabric was trimmed using serrated shears.

The laminating resin was then mixed. MGS L285/H285 was mixed in a 5:2 ratio (by mass); at this ratio, each blade required ten grams of resin and four grams of hardener. The resin was mixed for one minute, and the blade was wetted out for five to seven minutes (Fig. 2.24). Since this resin mixture has a pot life of over thirty minutes, small adjustments to the blade layup are possible during this time. The resin mixture was brushed into the blade to ensure that the entire assembly was wetted out, and then a paper towel was used to blot any excess resin from the blade. The wetted blade was then placed into the tool, and the cellophane was wrapped around the blade and secured with flash breaker tape (Fig. 2.25). The top half of the tool was placed atop this assembly (Fig. 2.26a), and the entire stack was secured to a benchtop using weights and clamps (Fig. 2.26b). The blade was allowed to cure at room temperature for sixteen hours.

After curing, the blade was removed from the tool. As shown in Fig. 2.27a, the tool has imparted the proper airfoil shape to the blade, and the blade now has an approximately trapezoidal planform. A (1:1 scale) drawing of the desired planform was created in SolidWorks, and was taped to the top of the blade as a template (Fig. 2.27b). Using a Zona razor saw (48 teeth per inch) and 300-grit sandpaper, the raw blade form was trimmed and sanded to match the planform template (Fig. 2.28a). Wrapping a cylindrical object (such as a can of WD-40) with sandpaper was found to be the easiest way to obtain the curvature of the blade root trailing edge. A #43 carbide drill bit was used to drill the mounting hole on the blade. Note that a #43 bit is used to tap a #4 machine screw, which is slightly smaller in major diameter than the M3 screw that secures the blade: a #4 hole is 0.112 in., or approximately 2.84 mm in diameter. This was intentional, and allows the M3 screw to re-bore (and consequently self-tap) the mounting hole during installation, ensuring a secure fit. A layer of waterproof clear coat was sprayed on all blade surfaces to prevent moisture absorption and preserve the skin finish. A finished blade is shown in Fig. 2.29.

The rotor blades were individually balanced after all three had been fabricated. Because the blades were hand-fabricated using wet layup, there were slight inconsistencies in mass and center of gravity between the three; if not balanced, these inconsistencies could cause severe vibration during wind tunnel testing. A simple balance methodology is to select the heaviest blade in the set as the master blade, and then to add mass (via balance tape or electrical tape) to the other
blades until their c.g. lies within an acceptable margin (typically 1 mm) of the master blade c.g. location. It is most useful to cut squares of tape of varying sizes prior to blade balancing, so that the total mass added to each blade can be readily calculated and recorded; however, areal density \((kg/m^2)\) varies between tape manufacturers and no standard value is presented here; standard thicknesses are 180 and 250 \(\mu m\), but gage weights will vary based on the chemical composition of the tape.

A UniShow digital scale (500 grams maximum capacity, 0.01 g resolution) was used to measure the blade masses. The chordwise c.g. location of the master blade can be measured by balancing the blade atop a knife edge as shown in Fig. 2.30; the chordwise c.g. location is then marked with a pencil, and the distance from the trailing edge of the blade may be measured with machinist’s calipers and recorded. This process is repeated with the other two blades, and the chordwise c.g. location could therefore be adjusted by placing pieces of tape at the leading or trailing edge of the blade as necessary; it should be noted that, prior to conducting the wind tunnel tests, the chordwise c.g. location did not vary by more than 0.007 in. between blades. The radial c.g. location was found by using a simple two-block balance setup, as shown in Fig. 2.31. A #4-40 threaded steel shaft was passed through the blades, which were then allowed to rotate freely; because the threaded shaft also taps the blades during insertion, this method ensures that the blades are not able to rotate about the shaft. Differences in radial c.g. locations will cause the blades to settle at an angle relative to the benchtop. Electrical tape was added to the tip of the lighter blade, and the process was repeated until the blades came to rest within 1° of parallel. After balancing was completed, the blades were marked as a matched set.
Figure 2.15: NACA 0011 section sketch on base plane.

Figure 2.16: NACA 0011 section sketch on base plane.
Figure 2.17: Blade female tool half and tool assembly for twisted blade.

Figure 2.18: Divinycell H80 blade core.
Figure 2.19: Core sanding to approximate airfoil section profile.

Figure 2.20: End-grain balsa blade root attachment.
Figure 2.21: Tool release film preparation.
Figure 2.22: Unidirectional fabric layup.

Figure 2.23: Twill layup and trim.
Figure 2.24: Blade wet-out.

Figure 2.25: Blade placement in tool.
Figure 2.26: Blade secured in tool (a) and clamped to table (b) for sixteen-hour cure at room temperature.

Figure 2.27: Raw blade (a) and template mounting (b).
Figure 2.28: Blade trimming, sanding, and drilling according to template.

Figure 2.29: Finished blade.
Figure 2.30: Rotor blade chordwise c.g. balancing assembly.

Figure 2.31: Rotor blade radial c.g. balancing assembly.
2.1.3 Wing design and fabrication

The Gen-1 model was able to achieve flutter only through the attachment of an extra mass aft of the trailing edge of the wing; this reduced the effective beamwise stiffness to the point where flutter could occur within the operating limits of the test facility. However, this aft-mass also caused the rotor gimbal to strike the rotor shaft during testing; this is evidenced by still frames of the slow-motion videos made during testing (Fig. 3.32) as well as physical damage (scoring) to the rotor shaft (Fig. 3.33). It is likely that these factors may have contributed to the discrepancies in predicted and experimental damping ratio values experienced by the Gen-1 model. Tuning the stiffness of the wing so that flutter testing could be conducted without the use of an aft-mass was one of the key improvements from the Gen-1 model. The section airfoil (Selig S4233), wing sweep (five degrees forward), and constant-chord planform of the wing were not changed from the Gen-1 model during the design of the Gen-3 model.

The monocoque structure of the Gen-1 wing also produced a very wide spread between the beam and chord modal frequencies; this frequency ratio was 5.40, which is much higher than the frequency spread of larger-scale whirl flutter models such as WRATS:

<table>
<thead>
<tr>
<th>Model</th>
<th>(f_{\text{beam}}, \text{Hz})</th>
<th>(f_{\text{chord}}, \text{Hz})</th>
<th>(f_{\text{beam}}/f_{\text{chord}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen-1 (Config. 4)</td>
<td>5.82</td>
<td>31.41</td>
<td>5.40</td>
</tr>
<tr>
<td>Gen-3 (target)</td>
<td>5.57</td>
<td>7.43</td>
<td>1.33</td>
</tr>
<tr>
<td>WRATS</td>
<td>5.43</td>
<td>8.11</td>
<td>1.49</td>
</tr>
</tbody>
</table>

It was immediately apparent that the use of another monocoque wing would not bring the ratio of modal frequencies any closer together, and was also unlikely to permit the model to experience an instability without the use of the aft-mass. This is not to say that a monocoque wing would _never_ experience flutter – and, indeed, one of the key conclusions from testing of the Gen-1 model (see [32]) was that demonstrating whirl flutter with a small-scale model was indeed possible; however, the only Gen-1 configuration that exhibited whirl flutter was only able to do so with a 1-lb. steel mass mounted aft of the wing trailing edge (aft-mass e.g. located 5.5 in. aft of the wing elastic axis). Large masses located aft of the wing trailing edge are not representative of full-scale aircraft, and thus one of the primary goals of the Gen-3 model was to tailor the wing stiffness such that the model would exhibit whirl flutter within the operating limits of the test facility and without the use of an added mass. As such, either a semi-monocoque wing, or a wing with a flexure, would be required.

Designing a true semi-monocoque wing would require extensive structural analysis and sizing of the requisite spars, stringers, ribs, and skin that would be required to resist the air loads at maximum tunnel speed. This would also present a number of practical fabrication challenges. The decision to use a composite wing, therefore, was made due to the fact that a composite wing could conceivably be designed and fabricated much more simply than a true semi-monocoque wing, and would have the added benefit of reusing the composites tools and chemicals already
acquired to fabricate the rotor blades. This method required the wing, spar, and nacelle to be redesigned as three independent pieces, which were then assembled during fabrication.

A composite wing by itself would have been far too stiff to allow for flutter testing in the current facility, so a spar was incorporated into the wing. If the spar is treated as a cantilevered beam, it may be seen that most of the first-mode bending occurs within the inboard 30% of the spar, whereas the rest of the mode shape is relatively linear (Fig. 2.34). This allows for the inner 30% of the spar to be treated as a flexure region, with the wing of the model spanning only the outer 70% of the semispan. Thus, the stiffness of the composite wing is itself irrelevant since it will not contribute to the overall root stiffness of the model. This assumption does not hold for higher-order modes, but as these modes would not be encountered during testing, they were deemed irrelevant.

An assembly of the Gen-3 wing is shown in Fig. 2.35. The spar was designed to ensure a clearance of at least 1.250 in. between the wind tunnel wall and the tip of the Gen-3b rotor blades. This meant that the rotor shaft centerline could be located no more than 14.20 in. from the mounting wall. One inch was required to secure the spar within the wing clamp, and a buffer of 0.125 in. was used for additional clearance. This constrained the cantilevered length of the spar to thirteen inches. Using the analytical model that had been developed for this project, target wing stiffness values were iterated until the Gen-3 model was predicted to exhibit whirl flutter at approximately the same tunnel speed as the Gen-1 model. From these target stiffness values, the modal frequencies of the Gen-3 model were calculated using the structural dynamics principles presented in Appendix A; these are the target modal frequencies presented in Table 2.2.

Since the overall spar length was already constrained, the spar modulus and cross-sectional inertia were the only variables left to iterate. Four possible spar materials were examined: cold-rolled steel (AISI 1080), aluminum (6061-T6), stainless steel (304), and graphite. Since the wing geometry was unchanged from the Gen-1 model, the cross-sectional geometry of the spar was constrained to a maximum of 1.000 x 0.8125 in. in order to reasonably fit within the wing. The material and geometric options created a matrix of 116 possible spars, of which only nine met the target stiffness requirements. Imposing the target frequency spread shown in Table 2.2 yielded the final spar dimensions. The maximum recorded flutter loads (a roll moment of approximately 7 Nm) presented in [32] were used to ensure that the spar would not yield during testing; the final spar design had a factor of safety of approximately 1.17. The geometric and material properties of the Gen-3 spar are given in Table 2.3. The bending and torsional stiffnesses of the spar were calculated using the methods presented in a standard text on strength of materials [55]. The raw material (0.375 x 0.375 in. billet) for the spar was machined at the Penn State Learning Factory. In order to minimize warping caused by residual stresses in the raw material, all four faces of the billet were milled with approximately the same number of passes and cut depths. A dimensioned drawing of the wing core (Fig. 2.36) was then emailed to Flying Foam, LLC for CNC hot-wire cutting from Spyderfoam stock.

Since the wing, spar, and nacelle were now independent pieces, a reliable method of maintain-
ing proper alignment between the three parts was necessary. For the spar, this meant cutting the top foam away from the wing core, which left a channel along the length of the wing; this is the rectangular cutout seen in Fig. 2.37. The oval cutout in Fig. 2.37 is a pass-through channel to accommodate the tubing for the excitation jets and the wiring for the servo. The top of the spar cutout was removed using a razor blade, which then created a U-channel for the spar (Fig. 2.38). The sides of the spar were sanded with 180-grit sandpaper to create a rough surface for adhesive bonding, and the spar and channel were both covered in Kwik Bond thirty-minute epoxy. The spar was placed within its channel, and the top of the wing core was replaced. The spar/wing assembly was allowed to cure overnight at room temperature.

The nacelle of the Gen-3 model was modified from the Gen-1 SolidWorks model; the Trim Entities and Extruded Cut commands in SolidWorks were used to remove unnecessary pieces of the Gen-1 model until only the nacelle and a portion of an attachment surface remained (Fig. 2.39). Two wing mounting holes were cut through this nacelle in SolidWorks and were sized for #8-32 machine screws, which were used to secure the nacelle to the wing (Fig.2.40). The machine screws are not the primary method of securing the nacelle, because over-tightening the nuts would crush the foam. The nacelle was bonded to the wing core using the same thirty-minute epoxy used to bond the spar to the wing; the machine screws thus serve only to align the nacelle, and as a secondary method of attachment.

The nacelle bonding was performed after the spar had set (but not cured) and after a fresh batch of epoxy had been mixed; this allowed both the spar and nacelle bonds to co-cure. Once the spar and nacelle had cured, the wing core was wrapped with a single layer of Toray T300 twill (areal density 5.7 oz./sq.yd.). The same fabrication procedure used to fabricate the rotor blades was repeated for the wing, with the primary difference being that the wing core was pre-cut and did not require additional shaping. This wing core came with a foam bed that served as a female tool, and provided flat outer surfaces for securing the layup during cure. Mylar sheets were also used for the wing release film, rather than the cellophane used for the rotor blades, which yielded a wing surface finish that was more consistent between samples. The wing overwrap provides both the aerodynamic shape for the wing and an extra measure of security in keeping the underlying components in place. The finished Gen-3 wing is shown mounted to the wind tunnel load cell in Fig. 2.41.

Rap tests were conducted to compare the wing modal frequencies with the rotor operating RPM in order to identify any regions of potential rotor/wing interaction. A Campbell diagram was constructed (Fig. 2.42) to determine an operating RPM that would not be near the wing modes. From this Campbell diagram, an operating speed of 2,000 RPM was selected for the Gen-3 rotor.
Table 2.3: Geometric and material properties of Gen-3 spar.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier</td>
<td>McMaster-Carr</td>
</tr>
<tr>
<td>Raw material</td>
<td>6061-T6 aluminum</td>
</tr>
<tr>
<td>Part number</td>
<td>89015K721</td>
</tr>
<tr>
<td>Spar width</td>
<td>0.252 in.</td>
</tr>
<tr>
<td>Spar thickness</td>
<td>0.190 in.</td>
</tr>
<tr>
<td>Cantilevered length</td>
<td>13.000 in.</td>
</tr>
<tr>
<td>$EI_{beam}$</td>
<td>4.069 $Nm^2$</td>
</tr>
<tr>
<td>$EI_{chord}$</td>
<td>7.234 $Nm^2$</td>
</tr>
<tr>
<td>$GJ$</td>
<td>17.42 $Nm^2$</td>
</tr>
</tbody>
</table>
Figure 2.32: Still frame of gimbal bucket striking rotor shaft.

Measured angle: 79.8°
Figure 2.33: Shaft damage at the conclusion of Gen-1 testing.
Figure 2.34: First mode shape of cantilevered beam.

Figure 2.35: Gen-3 wing assembly.
Figure 2.36: Dimensioned drawing of Gen-3 wing.

Figure 2.37: Gen-3 wing core.
Figure 2.38: Gen-3 wing core and spar mounted in foam bed.

Figure 2.39: Gen-3 nacelle.
Figure 2.40: Gen-3 nacelle secured to wing using machine screws.

Figure 2.41: Gen-3 wing mounted to wind tunnel wall.
Figure 2.42: Gen-3 wing frequencies versus rotor RPM.
2.2 Control System

Since the thrust of a rotor does not change the dynamics of a wing/rotor system in a significant way (see [13]), rotor thrust is not a useful parameter to vary for stability studies. A windmilling rotor is the slightly less stable case, and is far easier to construct than a powered rotor. The windmilling case has been validated for use in stability studies by [18], [20], [23], [25], [32], and [38]; because the rotor is unpowered, variation of collective pitch is used to control rotor RPM. All wind tunnel models examined in the current work accomplish this through the use of a rigid swashplate (viz. one which does not permit cyclic control) actuated by a digital servo mounted inside the nacelle. Shaft speed is measured using small disc magnets and a Hall effect sensor.

2.2.1 Rotor electromechanical control

The rotor actuation assembly is shown in Fig. 2.43. A bearing block machined from 6061 billet houses two five-millimeter ball bearings to support the shaft. The swashplate is divided into an upper and lower half; the lower half (Fig. 2.44) is drilled to accept two #2-56 threaded guide rods, and the top half secures the lower swashplate to two support bearings. The threaded guide rods pass through the bearing block and screw into the top half of the lower swashplate; these guide rods are secured using nuts and Loctite 242 thread locker. The pitch links are ball links threaded for M2.5x0.45 screws and connected to a variable-length pitch arm that is mounted to the servo. The pitch arm is attached to the bottom half of the swashplate with an M2 screw threaded through the ball link. The complete swashplate assembly and exploded view are shown in Fig. 2.45. Torque is transferred to the top swashplate using an alignment groove that was machined axially into the rotor shaft; this may be partially seen in Fig. 2.45b. An M3x0.5 hole was drilled and tapped through the upper swashplate to permit a socket head cap screw to be threaded into the alignment groove. This screw was filed down using a rotary cutting tool until it could be fully threaded into the upper swashplate without contacting the bottom of the alignment groove. This arrangement permits the swashplate assembly to slide axially along the rotor shaft, but does not permit rotation of the swashplate assembly relative to the rotor shaft.

Pitch actuation was controlled by a JR Servo DS8717, which is a common servo typically found in 600-size radio-controlled helicopters. Since the swashplate and rotor hub were based on a 600-size helicopter for the Gen-1 model, a new servo was not selected for the Gen-2 and Gen-3 models. The DS8717 is a lightweight, high-speed digital servo with a coreless motor and metal gearing; it is capable of 120° of rotation (60° in 0.07 sec.) and can produce over 196 oz.-in. (1 lb.-ft.) of torque when driven with a six-volt supply. Additional information on the DS8717 may be found in standard hobby references, such as [69].

Rotor rotational speed was measured using a Hall effect sensor and a small disc magnet (K&J Magnetics part number DH11) that was affixed to the set screw of the rotor shaft collar (shown adjacent to the bearing box in Fig. 2.43). Two magnets were used during wind tunnel testing, although the data acquisition code used for this project can theoretically accommodate any number of magnets. Cyanoacrylate adhesive was used to bond the Hall effect sensor to a
thin aluminum plate, which was then secured to the inside of the nacelle using five-minute epoxy. When supplied with power, the Hall effect sensor outputs a signal of approximately 0.5 volts; this output signal remains constant until a magnetic field crosses the sensor’s path, at which point the sensor output drops to zero. When a time-varying magnetic field is applied to the Hall effect sensor (such as the one caused by a magnet mounted on a rotating shaft), the output read by the data acquisition system is a time-varying square wave. Taking a fast Fourier transform (FFT) of this square wave shows the frequency of the magnetic field variation, which is then output by the DAQ software in the form of RPM.

The pitch control servo is controlled by a HiTec X1 multifunction controller (Fig. 2.46). The X1 can both power and control the servo, which eliminated the need for much of the support equipment that had previously been used during Gen-1 wind tunnel testing: the Gen-1 tests required a 24 VDC source for the solenoid valve, a 12 VDC source for the Hall effect sensor, and a 6 VDC source for the pitch servo; additionally, a separate Servo City controller - and a dedicated second computer - were needed to command the pitch servo. The current work simplified much of this by switching to a 12 VDC solenoid valve, and the X1 eliminated the need for a second power supply, a second computer, and a separate controller, as all of these functions are integrated within the unit. This not only improved the orderliness of the wind tunnel facility during flutter testing, but also permitted all testing and data acquisition to be performed by a single person safely and reliably. The use of the X1 controller greatly improved the resolution of RPM control: re-testing the Gen-1 model in October 2013 (using the original Servo City controller) revealed that the servo was not able to reliable maintain rotor speed to within less than approximately ±50 RPM, whereas the X1 was able to maintain rotor speed to within approximately ±5 RPM. This helped to reduce the risk of RPM variations affecting the correlation of experimental data with the computational predictions.
Figure 2.43: Rotor actuation assembly.

Figure 2.44: Swashplate bottom.
Figure 2.45: Swashplate assembly and exploded view, from [32].

Figure 2.46: HiTec X1 multifunction controller, used to power and command pitch control servo.
2.2.2 Operating range

The pitch control servo was controlled using a form of pulse-width modulation that is relatively unique to hobby-grade servos. The servo has a standard three-wire connection (Fig. 2.47) which connects to the X1 controller via four-foot lead extensions; two of these wires (red and brown) are for carrying DC power, and the third (orange) is used to carry the PWM signal. The PWM signal itself is a series of repeated pulses of varying width. The parameters for the pulse are minimum pulse width, maximum pulse width, and repetition rate, which are limited to a range that is predefined by the manufacturer. A PWM signal is shown diagrammatically in Fig. 2.48.

For a rotary servo, the neutral position (zero position, or 90°) is defined as the point where the servo arm has an equal amount of possible rotation in both the clockwise and counter-clockwise directions; the zero position for most digital servos is typically around 1.5 milliseconds of pulse width (sometimes known as the “zeroing pulse”). The angle of rotation of the servo is determined by the pulse width of the signal applied to the control wire, and the servo is controlled solely by varying this pulse width. The servo is programmed to expect a pulse at predetermined intervals, which is called the pulse window; the number of times that a signal is expected within this pulse window is known as the frame rate. For hobby-grade servos, this pulse window is approximately six milliseconds. The width of the pulse signal that is read during each pulse window will determine the amount of rotation exhibited by the servo motor.

Note that the type of PWM control used in hobby-grade servos differs slightly from the notion of duty cycles (“on” time vs. “off” time) that are typically used for PWM control. Most radio-control servos will move to the exact same position irrespective of the pulse window or the frame rate. For example, if the zeroing pulse width is 1.5 milliseconds, the servo will return to its zero position whether that zeroing pulse comes every ten milliseconds (a duty cycle of 15%) or every two milliseconds (a duty cycle of 75%). With most hobby servos, only the pulse width is of concern. This means that the total period of the signal can vary widely - and can also vary between individual pulses - without affecting the accuracy of the servo. When the pulse width is less than the zeroing pulse, the servo rotates to a fixed angle relative to the neutral position, and will hold that angle until the width of the pulse changes. When a pulse width is greater than the zeroing pulse, the shaft rotates in the opposite direction and continues to hold its position. Different servos will have different allowable pulse widths, although minimum and maximum pulse width values of one and two milliseconds, respectively, are common.

The selection criteria for the length of the servo arm are detailed in [32]; the servo arm length for the Gen-1 model was 10.75 mm, and was left unchanged for the Gen-2 and Gen-3 models. Benchtop testing was conducted to ensure linearity between the servo input signal and the shaft position. Since the rotor collective pitch cannot be directly measured during testing, it was necessary to develop a method by which the blade pitch could be calculated based upon the command signal of the servo controller. The X1 controller was a new feature to the wind tunnel testing of the Gen-2 and Gen-3 models, and thus the relationship between the command signal and the servo response first had to be analyzed to ensure linearity. To test the linearity of the new
controller, the Gen-1 model was mounted to the load cell of the Hammond wind tunnel, and the load cell window was rotated 90°; the shaft collar on the rotor shaft was relocated to the bottom of the gimbal, preventing blade flap and effectively locking the rotor plane into position. The pitch control servo was set to its minimum pulse width of 1.42 milliseconds, and the collective pitch was determined by using the same methodology presented in [32]: namely, measuring the distances from the leading and trailing edges of a given rotor blade to the bottom surface of the test section and determining the collective pitch as:

$$\theta_0 = \arcsin \left( \frac{d_{LE} - d_{TE}}{c_b} \right)$$

(2.12)

where \(d_{LE}, d_{TE}\), and \(c_b\) are defined in Fig. 2.49. A measuring tape was used to determine the values of \(d_{LE}\) and \(d_{TE}\) at each pitch setting. Three measurements of \(d_{LE}\) and \(d_{TE}\) (one for each blade) were taken at each pitch setting, and were then averaged together to obtain the \(d_{LE}\) and \(d_{TE}\) values used in Eq. 2.12. The pitch measurement results are shown in Fig. 2.50; as may be seen, the relationship between the servo controller command signal and the pitch setting is indeed linear. Equally evident is that several of the pitch measurements very nearly overlapped, and any differences between them are more likely due to measurement uncertainty than variations in the command signal. Once the pitch measurements were completed, the measurement process was repeated again, this time to determine the free play in the pitch control system, which was 2.1° on average; this is approximately 0.2° higher than what was measured during Gen-1 testing.

It should be noted that the X1 controller can vary the command signal by as little as 0.002 milliseconds, which would result in 138 possible positions throughout the range of the DS8717 servo; in the interest of expediency, the collective pitch was measured at only twelve command signals, which represented the range of collective pitch that would most likely be needed during flutter testing. The maximum pitch setting of the servo was found to be approximately 65°, which is ten degrees higher than the predicted pitch setting needed.
Figure 2.47: Servo connector diagram.

Figure 2.48: Diagram of pulse-width modulated signal.
Figure 2.49: Collective pitch measurement diagram, from [32].

Figure 2.50: Rotor root collective pitch versus servo PWM setting.
2.3 Inputs to Analytical Model

The stability predictions of the analytical model require several physical and environmental parameters as inputs. Where practical, values from SolidWorks were used as inputs for variables such as rotor radius and location of the wing elastic axis with respect to the wing leading edge; however, physically measured and experimentally-determined values were favored for variables such as mass, inertia, frequencies, etc.

In particular, the composite blades and composite wing required physical property testing, since there was a potential for variability in blade and wing properties - such as fiber volume fraction (a product of using a wet layup rather than pre-impregnated sheets for the fabrication of composite components), foam core density, etc. - that could impact the behavior of the wind tunnel model. The procedures used for obtaining the required physical measurements and the experimentally-determined values for the composite parts are detailed in Chapters 3 and 4. The inputs for the analytical model may be broadly categorized as operational (test environment and rotor RPM), rotor inputs, wing inputs, and nacelle inputs.

2.3.1 Operational inputs

The required operational inputs are tabulated below:

- $\rho$: freestream density, $slug/ft.^3$
- $\mu$: freestream dynamic viscosity, $slug/(ft. - sec.)$
- $\Omega$: rotor rotational speed, $rad/sec.$

Ideal gas relations were assumed to calculate density and viscosity from the pressure and temperature readings for a given test. The analytical model assumes constant atmospheric properties throughout all tunnel speeds, although this was not necessarily true during testing: specifically, prolonged use of the tunnel would cause the temperature of the air within the tunnel to increase. The rotor rotational speed was held at 1,600 RPM for the Gen-2 tests, and was increased to 2,000 RPM for the Gen-3 tests in order for the shaft speed of the rotor to be above the wing modal frequencies. Because Gen-2 used the same monocoque wing as Gen-1, there was no need to alter its operating speed.

2.3.2 Rotor inputs

The rotor properties required for the analytical model are detailed below:

- $a$: rotor section lift-curve slope, $/rad$
- $c$: rotor blade chord at 0.75 R, $ft.$
- $d_{mast}$: distance from the wing elastic axis to the c.g. of the rotor hub, $ft.$
• $I_b$: rotor flap inertia, slug $- ft.^2$

• $K_p$: rotor kinematic pitch-flap coupling, dimensionless

• $M_b$: rotor blade mass (including contribution from hub), slug

• $N$: number of blades

• $R$: rotor radius, $ft.$

These parameters were used to calculate two nondimensional parameters:

• $\bar{h}$: mast height normalized by rotor radius, $\bar{h} = \frac{d_{mast}}{R}$

• $M^*_b$: blade mass normalized by radius and flap inertia, $M^*_b = \frac{M_b R^2}{I_b}$

The blade lift-curve slope was assumed constant, since only one airfoil section is used throughout the blade span. Other aerodynamic coefficients are interpolated from polars based on the local blade angle of attack, which varies by blade station based on collective pitch, twist, freestream speed, and rotor RPM. The rotating cyclic flap frequency $\nu_\beta$ was set to unity, since the hub is gimbaled and lacks a hub spring. If the hub is assumed to be rigid, then the collective flap natural frequency is dominated by the blades, and the measured coning natural frequency of the blades ($\omega_1$) is used to estimate the rotating collective flap frequency:

$$\nu_{\beta_0} = \sqrt{1 + \left(\frac{\omega_1}{\Omega}\right)^2}$$  \hspace{1cm} (2.13)

The analytical model then calculates the rotor Lock number in the usual fashion, based on the aforementioned inputs:

$$\gamma = \frac{\rho ac R^4}{I_b}$$  \hspace{1cm} (2.14)

The key change in the code inputs concerns the Gen-3c configuration, for which the rotor blades are removed. For that configuration, the Lock number is manually set to a very small value (setting the Lock number to zero would cause an error in the analytical model). There is still hub mass and flap inertia (of the gimbal) in this configuration, so the $M_b$ and $I_b$ terms listed above are nonzero.

### 2.3.3 Wing and pylon inputs

The wing and pylon inputs required for the analytical model are:

• $c_w$: wing chord, $ft.$

• $e$: location of aerodynamic center from quarter-chord, positive aft, $ft.$

• $EI_b$: wing beam mode stiffness, $lb ft.^2$
• $EI_c$: wing chord mode stiffness, $lb \cdot ft$.\(^2\)

• $GJ$: wing torsional stiffness, $lb \cdot ft$.\(^2\)

• $L_n$: pylon element length, $ft$.

• $L_{\text{span}}$: wing semispan length, from wind tunnel wall to outside plane of nacelle, $ft$.

• $\Lambda$: wing sweep, positive aft, rad

• $m_{\text{tip}}$: nacelle tip mass (if present), slug

• $\zeta_{w_b}$: wing beam mode structural damping at zero tunnel speed, dimensionless

• $\zeta_{w_c}$: wing chord mode structural damping at zero tunnel speed, dimensionless

• $\zeta_{w_{\phi}}$: wing torsional mode structural damping at zero tunnel speed, dimensionless

The analytical model treats the nacelle as an individual element, and three total elements comprise the semispan of the wing. The nacelle element had a fixed length of 0.125 ft. These parameters were used to calculate the wing finite element properties and normalized nacelle properties:

• $I_n$: torsional inertia of nacelle, $slug - ft$.

• $L$: wing element length, $ft$.

• $m^{(n)}$: wing element mass, $slug - ft$.

• $S_{\alpha}^{(n)}$: normalized static imbalance of wing element, $slug$

• $I_n^*$: torsional inertia of nacelle normalized by element length, $slug - ft$

• $S_{\alpha_n}$: normalized static imbalance of nacelle, $slug$
Chapter 3

Experimental Setup and Stability Predictions

The methods used to characterize the measuring equipment and sensors that were necessary for flutter testing, as well as the procedures used for flutter testing and data processing, are presented in this chapter. The predictions of trim pitch settings and modal damping as a function of tunnel speed presented at the end of this chapter were generated by the analytical model used to guide the design and development of the wind tunnel models. This analytical model was based on the methods presented in Appendix C, with the necessary numerical inputs detailed in Chapter 2. The analytical model used herein differs from the methods of the 4DOF model summarized in Appendix C in the treatment of the tiltrotor wing: the method summarized in Appendix C addresses the wing dynamics using assumed modes, rather than the finite element method used by the analytical model developed for the present work.

3.1 Gram Scale Characterization

Mass measurements were taken using a UniShow digital scale, with a 500 gram capacity and a resolution of 0.01 grams. A set of jeweler masses were purchased from American Weigh Scales, which were certified to OIML Class M2 standards. The maximum allowable variation in mass (viz. actual versus claimed mass) are taken from OIML Recommendation R111 [70] and are tabulated in Table 3.1.

Note that the gram scale - and other measuring devices used - were characterized prior to flutter testing, not calibrated. The key difference between calibration and characterization, as the two terms are used in the present work, is that calibration is concerned with adjusting the behavior of a device in order for it to meet certain predefined performance targets; by contrast, characterization (or profiling) is the recording of how a device responds to a given input. Characterization results are only valid if the device is in the same state of calibration
as it was when it was last characterized, and thus certain tools (such as the gram scale) must be treated carefully, and not subjected to undue heat, humidity, or shock. Additionally, the certification compliance of the jeweler masses is time-sensitive, and cannot be assumed to remain compliant to OIML M2 standards past 2016.

The scale was tared on a laboratory benchtop, and masses were applied to the scale in one-gram increments; a maximum mass of 110 grams was applied to the scale, since no single part or subassembly of the wind tunnel models had mass higher than 110 grams. The scale readings were recorded, and are shown in Table 3.2 and Fig. 3.1. The variation between the applied and measured masses as a percentage (Fig. 3.2) show that the average error in measurement of the UniShow scale, for the mass range of interest (approximately 5 to 100 grams) is ± 0.0025%. The mean measurement error of the scale was calculated to be 0.1582%, with a standard deviation of 0.0014 grams. This is within the allowable limits of the Class M2 proof masses for the mass range of interest. The accuracy of the gram scale was considered sufficient for the present work, and possible variations in component mass were not factored into interpretation of the flutter test results.
Table 3.1: OIML Class M2 maximum allowable mass variations, from [70].

<table>
<thead>
<tr>
<th>Mass, g</th>
<th>Allowable Variation, mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>80</td>
</tr>
<tr>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>100</td>
<td>16</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>8.0</td>
</tr>
<tr>
<td>10</td>
<td>6.0</td>
</tr>
<tr>
<td>5</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
</tr>
<tr>
<td>1</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Table 3.2: Applied and recorded masses for UniShow gram scale.

<table>
<thead>
<tr>
<th>Mass, g</th>
<th>Scale, g</th>
<th>Mass, g</th>
<th>Scale, g</th>
<th>Mass, g</th>
<th>Scale, g</th>
<th>Mass, g</th>
<th>Scale, g</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>31</td>
<td>31.02</td>
<td>61</td>
<td>61.17</td>
<td>91</td>
<td>91.23</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>32</td>
<td>32.01</td>
<td>62</td>
<td>62.18</td>
<td>92</td>
<td>92.22</td>
</tr>
<tr>
<td>3</td>
<td>3.01</td>
<td>33</td>
<td>33.02</td>
<td>63</td>
<td>63.17</td>
<td>93</td>
<td>93.23</td>
</tr>
<tr>
<td>4</td>
<td>3.99</td>
<td>34</td>
<td>34.03</td>
<td>64</td>
<td>64.16</td>
<td>94</td>
<td>94.23</td>
</tr>
<tr>
<td>5</td>
<td>4.99</td>
<td>35</td>
<td>35.03</td>
<td>65</td>
<td>65.15</td>
<td>95</td>
<td>95.22</td>
</tr>
<tr>
<td>6</td>
<td>5.99</td>
<td>36</td>
<td>36.04</td>
<td>66</td>
<td>66.16</td>
<td>96</td>
<td>96.23</td>
</tr>
<tr>
<td>7</td>
<td>6.99</td>
<td>37</td>
<td>37.05</td>
<td>67</td>
<td>67.17</td>
<td>97</td>
<td>97.23</td>
</tr>
<tr>
<td>8</td>
<td>7.98</td>
<td>38</td>
<td>38.05</td>
<td>68</td>
<td>68.16</td>
<td>98</td>
<td>98.23</td>
</tr>
<tr>
<td>9</td>
<td>8.99</td>
<td>39</td>
<td>39.05</td>
<td>69</td>
<td>69.16</td>
<td>99</td>
<td>99.22</td>
</tr>
<tr>
<td>10</td>
<td>10.00</td>
<td>40</td>
<td>40.07</td>
<td>70</td>
<td>70.17</td>
<td>100</td>
<td>100.24</td>
</tr>
<tr>
<td>11</td>
<td>11.00</td>
<td>41</td>
<td>41.09</td>
<td>71</td>
<td>71.18</td>
<td>101</td>
<td>101.24</td>
</tr>
<tr>
<td>12</td>
<td>11.99</td>
<td>42</td>
<td>42.10</td>
<td>72</td>
<td>72.18</td>
<td>102</td>
<td>102.24</td>
</tr>
<tr>
<td>13</td>
<td>13.00</td>
<td>43</td>
<td>43.11</td>
<td>73</td>
<td>73.19</td>
<td>103</td>
<td>103.25</td>
</tr>
<tr>
<td>14</td>
<td>14.00</td>
<td>44</td>
<td>44.10</td>
<td>74</td>
<td>74.18</td>
<td>104</td>
<td>104.25</td>
</tr>
<tr>
<td>15</td>
<td>14.99</td>
<td>45</td>
<td>45.10</td>
<td>75</td>
<td>75.19</td>
<td>105</td>
<td>105.26</td>
</tr>
<tr>
<td>16</td>
<td>15.99</td>
<td>46</td>
<td>46.11</td>
<td>76</td>
<td>76.19</td>
<td>106</td>
<td>106.24</td>
</tr>
<tr>
<td>17</td>
<td>16.99</td>
<td>47</td>
<td>47.11</td>
<td>77</td>
<td>77.19</td>
<td>107</td>
<td>107.25</td>
</tr>
<tr>
<td>18</td>
<td>17.99</td>
<td>48</td>
<td>48.11</td>
<td>78</td>
<td>78.19</td>
<td>108</td>
<td>108.25</td>
</tr>
<tr>
<td>19</td>
<td>18.99</td>
<td>49</td>
<td>49.11</td>
<td>79</td>
<td>79.19</td>
<td>109</td>
<td>109.24</td>
</tr>
<tr>
<td>20</td>
<td>20.00</td>
<td>50</td>
<td>50.12</td>
<td>80</td>
<td>80.19</td>
<td>110</td>
<td>110.26</td>
</tr>
<tr>
<td>21</td>
<td>20.98</td>
<td>51</td>
<td>51.11</td>
<td>81</td>
<td>81.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>21.99</td>
<td>52</td>
<td>52.13</td>
<td>82</td>
<td>82.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>22.99</td>
<td>53</td>
<td>53.13</td>
<td>83</td>
<td>83.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>23.99</td>
<td>54</td>
<td>54.14</td>
<td>84</td>
<td>84.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>25.00</td>
<td>55</td>
<td>55.14</td>
<td>85</td>
<td>85.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>26.05</td>
<td>56</td>
<td>56.14</td>
<td>86</td>
<td>86.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>27.06</td>
<td>57</td>
<td>57.14</td>
<td>87</td>
<td>87.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>28.05</td>
<td>58</td>
<td>58.14</td>
<td>88</td>
<td>88.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>29.03</td>
<td>59</td>
<td>59.15</td>
<td>89</td>
<td>89.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>30.02</td>
<td>60</td>
<td>60.17</td>
<td>90</td>
<td>90.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.1: UniShow mass readings versus applied mass.

Figure 3.2: UniShow measurement error versus applied mass.
3.2 Wind Tunnel Facility Overview

Flutter testing was conducted in the Penn State Hammond Low-Speed Wind Tunnel. The Hammond tunnel is used extensively for both undergraduate and graduate research projects and laboratory assignments, and was originally designed for boundary layer development studies. The test section is 240 in. long, with a constant cross-section measuring 24 in. wide and 36 in. tall. Turbulence levels are minimized through the use of five screens, one perforated plate, and a honeycomb (6 in. length) upstream of the test section. Several test stations run the length of the tunnel, one of which is equipped with a six-channel ATI Industrial Automation Delta SI 660-60 load cell, capable of measuring lift, drag, and side forces and roll, pitch, and yaw moments. The wind tunnel models were connected to the load cell using four #8-32 socket head cap screws, flat washers, and threaded nuts. The load cell operating limits and resolutions are tabulated in Table 3.3 from values presented in [74].

Flow is produced by a twenty-five horsepower electric motor, with spins a 48 in. diameter fan. An auxiliary cooling fan helps to keep the motor temperature within acceptable levels, and was left on during flutter testing. The maximum allowable motor operating temperature was 100 °F, as suggested by the laboratory manager (Mr. Rick Auhl); however, testing was paused whenever the motor temperature exceeded 90 °F for an extra margin of safety. The maximum tunnel speed is 155 ft./sec. for short bursts, and the maximum sustained speed is 140 ft./sec. Above 140 ft./sec., maintaining a specific speed becomes difficult; flow speed was also difficult to maintain if the air within the tunnel exceeded approximately 75 °F, requiring testing to be periodically paused so that the tunnel doors could be opened to allow the air to cool. This cooling required approximately two to three hours. A schematic of the Hammond tunnel is shown in Fig. 3.3, and a CAD representation of the Gen-3 model mounted to the load cell is shown in Figs. 3.4 and 3.5.
Table 3.3: Operating ranges and resolutions of ATI Delta load cell, from [74].

<table>
<thead>
<tr>
<th></th>
<th>Lift, Draf ($F_x, F_y$)</th>
<th>Side ($F_z$)</th>
<th>Yaw, Roll, Pitch ($M_x, M_y, M_z$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>$\pm660,N$</td>
<td>$\pm1980,N$</td>
<td>$\pm60,Nm$</td>
</tr>
<tr>
<td>Resolution</td>
<td>$0.125,N$</td>
<td>$0.25,N$</td>
<td>$10/1333,Nm$</td>
</tr>
</tbody>
</table>

Figure 3.3: Hammond wind tunnel diagram.
Figure 3.4: Wind tunnel test section top view.

Figure 3.5: Hammond wind tunnel diagram.
3.3 Pressure Transducer Characterization

Two pressure transducers are installed in the wind tunnel, one in the contraction chamber (Transducer A) and one connected to a pitot-static probe (Transducer B). The tunnel speed may be obtained by relating the gas properties (pressure, $P$, density, $\rho$, and velocity, $V$) at the inlet and outlet of the contraction section (Fig. 3.6) by the Bernoulli equation,

$$P_i + \frac{1}{2} \rho V_i^2 = P_e + \frac{1}{2} \rho V_e^2$$  \hspace{1cm} (3.1)

where the subscripts $i$ and $e$ denote the entrance to and exit from the contraction section, respectively. The velocity at the exit of the contraction section is then

$$V_e = \sqrt{\frac{2 \left( P_i - P_e \right)}{\rho \left( 1 - \left( \frac{A_e}{A_i} \right)^2 \right)}}$$  \hspace{1cm} (3.2)

where $\frac{A_e}{A_i}$ is the contraction area ratio, which for the Hammond tunnel is approximately 10:1. The pressure drop through the contraction chamber, $P_i - P_e$, is measured by the pressure transducers.

The transducers were characterized using the analogue barometer installed in the wind tunnel facility, an inclined water manometer, and a hand pump; a diagram of the transducer characterization setup is shown in Fig. 3.7. The transducers were each wired to a Validyne CD23 display; prior to characterization the displays were zeroed and the gain knobs locked into position. The pump was used to increase the pressure in the manometer, decreasing the water level of the manometer incline; at each setting, the manometer water level was noted, and an in-house LabVIEW data acquisition program recorded the voltages of the two transducers. The barometric pressure and facility temperature were also input into the LabVIEW program for each test point. The pump handle was locked for one minute at each test point in order to ensure that the pressure in the test setup remained constant. The LabVIEW program then sampled the transducer voltages at 100 Hz for five seconds, recording the mean voltage value of each transducer as well as its standard deviation (labeled $\sigma_{\text{cent}}$ or $\sigma_{\text{pitot}}$, respectively). A total of fourteen data points were collected, which included a re-test of the original zero value to determine hysteresis. These data are tabulated in Table 3.4 and plotted in Fig. 3.8.

The errors of the transducer measurements shown in Table 3.4 were automatically calculated by the LabVIEW software; inspection of these errors shows that they are well within IEC 61298-2 standards of 0.5% for the range of the transducer span that is of interest. The slope of the linear regression line through the transducer voltage measurements was converted from volt/in.$H_2O$ to psf/volt to determine the transducer calibration factor, $K_t$. The pressure drop in Eq. 3.2 is then

$$P_i - P_e = K_t V_{\text{transducer}}$$  \hspace{1cm} (3.3)

A correction factor, $K_v$, is used to account for the growth of the boundary layer thickness along
the length of the test section, as well as velocity variations at different test stations along the length of the tunnel. The correction factor used for this project was 0.901, which had been calculated by the laboratory manager prior to the current work and was left unchanged. The velocity at the load cell is then

\[ V_{test} = K_v V_c \]  

(3.4)

The tunnel speed for each flutter test was determined by entering the desired velocity into an Excel spreadsheet, along with the static pressure and temperature of the tunnel air. This spreadsheet was set up to account for the transducer calibration factor and tunnel correction factor. The spreadsheet would then output the pitot-static transducer voltage required to obtain the desired test speed. The tunnel speed can then be manually increased or decreased until the pitot-static voltage matches the desired value.
Table 3.4: Pressure transducer characterization results.

<table>
<thead>
<tr>
<th>Test</th>
<th>Load, in.$H_2O$</th>
<th>Venturi</th>
<th>Pitot</th>
<th>$\sigma_{vent}$</th>
<th>$\sigma_{pitot}$</th>
<th>$\epsilon_{vent}$, %</th>
<th>$\epsilon_{pitot}$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.001</td>
<td>5.050E-04</td>
<td>6.070E-04</td>
<td>-53.16</td>
<td>59.55</td>
</tr>
<tr>
<td>2</td>
<td>0.001</td>
<td>-0.001</td>
<td>0.001</td>
<td>4.210E-04</td>
<td>5.950E-04</td>
<td>-43.15</td>
<td>55.85</td>
</tr>
<tr>
<td>3</td>
<td>0.500</td>
<td>0.832</td>
<td>0.845</td>
<td>6.100E-04</td>
<td>8.530E-04</td>
<td>0.073</td>
<td>0.101</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>1.637</td>
<td>1.655</td>
<td>1.146E-03</td>
<td>1.343E-03</td>
<td>0.070</td>
<td>0.081</td>
</tr>
<tr>
<td>5</td>
<td>1.500</td>
<td>2.505</td>
<td>2.524</td>
<td>1.778E-03</td>
<td>1.686E-03</td>
<td>0.071</td>
<td>0.067</td>
</tr>
<tr>
<td>6</td>
<td>2.000</td>
<td>3.327</td>
<td>3.343</td>
<td>2.189E-03</td>
<td>2.205E-03</td>
<td>0.066</td>
<td>0.066</td>
</tr>
<tr>
<td>7</td>
<td>2.500</td>
<td>4.173</td>
<td>4.183</td>
<td>2.050E-03</td>
<td>2.167E-03</td>
<td>0.049</td>
<td>0.052</td>
</tr>
<tr>
<td>8</td>
<td>3.000</td>
<td>4.995</td>
<td>4.995</td>
<td>2.067E-03</td>
<td>2.219E-03</td>
<td>0.041</td>
<td>0.044</td>
</tr>
<tr>
<td>9</td>
<td>3.500</td>
<td>5.834</td>
<td>5.821</td>
<td>3.848E-03</td>
<td>4.121E-03</td>
<td>0.066</td>
<td>0.071</td>
</tr>
<tr>
<td>10</td>
<td>4.000</td>
<td>6.677</td>
<td>6.647</td>
<td>2.054E-03</td>
<td>2.196E-03</td>
<td>0.031</td>
<td>0.033</td>
</tr>
<tr>
<td>11</td>
<td>4.500</td>
<td>7.516</td>
<td>7.466</td>
<td>4.492E-03</td>
<td>4.709E-03</td>
<td>0.060</td>
<td>0.063</td>
</tr>
<tr>
<td>12</td>
<td>5.000</td>
<td>8.346</td>
<td>8.272</td>
<td>1.822E-03</td>
<td>1.352E-03</td>
<td>0.022</td>
<td>0.016</td>
</tr>
<tr>
<td>13</td>
<td>5.500</td>
<td>9.175</td>
<td>9.075</td>
<td>2.656E-03</td>
<td>2.646E-03</td>
<td>0.029</td>
<td>0.029</td>
</tr>
<tr>
<td>14</td>
<td>0.000</td>
<td>0.002</td>
<td>0.004</td>
<td>2.139E-03</td>
<td>2.211E-03</td>
<td>57.23</td>
<td>57.23</td>
</tr>
</tbody>
</table>

SLOPE: 1.6532 volt/in.$H_2O$ 0.6049 psi/volt 3.147 psf/volt
Figure 3.6: Wind tunnel contraction section.

Figure 3.7: Pressure transducer characterization setup.
Figure 3.8: Pressure transducer voltages per load.
3.4 Wind Tunnel Characterization

After the transducer characterization tests, the transducers were reconnected to the venturi and pitot-static probes, and the pitot-static probe was reinstalled into the tunnel test section. A simple characterization of the wind tunnel was performed to determine if the presence of the wind tunnel model would require additional corrections due to blockages or wall effects. Some procedures for these corrections are given in [64], [66], [75], and [76], and are not repeated here.

The tunnel speed is controlled by a rotary potentiometer installed on a handheld control panel, which is connected to the tunnel circuit breaker. Turning the potentiometer dial one full turn increases the power supplied to the drive motor by approximately 10%, which increases the transducer voltage nonlinearly (Fig. 3.9). Stated another way, each turn of the potentiometer dial increases the tunnel operating percentage by 10%. Tunnel speed was measured as a function of operating percentage, and a LabVIEW code was used to measure the pitot-static transducer voltage; the transducer voltage output by the LabVIEW code was the average of 500 samples taken at that operating percentage. These voltage values were measured as tunnel speed was increased and decreased to check for hysteresis, and the voltage values were then used to calculate the tunnel speed as a function of operating percentage. The pitot-static transducer voltages are tabulated in Table 3.5 and plotted in Fig. 3.10. Although there were some slight differences noted in the transducer voltages during the ascending and descending portions of the test, the differences were not deemed to be of significance.

The wind tunnel was allowed to return to its initial temperature, and then the test was repeated with the wind tunnel model installed. The Gen-2a wind tunnel model was used for these blockage tests. The screws that attach the nacelle cap to the rest of the model were tied to the load cell mounting bracket with a length of heavy-duty string; this served as a safety wire, and was intended to prevent the model from experiencing any instabilities during these tests. Additionally, the rotor of the model was fully feathered during the blockage tests. The results of the blockage test are tabulated in Table 3.6 and plotted in Fig. 3.11. The rightmost column of Table 3.6 (ΔV) represents the difference in tunnel speed caused by the presence of the wind tunnel model. Given the likely accuracy of the pressure transducers, and given the range of tunnel speeds necessary for flutter testing (approximately 60 - 120 ft./sec.), the blockage effects caused by the presence of the wind tunnel model were deemed insignificant. Therefore, no additional correction factors were applied.
Table 3.5: Pitot-static transducer voltages by tunnel operating percentage, empty tunnel.

<table>
<thead>
<tr>
<th>% Pitot-Static, volts</th>
<th>$V_\infty$, ft./sec.</th>
<th>% Pitot-Static, volts</th>
<th>$V_\infty$, ft./sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0.003</td>
<td>0</td>
<td>100 8.772</td>
<td>155.4</td>
</tr>
<tr>
<td>10 0.073</td>
<td>14.16</td>
<td>90 7.386</td>
<td>142.6</td>
</tr>
<tr>
<td>20 0.330</td>
<td>30.12</td>
<td>80 5.859</td>
<td>127.1</td>
</tr>
<tr>
<td>30 0.781</td>
<td>46.33</td>
<td>70 4.522</td>
<td>111.7</td>
</tr>
<tr>
<td>40 1.427</td>
<td>62.63</td>
<td>60 3.337</td>
<td>95.95</td>
</tr>
<tr>
<td>50 2.260</td>
<td>78.80</td>
<td>50 2.287</td>
<td>79.44</td>
</tr>
<tr>
<td>60 3.300</td>
<td>95.22</td>
<td>40 1.438</td>
<td>63.00</td>
</tr>
<tr>
<td>70 4.486</td>
<td>111.0</td>
<td>30 0.799</td>
<td>46.97</td>
</tr>
<tr>
<td>80 5.856</td>
<td>126.9</td>
<td>20 0.339</td>
<td>30.60</td>
</tr>
<tr>
<td>90 7.420</td>
<td>142.8</td>
<td>10 0.079</td>
<td>14.79</td>
</tr>
<tr>
<td>100 8.772</td>
<td>155.4</td>
<td>0 0.013</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.6: Pitot-static transducer voltages by tunnel operating percentage, Gen-2a model tunnel.

<table>
<thead>
<tr>
<th>% Pitot-Static, volts</th>
<th>$V_\infty$, ft./sec</th>
<th>$\Delta V$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0.000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10 0.068</td>
<td>13.66</td>
<td>-3.53</td>
</tr>
<tr>
<td>20 0.318</td>
<td>29.47</td>
<td>-2.16</td>
</tr>
<tr>
<td>30 0.767</td>
<td>45.79</td>
<td>-1.16</td>
</tr>
<tr>
<td>40 1.402</td>
<td>61.90</td>
<td>-1.16</td>
</tr>
<tr>
<td>50 2.261</td>
<td>78.63</td>
<td>-0.22</td>
</tr>
<tr>
<td>60 3.245</td>
<td>94.18</td>
<td>-1.08</td>
</tr>
<tr>
<td>70 4.469</td>
<td>110.5</td>
<td>-0.46</td>
</tr>
<tr>
<td>80 5.746</td>
<td>125.3</td>
<td>-1.22</td>
</tr>
<tr>
<td>90 7.214</td>
<td>140.4</td>
<td>-1.69</td>
</tr>
</tbody>
</table>
Figure 3.9: Pitot-static transducer voltage versus tunnel speed, empty tunnel.

Figure 3.10: Measured tunnel speed versus percent tunnel speed commanded, empty tunnel.
Figure 3.11: Measured tunnel speed versus percent tunnel speed commanded, wind tunnel model installed.
3.5 Load Cell Characterization

The maximum roll moment measured by the load cell during flutter testing of the Gen-1 model, as presented in [32], was approximately 7 Nm; this was clearly adequate for the monocoque plastic wing, since no mention is made in [32] of the Gen-1 model suffering a structural failure during testing. Since the Gen-2 model uses the same monocoque wing design of the Gen-1 model and was fabricated in the same fashion, then by similitude the strength of the Gen-2 wing should also be acceptable if tested at the same tunnel speeds as the Gen-1 model. The narrow factor of safety of the Gen-3 wing (approximately 1.17, as discussed in Chapter 2) is another matter entirely, and the safe testing of the Gen-3 model required a way to ensure that the measurements of the load cell were sufficiently accurate. The roll moment channel of the load cell was the focus of the characterization effort, since this channel would serve as the primary method of measuring the modal behavior of the Gen-3 model.

The basic characterization test setup is shown in Fig. 3.12. A spare monocoque wing, which had never been tested in the wind tunnel, was bolted to the load cell; a thin plate of 6061-T6 aluminum was mounted as a spacer between the load cell and the wing. A 0.125 in. through-hole was drilled vertically through the elastic axis of the wing, through which was passed a length of safety string. A plain steel washer was used to secure the loose end of the string to the top of the wing; the other end of the string was tied to a mass hanger, onto which various slotted masses would be placed. The minimum mass applied to this setup was 50.79 grams, which was the mass of the bare hanger (Fig. 3.13); the maximum mass applied was 4,100 grams. The moment arm was composed of the distance between the wind tunnel wall plane to the center of the hole drilled through the wing (distance $d_1$, Fig. 3.14) and the distance from the load cell tool plane to the inner wind tunnel wall plane (distance $d_2$, Fig. 3.15). Both of these distances were measured using machinist calipers with 0.001 in. resolution and summed together, to which was added the manufacturer-specified offset distance from the load cell sensing plane to the tool plane (0.6 mm, Fig. 3.15). The total moment arm was calculated to be 12.093 in. (0.3072 m), giving an applied roll moment range of 0.1529 - 12.36 Nm. This was sufficient to cover the roll moments likely to be encountered during flutter testing. Smaller increments between the applied roll moments were taken within the range of interest for flutter testing (3 - 6 Nm).

The DAQ software used for the load cell characterization was tared prior to the installation of the items shown in Fig. 3.12. The total mass for a given test was applied to the wing, and the mass was stabilized as carefully as possible by hand to prevent it from swinging. The DAQ software was then set to record the measured roll moment at 10 Hz for 100 seconds; this resulted in test run of 1,000 samples. A total of ten test runs were performed for each applied mass, which formed a test case; a total of thirteen test cases were examined. The number of individual samples per test run ($n = 1,000$ each) and total samples per test case ($n = 10,000$) permitted the assumption of normality via Central Limit Theorem, which simplified the statistical analysis of the recorded data. Because the measured roll moment may be affected by the mass swinging, sampling too quickly might introduce additional uncertainty to the measurements,
since the samples would become more time-dependent (based on the position of the mass) as the sampling window narrowed; in an attempt to reduce this likelihood, test runs were conducted approximately five minutes apart.

Histograms and boxplots of the measured roll moments were constructed in MATLAB, and were seen (Figs. 3.16 and 3.17, respectively) to validate the assumption of normality. A standard deviation of approximately 0.004 Nm was present across all test cases, with a relatively constant coefficient of variation of approximately 0.002 within the roll moment range of interest. Each of these parameters were deemed too small to be significant, and were not considered during analysis of flutter test data. Measured moments exhibited a linear relationship (Fig. 3.18) and showed good correlation with the calculated values throughout the range of interest. The measurement error of the load cell was found to vary based on the applied roll moment of the test (Fig. 3.19); more importantly, at the maximum roll moment expected during flutter testing, the maximum measurement error of the load cell was found to be 0.36%. It was thus concluded that the calculated factor of safety of the Gen-3 wing was sufficient for flutter testing.

Additional measurements were taken to determine if there were time-dependent variations in roll moment measurements. These measurements were performed using the same applied roll moments and the aforementioned sampling frequency and test procedures. As shown in Fig. 3.20, no significant time-dependent variations in measured roll moments in any of the thirteen test cases were noticed.
Figure 3.12: Load cell characterization test setup.
Figure 3.13: Mass measurement for applied roll moment calculation.

Figure 3.14: Moment arm distance 1.
Figure 3.15: Moment arm distance 2.

Figure 3.16: Sample histogram of measured roll moment values; applied roll moment: 4.431 Nm.
Figure 3.17: Sample boxplot of measured roll moment values; applied roll moment: 4.431 Nm.

Figure 3.18: Observed mean roll moment versus applied roll moment.
Figure 3.19: Errors of measured roll moments.

Figure 3.20: Time histories of applied roll moments.
3.6 Flutter Testing Setup

Flutter testing was performed using custom LabVIEW software developed in-house. For all wind-on Gen-2 and Gen-3 tests, a Campbell-Hausfeld WL606001AJ compressor regulated to 100 psi provided the air supply used for perturbation of the wind tunnel models. This provided enough air for three to four test points before the compressor needed to be refilled. It is possible to refill the compressor while recording data, but this is not recommended: vibrations from the compressor motor are sufficient to be measured by the load cell even if the compressor is located an appreciable distance from the test setup; during Gen-2 testing, this vibration was noticed even with the compressor located more than twenty feet from the load cell. This additional vibration increases the noise floor of the recorded data, and makes data analysis more difficult.

The solenoid valve used for flutter testing (SMC Pneumatics VQD121 series) features a single input port and two output ports, to which were attached 1/16” NPT x barb fittings. This is an electromechanically-controlled valve that is capable of switching between outputs at up to 450 Hz, which is well above the 5 - 6 Hz required for flutter testing. The drawback of this type of valve is that at least one output port is always open, even when the valve is not powered. This necessitated a ball valve to be plumbed inline between the air compressor and the solenoid, or else the compressor would continually lose air and require more frequent pauses in testing to refill. This ball valve was only opened just before perturbation of the model, and was closed immediately after perturbation ceased. A bipolar junction transistor (style TO-126) was used to switch between output ports on the valve.

High-strength string was used as a safety wire to secure the model when testing at or near the predicted flutter speed; this string was tied in a knot around an exposed screw shank on the nacelle. The safety string may be tied to a secure support structure (such as the load cell mounting bracket), or may be wound tightly around a finger; the finger-wrap method is more flexible, since it permits the operator to quickly pull on the string (thus creating a brake) to reduce the motion of the wind tunnel model if an instability were to be encountered. In practice, the string was used as a brake only once during testing, and a rapid reduction in tunnel speed was found to be just as useful in reducing the oscillation amplitude of the model for unstable test cases. Tests were conducted at each tunnel speed three times to ensure repeatability. Flutter tests were conducted using the following steps:

1. Install model to wind tunnel load cell using #8-32 socket head cap screws.
2. Connect safety string to exposed nacelle screw shank.
3. Connect experiment electronics and pneumatics using the schematic provided in Fig. 3.21.
4. Connect servo and controller using the schematic provided in Fig. 3.22.
5. Obtain readings of static pressure and tunnel temperature from the barometer and thermocouple output.
6. Enter the pressure and temperature values into the transducer calibration spreadsheet.
7. Ensure that ball valve connecting the solenoid to the air compressor is closed.

8. Turn on air compressor and pressurize tank to 100 psi.

9. Ensure that tunnel control potentiometer is set to zero.

10. Turn on wind tunnel wall breaker.

11. Enter desired tunnel speed (in ft./sec.) into transducer calibration spreadsheet.

12. Enter excitation frequency, desired filename for the test, and file save destination into LabVIEW.

13. Turn on handheld tunnel speed controller.

14. Secure safety wire or safety string.

15. Adjust speed controller slowly to control tunnel speed as needed.

16. While visually monitoring the wind tunnel model, adjust blade pitch as necessary to maintain 2,000 RPM using the HiTec X1 multifunction controller.

17. Increase tunnel speed and blade pitch incrementally (and independently) as needed. Allow model to settle at each speed and pitch setting before adjusting.

18. Open ball valve and begin saving data.

19. Start excitation frequency at beamwise fundamental frequency. Excite model at beamwise frequency for three to five seconds.

20. Stop excitation.


22. Allow root forces and moments to decay to steady-state values.

23. End recording.

24. Reduce tunnel speed to zero.

25. Repeat test as needed.

26. Analyze data.
Figure 3.21: Equipment setup for whirl flutter testing.

Figure 3.22: Servo and controller wiring.
3.7 Stability Predictions

The stability predictions of the analytical model were used to guide the design of the Gen-3 wind tunnel model, especially in selecting the target stiffnesses of the wing spar. Trim collective pitch angles for Gen-3a and Gen-3b are shown in Figs. 3.23 and 3.25. The analytical model predicted instabilities for both the Gen-3a (untwisted blades) and the Gen-3b (twisted blades) models, although the first mode to reach instability is different between the two versions of the Gen-3 model: the chord mode was predicted to exhibit an instability first for the Gen-3a model (Fig. 3.24), whereas the torsion mode was predicted to be least stable in the Gen-3b model (Fig. 3.26). The lowest flutter speeds predicted was approximately 100 ft./sec. for Gen-3a and 98 ft./sec. for Gen-3b. The predicted stability for the final configuration tested, Gen-3c (no rotors), is shown in Fig. 3.27; for Gen-3c, the trim pitch angle term has no meaningful value, and therefore no predictions are presented.

The analytical model developed for the present work was not used to predict the performance and stability of the Gen-2 model. The entire purpose of the Gen-2 model was a proof-of-concept test to ensure that the composite rotor blades would be able to survive testing in the Hammond tunnel. As such, Gen-2 testing was more of a spot-check to determine the effect of higher-Lock number blades on the stability of the monocoque Gen-1 wing, rather than as a standalone test regimen. Flutter testing was performed on Gen-2 and compared to the results presented in [32]; these comparisons are presented in Chapter 4.
Figure 3.23: Predicted trim collective pitch versus tunnel speed, Gen-3a.

Figure 3.24: Predicted wing modal damping versus tunnel speed, Gen-3a.
Figure 3.25: Predicted trim collective pitch versus tunnel speed, Gen-3a.

Figure 3.26: Predicted wing modal damping versus tunnel speed, Gen-3b.
Figure 3.27: Predicted wing modal damping versus tunnel speed, Gen-3c.
3.8 Data Processing

After testing was completed, all data were analyzed using the moving-block method and bandpass filtering using a second-order Butterworth filter. Filtering was necessary to isolate the wing structural modes of interest, and is itself fairly common in aeroelastic testing. Complex filtering methods such as the Hilbert-Huang algorithm [77] and wavelet analyses for analyzing flight test data [78] have previously been used to successfully identify aircraft aeroelastic characteristics, and even real-time estimation of modal parameters [79] is becoming increasingly commonplace. The filtering methods used in the current work are not nearly this involved or mathematically elegant, primarily because the signals are fairly straightforward and increasing the computational efficiency of data analysis was not the primary focus of the project.

3.8.1 Moving-block method

The moving-block analysis is a common method to calculate the damping ratio of a time-varying signal, and has been widely used for computational aeroelasticity studies within the rotorcraft industry [80]. It was first developed in the 1970s by Lockheed, and is based on the FFT of a signal. Extensive details and nuances of the moving-block method, along with its strengths and limitations, are presented in [21] and [22].

Frequency domain analysis has the advantage of permitting the isolation of specific frequencies within a signal that is composed of both periodic contributions and random noise. A transient signal \( y(t) \) is typically composed of several independent sinusoids, which represent the modes of the structure. The infinite Fourier transform of the \( i^{th} \) mode of signal is

\[
Y(\omega_i) = \int_{-\infty}^{\infty} y_i(t) e^{i\omega_i t} \, dt
\]  

(3.5)

and \( Y(\omega_i) \) exists if \( \int_{-\infty}^{\infty} y_i(t) e^{i\omega_i t} \, dt < \infty \). Taking the Fourier transform over a specific time interval from time \( t_1 \) to time \( t + \tau \) (where \( \tau \) is the block time), known as the finite Fourier transform, becomes a function of the circular frequency \( \omega_i \) of the \( i^{th} \) mode and the time at which the transform is applied:

\[
Y(\omega_i, \tau) = \int_{t_1}^{t+\tau} y_i(t) e^{i\omega_i t} \, dt
\]  

(3.6)

The amplitude of Eq. 3.6 is referred to as the moving-block function. For lightly-damped modes (viz. \( \zeta_i \ll 1 \)), the natural logarithm of the moving-block function is

\[
\ln Y(\omega_i, \tau) = -\zeta_i \omega_i \tau + \frac{1}{2} \sin 2(\omega_i \tau + \phi_i) + \text{constant}
\]  

(3.7)

which is a superposition of the linear and oscillating functions of \( \tau \). The slope of the linear
portion is used to find the damping ratio for the $i^{th}$ mode.

An illustration of the moving-block method is shown in Fig. 3.28. The time series data is transformed to the frequency domain through the use of the fast Fourier transform. Once the frequency of interest has been identified, the initialization point for the moving-block method is selected; for the present work, this is the point in the time history at which excitation has ceased. The block size $B$ is chosen based on the number of data samples from the end of excitation until the mode has fully damped:

$$B = \frac{1}{2}t_{damp}$$  

(3.8)

This means the block size chosen may be different from one test case to the next. The block contains within it $N_b$ of the possible $N$ data points in the signal. The moving-block function is computed at the time point immediately after excitation has stopped, and the natural logarithm of the magnitude is calculated. The block then increments forward in time and the process is repeated, until the edge of the block reaches the end of the data (specifically, the process is repeated $N - N_b$ times). The stored values at each step is plotted with respect to time using a least-squares linear fit to estimate the slope and hence the damping ratio (Fig. 3.29). Note that only the linear portion of the data set containing the post-perturbation behavior is of interest, and visual inspection means that the end of the linear region is somewhat subjective. This can lead to variation in the damping ratio measured, although the damping ratio calculation is usually repeatable to within 0.05% [22].
Figure 3.28: Schematic of moving-block method, from [22].

Figure 3.29: Moving-block data curve fit, linear portion only.
3.8.2 Bandpass filtering

The calculation of modal damping ratios using the moving-block method is highly dependent upon having a very clearly-defined time history of the input signal. For systems with multiple modal frequencies spaced closely together, the time history can become cluttered, making a moving-block analysis difficult and inaccurate. To remedy this, the moving-block code used in [32] was augmented for the present work with a filtering subroutine that employs Butterworth filters to create a user-selectable passband.

A bandpass filter is a device that admits only those frequencies within a predefined range, called the passband, and attenuates frequencies that are outside of the passband (Fig. 3.30, from [79]). The difference between the upper and lower frequencies of the passband is known as the filter bandwidth. Bandpass filtering may be achieved either through electronic hardware (such as an RLC circuit), or through software such as MATLAB. For the present work, software filtering was used exclusively. Ideally, a bandpass filter would have a completely flat passband, and would completely reject all frequencies outside of the passband; in practice, the filter is unlikely to be able to reject all frequencies outside of the passband, or to attenuate unwanted frequencies to the same degree. There is also a range of frequencies on either side of the passband where unwanted frequencies are attenuated, but not rejected. These phenomena are known as roll-on and roll-off, respectively, and are usually expressed in dB per decade of attenuation. Therefore, one important filter performance metric is how narrow these roll-on and roll-off regions are, since if the input signal has multiple frequencies spaced very closely together, completely isolating the frequency of interest with poor roll-on and roll-off may be impossible.

Butterworth filters were used as needed to filter wind tunnel test data for moving-block analysis. For the current work, the second-order filter was found to have acceptable performance, although more filter elements can be added if roll-off performance needs to be improved. An example of the benefit of filtering is shown in Figs. 3.31 – 3.34, which were taken from a test case of the Gen-3 model tested well below flutter speed. The unfiltered time history (Fig. 3.31) clearly shows that multiple structural modes have undergone excitation at this test case; this is also seen in the frequency spectrum of the unfiltered signal (Fig. 3.32). The moving-block method can still analyze this signal, however the accuracy of the calculated damping ratio may be improved by isolating the modes of interest. A second-order Butterworth filter was used to create a passband from 0 - 6 Hz, in order to isolate the wing beam mode; plotting the filtered time history (Fig. 3.33) shows a much clearer picture of the wing beam mode, and the filtered frequency spectrum (Fig. 3.34) has completely eliminated all frequencies outside the passband. This permits a much more accurate analysis using the moving-block method alone.
Figure 3.30: Diagram of passband, from [79].
Figure 3.31: Sample unfiltered time history.

Figure 3.32: Sample unfiltered frequency spectrum.
Figure 3.33: Sample filtered time history.

Figure 3.34: Sample filtered frequency spectrum.
Results and Discussion

4.1 Composite Blade Mass and Lock Number

The composite blades were weighed after fabrication, and balanced via the methods discussed in Chapter 3. The measured masses, calculated standard deviations, and coefficients of variation for the untwisted and twisted composite blades are given in Table 4.1. These mass values are the masses of the blades after they had been balanced. The maximum mass variation of the composite blades was measured to be 0.03 grams; by comparison, the variation in masses between the Gen-1 blades was measured to be 0.15 grams - a fivefold increase in mass variation over the current blades. This mass variation may partially account for the 1/rev vibrations experienced by the Gen-1 model during wind tunnel testing.

The inertia of the composite blades was tested using the filar pendulum method described in Appendix A. First, the inertia of the pendulum plate was measured and recorded: the pendulum plate was rotated and allowed to complete ten oscillations, while a stopwatch was used to record the total period. This process was repeated ten times, with the average period used to calculate the plate inertia. The untwisted blades were then attached to the rotor hub subassembly, and the rotor shaft was passed through the central hole on the bottom plate of the filar pendulum. The process of measuring oscillation periods was repeated once more, and the rotational inertia of the combined rotor and plate assembly was calculated. The process was repeated a final time with the rotor blades removed from the rotor hub, in order to calculate the bare-hub inertia. To calculate the rotor Lock number, the chord at 0.75R was used, along with standard-day density and a lift-curve slope of 5.59 /rad [83]. This same process was then repeated for the twisted composite blades. The results of the Lock number testing are summarized in Table 4.2.
Table 4.1: Measured masses of composite blades.

<table>
<thead>
<tr>
<th>Blade Number</th>
<th>Untwisted Blades</th>
<th>Twisted Blades</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass, g</td>
<td>Mean, g</td>
</tr>
<tr>
<td>1</td>
<td>5.62</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.65</td>
<td>5.63</td>
</tr>
<tr>
<td>3</td>
<td>5.63</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Blade Number</th>
<th>Mass, g</th>
<th>Mean, g</th>
<th>Std. Dev., g</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.92</td>
<td>5.93</td>
<td>0.010</td>
<td>1.686E-03</td>
</tr>
<tr>
<td>3</td>
<td>5.94</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Lock number results of composite blades.

<table>
<thead>
<tr>
<th>Untwisted Blades</th>
<th>Twisted Blades</th>
</tr>
</thead>
<tbody>
<tr>
<td>c 0.0326 m</td>
<td>c 0.0257 m</td>
</tr>
<tr>
<td>R 0.2045 m</td>
<td>R 0.2172 m</td>
</tr>
<tr>
<td>$I_R$ experimental 1.208E-04 kgm$^2$</td>
<td>$I_R$ experimental 3.170E-04 kgm$^2$</td>
</tr>
<tr>
<td>$I_R$ Gen-1 3.995E-04 kgm$^2$</td>
<td>$I_R$ Gen-1 3.995E-04 kgm$^2$</td>
</tr>
<tr>
<td>$\gamma$ experimental 3.225</td>
<td>$\gamma$ experimental 3.702</td>
</tr>
<tr>
<td>$\gamma$ Gen-1 2.360</td>
<td>$\gamma$ Gen-1 2.360</td>
</tr>
</tbody>
</table>

Contributions to Total Inertia

<table>
<thead>
<tr>
<th>Untwisted Blades</th>
<th>Twisted Blades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blades 78.54%</td>
<td>Blades 97.90%</td>
</tr>
<tr>
<td>Hub assembly 21.46%</td>
<td>Hub assembly 2.10%</td>
</tr>
</tbody>
</table>
4.2  Composite Blade Static Stiffness Testing

The untwisted composite rotor blades were tested for static stiffness after fabrication. This static stiffness testing would serve to validate the stiffness predictions for the blades, which were calculated using the composite sandwich panel method presented in Appendix B, and build confidence that the blades would not be damaged during use in the wind tunnel. Because the purpose of the static stiffness test was merely to validate the blade stiffness prediction methodology, only the untwisted composite blades were tested.

The T-plate used during Gen-1 model development was modified to include a tapped 1/4-20 hole along its top surface (Fig. 4.1). This hole served as a mounting point for the blade, and provided a more secure mounting than the friction-fit channels used to test the Gen-1 blades. Two end-mass hangers (Fig. 4.2) were designed in SolidWorks for attachment to the end of the blades, and were cut from 0.032 in. aluminum sheet using a water jet; these plates are secured to the tip of the blade using #8-32 machine screws and threaded nuts (Fig. 4.3). The hole on the tongue of the end-mass hangers provides a point of attachment for a hooked mass hanger. The blade to be tested was attached to the top hole of the T-plate, then the T-plate itself was attached to a base plate (Fig. 4.4) with four 3/8-16 bolts, and the entire assembly was secured to a benchtop rail using C-clamps. Both the T-plate and the base plate were machined from 6061 billet. A closeup of the complete static stiffness test setup is shown in Fig. 4.5. Because the blade shanks were drilled to fit a 1/4-20 screw, all blade testing was performed prior to using the M3 screws to re-bore the blade root to enable it to fit securely within the yoke.

A linear variable displacement transducer (LVDT) was secured to an indicator stand with a magnetic base, the arm of the LVDT was placed at the tip of the blade, and a hooked mass hanger was attached to the end-mass hanger. The calibration value of the LVDT (0.0720 in./volt, in this case) was recorded, and a digital multimeter was used to record the zero-load voltage. Four masses (Table 4.3) were placed on the mass hanger in different combinations to load the blade, and the LVDT voltages were recorded. The measured LVDT voltages were converted to displacements (Table 5.4) by multiplying the LVDT voltages by the calibration value, and these displacements were then graphed versus the applied loads (Fig. 4.6). The inverse of the slope of each of these graphs was taken to be the flapwise bending stiffness of the blades. The static stiffness of each of the untwisted composite blades is tabulated in Table 4.4; the static stiffness of the Gen-1 blades are included for comparison.

Although the measured blade stiffnesses were reasonably similar, it is readily apparent that the difference in static stiffness between the blades was statistically significant. This may have been the result of the hand layup causing variances in the fiber volume fraction and void content of the blades. It is also possible that the clamping force applied to the blade mounting screw may have varied between tests. Since the applied torque of the screw was not measured during testing, this is a distinct possibility.
Table 4.3: Masses used for static stiffness testing.

<table>
<thead>
<tr>
<th>Test Mass</th>
<th>Mass, g</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.96</td>
</tr>
<tr>
<td>2</td>
<td>37.81</td>
</tr>
<tr>
<td>3</td>
<td>37.34</td>
</tr>
<tr>
<td>4</td>
<td>37.92</td>
</tr>
</tbody>
</table>

Table 4.4: Blade deflections versus applied loads.

<table>
<thead>
<tr>
<th>Blade</th>
<th>Applied Load, N</th>
<th>Measured Deflection, mm</th>
<th>$k, N/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3724</td>
<td>1.993</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.7433</td>
<td>2.835</td>
<td>324.2</td>
</tr>
<tr>
<td></td>
<td>1.1096</td>
<td>3.822</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.4816</td>
<td>4.791</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3724</td>
<td>1.370</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.7433</td>
<td>2.335</td>
<td>326.0</td>
</tr>
<tr>
<td></td>
<td>1.1096</td>
<td>3.761</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.4816</td>
<td>4.481</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3724</td>
<td>1.860</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.7433</td>
<td>2.756</td>
<td>320.6</td>
</tr>
<tr>
<td></td>
<td>1.1096</td>
<td>3.963</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.4816</td>
<td>4.718</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>323.6</td>
</tr>
<tr>
<td>Gen-1</td>
<td></td>
<td></td>
<td>10.52</td>
</tr>
<tr>
<td>CLPT</td>
<td></td>
<td></td>
<td>344.8</td>
</tr>
</tbody>
</table>
Figure 4.1: T-plate used for blade stiffness testing.
Figure 4.2: Dimensioned drawing of end-mass hanger.

Figure 4.3: End-mass hanger installed onto composite blade.
Figure 4.4: Dimensioned drawing of base plate.

Figure 4.5: Composite blade stiffness testing setup.
Figure 4.6: Composite blade deflection versus applied load.
4.3 Composite Blade Vibration Testing

Both the untwisted and twisted composite blades were subjected to vibration testing to obtain their fundamental coning frequencies. Because the analytical model treats the blades as rigid, it is necessary to ensure that the coning natural frequencies were higher than 1/rev at the rotor operating speed of 1,600 (Gen-2) or 2,000 (Gen-3) RPM. Vibration testing also assists in identifying rotor imbalances that might adversely affect the wind tunnel tests.

Vibration testing was conducted in the Penn State Center for Acoustics and Vibration (CAV) Laboratory using a single-axis shaker and custom frequency analysis software developed in-house by CAV. The complete blade and plate mounting assembly is shown in Fig. 4.7. The same T-plate/base plate assembly used for static stiffness testing was mounted to the shaker, and a strip of aluminum tape was affixed to the tip of the blade to serve as a reflective target for a Polytec OFC-534 laser Doppler vibrometer (LDV), which was used to measure the blade tip speed as a function of time. The LDV was mounted to a camera tripod, and the beam was aligned perpendicular to the laboratory floor by using a spirit level and adjusting the rotation of the LDV using the three rotation locks on the tripod. The tripod location was adjusted as needed to center the beam on the tape target, after which the LDV beam was focused. A Piezotronics 352C65 accelerometer was attached to the base plate using a piece of aluminum tape and cyanoacrylate adhesive. The 352C65 is an integrated electronic piezoelectric ("voltage-mode") accelerometer that incorporates built-in signal conditioning microelectronics; these accelerometers are less susceptible to noise from the surrounding environment [73]. The accelerometer sensitivity was given by the manufacturer and certification documents as 93.2 mV/g.

Two National Instruments cDAQ modules - NI9263 and NI9234 - were used, respectively, to control the Techron 7541 amplifier and to receive input from the measurements of shaker base acceleration and blade tip speed. The output and input ports for these cDAQ modules are shown in Fig. 4.8. The shaker was excited by using the in-house software to output a Gaussian white noise signal in continuous-repeat mode. The amplifier gain was adjusted until the LDV control box showed an acceptable signal strength level. The scaling factor of the input signal read by the NI9234 was set to 50 mm/sec./volt for clarity. The data acquisition software was then armed, and was set to collect data for five seconds, which it then averaged into a “hit.” The hit was saved, and the process was repeated. Each blade was tested three times, with the magnitude and phase of each tests frequency response function plotted using an in-house CAV MATLAB script.

The results of the untwisted blades is tabulated in Table 4.5. A finite element calculation using cubic-spline interpolation was used to calculate an “ideal” blade frequency (using CLPT and linear taper) of 120.0 Hz; this finite element calculation assumed that the blades had a fiber volume fraction of 0.60.

A sample frequency response function of an untwisted composite blade is shown in Fig. 4.9, and a boxplot (Fig. 4.10) of the untwisted blade frequencies compares the measured coning fundamental frequencies versus the finite element calculation. The standard deviations between the blades were deemed reasonably close to assume equal variances, and a single-factor analysis
of variance (ANOVA) was performed to determine if there was a statistically significant difference between the blade frequency values. The results of this ANOVA are shown in Table 4.6. The null hypothesis for this ANOVA was taken to be that there is no statistically significant difference in the mean blade frequencies \( H_0 : \mu_1 = \mu_2 = \mu_3 \). The p-value for these blade frequencies is greater than 0.05, which is typically the cutoff threshold to reject the null hypothesis. For vibration testing, failure to reject the null would indicate that there was no statistically significant difference between blade frequencies; this indicates that the blades were reasonably balanced with respect to each other. The untwisted blades all had coning natural frequencies well above 1/rev, indicating that the rigid-blade assumption used by the analytical model was justified.

The vibration tests were repeated with the twisted composite blades. For these blades, the finite element frequency was 130.0 Hz; this was higher than the calculated frequency for the untwisted blades due to the differences in the length of the unidirectional reinforcement layer: the unidirectional strip spanned approximately 0.75 R for the twisted blades, whereas for the untwisted blades it only spanned approximately 0.60 R. The results of the twisted blade vibration tests are tabulated in Table 4.7. As with the untwisted blades, a one-way ANOVA was performed for the twisted blades. The results of this ANOVA are tabulated in Table 4.8. As with the untwisted blades, the vibration results of the twisted blades showed that the blades had been balanced to a reasonable degree. A sample of a frequency response function for the twisted blades is shown in Fig. 4.11, and a boxplot of the coning natural frequencies is shown in Fig. 4.12.
Table 4.5: Vibration test results for untwisted composite blades.

<table>
<thead>
<tr>
<th></th>
<th>Blade 1</th>
<th>Blade 2</th>
<th>Blade 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$, Hz (test 1)</td>
<td>120.0</td>
<td>119.3</td>
<td>119.1</td>
</tr>
<tr>
<td>$f_n$, Hz (test 2)</td>
<td>121.1</td>
<td>118.5</td>
<td>119.5</td>
</tr>
<tr>
<td>$f_n$, Hz (test 3)</td>
<td>120.2</td>
<td>119.6</td>
<td>120.3</td>
</tr>
<tr>
<td>$f_n$, Hz (mean)</td>
<td>120.4</td>
<td>119.1</td>
<td>119.6</td>
</tr>
<tr>
<td>$f_n$, Hz (FEM)</td>
<td>120.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev., Hz</td>
<td>0.5859</td>
<td>0.5686</td>
<td>0.6110</td>
</tr>
<tr>
<td>CV</td>
<td>4.865E-03</td>
<td>4.773E-03</td>
<td>5.107E-03</td>
</tr>
</tbody>
</table>

Table 4.6: One-way ANOVA of untwisted composite blades, 95% confidence level.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>$F$</th>
<th>P-value</th>
<th>$F_{crit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>2.580</td>
<td>2</td>
<td>1.2900</td>
<td>3.721</td>
<td>0.0889</td>
<td>5.143</td>
</tr>
<tr>
<td>Within groups</td>
<td>2.080</td>
<td>6</td>
<td>0.3467</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.660</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.7: Vibration test results for twisted composite blades.

<table>
<thead>
<tr>
<th></th>
<th>Blade 1</th>
<th>Blade 2</th>
<th>Blade 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$, Hz (test 1)</td>
<td>124.8</td>
<td>124.3</td>
<td>124.1</td>
</tr>
<tr>
<td>$f_n$, Hz (test 2)</td>
<td>129.4</td>
<td>128.4</td>
<td>127.9</td>
</tr>
<tr>
<td>$f_n$, Hz (test 3)</td>
<td>129.8</td>
<td>129.6</td>
<td>129.0</td>
</tr>
<tr>
<td>$f_n$, Hz (mean)</td>
<td>128.0</td>
<td>127.4</td>
<td>127.0</td>
</tr>
<tr>
<td>$f_n$, Hz (FEM)</td>
<td></td>
<td>130.0</td>
<td></td>
</tr>
<tr>
<td>Std. Dev., Hz</td>
<td>2.804</td>
<td>2.779</td>
<td>2.571</td>
</tr>
<tr>
<td>CV</td>
<td>2.190E-03</td>
<td>2.181E-03</td>
<td>2.024E-03</td>
</tr>
</tbody>
</table>

Table 4.8: One-way ANOVA of twisted composite blades, 95% confidence level.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>$F_{crit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>1.5089</td>
<td>2</td>
<td>0.7544</td>
<td>0.1026</td>
<td>0.9040</td>
<td>5.146</td>
</tr>
<tr>
<td>Within groups</td>
<td>44.11</td>
<td>6</td>
<td>7.351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>45.62</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.7: Closeup of composite blade mounted to shaker.

Figure 4.8: Shaker data acquisition wiring.
Figure 4.9: Sample frequency response function magnitude (top) and phase (bottom) of untwisted composite blade.

Figure 4.10: Boxplot of coning natural frequencies, untwisted composite blades.
Figure 4.11: Sample frequency response function magnitude (top) and phase (bottom), twisted composite blade.

Figure 4.12: Boxplot of coning natural frequencies, twisted composite blades.
4.4 Wing Frequency Testing

The analytical model requires wing stiffness and wind-off damping values to calculate the flutter speed of the wind tunnel model. This meant that the Gen-3 model required physical testing to find the wing frequency, stiffness, and damping values. The beamwise and chordwise stiffnesses were calculated using the spar cross-section and modulus, with the wing and nacelle treated as a lumped end-mass; the torsional inertia of the spar was estimated using the approximation methods presented in a standard strength of materials textbook, such as [55]. The torsional inertia of the nacelle was calculated directly from the SolidWorks assembly.

The Gen-3a model was assembled and mounted to the load cell (Fig. 4.13) for a series of rap tests. The impact targets used for these rap tests were the outlets of the excitation jets for the beam mode and the center of the rotor shaft for the chord mode. To test the torsion mode, the nacelle was manually pitched and then released. All data were recorded at 1,000 Hz using the load cell and custom LabVIEW software. Time histories and frequency spectra of each of the wing modes are shown in Figs. 4.14 – 4.19. The moving-block method was used to calculate the damping of each of the wing modes. Each mode was tested three times, with the average of the damping ratios used as the zero-airspeed damping inputs to the analytical model. As shown in Table 4.9, the use of an independent spar allowed the Gen-3 frequency spread between the beam and chord modes (chord/beam = 1.416) to more closely approximate that of larger tiltrotor whirl flutter models. The torsion-beam frequency spread (torsion/beam = 4.983), however, remains higher than that of larger wind tunnel models.
Table 4.9: Gen-3 rap test frequency and damping results.

<table>
<thead>
<tr>
<th>Beam Mode</th>
<th>Test</th>
<th>Hz</th>
<th>ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>5.308</td>
<td>0.0137</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.308</td>
<td>0.0177</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.333</td>
<td>0.0133</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>5.316</td>
<td>0.0149</td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
<td>5.569</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chord Mode</th>
<th>Test</th>
<th>Hz</th>
<th>ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>7.500</td>
<td>0.0134</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.536</td>
<td>0.0156</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7.541</td>
<td>0.0127</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>7.526</td>
<td>0.0139</td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
<td>7.425</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Torsion Mode</th>
<th>Test</th>
<th>Hz</th>
<th>ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>25.93</td>
<td>0.0812</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>26.80</td>
<td>0.0872</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>26.73</td>
<td>0.0782</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>26.49</td>
<td>0.0822</td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
<td>27.50</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.13: Gen-3a model mounted for rap testing.
Figure 4.14: Beam mode rap test time history.

Figure 4.15: Beam mode rap test frequency spectrum.
Figure 4.16: Chord mode rap test time history.

Figure 4.17: Chord mode rap test frequency spectrum.
Figure 4.18: Torsion mode rap test time history.

Figure 4.19: Torsion mode rap test frequency spectrum.
4.5 Gen-2 Results and Discussion

As mentioned previously, the Gen-2 model was designed as an intermediary configuration, intended to build confidence in the composite fabrication process described herein by testing the untwisted blades on a wing assembly with known structural characteristics. To reduce the number of differences between Gen-1 and Gen-2, the original Servo City controller used for Gen-1 testing was used, rather than the more precise HiTec controller used for Gen-3 tests. Each of the Gen-2 configurations was tested over fifty times, from speeds of 65 ft./sec. to 120 ft./sec. The air compressor was pressurized to 100 psi for all of the Gen-2 tests. For each tunnel speed, each Gen-2 configuration was tested three times to ensure consistency and repeatability. Tunnel speed was increased in 5 ft./sec. increments for all Gen-2a tests, since the Gen-1 counterpart of Gen-2a had shown no indication of an instability. For testing of the Gen-2b model, tunnel speed was increased in 3 ft./sec. increments above 110 ft./sec., as the possibility of flutter was far greater.

4.5.1 Gen-2 Performance

The Gen-2 model consisted of the Gen-1 monocoque ABS wing with the untwisted composite rotor blades; two versions of the Gen-2 model were tested (Gen-2a and Gen-2b), which correspond, respectively, to Configurations 1 and 4 of Gen-1. The features of these models are summarized in Table 4.10.

As with the Gen-1 model, both of the Gen-2 models were tested at a constant 1,600 RPM. Experimentally measured pitch settings were determined based upon the known relationship between the servo PWM signal and the previously measured blade pitch. The maximum required pitch setting of approximately 55° was well within the 65° maximum throw of the servo. As was the case with Gen-1 testing, the flexibility of the control linkages was not accounted for. The experimental results were acceptably close to the calculated trim settings, and the differences between the calculated and measured trim values were found to be within the range of deviation of the servo. The Gen-2a pitch settings as a function of tunnel speed are summarized in Table 4.11 and shown in Fig. 4.20, the latter of which includes a maximum variation of ±2° that was present for all configurations.
Table 4.10: Comparison of Gen-1 and Gen-2 model properties. Aft-mass c.g. located 5.5 in. aft of wing elastic axis.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Rotor radius, in.</th>
<th>Twist</th>
<th>Taper</th>
<th>$\delta_1$</th>
<th>$\gamma$</th>
<th>$\sigma$</th>
<th>Added mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen-1 Config.1</td>
<td>7.55</td>
<td>0</td>
<td>0</td>
<td>$+47^\circ$</td>
<td>2.360</td>
<td>0.118</td>
<td>None</td>
</tr>
<tr>
<td>Gen-1 Config.4</td>
<td>7.55</td>
<td>0</td>
<td>0</td>
<td>$-47^\circ$</td>
<td>2.360</td>
<td>0.118</td>
<td>Aft</td>
</tr>
<tr>
<td>Gen-2a</td>
<td>8.05</td>
<td>0</td>
<td>0.7</td>
<td>$+47^\circ$</td>
<td>3.225</td>
<td>0.102</td>
<td>None</td>
</tr>
<tr>
<td>Gen-2b</td>
<td>8.05</td>
<td>0</td>
<td>0.7</td>
<td>$-47^\circ$</td>
<td>3.225</td>
<td>0.102</td>
<td>Aft</td>
</tr>
</tbody>
</table>

Table 4.11: Gen-2 trim pitch versus tunnel speed.

<table>
<thead>
<tr>
<th>$V_{\infty}$, ft./sec.</th>
<th>$\theta_{\text{predicted}}$, deg.</th>
<th>$\theta_1$, deg.</th>
<th>$\theta_2$, deg.</th>
<th>$\theta_3$, deg.</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>30.80</td>
<td>30.29</td>
<td>30.38</td>
<td>30.65</td>
<td>30.44</td>
<td>0.1873</td>
</tr>
<tr>
<td>65</td>
<td>33.53</td>
<td>33.41</td>
<td>33.28</td>
<td>33.34</td>
<td>33.34</td>
<td>0.0653</td>
</tr>
<tr>
<td>70</td>
<td>36.09</td>
<td>36.02</td>
<td>35.83</td>
<td>35.73</td>
<td>35.86</td>
<td>0.1453</td>
</tr>
<tr>
<td>75</td>
<td>37.33</td>
<td>36.57</td>
<td>36.72</td>
<td>36.83</td>
<td>36.71</td>
<td>0.1290</td>
</tr>
<tr>
<td>80</td>
<td>38.56</td>
<td>38.60</td>
<td>38.51</td>
<td>38.50</td>
<td>38.54</td>
<td>0.0556</td>
</tr>
<tr>
<td>85</td>
<td>40.81</td>
<td>41.01</td>
<td>41.10</td>
<td>41.08</td>
<td>41.06</td>
<td>0.0473</td>
</tr>
<tr>
<td>90</td>
<td>42.98</td>
<td>43.12</td>
<td>43.07</td>
<td>43.02</td>
<td>43.07</td>
<td>0.0480</td>
</tr>
<tr>
<td>95</td>
<td>44.11</td>
<td>44.77</td>
<td>44.64</td>
<td>44.45</td>
<td>44.62</td>
<td>0.1609</td>
</tr>
<tr>
<td>100</td>
<td>45.63</td>
<td>45.30</td>
<td>45.53</td>
<td>45.69</td>
<td>45.51</td>
<td>0.1938</td>
</tr>
<tr>
<td>105</td>
<td>46.86</td>
<td>47.07</td>
<td>47.13</td>
<td>47.16</td>
<td>47.12</td>
<td>0.0457</td>
</tr>
<tr>
<td>110</td>
<td>48.13</td>
<td>48.81</td>
<td>48.49</td>
<td>48.32</td>
<td>48.54</td>
<td>0.2478</td>
</tr>
<tr>
<td>115</td>
<td>49.40</td>
<td>50.01</td>
<td>49.68</td>
<td>49.67</td>
<td>49.78</td>
<td>0.1953</td>
</tr>
<tr>
<td>120</td>
<td>50.32</td>
<td>50.67</td>
<td>50.75</td>
<td>50.66</td>
<td>50.69</td>
<td>0.0493</td>
</tr>
</tbody>
</table>
Figure 4.20: Gen-2 calculated and experimental trim pitch values.
### 4.5.2 Gen-2 Stability

The Gen-2 damping was compared to the Gen-1 damping at several identical tunnel speeds. The damping values for Gen-1 were measured directly from [32]; as was the case with Gen-1, the Gen-2 tests only examined the beamwise mode of the wing. The addition of the aft-mass lowered the effective maximum beamwise damping of both Gen-1 Configuration 4 and Gen-2b, although this does not indicate a physical weakening of the wing: because the damping ratio is a function of its structural damping, mass, and stiffness, the addition of an end-mass will cause a lower total damping ratio for the model without affecting the structural integrity of the wing. It was noticed that Gen-2b was not capable of maintaining a stable RPM below a tunnel speed of 60 ft./sec. This was not noticed during testing of Gen-2a, nor is it thought to be a malfunction of the pitch control system; rather, the addition of the aft-mass caused the rotor hub to maintain visible contact with the shaft at lower tunnel speeds. It was only at a freestream speed of approximately 60 ft./sec. did the rotor achieve a continuous and stable “off-shaft” operating state. The measured damping of the Gen-2 models as a function of tunnel speed are tabulated in Tables 4.12.

The damping of Gen-2a was found to increase with tunnel speed, as was the case with its Gen-1 counterpart; however, the damping of Gen-2a was found to vary much more significantly above a tunnel speed of approximately 80 ft./sec., and was found to be slightly higher than the damping of Gen-1 Config.1 up to a tunnel speed of 130 ft./sec. The variation in damping is not surprising, since the rotor rotational speed degree of freedom (Ω) has a major effect on whirl flutter stability, and the in-plane hub force due to shaft speed is a principal mechanism of whirl flutter [3]. Beamwise deflection of the wing is accompanied by a corresponding shaft roll [82], and fixing the rotor RPM transfers the shaft roll motion to the rotor; if the rotor possesses high aerodynamic damping, this will stabilize the wing mode.

The possibility of the rotor RPM varying at different test cases must be considered, because during initial shakedown tests of Gen-1 in October 2013, the Servo City controller was demonstrably incapable of maintaining rotor speed to within ± 50 RPM; the maximum difference in rotor speed measured by the LabVIEW code was 147 RPM, which was confirmed with a stroboscopic lamp (although it must be made clear that these RPM excursions were transient in nature and difficult, if not impossible, to consistently reproduce). It is clear from [32] that small changes in pitch angle can have profound effects on the stability of the model, and the change in pitch angle of ±2° that was mentioned in that work - which caused appreciable differences in the predicted damping behavior of the model - is well within the position error of the Servo City controller. At the conclusion of Gen-2 testing, the servo controller remained the likely cause of pitch variations; it is unlikely that the torque required to maintain a given pitch angle exceeded the capabilities of the servo, since the same servo was used on the Gen-3 model (with a more precise controller) and rotor speed variations were greatly reduced.

The differences in the damping of Gen-2 may be due to the Lock number of Gen-2 being 37% higher than that of Gen-1. From [82], the moment exerted on a point a distance $h$ from the hub (viz. the mast height) is
\[
\Delta \left( \frac{2C_{Mx}}{\sigma \alpha^2} \right) = -h \left( H_\beta - R_\beta \right) \frac{16}{\gamma} \left( \frac{\dot{\alpha}_y}{\dot{\alpha}_x} \right)
\]  

(4.1)

For high inflow, \( H_\beta \) dominates \( \left( H_\beta \approx -\frac{1}{4} \cos \phi \right) \), and the moment coefficients in Eq. 4.1 are destabilizing; increasing the Lock number decreases the flapping required to produce a moment at the tip-path plane and in turn decreases the perturbation magnitude of the change in the stability coefficients \( C_{Mx} \) and \( C_{My} \); however, the stability of a system as complex as a tiltrotor wing/nacelle/rotor assembly cannot be explained by a single destabilizing term; for instance, the damping behavior of Gen-2a might be misleading unless the operating range of the tunnel is considered, and it is possible that at higher speeds the damping of Gen-2a might fall off more quickly than that of Gen-1. This is suggested in Fig. 4.21, where the damping of Gen-2a may be seen to decrease slightly below the value of its Gen-1 counterpart at tunnel speeds above 120 ft./sec. The Lock number effect is may be minimal for the Gen-2 model, but may still influence the stability of Gen-2 at speeds that could not be tested with the current facility. It is most likely, then, that the flutter stability of the Gen-2 model at these scales and test speeds is driven more by wing and pylon stiffnesses than Lock number.

An argument for this may be made via the Gen-2b damping results, which are plotted as a function of tunnel speed in Fig. 4.22. The only difference between Gen-2a and Gen-2b was the inclusion of the added mass extended aft of the trailing edge of the wing. As with Gen-2a, the stability of Gen-2b closely followed that of Gen-1 Config. 4 at tunnel speeds below 80 ft./sec., after which the Gen-2b damping was consistently (albeit slightly) lower than that of Gen-1 Config. 4. At a tunnel speed of approximately 95 ft./sec., the damping of Gen-2b began to deviate significantly from the Gen-1 values, and decreased in a roughly linear fashion from 100 to 118 ft./sec. The damping of the Gen-2b model stayed consistently negative after 113 ft./sec., whereas the Gen-1 damping increased briefly for one test point before the model exhibited an instability. The Gen-2b damping remained slightly more negative at the highest speed tested than Gen-1. Much like Gen-1 Config. 4, the damping of Gen-2b varied greatly near its flutter speed; however, the damping of Gen-2b was found to be more consistent than its Gen-1 counterpart - viz. there was less scatter in the results between test points. The 118 ft./sec. tunnel speed marked the conclusion of Gen-2b testing, as the model had already been tested to flutter and preserving the composite blades was necessary in order to test the Gen-3 model.
Table 4.12: Gen-2 damping versus tunnel speed. Flutter regime in red.

<table>
<thead>
<tr>
<th>$V_{\infty}$, ft./sec.</th>
<th>Gen-2a</th>
<th></th>
<th>Gen-2b</th>
<th></th>
<th>Gen-2b Avg.</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\zeta_{Gen-1Avg.}$</td>
<td>$\zeta_1$</td>
<td>$\zeta_2$</td>
<td>$\zeta_3$</td>
<td>$\zeta_{Gen-2Avg.}$</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.0106</td>
<td>0.0104</td>
<td>0.0115</td>
<td>0.0105</td>
<td>0.0108</td>
<td>0.0006</td>
</tr>
<tr>
<td>55</td>
<td>0.0111</td>
<td>0.0103</td>
<td>0.0118</td>
<td>0.0122</td>
<td>0.0144</td>
<td>0.0030</td>
</tr>
<tr>
<td>60</td>
<td>0.0116</td>
<td>0.0109</td>
<td>0.0129</td>
<td>0.0126</td>
<td>0.0121</td>
<td>0.0011</td>
</tr>
<tr>
<td>65</td>
<td>0.0128</td>
<td>0.0130</td>
<td>0.0125</td>
<td>0.0122</td>
<td>0.0126</td>
<td>0.0004</td>
</tr>
<tr>
<td>70</td>
<td>0.0125</td>
<td>0.0130</td>
<td>0.0138</td>
<td>0.0146</td>
<td>0.0138</td>
<td>0.0008</td>
</tr>
<tr>
<td>75</td>
<td>0.0132</td>
<td>0.0130</td>
<td>0.0128</td>
<td>0.0146</td>
<td>0.0135</td>
<td>0.0010</td>
</tr>
<tr>
<td>80</td>
<td>0.0148</td>
<td>0.0136</td>
<td>0.0167</td>
<td>0.0167</td>
<td>0.0156</td>
<td>0.0018</td>
</tr>
<tr>
<td>85</td>
<td>0.0156</td>
<td>0.0146</td>
<td>0.0174</td>
<td>0.0165</td>
<td>0.0162</td>
<td>0.0014</td>
</tr>
<tr>
<td>90</td>
<td>0.0165</td>
<td>0.0160</td>
<td>0.0158</td>
<td>0.0182</td>
<td>0.0167</td>
<td>0.0013</td>
</tr>
<tr>
<td>95</td>
<td>0.0170</td>
<td>0.0163</td>
<td>0.0185</td>
<td>0.0192</td>
<td>0.0180</td>
<td>0.0015</td>
</tr>
<tr>
<td>100</td>
<td>0.0180</td>
<td>0.0209</td>
<td>0.0205</td>
<td>0.0200</td>
<td>0.0205</td>
<td>0.0004</td>
</tr>
<tr>
<td>105</td>
<td>0.0179</td>
<td>0.0206</td>
<td>0.0205</td>
<td>0.0200</td>
<td>0.0204</td>
<td>0.0003</td>
</tr>
<tr>
<td>110</td>
<td>0.0206</td>
<td>0.0227</td>
<td>0.0230</td>
<td>0.0232</td>
<td>0.0230</td>
<td>0.0002</td>
</tr>
<tr>
<td>115</td>
<td>0.0195</td>
<td>0.0217</td>
<td>0.0218</td>
<td>0.0216</td>
<td>0.0217</td>
<td>0.0001</td>
</tr>
<tr>
<td>120</td>
<td>0.0233</td>
<td>0.0222</td>
<td>0.0226</td>
<td>0.0200</td>
<td>0.0216</td>
<td>0.0014</td>
</tr>
<tr>
<td>125</td>
<td>0.0254</td>
<td>0.0241</td>
<td>0.0235</td>
<td>0.0239</td>
<td>0.0238</td>
<td>0.0003</td>
</tr>
<tr>
<td>130</td>
<td>0.0278</td>
<td>0.0261</td>
<td>0.0257</td>
<td>0.0257</td>
<td>0.0258</td>
<td>0.0002</td>
</tr>
<tr>
<td>135</td>
<td>0.0300</td>
<td>0.0198</td>
<td>0.0224</td>
<td>0.0243</td>
<td>0.0221</td>
<td>0.0023</td>
</tr>
<tr>
<td>140</td>
<td>0.0310</td>
<td>0.0216</td>
<td>0.0229</td>
<td>0.0276</td>
<td>0.0240</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$V_{\infty}$, ft./sec.</th>
<th>$\zeta_{Gen-1Avg.}$</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$\zeta_{Gen-2Avg.}$</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.0090</td>
<td>0.0082</td>
<td>0.0090</td>
<td>0.0096</td>
<td>0.0089</td>
<td>0.0007</td>
</tr>
<tr>
<td>65</td>
<td>0.0095</td>
<td>0.0091</td>
<td>0.0097</td>
<td>0.0101</td>
<td>0.0096</td>
<td>0.0005</td>
</tr>
<tr>
<td>70</td>
<td>0.0094</td>
<td>0.0099</td>
<td>0.0091</td>
<td>0.0102</td>
<td>0.0097</td>
<td>0.0006</td>
</tr>
<tr>
<td>75</td>
<td>0.0100</td>
<td>0.0102</td>
<td>0.0106</td>
<td>0.0112</td>
<td>0.0107</td>
<td>0.0005</td>
</tr>
<tr>
<td>80</td>
<td>0.0102</td>
<td>0.0108</td>
<td>0.0114</td>
<td>0.0100</td>
<td>0.0107</td>
<td>0.0007</td>
</tr>
<tr>
<td>85</td>
<td>0.0087</td>
<td>0.0085</td>
<td>0.0092</td>
<td>0.0080</td>
<td>0.0086</td>
<td>0.0006</td>
</tr>
<tr>
<td>90</td>
<td>0.0085</td>
<td>0.0080</td>
<td>0.0073</td>
<td>0.0078</td>
<td>0.0077</td>
<td>0.0004</td>
</tr>
<tr>
<td>95</td>
<td>0.0087</td>
<td>0.0070</td>
<td>0.0081</td>
<td>0.0080</td>
<td>0.0077</td>
<td>0.0006</td>
</tr>
<tr>
<td>100</td>
<td>0.0091</td>
<td>0.0081</td>
<td>0.0083</td>
<td>0.0073</td>
<td>0.0079</td>
<td>0.0005</td>
</tr>
<tr>
<td>105</td>
<td>0.0084</td>
<td>0.0074</td>
<td>0.0080</td>
<td>0.0079</td>
<td>0.0078</td>
<td>0.0003</td>
</tr>
<tr>
<td>110</td>
<td>0.0036</td>
<td>0.0020</td>
<td>0.0028</td>
<td>0.0033</td>
<td>0.0027</td>
<td>0.0007</td>
</tr>
<tr>
<td>113</td>
<td>0.0062</td>
<td>0.0015</td>
<td>-0.0010</td>
<td>-0.0008</td>
<td>-0.0001</td>
<td>0.0014</td>
</tr>
<tr>
<td>115</td>
<td>-0.0025</td>
<td>-0.0012</td>
<td>-0.0018</td>
<td>-0.0015</td>
<td>-0.0015</td>
<td>0.0003</td>
</tr>
<tr>
<td>118</td>
<td>-0.0032</td>
<td>-0.0040</td>
<td>-0.0038</td>
<td>-0.0028</td>
<td>-0.0035</td>
<td>0.0006</td>
</tr>
</tbody>
</table>
Figure 4.21: Gen-2a and Gen-1 Config. 1 damping versus tunnel speed.

Figure 4.22: Gen-2b and Gen-1 Config. 4 damping versus tunnel speed.
4.6 Gen-3 Results and Discussion

The Gen-3 model variants all featured the composite semi-monocoque wing described previously, and the only difference between the variants is the rotor installed for testing. As with Gen-1 testing, the final configuration tested had the rotor blades removed in order to ensure that the instabilities measured during testing were due solely to the influence of the rotor rather than any inherent characteristic of the wing. Each of the Gen-3 models was tested at least three times at each tunnel speed to increase the repeatability of the data. The Gen-3 model properties are summarized in Table 4.13. The HiTec servo controller was used for all Gen-3 tests, and a safety string was installed onto the model for each test session. It was noted during several Gen-3 tests that the RPM readout on the LabVIEW code became fixed at 1,200 despite the conditions of the test making that impossible (e.g. tunnel speeds on the order of 20 ft./sec. with blades fully feathered); a stroboscopic flash was used to confirm the rotor RPM at each test point prior to excitation of the model.

Model fatigue was considered a potential source of error during testing; this possibility was first encountered during shakedown testing of Gen-1 in October 2013, which revealed that the beam and chord frequencies had decreased by approximately 20% from the values reported in [32]. Subsequent inspection of the Gen-1 model revealed that a crack had developed between the wing and the load cell mounting plate. The need to ensure the structural integrity of the model was paramount with Gen-3, which is composed of several independent parts that must be manually attached to create the complete wing/nacelle/rotor assembly. This created new failure modes for the semi-monocoque wing that were not present in the Gen-1 model, chief amongst them the potential for the chemical bond securing the spar within the wing to fail. This necessitated that the Gen-3 wing stiffnesses be re-characterized prior to each wind tunnel test session. The intent of this was to identify when the wing assembly had begun to weaken, and to avoid destroying the model by fabricating a new wing when the present wing was in need of replacement. Initially, a frequency boundary of 10% was used (viz. the wing would be replaced when any of its modal frequencies had decreased by at least 10%). In reality, the wing beam and chord mode never decreased by more than 2 - 3%, and the frequency boundary for these modes was found to be irrelevant. The torsional stiffness, however, was always the first to visibly weaken, and eventually the wing could be rotated up to 5° independently of the spar; this created a visible and audible cue that the wing was in need of replacement. The modular nature of the Gen-3 wing, however, meant that this replacement could be fabricated and fully cured in less than two days, which minimized downtime. The average lifecycle of a given wing was found to be on the order of 40 wind-on test points, although Gen-3c (which lacked a rotor) was found to have a slightly longer lifecycle of 50 - 60 wind-on test points.

4.6.1 Gen-3a performance

All Gen-3 models were tested at a constant rotor shaft speed of 2,000 RPM. The experimentally measured pitch settings were obtained in the fashion previously described. Gen-3a was not able
to maintain a stable RPM below a tunnel speed of 60 ft./sec. As was the case with the Gen-1 data presented in [32], the correlation between the calculated and measured trim pitch values improved with increasing tunnel speed; overall, the trim pitch values were acceptably close to the values calculated by the prediction code. The maximum trim pitch required for Gen-3a was 48.01° at a tunnel speed of 115 ft./sec. The Gen-3a trim pitch values are tabulated in Table 4.14 and plotted in Fig. 4.23, which includes the 2° maximum variation in servo arm position.

Table 4.13: Gen-3 model properties by configuration.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Rotor radius, in.</th>
<th>Twist</th>
<th>Taper</th>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen-3a</td>
<td>8.05</td>
<td>0</td>
<td>0.7</td>
<td>$-47^\circ$</td>
<td>3.225</td>
<td>0.102</td>
</tr>
<tr>
<td>Gen-3b</td>
<td>8.55</td>
<td>$-25^\circ/R$</td>
<td>0.7</td>
<td>$-47^\circ$</td>
<td>3.700</td>
<td>0.096</td>
</tr>
<tr>
<td>Gen-3a</td>
<td>None</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.14: Gen-3a trim pitch versus tunnel speed.

<table>
<thead>
<tr>
<th>$V_\infty$, ft./sec.</th>
<th>$\theta_{predicted}$, deg.</th>
<th>$\theta_1$, deg.</th>
<th>$\theta_2$, deg.</th>
<th>$\theta_3$, deg.</th>
<th>Average, deg.</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>28.29</td>
<td>28.29</td>
<td>28.38</td>
<td>28.65</td>
<td>28.44</td>
<td>0.1873</td>
</tr>
<tr>
<td>65</td>
<td>31.41</td>
<td>31.41</td>
<td>31.28</td>
<td>31.34</td>
<td>31.34</td>
<td>0.0653</td>
</tr>
<tr>
<td>70</td>
<td>34.02</td>
<td>34.02</td>
<td>33.83</td>
<td>33.73</td>
<td>33.86</td>
<td>0.1453</td>
</tr>
<tr>
<td>75</td>
<td>34.57</td>
<td>34.57</td>
<td>34.72</td>
<td>34.83</td>
<td>34.71</td>
<td>0.1290</td>
</tr>
<tr>
<td>80</td>
<td>36.60</td>
<td>36.60</td>
<td>36.51</td>
<td>36.50</td>
<td>36.54</td>
<td>0.0556</td>
</tr>
<tr>
<td>85</td>
<td>39.01</td>
<td>39.01</td>
<td>39.10</td>
<td>39.08</td>
<td>39.06</td>
<td>0.0473</td>
</tr>
<tr>
<td>90</td>
<td>41.12</td>
<td>41.12</td>
<td>41.07</td>
<td>41.02</td>
<td>41.07</td>
<td>0.0480</td>
</tr>
<tr>
<td>95</td>
<td>42.77</td>
<td>42.77</td>
<td>42.64</td>
<td>42.45</td>
<td>42.62</td>
<td>0.1609</td>
</tr>
<tr>
<td>97</td>
<td>43.05</td>
<td>43.05</td>
<td>43.26</td>
<td>43.14</td>
<td>43.15</td>
<td>0.1038</td>
</tr>
<tr>
<td>99</td>
<td>43.65</td>
<td>43.65</td>
<td>43.54</td>
<td>43.41</td>
<td>43.53</td>
<td>0.1178</td>
</tr>
<tr>
<td>101</td>
<td>43.30</td>
<td>43.30</td>
<td>43.53</td>
<td>43.69</td>
<td>43.51</td>
<td>0.1938</td>
</tr>
<tr>
<td>103</td>
<td>44.77</td>
<td>44.77</td>
<td>44.62</td>
<td>44.50</td>
<td>44.63</td>
<td>0.1348</td>
</tr>
<tr>
<td>105</td>
<td>45.07</td>
<td>45.07</td>
<td>45.13</td>
<td>45.16</td>
<td>45.12</td>
<td>0.0457</td>
</tr>
<tr>
<td>107</td>
<td>45.87</td>
<td>45.87</td>
<td>45.64</td>
<td>45.51</td>
<td>45.67</td>
<td>0.1815</td>
</tr>
<tr>
<td>110</td>
<td>46.81</td>
<td>46.81</td>
<td>46.49</td>
<td>46.32</td>
<td>46.54</td>
<td>0.2478</td>
</tr>
<tr>
<td>112</td>
<td>47.13</td>
<td>47.13</td>
<td>46.87</td>
<td>47.05</td>
<td>47.02</td>
<td>0.1343</td>
</tr>
<tr>
<td>115</td>
<td>48.01</td>
<td>48.01</td>
<td>47.68</td>
<td>47.67</td>
<td>47.78</td>
<td>0.1953</td>
</tr>
</tbody>
</table>
Figure 4.23: Calculated and experimental trim pitch values, Gen-3a.
### 4.6.2 Gen-3a stability

The Gen-3 damping as a function of airspeed was compared to the predictions of the analytical model. For each of the Gen-3a and Gen-3b tests, the model was perturbed at its beamwise natural frequency through the use of the compressed air excitation jets; however, the small cross-sectional geometry of the wing spar allowed all three wing modes (beam, chord, and torsion) to influence the stability of the wind tunnel model. These wing modes were measured using the roll (beam), yaw (chord), and pitch (torsion) channels of the load cell. The three modes for each test point were analyzed independently, since the moving-block code can only analyze the damping of one channel at a time. The analytical model predicted that the chord mode of Gen-3a would be the first wing mode to attain zero damping, at a tunnel speed of approximately 105 ft./sec. In reality, the model experienced one test point of negative damping at a tunnel speed of 99 ft./sec., but this could not be repeated; the model did, however, experience repeated negative damping in the chord mode beginning at 101 ft./sec., and this was considered to be the flutter speed of the model.

A sample time history of the Gen-3a stable case is shown in Fig. 4.24. The frequency spectra of Gen-3a (Fig. 4.25) showed that at lower tunnel speeds, the model experienced 1/rev vibrations that were significantly less than those experienced by the Gen-1 model: Gen-3a experienced 1/rev excitations at a maximum amplitude on the order of 0.1 in both the stable and unstable cases (Figs. 4.25 and 4.26), whereas for Gen-1, the 1/rev was on the order of 0.25 (Fig. 1.9). The noise floor of Gen-3a was also found to be significantly lower than that of Gen-1. This justifies the fabrication procedure used to manufacture the composite wing and rotor blades, as well as the effort undertaken to ensure accurate physical properties of the Gen-3 components.

It is also clear that the precision of the HiTec controller is not universal: at lower tunnel speeds, the controller did an acceptable job of maintaining the rotor shaft speed between 1,930 - 2,066 RPM (0.9648 - 1.033/rev, or approximately 3% of the commanded value); at higher tunnel speeds, the RPM drift increased to about 10% (1.107/rev, or 2,214 RPM). RPM drift at higher tunnel speeds was found to be one-sided (i.e. the RPM would only increase from the commanded value), whereas RPM drift at lower tunnel speeds was found to be two-sided (i.e. the RPM could increase or decrease from the commanded value). These variations were noticed near the end of Gen-3a testing, and indicated servo behavior that was significantly different from what was noticed during initial shakedown testing of Gen-3 (no perturbations), wherein the servo was demonstrably able to hold rotor speed to within ± 5 RPM of the commanded value. While the 500 or so wind-on tests accumulated between Gen-1, Gen-2, and Gen-3 demonstrated that the maximum torque of the servo is clearly adequate, there is the possibility of servo components beginning to experience wear. Metal-geared servos will typically sacrifice increased gear lash in favor of a longer wear life relative to nylon-geared servos, but the current servo was found to consistently chatter when connected to the controller, even in a wind-off state. Chatter is normal when a servo is under load, but chatter in a no-load condition is typically indicative of worn gearing or a fatigued potentiometer. The problem was not found to increase over time during
Gen-3 testing (viz. the RPM excursions experienced at the start of Gen-3a stability testing were virtually identical to the ones noticed at the end of Gen-3b testing), and thus the replacement of the servo is left as a future work item.

A high-speed camera was used to record two of the Gen-3a test points at 2,000 frames per second, using one 575-watt metal halide lamp (ARRISUN 5) as a hard light with a single sheet of Mylar used as a scrim. A still frame of the Gen-3a test is shown in Fig. 4.27, which is a composite frame of the model in its deflected state superimposed over the model in its undisturbed state. The maximum deflection of the model was determined by relating known geometric properties of the model to the deflection measured in a still frame. For Gen-1 (Fig. 4.28), this was done by relating the chord length and maximum deflection as measured in the still frame ($c_{\text{still}}$ and $\delta_{\text{still}}$, respectively) to the actual chord and deflection:

$$\frac{\delta_{\text{still}}}{c_{\text{still}}} = \frac{\delta_{\text{actual}}}{c_{\text{actual}}} \Rightarrow \delta_{\text{actual}} = \frac{\delta_{\text{still}}}{c_{\text{actual}}} c_{\text{still}}$$

(4.2)

Substituting in the appropriate values reveals that Gen-1 experienced a maximum deflection on the order of 1.614 in. during flutter. The camera angle used to record Gen-3 required Eq. 4.2 to be modified slightly, and the rotor radius of the model ($R$) was instead used to calculate the deflection:

$$\frac{\delta_{\text{still}}}{R_{\text{still}}} = \frac{\delta_{\text{actual}}}{R_{\text{actual}}} \Rightarrow \delta_{\text{actual}} = \frac{\delta_{\text{still}}}{R_{\text{actual}}} R_{\text{still}}$$

(4.3)

Substituting in the appropriate values reveals that Gen-3a experienced a maximum deflection on the order of 1.184 in. during flutter (approximately 26% less than Gen-1). This should not be taken to imply that the maximum deflection of Gen-3 during flutter was necessarily less than Gen-1: it must be noted that the Gen-1 tests allowed the model to oscillate for a prolonged period of time after its neutral stability boundary was attained or exceeded (in one case, the model was allowed to oscillate for 45 seconds after the excitation ceased), whereas during Gen-3 testing the tunnel speed was immediately reduced (and the safety string pulled taut) once the magnitude of the oscillations began to increase, in order to prevent damage to the model. It is therefore safe to presume that the 1.614 in. deflection of the Gen-1 wing was near the maximum deflection experienced during testing, whereas the 1.184 in. deflection of the Gen-3 wing was merely the maximum captured during testing. It is likely that, given the relative stiffnesses of the Gen-1 and Gen-3 wings, Gen-3 would have experienced a much higher deflection if the unstable oscillations had been allowed to continue. This is logical, given the relative stiffnesses of the Gen-1 and Gen-3 models; these relative stiffnesses are summarized in Table 4.15.

The wing modal frequencies and damping of the Gen-3a model are plotted as a function of tunnel speed in Fig. 4.30; overall, there is good agreement between the predicted and measured damping values, with approximately 65% of the measured damping values coming within 7% of the predicted values. Inspection of the Gen-3a high-speed video revealed that after perturbation had ceased, the torsional and transverse deflections were asynchronous (Fig. 4.31): the wing torsion mode was clearly seen first, followed by the beamwise and chordwise deflection of the
wing. This is in direct contradiction to what occurred during testing of Gen-1 Configuration 4 and presented in [32], wherein the high-speed video clearly shows the closely-coupled pitch-down/flap-up and pitch-up/flap-down motions of the wing (Fig. 4.28). This is explained by the difference in stiffnesses between the wing chord and torsion modes relative to the beam mode for each of the two models (Table 4.15): the beam and torsional stiffnesses of Gen-1 are nearly identical, whereas the Gen-3a wing stiffness have a wider spread and the motions are therefore decoupled. The modal damping of the Gen-3a model as a function of tunnel speed is given in Tables 4.16 – 4.18.
Table 4.15: Comparison of Gen-1 and Gen-3 measured wing stiffness values.

<table>
<thead>
<tr>
<th>Item</th>
<th>Gen-1 Config. 4</th>
<th>Gen-3a</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EI_{beam},lb\text{ft.}^2$</td>
<td>18.10</td>
<td>8.986</td>
</tr>
<tr>
<td>$EI_{chord},lb\text{ft.}^2$</td>
<td>400.0</td>
<td>18.01</td>
</tr>
<tr>
<td>$GJ,lb\text{ft.}^2$</td>
<td>19.70</td>
<td>42.10</td>
</tr>
<tr>
<td>$EI_{chord}/EI_{beam}$</td>
<td>22.10</td>
<td>2.011</td>
</tr>
<tr>
<td>$GJ/EI_{beam}$</td>
<td>1.088</td>
<td>4.685</td>
</tr>
</tbody>
</table>

Table 4.16: Gen-3a wing beamwise damping versus tunnel speed.

<table>
<thead>
<tr>
<th>$V_\infty$, ft./sec.</th>
<th>$\zeta_{\text{predicted}}$</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.0206</td>
<td>0.0205</td>
<td>0.0209</td>
<td>0.0207</td>
<td>0.0207</td>
<td>0.0002</td>
</tr>
<tr>
<td>70</td>
<td>0.0223</td>
<td>0.0229</td>
<td>0.0225</td>
<td>0.0212</td>
<td>0.0222</td>
<td>0.0009</td>
</tr>
<tr>
<td>75</td>
<td>0.0240</td>
<td>0.0246</td>
<td>0.0243</td>
<td>0.0241</td>
<td>0.0243</td>
<td>0.0003</td>
</tr>
<tr>
<td>80</td>
<td>0.0245</td>
<td>0.0258</td>
<td>0.0248</td>
<td>0.0235</td>
<td>0.0247</td>
<td>0.0011</td>
</tr>
<tr>
<td>85</td>
<td>0.0258</td>
<td>0.0256</td>
<td>0.0271</td>
<td>0.0264</td>
<td>0.0264</td>
<td>0.0007</td>
</tr>
<tr>
<td>90</td>
<td>0.0275</td>
<td>0.0272</td>
<td>0.0277</td>
<td>0.0270</td>
<td>0.0273</td>
<td>0.0004</td>
</tr>
<tr>
<td>95</td>
<td>0.0284</td>
<td>0.0286</td>
<td>0.0287</td>
<td>0.0283</td>
<td>0.0285</td>
<td>0.0002</td>
</tr>
<tr>
<td>97</td>
<td>0.0288</td>
<td>0.0287</td>
<td>0.0292</td>
<td>0.0285</td>
<td>0.0288</td>
<td>0.0004</td>
</tr>
<tr>
<td>99</td>
<td>0.0292</td>
<td>0.0290</td>
<td>0.0296</td>
<td>0.0293</td>
<td>0.0293</td>
<td>0.0003</td>
</tr>
<tr>
<td>101</td>
<td>0.0298</td>
<td>0.0302</td>
<td>0.0297</td>
<td>0.0296</td>
<td>0.0298</td>
<td>0.0003</td>
</tr>
<tr>
<td>103</td>
<td>0.0300</td>
<td>0.0293</td>
<td>0.0296</td>
<td>0.0290</td>
<td>0.0293</td>
<td>0.0003</td>
</tr>
<tr>
<td>105</td>
<td>0.0310</td>
<td>0.0303</td>
<td>0.0308</td>
<td>0.0301</td>
<td>0.0304</td>
<td>0.0004</td>
</tr>
<tr>
<td>107</td>
<td>0.0315</td>
<td>0.0308</td>
<td>0.0305</td>
<td>0.0313</td>
<td>0.0309</td>
<td>0.0004</td>
</tr>
<tr>
<td>110</td>
<td>0.0323</td>
<td>0.0339</td>
<td>0.0331</td>
<td>0.0334</td>
<td>0.0335</td>
<td>0.0004</td>
</tr>
<tr>
<td>112</td>
<td>0.0328</td>
<td>0.0314</td>
<td>0.0338</td>
<td>0.0326</td>
<td>0.0326</td>
<td>0.0012</td>
</tr>
<tr>
<td>115</td>
<td>0.0335</td>
<td>0.0337</td>
<td>0.0342</td>
<td>0.0341</td>
<td>0.0340</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Table 4.17: Gen-3a wing chordwise damping versus tunnel speed. Flutter regime in red.

<table>
<thead>
<tr>
<th>$V_∞$, ft./sec.</th>
<th>$ζ_{predicted}$</th>
<th>$ζ_1$</th>
<th>$ζ_2$</th>
<th>$ζ_3$</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.0109</td>
<td>0.0113</td>
<td>0.0115</td>
<td>0.0110</td>
<td>0.0113</td>
<td>0.0003</td>
</tr>
<tr>
<td>70</td>
<td>0.0082</td>
<td>0.0079</td>
<td>0.0086</td>
<td>0.0075</td>
<td>0.0080</td>
<td>0.0006</td>
</tr>
<tr>
<td>75</td>
<td>0.0058</td>
<td>0.0060</td>
<td>0.0054</td>
<td>0.0059</td>
<td>0.0058</td>
<td>0.0003</td>
</tr>
<tr>
<td>80</td>
<td>0.0052</td>
<td>0.0047</td>
<td>0.0053</td>
<td>0.0055</td>
<td>0.0052</td>
<td>0.0004</td>
</tr>
<tr>
<td>85</td>
<td>0.0037</td>
<td>0.0036</td>
<td>0.0038</td>
<td>0.0033</td>
<td>0.0036</td>
<td>0.0003</td>
</tr>
<tr>
<td>90</td>
<td>0.0019</td>
<td>0.0023</td>
<td>0.0016</td>
<td>0.0021</td>
<td>0.0020</td>
<td>0.0004</td>
</tr>
<tr>
<td>95</td>
<td>0.0011</td>
<td>0.0010</td>
<td>0.0015</td>
<td>0.0009</td>
<td>0.0011</td>
<td>0.0003</td>
</tr>
<tr>
<td>97</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0006</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0002</td>
</tr>
<tr>
<td>99</td>
<td>0.0004</td>
<td>-0.0001</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0003</td>
</tr>
<tr>
<td>101</td>
<td>0.0000</td>
<td>-0.0007</td>
<td>-0.0003</td>
<td>0.0000</td>
<td>-0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td>103</td>
<td>-0.0005</td>
<td>-0.0009</td>
<td>-0.0003</td>
<td>-0.0007</td>
<td>-0.0006</td>
<td>0.0003</td>
</tr>
<tr>
<td>105</td>
<td>-0.00010</td>
<td>-0.0012</td>
<td>-0.0011</td>
<td>-0.0013</td>
<td>-0.0012</td>
<td>0.0001</td>
</tr>
<tr>
<td>107</td>
<td>-0.0013</td>
<td>-0.0008</td>
<td>-0.0016</td>
<td>-0.0009</td>
<td>-0.0011</td>
<td>0.0004</td>
</tr>
<tr>
<td>110</td>
<td>-0.0018</td>
<td>-0.0019</td>
<td>-0.0021</td>
<td>-0.0016</td>
<td>-0.0019</td>
<td>0.0003</td>
</tr>
<tr>
<td>112</td>
<td>-0.0021</td>
<td>-0.0019</td>
<td>-0.0025</td>
<td>-0.0024</td>
<td>-0.0023</td>
<td>0.0004</td>
</tr>
<tr>
<td>115</td>
<td>-0.0025</td>
<td>-0.0027</td>
<td>-0.0023</td>
<td>-0.0029</td>
<td>-0.0026</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Table 4.18: Gen-3a wing torsional damping versus tunnel speed.

<table>
<thead>
<tr>
<th>$V_∞$, ft./sec.</th>
<th>$ζ_{predicted}$</th>
<th>$ζ_1$</th>
<th>$ζ_2$</th>
<th>$ζ_3$</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.0071</td>
<td>0.0065</td>
<td>0.0069</td>
<td>0.0069</td>
<td>0.0068</td>
<td>0.0002</td>
</tr>
<tr>
<td>70</td>
<td>0.0074</td>
<td>0.0069</td>
<td>0.0079</td>
<td>0.0069</td>
<td>0.0072</td>
<td>0.0006</td>
</tr>
<tr>
<td>75</td>
<td>0.0076</td>
<td>0.0072</td>
<td>0.0077</td>
<td>0.0074</td>
<td>0.0074</td>
<td>0.0002</td>
</tr>
<tr>
<td>80</td>
<td>0.0076</td>
<td>0.0085</td>
<td>0.0076</td>
<td>0.0075</td>
<td>0.0079</td>
<td>0.0005</td>
</tr>
<tr>
<td>85</td>
<td>0.0077</td>
<td>0.0082</td>
<td>0.0082</td>
<td>0.0081</td>
<td>0.0082</td>
<td>0.0001</td>
</tr>
<tr>
<td>90</td>
<td>0.0078</td>
<td>0.0082</td>
<td>0.0084</td>
<td>0.0080</td>
<td>0.0082</td>
<td>0.0002</td>
</tr>
<tr>
<td>95</td>
<td>0.0079</td>
<td>0.0081</td>
<td>0.0083</td>
<td>0.0081</td>
<td>0.0082</td>
<td>0.0001</td>
</tr>
<tr>
<td>97</td>
<td>0.0079</td>
<td>0.0077</td>
<td>0.0080</td>
<td>0.0082</td>
<td>0.0080</td>
<td>0.0003</td>
</tr>
<tr>
<td>99</td>
<td>0.0079</td>
<td>0.0082</td>
<td>0.0084</td>
<td>0.0081</td>
<td>0.0082</td>
<td>0.0002</td>
</tr>
<tr>
<td>101</td>
<td>0.0079</td>
<td>0.0082</td>
<td>0.0082</td>
<td>0.0076</td>
<td>0.0080</td>
<td>0.0003</td>
</tr>
<tr>
<td>103</td>
<td>0.0079</td>
<td>0.0075</td>
<td>0.0081</td>
<td>0.0082</td>
<td>0.0079</td>
<td>0.0004</td>
</tr>
<tr>
<td>105</td>
<td>0.0079</td>
<td>0.0082</td>
<td>0.0074</td>
<td>0.0081</td>
<td>0.0079</td>
<td>0.0004</td>
</tr>
<tr>
<td>107</td>
<td>0.0078</td>
<td>0.0076</td>
<td>0.0081</td>
<td>0.0079</td>
<td>0.0079</td>
<td>0.0003</td>
</tr>
<tr>
<td>110</td>
<td>0.0078</td>
<td>0.0082</td>
<td>0.0084</td>
<td>0.0081</td>
<td>0.0082</td>
<td>0.0002</td>
</tr>
<tr>
<td>112</td>
<td>0.0078</td>
<td>0.0083</td>
<td>0.0075</td>
<td>0.0079</td>
<td>0.0079</td>
<td>0.0004</td>
</tr>
<tr>
<td>115</td>
<td>0.0077</td>
<td>0.0082</td>
<td>0.0084</td>
<td>0.0081</td>
<td>0.0082</td>
<td>0.0002</td>
</tr>
</tbody>
</table>
Figure 4.24: Sample time history of stable Gen-3a test point, tunnel speed = 80 ft./sec.
Figure 4.25: Sample frequency spectra of stable Gen-3a test point, tunnel speed = 80 ft./sec.
Figure 4.26: Sample frequency spectra of unstable Gen-3a test point, tunnel speed = 115 ft./sec.
Figure 4.27: Composite still of Gen-3a experiencing beam and chord oscillation.

Figure 4.28: Deflection of Gen-1 Config. 4 as measured from video still, from [32]
Figure 4.29: Deflection of Gen-3a as measured from video still.
Figure 4.30: Gen-3a wing modal frequencies (a) and damping (b) versus tunnel speed.
Figure 4.31: Gen-3a exhibiting torsional deflection (a) prior to beam/chord deflection (b).
4.6.3 Gen-3b performance

The experimentally measured pitch settings for Gen-3b were obtained in an identical fashion to Gen-3a. The minimum tunnel speed for Gen-3b was 65 ft./sec., below which the rotor was unable to maintain a stable RPM at any pitch setting. Mean blade pitch was measured to be within \( \pm 0.5^\circ \) of the predicted values below tunnel speeds of 95 ft./sec., after which the measured pitch settings were increasingly higher than the calculated values. The maximum trim pitch for Gen-3b was 46.77\(^\circ\) at a tunnel speed of 115 ft./sec. The Gen-3b trim pitch values are presented in Table 4.19 and plotted in Fig. 4.32, which includes the maximum servo arm deviation of \( \pm 2^\circ \).

<table>
<thead>
<tr>
<th>( V_\infty, \text{ ft./sec.} )</th>
<th>( \theta_{\text{predicted}} )</th>
<th>( \theta_1, \text{ deg.} )</th>
<th>( \theta_2, \text{ deg.} )</th>
<th>( \theta_3, \text{ deg.} )</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>30.80</td>
<td>30.23</td>
<td>30.65</td>
<td>30.54</td>
<td>30.47</td>
<td>0.2178</td>
</tr>
<tr>
<td>70</td>
<td>32.17</td>
<td>31.95</td>
<td>32.09</td>
<td>32.13</td>
<td>32.06</td>
<td>0.0945</td>
</tr>
<tr>
<td>75</td>
<td>33.53</td>
<td>33.45</td>
<td>33.61</td>
<td>33.59</td>
<td>33.55</td>
<td>0.0872</td>
</tr>
<tr>
<td>80</td>
<td>36.09</td>
<td>35.98</td>
<td>36.12</td>
<td>36.05</td>
<td>36.05</td>
<td>0.0700</td>
</tr>
<tr>
<td>85</td>
<td>37.33</td>
<td>37.26</td>
<td>37.38</td>
<td>37.29</td>
<td>37.31</td>
<td>0.0624</td>
</tr>
<tr>
<td>90</td>
<td>38.56</td>
<td>38.49</td>
<td>38.52</td>
<td>38.58</td>
<td>38.53</td>
<td>0.0458</td>
</tr>
<tr>
<td>95</td>
<td>40.17</td>
<td>40.61</td>
<td>40.68</td>
<td>40.47</td>
<td>40.59</td>
<td>0.1082</td>
</tr>
<tr>
<td>97</td>
<td>40.81</td>
<td>41.24</td>
<td>41.28</td>
<td>41.32</td>
<td>41.28</td>
<td>0.0405</td>
</tr>
<tr>
<td>99</td>
<td>41.35</td>
<td>41.87</td>
<td>41.81</td>
<td>41.88</td>
<td>41.85</td>
<td>0.0383</td>
</tr>
<tr>
<td>101</td>
<td>41.89</td>
<td>42.85</td>
<td>42.79</td>
<td>42.78</td>
<td>42.81</td>
<td>0.0386</td>
</tr>
<tr>
<td>103</td>
<td>42.44</td>
<td>43.32</td>
<td>43.36</td>
<td>43.26</td>
<td>43.31</td>
<td>0.0513</td>
</tr>
<tr>
<td>105</td>
<td>42.98</td>
<td>43.96</td>
<td>43.98</td>
<td>44.01</td>
<td>43.99</td>
<td>0.0257</td>
</tr>
<tr>
<td>107</td>
<td>43.56</td>
<td>44.48</td>
<td>44.50</td>
<td>44.54</td>
<td>44.51</td>
<td>0.0312</td>
</tr>
<tr>
<td>110</td>
<td>44.43</td>
<td>45.48</td>
<td>45.51</td>
<td>45.46</td>
<td>45.49</td>
<td>0.0257</td>
</tr>
<tr>
<td>112</td>
<td>45.01</td>
<td>46.08</td>
<td>46.05</td>
<td>46.03</td>
<td>46.06</td>
<td>0.0257</td>
</tr>
<tr>
<td>115</td>
<td>45.70</td>
<td>46.77</td>
<td>46.75</td>
<td>46.68</td>
<td>46.73</td>
<td>0.0482</td>
</tr>
</tbody>
</table>

Table 4.19: Gen-3b trim pitch versus tunnel speed.
Figure 4.32: Gen-3b trim pitch versus tunnel speed.
4.6.4 Gen-3b stability

The Gen-3b damping as a function of tunnel speed was measured in a fashion identical to that described previously for the Gen-2 and Gen-3a models. Unlike Gen-3a, the torsion mode of Gen-3b was predicted to reach an instability first, at approximately 97 ft./sec. In reality, Gen-3b reached its instability slightly before the predicted value, at a tunnel speed of 94 - 95 ft./sec. (the pitot-static transducer voltage was found to vary repeatedly within this range, precluding a precise measurement); however, Gen-3b test points showed very low damping (at or near zero) at tunnel speeds as low as 90 ft./sec. It is possible that, should a method be devised to excite the wing modes independently, a more refined examination of the modal damping at these tunnel speeds could be made.

The deviations from the calculated values may be partially explained by the flutter behavior of Gen-3b and, by comparison, that of Gen-3a. Near the flutter speed of the model, Gen-3b exhibited very low damping, as expected - in some cases requiring on the order of ten seconds for the oscillations to dampen (Fig. 4.34); however, once the flutter speed was reached, the oscillations of Gen-3b quickly increased - especially in torsion. One of the milder instances of this instability is shown in Fig. 4.35. These “snap flutter” oscillations were actually sufficient to cause the chemical bond of two Gen-3 wings to fail, which required new wings to be fabricated before testing could continue. Gen-3b had a higher Lock number than Gen-3a (approximately 15% greater) and a much higher Lock number than Gen-1 (on the order of 50% greater), so the differences in flutter behavior between the models is not surprising. The fact that the untwisted blades of Gen-1, Gen-2, and Gen-3a had large regions of the blade span above the static stall angle of attack means that the forces imparted to the wing from the rotor were likely reduced; it stands to reason that the forces imparted to the wing by the twisted blades of the Gen-3b rotor were therefore significantly higher.
Table 4.20: Gen-3b modal damping versus tunnel speed. Flutter regime in red.

<table>
<thead>
<tr>
<th>$V_{\infty}$, ft./sec.</th>
<th>$\zeta_{\text{predicted}}$</th>
<th>$\zeta_1$, deg.</th>
<th>$\zeta_2$, deg.</th>
<th>$\zeta_3$, deg.</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beamwise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>0.0206</td>
<td>0.0213</td>
<td>0.0221</td>
<td>0.0211</td>
<td>0.0215</td>
<td>0.0005</td>
</tr>
<tr>
<td>70</td>
<td>0.0215</td>
<td>0.0231</td>
<td>0.0223</td>
<td>0.0221</td>
<td>0.0225</td>
<td>0.0005</td>
</tr>
<tr>
<td>75</td>
<td>0.0224</td>
<td>0.0234</td>
<td>0.0242</td>
<td>0.0232</td>
<td>0.0236</td>
<td>0.0005</td>
</tr>
<tr>
<td>80</td>
<td>0.0242</td>
<td>0.0254</td>
<td>0.0252</td>
<td>0.0242</td>
<td>0.0249</td>
<td>0.0007</td>
</tr>
<tr>
<td>85</td>
<td>0.0251</td>
<td>0.0265</td>
<td>0.0240</td>
<td>0.0247</td>
<td>0.0251</td>
<td>0.0013</td>
</tr>
<tr>
<td>90</td>
<td>0.0260</td>
<td>0.0294</td>
<td>0.0282</td>
<td>0.0281</td>
<td>0.0286</td>
<td>0.0007</td>
</tr>
<tr>
<td>95</td>
<td>0.0273</td>
<td>0.0305</td>
<td>0.0315</td>
<td>0.0314</td>
<td>0.0311</td>
<td>0.0006</td>
</tr>
<tr>
<td>97</td>
<td>0.0278</td>
<td>0.0296</td>
<td>0.0305</td>
<td>0.0294</td>
<td>0.0298</td>
<td>0.0006</td>
</tr>
<tr>
<td>99</td>
<td>0.0283</td>
<td>0.0306</td>
<td>0.0317</td>
<td>0.0304</td>
<td>0.0309</td>
<td>0.0007</td>
</tr>
<tr>
<td>101</td>
<td>0.0287</td>
<td>0.0316</td>
<td>0.0327</td>
<td>0.0314</td>
<td>0.0319</td>
<td>0.0007</td>
</tr>
<tr>
<td>Chordwise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>0.0096</td>
<td>0.0099</td>
<td>0.0103</td>
<td>0.0098</td>
<td>0.0100</td>
<td>0.0002</td>
</tr>
<tr>
<td>70</td>
<td>0.0083</td>
<td>0.0084</td>
<td>0.0087</td>
<td>0.0073</td>
<td>0.0081</td>
<td>0.0007</td>
</tr>
<tr>
<td>75</td>
<td>0.0070</td>
<td>0.0068</td>
<td>0.0061</td>
<td>0.0067</td>
<td>0.0065</td>
<td>0.0004</td>
</tr>
<tr>
<td>80</td>
<td>0.0050</td>
<td>0.0053</td>
<td>0.0035</td>
<td>0.0042</td>
<td>0.0043</td>
<td>0.0009</td>
</tr>
<tr>
<td>85</td>
<td>0.0041</td>
<td>0.0037</td>
<td>0.0028</td>
<td>0.0027</td>
<td>0.0031</td>
<td>0.0006</td>
</tr>
<tr>
<td>90</td>
<td>0.0032</td>
<td>0.0037</td>
<td>0.0038</td>
<td>0.0037</td>
<td>0.0038</td>
<td>0.0001</td>
</tr>
<tr>
<td>95</td>
<td>0.0023</td>
<td>0.0019</td>
<td>0.0012</td>
<td>0.0008</td>
<td>0.0013</td>
<td>0.0005</td>
</tr>
<tr>
<td>97</td>
<td>0.0019</td>
<td>0.0009</td>
<td>0.0008</td>
<td>0.0013</td>
<td>0.0010</td>
<td>0.0003</td>
</tr>
<tr>
<td>99</td>
<td>0.0016</td>
<td>0.0011</td>
<td>0.0021</td>
<td>0.0009</td>
<td>0.0014</td>
<td>0.0006</td>
</tr>
<tr>
<td>101</td>
<td>0.0013</td>
<td>0.0004</td>
<td>0.0008</td>
<td>-0.0007</td>
<td>0.0002</td>
<td>0.0008</td>
</tr>
<tr>
<td>Torsion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>0.0051</td>
<td>0.0053</td>
<td>0.0055</td>
<td>0.0057</td>
<td>0.0055</td>
<td>0.0002</td>
</tr>
<tr>
<td>70</td>
<td>0.0049</td>
<td>0.0054</td>
<td>0.0052</td>
<td>0.0045</td>
<td>0.0050</td>
<td>0.0005</td>
</tr>
<tr>
<td>75</td>
<td>0.0046</td>
<td>0.0039</td>
<td>0.0042</td>
<td>0.0053</td>
<td>0.0045</td>
<td>0.0008</td>
</tr>
<tr>
<td>80</td>
<td>0.0037</td>
<td>0.0029</td>
<td>0.0041</td>
<td>0.0032</td>
<td>0.0034</td>
<td>0.0006</td>
</tr>
<tr>
<td>85</td>
<td>0.0029</td>
<td>0.0020</td>
<td>0.0034</td>
<td>0.0025</td>
<td>0.0026</td>
<td>0.0007</td>
</tr>
<tr>
<td>90</td>
<td>0.0021</td>
<td>0.0010</td>
<td>0.0000</td>
<td>0.0009</td>
<td>0.0006</td>
<td>0.0006</td>
</tr>
<tr>
<td>95</td>
<td>0.0003</td>
<td>-0.0007</td>
<td>-0.0005</td>
<td>-0.0010</td>
<td>-0.0007</td>
<td>0.0003</td>
</tr>
<tr>
<td>97</td>
<td>-0.0004</td>
<td>-0.0009</td>
<td>-0.0012</td>
<td>-0.0016</td>
<td>-0.0012</td>
<td>0.0004</td>
</tr>
<tr>
<td>99</td>
<td>-0.0015</td>
<td>-0.0029</td>
<td>-0.0024</td>
<td>-0.0021</td>
<td>-0.0025</td>
<td>0.0004</td>
</tr>
<tr>
<td>101</td>
<td>-0.0025</td>
<td>-0.0037</td>
<td>-0.0040</td>
<td>-0.0030</td>
<td>-0.0036</td>
<td>0.0005</td>
</tr>
</tbody>
</table>
Figure 4.33: Gen-3b wing modal frequencies (a) and damping (b) versus tunnel speed.
Figure 4.34: Sample time history of stable Gen-3b test point, $V_\infty = 90\, ft./sec$.

Figure 4.35: Sample time history of unstable Gen-3b test point, $V_\infty = 99\, ft./sec$. 
4.6.5 Gen-3c stability

Gen-3c consisted of the same semi-monocoque composite wing as Gen-3a and Gen-3b, but with the rotor blades removed (Fig. 4.36). The servo, bearing block, wiring, and other components of the nacelle were left in place. Gen-3c was the final configuration tested, and served to demonstrate that the instabilities observed were indeed caused solely by the presence of the rotor. The sole excitation method for Gen-3c was the wing beam mode, as before; however, the removal of the rotor blades resulted in the amplitudes of the wing chord and torsion modes being barely registered by the load cell. As seen in Fig. 4.37, the wing beam mode is clearly identifiable, however the wing chord mode barely registered above the noise floor, and the wing torsion mode was indistinguishable from it. The lack of a rotor meant that these two wing modes were simply not sufficiently excited for meaningful measurements to be made, and as such are not examined here.

No instability was predicted for Gen-3c by the analytical model, and no instability was encountered during testing. The damping of Gen-3c, in fact, was found to increase with increasing tunnel speed in a relatively linear fashion (Fig. 4.38). This would certainly not remain the case if the tunnel speed continued to be increased indefinitely, and the model would eventually experience flutter; however, within the operating range of the test facility, it was not possible to make the blades-off configuration experience an instability. Thus, the instabilities exhibited by the Gen-3a and Gen-3b models were due solely to the influence of the rotor, and are thus indicative of whirl flutter. There appeared to be good correlation between the calculated and experimental damping ratios, with most (approximately 70%) of the test points showing a mean damping ratio that was within ±5% of the predicted values. The predicted and measured modal damping values of the Gen-3c model are presented in Table 4.21.
Table 4.21: Gen-3c wing beamwise damping versus tunnel speed.

<table>
<thead>
<tr>
<th>$V_\infty$, ft./sec.</th>
<th>$\zeta_{predicted}$</th>
<th>$\zeta_1$, deg.</th>
<th>$\zeta_2$, deg.</th>
<th>$\zeta_3$, deg.</th>
<th>Average</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.0319</td>
<td>0.0269</td>
<td>0.0262</td>
<td>0.0238</td>
<td>0.0257</td>
<td>0.0016</td>
</tr>
<tr>
<td>55</td>
<td>0.0328</td>
<td>0.0278</td>
<td>0.0290</td>
<td>0.0284</td>
<td>0.0284</td>
<td>0.0006</td>
</tr>
<tr>
<td>60</td>
<td>0.0347</td>
<td>0.0326</td>
<td>0.0328</td>
<td>0.0326</td>
<td>0.0327</td>
<td>0.0001</td>
</tr>
<tr>
<td>65</td>
<td>0.0364</td>
<td>0.0317</td>
<td>0.0333</td>
<td>0.0335</td>
<td>0.0328</td>
<td>0.0010</td>
</tr>
<tr>
<td>70</td>
<td>0.0374</td>
<td>0.0334</td>
<td>0.0337</td>
<td>0.0346</td>
<td>0.0339</td>
<td>0.0006</td>
</tr>
<tr>
<td>75</td>
<td>0.0384</td>
<td>0.0408</td>
<td>0.0328</td>
<td>0.0396</td>
<td>0.0377</td>
<td>0.0043</td>
</tr>
<tr>
<td>80</td>
<td>0.0404</td>
<td>0.0418</td>
<td>0.0409</td>
<td>0.0452</td>
<td>0.0426</td>
<td>0.0023</td>
</tr>
<tr>
<td>85</td>
<td>0.0414</td>
<td>0.0416</td>
<td>0.0421</td>
<td>0.0385</td>
<td>0.0407</td>
<td>0.0020</td>
</tr>
<tr>
<td>90</td>
<td>0.0424</td>
<td>0.0398</td>
<td>0.0424</td>
<td>0.0486</td>
<td>0.0436</td>
<td>0.0045</td>
</tr>
<tr>
<td>95</td>
<td>0.0434</td>
<td>0.0456</td>
<td>0.0445</td>
<td>0.0429</td>
<td>0.0443</td>
<td>0.0014</td>
</tr>
<tr>
<td>97</td>
<td>0.0445</td>
<td>0.0512</td>
<td>0.0401</td>
<td>0.0461</td>
<td>0.0458</td>
<td>0.0056</td>
</tr>
<tr>
<td>99</td>
<td>0.0450</td>
<td>0.0477</td>
<td>0.0469</td>
<td>0.0420</td>
<td>0.0455</td>
<td>0.0031</td>
</tr>
<tr>
<td>101</td>
<td>0.0455</td>
<td>0.0414</td>
<td>0.0459</td>
<td>0.0545</td>
<td>0.0473</td>
<td>0.0066</td>
</tr>
<tr>
<td>103</td>
<td>0.0460</td>
<td>0.0457</td>
<td>0.0450</td>
<td>0.0438</td>
<td>0.0448</td>
<td>0.0010</td>
</tr>
<tr>
<td>105</td>
<td>0.0465</td>
<td>0.0489</td>
<td>0.0503</td>
<td>0.0512</td>
<td>0.0501</td>
<td>0.0012</td>
</tr>
<tr>
<td>107</td>
<td>0.0471</td>
<td>0.0509</td>
<td>0.0493</td>
<td>0.0499</td>
<td>0.0500</td>
<td>0.0008</td>
</tr>
<tr>
<td>110</td>
<td>0.0480</td>
<td>0.0492</td>
<td>0.0478</td>
<td>0.0469</td>
<td>0.0480</td>
<td>0.0012</td>
</tr>
<tr>
<td>115</td>
<td>0.0494</td>
<td>0.0475</td>
<td>0.0511</td>
<td>0.0509</td>
<td>0.0498</td>
<td>0.0020</td>
</tr>
<tr>
<td>120</td>
<td>0.0506</td>
<td>0.0522</td>
<td>0.0531</td>
<td>0.0524</td>
<td>0.0526</td>
<td>0.0005</td>
</tr>
<tr>
<td>125</td>
<td>0.0527</td>
<td>0.0543</td>
<td>0.0556</td>
<td>0.0502</td>
<td>0.0534</td>
<td>0.0028</td>
</tr>
<tr>
<td>130</td>
<td>0.0538</td>
<td>0.0552</td>
<td>0.0543</td>
<td>0.0514</td>
<td>0.0536</td>
<td>0.0020</td>
</tr>
<tr>
<td>135</td>
<td>0.0548</td>
<td>0.0521</td>
<td>0.0563</td>
<td>0.0565</td>
<td>0.0550</td>
<td>0.0025</td>
</tr>
<tr>
<td>140</td>
<td>0.0558</td>
<td>0.0573</td>
<td>0.0529</td>
<td>0.0591</td>
<td>0.0565</td>
<td>0.0032</td>
</tr>
</tbody>
</table>
Figure 4.36: Gen-3c mounted to load cell (a) and testing (b).

Figure 4.37: Gen-3c FFTs for wing beam (a), chord (b), and torsion (c) modes.
Figure 4.38: Gen-3c damping versus tunnel speed.
Chapter 5

Conclusions and Recommendations for Future Work

5.1 Summary of Project Objectives

The present work focused on the experimental validation of a whirl flutter analytical model that had been previously developed in-house. This computational model focused on predicting the whirl flutter stability of a windmilling, semispan tiltrotor wind tunnel model. Several iterations of this wind tunnel model were performed, in order to create a design that was representative of full-scale tiltrotor and would be able to exhibit whirl flutter below a tunnel speed of 140 ft./sec., which is the current operating limit of the facility. The wind tunnel models tested for this work are evolutions of the first-generation wind tunnel model (“Gen-1”), which was developed and tested during the January 2012 – May 2013 timeframe. Two new generations of the wind tunnel model (“Gen-2” and “Gen-3”) were designed, fabricated, and tested; the key features of the wind tunnel models are summarized in Table 5.1. There were three areas of focus for the Gen-2 and Gen-3 models, which addressed the shortcomings of the Gen-1 model: the rotor, the wing, and the control system. The objectives of these model improvement efforts are detailed below.

The design and fabrication of new rotors was focused on reducing blade mass, increasing blade radius, and reducing the variations in balance between the independent blades. This allowed the rotor Lock number to increase while reducing the magnitude of 1/rev vibrations. The fabrication of new rotor blades also reduced the mass variations between individual blades: the Gen-1 blades exhibited a mass variation of 0.15 grams, whereas the Gen-2 and Gen-3 blades exhibited a mass variation of 0.03 grams. The rotor Lock number of the Gen-1 model was 2.36, which is unrealistically low for a tiltrotor; the use of composites permitted the fabrication of wind tunnel
models with rotor Lock numbers of 3.23 (Gen-2) and 3.70 (Gen-3) that are more representative of full-scale aircraft. A twist rate of \(-25^\circ/R\) was incorporated into one model variant (Gen-3b) to reduce the amount of rotor span that was above the static stall angle of attack of the rotor airfoil (NACA 0011) within the tunnel regime of interest (75 – 140 ft./sec.).

The Gen-1 model was only able to exhibit whirl flutter through the use of a one-pound steel mass mounted aft of the wing trailing edge; the c.g. of this aft-mass was located 5.5 in. aft of the wing elastic axis. High-speed video of the Gen-1 flutter tests revealed that during cases of instability, the gimbal would strike the rotor shaft during excitation. This was due to strongly-coupled beamwise bending and torsion motions of the wing. The monocoque design of the Gen-1 wing also created very high frequency ratios between the wing beam, chord, and torsion modes (viz. wing chord and torsion modes that were, respectively, approximately two to three times as stiff as the beam mode); this is in direct contrast to typical whirl flutter models, such as the Wing and Rotor Aeroelastic Test Stand (WRATS), which has wing chord and torsion modes less than twice as stiff as the wing beam mode. The wing spar of the Gen-3 wind tunnel model was tailored to permit the model to exhibit whirl flutter without the use of an aft-mass. Additionally, the use of an inboard flexure region reduced the frequency ratios between the wing modes, with the wing chord mode of the Gen-3 model having a stiffness of approximately 1.5 times that of the wing beam mode.

All of the wind tunnel models examined for the present work maintained a constant rotor RPM by varying the collective pitch of the rotor via servo control; however, the Gen-1 model required a separate power supply and secondary test computer, both of which were dedicated to controlling the collective pitch servo. The control board used to command the pulse-width modulation (PWM) signal of the pitch control servo during Gen-1 testing was also of limited resolution, and was only able to command the servo over a PWM range of approximately 150 milliseconds in one-millisecond increments. This limited the resolution of the PWM signal sent to the servo, and resulted in the servo being unable to maintain the shaft speed of the rotor to within less than \(\pm 50\) RPM. The servo control board was replaced with a multifunction controller that eliminated the need for a second power supply, and was able to command the pitch control servo to a PWM resolution of four microseconds. This permitted the wind tunnel models to maintain the rotor shaft speed, for most test cases, to within \(\pm 5\) RPM.
Table 5.1: Wind tunnel model features by generation.

<table>
<thead>
<tr>
<th></th>
<th>Gen-1</th>
<th>Gen-2</th>
<th>Gen-3a</th>
<th>Gen-3b</th>
<th>Gen-3c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing semispan, in.</td>
<td>9.40</td>
<td>9.40</td>
<td>14.00</td>
<td>14.00</td>
<td>14.00</td>
</tr>
<tr>
<td>Wing type</td>
<td>ABS plastic</td>
<td>ABS plastic</td>
<td>Composite</td>
<td>Composite</td>
<td>Composite</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>$\pm 47^\circ$</td>
<td>$\pm 47^\circ$</td>
<td>$-47^\circ$</td>
<td>$-47^\circ$</td>
<td>N/A</td>
</tr>
<tr>
<td>Blade twist</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-25^\circ/R$</td>
<td>N/A</td>
</tr>
<tr>
<td>Blade taper</td>
<td>0</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>N/A</td>
</tr>
<tr>
<td>Blade mass, grams</td>
<td>9.35</td>
<td>5.65</td>
<td>5.65</td>
<td>5.91</td>
<td>N/A</td>
</tr>
<tr>
<td>Rotor solidity</td>
<td>0.118</td>
<td>0.102</td>
<td>0.102</td>
<td>0.096</td>
<td>0</td>
</tr>
<tr>
<td>Lock number</td>
<td>2.36</td>
<td>3.23</td>
<td>3.23</td>
<td>3.70</td>
<td>0</td>
</tr>
</tbody>
</table>
5.2 Summary of Present Research

The present work served as an experimental validation of an analytical whirl flutter model previously developed in-house during the August 2014 - October 2015 timeframe, which employed a finite-element approach based on the WRATS and XV-15 test programs to predict the modal damping ratios of a semispan wing as a function of tunnel speed.

The design and fabrication of new composite rotor blades was a critical phase in the evolution of the wind tunnel model. Two sets of composite blades were fabricated: an untwisted set of 8.05 in. rotor radius, and a twisted set of 8.55 in. rotor radius. The maximum rotor radius was determined by the need to maintain an adequate clearance between the blade tip and the wind tunnel wall. Both sets of composite blades were tapered (taper ratio of 0.7) in order to maintain an approximately constant geometric solidity of 0.1 across all wind tunnel models. The implementation of blade twist reduced the region of the blade that would be above the static stall angle of attack, from approximately 45% (Gen-1) to less than 30% (Gen-3b). Blade element theory was used to determine the minimum amount of blade twist that would be required to achieve this, and a twist rate of \(-25^\circ/R\) was obtained. This twist rate is significantly lower than what is found on other tiltrotor wind tunnel models such as WRATS (which features a blade twist on the order of \(-47^\circ/R\)); however, it must be noted that many of these larger tiltrotor models were designed to test a variety of flight conditions, including hover, and are meant to accurately represent the dynamics of a full-scale vehicle. By contrast, the models examined for this project were meant to investigate only one flight condition – forward flight in airplane mode – for the purposes of experimental validation, and were not dynamically or geometrically scaled to represent any one particular vehicle. Therefore, although the blade twist chosen is a point design, it is justifiable given the focus of the current work.

The composite blades were fabricated using a Divinycell foam core, AS4-12k unidirectional carbon fiber strips, and a Toray T300 3x1 twill weave fabric; the blade shanks were constructed of end-grain balsa in order to prevent splintering of the blade during attachment to the yoke. The blades were fabricated using a wet-layup technique with MGS L285/H285 resin, and each blade was allowed to cure in its mold for sixteen hours at room temperature. After curing, the blades were statically balanced using electrical tape. The untwisted blades were subjected to static stiffness testing, where they showed good correlation with classical composite sandwich panel theory. The mean static stiffness of the untwisted blades was found to be within 6% of the theoretical value.

Both sets of composite blades (twisted and untwisted) were dynamically tested using a shaker and a laser Doppler vibrometer, and the fundamental coning frequency of the blades were determined using custom MATLAB scripts developed in-house. All of the composite blades were found to have fundamental coning frequencies above 3.6/rev, which validated the rigid-blade assumption employed by the computational model. Examination of the frequency spectrum of the Gen-2 and Gen-3 models showed that there was still a 1/rev excitation for all test cases, however these 1/rev excitations were 40% lower than those experienced during Gen-1 testing.
The Gen-2 model served as an intermediary step between the work done for the Gen-1 model and the all-new Gen-3 models developed specifically for this project. The primary purpose of the Gen-2 model was to validate the composite fabrication methods used by demonstrating the structural integrity of the composite rotor blades in a relevant test environment. A secondary objective of the Gen-2 model was to examine the effects of increased Lock number on the behavior of the monocoque Gen-1 wing. Two versions of the Gen-2 model were tested, which corresponded to the most and least stable configurations of Gen-1 (Configuration 1 and Configuration 4, respectively) that had been previously tested and presented in [32]. These versions were termed Gen-2a and Gen-2b. The key differences between these two versions was the sense of the kinematic pitch-flap coupling and the addition of a steel mass extended aft of the wing trailing edge.

Gen-2a showed similar behavior to Gen-1 Configuration 1, and neither model exhibited an instability within the operating limits of the Hammond tunnel. Gen-2a exhibited damping that increased with increasing tunnel speed up to approximately 120 ft./sec., in a fashion very similar to Gen-1 Config. 1. Above 120 ft./sec., the damping of Gen-2a was found to be consistently less than that of its Gen-1 counterpart, and at 140 ft./sec. (the maximum operating speed of the tunnel) the damping of Gen-2a was approximately 26% lower than that of Gen-1 Config. 1. Since the only key difference between these two models was the higher Lock number of Gen-2a, it is quite likely that Gen-2a would have experienced an instability at a lower tunnel speed than Gen-1; however, this was not testable due to the operating limits of the current facility.

Gen-2b was the follow-on counterpart to Gen-1 Configuration 4, which featured a negative sense of $\delta_3$ and the addition of a one-pound steel mass attached aft of the wing trailing edge. As was the case with Gen-2a, the stability of Gen-2b closely followed the Gen-1 results at lower tunnel speeds; above a tunnel speed of 80 ft./sec., the damping of Gen-2b was found to be consistently lower than that of its Gen-1 counterpart. Above 95 ft./sec., the damping of Gen-2b began to deviate from the Gen-1 values, and decreased in an approximately linear fashion from 110 to 118 ft./sec. The damping of Gen-2b remained slightly more negative at the highest speed tested than that of its Gen-1 counterpart, however both Gen-1 Config. 4 and Gen-2b exhibited whirl flutter at roughly the same tunnel speed (115 ft./sec. and 113 ft./sec., respectively).

Although the Gen-2 model contained some improvements over its Gen-1 counterparts, there were still some potential areas of improvement. Chief amongst these was the fact that neither Gen-1 Config. 4 nor Gen-2b were able to exhibit an instability without the addition of an aft-mass. The use of this mass caused the wing beam and torsion modes of the ABS monocoque wing to become strongly coupled. Examination of high-speed video of Gen-1 Config. 4 showed that the gimbal would repeatedly strike the steel rotor shaft during excitation of the wing; this repeated striking damaged the rotor shaft. These impacts may partially explain why the experimental results of Gen-1 Config. 4 did not match the predicted behavior of the model. The development of the Gen-3 model was intended to address this issue, and its wing was specifically designed to exhibit whirl flutter within the operating limits of the test facility.

The Gen-3 model was subdivided into three configurations: Gen-3a, Gen-3b, and Gen-3c. All
Gen-3 models featured a composite semi-monocoque wing, whereas the composite rotor blades were mounted only to Gen-3a (untwisted) and Gen-3b (twisted). The Gen-3c model was a blades-off variant, which was tested to prove that the instabilities encountered were due solely to the influence of the rotor, and thus indicative of whirl flutter. The Gen-3 wing featured an integral aluminum spar bonded to the foam core of the wing using thirty-minute epoxy. Approximately 35% of this spar was left exposed (viz. not housed within the wing), which served as a flexure. The maximum length of this spar was constrained by the width of the tunnel test section and the need to maintain an adequate clearance between the tunnel wall and the tip of the largest rotor blades to be tested. Once the spar length was constrained, the spar material and cross-sectional area were iterated until the final spar dimensions and material were selected from a matrix of over 100 possible combinations. The spar, wing core, and nacelle were bonded together, and the wing was wrapped in carbon fiber fabric and clear-coated to protect it against moisture absorption.

The Gen-3 models were mounted to the wind tunnel load cell and subjected to stick-rap testing to determine the wind-off wing beamwise, chordwise, and torsional frequencies and damping ratios. The damping values were directly input into the analytical model, and the frequency values were used to calculate the effective beamwise, chordwise, and torsional stiffnesses of the spar, which were also required inputs. The behavior of Gen-3 showed good correlation with the predicted values, with Gen-3a exhibiting an instability at approximately 101 ft./sec. and Gen-3b exhibiting an instability at approximately 97 ft./sec. The Gen-3 model variants were excited in a fashion identical to the Gen-1 model (viz. beamwise excitation via compressed air, exhausted through puffer jets mounted in the nacelle and located at the wing elastic axis), however the small spar of Gen-3 allowed all three wing modes to be excited and analyzed.

The modal damping was calculated using a moving-block MATLAB script, which was modified to incorporate bandpass filtering in order to isolate each of the wing modes. For Gen-3c, the wing chord and torsion modes were not clearly distinguishable from the signal noise floor. Gen-3c was not predicted to exhibit an instability, and indeed the wing beamwise damping ratio was found to increase in an approximately linear fashion with increasing tunnel speed. Thus, the Gen-3c model would never experience whirl flutter, but may experience conventional bending/torsion flutter at a higher airspeed than what was testable within the current facility. More importantly, the Gen-3c results demonstrate that the instabilities exhibited by the Gen-3a and Gen-3b models were due solely to the influence of the rotor, and are thus an indication of whirl flutter.

The work performed to date has further validated the feasibility of whirl flutter testing at a small scale. The Gen-3 model was able to exhibit an instability at a sufficiently low tunnel speed to permit preliminary testing of devices designed to enhance the whirl flutter speed of tiltrotors, such as tuned wing extensions, within the operating capabilities of the current facility.
5.3 Recommendations for Future Work

The majority of the experimental effort was predicated on the assumption that time and effort spent ensuring accurate and precise measurements of individual model components and assemblies would yield a wind tunnel model with superior capabilities for both the present experimental validation effort as well as future developments. In many ways, this assumption was justified, and certain pre-test measurement procedures resulted in demonstrable improvements over the Gen-1 model; for example, the time spent balancing and re-balancing the composite blades resulted in drastically reduced 1/rev vibration magnitude. Much of the effort to seek higher levels of geometric tolerance for all features of size were necessary purely due to the size of the model under development. For instance, a geometric variance of spar width on the order of 0.004 in. would likely be inconsequential on a larger wind tunnel model, but for the Gen-3 model this would equate to a 15% variance, with the attendant consequences to stiffness and hence stability. Note that the need to maintain tight tolerances has absolutely no bearing on how well the experimental results correlate with the predictions of the analytical model, because the input parameters to the analytical model can always be adjusted based on measured physical properties; instead, the need to maintain tight tolerances stems from the need to create a model that can flutter at a specific tunnel speed (or within a range of specific tunnel speeds), because follow-on work with stability-enhancing devices will require sufficient “tunnel margin” to demonstrate their effectiveness. For example, a stability-enhancing device intended to increase flutter speed by 15% can easily be tested in the current facility using a base model that flutters at 100 ft./sec., but this might not be the case for a base model that flutters at 120 ft./sec.

Despite the efforts undertaken to ensure high accuracy and high dimensional and material tolerances, there are several additional efforts that may be undertaken with future model iterations to improve the utility of small-scale testing within the Hammond tunnel. The first (and likely most beneficial) effort is the incorporation of precone and coning flexures into the rotor gimbal. Furthermore, the lag stiffness of the blades is still very high, and the blades are effectively stiff in torsion; increasing blade lag and torsional flexibility will not be possible using the composite fabrication techniques presented herein, but may be possible by using some combination of linear polyvinylchloride foam (e.g. Airex R63) – rather than the cross-linked polyvinylchloride foam used in the present work – for the blade cores, and flexible-matrix composites for the skin. These modifications will require that the analytical model be modified to account for the stiffness of the rotor control system, however the incorporation of these features will also permit the testing regime to be greatly expanded.

The current blade yoke features a single-point attachment for the blades. While this has proven to be adequate for the present work, this arrangement requires that the lagwise alignment of the blades be manually adjusted prior to testing. This increases the setup time for wind-on tests and may make incorporation of precone and coning flexures difficult. It is therefore recommended that a yoke adapter plate be fabricated that will permit two points of attachment for each blade. This may be accomplished through the use of CNC machining, or with a waterjet and sheet metal
bending equipment; most importantly, this will permit the current gimbal to continue being used. Since the gimbal is a highly labor- and time-intensive fabrication effort, it is recommended that it be modified (rather than replaced) if at all possible.

Switching from a wet-layup fabrication process to one that utilizes pre-impregnated (pre-preg) composites will create a more repeatable process method without the need for a very high level of fabrication skill on the part of the researcher. A correct and accurate estimation of fiber volume fraction is necessary in order to know precisely where within the failure envelope a given part at a specific loading will lie. This issue can be resolved by using pre-preg, for which the fiber fraction is known and is, typically, ideal. The use of pre-preg requires additional storage and logistics concerns, such as refrigeration, that wet-layup processing does not; however, many of these capabilities have become available within the VLRCOE since the conclusion of the present work. If the use of pre-preg is untenable, a more precise process such as vacuum bagging or vacuum-assisted resin transfer molding (VARTM) may be worth examining.

The rotor Lock number can still be greatly increased through the use of lighter-weight fabrics, since the rotor radius of the Gen-3b model is likely to be as large as is feasible within the current facility. The composite blades used in the present work - which used only two layers of carbon fiber - are demonstrably overbuilt based on the centrifugal and bending stresses expected during testing; it is therefore reasonable to assume that blades made from lighter-weight fabric can be safely used within the test facility. The carbon fabric used for the outer skin of the rotor blades in the current work had an areal density of 1.8 oz./sq.yd.; switching to fiberglass woven fabrics can yield significant mass savings for the same geometry: for instance, a 104-style woven glass fabric (0.58 oz./sq.yd.) has an areal density approximately one-third that of the blade fabric used, and the resulting reduction in blade mass (and hence inertia) could easily increase the Lock number. These fabrics were not examined because the current work is meant to serve as an evolutionary step between the Gen-1 model and future wind tunnel models; furthermore, the strength-to-weight ratio of a composite part is unlikely to be maximized unless the fiber fraction can be tightly controlled, which is difficult with a wet-layup but quite a simple matter with pre-preg or VARTM. A change in fabrication techniques may also permit the rotor blade twist rate to be increased to a level that is more conventional for tiltrotors (on the order of 45°/R). This may be difficult using the blade fabrication method presented herein, however the use of a CNC core (possibly made from Spyderfoam, rather than Divinycell) as a plug may mitigate this difficulty.

The flutter stability for a windmilling rotor has been examined both by [32] and the current work. For each case, the presence of the rotor - and its aerodynamic influence - was found to be a destabilizing influence on the wing; however, there has not (as of yet) been any work done in the current facility with wind tunnel models that feature rotors with zero aerodynamic contributions but nonzero inertial and gyroscopic properties. This might be examined by replacing the rotor blades with dowels sized to obtain a specific inertial value; doing so would require the nacelle to be modified to incorporate a mount for a small, hobby-scale motor, but this modification is well within the limits of feasibility. The X1 controller used for the current work can also be used to
power and control this motorized setup.

It has been shown that all three wing modes can be excited through the use of only beamwise excitation if a sufficiently small spar is employed in a composite wing. However, the incorporation of separate excitation jets into the nacelle would permit each wing mode to be excited independently, and would also permit the damping of each mode to be clearly identified when testing a blades-off configuration. This will likely require the nacelle and wing pass-through to be enlarged, in order to accommodate the extra fittings and pneumatic tubing necessary to accomplish this. The semi-monocoque wing design can also be altered slightly to incorporate an interchangeable flexure at the wing root, instead of relying upon the flexure region of the first mode shape of the spar; this may reduce the wing semispan (and allowing the rotor radius to be increased) while increasing the usefulness of the model.

The rotor RPM for the current work was held constant at 1,600 (Gen-2) and 2,000 (Gen-3); however, it is advisable to examine the effect of varying RPM upon the stability of the model; this is especially applicable to the analytical model used for the present work, which incorporates rotor RPM into its eigenanalysis. Additionally, examination into varying – and, specifically, decreasing – the magnitude of the $\delta_3$ angle is recommended. This will require the design and fabrication of a new rotor hub, as space for modifications is limited on the current hub.

The use of a composite wing might need to be reconsidered: while the arrangement was found to be adequate, the reliance on chemically bonding dissimilar materials (and the inherent fatigue of this chemical bond) was a point of concern throughout the present work. Fabricating a plastic wing that can be mechanically fastened to an internal spar might be a better option than relying on a chemical bond. During the present work, the Penn State Learning Factory has increased its capacity for rapid-prototyping parts, and now has new digital material machines that are capable of printing parts with a much smoother finish than what was available for the Gen-1 and Gen-2 wings. It is likely that a future wing printed from a digital material and mechanically fastened to an internal spar would result in a mechanical reliability that is superior to what was achieved for the Gen-3 model. This arrangement would also have the added benefit of an aerodynamically smooth outer mold line and a much sharper trailing edge than what was attainable through hand fabrication.

The goal of this project was to validate the predictions of analytical models that were being developed in-house, and to increase the testing capabilities of the current facility. In short, this project focused on developing the capability to answer “what if” in a flexible and affordable fashion. Therefore, an upper limit or “realistically high” target value for a given model parameter is likely of minimal usefulness for future work efforts, because it artificially constrains the type of model to be tested: what is a “realistically high” parameter today is likely to be an unreasonably low value for a next-generation aircraft. The computational tools that had been previously developed for this work (and the preceding work that created the Gen-1 model) are clearly adequate for predicting the flutter speed of a relatively conventional tiltrotor configuration in cruise flight. Evolving and expanding the capabilities of these tools – along with the necessary experimental validation – will provide a very fertile basis for future research.
Appendix A

System Dynamics

A review of the dynamics of single- and multiple-degree of freedom systems and the vibration of structures is presented in this appendix, including a brief derivation of the equations of motion. The custom filar pendulum used to measure the inertia of various model components is detailed, and the dynamics behind its operation are presented.

A.1 Vibration of Single-Degree of Freedom Systems

The particles that make up a mechanical system must be specified in space using generalized coordinates, which are the minimum group of parameters necessary to specify the configuration of the system; these coordinates are referred to as “generalized” because they are not restricted to being Cartesian and are not required to be measured from an inertial reference frame, although the kinetic and potential energy of the system must be computed with respect to an inertial reference frame. This requires the generalized coordinate to express an object’s velocity and displacement with respect to this inertial reference frame. Generally speaking, a system consisting of $N$ particles will require $3N$ Cartesian coordinates to be fully defined.

The canonical example of structural dynamics is that of the single-degree of freedom (SDOF) mechanical system, shown diagrammatically in Fig. A.1, which is composed of a mass $m$, a spring of stiffness $k$, and a viscous damper (an idealized dissipation device where the force developed is linearly proportional to the relative velocities of its ends) of coefficient $c$. The degrees of freedom of the system refer to the number of coordinates that can be independently varied by some small displacement: in the case of Fig. A.1, only the coordinate $x$ can be varied. A full derivation of the dynamics of SDOF systems is presented in [44], [45], [46], and [47] and is summarized here. The motion of the system as a function of time is defined by its displacement, $x(t)$, and its kinetic energy, $T$, is

$$T = \frac{1}{2} m \dot{x}^2$$  \hspace{1cm} (A.1)
where the overdot indicates the derivation of the displacement with respect to time. The strain energy of the spring is given by

$$V = \frac{1}{2} kx^2$$  \hspace{1cm} (A.2)

The viscous damper may be treated as a dissipative force, $D$, and is defined as

$$D = \frac{1}{2} c\dot{x}^2$$  \hspace{1cm} (A.3)

From the principle of virtual work, the incremental work $\delta W$ that is obtained when a force $f$ moves the particle an incremental displacement of $\delta x$ is

$$\delta W = f \delta x$$  \hspace{1cm} (A.4)

Lagrange’s equations are the differential equations of the system in generalized coordinates, and are typically written in the generalized coordinate $q$ as

$$\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial D}{\partial \dot{q}} + \frac{\partial V}{\partial q} = Q = \frac{\partial (\delta W)}{\partial (\delta x)}$$  \hspace{1cm} (A.5)

where $Q$ is the generalized force applied to the system; for a system in free vibration, $Q = 0$. For most non-rotating structures, the kinetic energy is dependent on the generalized velocities, $\dot{q}_i$, rather than the generalized displacements, $q_i$, and the $\frac{\partial T}{\partial q}$ term can be omitted. Substituting Eqs. A.1 – A.4 into A.5 for the coordinate $x$ yields the ordinary second-order differential equation

$$m\ddot{x} + c\dot{x} + kx = f(t)$$  \hspace{1cm} (A.6)

which is the well-known equation of motion for an SDOF system.

In free vibration, an initial condition is imposed upon the system, and motion occurs without an external force. The time history of this motion shows either oscillatory or non-oscillatory decay for a subcritical system; as the oscillatory condition corresponds to the low damping typically found in aircraft, it is the only case considered here. The system is solved by assuming a motion given by

$$x(t) = X e^{\lambda t}$$  \hspace{1cm} (A.7)

where $X$ is the oscillatory amplitude and $\lambda$ is the characteristic exponent defining the decay. Differentiating with respect to time to obtain $\dot{x}$ and $\ddot{x}$,

$$\dot{x} = \lambda X e^{\lambda t}$$  \hspace{1cm} (A.8)

$$\ddot{x} = \lambda^2 X e^{\lambda t}$$  \hspace{1cm} (A.9)

which can be substituted into Eq. A.6 to obtain
\[ m\lambda^2 X e^{\lambda t} + c\lambda X e^{\lambda t} + kX e^{\lambda t} = 0 \]  
(A.10)

Dividing through by \( X e^{\lambda t} \) yields the quadratic equation

\[ m\lambda^2 + c\lambda + k = 0 \]  
(A.11)

which is known as the characteristic equation of the system. The solution of the characteristic equation produces two complex roots,

\[ \lambda_{1,2} = -\frac{c}{2m} \pm i\sqrt{\frac{k}{m} - \frac{c^2}{2m}} \]  
(A.12)

which may be nondimensionalized as

\[ \lambda_{1,2} = -\zeta\omega_n \pm i\omega_n \sqrt{1 - \zeta^2} = -\zeta\omega_n \pm i\omega_d \]  
(A.13)

The damping ratio, \( \zeta \), is expressed as a proportion of the critical damping, \( c_{\text{crit}} \), which is the damping value at which the motion of the system becomes non-oscillatory.

The characteristic equation contains two roots, and the free vibration motion is expressed as

\[ x(t) = X_1 e^{\lambda_1 t} + X_2 e^{\lambda_2 t} \]  
(A.14)

Because the displacement of the system must be a real quantity, \( X_1 \) and \( X_2 \) must be complex conjugate pairs. Substituting Eq. A.13 into Eq. A.14 and simplifying, the motion becomes

\[ x(t) = e^{-\zeta\omega_n t} \left[ A_1 \sin(\omega_d t) + A_2 \cos(\omega_d t) \right] = A e^{-\zeta\omega_n t} \sin(\omega_d t + \phi) \]  
(A.15)

where \( \phi \) is the phase angle of the system. \( A_1 \) and \( A_2 \) are unknown and must be determined from the initial conditions for displacement and velocity. This motion is referred to as “underdamped,” and is characterized by the exponentially-decaying sinusoid shown in Fig. A.2.
Figure A.1: Diagram of an SDOF system.

Figure A.2: Oscillatory motion of underdamped SDOF system.
A.2 Vibration of Multi-Degree of Freedom Systems

The dynamics of the SDOF system may now be extended to a multi-degree of freedom (MDOF) system. The classical example of an MDOF system is that of the “mass train,” which is shown in Fig. A.3 as a two-degree of freedom system; it is demonstrable that any system described by multiple degrees of freedom will have its governing equation in an identical form to the classical mass train, albeit with different parameters. The mass train consists of masses $m_1, m_2, \cdots, m_n$, springs of stiffnesses $k_1, k_2, \cdots, k_n$, and viscous dampers of coefficients $c_1, c_2, \cdots, c_n$. The time-varying displacements of each of the masses is defined by $x_1(t), x_2(t), \cdots, x_n(t)$, and time-varying forces $f_1(t), f_2(t), \cdots, f_n(t)$ may be applied to the masses (these forces are omitted from Fig. A.3).

Although there are now multiple degrees of freedom (and therefore multiple equations of motion), the work and energy of the system are still additive scalars. For the system shown in Fig. A.3, the kinetic energy is given by

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$ (A.16)

The strain energy of the springs is dependent upon the relative extension or compression of each spring, and is given by

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2$$ (A.17)

Similarly, the damping terms are dependent upon the relative velocities of the masses,

$$D = \frac{1}{2}c_1\dot{x}_1^2 + \frac{1}{2}c_2(\dot{x}_2 - \dot{x}_1)^2$$ (A.18)

Lagrange’s equation (Eq. A.5) for an MDOF system can now be written as

$$\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} + \frac{\partial V}{\partial x_i} = Q = \frac{\partial (\delta W)}{\partial (\delta x_i)}$$ (A.19)

for $i = 1, 2, \cdots, n$. Using Lagrange’s equations will automatically yield as many equations of motion as there are degrees of freedom in the system, although some of these equations may be coupled. For the example case of free vibration with $n = 2$ degrees of freedom, as shown in Fig. A.3, the equations of motion are

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = 0$$ (A.20)

$$m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - k_2x_1 + k_2x_2 = 0$$ (A.21)

These equations are represented more clearly in matrix form as
\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2 \\
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} = 0
\] (A.22)

or as \([M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = 0\), where \([M],[C]\), and \([K]\) are the mass, damping, and stiffness matrices, respectively. Note that in this example, the diagonality of the mass matrix demonstrates that the system is uncoupled inertially, whereas the damping and stiffness matrices are coupled.

The equations of motion given in Eq. A.22 are in global coordinates; these may be transformed into modal coordinates, wherein coupling is absent, through the use of a modal matrix. As with an SDOF system, the natural frequencies of an MDOF system are found as solutions to the free vibration motions described by

\[x(t) = [X]\sin(\omega t)\] (A.23)

where \([X]\) is the vector of motion amplitudes. Substitution yields

\[[K - \omega^2 M][X] = 0\] (A.24)

For a nontrivial solution to Eq. A.24, the bracketed matrix must be singular (viz. have a determinant equal to zero). Setting this determinant equal to zero,

\[|K - \omega^2 M| = 0\] (A.25)

yields an \(n\)th-order polynomial in \(\omega^2\). The solution of this polynomial yields a vector of eigenvalues, which are roots \(\omega_i\), with \(i = 1, 2, \ldots, n\). These are the undamped natural frequencies of the system, and there exists one undamped natural frequency per degree of freedom of the system. The response of the system for each natural frequency is characterized by the displacement vector \(X_i\), and is given as the solution of

\[[K - \omega_i^2 M] X_i = 0\] (A.26)

for \(i = 1, 2, \ldots, n\). Equation A.26 yields the characteristic vectors of the system, although only the ratios of the vector elements are necessary. These vectors are the eigenvectors of the system, also known as the undamped mode shapes, which show the relative displacement of the physical coordinates when the system is vibrating at its corresponding undamped natural frequency. The eigenvectors are concatenated into a modal matrix, which contains each of the eigenvectors as columns:

\[
\phi = \begin{bmatrix}
  X_1 & X_2 & \cdots & X_n
\end{bmatrix}
\] (A.27)

The modal mass, damping, and stiffness matrices (as well as the modal force vector) may be
obtained via multiplication of the modal matrix and the corresponding global matrix:

\[
\begin{align*}
\vec{m} &= [\phi]^T [M] [\phi] \\
\vec{c} &= [\phi]^T [C] [\phi] \\
\vec{k} &= [\phi]^T [K] [\phi] \\
\vec{f} &= [\phi]^T [F] [\phi]
\end{align*}
\] (A.28)

The modal mass and stiffness matrices are uncoupled (viz. matrices are diagonal), with the diagonal elements equal to the modal mass \( m_i \) and modal stiffness \( k_i \) of the \( i^{th} \) mode. Diagonalization is possible because the vibration modes are all orthogonal with respect to the mass and stiffness matrices (Ref. 38). The nature of the modal damping matrix is less obvious: provided that the damping is proportional (viz. that the global damping matrix can be written as a linear combination of the global mass and stiffness matrices), then the modal damping matrix \( \vec{c} \) will also be diagonal; otherwise, the modal damping matrix will still be coupled [45]. The assumption of proportional damping for preliminary analysis is normal, and thus all modal matrices shown in Eqs. A.28 will be fully decoupled.

Note that the use of the modal matrix permits the MDOF system to be written as a collection of SDOF equations of motion:

\[
m_i \ddot{q}_i + c_i \dot{q}_i + k_i q_i = f_i(t)
\] (A.29)

for the \( i^{th} \) mode in generalized coordinate \( q \). These may be rewritten in nondimensional form:

\[
\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q = \frac{f_i(t)}{m_i}
\] (A.30)
Figure A.3: Diagram of an MDOF system.
A.3 Vibration of Structures

A structure may be defined as a collection of particles that can be analyzed as either a continuous or a discretized system. While some structures, such as uniform beams, can be treated as continuous systems, most are analyzed as discrete MDOF systems. This should not be surprising: if the discrete model of the system is made increasingly finer, by increasing the number of degrees of freedom, then a continuous system is obtained as $n \to \infty$. An eigenanalysis can still be performed, which for a continuous system will yield an infinite set of eigenvalues and corresponding eigenvectors obtained from differential (or integral) equations, rather than from the algebraic equations of the discrete systems presented in the previous section. For a finite body, an infinite number of discrete natural frequencies will be obtained. It may be argued that finite element methods combine both continuous and discretized analysis approaches: continuous within an individual element, but discretized overall.

In the previous section, the vibration of an MDOF system was characterized by the collection of a given number of the same repeated elements: springs, masses, and dampers. The simplifying assumptions were that the springs were massless and the masses had zero compliance; thus, the spring stiffness was the only parameter of interest, and the masses were incapable of deforming under load. This permitted the derivation of a mathematical model that considered $n$ coupled ordinary differential equations representing completely discretized system parameters. However, real structures are not perfectly rigid, nor are discrete masses and springs always present: every physical portion of the structure will likely possess both mass and elasticity, which may vary from one location in the structure to the next. An analysis of uniform beams and beams with lumped end-masses is presented first, as these analyses form the basis of the finite element methods used to analyze the composite rotor blades used for the Gen-2 and Gen-3 wind tunnel models.

A uniform cantilevered beam of length $L$ is shown in Fig. A.4. From elementary beam theory, the following apply for cases of static loading:

$$EI \left( \frac{d^2 y}{dx^2} \right) = M(x) \quad (A.31)$$

$$EI \left( \frac{d^4 y}{dx^4} \right) = S(x) \quad (A.32)$$

$$EI \left( \frac{d^4 y}{dx^4} \right) = P(x) \quad (A.33)$$

For cases of no external force, the load distribution will be simply the inertial loading due to the mass per unit length of the beam, taken to be $\mu$:

$$P(x) = -\mu \left( \frac{\partial^2 y}{\partial t^2} \right) \quad (A.34)$$

In this case, $y$ is a function of both position, $x$, and time, $t$. From Eqs. A.33 and A.34,
\[ EI \left( \frac{\partial^4 y}{\partial x^4} \right) = -\mu \left( \frac{\partial^2 y}{\partial t^2} \right) \quad (A.35) \]

Because there is no applied force, only simple harmonic motion is possible:

\[ y = \overline{y} \sin(\omega t) \quad (A.36) \]

\[ \frac{\partial^2 y}{\partial t^2} = -\omega^2 \overline{y} \sin(\omega t) \quad (A.37) \]

where \( \overline{y} \) is the mode shape factor and is a function solely of \( x \). The maximum occurs when \( \sin(\omega t) \) is equal to unity, and thus Eq. A.35 becomes

\[ \frac{\partial^4 y}{\partial x^4} = -\frac{\mu \omega^2}{EI} \overline{y} = \beta^4 y \quad (A.38) \]

where the circular natural frequency, \( \omega_n \), is found to be

\[ \beta^4 = \frac{\omega_n}{\sqrt{\frac{EI}{\mu}}} \quad (A.39) \]

The general solution of Eq. A.39 is

\[ y = A \sin(\beta x) + B \cos(\beta x) + C \sinh(\beta x) + D \cosh(\beta x) \quad (A.40) \]

The constants \( A, B, C, D \) are dependent upon the boundary conditions of the beam. Applying these boundary conditions yields a set of simultaneous equations equal to zero, the roots of which are given by the determinant. These roots are used to obtain the natural frequency of the \( i^{th} \) mode:

\[ \omega_i = \beta_i^2 \sqrt{\frac{EI}{\mu}} \quad (A.41) \]

with the value of \( \beta_i \) dependent on the boundary conditions of the beam.

Because the cantilevered beam serves as a simplified representation of two key components of the wind tunnel model (viz. the wing and the rotor blades), it is examined in greater detail here. A uniform cantilevered beam of length \( L \) has the following boundary conditions. At \( x = 0 \),

\[ \overline{y} = 0 \]
\[ \frac{d\overline{y}}{dx} = 0 \quad (A.42) \]
At \( x = L \),

\[
\frac{d^2 \tilde{y}}{dx^2} = 0
\]

\[
\frac{d^3 \tilde{y}}{dx^3} = 0
\]

Substituting these boundary conditions into Eq. A.40,

\[
A = -C
\]

\[
B = -D
\]

\[
C \left( \sin(\beta L) + \sinh(\beta L) \right) + D \left( \cos(\beta L) + \cosh(\beta L) \right) = 0
\]

\[
C \left( \cos(\beta L) + \cosh(\beta L) \right) + D \left( \sinh(\beta L) - \sin(\beta L) \right) = 0
\]

To find nontrivial solutions to Eqs. A.45, the determinant must equal zero:

\[
\begin{vmatrix}
\sin(\beta L) + \sinh(\beta L) & \cos(\beta L) + \cosh(\beta L) \\
\cos(\beta L) + \cosh(\beta L) & \sinh(\beta L) - \sin(\beta L)
\end{vmatrix} = 0
\]

which simplifies to

\[
1 + \cos(\beta L) \cosh(\beta L) = 0
\]

This equation can be solved numerically for \( \beta L \). The first four roots are 1.8751, 4.694, 7.855, and 11.00; these roots can then be substituted into Eq. A.41 to find the natural frequencies of the beam.

The wing of the wind tunnel models may be represented in simplified form as a cantilevered beam with an end-mass (Fig. A.5). Using this assumption, the nacelle components and rotor are all assumed to be part of a lumped end-mass located at the tip of the wing spar. Because the first natural frequency of the wing is used to find the stiffness value used as an input to the prediction method, it is only this fundamental frequency that needs to be considered. First, the beam is considered without the end-mass, and the governing differential equation (Eq. A.35) remains unchanged. The displacement of the wing is assumed to be a quarter-cosine wave:
The assumed mode meets all of the boundary conditions except for zero shear at the free end; however, it will be seen that this deficiency does not affect the solution for the fundamental frequency. Using the Rayleigh method to find the strain energy of the beam,

\[
y(x) = y_0 \left[1 - \cos\left(\frac{\pi x}{2L}\right)\right]
\]

\[
\frac{dy}{dx} = y_0 \left(\frac{\pi}{2L}\right) \sin\left(\frac{\pi x}{2L}\right)
\]

\[
\frac{d^2y}{dx^2} = y_0 \left(\frac{\pi}{2L}\right)^2 \cos\left(\frac{\pi x}{2L}\right)
\]

\[
\frac{d^3y}{dx^3} = -y_0 \left(\frac{\pi}{2L}\right)^3 \sin\left(\frac{\pi x}{2L}\right)
\]

(A.48)

By substitution,

\[
V = \frac{EI}{2} \int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx
\]

(A.49)

This simplifies to

\[
V = \frac{\pi^4 y_0^2}{64} \left[\frac{EI}{L^3}\right]
\]

(A.50)

Similarly, the Rayleigh method is used to find the kinetic energy of the beam:

\[
T = \frac{1}{2} \mu \omega_n^2 \int_0^L y^2 dx
\]

(A.52)

\[
T = \frac{1}{2} \mu \omega_n^2 \int_0^L y_0 \left[1 - \cos\left(\frac{\pi x}{2L}\right)\right]^2 dx
\]

(A.53)

This simplifies to

\[
T = \frac{1}{4} \mu \omega_n^2 y_0^2 L \left[3 - \frac{8}{\pi}\right]
\]

(A.54)

Equating the strain energy and kinetic energy terms,

\[
\frac{\pi^4 y_0^2}{64} \left[\frac{EI}{L^3}\right] = \frac{1}{4} \mu \omega_n^2 y_0^2 L \left[3 - \frac{8}{\pi}\right]
\]

(A.55)

Rearranging and solving for the natural frequency,

\[
\omega_n = \sqrt{\frac{\pi^4 \left(\frac{EI}{L^3}\right)}{16 \mu L \left(3 - \frac{8}{\pi}\right)}} = 2\pi f_n
\]

(A.56)
where \( f_n \) is the fundamental frequency, in Hertz, of the beam. Thus,

\[
f_n \approx \frac{3.664}{2\pi L^2} \sqrt{\frac{EI}{\mu}}
\]  

(A.57)

From standard handbooks on vibrations such as [46], the stiffness of a uniform cantilevered beam is \( k = \frac{3EI}{L^3} \), and the effective mass at the end of the beam is

\[
m_{\text{eff}} = \frac{k}{(2\pi f_n)^2} = \frac{3EI}{\mu (13.43)EI}
\]  

(A.58)

Or, simply, \( m_{\text{eff}} \approx 0.2234\mu L \). The total mass of the beam, \( M \), is then

\[
M \approx 0.2234\mu L + m
\]  

(A.59)

Recalling that \( \omega_n = \sqrt{\frac{k}{m}} \), the fundamental frequency of a cantilevered beam with an end-mass can then be approximated as

\[
f_n \approx \frac{1}{2\pi} \sqrt{\frac{3EI}{(0.2234\mu L + m) L^3}}
\]  

(A.60)

Table lookups of the characteristic equation and the roots of other common end-fixity conditions (e.g. pin-ended, free-free, etc.) are typically given in reference texts on structural dynamics, such as [44], [46], and [47].

It should be noted that the fundamental frequency calculation given in Eq. A.60 is only an approximation, and neglects to take into account the moment of inertia of the end-mass. Taking into account the rotational inertia, \( J_0 \), of the end mass changes the boundary conditions at the end of the beam slightly, and the moment and shear at \( L \) become

\[
EI \left( \frac{d^2y}{dx^2} \right) = J_0 \left( \frac{d^3y}{dxdl^2} \right)
\]

\[
EI \left( \frac{d^3y}{dx^3} \right) = m \left( \frac{d^2y}{dl^2} \right)
\]

(A.61)

both of which can be seen to be time-dependent. The decision to neglect the rotational inertia of the end-mass was justified given the compact nature of the assembly housed at the wing tip, and also has the added benefit of simplifying the analysis.
Figure A.4: Uniform cantilevered beam.

Figure A.5: Uniform cantilevered beam with end mass.
A.4 Filar Pendulum Design and Fabrication

The rotor Lock number is an important factor governing rotor dynamics, and thus plays an important role in the dynamic behavior of the wind tunnel model. While the aerodynamic and geometric contributions to Lock number are easily calculated, the flap inertia is more difficult to measure. The use of SolidWorks to calculate the mass properties of the rotor can be useful to obtain initial estimates of the rotor inertia, but uncertainty in the estimates can have significant effects on the correlation between the computational predictions and experimental results. A filar pendulum was designed and fabricated to determine the actual rotational inertia of the rotor assembly. For a gimbaled rotor, the rotational and flap axes are coincident and the mass distributions about these axes are similar, so the rotational inertia of the rotor can be assumed to be very close to its flap inertia.

The filar pendulum is a torsional pendulum used for the experimental measurement of the mass moment of inertia of a given object. Typically, this type of pendulum is named by the number of strings used to support the test object - viz. bifilar, trifilar, etc.; however, the generic name “filar pendulum” is used here. A detailed description of the filar pendulum’s use is given by [39] and [40], with a more complex model for measuring nonlinear systems given by [41]; the derivation presented here assumes small perturbations and ignores aerodynamic drag on the pendulum and the test object. The inertial contributions of the pendulum strings is also ignored, which is valid for strings that are very thin and lightweight relative to the test object.

Consider a mass of arbitrary shape mounted to a filar pendulum such that the c.g. of the test object is equally spaced between the pendulum strings (Fig. A.6); the distance between the pendulum strings is given as $D$. When subjected to a small angular perturbation $\theta$, the pendulum base will rotate by the amount $\theta$ and the pendulum strings will also twist by a small amount $\phi$. Letting $r = D/2$ be defined as the half-distance between the pendulum strings and $h$ be defined as the length of the strings (which are assumed to be massless and identical to each other), the relationship between the string twist angle and the perturbation angle is

$$ r\theta = h\phi \Rightarrow \phi = \frac{r\theta}{h} \quad (A.62) $$

As the test object is rotated, the rotation of the strings will cause it to climb a distance $y$ such that

$$ y = h - h \cos (\phi) = h [1 - \cos(\phi)] \quad (A.63) $$

With a known test object mass, $m$, and an unknown test object mass moment of inertia, $I$, the potential energy of the system is

$$ U = mgy = mgh [1 - \cos(\phi)] \quad (A.64) $$

and its kinetic energy is

$$ T = \frac{1}{2} I\dot{\theta}^2 \quad (A.65) $$
If aerodynamic drag is neglected and the object is assumed to be experiencing pure rotation, the system experiences zero damping; the free vibration of the system in the generalized coordinate $q$ is then

$$\frac{\partial}{\partial t} \left[ \frac{\partial T}{\partial \dot{q}} \right] + \frac{\partial U}{\partial q} = 0$$  \hspace{0.5cm} (A.66)

where the generalized coordinate $q$ is taken to be the initial rotation angle $\theta$:

$$\frac{\partial}{\partial t} \left[ I \ddot{\theta} \right] + \frac{\partial}{\partial \theta} \left[ mgh - mgh \cos \left( \frac{r\theta}{h} \right) \right] = 0$$  \hspace{0.5cm} (A.67)

and the equation of motion is therefore

$$I \ddot{\theta} + mgr \sin \left( \frac{r\theta}{h} \right) = 0$$  \hspace{0.5cm} (A.68)

Rearranging,

$$\ddot{\theta} + \frac{mgr}{h} \theta = 0$$  \hspace{0.5cm} (A.69)

For small perturbations, $\sin(\theta) \approx \theta$:

$$\ddot{\theta} + \frac{mgr^2}{h} \theta = 0$$  \hspace{0.5cm} (A.70)

which is of the standard form $\ddot{\theta} + \omega_n^2 \theta = 0$. Thus,

$$\omega_n^2 = \frac{mgr^2}{hI}$$

$$\frac{(2\pi f)^2}{(2\pi f)^2} = \frac{mgr^2}{hI}$$

$$\frac{4\pi^2}{T^2} = \frac{mgr^2}{hI}$$  \hspace{0.5cm} (A.71)

where $T$ is the period of oscillation (in seconds), and is defined as the inverse of the oscillation frequency. Rearranging and solving for the rotational inertia,

$$I = \frac{mgr^2T^2}{4\pi^2h}$$  \hspace{0.5cm} (A.72)

Equation A.72 will hold for any filar pendulum of $n$ vertical support wires, so long as each support wire has a length of $h$. The time required for the bare pendulum plate to oscillate a given number of times can be recorded to calculate the inertia of the pendulum plate. The combined inertia of the plate and test object is then calculated in the same fashion, and the inertia of the plate is subtracted from this value to obtain the inertia of the test object:

$$I_{\text{object}} = I_{\text{plate+object}} - I_{\text{plate}}$$  \hspace{0.5cm} (A.73)
The rotational inertia for the composite pendulum plate used for inertia testing of wind tunnel model components was measured to be \(1.881 \times 10^{-3} \text{kgm}^2\).

The filar pendulum used for this project was fabricated using an end-grain balsa core and uncoated Toray 1k plain weave face sheets. End-grain balsa (0.375 in. thick) was selected for its machinability (specifically, its resistance to splintering during drilling) and light weight. Six of the Toray sheets (thickness of 0.005 in. each) were applied to each face of the core, and an MGS L285/H285 laminating resin mix (ratio of 5:2, by mass, of resin to hardener) was used for the matrix. Mylar sheets sprayed with Polytek PolEase 2300 were used as release films, and an automotive lead-acid battery was used to compress the plate during cure. The entire assembly was laid up on a laboratory bench top and cured at room temperature for sixteen hours.

The cured plates were machined to a 6.000 x 6.000 in. square using a Bridgeport three-axis knee mill and a Niagara Cutter CVD diamond end mill at 4,000 RPM. A shop vacuum with a flexible hose and crevice attachment was used to contain any carbon fiber particulates released during the machining process; the use of the vacuum is critical, as composite particulates are usually very good electrical conductors (and can short out machine controllers) and are typically respirable, and can thus cause severe pulmonary damage if inhaled. A Dewalt CVD diamond drill bit was used to drill five 0.250 in. plain-bore holes into each plate; four of these holes served as mounts for the plates, and the central hole serves as a mount on the top plate and a pass-through on the bottom plate (Fig. A.7a). An anchor plate fabricated from 6061-T6 billet (Fig. A.7b) was bonded to the bottom plate using Kwik Bond thirty-minute epoxy and cured overnight. This anchor plate allows the user to slip-fit a hollow tube to the bottom plate, which will constrain the bottom plate to purely rotational motion. While this arrangement is not as accurate as using a bearing to constrain the motion of the plate, it requires no maintenance and is not susceptible to contamination.

The pass-through on the bottom plate is necessary to secure the rotor assembly during inertia testing; without the pass-through hole, the rotor assembly would need to be loosely placed on the bottom plate and balanced on its gimbal. In this arrangement, the only thing keeping the rotor assembly in alignment is the friction between the hub assembly and the composite plate; if the rotor is disturbed, the shaft will tilt from the vertical, which may cause the pitch of the individual blades to change due to the pitch-flap coupling of the hub. This change in blade pitch can cause the rotor to slide off of the plate, potentially damaging the components.

Four 1/4-20 threaded eye bolts were mounted to the plain-bore holes on pendulum each plate, and were secured with a threaded nut; a fifth eye bolt on the top plate allows the pendulum to be mounted to a bench top. The plates were connected with monofilament line (30 lb. test). The eye bolts and nuts are retained in their mounting holes purely by the tension in the monofilament lines, and may be adjusted as needed to ensure that the plates are level prior to performing inertia testing. Each plate was sprayed with water-resistant clear coat for protection from moisture and humidity; this step reduces the likelihood of the balsa cores warping due to moisture absorption. The pendulum was proof-tested to 50 lbs., which was more than 100 times the weight of any component or assembly used in the wind tunnel models.
Figure A.6: Test object mounted to filar pendulum, from [39].

Figure A.7: Filar pendulum plate, 6.000 x 6.000 x 0.440 in.
Appendix B

Mechanics of Composite Structures

The use of composites was central to the design, fabrication, and testing of the Gen-2 and Gen-3 wind tunnel models; as such, a brief overview of the basic principles governing the mechanics of composites is in order. This appendix covers the basics of composites, beginning with the mechanics of laminae and progressing to laminate plate theory and sandwich panel theory. For a more detailed discussion of composite materials (including strength criteria, discontinuous laminates, mechanical testing, and fracture), the reader is directed to [51] – [53]. An excellent reference of composites forming is [54], although this reference focuses mostly on industrial techniques, rather than the small-scale processes used for this work.

For the present work, hygrothermal effects and viscoelastic behavior were not considered. Material properties for strength predictions were taken directly from the manufacturer specification sheets when needed, although it is understood that these specifications are highly dependent on the fiber volume fraction of the composite and the cure process used; thus, the values presented by the specification sheets may be optimistic when compared to the material properties of the actual parts. Because all composite components were fabricated using wet layup and room-temperature cure, material properties are assumed to be on the order of 40% of the manufacturers’ published values. Examination of expected wind tunnel model air loads are compared with three common strength criteria – maximum stress, maximum strain, and Tsai-Hill – to ensure that the composite components would not fail during testing.

B.1 Mechanics of Continuous Fiber-Reinforced Laminae

An engineering composite is composed of two or more distinct constituents synthesized to obtain measurable and controllable mechanical properties that are superior to those of the constituent materials. The most common composite is composed of continuous-diameter textile fibers bonded
together by a matrix, which is itself selected based upon the operational and environmental constraints of the finished part. As with isotropic materials, the primary interest with composites is defining how stress relates to strain, via the constitutive equations.

The fundamental element of any composite structure is the lamina (pl. laminae), which consists of a single layer of fiber material bonded by a matrix. For a simplified mechanics analysis, a lamina comprised of constant-diameter, parallel, unidirectional fibers (“uni”) is considered first. In the most general case of a lamina’s stress-strain relationship, a composite element (Fig. B.1) experiences a three-dimensional state of stress, each component of which is related to the nine strain components of the element by a function $f_{ij}$, which may be nonlinear:

$$\sigma_{ij} = f_{ij}(\epsilon_{ij}) \tag{B.1}$$

for $i, j = 1 - 3$. For a linear-elastic material, the stress-strain relationship is given by

$$\{\sigma\} = [C] \{\epsilon\} \tag{B.2}$$

where the stress vector, $\{\sigma\}$, and the strain vector, $\{\epsilon\}$, are related by the fully-populated 9x9 stiffness matrix, $[C]$. Equation B.2 is the generalized Hooke’s law for anisotropic materials. Any book on elementary solid mechanics, such as [51], will demonstrate that stresses and strains are symmetric - viz. $\sigma_{ij} = \sigma_{ji}$ and $\epsilon_{ij} = \epsilon_{ji}$ – which reduces the number of stress and strain components to six. The stiffness matrix in Eq. B.2 may thus be reduced to a 6x6 matrix. If the lamina is assumed to be in a two-dimensional state of stress, the stress-strain relationship is simply

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} \tag{B.3}$$

where the compliances, $S_{ij}$, are directly related to the material properties of the lamina:

$$S_{11} = \frac{1}{E_L}$$
$$S_{22} = \frac{1}{E_T}$$
$$S_{12} = S_{21} = -\frac{\nu_{21}}{E_T} = -\frac{\nu_{12}}{E_L}$$
$$S_{66} = \frac{1}{G_{LT}} \tag{B.4}$$

The subscripts $L$ and $T$ in Eqs. B.4 refer, respectively, to the material properties of the lamina.
along and transverse to the fibers. Rearranging Eqs. B.4,

\[
\begin{align*}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} &=
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & 2Q_{66}
\end{bmatrix}
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\gamma_{12}
\end{bmatrix}
\end{align*}
\]  
(B.5)

Note that a factor of two is introduced to the tensor shear strain term. The components \(Q_{ij}\) of the stiffness matrix are defined as

\[
\begin{align*}
Q_{11} &= \frac{E_L}{1 - \nu_{12}\nu_{21}} \\
Q_{12} &= Q_{21} = \frac{\nu_{12}E_T}{1 - \nu_{12}\nu_{21}} \\
Q_{22} &= \frac{E_T}{1 - \nu_{12}\nu_{21}} \\
Q_{66} &= G_{LT}
\end{align*}
\]  
(B.6)

The material properties of the lamina will vary as the fiber orientation is misaligned relative to the structural axis (Fig. B.2). In certain cases, such as with unidirectional fiber, this variation in material properties can be extreme, as seen in Fig. B.3. Therefore, it is essential to know the “off-axis” properties of the lamina. The properties for a generally orthotropic lamina may be obtained from Eqs. B.6 through the use of a rotation matrix:

\[
[T] = 
\begin{bmatrix}
\cos^2(\theta) & \sin^2(\theta) & -2\cos(\theta)\sin(\theta) \\
\sin^2(\theta) & \cos^2(\theta) & 2\cos(\theta)\sin(\theta) \\
\cos(\theta)\sin(\theta) & -\cos(\theta)\sin(\theta) & \cos^2(\theta) - \sin^2(\theta)
\end{bmatrix}
\]  
(B.7)

where \(\theta\) is the angle between the fiber axis and the structural axis. For the present work, the sense of \(\theta\) is considered positive when there is a counter-clockwise rotation from the structural axis to the fiber axis (Fig. B.4). The stiffness matrix for a generally orthotropic lamina is then defined as

\[
[\mathcal{Q}] = [T]^{-1} [Q] [R] [T] [R]^{-1}
\]  
(B.8)

where the matrix

\[
[R] = 
\begin{bmatrix}
1 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]  
(B.9)

is used to transform engineering shear strain to tensor shear strain. Thus, the stress-strain relationship for a generally orthotropic lamina is

\[
\begin{bmatrix}
\sigma_x \\
\sigma_x \\
\tau_{xy}
\end{bmatrix} = [\mathcal{Q}]
\begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]  
(B.10)
Figure B.1: Three-dimensional state of stress, from [51].

Figure B.2: Generally-orthotropic lamina with fiber and structural axes shown, from [51].
Figure B.3: Variation of modulus with rotation angle.

Figure B.4: Sign convention for positive rotation angle of fibers relative to structural axes.
B.2 Strength Criteria of Lamina

The strength of a composite is derived from its fibers, but this strength is highly directional in nature. The longitudinal tensile strength of a fiber, for instance, is typically much greater than its compressive or transverse strength. The in-plane shear strength is another parameter that is usually independent of the others. These lamina properties form the basis of a simplified lamina strength analysis and multiaxial strength criteria; the strength criteria may then be used to ensure the structural integrity of the composite components fabricated.

Popular multiaxial composite failure criteria are predicated on the phenomenological methods of the World Wide Failure Exercise (detailed in [51]) and use the concept of failure surfaces or failure envelopes of stress and strain. Hypothetically, any combination of stresses that fall within these failure surfaces or failure envelopes will not cause failure; however, it is demonstrable that different models will be conservative to different degrees in the calculation of their failure envelope. Generally speaking, very conservative estimates of the failure boundary will result in very conservatively-designed structures, which will be heavier than required. The three most common strength criteria used are the maximum stress, maximum strain, and quadratic interaction criteria. Each of these criteria were used to determine whether the composite parts fabricated for the present work would be able to safely operate within the wind tunnel during testing.

B.2.1 Maximum stress criterion

The maximum stress criterion predicts failure of a lamina when any of the stresses along the material axes exceeds the corresponding strength. Meeting this criterion requires satisfaction of the following inequalities:

\begin{align}
-s_L^{-} < \sigma_1 < s_L^{+} \\
-s_T^{-} < \sigma_2 < s_T^{+} \\
|\tau_{12}| < s_{LT}
\end{align}

(B.11)

where the superscripts (+) and (−) denote tensile and compressive strengths, respectively. The strength values are typically obtained from the manufacturer or other tabulated source. For the maximum stress criterion, only the magnitude of the in-plane shear stress is of interest, although for other cases of off-axis loading the shear strength may depend on the direction of the loading. The failure envelope for the maximum stress criterion is given by a rectangle, as seen in Fig. B.5. As the strengths along the material axes are the inputs to this criterion, this criterion’s performance for stresses applied along the fiber axes is excellent; however, the lack of stress interaction may yield unacceptable correlations for cases of multiaxial stress.
B.2.2 Maximum strain criterion

The maximum strain criterion was proposed by [56] as an extension of maximum normal strain theory for isotropic materials. The strain criterion predicts failure when any strain along the material axes exceeds the associated ultimate strain. The inequalities that must be satisfied,

\[-\varepsilon_L^- < \varepsilon_1 < \varepsilon_L^+\]
\[-\varepsilon_T^- < \varepsilon_2 < \varepsilon_T^+\]
\[|\gamma_{12}| < \varepsilon_{LT}\]

are very similar to those of the maximum stress criterion. The \(\varepsilon\) values in Eqs. B.12 are the engineering ultimate strains, and are defined as

\[s_L^{(+)} = E_L \varepsilon_L^{(+)}\]
\[s_T^{(+)} = E_T \varepsilon_T^{(+)}\]
\[s_L^{(-)} = E_L \varepsilon_L^{(-)}\]
\[s_T^{(-)} = E_T \varepsilon_T^{(-)}\]
\[s_{LT} = G_{LT} \varepsilon_{LT}\]

As with the maximum stress criterion, the maximum strain criterion ignores the sense of the in-plane shear strain. The failure envelope of the maximum strain criterion in \((\sigma_1, \sigma_2)\) space is a parallelogram, as shown by the blue parallelogram in Fig. B.5.
B.2.3 Quadratic interaction criteria

Quadratic interaction criteria differ from the previous two strength criteria by their inclusion of interaction between stress components; these criteria tend to yield elliptical failure envelopes, which define the theoretical maximum stress that a lamina can experience before failure. The Tsai-Hill criterion is one of the most popular of these quadratic criteria, and was developed from Hill’s work in modifying the von Mises criterion for use with anisotropic materials. The extension of Hill’s theories to include transversely isotropic laminae was suggested by [57], and the resulting equation is referred to as the Tsai-Hill criterion:

\[
\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1 \sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} = 1 \tag{B.14}
\]

Failure is avoided for instances where the left-hand side of Eq. B.14 is less than unity.

The Tsai-Hill failure envelope is shown in Fig. B.5 by the red elliptical envelope. The Tsai-Hill criterion can be extended to the more complex Tsai-Wu criterion, which was not used for the analysis of the composite components fabricated for the present work, but is included here for the sake of completeness. The Tsai-Wu criterion is based on a series of uniaxial strength tests, and biaxial strength tests with \( \sigma_1 = \sigma_2 = \sigma \):

\[
(F_1 \sigma_1 + F_2 \sigma_2) + (F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \tau_{12}^2 + 2F_{12} \sigma_1 \sigma_2) = 1 \tag{B.15}
\]

where the \( F \) terms are derived from the lamina’s strength properties as follows:

\[
\begin{align*}
F_1 &= \frac{1}{s_L^{(+)}_L} - \frac{1}{s_L^{(-)}_L} \\
F_2 &= \frac{1}{s_T^{(+)}_T} - \frac{1}{s_T^{(-)}_T} \\
F_{66} &= \frac{1}{s_{LT}^2} \\
F_{11} &= \frac{1}{s_L^{(+)}_L s_L^{(-)}_L} \\
F_{22} &= \frac{1}{s_T^{(+)}_T s_T^{(-)}_T} \\
F_{12} &= \frac{1}{2\sigma^2} \left[ 1 - \sigma \left( \frac{1}{s_L^{(+)}_L} - \frac{1}{s_L^{(-)}_L} + \frac{1}{s_T^{(+)}_T} - \frac{1}{s_T^{(-)}_T} \right) + \sigma^2 \left( -\frac{1}{s_L^{(+)}_L s_L^{(-)}_L} - \frac{1}{s_T^{(+)}_T s_T^{(-)}_T} \right) \right]
\end{align*}
\]
Figure B.5: Plot of maximum stress (black), maximum strain (blue), and Tsai-Hill (red) criteria for Toray T300.
B.3 Classical Laminated Plate Theory

An understanding of lamina behavior is the foundation for composite mechanics, but unidirectional laminae are not always useful structures on their own. Realistic structures are likely to be composed of several laminae arranged as plies bonded together into laminates, with each ply oriented at an angle relative to the structural axis. The nearly limitless combinations of ply lamina materials, ply orientations, stacking sequences, and discrete reinforcements gives composites an inherently higher level of design flexibility relative to isotropic material.

Classical laminated plate theory (CLPT) permits the analysis of a laminated structure subjected to combined loading with any type of ply orientation and stacking sequence. CLPT also permits the analysis of the complex coupling effects that are sometimes present in laminates. Although a laminate is composed of multiple laminae, CLPT assumes that each ply is perfectly bonded and in a state of plane stress, with interlaminar stresses neglected; interfacial shear is also ignored. These assumptions treat the individual laminae as a unitary orthotropic plate, and many of the assumptions used in the development of isotropic plate theory may be used with minor modification. The coordinate system for CLPT is shown in Fig. B.6, and the stress resultants (force per unit width of laminate) are defined as

\[ N_{ij} = \int_{-h}^{h} \sigma_{ij} \partial z \]
\[ M_{ij} = \int_{-h}^{h} \sigma_{ij} z \partial z \]

For the present work, the stress resultant \( N \) and the moment resultant \( M \) correspond, respectively, to the radial blade stress caused by centrifugal loading and the flapwise bending stress caused by the freestream velocity. Note that in Fig. B.6, the coordinate system originates at the midplane of the laminate, and that the total thickness of the laminate is \( 2h \). The deflections of the plate (the sum of displacement plus rotation) in the \( x, y, \) and \( z \) directions are expressed, respectively, by \( u, v, \) and \( w \):

\[ u = u_0(x, y) + z\alpha(x, y) \]
\[ v = v_0(x, y) + z\beta(x, y) \]
\[ w = w(x, y) \]

where the subscript 0 indicates a displacement at the laminate centerline, and \( \alpha \) and \( \beta \) are the rotations. Neglecting transverse strain \( (\gamma_{xz} = \gamma_{yz} = 0) \), the rotations are the slopes of the deflections:

\[ \alpha = -\frac{\partial w}{\partial x} \]
\[ \beta = -\frac{\partial w}{\partial y} \]
If the centerline displacements and rotations are known, the deflections at any \((x, y, z)\) point are also known. The strains of the plate are then a combination of the midplane strains and plate curvatures:

\[
\begin{align*}
\epsilon_x &= \frac{\partial u}{\partial x} = \epsilon_0 + z\kappa_x = \epsilon_0 - z \left( \frac{\partial^2 w}{\partial x^2} \right) \\
\epsilon_y &= \frac{\partial v}{\partial y} = \epsilon_0 + z\kappa_y = \epsilon_0 - z \left( \frac{\partial^2 w}{\partial y^2} \right) \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_0 + z\kappa_{xy} = \gamma_0 - 2z \left( \frac{\partial^2 w}{\partial x \partial y} \right)
\end{align*}
\]  

\text{(B.20)}

Equations B.20 may be substituted into the stress-strain relationships to find the stresses of the \(k\)th ply:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_k = \begin{bmatrix} Q \end{bmatrix}_k \begin{bmatrix}
\epsilon_0 - z \left( \frac{\partial^2 w}{\partial x^2} \right) \\
\epsilon_0 - z \left( \frac{\partial^2 w}{\partial y^2} \right) \\
\gamma_0 - 2z \left( \frac{\partial^2 w}{\partial x \partial y} \right)
\end{bmatrix}_k
\]

\text{(B.21)}

Because each \(k\)th ply will have its own stress-strain relationship, it will also have its own stiffness matrix. Substituting the stress-strain relationship from Eq. B.21 into the definition of resultants given in Eqs. B.17 and simplifying:

\[
\begin{bmatrix} N \\
M \end{bmatrix} = \begin{bmatrix} A & B \\
B & D \end{bmatrix} \begin{bmatrix} \epsilon_0 \\
k \end{bmatrix}
\]

\text{(B.22)}

The extensional stiffness matrix of the laminate with \(n\) plates is given by

\[
A_{ij} = \int_{-h}^h [\overline{Q}_{ij}]_k dz = \sum_{k=1}^n [\overline{Q}_{ij}]_k (z_k - z_{k-1})
\]

\text{(B.23)}

where \(i, j = 1, 2, 6\) (cases of \(i, j = 4, 5\) are for transverse shear, and are dealt with separately). The terms \(A_{11}, A_{22},\) and \(A_{66}\) are the direct extensional stiffness terms of the laminate, and \(A_{12}\) is the laminate Poisson effect. Note that the entries in the \(A\) matrix are independent of the stacking sequence.

The out-of-plane behavior of the laminate is governed by the \(D\) matrix, which is given by

\[
D_{ij} = \int_{-h}^h [\overline{Q}_{ij}]_k z^2 dz = \frac{1}{3} \sum_{k=1}^n [\overline{Q}_{ij}]_k (z_k^2 - z_{k-1}^2)
\]

\text{(B.24)}

Unlike the \(A\) matrix, the \(D\) matrix is dependent upon the stacking sequence, and plies that are far from the plate centerline will have a large effect on the \(D\) matrix. The \(D_{11}\) and \(D_{22}\) entries are the direct bending stiffnesses of the laminate, and \(D_{66}\) is the torsional stiffness of the laminate. The \(D_{12}, D_{16},\) and \(D_{26}\) terms are the bending/torsion coupling terms of the laminate.

The \(B\) matrix couples the in-plane forces, \(\{N\}\), with the curvatures, \(\{k\}\), and the moments,
\{ M \}, with the midplane strains, \{ \epsilon_0 \}. The B matrix is defined as

\[
B_{ij} = \int_{-h}^{h} [\mathcal{Q}_{ij}] z \, dz = \frac{1}{2} \sum_{k=1}^{n} [\mathcal{Q}_{ij}]_k (z_k^2 - z_{k-1}^2)
\]

(B.25)

The \((z_k^2 - z_{k-1}^2)\) terms in the B matrix will change sense above and below the laminate centerline. Nonzero B matrix terms indicate that when bending or torsion is applied to the laminate, in-plane deformations will result; similarly, in-plane forces will result in bending and twist of the laminate.

If the composite plate is used as an approximation of a cantilevered wing or a helicopter rotor blade, it is immediately apparent that the coupling terms can have a profound effect on the aerodynamics of the aircraft. For instance, a nonzero \(B_{16}\) term indicates extension-torsion coupling, so that an in-plane tensile force (such as the centrifugal loading of a rotor blade) will cause the composite to twist. Nonzero \(B_{11}\) and \(B_{22}\) terms indicate that a tensile force will cause beamwise and/or chordwise bending of the laminate. A nonzero \(D_{26}\) term will cause torsion under chordwise bending, which can change the camber of the structure under load. Some of these effects can be intentionally designed into the structure, to enable effects such as passive extension-twist coupling in a rotor blade. Coupling effects can also be eliminated through careful selection of the laminate layup: for instance, all B matrix terms can be forced to zero through the use of a symmetric layup.
Figure B.6: Coordinate system and resultants for laminated plate, from [51].
B.4 Sandwich Panel Structures

Composite laminated plate theory can be extended to include the analysis of sandwich structures that consist of a core of thickness $h_c$ sandwiched between (and adhesively bonded to) orthotropic face sheets of thickness $t_f$; this arrangement is shown in Fig. B.7. This type of structure benefits from very high flexural strength-to-weight and stiffness-to-weight ratios. Examples of sandwich panel structures used in the present work are the composite rotor blades of the Gen-2 and Gen-3 models, as well as the composite wing used for the Gen-3 model. As with CLPT, the mechanics of composite sandwich panel structures are detailed in [51] and summarized here.

Sandwich panels are typically symmetric with respect to the centerline, making the coupling stiffnesses $B_{ij} = 0$. For sandwich panels with face sheet laminates that are thin relative to the core, the transverse stresses may be assumed to be constant through the face sheets, and are assumed to vary linearly through the core. The core is usually a very lightweight material of low density (e.g. foam, honeycomb, paper, etc.), and the in-plane stresses in the core are assumed negligible. The constitutive equations for transverse shear through the core are

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = h_c \begin{bmatrix} G_{TZ} & 0 \\ 0 & G_{LZ} \end{bmatrix} \begin{bmatrix} \beta + \frac{\partial w}{\partial y} \\ \alpha + \frac{\partial w}{\partial x} \end{bmatrix}$$ (B.26)

where the transverse shears $Q_{xz}$ and $Q_{yz}$ are defined as

$$Q_{xz}, Q_{yz} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xz}, \sigma_{yz}) \, dz$$ (B.27)

The $A,B,D$ matrices from CLPT are still used here; however, for thin face sheets ($h_c \gg t_f$), only the $A$ matrix of the face sheets is used in the analysis. The $A$ matrix for the entire panel, denoted as $\tilde{A}$, is calculated as

$$\tilde{A}_{ij} = \sum_{k=1}^{n} [\overline{Q}_{ij}]_k (z_k - z_{k-1})$$ (B.28)

$$\tilde{A}_{ij} = 2t_f \overline{[Q_{ij}]}_f + h_c \overline{[Q_{ij}]}_c$$

where the subscripts $f$ and $c$ denote the face sheet and the core, respectively. The in-plane/out-of-plane coupling matrix for the sandwich panel, denoted as $\tilde{B}$, is given as

$$\tilde{B}_{ij} = \frac{h_c}{2} [A_{ij}^{(top)} - A_{ij}^{(bottom)}]$$ (B.29)

which, for identical face sheets, is zero. The out-of-plane stiffness matrix of the sandwich panel, denoted as $\tilde{D}$, is given as

$$\tilde{D}_{ij} = \frac{h_c^2}{4} \left[ \left(1 + \frac{t_f^{(top)}}{h_c} \right) A_{ij}^{(top)} + \left(1 + \frac{t_f^{(bottom)}}{h_c} \right) A_{ij}^{(bottom)} \right]$$ (B.30)
The $A, B, D$ matrices of CLPT must now be adjusted to include the stiffness of the core:

$$
\tilde{C}_{ij} = \frac{h_c}{2} \left[ \left( 1 + \frac{t_f^{(\text{top})}}{h_c} \right) A_{ij}^{(\text{top})} - \left( 1 + \frac{t_f^{(\text{bottom})}}{h_c} \right) A_{ij}^{(\text{bottom})} \right]
$$

(B.31)

For identical face sheets, $\tilde{B} = \tilde{C} = 0$ and $\tilde{D}_{ij} = h_c C_{ij}^{(\text{top})}$. The $A, B, D$ matrix of the classical laminate (Eq. B.22) now becomes the $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ matrix of the sandwich panel:

$$
\begin{bmatrix}
N \\
M
\end{bmatrix} = 
\begin{bmatrix}
\tilde{A} & \tilde{B} \\
\tilde{C} & \tilde{D}
\end{bmatrix}
\begin{bmatrix}
\epsilon \\
\kappa
\end{bmatrix}
$$

(B.32)

For homogenous cores, such as foam, the core shear properties $G_{LZ}$ and $G_{TZ}$ will usually be equal. When designing a part to a target stiffness value, iteration of only $\tilde{A}$ and $h_c$ is usually sufficient for initial calculations.
Figure B.7: Geometry of composite sandwich panel.
B.5 Equivalent Properties of Composite Structures

The mechanics of composite structures is rooted in the analysis of beams made of isotropic materials, but must be adjusted to account for the orthotropy of the laminate. If the laminate properties can be converted to an “equivalent property” (such as an equivalent modulus), then the classical isotropic strength of materials methods can continue to be used. The analyses presented herein are not original derivations, but are summaries of the methods presented in [51], [53], and [84], among others.

B.5.1 Equivalent properties of symmetric laminates

Recall that for a symmetric laminate, all elements of the coupling matrix, $B_{ij}$, are equal to zero. This greatly simplifies finding the in-plane constants of the laminate. To find the $x$-direction modulus, the $x$-direction strains along the laminate midplane must be calculated. From the constitutive equations,

$$E_x = \frac{\sigma_x}{\epsilon_x} = \frac{N_x}{\epsilon_x} \quad (B.33)$$

where $h$ is the laminate thickness. Since all $B_{ij} = 0$, the constitutive equations become

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \quad (B.34)$$

To find the relationship between $N_x$ and $\epsilon_x$ for a load applied in the $x$-direction, from B.34:

$$N_x = A_{11} \epsilon_x^0 + A_{12} \epsilon_y^0 + A_{16} \gamma_{xy}^0 \quad (B.35)$$

$$0 = A_{12} \epsilon_x^0 + A_{22} \epsilon_y^0 + A_{26} \gamma_{xy}^0 \quad (B.36)$$

$$0 = A_{16} \epsilon_x^0 + A_{26} \epsilon_y^0 + A_{66} \gamma_{xy}^0 \quad (B.37)$$

From Eqs. B.36 and B.37,

$$\epsilon_y^0 = \epsilon_x^0 \left( \frac{A_{26} A_{16} - A_{12} A_{66}}{A_{22} A_{66} - A_{26}^2} \right) \quad (B.38)$$

and

$$\gamma_{xy}^0 = \epsilon_x^0 \left( -\frac{A_{16}}{A_{66}} + \frac{A_{26} A_{12} A_{66} - A_{26}^2 A_{16}}{A_{22} A_{66}^2 - A_{26}^2 A_{66}} \right) \quad (B.39)$$

Equations B.38 and B.39 can then be substituted into B.35:

$$\frac{N_x}{\epsilon_x^0} = A_{11} + A_{12} \left( \frac{A_{26} A_{16} - A_{12} A_{66}}{A_{22} A_{66} - A_{26}^2} \right) + A_{16} \left( -\frac{A_{16}}{A_{66}} + \frac{A_{26} A_{12} A_{66} - A_{26}^2 A_{16}}{A_{22} A_{66}^2 - A_{26}^2 A_{66}} \right) \quad (B.40)$$

Dividing Eq. B.40 by the laminate thickness, $h$, will yield the functional modulus in the $x$ direction, $E_x$. The same procedure is followed to find the equivalent modulus in the $y$ direction,
\( E_y \). The resulting equation is

\[
N_y = \frac{1}{\epsilon_y} = A_{12} \left( \frac{A_{16}A_{26} - A_{12}A_{66}}{A_{11}A_{66} - A_{16}} \right) + A_{22} + A_{26} \left( \frac{-A_{26}}{A_{66}} + \frac{A_{16}A_{12}A_{66} - A_{10}^2A_{26}}{A_{11}A_{66} - A_{16}A_{66}} \right) \quad (B.41)
\]

Dividing Eq. B.41 by the laminate thickness will yield the \( y \)-direction modulus, \( E_y \).

The equivalent shear modulus, \( G_{xy} \), is found in the same fashion. Its constitutive equations are

\[
0 = A_{11} \epsilon_x^0 + A_{12} \epsilon_y^0 + A_{16}\gamma_{xy} \quad (B.42)
\]

\[
0 = A_{12} \epsilon_x^0 + A_{22} \epsilon_y^0 + A_{26}\gamma_{xy} \quad (B.43)
\]

\[
N_{xy} = A_{16} \epsilon_x^0 + A_{26} \epsilon_y^0 + A_{66} \gamma_{xy} \quad (B.44)
\]

From Eqs. B.43 and B.44,

\[
\epsilon_x^0 = \gamma_{xy} \left( \frac{A_{12}A_{26} - A_{16}A_{22}}{A_{11}A_{22} - A_{12}^2} \right) \quad (B.45)
\]

and

\[
\epsilon_y^0 = \gamma_{xy} \left( \frac{-A_{26}}{A_{22}} + \frac{A_{16}A_{12}A_{22} - A_{12}^2A_{26}}{A_{11}A_{22}^2 - A_{12}^2A_{22}} \right) \quad (B.46)
\]

Substituting into Eq. B.44,

\[
\frac{N_{xy}}{\gamma_{xy}} = A_{66} - \frac{A_{26}^2}{A_{22}} + \frac{2A_{12}A_{16}A_{26}A_{22} - A_{12}^2A_{26}^2 - A_{16}^2A_{22}^2}{A_{11}A_{22}^2 - A_{12}^2A_{22}} \quad (B.47)
\]

Dividing Eq. B.47 by the laminate thickness will yield \( G_{xy} \).

To find Poisson’s ratio for the laminate, use Eqs. B.36 and B.37 to obtain

\[
0 = A_{12} \epsilon_x^0 + A_{22} \epsilon_y^0 + A_{26} \left( \frac{-A_{16}}{A_{66}} \epsilon_x^0 \frac{A_{26}}{A_{66}} \epsilon_y^0 \right) \quad (B.48)
\]

Rearranging,

\[
\mu_{xy} = \frac{A_{12} - \frac{A_{16}A_{26}}{A_{66}}}{A_{22} - \frac{A_{26}^2}{A_{66}}} \quad (B.49)
\]

and

\[
\mu_{yx} = \frac{\frac{A_{16}A_{26}}{A_{66}} - A_{12}}{A_{11}} \quad (B.50)
\]
B.5.2 Equivalent properties of nonsymmetric laminates

Since a nonsymmetric laminate will have nonzero $B_{ij}$ terms, the calculation of the in-plane engineering constants will of course be more involved. The same basic procedures are follow to derive the equivalent properties of the laminate, although now there are six constitutive equations rather than only three. Thus, it will be far easier to use matrix notation to present the constitutive equations in matrix form:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \epsilon^0_x \\ \epsilon^0_y \\ \gamma^0_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{B.51}$$

To find $E_x$, only the $x$-direction in-plane load is of consequence:

$$\begin{bmatrix} N_x \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \epsilon^0_x \\ \epsilon^0_y \\ \gamma^0_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{B.52}$$

Using Cramer’s rule to solve for $\epsilon^0_x$:

$$\epsilon^0_x = \frac{\begin{vmatrix} N_x & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ 0 & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ 0 & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ 0 & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ 0 & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ 0 & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}} {\begin{vmatrix} A & B \\ B & D \end{vmatrix}} \tag{B.53}$$
Using cofactor expansion on the numerator and simplifying, $E_x$ can be calculated as

\[
\frac{N_x}{\epsilon^0_x} = E_x = \frac{1}{\hbar} \begin{vmatrix} A & B \\ B & D \end{vmatrix}
\]

\[
\begin{pmatrix}
A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\
A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\
B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\
B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\
B_{26} & B_{66} & D_{16} & D_{26} & D_{66}
\end{pmatrix}
\]

(E.54)

$E_y$ can be found in a similar fashion:

\[
\frac{N_y}{\epsilon^0_y} = E_y = \frac{1}{\hbar} \begin{vmatrix} A & B \\ B & D \end{vmatrix}
\]

\[
\begin{pmatrix}
A_{11} & A_{16} & B_{11} & B_{12} & B_{16} \\
A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\
B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\
B_{12} & B_{26} & D_{12} & D_{22} & D_{26} \\
B_{16} & B_{66} & D_{16} & D_{26} & D_{66}
\end{pmatrix}
\]

(E.55)

Similarly, $G_{xy}$ is given by

\[
\frac{N_{xy}}{\gamma^0_{xy}} = G_{xy} = \frac{1}{\hbar} \begin{vmatrix} A & B \\ B & D \end{vmatrix}
\]

\[
\begin{pmatrix}
A_{11} & A_{12} & B_{11} & B_{12} & B_{16} \\
A_{12} & A_{22} & B_{12} & B_{22} & B_{26} \\
B_{11} & B_{12} & D_{11} & D_{12} & D_{16} \\
B_{12} & B_{22} & D_{12} & D_{22} & D_{26} \\
B_{16} & B_{26} & D_{16} & D_{26} & D_{66}
\end{pmatrix}
\]

(E.56)
Poisson’s ratio is determined in a fashion similar to symmetric laminates. For $\nu_{xy}$,

$$\nu_{xy} = \frac{-\epsilon_0^y}{\epsilon_0^x} = \frac{\begin{vmatrix} A_{12} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ -B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}$$  \hspace{1cm} (B.57)

Similarly,

$$\nu_{yz} = \frac{-\epsilon_0^x}{\epsilon_0^y} = \frac{\begin{vmatrix} A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{16} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}$$  \hspace{1cm} (B.58)
Appendix C

Whirl Flutter Mathematical Modeling

The analytical model of the rotor and wing dynamics used for the present work are based on the methods detailed in [12], [13], and [37]. The mathematics presented herein are summaries of these references and are not original derivations; however, a proper understanding of these mathematical models is critical to fully understand the flutter behavior of the wind tunnel models, and they are therefore included for completeness. An overview of blade element theory is also included in these derivations, as this was used to guide the design and fabrication of the rotor blades used on the Gen-2 and Gen-3 models.

C.1 Rotor Dynamics

A four-degree of freedom model of a tiltrotor wing assembly in cruise flight is shown in Fig. C.1. It will be shown through derivation that this model is sufficient to represent the coupled rotor/pylon motion of a tiltrotor wing with three or more blades. The rotor is attached to pivot point contained in the pylon, which has degrees of freedom $\alpha_y$ for pitch (positive for upward hub rotation) and $\alpha_x$ for yaw (positive for outward hub rotation). The pylon is assumed to be attached rigidly to the wing of the aircraft, and the aerodynamic characteristics of the pylon are neglected.

A set of $N$ rotor blades, each of radius $R$, are mounted to the hub, which is offset from the pivot by a mast of height $h$, which also serves as the driveshaft of the rotor. The rotor has a counter-clockwise rotation when viewed from the front, and the azimuth angle, $\psi$, is measured from the top of the rotation of a given blade; the angle between blades is given by $\Delta\psi = \frac{2\pi}{N}$. The rotor degrees of freedom are the out-of-plane flapping motion for each blade, which is defined as positive for forward flapping of a given blade (this correlates to the positive-up flapping motion in helicopter mode, as is the normal convention). For stiff-inplane rotors, such as the gimbaled rotor
used in the wind tunnel models of the present work, the dimensionless rotating natural frequency of the blade flapping is greater than 1/rev; a rigid-body mode is assumed for the blades, such that the flapping mode shape is linear and proportional with radial distance \( r \). The rotor exerts thrust, \( T \), axial force, \( H \), and side force, \( Y \), on the hub. A constant rotor rotational speed, \( \Omega \), is assumed throughout steady operation as well as perturbations; for the wind tunnel models of the present work, this constant RPM is controlled via the root pitch angle, \( \theta_0 \). The rotor operates in pure axial inflow of speed \( V \) during its normal state, and its nondimensional advance ratio, \( \mu = \frac{V}{\Omega R} \), is assumed to be near unity. It is assumed that the rotor forces are reacted by the pivot located at the root of the pylon, and that the pylon motion is constrained to the rotations \( \alpha_x \) and \( \alpha_y \). Blade flapping imparts a rotor pitch moment, \( M_y \), and rotor yaw moment, \( M_x \), to the hub.

Equilibrating the rotor forces and moments about the hub and assuming small perturbations, the linearized flap moment equation of motion for the \( m^{th} \) blade is given by

\[
I_b \left[ \ddot{\beta}^{(m)} + \nu^2 \dot{\beta}^{(m)} - (\ddot{\alpha}_y - 2\Omega \dot{\alpha}_x) \cos (\psi_m) + (\ddot{\alpha}_x + 2\Omega \dot{\alpha}_y) \sin (\psi_m) \right] = M_{\beta_m} \tag{C.1}
\]

If the mass of the rotor is treated as a point mass at the hub, the pylon yaw and pitch equations of motion are given, respectively, by

\[
I_x \ddot{\alpha}_x + c_x \dot{\alpha}_x + k_x \alpha_x = M_x - hY
\]

\[
I_y \ddot{\alpha}_y + c_y \dot{\alpha}_y + k_y \alpha_y = M_y - hH \tag{C.2}
\]

The rotor solidity, \( \sigma \), and rotor Lock number may now be included. The solidity is defined as the nondimensional ratio of blade area to rotor disk area,

\[
\sigma = \frac{A_{\text{blade}}}{A_{\text{rotor}}} \tag{C.3}
\]

which for a rotor consisting of rectangular blades of constant chord is simply \( \sigma = \frac{N_c \pi}{4R} \). The Lock number is defined as the nondimensional ratio of the aerodynamic and inertial forces of a rotor,

\[
\gamma = \frac{\rho ac R^4}{I_b} \tag{C.4}
\]

Dividing the flap equation of motion by \( I_b \),

\[
\ddot{\beta}^{(m)} + \nu^2 \dot{\beta}^{(m)} - (\ddot{\alpha}_y - 2\Omega \dot{\alpha}_x) \cos (\psi_m) + (\ddot{\alpha}_x + 2\Omega \dot{\alpha}_y) \sin (\psi_m) = \frac{\gamma C_{M_x}}{ac} \tag{C.5}
\]

Nondimensionalizing the pylon pitch and yaw equations of motion,

\[
I_x^* \ddot{\alpha}_x + c_x^* \dot{\alpha}_x + k_x^* \alpha_x = \gamma \left( \frac{2C_{M_x}}{\sigma a} - \frac{2hC_Y}{\sigma a} \right)
\]

\[
I_y^* \ddot{\alpha}_y + c_y^* \dot{\alpha}_y + k_y^* \alpha_y = \gamma \left( \frac{2C_{M_y}}{\sigma a} - \frac{2hC_H}{\sigma a} \right) \tag{C.6}
\]
where the asterisk quantities indicate normalization by \(I_b \) (e.g. \(I_x^* = \frac{I_x}{I_b} \)). The damping terms are normalized by \(\frac{N}{2} I_b \). The rotor axial force coefficient, \(C_H \), is the rotor axial force nondimensionalized by the rotor area, freestream density, and tip speed:

\[
C_H = \frac{H}{\rho \pi \Omega^2 R^4}
\]

where the rotor tip speed is defined as \(V_{tip} = \Omega R \), as normal. The hub moments, \(M_x \) and \(M_y \), and the hub side force, \(Y \), are nondimensionalized in a similar fashion. The aerodynamic flap moment coefficient, \(C_{M_{\beta}} \), is the aerodynamic flap moment, \(M_{\beta} \), normalized by \(\rho \Omega^2 R^5 \).

The pylon contributions, \(\ddot{\alpha}_y \) and \(\ddot{\alpha}_x \), to rotor flap are due to the total rotor flap angle being composed of the blade flap angle with respect to the hub plane, \(\beta \), plus the pylon angles, \(\alpha_x \) and \(\alpha_y \), which govern the tilt of the hub plane. The \(\ddot{\alpha}_x \) and \(2\dot{\alpha}_y \) terms in the flapping equation of motion are due to Coriolis acceleration: each blade element has linear velocity \(\Omega r \) in the hub plane; however, the hub itself has an angular velocity given by \(\dot{\alpha}_x \cos (\psi_m) + \dot{\alpha}_y \sin (\psi_m) \) due to the pylon motion. The cross-product of these reveals the Coriolis acceleration of the blade.

A Fourier coordinate transform is used to describe the blade motion of Eq. C.5 in the nonrotating frame, which describes the motion of the rotor tip-path plane. The transformed degrees of freedom for the \(m^{th} \) blade are

\[
\beta_0 = \frac{1}{N} \sum_{m=1}^{N} \beta^{(m)}
\]

\[
\beta_{nc} = \frac{2}{N} \sum_{m=1}^{N} \left( \beta^{(m)} \cos (n\psi_m) \right)
\]

\[
\beta_{ns} = \frac{2}{N} \sum_{m=1}^{N} \left( \beta^{(m)} \sin (n\psi_m) \right)
\]

The rotor flapping motion may now be described in terms of the coning angle (assumed constant) and the azimuth-dependent tilt of the tip-path plane:

\[
\beta^{(m)} = \beta_0 + \sum_n \left[ \beta_{nc} \cos (n\psi_m) + \beta_{ns} \sin (n\psi_m) \right]
\]

The summation of Eq. C.9 is from 1 to \(\frac{N-1}{2} \) for an odd number of blades, and from 1 to \(\frac{N-2}{2} \) for an even number of blades. A fourth blade degree of freedom, \(\beta_{N_2} \), appears only for an even number of blades, and is thus excluded from the present analysis. Converting the blade motion into the nonrotating frame has the advantage of creating constant inertial and aerodynamic coefficients for the axial-flow case, and only the \(\beta_{x1} \) and \(\beta_{x2} \) rotor terms (first longitudinal flapping and first lateral flapping, respectively) couple with the pylon degrees of freedom. The equations of motion in the nonrotating frame (for \(N = 3 \)) are then
\[
\ddot{\beta}_0 + \nu_0^2 \beta_0 = \gamma \left( \frac{M_{F_0}}{ac} \right)
\]
\[
\ddot{\beta}_{1c} + 2 \ddot{\beta}_{1s} + (\nu_0^2 - 1) \beta_{1c} - \ddot{\alpha}_y + 2 \dot{\alpha}_x = \gamma \left( \frac{M_{F_{1c}}}{ac} \right)
\]
\[
\ddot{\beta}_{1s} + 2 \ddot{\beta}_{1c} + (\nu_0^2 - 1) \beta_{1s} - \ddot{\alpha}_x + 2 \dot{\alpha}_y = \gamma \left( \frac{M_{F_{1s}}}{ac} \right)
\]
(C.10)

and the rotor collective, pitch, and yaw moments are, respectively,
\[
M_{F_0} = \frac{1}{N} \sum_m M_{F_m}
\]
\[
M_{F_{1c}} = \frac{2}{N} \sum_m M_{F_m} \cos(\psi_m)
\]
\[
M_{F_{1s}} = \frac{2}{N} \sum_m M_{F_m} \sin(\psi_m)
\]
(C.11)

Equations C.10 and C.11 do not change for \( N \geq 3 \). Thus, the 4DOF model is sufficient to describe the coupled rotor/pylon motion.
Figure C.1: 4DOF model of tilting rotor, from [12].
C.2 Rotor Aerodynamics and Blade Element Theory

A brief overview of rotor aerodynamics and blade element theory is now presented. A detailed description of actuator disk theory (momentum theory) as it applies to both rotor blades and propellers may be found in [2], [48], [49], and [50] and is not presented here; instead, the derivation of rotor aerodynamics presented in [2], [12], [13], and [82] are summarized.

A coordinate system fixed to the rotor hub is used; the origin of a blade is taken to be the hub center, and the blade extends outwards radially from the hub along the axis $y$. The blade element diagram for a positively-thrusting rotor (viz. a rotor that is actively driven by a propulsion system to produce thrust) is the normal case, and is shown in Fig. C.2. A blade section experiences perpendicular velocity $U_p$ (normal to the hub plane, positive toward the rear of the disk), which is composed of the axial velocity $V_c$ and the induced velocity $v_i$; the radial velocity $U_r$ (in the hub plane, positive radially outward along the blade); and the tangential velocity $U_t$ (in the hub plane, positive in the blade drag direction). For a tiltrotor in cruise flight, the perpendicular velocity $U_p$ may be replaced by the freestream velocity, $V_\infty$. The resultant of $U_p$ and $U_t$ is the relative velocity $U$ (sometimes given as $V_{rel}$): $U = \sqrt{U_p^2 + U_t^2}$. The perpendicular, tangential, and relative velocities form the “velocity triangle” that will be used for subsequent analysis.

Three angles of interest for each blade element are formed by the velocity triangle: the pitch angle, the inflow angle, and the angle of attack. The local blade pitch angle, $\theta$, is composed of the collective pitch, blade twist, and any incremental pitch due to cyclic control. The local inflow angle, $\phi$, is merely the inverse tangent of the perpendicular and tangential velocities, $\phi = \arctan \left( \frac{U_p}{U_t} \right) \approx \frac{U_p}{U_t}$. The local angle of attack, $\alpha$, is then merely the difference between these two: $\alpha = \theta - \phi$. Correct twisting of the blade will ensure that the local angle of attack across all blade stations will be at the best $L/D$ condition for that airfoil during cruise. The inflow angle also contributes to the inflow ratio, $\lambda$, which is simply $\lambda = \phi r$. It is immediately apparent that for the ideal condition of uniform inflow, $\phi$ must decrease monotonically with increasing $r$.

For tiltrotors in cruise flight, a uniform inflow assumption is usually justifiable since the induced velocity is much less than the freestream velocity ($v_i \ll V_\infty$) and the rotor has a low working thrust coefficient.

The lift and drag forces of the two-dimensional blade section are expressed as

$$
\begin{align*}
dL &= \frac{1}{2} C_l U^2 c dy \\
dD &= \frac{1}{2} C_d \rho U^2 c dy
\end{align*}
$$

where $C_l$ and $C_d$ are the section lift and drag coefficients, respectively, and $c$ is the local blade chord. The sectional lift and drag forces are then resolved into forces perpendicular and parallel to the hub plane:

$$
\begin{align*}
dF_z &= dL \cos \phi - dD \sin \phi \\
dF_x &= dL \sin \phi + dD \cos \phi
\end{align*}
$$
The incremental contributions of an individual blade section to the total thrust, torque, and power of an \( N \)-bladed rotor are, respectively,

\[
\begin{align*}
\frac{dT}{dL} &= N dF_z = N \int dL \cos \phi - dD \sin \phi \\
\frac{dQ}{dL} &= Ny dF_z = Ny \int dL \sin \phi + dD \cos \phi \\
\frac{dP}{dL} &= N y \Omega dF_x = N y \Omega \int dL \sin \phi + dD \cos \phi 
\end{align*}
\]  

(C.14)

The rotor will experience perturbation velocities due to the aerodynamic gusts as well as rotor and pylon degrees of freedom. The gust velocities (\( u_g \), here) are normalized by \( V_\infty \), as is the standard convention for aircraft stability analysis. Because the gust velocities are assumed to be small relative to the forward speed of the aircraft, this normalization reduces the vertical and lateral gust components to angles (\( \alpha_g \) and \( \beta_g \), respectively), and the longitudinal gust component to a fractional change in forward speed. The gust velocities do not affect the speed of the aircraft in an inertial reference frame, but only in an aerodynamic one; as such, the gusts will not appear in the inertial terms of the aircraft equations of motion. The tangential, perpendicular, and radial perturbation velocities are defined, respectively, as

\[
\delta U_t = -h (\dot{\alpha}_y \sin \phi_m + \dot{\alpha}_x \cos \phi_m) + (V_\infty + v_i) (\alpha_y \sin \phi_m + \alpha_x \cos \phi_m) + V_\infty (\beta_g \cos \phi_m + \alpha_g \sin \phi_m)
\]  

(C.15)

\[
\delta U_p = r (\dot{\beta}_g - \dot{\alpha}_y \cos \phi_m + \dot{\alpha}_x \sin \phi_m) + V_\infty u_g
\]  

(C.16)

\[
\delta U_r = h (-\dot{\alpha}_y \cos \phi_m + \dot{\alpha}_x \sin \phi_m) + (V_\infty + v_i) (\alpha_y \cos \phi_m - \alpha_x \sin \phi_m) + V_\infty (-\beta_g \sin \phi_m + \alpha_g \cos \phi_m)
\]  

(C.17)

The terms in the tangential and radial gust equations are, respectively, the inplane hub velocity, the inplane component of forward velocity due to pylon tilt, and the inplane velocity of the vertical and lateral gusts; the equation for the perpendicular velocity perturbation contains only the flapwise velocity (out of the hub plane) and the longitudinal perturbation of velocity due to the gust. The gust will also perturb the blade pitch,

\[
\delta \theta = \theta - K_p \beta
\]  

(C.18)

where the kinematic pitch-flap coupling term \( K_p \) is defined in terms of the pitch-flap coupling angle, \( \delta_3 \), as \( K_p = \tan \delta_3 \). Pitch perturbations are assumed to be uniform over the blade span, since it is a perturbation made through the control system.
The perturbations to the aerodynamic coefficients are defined by the following relationships:

\[ \delta C_l = \frac{\partial C_l}{\partial \alpha} \delta \alpha + \frac{\partial C_l}{\partial M} \delta M \]
\[ \delta C_d = \frac{\partial C_d}{\partial \alpha} \delta \alpha + \frac{\partial C_d}{\partial M} \delta M \]
\[ \delta \alpha = \delta \theta - \frac{U_l \delta U_p + U_p \delta U_p}{U} \]
\[ \delta M = M_{tip} \delta U \] (C.19)

where \( M_{tip} \) is the rotor tip Mach number. The resultant forces along the blade span may then be written as linear combinations of the blade velocity perturbations and pitch angle:

\[ \int_0^1 \frac{F_z}{ac} \, dt = T_0 + T_\mu \delta U_t + T_\beta \delta \left( \hat{\beta} - \hat{\alpha}_y \cos \phi_m + \hat{\alpha}_x \sin \phi_m \right) \]
\[ + T_\lambda \delta (V_\infty u_g) + T_\theta \delta \theta \] (C.20)

\[ \int_0^1 \frac{r F_z}{ac} \, dt = M_0 + M_\mu \delta U_t + M_\beta \delta \left( \hat{\beta} - \hat{\alpha}_y \cos \phi_m + \hat{\alpha}_x \sin \phi_m \right) \]
\[ + M_\lambda \delta (V_\infty u_g) + M_\theta \delta \theta \] (C.21)

\[ \int_0^1 \frac{F_x}{ac} \, dt = H_0 + H_\mu \delta U_t + H_\beta \delta \left( \hat{\beta} - \hat{\alpha}_y \cos \phi_m + \hat{\alpha}_x \sin \phi_m \right) \]
\[ + H_\lambda \delta (V_\infty u_g) + H_\theta \delta \theta \] (C.22)

\[ \int_0^1 \frac{F_r}{ac} \, dt = R_\mu \delta U_r - \beta \int_0^1 \frac{F_z}{ac} \, dr \] (C.23)

where \( R \) denotes blade radial forces. All of the coefficients in the foregoing four equations are constants, and are independent of azimuth in trimmed axial flight. The subscripts in Eqs. C.20-C.23 are defined as

- \( o \) = terms due to trim force or moment
- \( \mu \) = terms due to forces and moments caused by inplane hub velocity
- \( \hat{\beta} \) = terms due to blade flapwise velocity
- \( \lambda \) = terms due to axial velocity
- \( \theta \) = terms due to blade pitch control

These net blade forces and moments may be summed over all \( N \) blades of the rotor to find the resultant rotor forces and moments. Substituting the net blade forces into the equations for the net rotor forces reveals that all aerodynamic coefficients are independent of the blade index, \( m \).
The preceding derivation was for a positively-thrusting rotor; however, the wind tunnel models that are the focus of the present work all feature a windmilling rotor, and thus the blade element diagram must be modified to account for this (Fig. C.3). Without a power source to drive the rotor, the rotor must extract energy from the freestream flow in order to rotate; this requires some blade elements to have a net positive in-plane force (Fig. C.4). In order to accomplish this, the blade collective pitch must be adjusted until certain blade elements reach a negative angle of attack and are able to drive the rotor. The local angle of attack becomes a local negative angle of attack in order to reflect this; because the airfoil section used for all wind tunnel model blades was symmetric, it is not optimized for either powered or windmilling operation. The foregoing blade aerodynamic analysis, therefore, remains unchanged.
Figure C.2: Blade element diagram for positively-thrusting rotor, from [32].

Figure C.3: Blade element diagram for negatively-thrusting (windmilling) rotor, from [32].

Figure C.4: Tangential forces on windmilling rotor blade, from [32].
C.3 Wing Dynamics

The methods used to analyze the wing dynamics of the wind tunnel models are detailed in [26], [27], [28], and [48]; these methods are summarized here for the sake of completeness, and are not original derivations.

The wing is modeled as a discretized elastic beam; the continuous and discrete wing degrees of freedom are shown in Fig. C.5. The wing degree of freedom vector is given as \( \hat{\mathbf{q}} \), and is constructed as follows:

\[
\hat{\mathbf{q}} = \begin{bmatrix} w_1 & v_1 & \phi_1 & v'_1 & w_2 & v_2 & \phi_2 & v'_2 & w'_2 \end{bmatrix}^T
\]  

(C.24)

where \( w, v, \phi \) are the wing beamwise, chordwise, and torsional degrees of freedom, respectively. This vector is the multiplied by a matrix of Hermitian cubic shape functions, \([H]\), for bending and a linear Lagrangian polynomials, \(N^0\), for torsion to obtain a matrix of the continuous wing degrees of freedom, \( \hat{\mathbf{u}} \):

\[
[H] = \begin{bmatrix}
H^1_b & 0 & 0 & 0 & H^2_b & 0 & 0 & 0 & -H^3_b \\
0 & H^1_b & 0 & H^2_b & 0 & 0 & H^3_b & 0 & H^4_b \\
0 & 0 & N^1_b & 0 & 0 & 0 & 0 & N^2_b & 0 & 0 \\
\end{bmatrix}
\]  

(C.25)

\[
\hat{\mathbf{u}} = [H] \hat{\mathbf{q}} = \begin{bmatrix} w & v & \phi \end{bmatrix}^T
\]  

(C.26)

The term “Hermitian” in this instance refers to the use of an interpolation function named after French mathematician Charles Hermite. In cubic Hermitian interpolation, each spline is a third-degree polynomial specified by its values and first derivatives at the endpoints of the domain. The cubic spline is the one most commonly used; each spline is defined normally, as a function that is piecewise-definable by a polynomial function such that the interconnection points of the function (i.e. “knots”) are of sufficient smoothness. The structural mass and stiffness matrices of the entire wing – \([M_s]\) and \([K_s]\), respectively – are then computed by integrating the mass and stiffness matrices of each wing element – \([\hat{M}]\) and \([\hat{K}]\), respectively – along the wing half-span:

\[
[M_s] = \int_0^{\frac{1}{2}} [H]^T [\hat{M}] [H] \, dx
\]  

\[
[K_s] = \int_0^{\frac{1}{2}} [H]^T [\hat{K}] [H] \, dx
\]  

(C.27)
The elemental mass and stiffness matrices are defined as

$$\hat{M} = \begin{bmatrix} m & 0 & S_\alpha \\ 0 & m & 0 \\ S_\alpha & 0 & I_\theta \end{bmatrix}$$

$$\hat{K} = \begin{bmatrix} EI_b & 0 & 0 \\ 0 & EI_c & 0 \\ 0 & 0 & GJ \end{bmatrix}$$

(C.28)

where the subscripts $b$ and $c$ denote beamwise or chordwise bending, respectively.

The wing aerodynamics also contribute to the damping and stiffness of the wing; the wing geometry is given in Fig. C.6, with wing sweep defined as positive aft. Assuming quasisteady aerodynamics, the effective angle of attack, $\alpha_{eff}$, is defined as a function of the geometric angle of attack, the wing beamwise bending, wing torsion, wing sweep, and the freestream velocity:

$$\alpha_{eff} = \alpha_0 + \phi \cos \Lambda - \frac{\dot{w}}{V_\infty} - w' \sin \Lambda$$

(C.29)

where $\Lambda$ is the sweep angle of the wing, as measured along the elastic axis. The air loads on a given wing section - beamwise lift, chordwise lift, and pitching moment - are then defined as

$$L_w = C_{l_w} c q \alpha_{eff}$$

$$L_v = 0$$

$$M_\phi = L_w e$$

(C.30)

Because the effective angle of attack is a function of $w'$ due to wing sweep, it must be included in the aerodynamic contributions to the wing degrees of freedom:

$$\hat{u}_{aero} = \left\{ w \ v \ \phi \ v' \ w' \right\} = \left[H_{aero}\right] \hat{q}$$

(C.31)

where $\hat{q}$ is now redefined in order to include the aerodynamic contributions of the continuous wing degrees of freedom:

$$\hat{q} = \begin{bmatrix} w_1 & v_1 & \phi_1 & \theta_{z1} & \theta_{y1} & w_2 & v_2 & \phi_2 & \theta_{z2} & \theta_{y2} \end{bmatrix}$$

(C.32)

The aerodynamic matrix of shape functions is defined as

$$\left[H_{aero}\right] = \begin{bmatrix} H^1_b & 0 & 0 & 0 & -H^2_b & H^3_b & 0 & 0 & 0 & -H^4_b \\ 0 & H^1_b & 0 & H^2_b & 0 & 0 & H^3_b & 0 & H^4_b & 0 \\ 0 & 0 & N^1_\theta & 0 & 0 & 0 & 0 & N^2_\theta & 0 & 0 \\ 0 & \left(H^1_b\right)' & 0 & \left(H^2_b\right)' & 0 & 0 & \left(H^3_b\right)' & 0 & \left(H^4_b\right)' & 0 \\ \left(H^1_b\right)' & 0 & 0 & 0 & -\left(H^2_b\right)' & \left(H^3_b\right)' & 0 & 0 & 0 & -\left(H^4_b\right)' \end{bmatrix}$$

(C.33)
The sectional air loads given in Eqs. C.30 may be rewritten to include the contributions of wing sweep:

\[
\begin{bmatrix}
L_w \\
L_v \\
M_\phi \\
M_z \\
M_y
\end{bmatrix} = [A_1] \begin{bmatrix} \alpha_0 \\ 0 \\ 0 \end{bmatrix} + [A_2] \begin{bmatrix} w \\ v \\ v' \end{bmatrix} + [A_3] \begin{bmatrix} \dot{\phi} \\ \dot{v}' \\ \dot{w}' \end{bmatrix}
\]

(C.34)

where the coefficient matrices are defined as

\[
[A_1] = [A_2] = \begin{bmatrix}
0 & 0 & aqc \cos \Lambda & 0 & aqc \sin \Lambda \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & aqc \cos \Lambda & 0 & aqc \sin \Lambda \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(C.35)

\[
[A_3] = \begin{bmatrix}
-\frac{aqc}{V_\infty} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\frac{aqc}{V_\infty} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(C.36)

The virtual work done by the air loads along the half-span of the wing is defined as

\[
\delta W = \int_0^{b/2} \begin{bmatrix} \delta L_w \\ \delta L_v \\ \delta M_\phi \\ \delta M_z \\ \delta M_y \end{bmatrix} \begin{bmatrix} L_w \\ L_v \\ M_\phi \\ M_z \\ M_y \end{bmatrix} dx
\]

(C.37)

The aerodynamic damping and stiffness matrices, \([C_a]\) and \([K_a]\) are found by integrating along the wing half-span:

\[
[C_a] = \int_0^{b/2} [H_{aero}]^T [A_1] [H_{aero}] dx
\]

\[
[K_a] = \int_0^{b/2} [H_{aero}]^T [A_3] [H_{aero}] dx
\]

(C.38)

Finally, the mass, damping, and stiffness matrices for the entire wing – \([M_w]\), \([C_w]\), and \([K_w]\),
respectively – are formed as follows:

\[
[M_w] = [M_s] \\
[C_w] = [C_s] - [C_a] \\
[K_w] = [K_s] - [K_a]
\] (C.39)

The three diagonal elements of the structural damping matrix, \([C_s]\), were manually set to the experimentally-determined beamwise, chordwise, and torsion damping values at zero airspeed.
Figure C.5: Continuous and discrete wing degrees of freedom, from [48].

Figure C.6: Wing sweep geometry, from [48].
C.4 Wing/Rotor Coupling

The dynamics of the wing and rotor systems must now be coupled in order to complete the analysis. As described in [12], [26], [27], [28], [37], and [48] and summarized here, the equations of motion for the wing and rotor are coupled via the degree of freedom at the wing tip. This coupling occurs in two ways:

1. The rotor degrees of freedom at the pylon pivot
2. The rotor hub forces, which impart forces and moments to the wing tip

The aerodynamic and structural damping and stiffness matrices are constructed and the complex eigenvalues and eigenvectors are computed numerically. The degrees of freedom at the wing tip are given the general subscript \( c \) to denote the connection point with the rotor, rather than a specific node number. The vector of the wing tip degrees of freedom is then defined as

\[
\tilde{q}_{\text{tip}} = \begin{bmatrix} w_c & v_c & \phi_c & v'_c & w'_c \end{bmatrix}
\] (C.40)

The coupling between the wing and the rotor also requires a physical connection between the pylon and the rotor; for the purpose of the present work, it may assumed that the pylon is rigidly attached to the wing, which is an accurate representation of the physical wind tunnel models. Factoring in wing sweep and six pylon degrees of freedom \( x_p, y_p, z_p, \alpha_x, \alpha_y, \alpha_z \), as defined by the coordinate system in Fig. C.5 – the pylon degrees of freedom may be expressed in terms of the wing degrees of freedom at the connection point using the following coordinate transform:

\[
\begin{align*}
x_p &= w_c \\
y_p &= v_c \sin \Lambda \\
z_p &= v_c \cos \Lambda \\
\alpha_x &= v'_c \\
\alpha_y &= \phi_c \cos \Lambda - w'_c \sin \Lambda \\
\alpha_z &= -\phi_c \sin \Lambda - w'_c \cos \Lambda
\end{align*}
\] (C.41)
Applying the coordinate transform the rotor degrees of freedom vector, \( \mathbf{\hat{q}}_r \) (see [37]) includes the degrees of freedom of the connection point and may be redefined as \( \mathbf{\hat{q}}'_r \):

\[
\mathbf{\hat{q}}'_r = \begin{bmatrix}
    w_c \\
    v_c \\
    \phi_c \\
    v'_c \\
    w'_c \\
    \beta_{1c} \\
    \beta_{1s} \\
    \zeta_{1c} \\
    \zeta_{1s} \\
    \beta_0 \\
    \zeta_0
\end{bmatrix}^T
\]  

(C.42)

The work done on the wing by the rotor forces and blade perturbation motions is assumed to act only at the wing tip:

\[
\delta W = \begin{bmatrix}
    \delta w_c \\
    \delta v_c \\
    \delta \phi_c \\
    \delta v'_c \\
    \delta w'_c \\
    H \\
    T \cos \Lambda + Y \sin \Lambda \\
    (M_y + hH) \cos \Lambda + Q \sin \Lambda \\
    M_z - hY \\
    -Q \cos \Lambda + (M_y + hH) \sin \Lambda
\end{bmatrix}
\]  

(C.43)

The variational work is nondimensionalized by dividing through by \( \frac{N}{2} I_b \Omega^2 \), which simplifies Eq. C.43 to coefficient form:

\[
\frac{\delta W}{\frac{N}{2} I_b \Omega^2} = \delta \mathbf{\hat{q}}_c \left( \frac{2\gamma}{\sigma a} \right) \begin{bmatrix}
    C_H \\
    C_T \cos \Lambda + C_Y \sin \Lambda \\
    (C_{M_y} + \bar{t} C_H) \cos \Lambda + C_Q \sin \Lambda \\
    C_{M_z} - \bar{t} C_Y \\
    -C_Q \cos \Lambda + \left(C_{M_y} + \bar{t} C_H \right) \sin \Lambda
\end{bmatrix}
\]  

(C.44)

where \( \bar{t} \) is defined as the normalized mast height, \( \frac{h}{R} \). The nondimensionalized rotor aerodynamic forces are then defined as the product of the aerodynamic coefficients and perturbation velocities:

\[
\left( \frac{C_T}{\sigma a} \right)_{aero} = T_0 + T_\xi (\alpha^*_z - \zeta^*_0) + T_\beta + T_\beta^* \beta_0^* + T_\lambda z_p^* + T_\theta (\theta_0 - K_p \beta_0)
\]  

(C.45)
\[
\left(\frac{2C_H}{\sigma a}\right)_{\text{aero}} = H_\mu \left(-\bar{\tau}_x \alpha_x^* + \alpha_x V^* - y_p^*\right) - H_\zeta \left(-\zeta_{1c}^* + \zeta_{1c}\right)
\]
\[
+ H_\beta \left(\beta_{1s}^* + \alpha_y^*\right) + H_\theta \left(\theta_{1s} - K_p\beta_{1s}\right) - H_\beta \beta_{1s}
\]
\[\text{(C.46)}\]

\[
\left(\frac{2C_Y}{\sigma a}\right)_{\text{aero}} = -H_\mu \left(\bar{\tau}_x \alpha_x^* + \alpha_x V^* - y_p^*\right) - H_\zeta \left(-\zeta_{1c}^* + \zeta_{1c}\right)
\]
\[
- H_\beta \left(\beta_{1c}^* + \alpha_y^*\right) - H_\theta \left(\theta_{1s} - K_p\beta_{1s}\right) - H_\beta \beta_{1c}
\]
\[\text{(C.47)}\]

\[
\left(\frac{C_Q}{\sigma a}\right) = Q_0 + Q_\zeta (\alpha_z^* - \zeta_0^*) + Q_\beta \beta_0^* + Q_\theta \zeta_0^* + Q_\theta (\theta_0 - K_p\beta_0)
\]
\[\text{(C.48)}\]

where the subscripts are defined as in Eqs. C.20 – C.23. The thrust, hub force, side force, and torque coefficients \((C_T, C_H, C_Y, C_Q)\) are defined as normal and the asterisk superscript indicates a derivative with respect to azimuth angle \(\left(\frac{\partial}{\partial \psi}\right)\). The inertial contributions to the forces imparted to the wing by the rotor are resolved separately; the aerodynamic and inertial contributions of the nondimensionalized moment coefficients are given as:

\[
\left(\frac{C_T}{\sigma a}\right)_{\text{inertial}} = -\left(\frac{S_{bs}}{\gamma}\right) \beta_0^{**} - \left(\frac{M_b^*}{\gamma} \varsigma_p^{**}\right)
\]

\[
\left(\frac{2C_H}{\sigma a}\right)_{\text{inertial}} = -\frac{S_b^*}{\gamma} \zeta_{1s}^{**} - \frac{2}{\gamma} M_b^* \left(x_p^{**} + \bar{\tau}_x \alpha_y^{**}\right)
\]

\[
\left(\frac{2C_Y}{\sigma}\right)_{\text{inertial}} = -\frac{S_b^*}{\gamma} \zeta_{1c}^{**} - \frac{2}{\gamma} M_b^* \left(y_p^{**} - \bar{\tau}_x \alpha_x^{**}\right)
\]
\[\text{(C.49)}\]

\[
\left(\frac{C_Q}{\sigma a}\right)_{\text{inertial}} = -\frac{I_{\alpha_0}}{\gamma} \alpha_0^{**} - \frac{I_b^s}{\gamma} \alpha_z^{**}
\]

\[
\frac{2CM_x}{\sigma a} = -\frac{I_b^s}{\gamma} (\nu_\beta - 1) \beta_{1s}
\]

\[
\frac{2CM_y}{\sigma a} = \frac{I_b^s}{\gamma} (\nu_\beta - 1) \beta_{1c}
\]

where the double asterisk superscript \((**)\) indicates a second derivative with respect to the rotor azimuth angle \(\left(\frac{\partial^2}{\partial \psi^2}\right)\).

The coupled wing/rotor system is then assembled into a global matrix. Both the aerodynamic and inertial contributions from the wing and rotor are included, and the cantilevered boundary conditions on the wing are enforced for an eigenanalysis. The global mass, damping, and stiffness matrices are thus

\[
[M_g] = [M_w] + [M_r]
\]

\[
[C_g] = [C_w] + [C_r]
\]
\[\text{(C.50)}\]

\[
[K_g] = [K_w] + [K_r]
\]
Bibliography


doi: 10.2514/8.9417


http://www.servodatabase.com/servo/jr/ds8717


doi: 10.1155/2008/304362

doi: 10.2514/1.26706

