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ADDRESSING CAPACITANCE-RESISTANCE MODELING LIMITATIONS AND
INTRODUCING A NEW PRACTICAL FORMULATION

A Thesis in
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by
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ABSTRACT

Capacitance-resistance modeling (CRM) has become a popular and convenient tool that is used to model production rates and assist in reservoir characterization. Upon successful modeling, engineers would be able to link reservoir injected volumes to respective production wells, quantify injection signal dissipation times, and qualitatively infer presence of no-flow barriers and fractures. Requiring only well rates and bottom-hole pressures as inputs, CRM is able to mimic streamline simulation when seeking injected fluid allocation, and help guide reservoir engineers to educated decision making. However, this nonlinear semi-analytical tool, due to its simplicity, comes with a few documented limitations.

This study aims to build a robust capacitance-resistance model by both addressing modeling limitations, and introducing a new modeling equation. CRM limitations of extended producer shut-in times and new well introduction have been addressed by employing pseudo-injection rates. Aquifer influx cases are also successfully modeled by mapping influx rates to a new injector. If the aquifer influx cannot be modeled, the rates are inferred from the CRM's match quality, but are not a guarantee representation of actual aquifer influx rates. The limitation of highly compressible fluids, however, remains to be a modeling hurdle.

The new modeling equation encapsulates the compound heterogeneity effect on injection streams from near-producer and far injection-well volumes. The model is validated against a numerical reservoir simulator, and compared to another documented equation. The model produced acceptable results using fewer variables with physically-accepted values. Cases used for modeling include an inverted 9-spot injection pattern, a compartmentalized reservoir, and dual stacked reservoirs with cross-flow communication.
# TABLE OF CONTENTS

List of Figures .............................................................................................................. vi
List of Tables ................................................................................................................ xi
Acknowledgements ...................................................................................................... xii

Chapter 1 INTRODUCTION ......................................................................................... 1

Chapter 2 LITERATURE REVIEW .............................................................................. 3
  2.1 Historical work ...................................................................................................... 3
  2.2 Recent Developments .......................................................................................... 5
    2.2.1 Resistance Modeling ..................................................................................... 5
    2.2.2 Capacitance-Resistance Modeling ................................................................. 7
  2.3 Current Limitations .............................................................................................. 11

Chapter 3 PROBLEM STATEMENT ............................................................................. 12

Chapter 4 CRM LIMITATIONS AND SOLUTIONS .................................................... 14
  4.1 Extended Shut-in Period ...................................................................................... 16
    4.1.1 Problems ....................................................................................................... 16
    4.1.1 Solution ......................................................................................................... 18
  4.2 New Production Wells .......................................................................................... 20
    4.2.1 Problems ....................................................................................................... 20
    4.2.2 Solution ......................................................................................................... 22
  4.3 Presence of Aquifers ............................................................................................ 24
    4.3.1 Problems ....................................................................................................... 24
    4.3.2 Solution ......................................................................................................... 24
  4.4 Highly Compressible Fluid .................................................................................. 28
    4.4.1 Problems ....................................................................................................... 28

Chapter 5 CAPACITANCE-RESISTANCE MODEL DEVELOPMENT ......................... 29
  5.1 CRM Proof of Concept ........................................................................................ 29
    5.1.1 Homogeneous Single Phase 1P5I.................................................................... 29
    5.1.2 Inverted 9-Spot Pattern ............................................................................... 32
    5.1.3 Heterogeneous Two-Phase 12P10I ................................................................. 45
  5.2 New CRM Development ...................................................................................... 58
    5.2.1 Model Introduction ....................................................................................... 58
    5.2.2 Model Benchmarking .................................................................................... 62

Chapter 6 CONCLUSION AND RECOMMENDATIONS ........................................... 87

REFERENCES ............................................................................................................... 90
Appendix  MODEL DERIVATION .................................................................................................................. 92
NOMENCLATURE ...................................................................................................................................... 95
LIST OF FIGURES

Figure 1: Linear Variation of Injection Rate ................................................................. 9
Figure 2: Stepwise Variation of Injection Rate ............................................................... 10
Figure 3: Numerical model used to test CRM limitations ............................................... 14
Figure 4: Well-1 Base Case Match for Production Rate ................................................. 15
Figure 5: Well-2 Base Case Match for Production Rate ............................................... 15
Figure 6: Well-1 Extended Shut-in Case Rate Plot ......................................................... 17
Figure 7: Well-2 Extended Shut-in Case Rate Plot ......................................................... 17
Figure 8: Applying Solution to Well-1 Extended Shut-in Case ........................................ 19
Figure 9: Applying Solution to Well-2 Extended Shut-in Case ........................................ 20
Figure 10: Well-1 New Production Wells Case Rate Plot ............................................. 21
Figure 11: Well-2 New Production Wells Case Rate Plot ............................................. 21
Figure 12: Applying Solution to Well-1 New Production Wells Case ............................ 22
Figure 13: Applying Solution to Well-2 New Production Wells Case ............................ 23
Figure 14: Production Rate Match (Well-1, Horizontal Well) ......................................... 25
Figure 15: Rate Match (Well-2, Vertical Well) ............................................................... 26
Figure 16: Rate Match (Well-3, Vertical Well) ............................................................... 26
Figure 17: Rate Match (Well-4, Vertical Well) ............................................................... 27
Figure 18: Rate Match (Well-5, Vertical Well) ............................................................... 27
Figure 19: Water Injection Rates for 1P5I Example ......................................................... 30
Figure 20: Prd-1 Production Rate for 1P5I Example ...................................................... 30
Figure 21: Prd-1 Production Rate for 1P5I Example with Numerical Constraints .......... 31
Figure 22: Permeability Map of Inverted 9-Spot Injection Pattern ............................... 32
Figure 23: Well-1 Production Rate Match for Inverted 9-Spot. .............................................. 33
Figure 24: Well-2 Production Rate Match for Inverted 9-Spot. .............................................. 34
Figure 25: Well-3 Production Rate Match for Inverted 9-Spot. .............................................. 34
Figure 26: Well-4 Production Rate Match for Inverted 9-Spot. .............................................. 35
Figure 27: Well-1 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR). ................................................................. 36
Figure 28: Well-2 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR). ................................................................. 37
Figure 29: Well-3 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR). ................................................................. 37
Figure 30: Well-4 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR). ................................................................. 38
Figure 31: Compartmentalized Inverted 9-Spot Pattern.......................................................... 38
Figure 32: Well-1 Production Rate Match for Compartmentalized Inverted 9-Spot. ............... 40
Figure 33: Well-2 Production Rate Match for Compartmentalized Inverted 9-Spot. ............... 40
Figure 34: Well-3 Production Rate Match for Compartmentalized Inverted 9-Spot. ............... 41
Figure 35: Well-4 Production Rate Match for Compartmentalized Inverted 9-Spot. ............... 41
Figure 36: Well-1 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR)........ 43
Figure 37: Well-2 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR)........ 43
Figure 38: Well-3 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR)........ 44
Figure 39: Well-4 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR)........ 44
Figure 40: Injection Rates for Inverted 9-Spot. ........................................................................ 45
Figure 41: 3D Structure Map of Dual-Stacked Reservoir 12P10I Model. ............................... 46
Figure 42: East-West X-section of High Permeability Patches. ................................................. 46
Figure 43: North-South X-section of Fractures Connecting The Two Reservoirs. ................. 47
Figure 44: Water Injection Rates for 12P10I Example. ............................................................. 47
Figure 45: Total Production Rates for 12P10I Example. ......................................................... 48
Figure 46: CRMIP Production Rate Match for Wells 1,2,3,4 for 12P10I Example. .......... 49
Figure 47: CRMIP Production Rate Match for Wells 5,6,7,8 for 12P10I Example. .......... 49
Figure 48: CRMIP Production Rate Match for Wells 9,10,11,12 for 12P10I Example. ........ 49
Figure 49: PRA11 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 52
Figure 50: PRA12 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 52
Figure 51: PRA13 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 53
Figure 52: PRA21 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 53
Figure 53: PRA22 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 54
Figure 54: PRA23 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 54
Figure 55: PRB21 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 55
Figure 56: PRB22 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 55
Figure 57: PRB23 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 56
Figure 58: PRB31 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 56
Figure 59: PRB32 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 57
Figure 60: PRB33 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR) .................................................. 57
Figure 61: CRMIP Representation ............................................................................. 60
Figure 62: CRMIP Representation ............................................................................. 60
Figure 63: CRMIP Representation ............................................................................. 61
Figure 64: Well-1 Production Rate Match for Inverted 9-Spot .................................... 64
Figure 65: Well-2 Production Rate Match for Inverted 9-Spot .................................... 64
Figure 66: Well-3 Production Rate Match for Inverted 9-Spot .................................... 65
Figure 67: Well-4 Production Rate Match for Inverted 9-Spot .................................... 65
Figure 68: Well-1 $f_i$ vs $\tau_i$ Plot (CRMID) .............................................................. 66
Figure 69: Well-2 $f_i$ vs $\tau_i$ Plot (CRMID). ................................................................. 66
Figure 70: Well-3 $f_i$ vs $\tau_i$ Plot (CRMID). ................................................................. 67
Figure 71: Well-4 $f_i$ vs $\tau_i$ Plot (CRMID). ................................................................. 67
Figure 72: Well-1 Production Rate Match for Compartmentalized Inverted 9-Spot. .......... 69
Figure 73: Well-2 Production Rate Match for Compartmentalized Inverted 9-Spot. .......... 69
Figure 74: Well-3 Production Rate Match for Compartmentalized Inverted 9-Spot. .......... 70
Figure 75: Well-4 Production Rate Match for Compartmentalized Inverted 9-Spot. .......... 70
Figure 76: Well-1 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID). ....... 72
Figure 77: Well-2 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID). ....... 72
Figure 78: Well-3 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID). ....... 73
Figure 79: Well-4 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID). ....... 73
Figure 80: CRMID Production Rate Match for Wells 1,2,3,4 for 12P10I Example. .......... 74
Figure 81: CRMID Production Rate Match for Wells 5,6,7,8 for 12P10I Example. .......... 75
Figure 82: CRMID Production Rate Match for Wells 9,10,11,12 for 12P10I Example. ....... 75
Figure 83: Well 1 (CRMID) Match Improvement................................................................. 76
Figure 84: Well 2 (CRMID) Match Improvement................................................................. 77
Figure 85: Well 3 (CRMID) Match Improvement................................................................. 77
Figure 86: Well 6 (CRMID) Match Improvement................................................................. 78
Figure 87: Well 12 (CRMID) Match Improvement............................................................... 78
Figure 88: PRA11 $f_i$ vs $\tau_i$ Plot (CRMID)................................................................. 79
Figure 89: PRA12 $f_i$ vs $\tau_i$ Plot (CRMID)................................................................. 80
Figure 90: PRA13 $f_i$ vs $\tau_i$ Plot (CRMID)................................................................. 80
Figure 91: PRA21 $f_i$ vs $\tau_i$ Plot (CRMID)................................................................. 81
Figure 92: PRA22 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................. 81
Figure 93: PRA23 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 82
Figure 94: PRB21 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 82
Figure 95: PRB22 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 83
Figure 96: PRB23 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 83
Figure 97: PRB31 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 84
Figure 98: PRB32 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 84
Figure 99: PRB33 $f_i$ vs $\tau_i$ Plot (CRMID) ................................................................ 85
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression Variables for Example 1P5I</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>Regression Variables for Example 1P5I with Numerical Constraints</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>Initial Rate Values for Inverted 9-Spot</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>Productivity Index Values for Inverted 9-Spot</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>Initial Rate Values for Compartmentalized Inverted 9-Spot</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>Productivity Index Values for Compartmentalized Inverted 9-Spot</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>CRMIP R² Match Values for 12P10I Problem</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>Tabulated Values of $q_{ij}^0$ for 12P10I Problem</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>CRMIP and Equation [24] Comparison</td>
<td>59</td>
</tr>
<tr>
<td>10</td>
<td>Producer Time Constants and Productivity Indices for Base Case</td>
<td>62</td>
</tr>
<tr>
<td>11</td>
<td>Producer Time Constants and Productivity indices for Compartmentalized Case</td>
<td>68</td>
</tr>
<tr>
<td>12</td>
<td>CRMID R² Match Values for 12P10I Problem</td>
<td>75</td>
</tr>
<tr>
<td>13</td>
<td>CRMID Improved R² Match Values for 12P10I Problem</td>
<td>79</td>
</tr>
<tr>
<td>14</td>
<td>Producer Time Constants for 12P10I Problem</td>
<td>85</td>
</tr>
</tbody>
</table>
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Chapter 1

INTRODUCTION

Capacitance-resistance modeling is a method that utilizes signal processing analogous to that found in electric circuits. Injection support along with rate decay and bottom-hole pressure changes are used to infer the capacitance-resistance key variables in terms of injection weights and time constants. Injection streams are equivalent to electrical current, and the system pore volume and fluid compressibility describe a capacitor in which that energy is stored. This energy is then released to a resistor in the form of a production well.

A Capacitance Resistance Model (CRM) only requires rate and bottom-hole-pressure data as inputs. No fluid nor rock properties are needed as in the case of reservoir simulation. The outputs of CRMs are sets of time constants and injection weights including productivity indices in the case of variable bottom-hole-pressure producers. These outputs then can be used for production rate modeling and qualitative reservoir characterization. In terms of injection weights, capacitance-resistance modeling can be compared to streamline simulation (Cheng, 2007) where injected volumes are paired to production sinks. Unlike streamline simulation however, these weights are not influence by grid, fluid, nor rock properties that embed a lot of error and uncertainty.

In this report, a literature review in Chapter 2 will cover all significant work that has been done in: capacitance-resistance modeling, reported modeling limitations, and current CRM utilizations. Chapter 3 will clearly state the problem statement of this report and what the goals of
this study are. Chapters 4 and 5 will be the core of all accomplished work. Chapter 4 will cover CRM limitations and solutions while Chapter 5 will be focused on modeling different cases and presenting a new equation form. Finally Chapter 6 will end this report with some conclusions and recommendations.
Chapter 2

LITERATURE REVIEW

Work on employing electric circuits to help understand hydrocarbon reservoirs has been one of the oldest analytical methods that were adopted due to the weakness of computing powers. After the development of fast computers, it became convenient to solve reservoir partial differential equations that better represented the physical behavior of that system. Today, engineers are revisiting old reservoir engineering concepts, and redeveloping equations that will save time and provide insights to be used in further assessments.

Capacitance-resistance modeling is an analytic method to model production rate using problem-specific constants. The naming comes from its analogous approach to an electric circuit whereby the resistors are production wells, reservoir rock and fluids represent a capacitor, and the injectors are the source for electric current.

2.1 Historical work

Resistance-Capacitance (RC) networks were used as an early analytical method to model fluid flow in porous media (Bruce, 1942). The analogy come from the reservoir's storage of reservoir fluid (capacitance) and production well's consumption of that energy (resistance).

This analogy can be presented by the conservation of mass for a single phase:

\[ Q_{in}^* - Q_{out}^* = V_t \frac{d(\rho t)}{dt} \]  [1]
To model fluid flow in porous media, the governing Darcy's law is used:

\[ Q = \frac{Q^*}{\rho} = -\frac{kA}{\mu B} \cdot \frac{dP}{L} \]  \[2\]

Solving for the pressure drop:

\[ dP = -Q \frac{\mu B L}{k A} \]  \[3\]

Introducing the known definition of electrical resistance:

\[ R = \frac{\bar{V}}{\bar{I}} = \frac{rL}{A} \]  \[4\]

We can now compare the electrical resistance to Darcy's law through the analogy of voltage to \( \Delta P \), and flow rate to electrical current.

Hence:

\[ \left( R \equiv \frac{dP}{Q} = \left[\frac{1}{\bar{I}}\right]^{-1} \right) \text{ and } \left( r \equiv \frac{\mu B}{k} \right) \]  \[5\]

This gives us our analogous system resistance and resistivity definitions.

Introducing the definition of oil density for under-saturated system

\[ \rho = \rho_0 e^{c(P - P_0)} \]  \[6\]

Hence:

\[ \frac{d\rho}{dt} = c\rho \frac{dP}{dt} \]  \[7\]
Substituting this back into mass equation we get:

\[ Q_{in} - Q_{out} = V_t c_l \frac{dP}{dt} \]  \[ \text{[8]} \]

Which is equivalent to:

\[ \dot{I} = V_t c_l \frac{dV}{dt} \]  \[ \text{[9]} \]

This brings us to the current-voltage relationship for a capacitor:

\[ \dot{I} = C \frac{dE}{dt} \]  \[ \text{[10]} \]

Hence:

\[ (C \equiv V_p S_t c_l) \]  \[ \text{[11]} \]

Which gives us the analogous system capacitance definition.

\[ \text{2.2 Recent Developments} \]

\[ \text{2.2.1 Resistance Modeling} \]

Albertoni and Lake (Albertoni & Lake, 2003) introduced a linear multivariate regression tool which was strictly a resistance model that used injection and production rates to solve for weighting factors. The idea behind it was that production rates could be modeled as a sum of linearly-weighted injection rates. Albertoni also incorporated diffusivity filters to improve model matching.
Two different equations were used by Albertoni and Lake based upon if, or if not, the field injection and production volumes were balanced. The balanced multivariate linear regression equation is:

\[ q_j^n = \sum_{i=1}^{NI} \{ \beta_{ij} l_i^n \} \]  \hspace{1cm} [12]

Whereas the unbalanced multivariate linear regression equation is:

\[ q_j^n = \beta_{0j} + \sum_{i=1}^{NI} \{ \beta_{ij} l_i^n \} \]  \hspace{1cm} [13]

As it appears, the equations look to better represent steady-state incompressible flow. This would not agree well with all tested data due to an expected time lag between injection support and production rate. To fix this issue in presence of dissipation, diffusivity filters were incorporated as follows:

\[ q_j^n = \sum_{i=1}^{NI} \left\{ \beta_{ij} \sum_{k=0}^{n} (\alpha^k l_i^{n-k}) \right\} \]  \hspace{1cm} [14]

Where

\[ \alpha^k = \frac{\int_{t=k}^{t=k+1} \Delta q dt}{\int_{t=0}^{t=n} \Delta q dt} \]  \hspace{1cm} [15]

Gentil later explored the physical meaning of injection weights and incorporated bottom-hole pressure values (Gentil, 2005). He defined an injection weight to be the ratio of the transmissibility between an injector and the respective producer over the sum of transmissibilities between the same injector and all other benefiting producers.
2.2.2 Capacitance-Resistance Modeling

Al-Yousef transformed the linear model to a nonlinear tool that incorporated capacitance to make the model more capable of characterization (Al-Yousef, 2006). This was done by introducing a time constant that is a function of compressibility, pore volume, and well productivity-index. These sets of parameters (injection weights and time constants) were then utilized in flow-capacity and log-log plots to infer the presence of geological features such as highly conductive flow channels and no-flow barriers.

His final CRM equation has the following form:

\[
q^{(t)}_{ij} = q^{(t_0)}_{ij} \cdot e^{-\frac{(t-t_0)}{\tau}} + \frac{e^{-t}}{\tau} \int_{\xi=t_0}^{\xi=t} f^{(\xi)}_i e^{\frac{\xi}{\tau}} d\xi \\
+ J \left[ \frac{e^{-t}}{\tau} \int_{\xi=t_0}^{\xi=t} e^{\frac{\xi}{\tau}} P_{wf}^{(\xi)} d\xi - P_{wf}^{(t)} + e^{-\frac{(t-t_0)}{\tau}} P_{wf}^{(t_0)} \right]
\]

[16]

After discretizing (Equation [16]), extending it to multiple wells, and incorporate their interaction through bottom-hole pressures, the final form for a balanced capacitance model (BCM) is:

\[
q_j(n) = \lambda_p q(n_0) e^{-\frac{(n-n_0)}{\beta}} + \sum_{i=1}^{i=1} \lambda_{ij} j_i^{(n)} + \sum_{k=1}^{k=K} \nu_k \left[ P_{wf}^{n_0}(n_0) e^{-\frac{(n-n_0)}{\beta}} - P_{wf}^{n_0}(n) + P_{wf}^{n_0}(n) \right]
\]

[17]
Where

\[ i_{ij}^{(n)} = \sum_{m=0}^{\Delta n-1} \frac{\Delta n}{\tau_{ij}} e^{-\frac{(m-n)}{\tau_{ij}}} i_{ij}^{(m)} \]  \hspace{1cm} \text{[18]}

And

\[ p'_{\alpha j}^{(n)} = \sum_{m=0}^{\Delta n-1} \frac{\Delta n}{\tau_{j}} e^{-\frac{(m-n)}{\tau_{j}}} p_{\alpha j}^{(m)} \]  \hspace{1cm} \text{[19]}

The final form of the unbalanced capacitance model (UCM) is achieved by adding a constant rate term to (Equation [17]).

Sayarpour later improved the final representation of the equation by first analytically solving the integrals of an equation similar to (Equation [16]), presented in the Appendix, then applying superposition in time and space which resulted in an equation used for both cases of balanced and unbalanced rates. Furthermore, different sets of discretized equations were introduced to incorporate newly implemented control volumes (Sayarpour M., 2008). These control volumes are CRMIP for injector/producer intra-volume, CRMP for producer near-by volume, and CRMT for a tank representation of the entire system. The discretized version of the CRMIP model assuming linearly varying injection rates (LVIR) between any two adjacent data points is given in the following equation:

\[
q_j^n = \sum_{i=1}^{N_I} \left( q_j^0 e^{-\frac{(n-1)}{\tau_{ij}}} + f_{ij} \left( I_j^n - I_j^0 e^{-\frac{(n-1)}{\tau_{ij}}} \right) \right)
- \tau_{ij} \sum_{k=1}^{n} \left( e^{-\frac{-(n-k)}{\tau_{ij}}} - e^{-\frac{-(n-k-1)}{\tau_{ij}}} \right) \left( f_{ij} \frac{\Delta f_k}{\Delta t_k} + f_j \frac{\Delta P_k^{ij}}{\Delta t_k} \right) \right)
\]  \hspace{1cm} \text{[20]}
If injection rates are assumed to vary in a stepwise fashion (SVIR), the equation has this simpler form:

\[
q_j^n = \sum_{i=1}^{NI} q_{ij}^0 e^{-\frac{(t_n-t_0)}{\tau_{ij}}} + \sum_{k=1}^{n} \left\{ \left( e^{\frac{-(t_n-t_{k})}{\tau_{ij}}} - e^{\frac{-(t_n-t_{k-1})}{\tau_{ij}}} \right) \left[ f_{ij} l_i^k - \tau_{ij} f_{ij} \Delta p_{wj}^k \Delta t_k \right] \right\} \tag{21}
\]

The difference between the two approaches is evident when injector boundary conditions are based on pressure, and even more so if we are dealing with data points sampled at large time steps (Figure 1 and Figure 2). The two figures both show two sets of data, one with small time steps and another with larger time steps.

![Figure 1: Linear Variation of Injection Rate](image)
Capacitance-resistance models have been used for many implementations including utilization for injection strategy optimization (Weber D. B., 2009), estimation of waterflood performance (Sayarpour, Zuluaga, Kabir, & Lake, 2007; Chen, Zhang, Sun, & Li, 2010), reservoir characterization (Yousef, Lake, & Jensen, 2006; Nguyen, Kim, Lake, Edgar, & Haynes, 2011), optimization of oil production rates (Liang, et al., 2007), and estimation of fracture distribution (M. Delshad & Pourafshary, 2009).
2.3 Current Limitations

With capacitance resistance models being heuristic and requiring only few and simple data inputs, many limitations are expected. Some of these limitations are attributed to the model itself while others are different cases of data input. Examples of limitations due to the nature of the model include highly varying fluid compressibility. This restricts reservoir fluids to water and oil due to these fluids being incompressible and slightly compressible, respectively. Another limitation is altering well productivity index. This restricts the use of data whereby wells are recompleted and/or well skin values are altered. Data restrictions, however, are experienced in modeling a time interval whereby new production wells are introduced, producers are frequently shut-in, and presence of external sinks/sources (Al-Yousef, 2006).

Varying fluid compressibility and well productivity index will create variable time constants (τ), making it impossible to form a solution with the current CRM foundations. Similarly does changing well completions, skin values, and number of active producers on the constant injection weights. As for presence of aquifers (external sink/source), a CRM requires all contributing injection rates to be known inputs which is difficult if aquifer flux rate is unknown.
Chapter 3

PROBLEM STATEMENT

This study is focused on developing a stand-alone capacitance-resistance model that is capable of quantifying injector-producer communication. After analyzing model outputs, it becomes possible to identify presence of fractures and no-flow boundaries between injector-producer pairs, and predict total production rates. The study will also attempt to address currently present CRM limitations by providing either solutions or assessments on how these limitations can be addressed. Finally, the CRMIP equation will be re-examined and enhanced to provide better models.

The need for this study stems from an attempt to strengthen CRM's position when compared to alternatives, namely numerical reservoir and streamline simulators. Reservoir and streamline simulation require a vast amount of data that is accompanied by large error and uncertainty, hence the need for history matching. Luckily, a CRM only requires observed and measured data in well rates and bottom-hole pressures. In that sense, CR modeling has the upper hand in avoiding the need for a static geological model, and requiring less data. However, there are a lot of limitations that prevent CRM from breaking into the competition that numerical simulators are in. CRM limitations restrict the model from reaching greater support. Therefore, it is of paramount importance to address these limitations and try to produce a CRM that is as versatile as possible. These limitations include high fluid compressibility, extended shut-in times for production wells, presence of external flow sinks/sources (e.g. open boundary reservoirs), and significant changes in well productivity index (e.g. well workover and recompletion).

As for changing the currently present CRM, this was done after witnessing that present models have a somewhat large spectrum of non-unique solutions due to existence of redundantly
unknown variables. Furthermore, these sets of unknowns are rarely in physical agreement, and are very difficult to validate. The new formulation takes care of that by reducing the number of unknowns by a minimum of $NP^*NI-NP$ to a maximum of twice that number depending on the percentage of variable BHP producers among total production wells. Reducing the number of non-unique solutions will play a great role in placing more emphasis on the essential variables.

To summarize the objectives of this research study, the target is to construct a stand-alone CRM tool, re-evaluate current models, and clearly state the limitations of capacitance-resistance modeling while providing solutions to some of these limitations. Initially, the already developed equations for CRM will be utilized as a proof of concept in matching observed rate data and testing the capability of solved weights and time constants in reservoir characterization. A new CRM will be presented, and CRM limitations will be addressed. All numerical data was generated by IMEX black-oil simulator developed by Computer Modeling Group (CMG).
Capacitance-resistance modeling is possible only when all model unknowns remain constant. When modeling below the bubble-point pressure, energy and compressibility of the system vary with production and injection making it difficult to represent the control volume with a single capacitor. Limitations of extended shut-in periods and new production wells will be tested on a 10x15x7 grid with two water injectors and two producers. The numerical model used for data input along with the CRM results for the base case (no limitations) are displayed in the few following figures.

Figure 3: Numerical model used to test CRM limitations
Figure 4: Well-1 Base Case Match for Production Rate.

Figure 5: Well-2 Base Case Match for Production Rate.

The limitation of aquifer presence will be tested on a different two-phase model containing four vertical and one horizontal producers in a reservoir with bottom aquifer support.
Before leaping into any of these limitations, we must touch upon some of the data issues that can cause difficulties in modeling. These sources of error include few data points, collinearity, choice of initial values, poor data quality, constant production rates, poor/constant injection signals, and transient-flow dominated data (Al-Yousef, 2006; Sayarpour M., 2008). Although these cases can result in poor modeling quality, they will not be analyzed in this report.

4.1 Extended Shut-in Period

In this section, an investigation of problems caused by data containing shut-in production wells will be conducted.

4.1.1 Problems

The problems associated with extended shut-in periods for production wells are due to their effect on varying injection weights. As mentioned in previous sections, injection weights are treated as constants. When a producer is shut in, injection flow streams regrouping to join new paths towards lower pressure points. All other online producing wells will inherit the weights of the shut-in producers resulting in CRM errors during that shut-in period. Below are rate plots of the two producers for the entire time period in which Well-1 is shut-in from time=1,090 days to time=1,400 days.
It can be clearly seen in (Figure 7) that the model faces difficulty in matching the high rate surge due to the newly inherited injection streams resulting from shutting-in Well-1. In
general, this effect is witnessed in all producers that are mutually supported by injectors when some of these producers are shut-in.

4.1.1 Solution

The solution to problems that arise from shut-in producers is somewhat simple, yet very effective. All what is required is to deceive the online producers into thinking that none of the production wells are shut-in. To do that, the injection rates supporting the online producers must be modified to include the support initially allocated to these shut-in wells. Also, injection rates supplied to producers during their shut-in periods should be replaced by zero. Hence both online and offline producers will be supplied with two different sets of pseudo-injectors during well shut-in times in a fashion that will honor the material balance of the system.

Pseudo injection-rate for online producers:

\[ I_i^* = I_i^1 \times \left(1 + \sum_{j}^{NP_{\text{shut-in}}} f_{ij} \right), \quad \text{for all } i = 1 \text{ to } NI \]  \[ 22 \]

Pseudo-injection rates for offline producers:

\[ I_i^* = 0, \quad \text{for all } i = 1 \text{ to } NI \]  \[ 23 \]

Adjusting injection rates to zero for shut-in wells is not done to improve our model match as we can see from Well-1’s match quality. However, it is needed because the presence or absence of injection streams will affect the values needed of model variables to equate the CRM
equation to zero during shut-in periods, along with the need to preserve the material balance. After applying these adjustments to injection rates, the match of the online producer improves significantly as seen below (Figure 9).

Figure 8: Applying Solution to Well-1 Extended Shut-in Case.
4.2 New Production Wells

Problems caused by introducing new production wells will be discussed in this section alongside practical solutions.

4.2.1 Problems

The weakness of capacitance-resistance models in introducing new production wells during modeling is similar to that encountered in producer shut-in periods. Introducing producers that are newly drilled, or shut-in at modeling start time, does not agree with the modeling rule of constant injection weights. After introducing a new production well, injection weights must change to accommodate the new draw-down effect on established injection streams. Using our
same numerical model, the first producer is put on stream at a later stage. The negative results of this behavior can be seen in the production match plots below.

**Figure 10:** Well-1 New Production Wells Case Rate Plot.

**Figure 11:** Well-2 New Production Wells Case Rate Plot.
As witnessed in (Figure 11), the CRM did not perform well in matching the production profile of Well-2. It appears that the model over-predicts the well's production rate after the introduction of Well-1.

4.2.2 Solution

The same adjustments made to the shut-in producers in the previous section can be applied to this limitation. That would mean adjusting injection rates to zero for times preceding Well-1’s introduction, solving for the Well-1’s injection weights, and finally applying (Equation [22]) to solve for Well-2’s unknown variables. Improvements on the base case can be seen in the plots below where the Well-2 match improved from $R^2=0.55$ to 0.88.

Figure 12: Applying Solution to Well-I New Production Wells Case.
It must be noted that when applying this to a large number of newly introduced wells, it needs to be done in a systematic procedure. This would involve starting from the most recent producer then adjusting prior injection rates, and proceeding in a backward fashion. The biggest pitfall to this solution method obviously is when we fail to match one of these new wells because the weights are needed with low error to ensure good matches to other production wells. To avoid this pitfall, instead of using a weight with great error, one could include it as an unknown in the CRM equation.
4.3 Presence of Aquifers

4.3.1 Problems

Aquifer support during modeling represents an additional injector with unknown injection rate. CRM requires all supporting rate data to be provided, hence missing that piece of information will affect the quality of our model.

4.3.2 Solution

If the aquifer influx rate can be correctly modeled, mapping the influx rate to a new injector should be an easy task. However, if the rate cannot be modeled for the studied time interval, the task becomes very difficult. Two approaches can help in modeling production rates supported by aquifer influx.

The first approach applies to models that achieve acceptable match quality without additional injectors. The model should not apply constraints on any injection weights other than \( f_{ij} \geq 0 \). The approximated aquifer influx would then be modeled as an additional injector with injection rates equal to the sum of injection rate support resulting from all weights exceeding the value of one. This approach is weak if there are very few wells because the influx profile would almost be identical to a few injection rates. The second approach is therefore recommended to be used instead, or alongside the first method.

The second approach is more tedious and requires some engineering analysis. If the boundary condition of the aquifer is of constant influx rate, an injector with an arbitrary constant injection rate is added. The rate is then increased, or decreased, until an acceptable match is achieved with \( \sum f_{\text{Aquifer},j} \leq 1 \). Alternatively if the aquifer support is of constant pressure, the influx rate should be modeled to be the sum of all well production-rates supported by the aquifer. The
influx rate is then reduced to equate to the fraction of influx supporting the production wells as per CRM aquifer weights. Many injector wells can be added to simulate different sectors of the aquifer. However, it must be noted that having a perfectly matched model does not guarantee that the injection rate used is an exact match to actual aquifer influx rate. Furthermore, if the aquifer influx is not of constant rate, the model parameters cannot be used to predict future rates after the training period.

Below we are testing a reservoir with five producers, with Well-1 being the only horizontal well among four other vertical producers. The reservoir is supported by a bottom aquifer across the entire area of the reservoir. As seen below, the CRM matched all well profiles along with good indications to its recognition of the horizontal well. The horizontal well (Well-1) was given a CRM productivity index of 428.7 BPD/psi where the average productivity index of the four vertical wells was only 42.2 BPD/psi.

![Well-1 (R²=0.996)](image)

**Figure 14:** Production Rate Match (Well-1, Horizontal Well).
Figure 15: Rate Match (Well-2, Vertical Well).

Well-2 ($R^2=0.941$)

![Graph showing production rate vs. time for Well-2 with $R^2=0.941$.]

Figure 16: Rate Match (Well-3, Vertical Well).

Well-3 ($R^2=0.991$)

![Graph showing production rate vs. time for Well-3 with $R^2=0.991$.]
The aquifer support in the previous plots was modeled using the second approach by summation of production rates. An attempt to model it using constant influx was unsuccessful as
the CRM did not converge, and the first approach was not an option due to the absence of injection wells.

4.4 Highly Compressible Fluid

4.4.1 Problems

Saturated reservoirs, or fluids of high compressibility, are considered the greatest weakness to capacitance-resistance models. In a CRM, all values of \( \tau \) are considered constant where \( \tau = (V_p c_f)/J \). Water and oil are only incompressible and slightly compressible fluids respectively whereas gas is a fluid of high compressibility leading to very large errors in capacitance-resistance modeling. Simple cases of modeling can only be accomplished during a time interval when the reservoir becomes under-saturated (Al-Yousef, 2006). There are not yet any CRMs to date that incorporate dynamic \( \tau \) values which are needed to manage fluid compressibility changes.
Chapter 5
CAPACITANCE-RESISTANCE MODEL DEVELOPMENT

5.1 CRM Proof of Concept

A CRM was developed using CRMIP LVIR (Equation 20) and tested through reproducing production rate profiles and, in some cases, characterizing reservoir volumes between injector-producer pairs.

5.1.1 Homogeneous Single Phase 1P5I

This example demonstrates applying CR modeling to match the production rate of a single well in a homogeneous and isotropic water reservoir by solving for an initial rate, injection weight, and time constant for each injector-producer pair. With the production well operating at constant bottom-hole pressure, and the presence of five injectors, this leads to solving for a total of 15 unknowns. The injection rates for all water injectors are presented in (Figure 19) along with the observed and modeled production rate for the single producer in (Figure 20). The $R^2$ (coefficient of determination) for this match is a perfect 0.999, with all the variables leading to this match listed in (Table 1). If we do restrict the injection weights to their physical constraints of $(\sum_i(f_i)\leq1)$, the model still maintains a healthy $R^2$ of 0.998 (Figure 21 & Table 2).
Figure 19: Water Injection Rates for 1P5I Example.

Figure 20: Prd-1 Production Rate for 1P5I Example
Table 1: Regression Variables for Example 1P5I

<table>
<thead>
<tr>
<th>Injector</th>
<th>$q_{ij}^0$</th>
<th>$\tau_{ij}$</th>
<th>$f_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inj-1</td>
<td>4,648</td>
<td>60.2</td>
<td>1.01</td>
</tr>
<tr>
<td>Inj-2</td>
<td>4,745</td>
<td>47.5</td>
<td>0.65</td>
</tr>
<tr>
<td>Inj-3</td>
<td>15,672</td>
<td>54.7</td>
<td>1.27</td>
</tr>
<tr>
<td>Inj-4</td>
<td>13,494</td>
<td>33.0</td>
<td>1.17</td>
</tr>
<tr>
<td>Inj-5</td>
<td>3,757</td>
<td>1.4</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Figure 21: Prd-1 Production Rate for 1P5I Example with Numerical Constraints

Table 2: Regression Variables for Example 1P5I with Numerical Constraints

<table>
<thead>
<tr>
<th>Injector</th>
<th>$q_{ij}^0$</th>
<th>$\tau_{ij}$</th>
<th>$f_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inj-1</td>
<td>2,826</td>
<td>70.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Inj-2</td>
<td>7,484</td>
<td>61.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Inj-3</td>
<td>13,119</td>
<td>57.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Inj-4</td>
<td>14,462</td>
<td>53.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Inj-5</td>
<td>4,543</td>
<td>1.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>
5.1.2 Inverted 9-Spot Pattern

This section will present a two-phase reservoir (water, oil) containing five injectors and four producers in an inverted 9-spot injection pattern with the production wells operating on variable bottom-hole pressure. First part will be a base case while the second will implement no-flow barriers to construct separate compartments.

5.1.2.1 Base Case

![Permeability Map of Inverted 9-Spot Injection Pattern.](image)

This case (Figure 22) is an 11x11 two phase (water, oil) 2D model containing an inverted nine-spot injection pattern. Permeability is isotropic and homogeneous (k=50 md) throughout the grid except for the cells connecting Inj-1 and Well-1 (k=200 md), and between Inj-4 and Well-2 (k=100 md). The CRM achieved reasonable match quality for the four producers as presented in the following plots. The variables solved for also come into terms with the physics presented by the parent reservoir when analyzing the figures of $f_{ij}$ versus $\tau_{ij}$ (Figure 27 to Figure 30). We can observe in those figures that both Well-1 and Well-2 have good overall communication with
reservoir injectors compared to the other remaining producers. This is evident by the small time-
constant and large weight values associated to their respective injectors especially between Well-
1 & Inj-1 and Well-2 & Inj-4.

Figure 23: Well-1 Production Rate Match for Inverted 9-Spot.
Figure 24: Well-2 Production Rate Match for Inverted 9-Spot.

Figure 25: Well-3 Production Rate Match for Inverted 9-Spot.
Figure 26: Well-4 Production Rate Match for Inverted 9-Spot.

Table 3: Initial Rate Values for Inverted 9-Spot.

<table>
<thead>
<tr>
<th>$q^0_{ij}$</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector</td>
<td>Well-1</td>
</tr>
<tr>
<td>Inj-1</td>
<td>3185.53</td>
</tr>
<tr>
<td>Inj-2</td>
<td>2.48</td>
</tr>
<tr>
<td>Inj-3</td>
<td>6.64</td>
</tr>
<tr>
<td>Inj-4</td>
<td>3818.76</td>
</tr>
<tr>
<td>Inj-5</td>
<td>4.36</td>
</tr>
<tr>
<td>sum:</td>
<td>7017.76</td>
</tr>
<tr>
<td>data:</td>
<td>8343.26</td>
</tr>
<tr>
<td>error:</td>
<td>-16%</td>
</tr>
</tbody>
</table>
Table 4: Productivity Index Values for Inverted 9-Spot.

<table>
<thead>
<tr>
<th>$J_{ij}$</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector</td>
<td>Well-1</td>
</tr>
<tr>
<td>Inj-1</td>
<td>2.26</td>
</tr>
<tr>
<td>Inj-2</td>
<td>4.33</td>
</tr>
<tr>
<td>Inj-3</td>
<td>0.00</td>
</tr>
<tr>
<td>Inj-4</td>
<td>0.00</td>
</tr>
<tr>
<td>Inj-5</td>
<td>4.20</td>
</tr>
</tbody>
</table>

Figure 27: Well-1 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
Figure 28: Well-2 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 29: Well-3 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
5.1.2.2 No-Flow Barriers

Figure 30: Well-4 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 31: Compartmentalized Inverted 9-Spot Pattern.
Here we apply zero transmissibility multipliers to form three separate compartments to the same previously introduced, inverted 9-spot pattern. Wells Well-1 and Well-4 are together with injectors Inj-1 and Inj-2 in an isolated compartment. Similarly is Well-3 with injectors Inj-4 and Inj-5 while Well-2 is separated on its own with Inj-3 (Figure 31). We would expect the CRM to correctly predict injector-producer communication as grouped in these three compartments.

In the following plots (Figure 32 to Figure 35), we can see very successful models that match the compartmentalized production rates. Nevertheless, variables achieving these matches must also correctly represent the actual reservoir and supporting injectors. For example, in (Figure 37) we can see that Well-2 is only communicating with Inj-3 because all other injection weights are zero. This correctly agrees with the reservoir compartment containing these two wells. Well-3 also shows very good communication with injectors located in the same compartment, however, other injectors seem to provide some support although their respective injection weights are small compared to Inj-4 and Inj-5 (Figure 38). As for the last compartment containing producers Well-1 and Well-4, both display connectivity with Inj-1 and Inj-2, and zero communication with Inj-4. However, Well-1 and Well-4 indicate support from Inj-5 which is not part of their group. The only injector to demonstrate exclusive support to its relevant producers is Inj-4. This and whatever misinforming results presented are products of the similarities in injection pulse times that can mislead the CRM (Figure 40).
Figure 32: Well-1 Production Rate Match for Compartmentalized Inverted 9-Spot.

Figure 33: Well-2 Production Rate Match for Compartmentalized Inverted 9-Spot.
Figure 34: Well-3 Production Rate Match for Compartmentalized Inverted 9-Spot.

Figure 35: Well-4 Production Rate Match for Compartmentalized Inverted 9-Spot.
Table 5: Initial Rate Values for Compartmentalized Inverted 9-Spot.

<table>
<thead>
<tr>
<th>$q_{ij}^0$</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector</td>
<td>Well-1</td>
</tr>
<tr>
<td>Inj-1</td>
<td>6078.58</td>
</tr>
<tr>
<td>Inj-2</td>
<td>0.00</td>
</tr>
<tr>
<td>Inj-3</td>
<td>0.04</td>
</tr>
<tr>
<td>Inj-4</td>
<td>1259.46</td>
</tr>
<tr>
<td>Inj-5</td>
<td>0.00</td>
</tr>
<tr>
<td>sum:</td>
<td>7338.08</td>
</tr>
<tr>
<td>data:</td>
<td>8341.83</td>
</tr>
<tr>
<td>error:</td>
<td>-12%</td>
</tr>
</tbody>
</table>

Table 6: Productivity Index Values for Compartmentalized Inverted 9-Spot.

<table>
<thead>
<tr>
<th>$J_{ij}$</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector</td>
<td>Well-1</td>
</tr>
<tr>
<td>Inj-1</td>
<td>0.00</td>
</tr>
<tr>
<td>Inj-2</td>
<td>4.28</td>
</tr>
<tr>
<td>Inj-3</td>
<td>4.07</td>
</tr>
<tr>
<td>Inj-4</td>
<td>6.39</td>
</tr>
<tr>
<td>Inj-5</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Figure 36: Well-1 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR).

Figure 37: Well-2 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR).
Figure 38: Well-3 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR).

Figure 39: Well-4 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMIP LVIR).
5.1.3 Heterogeneous Two-Phase 12P10I

The following case is a 3D two-phase (water, oil) model with twelve producers and ten flank water injectors (Figure 41). The model represents two low-permeability stacked reservoirs with a thin impermeable layer in-between. Both reservoirs contain patches of high permeability and separate water-oil contacts (Figure 42). The impermeable layer is fractured in the middle to allow fluid communication between the two reservoirs (Figure 43). All twenty-two wells are completed in the low-permeability matrix.
Figure 41: 3D Structure Map of Dual-Stacked Reservoir 12P10I Model.

Figure 42: East-West X-section of High Permeability Patches.
Figure 43: North-South X-section of Fractures Connecting The Two Reservoirs.

Reservoir A (top reservoir) contains three injectors in the west flank and two injectors in the east flank. On the other hand, reservoir B (bottom reservoir) contains two injectors in the west flank and three injectors in the east. As for production wells, reservoir A has three producers in the first layer and two in the second layer while reservoir B has two producers in its second layer and three in its third layer. All wells operate on constant bottom-hole pressure.

Figure 44: Water Injection Rates for 12P10I Example.
Using the injection rates (Figure 44) and production rates (Figure 45) of the system, the CRMIP LVIR model was able to perform reliable matches with $R^2$ values ranging from a low of 0.737 to a high of 0.99 (Table 7). Plots of total reservoir production rates for both observed data and CRM are displayed in the following plots.
Figure 46: CRMIP Production Rate Match for Wells 1,2,3,4 for 12P10I Example.

Figure 47: CRMIP Production Rate Match for Wells 5,6,7,8 for 12P10I Example.

Figure 48: CRMIP Production Rate Match for Wells 9,10,11,12 for 12P10I Example.
Table 7: CRMIP $R^2$ Match Values for 12P10I Problem.

<table>
<thead>
<tr>
<th>Producer</th>
<th>$R^2$ Match</th>
<th>Producer</th>
<th>$R^2$ Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prd-1</td>
<td>0.837</td>
<td>Prd-7</td>
<td>0.755</td>
</tr>
<tr>
<td>Prd-2</td>
<td>0.939</td>
<td>Prd-8</td>
<td>0.968</td>
</tr>
<tr>
<td>Prd-3</td>
<td>0.797</td>
<td>Prd-9</td>
<td>0.737</td>
</tr>
<tr>
<td>Prd-4</td>
<td>0.990</td>
<td>Prd-10</td>
<td>0.790</td>
</tr>
<tr>
<td>Prd-5</td>
<td>0.984</td>
<td>Prd-11</td>
<td>0.886</td>
</tr>
<tr>
<td>Prd-6</td>
<td>0.951</td>
<td>Prd-12</td>
<td>0.923</td>
</tr>
</tbody>
</table>

Upon analysis of CRM initial rate match variables (Table 8), we can see that only three producers had initial-rate ($q_0^i$) matches within 15% error of the observed data values. Having lower initial rates compared to observed data would force the model to high injection weights to make up for lower projected rates and vice versa. This essentially means that both sets of initial rates and injection weights should work together for matching the observed data which is discussed in the new CRM development section.

Table 8: Tabulated Values of $q_0^i$ for 12P10I Problem

<table>
<thead>
<tr>
<th>$q_0^i$</th>
<th>PRA11</th>
<th>PRA12</th>
<th>PRA13</th>
<th>PRA21</th>
<th>PRA22</th>
<th>PRA23</th>
<th>PRB21</th>
<th>PRB22</th>
<th>PRB23</th>
<th>PRB31</th>
<th>PRB32</th>
<th>PRB33</th>
</tr>
</thead>
<tbody>
<tr>
<td>INAE1</td>
<td>0.01</td>
<td>2.39</td>
<td>28.68</td>
<td>15.9</td>
<td>0.00</td>
<td>0.00</td>
<td>480.46</td>
<td>80.5</td>
<td>0.12</td>
<td>1.37</td>
<td>0.16</td>
<td>219.25</td>
</tr>
<tr>
<td>INAE2</td>
<td>25.38</td>
<td>195.32</td>
<td>10.87</td>
<td>75.86</td>
<td>0.00</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>161.71</td>
<td>28.99</td>
<td>5.12</td>
<td></td>
</tr>
<tr>
<td>INAW1</td>
<td>0.01</td>
<td>41.58</td>
<td>5.44</td>
<td>160.85</td>
<td>23.44</td>
<td>158.83</td>
<td>12.46</td>
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<td>2.28</td>
<td>50.29</td>
<td>68.20</td>
<td></td>
</tr>
<tr>
<td>INAW2</td>
<td>0.11</td>
<td>203.84</td>
<td>28.18</td>
<td>48.71</td>
<td>268.63</td>
<td>0.00</td>
<td>49.55</td>
<td>1.62</td>
<td>26.01</td>
<td>7.79</td>
<td>56.47</td>
<td>0.23</td>
</tr>
<tr>
<td>INAW3</td>
<td>32.80</td>
<td>0.97</td>
<td>128.58</td>
<td>251.77</td>
<td>327.56</td>
<td>3.67</td>
<td>2.13</td>
<td>0.00</td>
<td>3.43</td>
<td>2.34</td>
<td>40.30</td>
<td>0.13</td>
</tr>
<tr>
<td>INBE1</td>
<td>62.04</td>
<td>97.58</td>
<td>121.06</td>
<td>69.05</td>
<td>58.87</td>
<td>196.54</td>
<td>648.37</td>
<td>0.01</td>
<td>44.74</td>
<td>122.79</td>
<td>21.40</td>
<td>2.06</td>
</tr>
<tr>
<td>INBE2</td>
<td>73.38</td>
<td>109.14</td>
<td>31.47</td>
<td>123.32</td>
<td>189.53</td>
<td>0.02</td>
<td>131.74</td>
<td>0.01</td>
<td>5.08</td>
<td>6.68</td>
<td>7.12</td>
<td>2.98</td>
</tr>
<tr>
<td>INBE3</td>
<td>186.68</td>
<td>1.33</td>
<td>8.78</td>
<td>0.04</td>
<td>151.09</td>
<td>0.00</td>
<td>24.12</td>
<td>121.95</td>
<td>2.71</td>
<td>5.81</td>
<td>1.46</td>
<td>0.04</td>
</tr>
<tr>
<td>INBW1</td>
<td>113.88</td>
<td>328.21</td>
<td>48.9</td>
<td>55.92</td>
<td>101.19</td>
<td>105.32</td>
<td>27.2</td>
<td>250.39</td>
<td>115.56</td>
<td>0.03</td>
<td>25.43</td>
<td>112.70</td>
</tr>
<tr>
<td>INBW2</td>
<td>60.58</td>
<td>0.39</td>
<td>20.04</td>
<td>55.24</td>
<td>98.09</td>
<td>69.88</td>
<td>1087.02</td>
<td>0.00</td>
<td>102.66</td>
<td>2.46</td>
<td>34.49</td>
<td>0.11</td>
</tr>
<tr>
<td>sum:</td>
<td>554.87</td>
<td>980.75</td>
<td>432.00</td>
<td>856.66</td>
<td>1218.40</td>
<td>534.26</td>
<td>2464.10</td>
<td>589.83</td>
<td>486.16</td>
<td>334.03</td>
<td>267.01</td>
<td>410.82</td>
</tr>
<tr>
<td>data:</td>
<td>392.9548</td>
<td>396.5747</td>
<td>392.5411</td>
<td>1870.54</td>
<td>576.1797</td>
<td>594.2173</td>
<td>1803.207</td>
<td>576.4764</td>
<td>606.9466</td>
<td>785.8226</td>
<td>153.3088</td>
<td>332.208</td>
</tr>
<tr>
<td>error:</td>
<td>41.2%</td>
<td>147.3%</td>
<td>10.1%</td>
<td>-54.2%</td>
<td>111.5%</td>
<td>-10.1%</td>
<td>36.7%</td>
<td>2.3%</td>
<td>-19.9%</td>
<td>-57.5%</td>
<td>74.2%</td>
<td>23.7%</td>
</tr>
</tbody>
</table>
The five-character naming convention used for this example is as follows:

- First two letters indicate well type. PR for producer and IN for injector.
- Third letter is for reservoir. Reservoir "A" is the top reservoir and "B" is the bottom reservoir.
- The fourth character for injector naming is a letter indicating which flank the well is located at whereas for producer naming it is a number indicating which layer the well is completed in. All injectors are completed in the third layer of their respective reservoirs.
- The last character is utilized for numbering.

Naming (Prd-1..Prd-12 and Inj-1...Inj-10) follow the order injectors and producers appear in as in the previous table.

The next figures are plots for injection weights \(f_{ij}\) versus time constants \(\tau_{ij}\) for all twelve producers. The location of a data point in the plot describes how well an injector communicates with the producer and what percentage of its injected fluids is allocated to that producer. For example, we can see from (Figure 49) that PRA11 is only supported by two of the three reservoir A west-flank injectors. More analysis on certain plots is done in the new CRM development section.
Figure 49: PRA11 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 50: PRA12 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
Figure 51: PRA13 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 52: PRA21 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
Figure 53: PRA22 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 54: PRA23 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
Figure 55: PRB21 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 56: PRB22 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
Figure 57: PRB23 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).

Figure 58: PRB31 $f_i$ vs $\tau_i$ Plot (CRMIP LVIR).
Figure 59: PRB32 \( f_i \) vs \( \tau_i \) Plot (CRMIP LVIR).

Figure 60: PRB33 \( f_i \) vs \( \tau_i \) Plot (CRMIP LVIR).
5.2 New CRM Development

5.2.1 Model Introduction

This new model is introduced to address a few observations in the CRMIP models; namely the number of non-unique solutions, and behavior of model variables. This can be fixed by decreasing the number of variables. The CRMIP introduced earlier (Equation [20]) has \( N_P \times N_I \times 3 \) unknowns for constant bottom-hole-pressure producers and \( N_P \times N_I \times 4 \) unknowns for variable bottom-hole-pressure producers. On the other hand, the new formulation reduces the number of unknowns to \( N_P \times (N_I \times 2 + 1) \) for constant bottom-hole pressure and \( N_P \times (N_I \times 2 + 2) \) for variable bottom-hole pressure producers.

The main idea behind this new form (Equation [24]) is to reduce the number of unknowns and allow all these variables to have a more concrete meaning we can relate to. Convoluted partial initial rates and partial productivity indices are replaced by one known initial rate and one unknown productivity index (Table 9). This formulation can simply be thought of as a hybrid between CRMIP (Figure 61) and CRMP (Figure 62) that contains producer near-by volume and connecting injection stream paths. For convenient referencing, the new formulation with be referred to as CRMID in this document.

\[
q_j^n = q_j^0 e^{-\frac{(t_n-t_0)}{\tau_j}} + \sum_{l=1}^{N_I} \left\{ f_{ij} \left[ l^n - l_i^0 e^{-\frac{(t_n-t_0)}{\tau_j}} \right] - \tau_j \sum_{k=1}^{n} \left\{ \left( e^{-\frac{(t_n-t_k)}{\tau_j}} - e^{-\frac{(t_n-t_k-1)}{\tau_j}} \right) f_{ij} \frac{\Delta l_k}{\Delta t_k} \right\} \right\} \\
- \tau_j \sum_{k=1}^{n} \left\{ \left( e^{-\frac{(t_n-t_k)}{\tau_j}} - e^{-\frac{(t_n-t_k-1)}{\tau_j}} \right) f_{ij} \frac{\Delta P^k_w}{\Delta t_k} \right\} 
\]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation 20 (CRMIP LVIR)</th>
<th>Equation 24</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Rate</td>
<td>$q_{ij}^0$</td>
<td>$q_j^0$</td>
<td>• Reduced $q_{ij}^0$ unknowns from $NI*NP$ to zero.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Initial rate is a known value, on the other hand it is very difficult to validate $q_{ij}^0$ values.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• When $\sum q_{ij}^0 \neq q_j^0$, other variable values will be affected when performing a match.</td>
</tr>
<tr>
<td>Productivity Index</td>
<td>$J_{ij}$</td>
<td>$J_j$</td>
<td>• Reduced $J$ unknowns from $NI*NP$ to $NP$.</td>
</tr>
<tr>
<td>Time Constant</td>
<td>$\tau_{ij}$</td>
<td>$\tau_{ij} &amp; \tau_j$</td>
<td>• Increased $\tau$ unknowns by $NP$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Separate between media of injection stream path and producer drainage area because initial rate and productivity index should not be affected by injector parameters</td>
</tr>
<tr>
<td>Injection Weights</td>
<td>$f_{ij}$</td>
<td>$f_{ij}$</td>
<td></td>
</tr>
<tr>
<td>Total Unknowns</td>
<td>$NP*(NI*4)$</td>
<td>$NP*(NI*2+2)$</td>
<td>Variable bottom-hole-pressure producers.</td>
</tr>
<tr>
<td></td>
<td>$NP*(NI*3)$</td>
<td>$NP*(NI*2+1)$</td>
<td>Constant bottom-hole-pressure producers.</td>
</tr>
</tbody>
</table>
Figure 61: CRMIP Representation.

Figure 62: CRMP Representation.
This reduction of number of unknowns in CRMID should not be confused with an expected improvement in the CRM and observed data match. CRMIP models have greater freedom in fine-tuning complete sets of $q^0_{ij}, f_{ij}, \tau_{ij},$ and $J_{ij}$ values to match an observed rate. This behavior is frequently seen when a certain injector-producer relationship has an initial rate, injection weight, and productivity index that do not complement each other (Sayarpour, Zuluaga, Kabir, & Lake, 2007). In (Equation [24]), the contribution of an injector is entirely dependent on the injection weight; if the weight is zero, the injection contribution is exactly zero. However, in CRMIP equation an injector can have zero weight, but still play a role in a well's production rate through $q^0_{ij}$ and $J_{ij}$. 

Figure 63: CRMID Representation.
5.2.2 Model Benchmarking

Comparing CRMID to CRMIP, the previous multi-producer models will be utilized using the same input data and initial values. The CRM output variables of the two equations will then be evaluated against each other.

5.2.2.1 Inverted 9-Spot Pattern

Presenting the matches for the base case inverted 9-spot model, the producer time constants and productivity indices of CRMID model fully support the parent reservoir and well locations. Examining (Table 10), Well-1 has the greatest productivity index followed by Well-2. These two producers are completed in enhanced permeability blocks, and all producer productivity indices are almost proportional to the hosting-cell permeability value. Productivity index of Well-1 is almost twice that of Well-2, and Well-2’s productivity index is also nearly double the value of Well-3 or Well-4. As for producer time constants, Well-1 has the smallest value followed by Well-4, Well-2, and Well-3. CRM match quality can be seen in (Figure 64) to (Figure 67).

<table>
<thead>
<tr>
<th>Producer</th>
<th>τ</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well-1</td>
<td>62.61</td>
<td>9.57</td>
</tr>
<tr>
<td>Well-2</td>
<td>114.19</td>
<td>5.52</td>
</tr>
<tr>
<td>Well-3</td>
<td>135.60</td>
<td>2.32</td>
</tr>
<tr>
<td>Well-4</td>
<td>108.86</td>
<td>2.20</td>
</tr>
</tbody>
</table>
The CRMIP overall match quality was superior to what the CRMID equation was able to achieve. However, the previously mentioned inconsistencies of injector-producer pair values can still be witnessed in CRMIP output. Going back to the CRMIP partial initial-rates and productivity-indices in (Table 3) and (Table 4), we can see examples of incoherent $q_{ij}$ and $J_{ij}$ pairs along with their $f_{ij}$ and $\tau_{ij}$ values. This expectedly helps in achieving better match quality as the CRMIP has more flexibility in terms of the number of parameters. Moving on to CRMID injector-producer time constants and weights, Well-1 does not show preferable communication with Inj-1 although the other injectors do appear in better communication (Figure 68). Well-2 emerges with (Figure 69) to have no communication with Inj-2, but does receive the greatest support from Inj-4. Well-3 is not in communication with any of the two furthest injectors Inj-2 and Inj-3, and its main support comes from Inj-5 (Figure 70). Finally, (Figure 71) shows that Well-4 collects a small portion of all injection streams.

As for the earlier CRMIP results, Well-1 is in good contact with Inj-1 followed by Inj-4 although it takes up about half the injection support from wells Inj-2 and Inj-5. Well-2 is also greatly supported by Inj-4, however, Inj-3 appears to be the furthest away although it is one of the closest two injectors to Well-2. CRMIP shows Well-3 receiving about the same weighted support from all injectors with Inj-1 being the closest and injectors Inj-2 and Inj-3 being the furthest. Finally, Well-4 injection weights are almost in agreement with CRMID although the injector time constants are not.
Figure 64: Well-1 Production Rate Match for Inverted 9-Spot.

Figure 65: Well-2 Production Rate Match for Inverted 9-Spot.
Figure 66: Well-3 Production Rate Match for Inverted 9-Spot.

Figure 67: Well-4 Production Rate Match for Inverted 9-Spot.
Figure 68: Well-1 \( f_i \) vs \( \tau_i \) Plot (CRMID).

Figure 69: Well-2 \( f_i \) vs \( \tau_i \) Plot (CRMID).
Figure 70: Well-3 $f_i$ vs $\tau_i$ Plot (CRMID).

Figure 71: Well-4 $f_i$ vs $\tau_i$ Plot (CRMID).
Now comparing the compartmentalized model results, CRMIP models are still superior in matching the observed well production rates especially for Well-4 (Figure 75). The producer productivity indices for CRMID are comparable to the previous base case which is an interesting observation. Producer time constants, however, are of expected smaller values due to the smaller volumes they reside in. That is true except for Well-3 which now has a larger time constant due to its slightly smaller productivity index (Table 11). Looking back on CRMIP’s $J_{ij}$ values (Table 6), we experience the same problem with inconsistent values when comparing paired $q_{ij}^0$, $f_{ij}$, and $\tau_{ij}$ values. For example, looking at the variables for Well-1, it can be seen that Inj-1 and Inj-2 have the largest weights among all other injectors yet Inj-2 has zero initial rate, Inj-4 has a high initial rate and the greatest productivity index although its weight is zero while the productivity index of the highly contributing Inj-1 is zero. These values of zero productivity index, similarly found in the base case, should theoretically result in infinite time-constant values since $\tau = V_p c_i / J$.

<table>
<thead>
<tr>
<th>Producer</th>
<th>$\tau$</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well-1</td>
<td>48.70</td>
<td>9.79</td>
</tr>
<tr>
<td>Well-2</td>
<td>55.83</td>
<td>4.87</td>
</tr>
<tr>
<td>Well-3</td>
<td>177.67</td>
<td>1.85</td>
</tr>
<tr>
<td>Well-4</td>
<td>88.20</td>
<td>2.16</td>
</tr>
</tbody>
</table>
Figure 72: Well-1 Production Rate Match for Compartmentalized Inverted 9-Spot.

Figure 73: Well-2 Production Rate Match for Compartmentalized Inverted 9-Spot.
Figure 74: Well-3 Production Rate Match for Compartmentalized Inverted 9-Spot.

Figure 75: Well-4 Production Rate Match for Compartmentalized Inverted 9-Spot.
Observing the plots below that describe injector-producer communication, Well-1 shows great support from the only injectors located in the same compartment Inj-1 and Inj-2, and nearly zero and extremely far-off support from Inj-4. However, injectors Inj-3 and Inj-5 do not indicate zero communication which is similar to CRMIP results although in this model they both have larger time-constant values. Well-2 presents Inj-3 with significant support, but the rest of injectors with much larger time constants and very low injection weights. The corresponding CRMIP figure might show more accurate injection weights; however, these injectors with very small weights apply great contribution with their respective time constants, initial rates, and productivity indices that have a major role in achieving CRMIP's better match. CRMID's Well-3 numbers prove the best when compared to CRMIP. Weights of Inj-4 and Inj-5 are both close to one with the other three injectors having very small values. Although the CRMIP model for Well-3 had less convincing numbers, it was still a better match than the CRMID model. Finally with Well-4, both formulations have almost similar injection weight values although CRMIP presents Inj-1 and Inj-2 with smaller time constants. It must be reiterated that although some CRMIP injector-producer pair weight values are close or equal to zero, the pair could still have a considerable effect through $q_{ij}^0$, $J_{ij}$, and $\tau_{ij}$ values whereas in CRMID all injector contribution is judged solely by injection weights.
Figure 76: Well-1 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID).

Figure 77: Well-2 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID).
Figure 78: Well-3 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID).

Figure 79: Well-4 $f_i$ vs $\tau_i$ Plot for Compartmentalized Inverted 9-Spot (CRMID).
5.2.2.2 Heterogeneous Two-Phase 12P10I

The CRMID results for this problem are displayed in the next three figures. Straight away we can see the improvements to the matches bearing in mind that now there is no error in any initial rates unlike the previous results of CRMIP. Another feature of this formulation is the quick indications it gives for non-contributing injectors. All injectors that do not provide support to a specific producer will most likely give very high $\tau_{ij}$ values and/or zero $f_{ij}$ values. Modeling a production rate using more injectors than the ones actually contributing can have negative implications on the model results and output. There are a few techniques in identifying these non-contributing injectors that become more apparent in large reservoirs. The techniques include injector removal based on distance to producers, and based on a weight cutoff value. Hence, an injector will not be considered if it is more than a distance away from a respective producer, or if the injection weight between these two wells is less than a certain small value (Weber, Edgar, Lake, Lasdon, Kawas, & Sayarpour, 2009).

Figure 80: CRMID Production Rate Match for Wells 1,2,3,4 for 12P10I Example.
Figure 81: CRMID Production Rate Match for Wells 5,6,7,8 for 12P10I Example.

Figure 82: CRMID Production Rate Match for Wells 9,10,11,12 for 12P10I Example.

Table 12: CRMID $R^2$ Match Values for 12P10I Problem.

<table>
<thead>
<tr>
<th>Producer</th>
<th>R2 Match</th>
<th>Producer</th>
<th>R2 Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRA11</td>
<td>0.748</td>
<td>PRB21</td>
<td>0.990</td>
</tr>
<tr>
<td>PRA12</td>
<td>0.962</td>
<td>PRB22</td>
<td>0.992</td>
</tr>
<tr>
<td>PRA13</td>
<td>0.828</td>
<td>PRB23</td>
<td>0.764</td>
</tr>
<tr>
<td>PRA21</td>
<td>0.990</td>
<td>PRB31</td>
<td>0.904</td>
</tr>
<tr>
<td>PRA22</td>
<td>0.990</td>
<td>PRB32</td>
<td>0.990</td>
</tr>
<tr>
<td>PRA23</td>
<td>0.925</td>
<td>PRB33</td>
<td>0.896</td>
</tr>
</tbody>
</table>
After removal of injectors that had very large time constants and/or zero weights, CRMID showed much improvement in the overall match. The same was done to CRMIP, however, a slightly better match was achieved in only one producer. Well 1 in CRMID showed that the time constant for injector 10 was 6.5E151 whereas injector 1 appeared with a time constant of 1.6E17 for CRMIP. After removing these wells from their respective models, the CRMIP match declined from $R^2=0.837$ to 0.767 with no more extremely large time constants nor zero injection weights. CRMID match also decreased, but three injectors appeared with zero injection weights. By iteratively removing these injectors, the remaining injectors ended with contributing values and the overall match improved and surpassed CRMIP's initially greater $R^2$ match. Below are the plots of production well rates after removal of non-contributing injectors for CRMID.

![Well 1 (CRMID) Match Improvement](image)

Figure 83: Well 1 (CRMID) Match Improvement.
Figure 84: Well 2 (CRMID) Match Improvement.

Figure 85: Well 3 (CRMID) Match Improvement.
Figure 86: Well 6 (CRMID) Match Improvement.

Figure 87: Well 12 (CRMID) Match Improvement.
Table 13: CRMID Improved $R^2$ Match Values for 12P10I Problem.

<table>
<thead>
<tr>
<th>Producer</th>
<th>$R^2$ Match</th>
<th>Producer</th>
<th>$R^2$ Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRA11</td>
<td>0.959</td>
<td>PRB21</td>
<td>0.990</td>
</tr>
<tr>
<td>PRA12</td>
<td>0.990</td>
<td>PRB22</td>
<td>0.992</td>
</tr>
<tr>
<td>PRA13</td>
<td>0.987</td>
<td>PRB23</td>
<td>0.807</td>
</tr>
<tr>
<td>PRA21</td>
<td>0.990</td>
<td>PRB31</td>
<td>0.904</td>
</tr>
<tr>
<td>PRA22</td>
<td>0.990</td>
<td>PRB32</td>
<td>0.990</td>
</tr>
<tr>
<td>PRA23</td>
<td>0.943</td>
<td>PRB33</td>
<td>0.997</td>
</tr>
</tbody>
</table>

A good feature that presented itself during injector removal was that it was a commutative operation. Eliminating non-contributing injectors first by large time constants followed by zero weights would lead to the same final result as removing zero-weight injectors then ones with large time constants. Below are plots for injection weights ($f_{ij}$) versus time constants ($\tau_{ij}$) for all twelve producers along with tabulated producer time constants ($\tau_p$).

![Figure 88: PRA11 $f_i$ vs $\tau_i$ Plot (CRMID).](image-url)
Figure 89: PRA12 $f_i$ vs $\tau_i$ Plot (CRMID).

Figure 90: PRA13 $f_i$ vs $\tau_i$ Plot (CRMID).
Figure 91: PRA21 $f_i$ vs $\tau_i$ Plot (CRMD).

Figure 92: PRA22 $f_i$ vs $\tau_i$ Plot (CRMD).
Figure 93: PRA23 $f_1$ vs $\tau_1$ Plot (CRMID).

Figure 94: PRB21 $f_1$ vs $\tau_1$ Plot (CRMID).
Figure 95: PRB22 $f_i$ vs $\tau_i$ Plot (CRMID).

Figure 96: PRB23 $f_i$ vs $\tau_i$ Plot (CRMID).
Figure 97: PRB31 $f_i$ vs $\tau_i$ Plot (CRID).

Figure 98: PRB32 $f_i$ vs $\tau_i$ Plot (CRID).
Figure 99: PRB33 $f_i$ vs $\tau_i$ Plot (CRMD).

Table 14: Producer Time Constants for 12P10I Problem.

<table>
<thead>
<tr>
<th>Producer</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRA11</td>
<td>15.17</td>
</tr>
<tr>
<td>PRA12</td>
<td>4.80</td>
</tr>
<tr>
<td>PRA13</td>
<td>4.61</td>
</tr>
<tr>
<td>PRA21</td>
<td>6.15</td>
</tr>
<tr>
<td>PRA22</td>
<td>6.35</td>
</tr>
<tr>
<td>PRA23</td>
<td>16.48</td>
</tr>
<tr>
<td>PRB21</td>
<td>5.55</td>
</tr>
<tr>
<td>PRB22</td>
<td>13.32</td>
</tr>
<tr>
<td>PRB23</td>
<td>51.74</td>
</tr>
<tr>
<td>PRB31</td>
<td>2.44</td>
</tr>
<tr>
<td>PRB32</td>
<td>9.12</td>
</tr>
<tr>
<td>PRB33</td>
<td>10.05</td>
</tr>
</tbody>
</table>
From the previous plots (Figure 88 to Figure 99) we can infer producer-injector connectivity and how well they communicate as done in previous sections. Small time constants and high injection weights are good signs of strong injector-producer bonds. Hence, these figures state that producer PRA11 (reservoir A, first layer) is only supported by wells in the same reservoir and more so by injectors located in the west flank. Well PRB23 seems to be in communication with only west-flank reservoir A injectors due to its location being closer to these injectors and near the fractures that allow reservoir fluid cross-flow. Producer PRB32 (located in the bottom layer of reservoir B) detects that all reservoir B injectors come from within the same distance due to similarly equal time constants. However, stronger support is provided by the east flank.

Comparing these observations with CRMIP outputs, PRA11 is only supported by reservoir A west flank injectors and not by the east injectors as CRMID model claims. PRB23 communicates with one injector from each reservoir B flank, two west-flank and one east-flank reservoir A injectors. The injector with the strongest signal is a reservoir A-east injector whereas the one with the greatest support is a reservoir B-east injector. This conflicts with the CRMID claim that support is only due to west injectors. As for PRB32, CRMIP shows virtually no communication with any of the reservoir A west injectors and INBW2, however, there is some support with a degree of contrast among time constants of reservoir B injectors, and support from INAE1.

Another diagnostic tool to analyze is the producer time constants in (Table 14). It gives us a good evaluation of area the producer is completed in, and a general overall evaluation. When comparing values for producers completed in the same reservoir/layer, we notice that wells PRA12 and PRA13 are in good locations compared to PRA11, furthermore, producer PRB31 has the best value amongst all twelve producers. These values are successfully supported by the $f_{ij}$ versus $\tau_{ij}$ plots.
Chapter 6

CONCLUSION AND RECOMMENDATIONS

The objectives of this study were to build a capacitance-resistance model, test it against various reservoir data, and investigate CRM limitations. A CRMIP LVIR was constructed and tested against reported CRM limitations. Limitations of extended producer shut-in periods, and new production well introduction were effectively addressed by using pseudo-injection rates to substitute the behavior of injection-weight variation. Presence of aquifer support was also addressed to achieve acceptable match quality, however, the presented solution does not guarantee perfect modeling of aquifer influx rates. The final limitation of highly varying compressibility (varying time constants) still remains a hurdle to reach a fully robust capacitance-resistance model.

The built CRM was tested with a single phase reservoir with one producer and five injectors, a two-phase inverted 9-spot injection pattern, and a dual stacked reservoir with twelve producers and ten injectors. The model produced acceptable match quality for all tested data. Finally, a new form of the model equation was constructed and compared to CRMIP LVIR outputs. The new model fared well when compared to the previous CRMIP results, reaching high quality matches and acceptable physical conclusions. Its main advantages are less unknowns, zero initial rate error, and no inconsistency as witnessed in CRMIP's \( q_0, f_i, J_i, \) and \( \tau_i \) values. The CRMIP model can apply constrains to match initial rate, but it would be difficult to validate partial initial rates as is validating partial productivity indices.

In conclusion, capacitance-resistance models show considerable ability in predicting inter-well connectivity and modeling total production rates using little and readily available data.
Our introduction of pseudo injection-rates also drastically helped capacitance-resistance models apply themselves to more scenarios and eliminate the limitations of introducing new producers, presence of external support, and frequent/extended shut-in of production wells. Limitations of altering well completions, or skin value, can also be avoided via splitting the producer into two wells; one shut-in well and one newly introduced. The new control volume (CRMID) offered more convincing values for variables accompanying injector-producer time constants and injection weights when compared to CRMIP output. Most documented examples show disagreement between the strength of an injection stream, and initial rate and/or productivity index. Nevertheless, CRMIP in most cases reaches better match quality which is encouraging. However, this is a product of having more variables which might lead to inconsistencies and parameter disagreement. Another characteristic of the newly introduced equation is that it can still be used without injectors unlike CRMIP which cannot be applied without any injectors. When no injectors exist, the CRMID model forms into an equivalent equation to CRMP.

Recommendations for future work:

- In case of a large reservoir with many wells, injectors with a very small weight and/or a very large time constant should be removed. However, there should be an analysis on what are reasonable cut-off values to consider injectors as non-contributing.

- Find a way to model below bubble point and overcome the limitations of varying compressibility. This might require a completely different formulation, or a simple inclusion of new adjusting variables to the current CRM.

- Test these CRMs and compare them using real field data. Also, a comprehensive comparison between CRMs and streamline simulation needs to be evaluated.
With the success of capacitance-resistance modeling, one might consider a completely heuristic graphical approach to either match observed data or obtain good initial values to be used for capacitance-resistance modeling. In some cases during modeling, the action-reaction between injection and production rates is visually clear. It would be interesting to see the performance of artificial neural networks in forecasting production rates based on historic data used in CRM input.

- Find out more about signal detection using time constant and injection weight values. Look more into how strong an injection pulse must be for a producer to detect it, and how long it must be sustained.

- Test the sensitivity of the model to rock compressibility.

- Test modeling a single producer under natural depletion. Then add an injector and see if the producer time constant and productivity index change from the natural depletion values.

- Provide sample cases of utilizing CRM output to aid in reservoir simulation history matching.

- Compare CRM results to well-testing reports in cases of fractures and faults, and investigate if time constants agree with the time required for a well-test signature to develop.

- If using CRMIP, apply measures on initial rate and productivity index values so that they complement indications we get from weights and time constants.
REFERENCES


Appendix

MODEL DERIVATION

Starting with an elementary volume mass balance equation for a certain fluid (water)

\[ Q^*_{in} - Q^*_{out} = mass_{t+dt} - mass_t \]

\[ (I_w - q_w) \cdot \rho_w \cdot \Delta t = \Delta mass \]

Dividing by \( \rho_w \cdot \Delta t \) and substituting mass=\( V_p \cdot S_w \cdot \rho_w \)

\[ (I_w - q_w) = S_w \cdot \frac{dV_p}{dt} + S_w \cdot \frac{\rho_w}{\rho_w} \cdot \frac{dP_{avg}}{dt} + V_p \cdot \frac{dS_w}{dt} \]

Assuming constant pore volume

\[ (I_w - q_w) = S_w \cdot \frac{V_p}{\rho_w} \cdot \frac{d\rho_w}{dP_{avg}} \cdot \frac{dP_{avg}}{dt} + V_p \cdot \frac{dS_w}{dt} \]

Substituting water compressibility \( c_w=1/\rho_w \cdot d\rho_w/dP_{avg} \)

\[ (I_w - q_w) = S_w \cdot V_p \cdot c_w \cdot \frac{dP_{avg}}{dt} + V_p \cdot \frac{dS_w}{dt} \]

The same equation can be derived for the other two phases, oil and gas. After summing the other two equations and knowing that \([S_w + S_o + S_g=1]\) and \([dS_o/dt + dS_w/dt + dS_g/dt =0]\), we get:

\[ I_t - q_t = V_p \cdot c_f \cdot \frac{dP_{avg}}{dt} \]
Substituting $P_{avg}$ from definition of productivity index, and assuming constant $J$.

\[ J = \frac{q}{P_{avg} - P_{wf}} \rightarrow P_{avg} = \frac{q}{J} + P_{wf} \]

\[ I_t - q_t = V_p * c_f \frac{dP_{wf}}{dt} + \frac{V_p * c_f}{J} \frac{dq_{prod}}{dt} \]

Introducing $\tau = V_p * c_f / J$ and dividing by $\tau$ with a little rearrangement

\[ \frac{dq_t}{dt} + \frac{q_t}{\tau} = \frac{l_t}{\tau} - J * \frac{dP_{wf}}{dt} \]

Solving this PDE using integrating factor $e^{\int_0^t dt / \tau}$

\[ \frac{d}{dt} (q_t * e^{t / \tau}) = \frac{e^{t / \tau}}{\tau} l_t - J e^{t / \tau} * \frac{dP_{wf}}{dt} \]

Integrating:

\[ q_t * e^{t / \tau} = \frac{1}{\tau} \int_{\xi = t_0}^{\xi = t} l_\xi e^{\xi / \tau} d\xi - J \int_{\xi = t_0}^{\xi = t} e^{\xi / \tau} * \frac{dP_{wf}}{dt} d\xi + C \]

When $t = t_0$, constant of integration $C = q^0 * e^{t_0 / \tau}$, hence

\[ q_t * e^{t / \tau} = \frac{1}{\tau} \int_{\xi = t_0}^{\xi = t} l_\xi e^{\xi / \tau} d\xi - J \int_{\xi = t_0}^{\xi = t} e^{\xi / \tau} * \frac{dP_{wf}}{dt} d\xi + q^0 t * e^{t_0 / \tau} \]

Divide by $e^{t / \tau}$

\[ q_t = q^0 t * e^{-\frac{(t-t_0)}{\tau}} + \frac{e^{t / \tau}}{\tau} \int_{\xi = t_0}^{\xi = t} l_\xi e^{\xi / \tau} d\xi - J e^{-t / \tau} \int_{\xi = t_0}^{\xi = t} e^{\xi / \tau} * \frac{dP_{wf}}{dt} d\xi \]

Applying integration by parts for first integral we get
\[ q_t = q_0^\tau e^{-\frac{t-t_0}{\tau}} + l_t - p_0^\tau e^{-\frac{t-t_0}{\tau}} - \int_{t=t_0}^{t=t} \frac{dI_t}{dt} d\xi - J e^{-\frac{t}{\tau}} \int_{t=t_0}^{t=t} e^{-\frac{t}{\tau}} dP_{wf} d\xi \]

Giving a final form of the equation to be (assuming injection rates and bottom-hole pressures vary linearly between \( t_k \) and \( t_{k+1} \)):

\[ q_t = q_0^\tau e^{-\frac{t-t_0}{\tau}} + l_t - p_0^\tau e^{-\frac{t-t_0}{\tau}} - \tau \left(1 - e^{-\frac{t-t_0}{\tau}}\right) \left(\frac{dI_t}{dt} - J \frac{dP_{wf}}{dt}\right) \]

Now this equation can only be used for one injector and one producer in only one time step. To apply it to more wells and data points, a control volume must be chosen to then implement superposition in space and time to acquire something similar to (Equation [20]).
## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>viscosity</td>
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<td>$A$</td>
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<td>total production flow rate</td>
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<tr>
<td>$Q^*$</td>
<td>mass flow rate</td>
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<tr>
<td>$R$</td>
<td>Resistance</td>
</tr>
</tbody>
</table>
\( r = \) resistivity
\( S = \) saturation
\( t = \) time
\( \dot{V} = \) voltage
\( V_b = \) bulk volume
\( V_p = \) pore volume
\( \alpha_{ij} = \) diffusivity filter coefficient
\( \lambda = \) weight
\( \rho = \) density
\( \tau = \) time constant

**Subscript**

\( 0 = \) property at bubble point pressure
\( f = \) fluid
\( g = \) gas
\( i = \) injector \( 'I' \)
\( j = \) producer \( 'j' \)
\( l = \) liquid
\( o = \) oil
\( t = \) total
\( w = \) water

**Superscript**

\( 0 = \) time \( t=t_0 \)
\( k = \) time step \( k \)
\( n = \) time step \( n \)