ESSAYS ON FINANCIAL FRAGILITY

A DISSERTATION IN ECONOMICS
BY
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ABSTRACT

Each of the three chapters in this dissertation is based on a different research paper. The papers are linked by their topic. Each one develops a model to study the causes of bank and financial fragility, as well as policies to prevent them.

Diamond and Dybvig (1983) describe bank runs as a multiple equilibria phenomenon. However, a simple modification on their bank mechanism, namely, a suspension of convertibility when withdrawals are too numerous, is enough to eliminate the bank-run equilibrium. In their 1983 paper, Diamond and Dybvig argue that such arrangement would not work in a realistic environment where the level of withdrawals is uncertain; but the authors never designed the optimal deposit mechanism for this environment. Peck and Shell (2003) and Ennis and Keister (2009) characterize the optimal mechanism in economies with aggregate uncertainty and provide examples with a small number of depositors where the economy has a bank-run equilibrium. The difficulty of having aggregate uncertainty and a large number of depositors is that the model becomes intractable. In “Optimal Diamond-Dybvig mechanism in large economies with aggregate uncertainty” (chapter 1) I address this issue by providing a specification of aggregate uncertainty that allows me to use results from variational calculus in order to characterize the optimal deposit contract.

In traditional bank contracts depositors decide only on one dimension; how much to deposit in, or to withdraw from, their account. In terms of economic theory, this univariate choice translates into a direct mechanism where each depositor reveals only his liquidity need. A large literature, built on Diamond-Dybvig, shows that such type of mechanism is vulnerable to bank runs even when optimally designed. In “Preventing bank-runs” (chapter 2), David Andolfatto, Ed Nosal and I show that in the Diamond-Dybvig environment a bank can prevent runs by using an indirect mechanism. The mechanism
gives depositors the option of moving their funds between accounts with different levels of guarantees. This feature allows the bank to identify whether depositors believe a run is on or not. And a suspension of convertibility conditional on this information is capable of stopping depositors from running against the bank. At the end of the day, we are able to construct an indirect mechanism that implements the constrained-efficient allocation in iterated elimination of strictly dominated strategies.

New developments in the financial industry allow financial institutions to transform a pool of investments, such as loans to individual consumers or firms, into tradable financial assets—a process known as securitization. Once these assets are created, they are traded in an over-the-counter market. However, the literature on financial panics do not take into account that nowadays banks’ portfolio consist of tradable financial assets. In “Financial fragility and over-the-counter markets” (chapter 3) I propose a model of financial panics where banks trade financial assets in an over-the-counter market. The model builds on Duffie, Garleanu and Pedersen (2005) and Lagos and Rocheteau (2009). I show that Athey-Segal dynamic mechanism is an optimal mechanism in this environment. That is, it supports the constrained Pareto efficient allocation as an equilibrium outcome. Then I construct numerical examples where the economy also has an equilibrium which resembles a financial panic. In this equilibrium all depositors demand short term payments, assets price falls and the trade volume collapses.

I find that the over-the-counter market frictions are essential in generating bank panics under Athey-Segal mechanism. This finding implies that reducing trade frictions has a twofold benefit: it improves welfare by allowing for a better allocation of financial assets, and it also has the power to eliminate inefficient equilibria such as bank panics. Of course, in many cases regulators are not able to reduce trade frictions. In these cases, I propose two alternative policies to eliminate bank panics. The first is a suspension scheme and the second is the opening of a centralized exchange facility similar to the one opened by the Federal Reserve Bank during the 2007/08 financial crisis.
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Optimal Diamond-Dybvig mechanism in large economies with aggregate uncertainty

Abstract This paper characterizes the direct mechanism which implements the constrained optimal outcome in a version of Diamond and Dybvig (1983) with aggregate uncertainty and a continuum of agents. Using this result, numerical examples where the best direct mechanism has a bank-run-equilibrium are easily obtained.

Keywords Diamond-Dybvig model, bank-run, deposit contracts.

At least since Diamond and Dybvig (1983) (hereafter DD), bank runs have been thought of as arising from multiple equilibria. However, it has not been easy to show that the direct mechanism which has the best no-run-equilibrium also has a run-equilibrium in settings where the bank can commit to a mechanism. In environments without aggregate uncertainty, in which the number of impatient agents is known, a suspension scheme strongly implements the best outcome. In environments with aggregate uncertainty, the result depends on the details of the model. Green and Lin (2003) (hereafter GL) provide a negative answer in an environment with a finite number of agents and i.i.d. preference shocks when agents are informed about their position in the queue before deciding whether to withdraw or not. Peck and Shell (2003) and Ennis and Keister (2009c) (hereafter PS and EK) provide examples with a finite (and small) number of agents where the best direct mechanism has a run-equilibrium. While agents are uninformed about their position in

---

3Here and throughout the paper I consider only environments with commitment. For a discussion on bank runs in a setting without commitment see Ennis and Keister (2009a).
the queue in PS, EK consider both cases. EK also show by examples that the result obtained by GL does not hold if the preference shocks are correlated. In a setting similar to PS, Cavalcanti et al. (2011) (hereafter CBM) show that a run-equilibrium exists under the best mechanism if the population is large enough. However, they restrict their analysis to an environment with i.i.d. preference shocks. The law of large numbers implies that this economy is well approximated by an economy without aggregate uncertainty when the population is large. Therefore, CBM is not suitable for the analysis of aggregate uncertainty.

My model has aggregate risk, a nonatomic measure of agents, and the information structure in PS. The uncertainty is modelled as follows. The impatient fraction of the population is a random variable. Conditional on its realization, the event that a person is patient or impatient is i.i.d. across the population. In this setting the preference shocks are unconditionally correlated across people. The virtue of this specification is that results from variational calculus can be used to characterize the direct mechanism which has the best no-run-equilibrium. In addition, it is easy to check whether the mechanism has a run-equilibrium.

Previous papers have shown similar features in economies with a finite number of agents. However, in those papers the number of equations and unknowns is proportional to the number of agents, which is why PS only provide examples with few people. EK show that dynamic programming can be used to compute the constrained optimal allocation in large economies if the constrained optimal allocation coincides with the unconstrained one. However, for some economies, the unconstrained optimal allocation is not the constrained optimal one. In this case, the EK dynamic programming approach cannot be used while the method I propose here is still valid.

2PS have an example with 300 agents, but they restrict the number of impatient types to be either 100, 200 or 300. This restriction implies that this economy is equivalent to one with only 3 agents.
1.1. The model

In this section I describe the environment, the class of mechanisms, and the kind of equilibrium that may arise given a mechanism.

1.1.1. Environment

There are three periods, 0, 1, 2. In period 0, agents have $y$ units of wealth which they invest to consume in periods 1 and 2. The investment technology is the same of DD, an amount $x$ invested in period 0, pays gross return 1 if liquidated in period 1 and gross return $R > 1$ if liquidated in period 2. There is a continuum of people with nonatomic unit measure and they can be either patient or impatient. The utility of an individual of type impatient is $u(c_1)$, while that of type patient is $v(c_1 + c_2)$, where $(c_1, c_2)$ is consumption in periods 1 and 2 respectively. The functions $u, v : \mathbb{R}_+ \to \mathbb{R}$ are strictly increasing, strictly concave and twice continuously differentiable. Let $\alpha \in [0, 1]$ denote the fraction of agents who are impatient, where $\alpha$ is a random variable with cumulative distribution function $F$ and a continuous density function $f$ which satisfies $f(x) > 0$ for all $x \in (0, 1)$. Conditional on the realization of $\alpha$, the event that an individual is impatient follows a Bernoulli distribution with parameter $\alpha$.

I study what PS call the post-deposit game. That is, at the outset (i.e. at the beginning of period 0), I assume that all individuals deposit their resources in the bank. Then I focus solely on the agent’s decision to withdraw or not once they know the realization of their types. In period 0, individuals face uncertainty about the value of $\alpha$ and their types. They only know the distribution of $\alpha$, which is common knowledge. In the beginning of period 1, each individual observes his own type (which is private information). No one observes the realization of $\alpha$. Then, agents simultaneously decide whether to withdraw or not. The bank serves the withdrawal requests of the individuals in a random sequence, which the literature refers to as the sequential service constraint. An individual’s position in
the queue is uniformly distributed among people who decide to withdraw. Although they arrive at the bank in sequence, the rate of arrivals cannot be measured by the bank.\(^3\) After all withdrawal payments are made, what is left in the bank pays a gross return of \(R\). In period 2 the bank distributes the amount left to those who did not withdraw in period 1. Figure 1.1 depicts the sequence of actions.

![Figure 1.1 Sequence of actions.](image)

An important difference between this paper and part of the literature is the assumption that only people who decide to withdraw report to the bank. The purpose of this assumption is twofold. First, it seems realistic. We don’t observe depositors without liquidity needs contacting the bank to announce that they are not withdrawing. The second reason is technical. If all agents contact the bank, it is possible to design a mechanism that uses an arbitrarily small fraction of the population to exactly estimate the realization of the aggregate uncertainty. I rule out this possibility in order to study the effect of aggregate uncertainty on \textit{bank runs} in large economies.

1.1.2. \textit{Direct mechanisms}

I focus on direct mechanisms. The revelation principle implies that this is without loss of generality with respect to finding the best weakly implementable outcome. Because withdrawing in period 1 is equivalent to claiming to be of type \textit{impatient}, I will not distinguish between those two actions. I also do not distinguish between withdrawing

\(^3\)This assumption can be rationalized by assuming that the time interval in which agents arrive varies proportionally to the number of agents visiting the bank.
in period 2 or revealing to be of type \textit{patient}. Because the bank does not observe \(\alpha\), the mechanism describes only how much an individual consumes at date 1 as a function of the position \(z \in [0, 1]\) in the queue.

\textbf{Definition 1.1} A direct mechanism is a pair of continuous\(^4\) functions, \(m = (c_1, c_2)\), with \(c_1 : [0, 1] \to \mathbb{R}_+\), and \(c_2 : [0, 1] \to \mathbb{R}_+\). The function \(c_1(z)\) is the payment to a person who withdraws in period 1 and has position \(z\) in the queue. The function \(c_2(\kappa)\) is the payment to a person who does not withdraw when the fraction of people who withdrew is \(\kappa \in [0, 1]\).

\textbf{Definition 1.2} A direct mechanism \(m = (c_1, c_2)\) is feasible if

\[
c_2(\kappa) = \frac{y - \int_0^\kappa c_1(z) \, dz}{1 - \kappa} R \quad \text{for all} \quad \kappa \in (0, 1), \quad \text{and} \quad \int_0^1 c_1(z) \, dz = y. \tag{1.1}
\]

There are two restrictions imposed here. First, I require everyone to consume the same amount in period 2. Second, the feasibility constraints are required to hold at equality. Both restrictions are without loss of generality with respect to implementing the best outcome because utility functions are strictly increasing and concave.

A mechanism \(m\) and the sequence of actions induce a Bayesian game where each player has only two types: \textit{patient} or \textit{impatient}; and two actions: either withdraw in period 1 or period 2. A strategy profile is a function \(s\) that maps types \(\theta \in \{\text{patient, impatient}\}\) into probability measures over the periods of withdrawing, \(\{1, 2\}\). I consider only symmetric Bayesian Nash equilibria of this game, where symmetric means that players of the same type follow the same strategy.

\(^4\)The continuity on \(m\) is without loss of generality with respect to finding the constrained optimal outcome. It suffices that \(c_1\) is a piecewise continuous function. In this case \(c_1\)'s continuity can be established as a necessary condition of optimality.
1.2. The best weakly implementable outcome

By the revelation principle, we know that the best weakly implementable outcome is achieved by the truth-telling equilibrium of some direct mechanism.

Definition 1.3 A direct mechanism $m = (c_1, c_2)$ is incentive compatible if the game induced by $m$ has a truth-telling equilibrium.

Because the relevant game here is among *patient* types (note that withdraw in period 1 is weakly dominant for the *impatient* types), a truth-telling equilibrium exists if and only if the *patient* type wants to withdraw in the second period. The following lemma gives the condition for existence of such an equilibrium.²

Lemma 1.1 Let $f_p(\alpha) = (1 - \alpha)f(\alpha)/\int_0^1 (1 - z)f(z)dz$, the density of $\alpha$ conditional on the agent being of type patient. A feasible direct mechanism $m = (c_1, c_2)$ is incentive compatible if and only if it satisfies

$$\int_0^1 \int_0^\alpha \frac{v(c_1(z))}{\alpha} dz f_p(\alpha) d\alpha \leq \int_0^1 v(c_2(\alpha)) f_p(\alpha) d\alpha. \tag{1.2}$$

After observing his type, the agent uses Bayes’ rule in order to update his beliefs about the distribution of $\alpha$. The left-hand side of the above inequality is the expected utility of a *patient* type if he claims to be *impatient* and the right-hand side is his expected utility if he truthfully reveals his type — all conditional on truth-telling by other agents.

Given that the truth-telling equilibrium is played, the *ex-ante* welfare implied by a direct mechanism $m = (c_1, c_2)$ is

$$W(m) = \int_0^1 \left[ \int_0^\alpha u(c_1(z))dz + (1 - \alpha)v(c_2(\alpha)) \right] f(\alpha) d\alpha. \tag{1.3}$$

²See the appendix for all proofs.
Therefore, we have

**Definition 1.4** A direct mechanism is an optimal bank mechanism if it achieves the maximum of $W(\cdot)$ among all feasible mechanisms that are incentive compatible (satisfy (1.2)).

In order to find an optimal bank mechanism, I first write this problem as a function of $(w, w')$ where $w(\kappa) := \int_0^\kappa c_1(z)dz$ and $w'(\kappa) := c_1(\kappa)$. Then I use variational calculus to find the necessary and sufficient conditions for optimality. Propositions 1.2 and 1.3 provide the characterization.

**Proposition 1.2** Let $m = (c_1, c_2)$ be a direct mechanism, let $w(\alpha) = \int_0^\alpha c_1(z)dz$, and let $Z(\alpha) = \int_\alpha^1 \frac{1-z}{z} f(z)dz$. If $m = (c_1, c_2)$ is an optimal bank mechanism and $c_1$ is strictly positive on $[0, 1]$ then there exists $\lambda \geq 0$ such that $w$ satisfies

$$ w'' \left( u''(w') [1 - F] - \lambda v''(w') Z \right) = \left[ u'(w') - (1 + \lambda) R v' \left( \frac{y - w}{1 - \alpha} R \right) - \lambda v'(w') \frac{1 - \alpha}{\alpha} \right] f $$

with boundary conditions: $w(0) = 0$ and $w(1) = y$.

In the differential equation (1.4), $\lambda$ is the Lagrange multiplier of the incentive compatibility constraint. Because this constraint does not define a convex set, the necessary condition is not sufficient for a maximum. However, if the incentive compatibility constraint is not binding, then the Lagrange multiplier is zero and we have sufficient conditions for a maximum.

**Proposition 1.3** Let $m = (c_1, c_2)$ be a feasible direct mechanism that is incentive compatible (satisfy (1.2)). If $w(\alpha)$ satisfies (1.4) with $\lambda = 0$ then $m$ is an optimal bank mechanism.
1.3. Bank runs

A run-equilibrium is an equilibrium in which all agents withdraw independently of their types. The following inequality is necessary and sufficient for existence of this kind of equilibrium:

\[ \int_0^1 v(c_1(\alpha))d\alpha \geq v(c(2)) = v(c_1(1)R). \]

The left hand side of the inequality is the expected utility of a patient type if he decides to withdraw in period 1, while the right hand side of the inequality is his expected utility if he decides to withdraw in period 2 — all conditional on everyone else withdrawing in the first period. The equality \( v(c_2(1)) = v(c_1(1)R) \) comes from equation (1.1) and continuity of \( c_2 \) which implies:

\[
c_2(1) = \lim_{\kappa \to 1} c_2(\kappa) = \lim_{\kappa \to 1} \frac{y - \int_0^\kappa c_1(z)dz}{1 - \kappa} R = \lim_{\kappa \to 1} \frac{\int_0^1 c_1(z)dz - \int_0^\kappa c_1(z)dz}{1 - \kappa} R = c_1(1)R.
\]

In what follows I provide numerical examples where a run-equilibrium exists under the optimal bank mechanism. Such examples were extremely easy to find. For most of the examples I studied, a run-equilibrium exists. Indeed it was not easy to find examples where a run does not exist.

1.3.1. Numerical examples

In order to find the optimal bank mechanism, I use the following strategy. First I solve (1.4) with \( \lambda = 0 \) and check whether the solution is incentive compatible. If it is, then the implied mechanism gives the unique solution of the planner’s problem. Otherwise, equation (1.4) should be solved for different values of \( \lambda \) until a solution is found with the IC constraint binding. For the specifications I chose, the solution to (1.4) with \( \lambda = 0 \) is incentive compatible. Concerning the numerical algorithm, I transform the second-order differential equation (1.4) into a first order differential equation in two variables \((w, c_1)\),
where by construction $w' = c_1$. In order to solve it, I apply a second-order Runge-Kutta method combined with a shooting algorithm for the boundary. The code is implemented in C++ and is available upon request.

With income $y$ normalized to be one, the model has four primitives: the return $R$, the utility functions $u$ and $v$, and the distribution $F$. I set $R = 1.2$ and $u(c) = v(c) = (c^{1-a} - 1) / (1 - a)$ with $a = 3$. As regards $F$, it has density function $f(x) = (1 - \varepsilon)\phi(x) + \varepsilon$, where $\phi$ is the density of a normal distribution truncated on $[0, 1]$ with mean $1/2$ and standard deviation $\sigma$ and $\varepsilon = 10^{-6}$. I computed examples for $\sigma = 0.1, 0.01, 0.001$. I set $\varepsilon > 0$ because the program works better with a sufficiently high lower bound on $f$. Figure 1.2 shows the optimal payment in the first period for different values of $\sigma$. As a point of reference, the last panel shows the unique optimum with suspension in the model without aggregate uncertainty. In all examples, but the one without aggregate uncertainty, the optimal mechanism has a run-equilibrium. This is consistent with the observation made by EK that different marginal utilities between types is not necessary to generate bank runs at the optimal contract.

In these examples $c_1$ is strictly decreasing. Using equation (1.4) with $\lambda = 0$, one can show that $c'_1(\alpha) < 0$ if and only if $u'(c_1(\alpha)) > Rv'(c_2(\alpha))$. That is, for a given realization of $\alpha$ the last person in the queue is always under-insured with respect to the preference shock. Because there is aggregate and idiosyncratic uncertainty, the goal of the bank is to provide insurance against both sources of risk. In order to provide perfect insurance against the aggregate shock the payments in the first and second period should not depend on the fraction of impatient. But the only feasible mechanism with this property is autarky, which does not provide insurance against the idiosyncratic shock. Therefore, there is a trade-off between insurance against idiosyncratic and aggregate uncertainty.

It is well known that in the absence of aggregate uncertainty the ex-ante optimum can be strongly implemented by a mechanism with suspension. The mechanism pays the optimal insurance amount until the fraction of the population which has withdrawn is less
than or equal to the known fraction of impatient people. Then, as shown in graph (d) of figure 1.2, payments are suspended. This suspension scheme guarantees that bank runs do not occur in equilibrium. In these examples the expected fraction of impatient is $1/2$. As I reduce the variance of $\alpha$, the consumption of people with a position in the queue $z < 1/2$ approximates the optimal payment with suspension in the environment with no aggregate risk. This suggests that if the aggregate uncertainty is small a suspension scheme can be used to avoid bank runs with low welfare cost. A similar result is obtained by CBM in a different way. In their setting, there is a finite number of agents and preference shocks are i.i.d. across people. They show how to build a strongly implementable allocation for which the implied welfare approximates the optimal one in large economies. The idea

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6If $u(0) = -\infty$ then a total suspension will lead to welfare $-\infty$. In this case we should suspend to a level $c_1 > 0$ where $c_1$ is small enough to avoid runs.
behind their result is that the aggregate uncertainty vanishes as the number of agents goes to infinity. Therefore, one can use the optimal mechanism with no aggregate risk to build a strongly implementable allocation with implied welfare close to the optimum.

## 1.4. Partial bank runs

A partial run-equilibrium is an equilibrium where a fixed fraction of the agents withdraw independently of their types while the remaining agents withdraw only if they are of type \textit{impatient}. A partial run-equilibrium exists if, and only if:

\[
\int_0^1 \int_0^{\kappa+(1-\kappa)\alpha} \frac{v(c_1(z))}{\kappa + (1 - \kappa)\alpha} dz f_p(\alpha) d\alpha = \int_0^1 v(c_2(\kappa + (1 - \kappa)\alpha)) f_p(\alpha) d\alpha \tag{1.6}
\]

for some \(\kappa \in (0, 1)\). The left hand side of the inequality is the expected utility of a \textit{patient} type if he decides to withdraw in period 1, while the right hand side of the inequality is his expected utility if he decides to withdraw in period 2 — all conditional on a fixed fraction \(\kappa\) of agents running against the bank while the remaining \(1-\kappa\) fraction follows the truth-telling strategy. Since the \textit{patient} agents who withdraw in the first and second period are identical, their expected utility needs to be the same in equilibrium.

**Proposition 1.4** Let \(m = (c_1, c_2)\) be a feasible direct mechanism which satisfies (1.2) and (1.5) with strict inequality. Then \(m = (c_1, c_2)\) admits a partial run-equilibrium.

The conditions of proposition 1.4 are satisfied in the examples (a)-(c) displayed in subsection 1.3.1. Therefore, those mechanisms are vulnerable to partial bank runs.\(^7\)

\(^7\)Because proposition 1.4 only establishes sufficient conditions for the existence of partial bank runs, there is no obvious way of extending the method in this paper to compute the best outcome implementable by a mechanism immune to bank runs. This difficulty implies that I cannot replicate some of the exercises found in the existing literature. In particular, I cannot replicate the exercise in section 4 of PS: finding the best mechanism conditional on agents playing a sunspot equilibrium.
1.5. Conclusion

I use variational calculus to characterize the optimal bank mechanism by a second-order differential equation in an environment with a continuum of agents and aggregate uncertainty. Applying this result, I am able to show that the main result in PS holds for large economies: for many such economies, the optimal direct mechanism admits a run-equilibrium. In particular, there is a run-equilibrium in settings with a high level of aggregate uncertainty — settings in which the attainment of uniqueness through a suspension scheme would be costly.

1.6. Acknowledgements

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1.A. Appendix

Proof of Lemma 1.1. First I describe how a person update his beliefs over the distribution of \( \alpha \) after he observes he is patient. By the definition of conditional probability and Bayes’ rule, for any \( x \in [0, 1] \),

\[
F_p(x) = P(\alpha \leq x | \text{patient}) = \frac{P(\text{patient} | \alpha \leq x) P(\alpha \leq x)}{P(\text{patient})} = \frac{\int_0^x (1-z) f(z) dz}{\int_0^1 (1-z) f(z) dz}.
\]

Thus, the conditional density function is

\[
f_p(x) = \frac{(1-x)f(x)}{\int_0^1 (1-z)f(z)dz}.
\]
Therefore, the expected utility of an individual who is \textit{patient} and pretends he is \textit{impatient} when other players truthfully reveal their types is
\[
\int_0^1 \left[ \int_0^\alpha \frac{v(c_1(z))}{\alpha} \, dz \right] f_p(\alpha) \, d\alpha,
\]
while the expected utility if he truthfully reveals his type is
\[
\int_0^1 v(c_2(\alpha)) f_p(\alpha) \, d\alpha.
\]
Since truth-telling is a dominant strategy for \textit{impatient} types, we can conclude that a mechanism has a truth-telling equilibrium if and only if (1.2) holds. \hfill \blacksquare

\textbf{Proof of Proposition 1.2.} The optimal bank mechanism solves the problem
\[
\max_m \left\{ \int_0^1 \left[ \int_0^\alpha u(c_1(z)) \, dz + (1 - \alpha)v(c_2(\alpha)) \right] f(\alpha) \, d\alpha \right\}
\text{s.t. } \left\{ \begin{array}{l}
\int_0^1 \int_0^\alpha \frac{v(c_1(z))}{\alpha} \, dz f_p(\alpha) \, d\alpha \leq \int_0^1 v(c_2(\alpha)) f_p(\alpha) \, d\alpha \\
c_1(\alpha) \geq 0, \ c_2(\alpha) = \frac{y - \int_0^\alpha c_1(z) \, dz}{1 - \alpha} R \ \text{and} \ \int_0^1 c_1(\alpha) \, d\alpha = y.
\end{array} \right.
\]

Note that
\[
\int_0^1 \int_0^\alpha u(c_1(z)) f(\alpha) \, dz \, d\alpha = \\
\int_0^1 \int_z^1 u(c_1(z)) f(\alpha) \, d\alpha \, dz = \int_0^1 u(c_1(\alpha)) [1 - F(\alpha)] \, d\alpha,
\]
where we have just changed the order of integration. Therefore,
\[
W(m) = \int_0^1 u(c_1(\alpha)) [1 - F(\alpha)] + (1 - \alpha)v(c_2(\alpha)) f(\alpha) \, d\alpha.
\]
In a similar way, the incentive compatibility constraint can be written as

\[
\int_0^1 v(c_1(\alpha)) Z(\alpha) d\alpha \leq \int_0^1 v(c_2(\alpha)) (1 - \alpha) f(\alpha) d\alpha
\]

where the function \( Z(\alpha) \) is defined by \( Z(\alpha) = \int_1^\alpha \frac{1 - z}{z} f(z) dz \). Now we can replace \( w(\alpha) := \int_0^\alpha c_1(z) dz \) and \( w'(\alpha) := c_1(\alpha) \) and write the problem as

\[
\max_w \left\{ \int_0^1 u(w'(\alpha)) \left[ 1 - F(\alpha) \right] + (1 - \alpha)v \left( \frac{y - w(\alpha)}{1 - \alpha} - R \right) f(\alpha) d\alpha \right\}
\]

s.t.

\[
\int_0^1 v(w'(\alpha)) Z(\alpha) d\alpha \leq \int_0^1 v \left( \frac{y - w(\alpha)}{1 - \alpha} - R \right) (1 - \alpha) f(\alpha) d\alpha
\]

\[
\forall \alpha \in (0, 1) : w'(\alpha) \geq 0, \ w(0) = 0 \ \text{and} \ \ w(1) = y.
\]

Therefore, the objective function is given by \( \int_0^1 \Phi(\alpha, w, w') d\alpha \), where

\[
\Phi(\alpha, w, w') = u(w') \left[ 1 - F \right] + (1 - \alpha)v \left( \frac{y - w}{1 - \alpha} - R \right) f.
\]

And the constraint is \( \int_0^1 \Psi(\alpha, w, w') d\alpha \leq 0 \), where

\[
\Psi(\alpha, w, w') = v(w') Z - (1 - \alpha)v \left( \frac{y - w}{1 - \alpha} - R \right) f.
\]

Because we are assuming that the optimal \( c_1 \) is strictly positive, the restriction \( w' \geq 0 \) is not binding and can be ignored. A necessary condition for some \( w \) to achieve the maximum is that it solves the Euler equation:

\[
\Phi_w(\alpha, w, w') - \lambda \Psi_w(\alpha, w, w') = \frac{d}{d\alpha} \left[ \Phi_w'(\alpha, w, w') - \lambda \Psi_w'(\alpha, w, w') \right] \tag{1.9}
\]

for some non-negative real number \( \lambda \). This result is implied by Theorem 1, page 15; Theorem 1, page 43; and Theorem 2, page 100 of Gelfand and Fomin (1963).
For the function \( \Phi \) we have,

\[
\Phi_w(\alpha, w, w') = -Rv' \left( \frac{y - w}{1 - \alpha} R \right) f; \quad \Phi_{w'}(\alpha, w, w') = u'(w') \left[ 1 - F \right]; \quad \text{and}
\]

\[
\frac{d}{d\alpha} \Phi_{w'}(\alpha, w, w') = w'' u''(w') \left[ 1 - F \right] - u'(w') f.
\]

And for \( \Psi \) the derivatives are

\[
\Psi_w(\alpha, w, w') = Rv' \left( \frac{y - w}{1 - \alpha} R \right) f; \quad \Psi_{w'}(\alpha, w, w') = v'(w') Z; \quad \text{and}
\]

\[
\frac{d}{d\alpha} \Psi_{w'}(\alpha, w, w') = w'' v''(w') Z - v'(w') \frac{1 - \alpha}{\alpha} f.
\]

Substituting these into the Euler equation (1.9) we get equation (1.4). Since (1.4) is a second-order differential equation we need two boundary conditions. They are \( w(0) = 0 \) and \( w(1) = y \). Those are the boundary conditions required in the problem to make the mechanism implied by \( w \) feasible. The conditions above were derived assuming that \( c_1 \geq 0 \) was not binding. Hence, our result is a necessary condition for a solution under this assumption.

\[\square\]

**Proof of Proposition 1.3.** Consider the problem:

\[
\max_m \left\{ \int_0^1 \left[ \int_0^\alpha u(c_1(z)) dz + (1 - \alpha)v(c_2(\alpha)) \right] f(\alpha) d\alpha \right\}
\]

s.t. \( c_1(\alpha) \geq 0 \), \( c_2(\alpha) = \frac{y - \int_0^\alpha c_1(z) dz}{1 - \alpha} R \) and \( \int_0^1 c_1(\alpha) d\alpha = y \).

A solution to this problem achieves the maximum of \( W(\cdot) \) among all feasible mechanisms. Therefore, if this solution is incentive compatible, it also achieves the maximum of \( W(\cdot) \) among all feasible mechanisms that are incentive compatible. Let us derive sufficient conditions for a maximum of this problem. By the same arguments used in the proof of
proposition 1.2 The problem can be written as

$$\max_w \left\{ \int_0^1 u(w'(\alpha)) [1 - F(\alpha)] + (1 - \alpha) v \left( \frac{y - w(\alpha)}{1 - \alpha} R \right) f(\alpha) d\alpha \right\}$$

s.t. \( \forall \alpha \in (0, 1) : w'(\alpha) \geq 0, \ w(0) = 0 \text{ and } w(1) = y. \)

The objective function is

$$\Phi(\alpha, w, w') = u(w') [1 - F] + (1 - \alpha) v \left( \frac{y - w}{1 - \alpha} R \right) f$$

and the boundaries are \( w(0) = 0 \) and \( w(1) = y. \) The following are sufficient condition for a non-decreasing \( w \) to solve the above problem:

i - \( w \) solves the differential equation \( \Phi_w(\alpha, w, w') = \frac{d}{d\alpha} \Phi_w(\alpha, w, w') \) with boundary constraint \( w(0) = 0 \) and \( w(1) = y; \) and

ii - for each \( \alpha \in (0, 1), \Phi(\alpha, \cdot, \cdot) \) is strictly concave.

See Gelfand and Fomin (1963) chapter 5 for details about this claim. Condition (i) is the same of (1.4) with \( \lambda = 0. \) Condition (ii) is satisfied because \( u \) and \( v \) are strictly concave functions. Hence, if a feasible mechanism that is incentive compatible solves equation (1.4) with \( \lambda = 0, \) then it achieves the maximum of \( W(\cdot) \) among all feasible mechanisms and, in particular, it achieves the maximum of \( W(\cdot) \) among all feasible mechanisms that are incentive compatible.

\[\blacksquare\]

**Proof of proposition 1.4.** Define \( H : [0, 1] \to \mathbb{R} \) as

$$H(\kappa) = \int_0^1 \int_0^{\kappa + (1 - \kappa)\alpha} \frac{v(c_1(z))}{\kappa + (1 - \kappa)\alpha} dz f_p(\alpha) d\alpha - \int_0^1 v(c_2(\kappa + (1 - \kappa)\alpha)) f_p(\alpha) d\alpha.$$
Since \( m = (c_1, c_2) \) is continuous, the function \( H \) is continuous. Note that

\[
H(0) = \int_0^1 \int_0^\alpha \frac{v(c_1(z))}{\alpha} dz f_p(\alpha) d\alpha - \int_0^1 v(c_2(\alpha)) f_p(\alpha) d\alpha < 0
\]

because (1.2) is satisfied with strictly inequality. And

\[
H(1) = \int_0^1 v(c_1(\alpha)) d\alpha - v(c_2(1)) > 0
\]

because (1.5) is satisfied with strictly inequality. Since \( H \) is continuous, \( H(0) < 0 \) and \( H(1) > 0 \), the intermediate value theorem implies that there exists \( \kappa^* \in (0, 1) \) such that \( H(\kappa^*) = 0 \). Which implies that,

\[
\int_0^1 \int_0^{\kappa^*+(1-\kappa^*)\alpha} \frac{v(c_1(z))}{\kappa^*+(1-\kappa^*)\alpha} dz f_p(\alpha) d\alpha = \int_0^1 v(c_2(\kappa^*+(1-\kappa^*)\alpha)) f_p(\alpha) d\alpha.
\]

Hence, \( m = (c_1, c_2) \) admits a partial run-equilibrium. \( \blacksquare \)
Preventing bank runs

with David Andolfatto and Ed Nosal

Abstract Diamond and Dybvig (1983) is commonly understood as providing a formal rationale for the existence of bank-run equilibria. It has never been clear, however, whether bank-run equilibria in this framework are a natural byproduct of the economic environment or an artifact of suboptimal contractual arrangements. In the class of direct mechanisms, Peck and Shell (2003) demonstrate that bank-run equilibria can exist under an optimal contractual arrangement. The difficulty of preventing runs within this class of mechanism is that banks cannot identify whether withdrawals are being driven by psychology or by fundamentals. Our solution to this problem is an indirect mechanism with the following two properties. First, it provides depositors an incentive to communicate whether they believe a run is on or not. Second, the mechanism threatens a suspension of convertibility conditional on what is revealed in these communications. Together, these two properties can eliminate the prospect of bank-run equilibria in the Diamond-Dybvig environment.

Keywords bank runs, optimal deposit contract, financial fragility.

Banking is the business of transforming long-maturity illiquid assets into short-maturity liquid liabilities. The demandable debt issued by commercial banks constitutes the quintessential example of this type of credit arrangement. The use of short-maturity debt to finance long-maturity asset holdings is also prevalent in the shadow-banking sector. \(^1\) Demandable debt or short-maturity debt in general has long been viewed by economists and regulators as an inherently fragile financial structure—a credit arrangement that is susceptible to runs or roll-over risk. The argument is a familiar one. Suppose that depositors expect

\(^1\) This sector includes, but is not limited to, structured investment vehicles (SIVs), asset-backed commercial paper (ABCP) conduits, money market funds (MMFs), and markets for repurchase agreements (repos).
a run—a wave of early redemptions driven by fear, rather than by liquidity needs. By the definition of illiquidity, the value of what can be recouped in a fire-sale of assets must fall short of existing obligations. Because the bank cannot honor its promises in this event, it becomes insolvent. In this manner, the fear of run can become a self-fulfilling prophecy.

If demandable debt is run prone, then why not tax it, or better yet, legislate it out of existence? Bryant (1980) suggests that the American put option embedded in bank liabilities is a way to insure against unobservable liquidity risk. In short, banking is an efficient risk-sharing arrangement when assets are illiquid, depositors are risk averse, and liquidity preference is private information. But if this is the case, then the solution to this one problem seems to open the door to another. Indeed, the seminal paper by Diamond and Dybvig (1983) on bank runs demonstrates precisely this possibility: Demandable debt as an efficient risk-sharing arrangement is also a source of indeterminacy and financial instability.

Diamond and Dybvig (1983) is most often viewed as a theory of bank runs, but it also offers a prescription for how to prevent bank runs for the case in which aggregate risk is absent. The prescription entails embedding bank liabilities with a suspension clause that is triggered when redemptions exceed a specified threshold. This simple fix prevents bank runs.

As Diamond and Dybvig (1983) point out, a full suspension of convertibility—conditional on a threshold level of redemption activity being breached—is not likely to be optimal in the presence of aggregate risk. In the absence of aggregate uncertainty, redemptions exceeding the appropriate threshold constitutes a signal that a run is occurring. With aggregate uncertainty, the optimal redemption schedule is state contingent. As a consequence, it is not possible to confirm whether heavy redemptions are driven by fundamentals or by

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2 If the underlying assets are not illiquid, the demand for maturity transformation would be absent.
3 This is essentially the recommendation recently put forth by Cochrane (2014).
4 Diamond and Dybvig (1983) do not actually characterize the optimal contract for the case in which aggregate risk is present.
5 This property was suggested by Wallace (1988) and later confirmed by Green and Lin (2003).
psychology. Threatening full suspension is desirable in the latter case, but not the former.

Our proposed solution to the bank-run problem under aggregate uncertainty is to exploit the idea that while the bank may not know whether a run is on, there are agents in the economy that do. That is, in equilibrium, the beliefs of agents in the economy are consistent with the reality unfolding around them. Can the bank somehow elicit this information in an incentive-compatible manner? If it can, then might the threat of suspensions conditional on such information—and not on withdrawals—serve to eliminate run equilibria?

We provide a positive answer for both these questions, and by so doing depart from the direct mechanism approach usual in the literature. In a direct mechanism, a depositor in the sequential service queue simply requests to withdraw or not. That is, the depositor communicates only his type; impatient if he withdraws or patient if he does not. Our indirect mechanism expands the message space to accommodate additional communications. In this way, we permit a depositor to communicate his belief that a run is on. We can show that the threat of suspension conditional on this communication eliminates the possibility of a run equilibrium.

In practice, such information could be gleaned by introducing a separate financial instrument, the choice of which implicitly reveals what the depositor believes.\(^6\) Our mechanism rewards the depositor for delivering such a message when a run is on. The reward is such that his payoff is higher compared to the payoff associated with concealing his belief that a run is on and making an early withdrawal—that is, misrepresenting his type and running with the other agents. Upon receiving such a message, the mechanism fully suspends all further redemptions. The design of our mechanism ensures that a patient agent never has an incentive to either run when a run is on or announce that he believes a run is on when it is not. At the end of the day, we are able to construct an indirect mechanism that implements the constrained-efficient allocation in iterated elimination

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\(^6\)We elaborate on this in Section 3.9.
of strictly dominated strategies.

\section*{2.1. Literature review}

A number of papers have studied bank fragility under optimal arrangements in the Diamond and Dybvig (1983) setting. Green and Lin (2003) were the first to characterize an optimal bank contract under private information, sequential service, and aggregate uncertainty. In their version of the Diamond-Dybvig model, the first-best allocation is implementable as a \textit{unique} Bayes-Nash equilibrium of a direct revelation game.

The allocation rule in Green and Lin (2003) allows early withdrawal payments in the sequential service queue to depend on the history of announcements—"I want to withdraw" or "I do not want to withdraw"—and payments to that point. The maximum withdrawal amount faced by an agent in the service queue is lower the larger is the number of preceding withdrawals. This partial suspension scheme is in stark contrast to Diamond and Dybvig (1983), who restrict the maximum withdrawal amount to be insensitive to realized withdrawal demand, so that resources are necessarily exhausted in the event of a run.\footnote{Wallace (1990) reports that partial suspensions were prevalent in the banking panic of 1907, and that in one form or another must have been a feature of other suspension episodes as well.}

Peck and Shell (2003) modify the Green and Lin (2003) environment in at least two important ways. First, they alter the preferences so that incentive-compatibility constraints bind at the optimum. This implies, among other things, that the first-best allocation cannot be implemented. Second, they assume that depositors do not know (or are not told) their position in the service queue. If depositors do not know their queue position, then it is not possible to use backward induction argument of Green and Lin (2003) to eliminate a bank-run equilibrium. It also turns out—and this was not recognized at the time—not revealing queue positions to depositors is part of an optimal mechanism when
incentive-compatibility constraints bind.\footnote{By not revealing or knowing queue positions, multiple incentive compatibility constraints can be replaced by a single incentive compatibility constraint. As a result, the set of implementable incentive compatible allocations expands.} Peck and Shell (2003) use a direct revelation mechanism and demonstrate by example that the optimal direct mechanism can have a bank-run equilibrium.

Ennis and Keister (2009b) modify the Green-Lin environment by assuming the distribution of depositors types is correlated; Green and Lin (2003) assume independence. Using a direct revelation mechanism, Ennis and Keister (2009b) demonstrate that a bank-run equilibrium can exist. But, it is no longer obvious, that a direct revelation mechanism is the “best” mechanism since it does not deliver a uniqueness result. Indeed, Cavalcanti and Monteiro (2011) examine indirect mechanisms in the Ennis and Keister (2009b) environment and demonstrate that the best allocation can be uniquely implemented in dominant strategies. Unfortunately, the backward induction argument implicitly embedded in their mechanism—which is key to their uniqueness proof—will not work in the more general Peck and Shell (2003) environment since depositors do not know their positions in the queue.

There is a mechanism design literature that studies how indirect mechanisms can help to implement optimal outcomes uniquely. Demski and Sappington (1984) examine a principal-two-agent setting where agents separately make production decisions and their costs are private and correlated. The optimal direct mechanism has two equilibria: A truth-telling equilibrium and a “cheating” equilibrium, where the cheating equilibrium leaves both agents better off and the principal worse off compared to the truth-telling equilibrium. Ma et al. (1988) shows how an indirect mechanism can prevent agents from misrepresenting their types—or stop agents from cheating—in the Demski and Sappington (1984) model.\footnote{Postlewaite and Schmeidler (1986) also produced an example where an indirect mechanism has a unique equilibrium yielding the optimal outcome while the corresponding direct mechanisms possess multiple equilibria.} Mookherjee and Reichelstein (1990) generalizes this approach. Unfortu-
nately, these results cannot be directly applied to the banking problem because sequential service, which is absent in the mechanism design models, complicates the analysis.

The paper is organized as follows. The next section describes the economic environment. Section 3 characterizes the best weakly implementable allocation. In Section 4 we provide a stripped down version of model to illustrative the key features of our mechanism. In Section 5 we construct an indirect mechanism and provide sufficient conditions for unique implementation of the best weakly implementable allocation. In Section 6, we examine examples for which the sufficient conditions are not valid and Section 7 examines an alternative indirect mechanism that addresses these examples. Some policy remarks are offered in the final section.

### 2.2. Environment

There are three dates: 0, 1 and 2. The economy is endowed with $Y > 0$ units of date-1 goods. A constant returns to scale investment technology transforms $y$ units of date-1 goods into $yR > y$ units of date-2 goods. There are $N$ ex-ante identical agents who turn out to be one of two types: $t \in T = \{1, 2\}$. We label a type $t = 1$ agent “impatient” and a type $t = 2$ agent “patient”. The number of patient agents in the economy is drawn from the distribution $\pi = (\pi_0, \ldots, \pi_N)$, where $\pi_n > 0, n \in \mathbb{N} \equiv \{0, 1, \ldots, N\}$, is the probability that there are $n$ patient agents. $^\text{10}$ A queue is a vector $t^N = (t_1, \ldots, t_N) \in T^N$, where $t_k \in T$ is the type of the agent that occupies the $k$th position or coordinate in the queue. Let $P_n = \{t^N \in T^N | n_{t^N} = n\}$ and $Q_n(t^N) = \{j | t_j = 2$ for $t^N \in P_n\}$, where $n_{t^N}$ denotes the number of patient agents in the queue $t^N$. $P_n$ is the set of queues with $n$ patient agents and $Q_n(t^N)$ is the set of queue positions of the $n$ patient agents in queue $t^N \in P_n$. $^\text{11}$ The probability of a queue $t^N \in P_n$ is $\pi_n / \binom{N}{n}$, where the binomial coefficient,

$^\text{10}$The full support assumption is not crucial to any result. It is imposed only for simplicity.

$^\text{11}$We omit the argument of $Q_n(t^N)$ throughout the paper to reduce notational burden.
\( (N_n) \), is the number of queues \( t^N \in P_n \). In other words, all queues with \( n \) patient agents are equally likely. Agents are randomly assigned to a queue position, where the unconditional probability that an agent is assigned to position \( k \) is \( 1/N \). Label an agent assigned to position \( k \) agent \( k \). The queue realization, \( t^N \), is observed by no one; not by any of the agents nor the planner. Agent \( k \) does not observe his queue position, \( k \), but does privately observe his type \( t \in T \). The utility function for an impatient agent is \( U(c^1, c^2; 1) = u(c^1) \) and the utility function of a patient agent is \( U(c^1, c^2; 2) = \rho u(c^1 + c^2) \), where \( c^1 \) is date-1 consumption and \( c^2 \) is date-2 consumption. The function \( u \) is increasing, strictly concave and twice continuously differentiable, and \( \rho > 0 \) is a parameter.\(^{12}\) Agents maximize expected utility.

The timing of events and actions are as follows. At date 0, the planner constructs a mechanism that determines how date-1 and date-2 consumption are allocated among the \( N \) agents. A mechanism consists of a set of announcements, \( M \), and an allocation rule, \( c = (c^1, c^2) \), where \( c^1 = (c^1_1, \ldots, c^1_N) \) and \( c^2 = (c^2_1, \ldots, c^2_N) \). The planner can commit to the mechanism.\(^{13}\) The queue \( t^N \) is realized at the beginning of date 1. Then agents meet the planner sequentially, starting with agent 1. Each agent \( k \) makes an announcement \( m_k \in M \).\(^{14}\) Only agent \( k \) and the planner can directly observe \( m_k \). There is a sequential service constraint at date 1, which means the planner allocates date-1 consumption to agent \( k \) based on the announcements of agents \( j \leq k, (m^{k-1}, m_k) \), where \( m^{k-1} = (m_1, \ldots, m_{k-1}) \), and each agent \( k \) consumes \( c_k^1(m^{k-1}, m_k) \) at his date-1 meeting with the planner. Date 1 ends after all agents meet the planner. In between dates 1 and 2 the planner’s resources

\(^{12}\)These preferences are identical to the ones in Diamond and Dybvig (1983). In addition, they assume that \( \rho R > 1 \) and \( \rho \leq 1 \).

\(^{13}\)For a discussion of bank fragility in a setting without commitment, see Ennis and Keister (2009a).

\(^{14}\)One could imagine that the planner makes announcement \( a_k \) to agent \( k \) before \( k \) makes his announcement. For example, the planner could tell agent \( k \) his queue position, as in Green and Lin (2003), or the set of all messages sent in the previous \( k - 1 \) planner-agent meetings, as in Andolfatto et al. (2007), or “nothing”, \( a_k = \emptyset \), as in Peck and Shell (2003). The optimal mechanism, however, will have the planner announcing nothing. To reduce notation, and without loss of generality, we assume that the planner cannot make announcements to agents, unless otherwise specified. See footnote 16 for a discussion.
are augmented by a factor of $R$. At date 2, the planner allocates the date-2 consumption good to each agent based on the date-1 announcements, i.e., agent $k$ receives $c^2_k(m^N)$, where $m^N = (m_1, \ldots, m_N) \in M^N$. Figure 2.1 depicts the sequence of actions.

![Figure 2.1 Sequence of actions.](image)

### 2.3. The best weakly implementable outcome

An allocation is weakly implementable if it is an equilibrium outcome of a mechanism; it is strongly or uniquely implementable if it is the unique equilibrium outcome of a mechanism. Among the set of weakly implementable allocations, the best weakly implementable allocation provides agents with the highest expected utility. To characterize the best weakly implementable allocation, it is without loss of generality to restrict the planner to use a direct revelation mechanism, where agents only announce $m_k = t_k \in T = \{1, 2\}$. The welfare—which we measure as the expected utility of an agent before he learns his type—associated with allocation rule $c$ when agents use strategies $m_k \in T$ is

$$\sum_{n=0}^{N} \frac{\pi_n}{(N)} \sum_{t^N \in \mathcal{P}_n} \sum_{k=1}^{N} U \left[ c^1_k \left( m^{k-1}, m_k \right), c^2_k \left( m^1_N \right) ; t_k \right]. \quad (2.1)$$

The allocation rule $c = (c_1, c_2)$ is **feasible** if for all $m^N \in T^N$

$$\sum_{k=1}^{N} \left[ Rc^1_k \left( m^{k-1}, m_k \right) + c^2_k \left( m^M \right) \right] \leq RY. \quad (2.2)$$
The best weakly implementable allocation has all agents \( k \) announcing truthfully, i.e., \( m_k = t_k \). Allocation rule \( c \) must be incentive compatible in the sense that agent \( k \) has no reason to announce \( m_k \neq t_k \). Since impatient agents \( k \) only value date-1 consumption, they always announce \( m_k = 1 \). Patient agent \( k \) has no incentive to defect from the strategy \( m_k = 2 \), assuming that all other agents announce truthfully, if

\[
\sum_{n=1}^{N} \hat{\pi}_n \sum_{i \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho \left\{ u \left[ c^1_k \left( t^{k-1}, 2 \right) + c^2_k \left( t^N \right) \right] - u \left[ c^1_k \left( t^{k-1}, 1 \right) + c^2_k \left( t^{k-1}, 1, t_k^{N+1} \right) \right] \right\} \geq \delta
\]

(2.3)

where, for any vector \( x^N = (x_1, \ldots, x_N) \), \( x^i \) denotes \((x_i, \ldots, x_j)\), \( \delta > 0 \) is a parameter, and

\[
\hat{\pi}_n = \frac{\pi_n / \binom{N}{n}}{\sum_{n=1}^{N} \pi_n / \binom{N}{n}}
\]

is the conditional probability that agent \( k \) is in a specific queue with \( n \) patient agents.\(^\text{16}\) The \( 1/n \) term that appears in (2.3) reflects that a patient agent has a \( 1/n \) chance of occupying each of the patient queue positions in \( Q_n \).

The best weakly implementable allocation is given by the solution to

\[
\text{max (2.1) subject to (2.2) and (2.3),}
\]

(2.4)

\(^{15}\) This anticipates the result that the best weakly implementable allocation provides zero date-1 consumption to agents who announce that they are patient, which implies that the incentive compatibility constraint for impatient agents is always slack.

\(^{16}\) To characterize the best weakly implementable allocation, one wants to choose from the largest possible set of incentive compatible allocations. This implies the planner should not make any announcements, as noted in footnote 14. In particular, if the planner does not make any announcements, then there is only one incentive compatibility constraint for all patient agents, (2.3). If, however, the planner did make an announcement \( a_k \) to agent \( k \), there will be additional incentive constraints for the agent who received the announcement. For example, suppose that \( a_k = k \) for all \( k \), i.e., the planner announces to each agent his place in the queue. Then there would be \( N \) incentive compatibility constraints for patient agents, one for each queue position. Since an appropriately weighted average of these distinct incentive constraints implies (2.3), the set of incentive compatible allocations when the planner makes announcements is a subset of the set of incentive compatible allocations when he does not. By not making any announcements, the planner is able to choose from a larger set of incentive feasible allocations.
where \( m_k = t_k \) for all \( k \in \mathbb{N} \). We restrict \( \delta > 0 \) to those values that admit a solution to problem (2.4). Let \( c^\ast(\delta) = (c^{1\ast}(\delta), c^{2\ast}(\delta)) \) be a solution to problem (2.4) and let \( W^\ast(\delta) \) denote its maximum. We consider \( \delta > 0 \) to guarantee that the incentive compatibility holds in an open neighbourhood of \( c^\ast \). The existence of such neighbourhood is necessary for our uniqueness result but \( \delta > 0 \) can be made arbitrarily small. Therefore, we can apply Berge’s maximum theorem, which says that \( W^\ast(0) \) is approximated by \( W^\ast(\delta) \) when \( \delta \) is close to zero. The allocation rule \( c^\ast(\delta) \) has the following features: (i) an agent \( k \) who announces \( m_k = 1 \) consumes only at date 1, that is, \( c^{2\ast}_k(m_1, \ldots, m_{k-1}, 1, m_{k+1}, \ldots, m_N) = 0 \) for all \( k \in \mathbb{N} \); (ii) an agent \( k \) who announces \( m_k = 2 \) consumes only at date 2, that is, \( c^{1\ast}_k(m_1, \ldots, m_{k-1}, 2) = 0 \) for all \( k \in \mathbb{N} \); and (iii) all agents \( j \) and \( k \) announcing \( m_j = m_k = 2 \) consume identical amounts at date 2, that is, \( c^{2\ast}_j(m_N) = c^{2\ast}_k(m_N) \) for all \( m_j = m_k = 2 \).

The best-weakly implementable allocation is \( c^\ast(0) \), which corresponds to the allocation rule derived in Peck and Shell’s (2003) Appendix B.

Define a bank run as a non-truth-telling equilibrium for the mechanism \( \{ M, c \} \), where some and possibly all \( k \in Q_n(t^N) \) announce \( m_k = 1 \). Both Peck and Shell (2003) and Ennis and Keister (2009b) demonstrate, by example, that the direct mechanism \( \{ T, c^\ast(0) \} \) can have two equilibria: one where agents play truth-telling strategies, \( m_k = t_k \) for all \( k \), and another where all patient agents \( k \) play bank-run strategies, \( m_k = 1 \).\(^{17}\) We claim that bank-run equilibria arise in these examples because the direct revelation mechanism they use, \( \{ T, c^*(0) \} \), is not an optimal one; there exists an indirect mechanism that strongly implements the best weakly implementable allocation. Before we demonstrate this result, we provide a simple example that illustrates the basic intuition underlying our optimal mechanism.

\(^{17}\) The Ennis and Keister (2009b) bank-run example is in section 4.2 of their paper. There, agents do not know their position in the queue, as in Peck and Shell (2003), and the utility functions of patient and impatient agents are the same, \( \rho = 1 \), as in Green and Lin (2003).
2.4. A simple example

Consider a stripped-down version of a Diamond-Dybvig model where there are only 2 agents—column and row—and both agents are patient. Agents simultaneously announce that they are either patient, \( m = 2 \), or impatient, \( m = 1 \). The payoffs to agents for this game are given by

\[
\begin{array}{cc}
m = 1 & m = 2 \\
1, 1 & 2, 0 \\
0, 2 & 3, 3
\end{array}
\]

Figure 2.2 Payoffs of the initial game.

This simple normal form game captures two important insights of the Diamond-Dybvig model. First, there are multiple equilibria: one where both agents announce the truth, \( m = 2 \), one where both agents announce they are impatient, \( m = 1 \), and another where both agents randomize between each strategy with probability half. And second, the truth-telling equilibrium generates the higher payoffs for agents than a bank-run equilibrium.

Consider now a normal form game that simply augments the announcement space of the original game from \( \{1, 2\} \) to \( \{1, 2, g\} \), with associated payoffs

\[
\begin{array}{ccc}
m = 1 & m = 2 & m = g \\
1, 1 & 2, 0 & 0, 1 + \epsilon \\
0, 2 & 3, 3 & 3, 2 + \epsilon \\
1 + \epsilon, 0 & 2 + \epsilon, 3 & \epsilon, \epsilon
\end{array}
\]

Figure 2.3 Payoffs of the augmented game.

There are three features of the augmented game that we would like to highlight. First,
when agents restrict their announcements to \( \{1, 2\} \), the payoffs they receive are identical to the original game. Second, announcement \( m = g \) strictly dominates announcement \( m = 1 \). And finally, the payoff to an agent who announces \( m = 2 \) is the same regardless if his opponent announces \( m = 2 \) or \( m = g \).

Since agents never play \( m = 1 \) in the augmented game—it is strictly dominated by playing \( m = g \)—the relevant augmented game that agents play is

\[
\begin{array}{ccc}
  m = 2 & m = g \\
  m = 2 & 3, 3 & 2, 0 \\
  m = g & 2 + \varepsilon, 3 & \varepsilon, \varepsilon
\end{array}
\]

Figure 2.4 Payoffs after elimination of strictly dominated strategies.

But in this relevant augmented game, announcement \( m = g \) is strictly dominated by announcement \( m = 2 \). Therefore, the unique iterated strict dominant equilibrium to the augmented game is one of truthtelling, \( m = 2 \). Hence, by modifying the game that agents play, we get rid of the “bad” bank-run equilibria that existed in the original game.

The best weakly implementable allocation described in Section 2.3, \( c^*(\delta) \), is somewhat more complicated than the payoff structure in the stripped-down example. Nevertheless, our approach to eliminate the bad equilibria is the same: We construct an indirect mechanism \( \{\hat{M}, \hat{c}\} \) with the properties: (i) \( \hat{M} = \{1, 2, g\} \); (ii) announcing \( \hat{m}_k = 1 \) is strictly dominated by announcing \( \hat{m}_k = g \) for patient agents; and (iii) after announcement \( \hat{m}_k = 1 \) is eliminated for patient agents announcing \( \hat{m}_k = 2 \) strictly dominates announcing \( m_k = g \). The uniqueness result is a bit more tricky to prove because we need enough resources to construct an allocation rule \( \hat{c} \) that provides sufficiently high payoffs to patient agents so that announcing truthfully is the unique rational strategy. In the subsequent section, we characterize an indirect mechanism and provide sufficient conditions under which this mechanism uniquely implements the best weakly implementable
allocation using dominance arguments similar to the simple example.

2.5. An indirect mechanism

Consider an indirect mechanism \( \hat{M}, \hat{c} \), where \( \hat{M} = \{1, 2, g\} \) and \( \hat{c} \) is described below. The basic construction of the allocation rule \( \hat{c} \) uses \( c^{*}(\delta) \). If agent \( j \) announces \( \hat{m}_{k} = 1 \), then

\[
\hat{c}^{1}_{k}(\hat{m}^{k-1}, 1) = \begin{cases} 
  c^{1*}_{k}(\delta)(\hat{m}^{k-1}, 1) & \text{if } \hat{m}_{j} \in \{1, 2\} \text{ for all } j < k \\
  0 & \text{if } \hat{m}_{j} = g \text{ for some } j < k
\end{cases}
\]

and \( \hat{c}^{2}_{k}(\hat{m}^{k-1}, 1, \hat{m}_{k+1}^{N}) = 0 \).

\( (2.5) \)

An agent \( k \) announcing \( \hat{m}_{k} = 1 \) receives the date-1 consumption payoff under the direct revelation mechanism \( \{T, c^{*}(\delta)\} \) only if all earlier agents \( j < k \) announce either \( \hat{m}_{j} = 1 \) or \( \hat{m}_{j} = 2 \); otherwise he receives zero. That is, there is a suspension of first-period payments after an agent \( j < k \) announces \( \hat{m}_{j} = g \). The date-2 consumption payoff associated with the announcement \( \hat{m}_{k} = 1 \) is zero, as in the direct revelation mechanism \( \{T, c^{*}(\delta)\} \). If agent \( k \) announces \( \hat{m}_{k} = g \), then

\[
\hat{c}^{1}_{k}(\hat{m}^{k-1}, g) = 0 \quad \text{and} \quad \hat{c}^{2}_{k}(\hat{m}^{k-1}, g, \hat{m}_{k+1}^{N}) = \hat{c}^{1}_{k}(\hat{m}^{k-1}, 1) + \varepsilon,
\]

where \( \varepsilon > 0 \) is an arbitrarily small number. To keep the presentation simple, we assume throughout the paper that \( \varepsilon \) is taken small enough so all results hold. If agent \( k \) announces \( \hat{m}_{k} = g \), then he receives a zero payoff at date 1. At date 2, he receives a payoff that is slightly bigger than the date-1 payoff he would receive by announcing \( \hat{m}_{k} = 1 \)—see \( (2.5) \)—which implies that \( \hat{c}^{2}_{k}(\hat{m}^{k-1}, g, \hat{m}_{k+1}^{N}) = \hat{c}^{1}_{k}(\hat{m}^{k-1}, 1) + \varepsilon \). Hence, announcing \( \hat{m}_{k} = g \) strictly dominates announcing \( \hat{m}_{k} = 1 \) for any patient agent \( k \). Finally, if agent \( k \)
announces $\hat{m}_k = 2$, then

$$
\hat{c}_k^1(\hat{m}^{k-1}, 2) = 0 \text{ and } \hat{c}_k^2(\hat{m}^{k-1}, 2, \hat{m}_N^k) = \frac{R \left[ Y - \sum_{j=1}^N \hat{c}_j^1(\hat{m}_j) \right]}{n_{\hat{m}_N}} - \sum_{j=1}^N \hat{c}_j^2(\hat{m}_N) \mathbb{1}_{\hat{m}_j = g}
$$

(2.7)

where $n_{\hat{m}_N}$ represents the number of agents who announced $\hat{m} = 2$ in the announcement vector $\hat{m}_N$ and $\mathbb{1}_{\hat{m}_j = g}$ is an indicator function, where $\mathbb{1}_{\hat{m}_j = g} = 1$ if $m_j = 1$ and 0 otherwise.

If agent $k$ announces $\hat{m}_k = 2$, then he receives a $1/n_{\hat{m}_N}$ share of the total date-2 output that remains after payments to agents $j$ who announced either $m_j = 1$ or $m_j = g$ are made. Since the allocation rule $\hat{c}$, given by (2.5)-(2.7), depends on $\delta$ and $\epsilon$, we will denote it as $\hat{c}(\delta, \epsilon)$.

Generally speaking, a patient agent $j$ who announces $m_j = 1$ adversely affects the payoffs of truthfully announcing patient agents in two ways. First, the payments to an agent who announces $m_j = 1$ are made in period 1 which implies that these resources cannot benefit from the investment opportunity, $R$, available between dates 1 and date 2. Second, if impatient agents have a relatively high marginal utility of consumption compared to patient agents, i.e., $\rho$ is small, then, due to risk-sharing considerations, payments to agents who announce $m_j = 1$ can be quite high, leading to less resources available to the patient agents. Interestingly, the story is a bit different when patient agent $j$ announces $\hat{m}_j = g$ and impatient agents have a relatively low marginal utility of consumption compared to patient agents. Following a $g$ announcement there is a suspension of date 1 payments and agents who announce $g$ receive their payments at date 2. Hence, all suspended payments benefit from the investment opportunity that is available between dates 1 and 2, and patient agents who announced truthfully will receive a fraction of the investment return, $R$. In addition, if $\rho$ is relatively large, then the date-2 payment to agent $j$ will be relatively low, which benefits truth-telling patient agents.

Patient agent $k$ who announces truthfully will benefit from announcement $m_j = g$ if
allocation rule \( \hat{c}(\delta, \varepsilon) \) has the following property

\[
\hat{c}_k^2(\delta, \varepsilon) (\hat{m}_k^{k-1}, 2, \hat{m}_{k+1}^N) \geq c_k^2(\delta, \varepsilon) (\hat{t}_k^{k-1}, 2, \hat{t}_{k+1}^N) = c^*_k(\delta)(\hat{t}_k^{k-1}, 2, \hat{t}_{k+1}^N), \tag{P1}
\]

where \( \hat{t}_i \in T_i \) (\( \hat{t}_i^N \in T_i^N \)) is a vector of length \( i \) (\( T - i \)) such that for each \( j \leq i \) (\( i \leq j \leq N \)), \( \hat{t}_j = 1 \) if \( \hat{m}_j = 1 \) and \( \hat{t}_j = 2 \) if either \( \hat{m}_j = 2 \) or \( \hat{m}_j = g \). In words, vector \( \hat{t}_i \) (\( \hat{t}_i^N \)) is constructed from the message vector \( \hat{m}_i \) (\( \hat{m}_i^N \)) by replacing all of the \( g \)'s with 2's. The first term in (P1) is the payoff to a truthfully announcing patient agent when some (patient) agents announce \( g \). The second term is the payoff to patient players when those \( g \) announcements are replaced by 2, which, by construction, also equals the payment from the best implementable allocation. If the contract \( \hat{c}(\delta, \varepsilon) \) is characterized by property (P1), then, clearly, a truthfully announcing patient agent benefits if some other (patient) agent announces \( g \). In fact, his payoff will exceed that associated with the best weakly implementable allocation, \( c^*(\delta) \).

Under what circumstances will the allocation rule \( \hat{c}(\delta, \varepsilon) \) have property (P1)? The above discussion suggests that truthfully announcing patient agents benefit from a \( m_j = g \) announcement the larger is \( R \) and/or the larger is \( \rho \). (Recall that the higher is \( \rho \), the smaller will be the payments to impatient agents.) Our first proposition verifies this intuition.

**Proposition 2.1** If \( \rho R > 1 \), then property (P1) holds.

**Proof.** See Appendix. \( \square \)

Property (P1) seems to imply that, since more resources are available to patient players who announce truthfully and less to patient players who announce \( g \), it is rational for patient players to announce truthfully. Our main proposition demonstrates that this intuition is, in fact, correct.
Proposition 2.2 If property (P1) holds, then the indirect mechanism \( \{ \hat{M}, \hat{c}(\delta, \epsilon) \} \) strongly implements allocation \( c^*(\delta) \) in rationalizable strategies.

Proof. The mechanism \( \{ \hat{M}, \hat{c}(\delta, \epsilon) \} \) induces a symmetric Bayesian game \( \Gamma = \{ T, S \} \) where, \( T = \{ 1, 2 \} \) is the set of types, \( s_t \in \hat{M} \) is the player’s message contingent on his type \( t \in T \) and \( S = \{(s_1, s_2) \in \hat{M}^2 \} \) is the set of pure strategies. We solve the game by iterated elimination of strictly dominated strategies in two rounds.

Round 1 - Any strategy \( (s_1, s_2) \in S \), with \( s_1 \neq 1 \), is strictly dominated by \( (1, s_2) \) since, contingent on being impatient, an agent only derives utility from period 1 consumption. Additionally, any strategy \( (s_1, 1) \) is strictly dominated by \( (s_1, g) \) since, contingent on being patient, agents are indifferent between period 1 or period 2 consumption and announcing \( g \) always gives a total payment that is \( \epsilon \) higher than announcing 1. Let \( S^1 = \{(1, 2), (1, g)\} \) denote the set of strategies that survive the first round of elimination of strictly dominated strategies.

Round 2 - When strategies are restricted to \( S^1 \), impatient agents announce 1 and patient agents announce either 2 or \( g \). From property (P1), the lower bound on the expected payoff to a patient player who announces 2 is

\[
\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left( c_1^{2*}(\delta) (t_k^{k-1}, 2, t_{k+1}^N) \right).
\]

Since the payment to agent \( k \) who announces \( m_k = g \) is either \( c_1^{1*}(t_k^{k-1}, 1) + \epsilon \) or \( \epsilon \), the expected payoff to a patient player who announces \( g \) is bounded above by

\[
\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left( c_1^{1*}(\delta) (t_k^{k-1}, 1) + \epsilon \right).
\]
Since $u$ is continuous, there exists an $\varepsilon > 0$ sufficiently small so that

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t \in P_n} \frac{1}{n} \sum_{k \in Q_n} \left\{ \rho u\left(c^1_k(\delta)(t^{k-1}, 1) + \varepsilon\right) - \rho u\left(c^1_k(\delta)(t^{k-1}, 1)\right) \right\} < \delta.$$ 

The incentive compatibility condition (2.3) can be rewritten as

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u\left(c^2_k(\delta)(t^{k-1}, 2, t_k^{N+1})\right) \geq \sum_{n=1}^{N} \hat{\pi}_n \sum_{t \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u\left(c^1_k(\delta)(t^{k-1}, 1)\right) + \delta.$$ 

Combining the above two inequalities, we get

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u\left(c^2_k(\delta)(t^{k-1}, 2, t_k^{N+1})\right) > \sum_{n=1}^{N} \hat{\pi}_n \sum_{t \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u\left(c^1_k(\delta)(t^{k-1}, 1) + \varepsilon\right).$$

(2.8)

Therefore, the strategy $(1, g)$ is strictly dominated by the strategy $(1, 2)$ in $S^1$. Let $S^2$ be the set of strategies that survive the second round of elimination of strictly dominated strategies. Since $S^2 = \{(1, 2)\}$ is a singleton, the game is iterated strict dominance solvable. The unique equilibrium strategy is the truth-telling $s = (1, 2)$, which implies the same outcome as the truth-telling equilibrium of the direct mechanism $\{T, c^*(\delta)\}$. ■

If allocation $\hat{c}(\delta, \varepsilon)$ has property (P1), then, just as in the stripped-down example from Section 2.4, mechanism $\{\hat{M}, \hat{c}(\delta, \varepsilon)\}$ admits only one equilibrium characterized by truth telling for all agents. Hence, mechanism $\{\hat{M}, \hat{c}(\delta, \varepsilon)\}$ does not allow bank runs. In addition, the allocation delivered by the mechanism, $\hat{c}(\delta, \varepsilon)$, can be made arbitrarily close to the best weakly implementable allocation $c^*(0)$ by choosing $\delta$ arbitrarily close to zero.

Together, Propositions 2.1 and 2.2 imply that a sufficient condition for unique implementation is $\rho R > 1$. This is quite interesting and, perhaps, even remarkable. Diamond and Dybvig (1983) construct a model where fractional reserve banks endogenously arise
and use the model to help us understand the notion that banks are inherently unstable. Their 1983 article requires that $\rho R > 1$. Propositions 2.1 and 2.2 in this article, however, indicates that for this parametrization banks are always stable.

We want to emphasize that conditions stated in Propositions 2.1 and 2.2 are only sufficient conditions. Regarding Proposition 2.1, one can see from the proof that if incentive compatibility condition (2.3) does not bind, then condition the $\rho R > 1$ is not necessary. This means that contract $\hat{c}(\delta, \varepsilon)$ can be consistent with property (P1) even if $\rho R < 1$. In the subsequent section, we provide an example of this (even when the incentive compatibility condition (2.3) binds). Regarding Proposition 2.2, property (P1) allows us to derive a lower bound on the expected payoff of a patient agent announcing $m = 2$ and, therefore, to use dominance arguments to demonstrate uniqueness. But neither, such lower bound or dominance arguments, are necessary for uniqueness. In the subsequent section we provide an example where contract allocation $\hat{c}(\delta, \varepsilon)$ does not have property (P1) but the indirect mechanism $\{\hat{M}, \hat{c}(\delta, \varepsilon)\}$ uniquely implements $\hat{c}(\delta, \varepsilon)$.

### 2.6. Some examples

In this section we provide some examples that show the sufficient conditions described in Propositions 2.1 and 2.2 are not necessary for unique implementation of the allocation rule $c^*(\delta)$. The first example shows that property (P1) can hold when $\rho R < 1$. A second example shows that allocation rule $c^*(\delta)$ can be uniquely implemented when property (P1) is violated.

Common to all examples are: (i) $R = 1.05$; (ii) $Y = 6$; (iii) $\rho R < 1$; (iv) $\delta = 10^{-10}$; and (v) the general structure of preferences is given by

$$u(x) = \frac{(x + 1)^{1-\gamma} - 1}{1 - \gamma}, \quad \gamma > 1.$$  \(2.9\)

\(^{18}\) Notice that $u(0) = 0$. 

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In the first example, \( N = 2, \, \rho = 0.9, \, \gamma = 1.01 \) and \((\pi_0, \pi_1, \pi_2) = (0.005, 0.4975, 0.4975)\). Notice that \( \rho R < 1 \). The best weakly implementable allocation, \( c^*(0) \), which is obtained by solving (2.4), has \( c^*_1(1) = 3.1487 \) and \( c^*_2(2, 1) = 3.1481 \). The other payments can be derived from the resource constraint (2.2) holding at equality. It is straightforward to show that the direct mechanism \( \{ T, c^*(0) \} \) admits a bank-run equilibrium for this example. For \( \epsilon \) arbitrarily small, property (P1) holds, even though \( \rho R < 1 \). Therefore, although \( \rho R > 1 \) is a sufficient condition for property (P1), it is not a necessary one. Since property (P1) is satisfied in this example, Proposition 2.2 implies that \( \{ \hat{M}, \hat{c}(\delta, \epsilon) \} \) uniquely implements allocation \( c^*(\delta) \) for \( \delta \) and \( \epsilon \) small. In this example, constraint (2.3) binds. This implies that incentive constraints in the Green and Lin (2003) environment—where agents know their queue positions—will also bind and that the best implementable allocation from that environment is not equal to \( c^*(0) \).\(^{19}\) Hence, the Green and Lin (2003) mechanism is unable to even weakly implement the allocation \( c^*(\delta) \), where \( \delta \) is arbitrarily small.

The second example replicates the Peck and Shell (2003) example in Appendix B. The only difference between the examples is the specification of preferences. Peck and Shell (2003) assume that \( u(x) = c^{1-\gamma}/(1-\gamma) \), which implies that \( u(0) = -\infty \). For these preferences, our mechanism trivially uniquely implements allocation \( c^*(\delta) \), since patient agent \( k \) will never announce \( m_k = g \) if there is a probability, however small, that some other agent \( j \) will announce \( m_j = g \). The parameters for our second example are \( N = 2, \, \rho = 0.1, \, \gamma = 2 \) and \((\pi_0, \pi_1, \pi_2) = (0.25, 0.5, 0.25)\). Notice that \( \rho R < 1 \). The best weakly implementable allocation, \( c^*(0) \), is characterized by \( c^*_1(1) = 3.0951 \) and \( c^*_2(2, 1) = 3.1994 \). Allocation \( c^*(0) \) features bank runs and a binding incentive constraint (2.3). (This implies that a Green and Lin (2003) mechanism cannot weakly implement \( c^*(0) \).) It is straightforward to demonstrate that the mechanism \( \{ \hat{M}, \hat{c}(\delta, \epsilon) \} \)

\(^{19}\)Our environment can be turned into the Green and Lin (2003) environment by allowing the planner to tell agent \( k \) his queue position, \( k \), before agent \( k \) makes sends his message.
uniquely implements allocation $c^*(\delta)$, for $\delta$ and $\varepsilon$ arbitrarily close to zero. For this example $c_1^*(2, 1) + c_2^*(2, 2) > RY$, which implies that property (P1) is not satisfied for all $\hat{m}^N \in \hat{M}^N$. Hence, property (P1) is not necessary for unique implementation. We are not aware of any mechanism in the literature that can implement the best weakly implementable allocations from these two examples. We have experimented with many combinations of model parameters. We are unable to find a set of parameters for which the indirect mechanism $\{\hat{M}, \hat{c}(\delta, \varepsilon)\}$ cannot uniquely implement an allocation that is arbitrarily close to the best weakly implementable allocation. Our search, however, was restricted to $N \in \{2, 3\}$. It is, of course, possible that the indirect mechanism $\{\hat{M}, \hat{c}(\delta, \varepsilon)\}$ does not uniquely implement the best weakly implementable allocation for some set of parameters—that we were unable to recover—when $\rho R \leq 1$. In the next section, we propose an alternative indirect mechanism to deal with this case.

### 2.7. An alternative mechanism

The indirect mechanism $\{\hat{M}, \hat{c}(\delta, \varepsilon)\}$ uniquely implements allocation $c^*(\delta)$ for the $N \in \{2, 3\}$ examples we considered, but there may exist primitives for which it does not. To address this issue, we construct an alternative mechanism that uniquely implements $c^*(\delta)$ in pure and symmetric strategies. The mechanism, however, does rule out the existence of mixed or asymmetric Nash equilibria.\(^{20}\)

The alternative indirect mechanism is denoted by $\{\hat{M}, \hat{c}\}$, where $\hat{M} = \{1, 2, g\}$ and $\hat{c}$ is described below. For a given $\hat{m}^{k-1} \in \hat{M}^{k-1}$, define $\hat{t}^{k-1} \in T^{k-1}$ as a vector of length $k - 1$, where for each $j \leq k - 1$, $\hat{t}_j = 1$ if either $\hat{m}_j = 1$ or $\hat{m} = g$; and $\hat{t}_j = 2$ if $\hat{m}_j = 2$. It is important to emphasize that the relationship between $\hat{m}_j$ and $\hat{t}_j$ is different from that of $\hat{m}_j$ and $\hat{t}_j$. Specifically, the vector $\hat{t}^{k-1}$ is constructed from $\hat{m}^{k-1}$ by replacing any $g$'s

\(^{20}\) Mechanism $\{\hat{M}, \hat{c}(\delta)\}$ does rule out these equilibria when $\rho R > 1$. It is interesting to note, however, that in the literature virtually all of the analyses of the Diamond-Dybvig model focus on pure and symmetric equilibria.
with 1’s, while vector $\tilde{i}^{k-1}$ is constructed from $\tilde{m}^{k-1}$ by replacing any $g$’s with 2’s.

The construction of the allocation rule $\tilde{c}$ uses the best weakly implementable allocation rule, $c^\ast(0)$. If agent $k$ announces $\tilde{m}_k = 1$, then

$$\tilde{c}_k^1(\tilde{m}^{k-1}, 1) = c_k^1(0) \left( \tilde{y}^{k-1}, 1 \right) \text{ and } \tilde{c}_k^2(\tilde{m}^{k-1}, 1, \tilde{m}_{k+1}^N) = 0. \quad (2.10)$$

When agent $k$ announces $\tilde{m}_k = 1$ he receives the consumption associated with announcing $m_k = 1$ in the direct revelation mechanism $\{T, c^\ast(0)\}$, where announcement $\tilde{m}_j = g$ in the indirect mechanism is treated as if it is $\tilde{m}_j = 1$. If agent $k$ announces $\tilde{m}_k = g$, then

$$\tilde{c}_k^1(\tilde{m}^{k-1}, g) = 0 \text{ and } \tilde{c}_k^2(\tilde{m}^{k-1}, g, \tilde{m}_{k+1}^N) = \begin{cases} 
    c_k^1(0) \left( \tilde{y}^{k-1}, 1 \right) + \epsilon & \text{if } \tilde{m}_j = 1 \text{ for all } j \neq k, \\
    0 & \text{otherwise}
\end{cases}, \quad (2.11)$$

where $\epsilon > 0$ is arbitrarily small. If agent $k$ announces $\tilde{m}_k = g$, he receives a zero date 1 payoff. His date 2 payoff is slightly bigger than what he would receive by announcing $\tilde{m}_k = 1$ but only if all other agents $j$ announce $m_j = 1$; otherwise, he receives a payoff of zero. Finally, if agent $k$ announces $\tilde{m}_k = 2$, then

$$\tilde{c}_k^1(\tilde{m}^{k-1}, 2) = 0 \text{ and } \tilde{c}_k^2(\tilde{m}^{k-1}, 2, \tilde{m}_{k+1}^N) = \frac{R \left[ Y - \sum_{j=1}^N \tilde{c}_j^1(\tilde{m}^k) \right] - \sum_{j=1}^N \tilde{c}_j^2(\tilde{m}^N) \mathbb{1}_{\tilde{m}_j = g}}{n_{\tilde{m}^N}}, \quad (2.12)$$

where $n_{\tilde{m}^N}$ represents the number of agents $j$ who announced $\tilde{m}_j = 2$. If agent $k$ announces $\tilde{m}_k = 2$, then he receives an equal share of date-2 output net of any payments made to agent $j$ who announce $\tilde{m}_j = g$. Since the allocation rule $\tilde{c}$ given by (2.10)–(2.12) depends on $\delta$, we will denote it as $\tilde{c}(\epsilon)$.

When considering only pure and symmetric equilibria, the indirect mechanism $\{\tilde{M}, \tilde{c}(\epsilon)\}$ is quite powerful. Specifically,
Proposition 2.3  The indirect mechanism \( \{ \bar{M}, \bar{c}(\varepsilon) \} \) uniquely implements the best weakly implementable allocation \( c^*(0) \) in pure and symmetric Nash equilibrium.

Proof. All impatient agents \( k \) announce truthfully since announcing \( \bar{m}_k = 1 \) results in a strictly positive date-1 payoff and announcing \( \bar{m}_k \neq 1 \) results in a date-1 payoff equal to zero.

First, there cannot exist an equilibrium where all patient agents \( k \) announce \( \bar{m}_k = 1 \). Suppose such an equilibrium exists. Then some patient agent \( j \) can defect from proposed equilibrium and announce \( \bar{m}_j = g \). Agent \( j \)'s payoff is strictly greater than the payoff associated with announcing \( \bar{m}_j = 1 \) by the amount \( \varepsilon > 0 \); a contradiction.

Second, there cannot be an equilibrium where all patient players \( k \) announce \( \bar{m}_k = g \). To see this, note that if agent \( k \) announces \( \bar{m}_k = g \), then his payoff will be zero if there are other patient agents in the economy. The (proposed) equilibrium payoff, therefore, is

\[
\hat{\pi}_1 \sum_{k=1}^{N} \rho u \left[ c^*_1(0) \left( 1^{k-1}, 1 \right) + \varepsilon \right] + (1 - \hat{\pi}_1) \rho u(0).
\]  \tag{2.13}

If instead, agent \( k \) defects from proposed play announces \( \bar{m}_k = 1 \), his payoff will be

\[
\frac{1}{N} \sum_{k=1}^{N} \rho u \left[ c^*_k(0) \left( 1^{k-1}, 1 \right) \right].
\]  \tag{2.14}

Since \( \hat{\pi}_1 < 1 \), for \( \varepsilon > 0 \) sufficiently small (2.14) exceeds (2.13); a contradiction.

Third, there is an equilibrium where all patient agents \( k \) announce \( \bar{m}_k = 2 \). By construction, patient agent \( j \) has no incentive to announce \( \bar{m}_j = 1 \) when all other agents announce truthfully, i.e., allocation rule \( c^*(0) \) is incentive compatible for patient agents when \( \bar{m}_j \) is restricted to the set \( \{1, 2\} \). Suppose, instead, that patient agent \( j \) defects from equilibrium play and announces \( \bar{m}_j = g \). In this case, his payoff will be only slightly greater than the payoff associated with announcing \( \bar{m}_j = 1 \) if and only if he is the only
patient agent in the economy—an event that occurs with probability $\hat{\pi}_1$. With probably $1 - \hat{\pi}_1$, there are other patient agents $k$ who announce $\hat{m}_k = 2$, which implies that patient agent $j$ receives a zero payoff. For any $\hat{\pi}_1 < 1$, there exists an $\epsilon > 0$ sufficiently small so that the expected payoff associated with announcing $\hat{m}_j = g$ is strictly less than that associated with announcing $\hat{m}_j = 1$, when all other agents announce truthfully.

The unique symmetric and pure equilibrium strategy for mechanism $\{\tilde{M}, \tilde{c}\}$ is characterized by truth-telling, i.e., $\hat{m}_k = t_k$ for all $k$. By construction, these strategies implement the best weakly implementable allocation in $c^*(0)$.

There is an interesting tradeoff between the two indirect mechanisms that have been studied. Mechanism $\{\hat{M}, \hat{c}\}$ has a very weak equilibrium concept, rationalizability. However, unique implementation is guaranteed only if the restriction $\rho R > 1$ is satisfied. Unique implementation is possible when $\rho R \leq 1$, as our examples demonstrate, but it has to be verified on a case-by-case basis. Mechanism $\{\tilde{M}, \tilde{c}\}$ has a very strong equilibrium concept, pure and symmetric Nash equilibria. However, no restrictions are required on model parameters to guarantee unique implementation. Unique implementation is possible for when mix strategies are allowed, but it has to be verified on a case-by-case basis.

Finally, the indirect mechanism $(\hat{M}, \hat{c})$ relies on both punishments and suspension for unique implementation. Since strategies are restricted to be pure and symmetric, indirect mechanism $(\tilde{M}, \tilde{c})$ only relies on punishments for unique implementation.

### 2.8. Policy discussion

The most common prescription for enhancing the stability of demandable debt is to modify the contract to include a partial suspension clause. For example, Cochrane (2014), suggests that if securities are designed so debtors have the right to delay payment, suspend convertibility, or pay in part, then it is much harder for a run to develop. Santos and Neftci (2003) recommend the use of extendable debt—which is a suspension in payments—
in the sovereign debt market to help mitigate the frequent debt crises that have afflicted emerging economies and, recently, more advanced economies as well. In June 2013, the Securities and Exchange Commission (SEC) announced a set of proposals to help stabilize money market funds (MMFs). One of the key proposals recommends that the MMF board of directors have the discretion to impose of penalty redemption fees and redemption gates—or suspension of payments—in times of heavy redemption activity.

The effect of such proposals is to render demandable debt more state-contingent. In this sense, the proposals above are consistent with the properties of the optimal debt contracts described in Diamond and Dybvig (1983), Green and Lin (2003), and Peck and Shell (2003). But given that bank-run equilibria remain a possibility in the latter model, one is led to question whether the use of such measures constitute only necessary, and not sufficient conditions, for stability.

The key question concerns the issue of precisely what information is used to condition the suspension/extension clause. In the Diamond and Dybvig (1983) model without aggregate risk, suspension is triggered when “reserves” are reach a well-specified critical level. Evidently, this conditioning factor is sufficient to prevent runs in that environment. Similarly, the partial suspension schedules described in Green and Lin (2003) and Peck and Shell (2003) are triggered by measures of reserve depletion (more precisely, the history of reported types). In reality, the volatility of redemption rates varies across different classes of MMFs. Schmidt et al. (2013), for example, report that MMFs with volatile flow rates prior to the financial crisis of 2008 were more likely to experience runs during the crisis. How are directors of these funds to ascertain whether a spike in redemptions is attributable to fear rather than fundamentals? Our indirect mechanism suggests that information beyond some measure of redemption activity or resource availability is needed to prevent the possibility of a bank-run. We need to know why depositors are exercising their redemption option. For better or worse, this information is private and must therefore be elicited directly—as in our model—or inferred indirectly—through some
other means. Of course, information revelation must be incentive compatible.

Just how realistic is this idea? There is, in fact, historical precedence for the practice of soliciting additional information in periods of heavy redemption activity. For example, banks would sometimes permit limited redemptions to occur for depositors that could demonstrate evidence of impatience, e.g., a need to meet payroll. Gorton (1985, fn 7) reports that 19th century clearinghouses would regularly investigate rumors pertaining to the financial health of member banks.

As a practical matter, the spirit of our mechanism could be implemented in several different ways. One way would be to permit depositors to pay a small fee for the right to have their funds diverted to a segregated, priority account.\textsuperscript{21} Such an action could be interpreted as a communication of an impending run. The priority debt differs from other debt only in the event of failure and the ratio of priority to non-priority debt outstanding informs the issuer on the degree to which depositors expect the bank to fail. In principle, the suspension clause could be made conditional on this ratio hitting some specified threshold. It does not need to be official as long as there is a mutual understanding that it will be used. And along the lines suggested by our mechanism, if one knows that the bank will suspend before any rumor-induced trouble affects their balance sheet, then depositors know that there will be no reason, in equilibrium, to actually exercise the option of converting their claims to priority debt.

To summarize, current policy proposals designed to prevent, or at least mitigate, bank runs in demandable debt structures focus on enhancing state-contingency, with contingencies dictated by some measure of redemption activity or resource depletion. Our analysis suggests that while state contingency is necessary, it may not be sufficient to prevent bank runs. Suspension clauses should be conditioned on information relating to depositor beliefs about what they perceive to be happening around them. The desired information could be elicited in an incentive compatible manner through an appropriate modification.

\textsuperscript{21}This is effectively what happens in our mechanism when a depositor reports $m = g$. 42
of the deposit contract—an example of which we described above. If we are wrong in our present assessment, the inclusion of such a clause would be inconsequential. But if we are correct, then the inclusion of such a clause may help to prevent bank runs in debt structures that are presently run prone.

2. A. Appendix

In order to prove proposition 2.1 we first establish the following result.

Lemma 2.4 If \( \rho R > 1 \) then \( c^1_k(\delta)(\bar{t}^{k-1}, 1) < c^2_k(\delta)(\bar{t}^{k-1}, 2^{N-k+1}) \) for all \( k \in \mathbb{N} \) and \( \bar{t}^{k-1} \in T^{k-1} \). Where \( 2^n \) denotes the \( n \)–dimensional vector of twos.

Proof. Since \( c^*(\delta) \) solves problem (2.4), it satisfies the implied first-order conditions.\(^{22}\)

Let \( \lambda_{tn} \) denote the Lagrange multiplier of the feasibility constraint (2.2) for each \( t^n \in T^n \) and \( \mu \) denotes the Lagrange multiplier of the incentive compatibility (2.3). By simplicity, \( \lambda_{tn} \) is normalized by \( \pi_{nN} / (\begin{pmatrix} N \\ n \end{pmatrix}) \), where \( n_{tN} \) denotes the number of type 2 players in queue \( t^n \). And \( \mu \) is normalized by \( \bar{\pi} = \sum_{n=1}^{N} \pi_n / (\begin{pmatrix} N \\ n \end{pmatrix}) \). Since \( u'(0) = \infty \) the constraint \( c^1 \geq 0 \) and \( c^2 \geq 0 \) are not binding and the respective Lagrange multipliers can be ignored. The first order conditions of the problem are given below.

\[
\begin{bmatrix}
 c^1_k(\bar{t}^k) \\
 \sum_{n=0}^{N} \frac{\pi_n}{(\begin{pmatrix} N \\ n \end{pmatrix})} \sum_{t^n \in P_n} \left\{ u'\left[c^1_k(\bar{t}^k) - \lambda_{tn} R\right]\right\} - \sum_{n=1}^{N} \frac{\pi_n}{(\begin{pmatrix} N \\ n \end{pmatrix})} \sum_{t^n \in P_n} \frac{\mu \rho}{n_{tN}} u'\left[c^1_k(\bar{t}^k)\right]
\end{bmatrix} = 0
\]

(2.15)

\(^{22}\) From now on we will denote \( c^*(\delta) \) just by \( c \) in order to keep the notation short.
for all $k \in \mathbb{N}$ and $\bar{t}^{k-1} \in T^{k-1}$ such that $\bar{t}_k = 1$; and

$$\left[ c^2(t^N) \right] : \frac{\pi n}{(N)} \left\{ \rho u' \left[ c^2(t^N) \right] - \lambda_{t^N} + \frac{\mu \rho}{n t^N} u' \left[ c^2(t^N) \right] \right\} = 0 \quad (2.16)$$

for all $t^N \in T^N$ such that $n_{t^N} > 0$. We can solve the above equations for $\lambda_{t^N}$ and obtain

$$\lambda_{t^N} = \begin{cases} \rho \left( 1 + \frac{\mu}{n_{t^N}} \right) u' \left[ c^2(t^N) \right] & \text{if } n_{t^N} > 0 \\ \frac{1}{R} u' \left[ c^1(k^N) \right] & \text{if } n_{t^N} = 0 \end{cases}.$$  

Note that $c^2(t^N)$ is not defined if $t^N = 1^N = (1, 1, \ldots, 1)$—there is no second period payments when every depositor announces to be of type impatient in the first period. In order to keep the notation short, let us define $u' \left[ c^1(k^N) \right] = \rho R u' \left[ c^2(1^N) \right]$ and $1/n_{1^N} = 0$. Then, $\lambda_{t^N}$ is given by

$$\lambda_{t^N} = \rho \left( 1 + \frac{\mu}{n_{t^N}} \right) u' \left[ c^2(t^N) \right]. \quad (2.17)$$

After replace equation (2.17) in equation (2.15) we obtain that for all $k \in \mathbb{N}$ and $\bar{t}^k = (\bar{t}^{k-1}, 1) \in T^{k-1}$:

$$\sum_{n=0}^{N} \frac{\pi n}{(N)} \sum_{t^N \in p_n} u' \left[ c^1_k(\bar{t}^k) \right] - \sum_{n=0}^{N} \frac{\pi n}{(N)} \sum_{t^N \in p_n} \frac{\mu \rho}{n_{t^N}} u' \left[ c^1_k(\bar{t}^k) \right] = \sum_{n=0}^{N} \frac{\pi n}{(N)} \sum_{t^N \in p_n} R \rho \left( 1 + \frac{\mu}{n_{t^N}} \right) u' \left[ c^2(t^N) \right]$$

which is equivalent to

$$\left\{ P \left[ t^k = (\bar{t}^{k-1}, 1) \right] - \sum_{n=0}^{N} \frac{\pi n}{(N)} \sum_{t^N \in p_n} \frac{\mu \rho}{n_{t^N}} \right\} u' \left[ c^1_k(\bar{t}^k) \right] = \sum_{n=0}^{N} \frac{\pi n}{(N)} \sum_{t^N \in p_n} R \rho \left( 1 + \frac{\mu}{n_{t^N}} \right) u' \left[ c^2(t^N) \right].$$
Therefore, we know that the resources constraints holds at equality because $u$.

And after reorganize the equation above we have that $k = n$ starting from $k = N$ and going down until $k = 1$.

**Proof for $k = N$:** Fix any $i^N = (i^{N-1}, 1)$. From equation (2.18) we have that

$$
\left[ 1 - \gamma(i^{k-1}) \right] u\left[ c_1^k(i^k) \right] = \mathbb{E}_{i^N \mid i^k = (i^{k-1}, 1)} \left\{ R\rho \left( 1 + \frac{\mu}{n_{i^N}} \right) u\left[ c^2(t^N) \right]\right\}
$$

(2.18)

where $\gamma(i^{k-1}) = \mathbb{P}\left[ t^k = (i^{k-1}, 2) \mid i^N \right] / \mathbb{P}\left[ t^k = (i^{k-1}, 1) \right]$. The result will be derived from equation (2.18). Let us use induction on $k \in \mathbb{N}$.

We can also write the equation in expectations, which yields to the formula

$$
\bar{\gamma}(i^k) = \mathbb{E}_{i^N \mid i^k = (i^{k-1}, 1)} \left\{ R\rho \left( 1 + \frac{\mu}{n_{i^N}} \right) u\left[ c^2(t^N) \right]\right\}
$$

(2.18)

which implies that $u\left[ c_1^k(i^k) \right] > u\left[ c^2(i^k) \right]$. Thus, $c_1^k(i_{N-1}, 1) < c^2(i_{N-1}, 1)$.

We know that the resources constraints holds at equality because $u$ is strictly increasing. Therefore,

$$
n_{(i^{N-1}, 2)} c^2(i^{N-1}, 2) = [n_{i^N} + 1] c^2(i^{N-1}, 2) = n_{i^N} c^2(i^{N-1}, 1) + R c_1^N(i^{N-1}, 1)
$$

And after reorganize the equation above we have that

$$
c^2(i^{N-1}, 2) = \frac{n_{(i^{N-1}, 1)}}{n_{(i^{N-1}, 1)} + 1} c^2(i^{N-1}, 1) + \frac{1}{n_{(i^{N-1}, 1)} + 1} R c_1^N(i^{N-1}, 1) > c_1^N(i^{N-1}, 1).
$$

Hence, for the case $k = N$, we can conclude that $c_1^k(i^{k-1}, 1) < c^2(i^{k-1}, 2^{N-k+1})$.

**Proof for $k < N$:** Assume the result holds for all $j > k$ and $\Bar{i} = (\Bar{i}^{j-1}, 1) \in T^j$. That is, for all $j > k$ we have $c_1^j(\Bar{i}^{j-1}, 1) < c^2(\Bar{i}^{j-1}, 2^{N-j})$. Let us show it also holds for $k$. Fix some $\Bar{i}^k = (\Bar{i}^{k-1}, 1) \in T^{k-1}$, then equation (2.18) is given by

$$
u\left[ c_k^1(\Bar{i}^k) \right] = \frac{1}{1 - \gamma(\Bar{i}^{k-1})} \mathbb{E}_{i^N \mid i^k = (\Bar{i}^{k-1}, 1)} \left\{ R\rho \left( 1 + \frac{\mu}{n_{i^N}} \right) u\left[ c^2(t^N) \right]\right\}.
$$
Note that, for any function $X : T^N \to \mathbb{R}$, the conditional expectation can be decomposed as

$$
\mathbb{E}_{t^N | t^k = \bar{t}^k} \left\{ X(t^N) \right\} = \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{t}^k, 2^{j-k-1}, 1) \mid t^k = \bar{t}^k \right] \mathbb{E}_{t^N | t^j = (\bar{t}^k, 2^{j-k-1}, 1)} \left\{ X(t^N) \right\} + \\
\mathbb{P} \left[ t^N = (\bar{t}^k, 2^{N-k}) \mid t^k = \bar{t}^k \right] X(\bar{t}^k, 2^{N-k}).
$$

Applying this decomposition to equation (2.18) we obtain

$$
u' \left[ c^1_k (\bar{t}^k) \right] = \left\{ \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{t}^k, 2^{j-k-1}, 1) \mid t^k = \bar{t}^k \right] \mathbb{E}_{t^N | t^j = (\bar{t}^k, 2^{j-k-1}, 1)} \left\{ R \rho \left( 1 + \frac{\mu}{n_{(\bar{t}^k, 2^{N-k})}} \right) u' \left[ c^2 (t^N) \right] \right\} + \\
\mathbb{P} \left[ t^N = (\bar{t}^k, 2^{N-k}) \mid t^k = \bar{t}^k \right] R \rho \left( 1 + \frac{\mu}{n_{(\bar{t}^k, 2^{N-k})}} \right) u' \left[ c^2 (\bar{t}^k, 2^{N-k}) \right] \right\} \frac{1}{1 - \gamma (\bar{t}^k - 1)}. \]

By equation (2.18) we know that

$$
\left[ 1 - \gamma (\bar{t}^k, 2^{j-k-1}) \right] \nu' \left[ c^1_j (\bar{t}^k, 2^{j-k-1}, 1) \right] = \mathbb{E}_{t^N | t^j = (\bar{t}^k, 2^{j-k-1}, 1)} \left\{ R \rho \left( 1 + \frac{\mu}{n_{(\bar{t}^k, 2^{N-k})}} \right) u' \left[ c^2 (t^N) \right] \right\}
$$

for $j = k+1, \ldots, N$. Hence,

$$
u' \left[ c^1_k (\bar{t}^k) \right] = \left\{ \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{t}^k, 2^{j-k-1}, 1) \mid t^k = \bar{t}^k \right] \left[ 1 - \gamma (\bar{t}^k, 2^{j-k-1}) \right] u' \left[ c^1_j (\bar{t}^k, 2^{j-k-1}, 1) \right] + \\
\mathbb{P} \left[ t^N = (\bar{t}^k, 2^{N-k}) \mid t^k = \bar{t}^k \right] R \rho \left( 1 + \frac{\mu}{n_{(\bar{t}^k, 2^{N-k})}} \right) u' \left[ c^2 (\bar{t}^k, 2^{N-k}) \right] \right\} \frac{1}{1 - \gamma (\bar{t}^k - 1)}. \]

By the inductive hypothesis we know that $c^1_j (\bar{t}^k, 2^{j-k-1}, 1) < c^2 (\bar{t}^k, 2^{N-k})$, which implies that

$$
u' \left[ c^1_k (\bar{t}^k) \right] > \frac{1}{1 - \gamma (\bar{t}^k - 1)} \left\{ \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{t}^k, 2^{j-k-1}, 1) \mid t^k = \bar{t}^k \right] \left[ 1 - \gamma (\bar{t}^k, 2^{j-k-1}) \right] u' \left[ c^2 (\bar{t}^k, 2^{N-k}) \right] + \\
\mathbb{P} \left[ t^N = (\bar{t}^k, 2^{N-k}) \mid t^k = \bar{t}^k \right] R \rho \left( 1 + \frac{\mu}{n_{(\bar{t}^k, 2^{N-k})}} \right) u' \left[ c^2 (\bar{t}^k, 2^{N-k}) \right] \right\}
$$
\[
\frac{1}{1 - \gamma(\bar{p}_k^{(k-1)})} \left\{ \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1) \mid t^k = \bar{p}_k \right] \left[ 1 - \gamma(\bar{p}_k^{(k-j-1)}) \right] + \mathbb{P} \left[ t^N = (\bar{p}_k^{(k-N)}, 1) \mid t^k = \bar{p}_k \right] R\rho \left( 1 + \frac{\mu}{n_{(\bar{p}_k, 2^{N-k})}} \right) \right\} u' \left[ c^2(\bar{p}_k^{(k)}, 2^{N-k}) \right] \\
= \frac{1}{1 - \gamma(\bar{p}_k^{(k-1)})} \left\{ \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1) \mid t^k = \bar{p}_k \right] \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1) \mid t^k = \bar{p}_k \right] \mathbb{E}_{t^N \mid t^j = (\bar{p}_k^{(k-j)}, 1)} \left[ \frac{\mu \rho}{n_{(\bar{p}_k^{(k-j)}, \bar{p}_k^{(k-j)})}} \right] u' \left[ c^2(\bar{p}_k^{(k)}, 2^{N-k}) \right] \\
- \sum_{j=k+1}^{N} \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1) \mid t^k = \bar{p}_k \right] \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1) \mid t^k = \bar{p}_k \right] \mathbb{E}_{t^N \mid t^j = (\bar{p}_k^{(k-j)}, 1)} \left[ \frac{\mu \rho}{n_{(\bar{p}_k^{(k-j)}, \bar{p}_k^{(k-j)})}} \right] u' \left[ c^2(\bar{p}_k^{(k)}, 2^{N-k}) \right] \\
+ \mathbb{P} \left[ t^N = (\bar{p}_k^{(k-N)}, 1) \mid t^k = \bar{p}_k \right] R\rho \left( 1 + \frac{\mu}{n_{(\bar{p}_k^{(k-N)}, \bar{p}_k^{(k-N)})}} \right) \right\} u' \left[ c^2(\bar{p}_k^{(k)}, 2^{N-k}) \right].
\]

After simplify the above equation we obtain

\[
u' \left[ c_k(\bar{p}_k^{(k)}) \right] > \frac{1}{1 - \gamma(\bar{p}_k^{(k-1)})} \left\{ 1 - \mathbb{P} \left[ t^N = (\bar{p}_k^{(k-N)}, 1) \mid t^k = \bar{p}_k \right] \right\} u' \left[ c^2(\bar{p}_k^{(k)}, 2^{N-k}) \right] \tag{2.19}
\]

The fact that the queue position is withdrawn uniformly implies that

\[
\mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1) \mid t^k = \bar{p}_k \right] = \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-j)}, 1, 1) \mid t^k = \bar{p}_k \right] = \mathbb{P} \left[ t^j = (\bar{p}_k^{(k-1)} - 1, 1) \mid t^k = \bar{p}_k \right]
\]

and

\[
\mathbb{E}_{t^N \mid t^j = (\bar{p}_k^{(k-j)}, 1, 1)} \left[ \frac{\mu \rho}{n_{t^N}} \right] = \mathbb{E}_{t^N \mid t^j = (\bar{p}_k^{(k-1)} - 1, 1)} \left[ \frac{\mu \rho}{n_{t^N}} \right] = \mathbb{E}_{t^N \mid t^j = (\bar{p}_k^{(k-1)} - 1, 1)} \left[ \frac{\mu \rho}{n_{t^N}} \right].
\]

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This implies that

\[
\sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{R}, 2^{j-k})]}{\mathbb{P}[t^k = \bar{R}]} \mathbb{E}_{t^N|t^j = (\bar{R}, 2^{j-k})} \left[ \frac{\mu \rho}{n_{t^N}} \right] = \sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{R} - 1, 1, 2^{j-k})]}{\mathbb{P}[t^k = (\bar{R} - 1, 1)]} \mathbb{E}_{t^N|t^j = (\bar{R} - 1, 1, 2^{j-k})} \left[ \frac{\mu \rho}{n_{t^N}} \right] 
\]

\[(2.20)\]

\[
= \sum_{j=k+1}^N \frac{\mathbb{P}[t^j = (\bar{R} - 1, 2^{j-k})]}{\mathbb{P}[t^k = (\bar{R} - 1, 1)]} \mathbb{E}_{t^N|t^j = (\bar{R} - 1, 2^{j-k})} \left[ \frac{\mu \rho}{n_{t^N}} \right] + \frac{\mathbb{P}[t^k = (\bar{R} - 1, 2^{N-k})]}{\mathbb{P}[t^k = (\bar{R} - 1, 1)]} \mathbb{E}_{t^N|t^j = (\bar{R} - 1, 2^{N-k})} \left[ \frac{\mu \rho}{n_{t^N}} \right] - \frac{\mathbb{P}[t^k = (\bar{R} - 1, 2^{N-k})]}{\mathbb{P}[t^k = (\bar{R} - 1, 1)]} \mathbb{E}_{t^N|t^j = (\bar{R} - 1, 2^{N-k})} \left[ \frac{\mu \rho}{n_{t^N}} \right] \]

\[
= \gamma(\bar{R} - 1) - \frac{\mathbb{P}[t^k = (\bar{R} - 1, 2^{N-k})]}{\mathbb{P}[t^k = (\bar{R} - 1, 1)]} \mathbb{E}_{t^N|t^j = (\bar{R} - 1, 2^{N-k})} \left[ \frac{\mu \rho}{n_{t^N}} \right].
\]

Replacing equation (2.20) in inequality (2.19) and reorganising the terms in the inequality, we obtain

\[
u' \left[ c_k^1(\bar{R}) \right] > \frac{1}{1 - \gamma(\bar{R} - 1)} \left\{ 1 - \gamma(\bar{R} - 1) + \frac{\mathbb{P}[t^k = (\bar{R} - 1, 2^{N-k})]}{\mathbb{P}[t^k = (\bar{R} - 1, 1)]} \frac{\mu \rho}{n_{(\bar{R} - 1, 2^{N-k})}} \right\} + \mathbb{P}[t^N = (\bar{R} - 1, 2^{N-k})] R \rho \left( 1 + \frac{\mu}{n_{(\bar{R}, 2^{N-k})}} - \frac{1}{R \rho} \right) u' \left[ c^2(\bar{R}, 2^{N-k}) \right].
\]

\[(2.21)\]

Because \( R \rho > 1 \), the inequality (2.21) implies that

\[
u' \left[ c_k^1(\bar{R} - 1, 1) \right] = \nu' \left[ c_k^1(\bar{R}) \right] > \nu' \left[ c^2(\bar{R}, 2^{N-k}) \right] = \nu' \left[ c^2(\bar{R} - 1, 1, 2^{N-k}) \right].
\]
And since $u$ is concave, it implies that $c_k^1(\bar{t}^{k-1},1) < c^2(\bar{t}^{k-1},1,2^{N-k})$. The resources constraint implies that

$$\left[n_{(\bar{t}^{k-1},1,2^{N-k})} + 1\right] c^2(\bar{t}^{k-1},2^{N-k+1}) = n_{(\bar{t}^{k-1},1,2^{N-k})} c^2(\bar{t}^{k-1},1,2^{N-k}) + Rc_k^1(\bar{t}^{k-1},1).$$

And finally we can conclude that

$$c^2(\bar{t}^{k-1},2^{N-k+1}) = \frac{n_{(\bar{t}^{k-1},1,2^{N-k})}}{n_{(\bar{t}^{k-1},1,2^{N-k})} + 1} c^2(\bar{t}^{k-1},1,2^{N-k}) + \frac{1}{n_{(\bar{t}^{k-1},1,2^{N-k})} + 1} Rc_k^1(\bar{t}^{k-1},1) > c_k^1(\bar{t}^{k-1},1).$$

We have shown that the result holds for $k = N$ and that if it holds for all $j \in \{k+1, \ldots, N\}$ it holds for $k$. Therefore, by induction, we can conclude that the result holds for all $k \in \mathbb{N}$.

2.A.1. Proposition 2.1

**Proof.** We know that for any vector of announcements $\hat{m}^N \in \hat{M}^N$, if either $\hat{m}^N \in T^N$ or $\hat{m}_k \neq 2$ for all $k$, the result is trivial. Consider a realized vector of announcements $\hat{m}^N \in \hat{M}^N$, with $\hat{m}^N \notin T^N$, $\hat{m}_k = 2$, and let $j$ be the queue position of the first agent to announce $\hat{g}$. As before, $\hat{t}^N \in T^N$ denotes the vector $\hat{m}^N$ we replace all $\hat{g}$’s with $2$’s. When agent $j$ announced $\hat{g}$ the in the first period payments were suspended, hence, the total resources in the beginning of period 2 is

$$R \left[ Y - \sum_{i=1}^{N} \hat{c}_i^1(\hat{m}^i) \right] = R \left[ Y - \sum_{i=1}^{j} \hat{c}_i^{*1}(\hat{t}^i) \right] = n_{(\hat{t}^{j-1},2^{N-j+1})} c_k^{*2}(\hat{t}^{j-1},2^{N-j+1}).$$

Where $n_{(\hat{t}^{j-1},2^{N-j+1})}$ is the number of $2$’s in the vector $(\hat{t}^{j-1},2^{N-j+1})$. Let $d_{\hat{m}^N}$ denote the number of agents who have announced $\hat{g}$ and $n_{\hat{m}^N}$ the number of agents who announced 2. The total payments in the second period to agents who announced $\hat{g}$ is given by

$$\sum_{k=1}^{N} c_k^2(\hat{m}^N) \mathbb{I}_{\hat{m}_k = \hat{g}} = c_j^{*1}(\hat{t}^{j-1},1) + d_{\hat{m}^N} \varepsilon.$$
Hence, payment to agent $k$ is

$$\hat{c}_k^2 \left( \hat{m}^{k-1}, 2, \hat{m}^N_{k+1} \right) = \frac{R \left[ Y - \sum_{k=1}^N \hat{c}_k^1(\hat{m}^k) \right] - \sum_{k=1}^N \hat{c}_k^2(\hat{m}^N) \mathbb{1}_{\hat{m}_k = \hat{m}}}{n_{\hat{m}^N}}$$

$$= \frac{n_{(i-1,2^{N-j+1})} \hat{c}_k^2(\hat{i}^{-1}, 2^{N-j+1}) - c_j^1(\hat{i}^{-1}, 1) - d_{\hat{m}^N} \epsilon}{n_{\hat{m}^N}}.$$ 

By lemma (2.4) we know that $c_j^1(\hat{i}^{-1}, 1) < c_j^2(\hat{i}^{-1}, 2^{N-j+1})$. Thus, by taking $\epsilon > 0$ small enough, we have that,

$$\hat{c}_k^2 \left( \hat{m}^{k-1}, 2, \hat{m}^N_{k+1} \right) \geq \frac{[n_{(i-1,2^{N-j+1})} - 1] \hat{c}_k^2(\hat{i}^{-1}, 2^{N-j+1})}{n_{\hat{m}^N}}.$$

By construction we have that

$$\hat{c}_k^2 \left( \hat{i}^{k-1}, 2, \hat{i}^N_{k+1} \right) = \frac{R \left[ Y - \sum_{l=1}^N \hat{c}_l^1(\hat{i}^l) \right]}{n_{\hat{m}^N} + d_{\hat{m}^N}} \leq \frac{R \left[ Y - \sum_{l=1}^l \hat{c}_l^1(\hat{i}^l) \right]}{n_{\hat{m}^N} + 1} = \frac{n_{(i-1, 2^{N-j+1})} \hat{c}_k^2(\hat{i}^{-1}, 2^{N-j+1})}{n_{\hat{m}^N} + 1}.$$

Note that,

$$\frac{n_{(i-1, 2^{N-j+1})} - 1}{n_{\hat{m}^N}} \geq \frac{n_{(i-1, 2^{N-j+1})}}{n_{\hat{m}^N} + 1} \iff n_{(i-1, 2^{N-j+1})} n_{\hat{m}^N} + n_{(i-1, 2^{N-j+1})} - n_{\hat{m}^N} - 1 \geq n_{(i-1, 2^{N-j+1})} n_{\hat{m}^N} \iff n_{(i-1, 2^{N-j+1})} \geq n_{\hat{m}^N} + 1.$$

The last inequality holds because $n_{(i-1, 2^{N-j+1})} \geq n_{\hat{m}^N} = n_{\hat{m}^N} + d_{\hat{m}^N}$. Hence,

$$\hat{c}_k^2 \left( \hat{m}^{k-1}, 2, \hat{m}^N_{k+1} \right) \geq \frac{[n_{(i-1,2^{N-j+1})} - 1] \hat{c}_k^2(\hat{i}^{-1}, 2^{N-j+1})}{n_{\hat{m}^N}} \geq \frac{n_{(i-1,2^{N-j+1})} \hat{c}_k^2(\hat{i}^{-1}, 2^{N-j+1})}{n_{\hat{m}^N} + 1} \geq \hat{c}_k^2 \left( \hat{i}^{k-1}, 2, \hat{i}^N_{k+1} \right).$$
Which concludes the proof.
Financial fragility and over-the-counter markets

Abstract I propose a model to study whether over-the-counter market frictions generates financial fragility. I model the financial sector as a large number of finite size groups of investors, where each investors is subject to privately observed liquidity shocks. The groups’ problem is to maximize the welfare of their investors by implementing the efficient allocation of assets among them. I show that when the balanced team mechanism, proposed by Athey and Segal (2013), is used, there is always a truth-telling equilibrium which supports the constrained Pareto efficient allocation. When the frictions in the over-the-counter market are small, this equilibrium is unique. However, I provide numerical examples in which these frictions are severe and the economy has other equilibria. In one equilibrium investors claim high liquidity needs, asset price falls, the trade volume collapses and the equilibrium allocation is not constrained Pareto efficient—characteristics of a self-fulfilling financial crisis. I interpret the existence of such type of equilibria as a fragility of the financial sector.

Keywords: Over-the-counter markets, financial intermediation, financial fragility, financial crisis, dynamic mechanism design, weak implementation.

3.1. Introduction

Investors in well developed financial systems usually participate in financial markets as part of a group/coalition which trade financial assets (often in over-the-counter markets) on behalf of members and also provide liquidity (withdrawal options) to them.
Examples of such groups are financial institutions such as bank conduits, structured investment vehicles and mutual funds. In this paper I study whether these features can generate financial fragility. Specifically, if they can generate self-fulfilling financial crisis equilibrium.

To answer my question I propose a new model of financial fragility where financial assets are traded over the counter by groups of investors. The model builds on a discrete time version of Duffie et al. (2005) and Lagos and Rocheteau (2009), hereafter DGP and LR, with three important modifications. First, investors receive privately observed preference shocks over time which generates stochastic liquidity needs (in DGP and LR these shocks are public). Second, each investor belongs to a group and participates in risk-sharing arrangement with the other group members. Third, with private information there is an issue of how to achieve truth-telling among investors. I investigate balanced team mechanism, proposed by Athey and Segal (2013), which supports truth-telling as one equilibrium outcome.\footnote{It is worth emphasizing that truth-telling is one possible equilibrium of the balanced team mechanism, but this equilibrium is not unique. And to the best of my knowledge there is no alternative mechanism able to implement an efficient outcome and which features a unique equilibrium.}

I show that there always exists an equilibrium which supports the constrained Pareto efficient allocation in the economy and, when investors trade opportunities are frequent, this equilibrium is unique. However, I provide numerical examples showing that when trade opportunities are infrequent, there may exist equilibria that are inefficient and resemble a financial crisis. In the inefficient equilibrium investors announce a high demand for liquidity because they believe that the other investors are doing the same—a form of self-fulfilling crisis which I label a panic. During a panic, the asset price falls and the trade volume collapses.

Having multiple equilibria in a model of over-the-counter markets can shed some

\footnote{The balanced team mechanism extends the mechanism proposed by Arrow (1979) and d’Aspremont and Gérard-Varet (1979), known as the AGV-Arrow mechanism, to dynamic environments. See Fudenberg and Tirole (1991) for details and results regarding the AGV-Arrow mechanism.}
light on our understanding of financial crisis. A growing empirical literature suggests that part of the events in the recent financial crisis were due to strategic complementarity within investors and the consequent multiple equilibria. See, for example, Kacperczyk and Schnabl (2010), Gorton and Metrick (2012), Schmidt et al. (2013) and Covitz et al. (2013). This paper provides a benchmark model to think about how the market structure relates with financial panics and financial fragility broadly speaking.

In particular, this paper suggests that the benefits of reducing trade frictions in an over-the-counter market are twofold. The first reason is present in previous models of over-the-counter markets. Since agents face idiosyncratic preference shocks, there are gains from trading with each other. As a result, with less trading frictions, they explore the trading opportunities more efficiently. The second reason is a novelty in my model. When the over-the-counter market is efficient, the economy features only an efficient equilibrium. Therefore, by eliminating, or substantially reducing, trade frictions, policy makers can enhance financial stability.

Of course, in many cases policy makers are unable to reduce trade frictions and, as an alternative, I investigate two policies to enhance financial stability: a suspension scheme and the opening of a centralized exchange facility where groups can trade financial assets. I consider an arrangement where either one of above policies is implemented once the aggregate distribution of announcements does not coincide with the true distribution of preference shocks, which is known since there is no aggregate uncertainty in the model. Conditional on either policy being implemented, truth-telling is a best response because there is no trade among investors in the same group—either trade occurs in a centralized market or it doesn’t occur at all. Since investors anticipate that they won’t have incentives to misrepresent their types ex-post, they also don’t have incentives to misrepresent

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3 This policy provides a rationale for a Fed intervention in the aftermath of the crisis, namely, the creation of the Term Asset-Backed Securities Loan Facility (TALF). The model suggests that the Fed should use such facility to eliminate the trade frictions by operating as a market-maker. Worth mentioning that the Fed also lent over $1 trillion dollars taking ABSs as collateral, which is inconsistent with the policy recommendation in my model.
their preference shocks *ex-ante*. As a result, there cannot exist an equilibrium where the distribution of investors’ announcements differs from the true distribution of types.

**Literature review**

Related to this paper, a large literature studies optimal bank mechanisms in the Diamond-Dybvig model to understand under which assumptions the model features multiple equilibria.\(^4\) This literature shows that for Diamond-Dybvig banks to have multiple equilibria under an optimal direct mechanism, banks need to face aggregate uncertainty and a sequential service constraint—payments must respect a first-come, first-served rule. Uncertainty is also a necessary condition for fragility in my model; however, the sequential service constraint is not. Several financial institutions considered part of the shadow bank sector finance its assets by issuing debt with specific due dates. Hence, sequential service does not seem a relevant constraint for those institutions and it is appropriate to have a model that explains financial fragility without imposing sequential service. A second finding of this literature is that an indirect mechanism can be used to prevent runs.\(^5\) Although I consider this an interesting possibility, I do not address it in this paper.

The fact that some form of trade friction is an essential element of a fragile financial sector is not new. Jacklin (1984) studies a version of the Diamond-Dybvig model where there is a centralized market to trade financial assets. In this case, they show that the equilibrium is unique and the economy is not fragile. In the limit case of my model where the market for financial assets is a centralized exchange, I obtain this same result. One may ask why not study only the limit cases where either there is a complete market or not—as most of the Diamond-Dybvig literature does. In other words, what do we gain by considering an incomplete market? The main advantage of studying a model which takes

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\(^4\) This literature includes, but is not limited to, Wallace (1988), Peck and Shell (2003), Green and Lin (2003), Andolfatto et al. (2007), Ennis and Keister (2009b), Cavalcanti and Monteiro (2011) and Andolfatto et al. (2014).

\(^5\) See Cavalcanti and Monteiro (2011) and Andolfatto et al. (2014) for a discussion of indirect mechanisms in the context of the Diamond-Dybvig model.
into account incomplete market structures is that we can evaluate market interventions such as purchase of financial assets by a policy maker. In Diamond-Dybvig model without market, such interventions cannot be studied. And in Diamond-Dybvig model with a centralized and complete asset market, such interventions are irrelevant. This evaluation is one of the contributions of my model.

Allen and Gale (2000) and Allen and Gale (2004), here after AG, study the implications of bank fragility in a setting were Diamond-Dybvig banks trade contingent claims in a static inter-bank market. There are two important differences between my work and AG. First, AG focus on understanding the implications of bank fragility in the market outcomes given different market structures—complete vs incomplete markets. But they don’t ask the question of why banks are fragile. On the other hand, the main focus of my paper is exactly to understand why groups are fragile and how it relates with different market structures. The second difference is that AG study a static Walrasian market. While I study a dynamic over-the-counter market, which allows me to analyse how market outcomes evolve over time.

Lagos et al. (2011) study financial crises in the context of an over-the-counter market where dealers provide liquidity to the economy. The financial crisis is modelled as an exogenous aggregate shock that makes all agents have a low valuation of the underlying financial asset. There is evidence that some form of aggregate shock decreased the value of mortgage-backed securities during the 2007/08 crisis period. However, in practice, it is hard to differentiate whether such shock was exogeneous, as in Lagos et al. (2011), the result of panics, as in my model, or both.

Trejos and Wright (2014) generalize preferences in the DGP model to preferences that are separable but not quasi-linear. They show that, for some parameters, there are multiple equilibria and the equilibrium dynamics can be the outcome of self-fulfilling prophecies—sunspots. The main reason for multiplicity in Trejos and Wright (2014) is

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6 Their work also integrates DGP and the monetary economy of Shi (1995) and Trejos and Wright (1995).
that the asset can also be used as a means of exchange. As a result, beliefs over whether people will accept the asset as payments in the future change the value of the asset in the present generating multiple equilibria. In my model assets do not have value as a means of exchange, instead, I explore agents long-term relationships as in institutional arrangements. My understanding is that both approaches are complementary to each other.

The rest of the paper is organized as follows. Section 2 studies a simplified version of the model where there is only one group of investors. I use this simple version to introduce the AGV-Arrow mechanism in a simple context and to provide intuition of why the model has multiple equilibria. Section 3 extends this version with one group to an infinite-horizon model. I use this one-group infinite-horizon version to introduce the Athey and Segal (2013) balanced team mechanism mechanism and to discuss a different type of equilibria that emerges in a dynamic setting. Section 4 introduces the complete model, where there are a large number of groups which interact in an over-the-counter market. Section 5 describes the balanced team mechanism for the complete version; characterizes the constrained Pareto efficient allocation; shows that this allocation is supported by an equilibrium; and also shows that, if over-the-counter frictions are small, the equilibrium that supports the constrained Pareto efficient allocation is the unique equilibrium. Section 6 provides a numerical example of an equilibrium resembling a financial crisis. Section 7 considers an extension where investors have no commitment. Section 8 studies policies to enhance stability in the financial system. And section 9 discusses the results and possible extensions.

3.2. A single group one-period model

In this section I study the simplest version of the model, where there is one group of investors who live for one period. An advantage of this version is that the implementation
result of Arrow (1979) and d’Aspremont and Gérard-Varet (1979) applies. Namely, the direct mechanism proposed by them, known as the AGV-Arrow mechanism, implements the efficient outcome in Bayesian equilibrium. However, I provide an example where the economy also has an inefficient equilibrium under this same mechanism. I label this equilibrium a financial panic equilibrium.

### 3.2.1. Environment

The economy consists of a group of $N \in \mathbb{N}$ ex-ante identical investors. There are two consumption goods: a numéraire good and fruits. There is an aggregate endowment $\bar{A} > 0$ of a financial asset, where one unit of the asset bears one unit of fruit.

Investors receive a preference type $\theta$ which is private information. The total utility of an investor of type $\theta$ is given by $u(a; \theta) + m$, where $a \in \mathbb{R}_+$ is the consumption of fruits and $m \in \mathbb{R}$ is the consumption of the numéraire good. Note that preferences are quasi-linear in the numéraire good and it can be either positive or negative. Preference types are drawn from a known distribution $\pi$ with finite support $\Theta \subset \mathbb{R}_+$, and are i.i.d. across investors. For each $\theta \in \Theta$, I assume $u(\cdot; \theta)$ is twice continuous differentiable, strictly increasing, strictly concave, $u'(0; \theta) = \infty$ and $u'(\infty; \theta) = 0$.

### 3.2.2. Direct mechanisms and equilibrium

I focus on the class of direct mechanisms, where each investor only announces his type, and I assume that investors can commit with a direct mechanism. The time of actions is the following. First, investors observe their types and then they simultaneously make an announcement of it. Label $\theta = (\theta^1, \ldots, \theta^N) \in \Theta := \Theta^N$ the announcement vector, which I assume is publicly observable. After announcements are made, the mechanism transfers numéraire goods and allocates financial assets—all contingent on the announcement vector.

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7 In section 3.7 I discuss an extension of the model with no commitment.
Formally, a direct mechanism is a pair of asset and numéraire policies, \( \mu = \{ \chi, \psi \} \). An asset policy is a function \( \chi = (\chi^1, \ldots, \chi^N) : \Theta \to \mathbb{R}_+^N \) which assigns assets to each investor contingent on the announcement vector. And a numéraire policy is a function \( \psi = (\psi^1, \ldots, \psi^N) : \Theta \to \mathbb{R}_+^N \) which assigns numéraire good to each investor contingent on the announcement vector. An asset policy \( \chi \) is feasible if \( \sum_n \chi^i(\theta) = \bar{A} \) for every \( \theta \). That is, if the aggregate asset holdings is consistent with the total amount of assets in the economy. Analogously, a numéraire policy \( \psi \) is feasible if it satisfies \( \sum_n \psi^i(\theta) = \bar{M} \) for every \( \theta \). I label this the budget balanced condition. A direct mechanism is feasible if both its policies are feasible. Label \( \mathcal{M} \) the set of feasible direct mechanisms. Figure 3.1 depicts the sequence of actions.

<table>
<thead>
<tr>
<th>investors observe types ( \theta^n \in \Theta )</th>
<th>investors make type announcements</th>
<th>group transfers numéraire ( \psi^n(\theta) ) and allocates assets ( \chi^n(\theta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
</tr>
</tbody>
</table>

Figure 3.1 Sequence of actions

A direct mechanism \( \mu = \{ \chi, \psi \} \) should be interpreted as a trade mechanism. An announcement \( \theta^n \) gives a investor \( n \) a claim \( \chi^n(\theta^n, \theta^{-n}) \) on the assets, where \( \theta^{-n} \) denotes the announcements from investors \( i \neq n \). The price the investor pays/receives for this claim in terms of the numéraire good is \( \frac{\bar{M}}{N} - \psi^n(\theta^n, \theta^{-n}) \). Ultimately, the mechanism establishes trade quantities and prices contingent on the vector of announcements.

A feasible direct mechanism \( \mu \in \mathcal{M} \) is associated with a Bayesian game for investors. A investor’s pure strategy is an announcement contingent on his preference type, \( \sigma(\theta) \in \Theta \). Label \( \Sigma \) the set of pure strategies, which is the same for every investor. I restrict attention to equilibria in pure strategies, which is without loss of generality with respect to implementing the efficient outcome. The payoff of a investor \( n \), when he is of type \( \theta^n \)
and the announcement vector is $\hat{\theta}$, is

$$v^n(\hat{\theta}; \theta^n) = u \left( \chi^n(\hat{\theta}); \theta^n \right) + \psi^n(\hat{\theta}).$$  \hfill (3.1)

I label this game the investors-game and I focus on its Bayesian equilibria.

### 3.2.3 The AVG-Arrow mechanism

The group objective is to implement the efficient distribution of assets across investors given any type realization. That is, the group’s problem is

$$\max \left\{ \sum_n u(\chi^n; \theta^n); \sum_n \chi^n = \bar{A} \right\}$$  \hfill (3.2)

given any type vector $\theta$. Label $\chi^*(\theta)$ the asset policy associated with the solution of problem (3.2).

The group designs a numéraire policy $\psi$ in order to generate incentives for investors to truthfully announce their preference type, so $\chi^*(\theta)$ can be implemented. It is known that the VCG mechanism implements the efficient outcome in dominant strategies in this environment, but its transfer scheme is not budget balanced. As an alternative, I use the AGV-Arrow mechanism, which is a budget balanced mechanism.

Let the numéraire policy $\bar{\psi}^*$ be defined by

$$\bar{\psi}^*(\theta) = \frac{M}{N} + \gamma^n(\theta^n) - \frac{1}{N-1} \sum_{i \neq n} \gamma^i(\theta^i),$$  \hfill (3.3)

where

$$\psi^*(\theta) = \sum_{i \neq n} u(\chi^i(\theta); \theta^i),$$  \hfill (3.4)

$$\gamma^n(\theta^n) = \mathbb{E}[\psi^*(\theta)|\theta^n] - \mathbb{E}[\psi^*(\theta)].$$  \hfill (3.5)
The AGV-Arrow mechanism is given by the pair $\mu^* = \{\chi^*, \psi^*\}$. The term $\psi^{\ast n}(\theta)$ is the utility of investors other than investor $n$ in state $\theta$. In order to generate incentives for truth-telling, investor $n$ needs to internalize this term through transfers. However, generically, we cannot make transfers associated with $\psi^{\ast n}(\theta)$ and balance the budget for every state $\theta$. AVG-Arrow solve this problem by working with $\gamma^{n}(\theta^n)$, which is the change in the expected $\psi^{\ast n}$ implied by investor $n$’s type. When the other investors are announcing truthfully, the transfer $\gamma^{n}$ is associated with the expected $\psi^{\ast n}$ and, therefore, $\gamma^{n}$ provides the same incentives for truth-telling as $\psi^{\ast n}$. For this reason, $\gamma^{n}$ is labelled the incentive term of investors $n$. Since $\gamma^{n}(\theta^n)$ depends only on $\theta^n$, the budget can be balanced by making investor $n$ pays an equal share of investors $i \neq n$ transfers without distorting incentives. Which leads to the transfer $\bar{\psi}^{\ast n}(\theta)$.

A strategy $\sigma$ is a truth-telling strategy if $\sigma(\theta^n) = \theta^n$ for all $\theta^n \in \Theta$. Truth-telling is a Bayesian equilibrium of the game implied by the AVG-Arrow mechanism and the implied outcome achieves the maximum of problem (3.2). See Fudenberg and Tirole (1991) chapter 7 for a proof.

3.2.4. A panic example

The AVG-Arrow mechanism is an optimal mechanism in the sense that it has a Bayesian equilibrium associated with the efficient allocation of assets across investors. Unfortunately, truth-telling is not necessarily the unique equilibrium. Consider the following illustrative example. There are $N = 3$ investors and the total endowments are of $\bar{M} = 3.0$ and $\bar{A} = 3.0$. The utility function is a constant relative risk aversion $u(a; \theta) = \theta^{(1-\delta)-1}$ with parameter $\delta = 6.0$. The type space is $\Theta = \{\theta_L, \theta_H\} = \{1.0, 1.5\}$. The probability of type $\theta_L$ is $\pi_L = 0.1$ and the probability of type $\theta_H$ is $\pi_H = 0.9$.

Figure 3.2 depicts the investors-game associated with the AVG-Arrow mechanism in this example. The first table contains the payoff of investor $n$ for each possible vector of announcements when his true type is $\theta_L$. And the second table contains the payoff of
investor \( n \) for each possible vector of announcements when his true type is \( \theta_H \). The rows represent investor \( n \)'s announcement and the columns the possible combinations of the other investors’ announcements. Since the environment is symmetric, the AVG-Arrow mechanism is symmetric and, therefore, these two tables fully characterize the investors-game.

The AVG-Arrow mechanism has a truth-telling equilibrium, but in this example it also has another equilibrium. Consider the strategy profile in which every investor announces type \( \theta_L \) independent of their true type. If an investor is of type \( \theta_L \) and deviate from the proposed equilibrium by announcing \( \theta_H \), his payoff is 0.9881 instead of 1.0000, which is not a profitable deviation. If an investor is of type \( \theta_H \) and deviate from the proposed equilibrium by announcing \( \theta_H \), his payoff is 0.9988 instead of 1.0000, which is also not a profitable deviation. Therefore, announcing type \( \theta_L \) independent of your type is a Bayesian equilibrium of the game. I call this equilibrium a panic equilibrium because investors announce a low demand for the assets anticipating that other investors are going to do the same, which a form of self-fulfilling crisis.

The reason a panic equilibrium exists in this example is that, during a run, assets are relatively “cheap”. The price a depositor pays for the asset is associated with the impact of his announcement on the expected utility of other investors. When a investor announces \( \theta_H \), the mechanism allocates more assets to him which causes a big reduction on the utility of the other investors. Therefore, the price he pays when announcing \( \theta_H \) is high. A investor is willing to pay this high price under a truth-telling equilibrium.
because, if he misrepresent his type, he expects that the mechanism will allocate very little
assets to him—in which case his marginal utility of holding more assets is high. Once an
investor believes everyone else will announce type $\theta_L$, he believes the group will allocate
a reasonable amount of assets to him independently of his announcement. In this sense
assets are “cheap”. As a consequence, the investor has a low marginal utility of holding
more assets and he has no incentives to announce $\theta_H$ independently of his true type.

This form of panic is in some sense opposite to the panics presented in Diamond and
Dybvig (1983). In the Diamond-Dybvig model investors believe that resources are going
to be scarce due to overpayments. Therefore, they run to the bank in order to get as
much resources out of the bank as they can. In my model investors believe that resources
(financial assets) are going to be abundant due to a low demand for it. Therefore, they
run to the group in order to pay a lower price for those resources.

3.3. A single group infinite-horizon model

An advantage of my model is that it can easily be extended to an infinite-horizon
model. In a version with a single group and infinite-horizon, the model is a special case of
the model studied by Athey and Segal (2013) and I can use their results of implementation
in perfect Bayesian equilibrium. This extension is useful because it highlights a type of
equilibrium which doesn’t exist in static models. In dynamic settings, inefficient actions
in the present can be supported as an equilibrium outcome by fear of even worse actions
in the future. I label this form of inefficient equilibria a dynamic bank run. This is the
type of panics I use to construct the financial crisis example in section 3.6.

3.3.1. Environment

The environment is an infinite repetition of the one discussed in the previous section.
Time is discrete and goes from zero to infinite. All investors discount the future at rate
Each preference type is drawn from a known distribution $\pi^0$, in date $t = 0$, and from $t = 1$ forwards it follows a Markov process with transition $Q : \Theta \times \Theta \rightarrow [0, 1]$, which I assume has a unique ergodic distribution. As before, types are i.i.d. across investors.

3.3.2. Direct mechanisms and equilibrium

In this setting, since types change over time, investors announce their type at every date. Label $\theta_t = (\theta^1_t, \ldots, \theta^N_t) \in \Theta := \Theta^N$ the date $t$ announcement vector of investors, and $\theta^t = (\theta_0, \ldots, \theta_t) \in \Theta^t$ the history of announcement vectors from period zero up to period $t$.

A direct mechanism is a pair of asset and numéraire policies, $\mu = \{\chi, \psi\}$. An asset policy is a sequence $\chi = \{\chi_t\}_t$ of functions $\chi_t = (\chi^1_t, \ldots, \chi^N_t) : \Theta^t \rightarrow \mathbb{R}^N_+$ which assigns assets to each investor contingent on the history of announcement vectors $\theta^t$. A numéraire policy is a sequence $\psi = \{\psi_t\}_t$ of functions $\psi_t = (\psi^1_t, \ldots, \psi^N_t) : \Theta^t \rightarrow \mathbb{R}^N_+$ which assigns numéraire good to each investor contingent on the history of announcement vectors $\theta^t$. An asset policy $\chi$ is feasible if $\sum_n \chi^t_n(\theta^t) = \bar{A}$ for every date $t$ and history $\theta^t$. That is, if the aggregate consumption asset holdings is consistent with the total amount of assets in the economy. Analogously, a numéraire policy $\psi$ is feasible if it satisfies the budget balanced condition, $\sum_n \psi^t_n(\theta^t) = \bar{M}$, for every date $t$ and history $\theta^t$. A direct mechanism is feasible if both its policies are feasible. I label $\mathcal{M}$ the set of feasible direct mechanisms.

A investor’s pure strategy is a sequence of announcements contingent on his type and the history of announcements. Formally, a pure strategy for a investor $n$ is a sequence $\sigma = \{\sigma_t\}_t$, where $\sigma_t$ maps $(\theta^{t-1}, \theta^t_n)$ into an announcement $\sigma_t(\theta^{t-1}, \theta^t_n) \in \Theta$. Label $\Sigma$ the set of pure strategies.

A feasible direct mechanism $\mu \in \mathcal{M}$ is associated with a dynamic Bayesian game for investors. The strategy set, $\Sigma$, is the same for every investor and, given a history of
announcement vectors $\hat{\theta}_t^n$, the date $t$ payoff of a investor $n$ when he is of type $\theta^n_t$ is 

$$v^n(\hat{\theta}_t^n; \theta^n_t) = u(\chi^n_t(\hat{\theta}_t^n); \theta^n_t) + \psi^n_1(\hat{\theta}_t^n).$$  \hspace{1cm} (3.6)$$

As in the previous section, I label this game the investors-game and consider its perfect Bayesian equilibria, from now on PBE.

3.3.3. The balanced team mechanism

The AVG-Arrow mechanism is designed for one-period environments, not dynamic ones. In dynamic environments types change over time and investors are required to announce their type in every period. In this case, when a investor misrepresent his type, his belief over the distribution of future types may differs from the bank’s belief. Hence, it is possible that a investor have incentive to lie in order to manipulate the group’s belief. Athey and Segal (2013) proposed an extension of the AGV-Arrow mechanism, which they called the balanced team mechanism, to deal with this case.

Let me start introducing the optimal asset policy. The group objective is to implement the efficient allocation of assets across investors in every period and every realization of types. That is, the group’s problem is

$$\max \left\{ \sum_n u(\chi^n_t; \theta^n_t) ; \sum_n \chi^n = A \right\}. \hspace{1cm} (3.7)$$

for all date $t$ and history $\theta^t$. Label $\chi^*$ the asset policy associated with the solution to (3.7).

The balanced team mechanism makes transfers so each investor internalises the welfare of the other investors in the group, pretty much in the same way the AVG-Arrow mechanism does. The crucial difference is that in a dynamic setting each investor needs to internalise not only the utility of others in the current period, but also the expect present value of their future utilities. In this way, investors do not have incentives to manipulate the group future beliefs. Formally, for all period $t$ and history $\theta^t$, the transfer to an
investor $n$ in the balanced team mechanism is

$$\psi^*_n(\theta^t) = \frac{\bar{M}}{N} + \gamma^*_n(\theta^{t-1}, \theta^n_t) - \frac{1}{N-1} \sum_{i \neq n} \gamma^i_t(\theta^{t-1}, \theta^n_t), \quad (3.8)$$

where

$$\psi^*_i(\theta^t) = \sum_{i \neq n} u \left( \chi^*_i(\theta^t); \theta^n_t \right), \quad (3.9)$$

$$\Psi^*_i(\theta^t) = \mathbb{E} \left\{ \sum_s \beta^{s-t} \psi^*_s(\theta^s) \mid \theta^t \right\}, \text{ and}$$

$$\gamma^*_i(\theta^{t-1}, \theta^n_t) = \mathbb{E} \left[ \Psi^*_i(\theta^t) \mid \theta^{t-1}, \theta^n_t \right] - \mathbb{E} \left[ \psi^*_i(\theta^t) \mid \theta^{t-1} \right]. \quad (3.11)$$

The balance team mechanism is given by $\hat{\mu}^* = \{\chi^*, \psi^*\} \in \mathcal{M}$.

The environment here is a special case of Athey and Segal (2013). They show that truth-telling is a perfect Bayesian equilibrium of the balanced team mechanism, where a strategy $\sigma = \{\sigma_t\}_t$ is a truth-telling strategy if $\sigma_t(\theta^{t-1}, \theta_t) = \theta_t$ for all $t$ and $(\theta^{t-1}, \theta_t)$.

Consequently, for each history of type realizations $\theta^t$, the implied outcome achieves the maximum of problem (3.7).

### 3.3.4. A panic example

A dynamic environment generates an additional source of financial fragility. Once an investor believes his announcement will trigger future “bad” behaviour among other investors, he may have incentives to misrepresent his type even if it implies a welfare loss in the present. Consider the following illustrative example. There are $N = 3$ investors and the total endowments are of $\bar{M} = 3.0$ and $\bar{A} = 3.0$. The utility function is a constant relative risk aversion $u(a; \theta) = \theta^\frac{1-\delta}{\delta-1}$ with parameter $\delta = 6.0$. The type space is $\Theta = \{\theta_L, \theta_H\} = \{1.0, 1.5\}$. For simplicity, instead of consider a Markov process for types, I will look at the particular case where types are i.i.d. over time. The probability of type $\theta_L$
is \( \pi_L = 0.7 \) and the probability of type \( \theta_H \) is \( \pi_H = 0.3 \). When types are i.i.d. over time, the balanced team mechanism is reduced to a repetition of an AVG-Arrow mechanism at every period, which simplifies the presentation of the investors-game. This simplification is not essential in order to construct bank run examples.

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<table>
<thead>
<tr>
<th>( \theta_L )</th>
<th>( \theta_L, \theta_L )</th>
<th>( \theta_H, \theta_L )</th>
<th>( \theta_L, \theta_H )</th>
<th>( \theta_H, \theta_H )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.0020</td>
<td>1.0020</td>
<td>1.0016</td>
</tr>
<tr>
<td>( \theta_H )</td>
<td>0.9871</td>
<td>0.9945</td>
<td>0.9945</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

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<tr>
<th>( \theta_L )</th>
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<th>( \theta_H, \theta_H )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9898</td>
<td>0.9898</td>
<td>0.9761</td>
</tr>
<tr>
<td>( \theta_H )</td>
<td>1.0071</td>
<td>1.0049</td>
<td>1.0049</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Figure 3.3 The investors-game

Figure 3.3 depicts the investors-game associated with the balanced team mechanism for this example. As before, the first table represents the payoff of investor \( n \) for each possible vector of announcements when his true type is \( \theta_L \). While the second table represents the payoff of investor \( n \) for each possible vector of announcements when his true type is \( \theta_H \). The rows represent investor \( n \)'s announcement and the columns the possible combinations of the other investors' announcements. While the second table is the payoff of investor \( n \) for each possible vector of announcements when his true type is \( \theta_H \). Since types are i.i.d. over time, the stage game is the same in every date \( t \) and, therefore, it can be described with only two tables. If types followed a Markov process, the stage game would be contingent on the previous announcement vector.

This investors-game has two interesting features. First, truth-telling at every period is a dominant strategy when \( \beta \) is close to zero. To see this note that, in the first table, when the investor is type \( \theta_L \), his payoff of announcing \( \theta_L \) is always higher than the payoff of announcing \( \theta_H \). And in the second table, when the investor is type \( \theta_H \), his payoff of announcing \( \theta_H \) is always higher than the payoff of announcing \( \theta_L \).

The second feature is that the dominant strategy implementation result does not hold when \( \beta \) is close to one. In this case, there are equilibria where investors misreport their
types in at least one period and the implied welfare is strictly lower than truth-telling. For example, consider the following strategy profile:

1. In date \( t = 0 \) announce \( \theta_L \) independent of the true type, then go to 2.

2. If in date \( t = 0 \) all investor have announced \( \theta_L \), then go to 4, otherwise, go to 3.

3. Announce \( \theta_H \) until all investors have announced \( \theta_H \) for 5 consecutive periods, then go to 4.


Label \( \sigma_u \) the strategy profile described above.

**Claim 3.1** There exists a \( \bar{\beta} \in (0, 1) \) such that, for all \( \beta \in [\bar{\beta}, 1) \), the strategy profile \( \{\sigma^n\}_{n} \), with \( \sigma^n = \sigma_u \) for all investor \( n \), constitutes a perfect Bayesian equilibrium of the investors-game depict in Figure 3.3. Furthermore, in the equilibrium path, all investors announce \( \theta_L \) in date zero.\(^8\)

The gains from trade within a group occurs when investors are from different types. In this case, the gains of trade are shared following the rules the balanced team mechanism in order to generate incentives for truth-telling. When all investors misrepresent their types and announce \( \theta_L \), the gains from trade are not realized. This “inefficient” action can be supported as an equilibrium by the threaten of having investors misrepresenting their type in the future. In other words, the one-period loss of announcing \( \theta_L \) is lower than the five-periods loss of announcing \( \theta_H \). One may ask why to stay five periods announcing \( \theta_H \) is a credible threat. It is credible because if some investor deviate the punishment will be to stay at least one more period announcing \( \theta_H \). Of course, \( \sigma_u \) is only an equilibrium when investors have high valuation of future utilities—\( \beta \) is close to one.

\(^8\) For this proof see Appendix 3.A.
3.4. The complete model

In this section I introduce a version of the model where groups trade assets in an over-the-counter market. I emphasize one feature of over-the-counter markets, namely, trade happens with delay. When the trade delay is short, the inter-group market resembles a centralized market like the NYSE, when the trade delay is long, it resembles an over-the-counter market like the ABS market.

3.4.1. Environment

There is a non-atomic unit measure of groups. Each group is the same described in section 3.3, and preference shocks across investors in different groups are independent. From period $t = 1$ forwards, a group access a centralized Walrasian market with probability $\alpha \in (0, 1]$. This assumption is equivalent to introducing a dealer, as as in DGP and LR, but giving all the bargaining power to the group. When dealers have bargaining power, the Nash bargaining solution does not apply because investors have private information. And, in order to keep the analysis simple, I do not consider this version of the model. Let $\iota_t \in \{0, 1\}$ denotes whether a group had access to the market, $\iota_t = 1$, or not, $\iota_t = 0$, in date $t$. I assume that $\iota_t$ is independent across groups and over time.

The price of assets in terms of numéraire good in the centralized is given by a stochastic process $p = \{p_t\}$. For this section I assume that investors observe a sunspot variable $x_t$, which is the same variable for all investors in all groups. The sunspot random variables are i.i.d. over time with distribution $F$ and finite support $S$. Since there is a non-atomic measure of groups, and preference types and market accesses are independent across groups, the only source of aggregate uncertainty are the sunspots. Therefore, the price process is a deterministic function of sunspots. That is, a price is a function $p_t : S^t \rightarrow \mathbb{R}_+$. 
3.4.2. Direct mechanisms

Before formally define a direct mechanism, let me introduce some notation. I label \( \theta_t = (\theta_1^t, \ldots, \theta_N^t) \in \Theta := \Theta^N \) the period \( t \) announcement vector, \( \iota_t \in \{0, 1\} \) whether the groups has access to the centralized market in period \( t \) or not, and \( p_t \) the period \( t \) price of assets in terms of numéraire good in the Walrasian market. The vector with the realization of these variables is denoted by \( h_t = (\theta_t, \iota_t, p_t) \in H := \Theta \times \{0, 1\} \times \mathbb{R}_+ \). As usual, I use superscript \( t \) to denote the history of variables from period zero up to period \( t \).

A direct mechanism is a pair of asset and numéraire policies denoted by \( \mu = \{\chi, \psi\} \).

An asset policy is a sequence \( \chi = \{\chi_t\}_t \), where \( \chi_t = (\chi_1^t, \ldots, \chi_N^t) : H^t \to \mathbb{R}_+^N \) denotes the amount of assets allocated to investor in the group contingent on the history \( h_t \). A numéraire policy is a sequence \( \psi = \{\psi_t\}_t \), where \( \psi_t = (\psi_1^t, \ldots, \psi_N^t) : H^t \to \mathbb{R}_+^N \) denotes the transfer of numéraire good to each investor contingent on the history \( h_t \).

The difference between groups in this section and in the previous ones is that it can adjust its aggregate asset holdings when accessing the inter-group market. In practice, this only changes the feasibility conditions of the direct mechanisms. That is, an asset policy \( \chi = \{\chi_t\}_t \) is feasible if \( \sum_n \chi_0^n = \bar{A} \) and \( (1 - \iota_t) \sum_n [\chi_n^t(h^t) - \chi_n^t(h^{t-1})] = 0 \). The first condition is that the aggregate asset holding in a group needs to be consistent with the initial distribution of assets among investors. The second condition is that the group can only adjust its aggregate asset holdings if it accesses the centralized market.

Label \( \Gamma \) the set of all feasible asset policies. A numéraire policy \( \psi = \{\psi_t\}_t \) is feasible if it satisfies the budget balanced condition, \( \sum_n \psi_n^t(h^t) = \bar{M} + p_t \sum_n [\chi_n^{t-1}(h^{t-1}) - \chi_n^t(h^t)] \). That is, the aggregate numéraire transfers among investors equals the aggregate transfer with the market. A direct mechanism is feasible if both policies are feasible. Label \( \mathcal{M} \) the set of all feasible direct mechanisms.

A pure strategy is a sequence of announcements contingent on these variables. For-
mally, a investor $n$ strategy is a sequence $\sigma^n = \{\sigma^n_i\}$ of measurable functions, where $\sigma^n_i$ maps a vector $(h^{t-1}, \theta^n_i, x^t)$ into a announcement $\sigma^n_i(h^{t-1}, \theta^n_i, x^t) \in \Theta$. Label $\Sigma$ the set of all pure strategies.

A feasible direct mechanism, $\mu \in \mathcal{M}$, and a price process, $p$, is associated with a stochastic game of incomplete information to investors. Where the strategy set of each investor $n$ is $\Sigma$ and his period utility in date $t$ given a history $h^t$ when he is of type $\theta^n_i$ is

$$v^n_t(h^t; \theta^n_i) = u(\chi^n_t(h^t); \theta^n_i) + \psi^n_t(h^t). \quad (3.12)$$

I label this game the investors-game.

### 3.4.3. Equilibrium

I restrict attention to symmetric equilibria, meaning, every group adopts the same mechanism and all investors adopt the same strategy. Note that investors from different groups can still make different announcements because they face different history of type realizations.

The only aggregate uncertainty in the economy comes from the public sunspot. Therefore, the price in a period $t$ is a deterministic function of $x^t = (x_0, x_1, \ldots, x_t)$. The market clearing condition requires that, for every period $t$ and public sunspot realization $x^t$, the demand of assets equal the total offer of assets in the economy.

Formally, the transition probability of types, $Q$, the probability of accessing the centralized market, $\alpha$, the distribution of sunspot, the price process, $p$, a feasible mechanism, $\mu \in \mathcal{M}$, and a strategy profile $\{\sigma^n\}_n \in \Sigma^n$, generate a sequence of measures $\eta = \{\eta_t\}_t$ over the space of histories $H^t$. These measures are defined in the usual way. The aggregate demand for assets in the centralized market in period $t$ is

$$D_t(x^t) := \int \sum_n \chi^n_t(h^t) \eta_t(h^t|x^t). \quad (3.13)$$
Note that the aggregate demand for assets is a function of the sunspot history $x^t$. Markets clear at a period $t$ if $D_t(x^t)$ equals $\bar{A}$ almost surely.

**Definition 3.1** Given a feasible direct mechanism $\mu \in \mathcal{M}$, a symmetric equilibrium is a pair $\{\sigma, p\}$ such that: (i) the strategy profile $\{\sigma^n\}_n$, with $\sigma^n = \sigma$ for all $n$, is a perfect Bayesian equilibrium of the investors-game associated with $\mu$ and $p$; and (ii) markets clear at every period.

### 3.5. Efficiency results

In this section I show that there exists a price process and a direct mechanism which supports the constrained Pareto efficient allocation as an equilibrium outcome. Where constrained means constrained by the market access ($\iota$ equal to zero or one).

#### 3.5.1. The balanced team mechanism

Let me start characterizing the optimal asset policy. Given a price process $p$, the expected aggregate utility of a group implied by an asset policy $\chi$ is

$$W_p(\chi) = \mathbb{E} \left\{ \sum_t \beta^t \left[ \sum_n u(\chi^n_t, \theta^n) + \bar{M} + p_t \sum_n (\chi^n_{t-1} - \chi^n_t) \right] \right\}. \quad (3.14)$$

Label $\chi^*_p$ the asset policy that maximizes $W_p$ among all feasible asset policies. The policy $\chi^*_p$ is only defined for a given price process $p$, but throughout the text I will omit the argument $p$ to keep the notation short whenever it is convenient. Note that $\chi^*_p$ may not exist for a particular price process. However, if $\chi^*_p$ exists it is unique since $u(\cdot, \theta)$ is strictly concave for all $\theta$.

Before I proceed, let me introduce some notation. Label $\mathcal{U}(A, \theta)$ the maximum aggregate period utility of a group with total assets equal to $A$ and vector type $\theta$. That
is,

$$U(A; \theta) = \max \left\{ \sum_n u(a^n; \theta^n); \sum_n a^n = A \right\}.$$  \hspace{1cm} (3.15)

The function $U(\cdot; \theta)$ inherit all the properties of $u(\cdot; \theta)$. Namely, it is twice continuous differentiable, strictly increasing, strictly concave, $U'(0; \theta) = -\infty$ and $U'(\infty; \theta) = 0$. Label $\{t_k\}_{k=1}^\infty$ the random sequence of periods in which a group access the centralized market, and $d_k = t_{k+1} - t_k$ the time length between the accesses. Note that $d_k$ follow a geometric distribution with parameter $\alpha$. That is, the probability of the next access to the market be in $d$ periods is $(1 - \alpha)^{d-1}\alpha$.

**Proposition 3.1** Consider a feasible asset policy, $\chi$, and a sequence $A = \{A_t\}_t$ satisfying $A_t(h^t) = \sum_n \chi^n_t(h^t)$ almost surely. A sufficient condition for $\chi$ to solve problem (3.14) is that for all $t$ and $h^t$: (i) the policy $\chi_t(h^t)$ solves problem (3.15) for $\theta = \theta_t$ and $A = A_t(h^t)$ almost surely; (ii) if the group has its $k$-th access to the centralized market in period $t = t_k$, then $A_t(h^t)$ satisfies

$$p_t - \mathbb{E} \left\{ \beta^{d_k} p_{t+d_k} \mid h^t \right\} = \mathbb{E} \left\{ \sum_{d=0}^{d_k-1} \beta^d U'(A_t; \theta_{t+d}) \mid h^t \right\} \hspace{1cm} (3.16)$$

almost surely; and (iii) the transversality condition, $\lim_{K \to \infty} \mathbb{E} \left\{ \beta^{t_K} p_{t_K} A_{t_K} \right\} = 0$, holds.$^9$

Proposition 3.1 provides sufficient conditions for a feasible asset policy to be optimal, which are given by the first order conditions of the group problem. I use this conditions later when I show that there exists an equilibrium which supports the constrained Pareto efficient allocation.

$^9$ See Appendix 3.B for all the proofs in this section.
The numéraire policy in the balanced team mechanism is constructed so a investor internalizes the aggregate welfare of the group. Label $\psi_{*}^{n}(h^t)$ the difference between investor $n$ utility and the aggregate utility of the group in date $t$ given history $h^t$, net from the cost of investor $n$’s own assets $p_t(\chi_{t-1}^n - \chi_t^n)$. That is,

$$
\psi_{*}^{n}(h^t) = \sum_{i \neq n} \left[ u\left( \chi^*_i(h^t); \theta_i^t \right) + p_t(\chi_{t-1}^i(h^t-1) - \chi_i^t(h^t)) \right].
$$

(3.17)

Label $\Psi_{*}^{n}(h^t)$ the period $t$ expected present value of the sequence $\{\psi_{*}^{s}(h^s)\}_{s=t}^{\infty}$ given $h^t$. That is,

$$
\Psi_{*}^{n}(h^t) = \mathbb{E}\left\{ \sum_{s=t}^{\infty} \beta^{s-t} \psi_{*}^{s}(h^s) \mid h^t \right\}.
$$

(3.18)

The agent $n$ incentive term of announcing $\theta^n_i$ in period $t$ is given by

$$
\gamma_{*}^{n}(h^{t-1}, p_t, \theta_{*}^{n}) = \mathbb{E}\left[ \Psi_{*}^{n}(h^t) \mid h^{t-1}, p_t, \theta_{*}^{n} \right] - \mathbb{E}\left[ \Psi_{*}^{n}(\theta_{*}^{t}) \mid h^{t-1}, p_t \right].
$$

(3.19)

For all period $t$ and history $h^t$, the transfer to a investors $n$ in the balanced team mechanism is

$$
\tilde{\psi}_{*}^{n}(h^t) = \frac{\tilde{\mu}}{N} + p_t(\chi_{t-1}^n(h^t-1) - \chi_t^n(h^t)) + \gamma_{*}^{n}(h^{t-1}, p_t, \theta_{*}^{n}) - \frac{1}{N-1} \sum_{i \neq n} \gamma_{*}^{i}(h^{t-1}, p_t, \theta_{*}^{i}).
$$

(3.20)

The balance team mechanism is given by $\tilde{\mu}_p^* = \{\chi_{p}^*, \tilde{\psi}_p^*\} \in \mathcal{M}$. As in the case with only one group, the investors-game implied by the balance team mechanism has a perfect Bayesian equilibrium in truth-telling strategies. See Athey and Segal (2013) for a proof.
In this class of models the numéraire good allows investors to transfer utility with a linear technology. Therefore, an outcome is constrained Pareto efficient if, and only if, it maximizes investors aggregate welfare. In this subsection I characterize this allocation. Note that constrained here refers to the constraint on market accesses, not the incentive compatibility constraint.

An allocation is a sequence \( a = \{ a_t \}_t \) of measurable functions \( a_t = (a^1_t, \ldots, a^N_t) : \Theta^t \times \{0, 1\}^t \rightarrow [0, B]^N \), where \( B > 0 \) is an upper bound on investors asset holdings. I impose two restrictions on the allocation: there is an upper bound on asset holdings and the allocation is symmetric. Both restrictions are without loss of generality with respect to maximize aggregate welfare. The upper bound \( B \) can be taken large enough so it does not bind.\(^{10}\) And the strictly concavity of \( u(\cdot; \theta) \) implies that the welfare maximizing allocation is symmetric.

A symmetric allocation \( a \) is feasible if for all \((\theta^t, i^t)\) it satisfies

\[
\sum_n a^n_t(\theta^t, i^{t-1}, 0) = \sum_n a^n_{t-1}(\theta^{t-1}, i^{t-1}), \quad (3.21)
\]

\[
\sum_{\theta^t} \sum_{i^t} \mathbb{P}(\theta^t, i^t) \sum_n a^n_t(\theta^t, i^t) = \bar{A}, \quad (3.22)
\]

\[
\sum_n a^n_0 = \bar{A}. \quad (3.23)
\]

The first equation means that, in case the group does not access the market, its total amount of assets is the same of the previous period. The second equation means that

\(^{10}\) A necessary condition for an allocation \( a \) to maximize welfare is that, \( \forall(\theta^t, i^t), (\theta'^t, i'^t) \) such that \( i^t = i'^t = 1 \) and \( \theta^t = \theta'^t \), we have \( a_t(\theta^t, i^t) = a_t(\theta'^t, i'^t) \). This result comes from the fact that \( u(\cdot; \theta) \) is strictly concave and that the stochastic type process is Markov. In addition, \( u(\infty; \theta) = 0 \) for each \( \theta \in \Theta \) and the measure of groups with type vector \( \theta \in \Theta \) is bounded away from zero at any period \( t \). Combined with the fact that assets exist in finite supply \( \bar{A} \), these features imply that there is a positive real number, \( B \), such that for all feasible \( a \), if \( a_t(\theta^t, i^t) > B \) for some \( (\theta^t, i^t) \), then there exists a Pareto superior and feasible outcome \( a' \) that is bounded above by \( B \).
the total amount of assets held by investors equals the total amount of assets existent in the economy. The last equation is the requirement that, since the initial distribution of assets is uniform, every group should hold exactly $\bar{A}$ assets at period zero. Let $\mathcal{F}$ denotes the set of all feasible allocations $a$.

The aggregate welfare of the economy implied by an allocation $a \in \mathcal{F}$ is

$$W(a) = \mathbb{E} \left\{ \sum_t \beta_t \sum_n u(a^n_t; \theta^n_t) \right\}. \quad (3.24)$$

An allocation is constrained Pareto efficient if it achieves the maximum aggregate welfare among all feasible allocations. It is easy to show that such allocation exists and it is unique. Label this allocation $a^*$. The following proposition characterizes $a^*$.

**Proposition 3.2** Consider a feasible allocation, $a \in \mathcal{F}$, and a sequence $A = \{A_t\}_t$ defined by $A_t(\theta^t, \iota^t) = \sum_n a^n_t(\theta^t, \iota^t)$. A necessary and sufficient condition for $a$ to maximize (3.24) is that for all $(\theta^t, \iota^t)$: (i) the allocation rule $a_t(\theta^t, \iota^t)$ solves (3.15) for $\theta = \theta^t$ and $A = A_t(\theta^t, \iota^t)$; and (ii) there exists a sequence of Lagrange multipliers, $\lambda = \{\lambda_t\}_t$, such that, if $\iota_t = 1$, then $A_t(\theta^t, \iota^t)$ satisfies

$$\lambda_t = \mathbb{E} \left\{ \sum_{d=0}^{d_t-1} \beta^{t+d} \nu(A_t; \theta_{t+d}) \right\}. \quad (3.25)$$

Proposition 3.2 provides sufficient conditions for a feasible asset allocation to be constrained Pareto efficient, which are given by the first order conditions implied by the maximization of (3.24) constrained by equations (3.21)-(3.23).

3.5.3. The constrained Pareto efficient equilibrium

The balanced team mechanism weakly implements the optimal outcome within a group for a given price. In addition, as long as investors follow truth-telling strategies, it also sustain the constrained optimal outcome as an equilibrium outcome.
Proposition 3.3  There exists a price process $p$, a strategy $\sigma$ and a feasible mechanism $\bar{\mu}^* = \{\chi^*, \psi^*\} \in \mathcal{M}$ such that: (i) $\bar{\mu}^* = \bar{\mu}_p^*$ is the balanced team mechanism associated with the price process $p$; (ii) $\sigma$ is the truth-telling strategy; (iii) $\{\sigma, p\}$ is an equilibrium associated with the direct mechanism $\bar{\mu}^*$; and (iv) the implied asset allocation is constrained Pareto efficient.

The proof of proposition 3.3 follow two steps. First, I use the Lagrange multipliers of proposition 3.2 in order to build a price sequence such that the optimal asset policy is associated with the constrained Pareto efficient allocation. This is done by equalizing the price in the first order conditions of the group, equation (3.16), to the Lagrange multiplier in equation (3.25). The second step is to generate incentives for truth-telling. I use the balanced team mechanism which always has a truth-telling equilibrium.

3.5.4. A uniqueness result when the inter-group market is efficient

From previous sections, we know that groups are fragile when they are isolated—$\alpha = 0$. Is the inter-group market able to eliminate panics? I am able to provide a positive answer to this question when the over-the-counter market frictions are small.

Proposition 3.4  There exist $\bar{\alpha} \in (0, 1)$ such that, for all $\alpha \in [\bar{\alpha}, 1]$, the constrained Pareto efficient allocation is the unique equilibrium outcome associated with the balanced team mechanism.

When $\alpha$ is close to one, there is very little risk-sharing among a group investors because the group adjusts its portfolio immediately after a investor announcement. As a result, the balanced team mechanism only replicates the allocation of assets that each investors would choose if they were by themselves accessing the market. And, because this is the Walrasian demand, investors cannot get better by misrepresent their types.
3.6. A financial crisis example

I have shown that the model with a single group generates panic equilibria, and that when a large numbers of groups trade assets in a efficient inter-group market such equilibria disappears. But what happens when the inter-group market features severe over-the-counter trade frictions?

In this section I provide a numerical example of an equilibrium which resembles a financial turmoil in an economy where the over-the-counter market frictions are severe. Consider the following parametrization. There are \( N = 3 \) investors and the total endowments are \( \overline{M} = 3.0 \) and \( \overline{A} = 3.0 \). The utility function is a constant relative risk aversion \( u(a; \theta) = \theta a^{1-\delta-1} \) with parameter \( \delta = 6.0 \). The type space is \( \Theta = \{ \theta_L, \theta_H \} = \{ 1.0, 1.5 \} \). The distribution of types over time is driven by a Markov process \( Q \), where \( Q(\theta_L, \theta_L) = 0.95 \) and \( Q(\theta_H, \theta_H) = 0.2 \), and the economy starts in the steady state distribution of \( Q \). Investors discount the future at rate \( \beta = 0.98 \). The probability of accessing the market is \( \alpha = 0.5 \). And assume that there exists \( \bar{x} \in S \) such that \( F(\bar{x}) = 10^{-8} \), where \( F \) is the distribution of sunspots.

Let me construct the strategy profile that will be supported in equilibrium. Label \( t_u \) the first time period in which the sunspot, \( x_t \), is in the interval \([0, \bar{x}]\). Consider the following strategy:

1. Announce truthfully in all periods \( t < t_u \).
2. In period \( t_u \) announce \( \theta_L \).
3. In period \( t > t_u \)
   - if in period \( t - 1 \) all investors have announced \( \theta_L \), then keep announcing \( \theta_L \).
   - otherwise, announce \( \theta_H \) until all investors have announced \( \theta_H \) for 2 con-
secutive periods. Then announce $\theta_L$.

Label this strategy $\sigma_{ul}$. To summarize, investors announce truthfully until a sunspot hits the economy, which happens with probability $10^{-8}$. The period of this shock is labelled $t_u$ and I interpret it as the period the crisis starts. From $t_u$ forwards, investors “run” against the group by announcing $\theta_L$ independent of their types. Having investors announcing $\theta_L$ independent of their types can be supported as an equilibrium outcome by the fear of going to a cycle of having every investors announcing $\theta_H$. That is, this is a dynamic panic equilibrium.

I numerically verify that there exists an equilibrium price process, $p$, such that $\{\sigma_{ul}, p\}$ constitutes an equilibrium associated with the balanced team mechanism. This equilibrium display interesting features. The price is constant until period $t_u$, when the crisis occurs, and then it drops by 9.94 percent. The trade volume has a more interesting dynamic, as depicted in Figure 3.4. At the moment of the crisis the trade volume increases by 168 percent. This increase is due to a “fire-sale” effect: every group try to reduce their asset holdings and, as a result, price drops and the trade volume increases. As time pass by, assets are reallocated and the trade volume converges to zero, which I interpret as a collapse of the over-the-counter asset market.

![Figure 3.4 Trade volume](image)

Figure 3.4 Trade volume
3.7. The non-commitment case

A weakness of the results in the previous sections is that investors need to be able to commit with a long term relationship, which is a strong assumption. For instance, in a mutual fund there is nothing preventing an investor from selling his respective share and permanently leave the fund. In this section, I consider the non-commitment version of the model, where investors cannot commit with future actions. I show that the balanced team mechanism implements the constrained Pareto optimal allocation if \( \beta \) is high. That is, there exists \( \bar{\beta} \in (0, 1) \) such that for all \( \beta \in [\bar{\beta}, 1) \) the constrained Pareto efficient allocation is an equilibrium outcome. I also show that there exists at least one inefficient equilibrium, which is autarky.\(^\text{11}\)

The sequence of actions is the following. Investors start the period with whatever assets they finish in the period before. Then they announce their types. After announcements are made, the mechanism will suggest how much assets each investor should transfer to each other, how much assets to trade with the market in case there is access, and how much of the numéraire good to transfer. The key difference here is that the mechanism will only suggest the transfers, whether they are made or not depends on investors’ decision. That is, after observe the suggested transfers, each investor have the option to deviate from what was suggested (transferring whatever amounts he so desires) and go to autarky, or to sticky with the transfers suggested by the mechanism. I assume that, once a investor goes to autarky, he cannot get back into the group. This is without loss of generality with respect to implement the best group outcome since, in order to generate incentives, we always want to give a investor the worse punishment for a deviation. The probability of accessing the market once in autarky is the same probability of the group accessing the market.

The balanced team mechanism need to be extended to account for the possibility of

\(^{11}\) See Appendix 3.C for a formal discussion.
some investor deviating from the mechanism and going to autarky. The modification I propose is simple. The mechanism follow the balanced team mechanism until a investor deviate. After a deviation the mechanism recommend autarky to all remaining investors. Label this mechanism $\tilde{\mu}^*$ and call it the modified balanced team mechanism.

**Proposition 3.5** There exist $\tilde{\beta}$ such that, for all $\beta \in [\tilde{\beta}, 1)$, the constrained Pareto efficient allocation is an equilibrium outcome associated with the modified balanced team mechanism $\tilde{\mu}^*$.

The intuition for this result is that going to autarky prevents investors from engaging in risk sharing. Even though there may be some gains from doing this in the short run, due to some particular shocks, in the long run it definitely represents a cost. If investors are patient enough, the long run cost exceeds any short term benefit from a deviation.

Proposition 3.5 shows that truth-telling is an equilibrium when $\beta$ is high. On the other hand, autarky is always an equilibrium associated with any feasible mechanism and any $\beta$. Given any feasible mechanism, investors are indifferent between deviating from the mechanism or not when all other investors are going to autarky. As a result, autarky is always an equilibrium.

### 3.8. Optimal policy

The model I developed here has a fragile economy in the sense that different equilibria arise and some are associated with low welfare.\(^{12}\) In this section I consider again the commitment version of the model which I extend to study two alternative policies to

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\(^{12}\) One may suggest that the use of a different mechanism, instead of the balanced team mechanism, can eliminate the inefficient equilibria. Unfortunately, the recent developments in implementation theory do not provide such mechanism. Unique, or full, implementation is only guaranteed under a particular set of assumptions and as a limit result as the discount rate, $\beta$, goes to one. See, for example, Renou and Tomala (2013).
reduce financial fragility.

3.8.1. Suspension

Assume that groups can monitor the distribution of announcement vectors in the whole economy. Label $\hat{\Pi}_t$ the distribution of announcement vectors in period $t$ and $\hat{\Pi}^t$ the history of $\hat{\Pi}_t$ realizations. Note that I use capital pi to denote distribution over vector types and non-capital pi to denote distribution over types.

The relevant state for a group, $h_t$, needs to be extended in order to account for $\hat{\Pi}_t$. That is, let $h_t = (\theta_t, \iota_t, p_t, \omega_t, \bar{p}_t)$. The asset policy, the transfer policy, the investors strategies and the equilibrium definition are extended to capture the extension in the space of histories.

I consider a very simple modification of the balanced team mechanism. Let $t_u$ denotes the first period in which $\hat{\Pi}_t \neq \Pi_t$, where $\Pi_t$ is the true distribution of group type vectors. The modified mechanism follows exactly the balanced team mechanism until $t = t_u$. For $t \geq t_u$, the modified mechanism maximize each investor utility separately, which is a reversion to DGP and LR where investors are isolated. That is, for $t \geq t_u$ the group maximizes

$$E \left\{ \sum_t \beta^{t-t_u} \left[ u(\chi^n_t, \theta^n) + \frac{M}{N} + p_t(\chi^n_{t-1} - \chi^n_t) \right] \right\}$$

(3.26)

for each investor $n$. And the numéraire consumption is $\bar{\Psi}^n = \bar{M} + p_t(\chi^n_{t-1} - \chi^n_t)$. Label $\bar{\mu}^*_{mod}$ the modified balanced team mechanism. I refer to this policy as a suspension because from $t_u$ forward the group suspend all risk-sharing among investors.

Proposition 3.6 Consider a price process, $p$, and a associated modified balanced team mechanism, $\bar{\mu}^*_{mod}$. If a pair \{\sigma, p\} is an equilibrium for the mechanism $\bar{\mu}^*_{mod}$ then $\sigma$ generates a distribution of type vectors that equals the true distribution $\Pi_t$ at every
The reason proposition 3.6 does not guarantee that the equilibrium is truth-telling is because suspension only occurs when the group knows investors are misrepresenting. If investors are playing an strategy that generates the true distribution of announcements, but is not truth-telling, the group cannot differentiate this from truth-telling. Consequently, there is not threat of suspension. Fortunately, generically this cannot happen, where generically here means on a full measure set of the space of initial distribution of types, $\pi_0$, Markov processes, $Q$, and sunspot distributions $F$.

Proposition 3.7  Generically, given a price process $p$ and associated balanced team mechanism, $\tilde{\mu}_p$, if a pair $\{\sigma, p\}$ is an equilibrium for the mechanism $\tilde{\mu}^*$ then $\sigma$ is a truth-telling strategy.

Suspension prevents group runs for a different reason that it does in Diamond and Dybvig (1983). In Diamond-Dybvig agents don’t run against the group during a suspension regime because they know there will be enough resources for later investors. While in my setting suspension works because it kills risk-sharing and, therefore, any possibility for strategic complementarity.

3.8.2. Trading facilities

Assume that there exists a benevolent policy-maker with access to a commitment technology and a trading facility. The policy-maker can produce or consume the numéraire good with a linear technology as other investors, but he derives no utility from holding assets. A trading facility is a place that can be open or closed. When the facility is closed, the physical environment is that same discussed before and no one can trade with the policy-maker. When the facility is open, every group has access to it and they can trade

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13 See Appendix 3.D for the proofs in this section.
14 This assumption can be replaced by the assumption that the market-maker can collect taxes.
with the policy-maker. There is an operational cost \( c \geq 0 \) in terms of the numéraire good for the policy-maker to open the facility.

The sequence of actions within a period are the following. First, investors make their announcements in the group. Second, each group reports the announcement vectors to the policy-maker. Without loss of generality, I assume that groups cannot misreport investors announcements. This assumption is without loss of generality because, under the policy I consider, groups will have no incentives to misreport. For the last, the policy-maker decides whether to pay the operational cost and trade occurs.

A policy-maker decision is family \( f = \{f_t\}_t \), where \( f_t(\Pi_t) = (\omega_t(\Pi_t), \bar{p}_t(\Pi_t)) \). \( \omega_t(\Pi_t) \in \{0, 1\} \) denotes whether the policy-maker paid the operational cost \( c \geq 0 \) in order to trade with the groups, and \( \bar{p}_t(\Pi_t) \in \mathbb{R}_+ \) is the price the policy-maker is willing to buy and sell assets. Which means the policy-maker acts as a market-maker. Let \( W^M \) denotes the maximum welfare attained if the policy-maker operates at every period. Formally, we have that

\[
W_M = \sum_t \beta^t \sum_{\theta} \pi_t(\theta) u(a^M_t(\theta)), \tag{3.27}
\]

where \( \{\pi_t\} \) is the true distribution of types and \( \{a^M_t(\theta)\}_t \) satisfies

\[
\sum_{\theta} \pi_t(\theta) a^M_t(\theta) = \tilde{A}, \quad \text{and} \quad u'(a^M_t(\theta); \theta) = u'(a^M_t(\tilde{\theta}); \tilde{\theta}) \quad \text{for all } \theta, \tilde{\theta} \in \Theta. \tag{3.28}
\]

Since there is a non-atomic measure of investors and shock are \textit{i.i.d.}, there is no uncertainty over \( \pi_t \). That is, \( \{\pi_t\}_t \) is a known deterministic sequence.

When the operational cost is low enough such that \( W_M - \frac{c}{1-\beta} \geq W(a^*) \), the optimal policy is to pay the operational cost at every period and set the price \( \bar{p}_t \) to satisfy the
difference equation

\[ u'(a_t^M(\theta); \theta) = \bar{p}_t - \beta \bar{p}_{t+1} \text{ for all period } t \]  

(3.30)

for all \( \theta \). Label this decision policy \( f^M \). For this decision policy there will be a unique equilibrium associated with the balanced team mechanism.

**Proposition 3.8** Given \( f^M \), for any price process, \( p \), and associated balanced team mechanism, \( \bar{\mu}^* \), the implied investors-game features a unique equilibrium which is truth-telling.

Proposition 3.8 implies that in this setting we can always guarantee stability and a welfare of \( W_M - \frac{c}{1-\beta} \). On the other hand, if the operational cost \( c \) is so high that \( W_M - \frac{c}{1-\beta} < W(a^*) \), using \( f^M \) leads to a welfare loss. Is it possible to achieve stability under this circumstances without sacrificing welfare?

The answer to the above question is yes. The simple threat of implementing \( f^M \) is enough to guarantee that the equilibrium will be unique. Consider the following policymaker policy decision. Let \( t_u \) denotes the first period in which \( \bar{\Pi}_t \neq \Pi_t \), where \( \Pi_t \) is the true distribution of group type vectors. The policy-maker decision is \( \omega_t = 0 \) for all \( t < t_u \), and, the policy \( f^M \) from \( t_u \) forward. Label this decision policy \( f^S \).

**Proposition 3.9** Given \( f^S \), for any price process, \( p \), and associated balanced team mechanism, \( \bar{\mu}^*_p \), if a pair \( \{\sigma, p\} \) is an equilibrium for the mechanism \( \bar{\mu}^*_p \) then \( \sigma \) generates a distribution of type vectors that equals the true distribution \( \Pi_t \) at every period \( t \) with probability one.

The reason proposition 3.9 does not guarantee that the equilibrium is truth-telling is the same of 3.6. If investors are playing a strategy that generates the true distribution of
announcements, the policy-maker cannot differentiate this from truth-telling, so there is not threat of intervention. But again this cannot happen in general.

**Proposition 3.10** Generically, given $f^S$, a price process, $p$, and associated balanced team mechanism, $\bar{\mu}^*$, if a pair $\{\sigma, p\}$ is an equilibrium associated with the mechanism $\bar{\mu}^*$ then $\sigma$ is a truth-telling strategy.

As before, generically here means on a full measure set of the space of initial distribution of types, $\pi^0$, Markov processes, $Q$, and sunspot distributions $F$.

### 3.9. Discussion and future extensions

My model helps us to identify one of the causes for the 2007/08 financial crisis. In recent years the market for asset-backed securities (ABSs) has expanded extremely fast. Annual issuance of ABS went from $10$ billions in 1986 to $893$ billions in 2006, as reported by Agarwal et al. (2010). And a growing shadow bank sector has purchased most of these assets, which are usually traded in an over-the-counter fashion. Moreover, ABSs are complex financial instruments, which make it hard to trade since only very specialized traders are able to evaluate those assets. As a result, in 2007 the financial sector featured a large number of financial institutions operating a market with severe over-the-counter market frictions—all important elements for a fragile bank sector, as suggested by my model.

The most common prescription for enhancing financial stability is to regulate the contracts offered by financial institutions. For example, recently, the Securities and Exchange Commission (SEC) announced a set of proposals to enhance financial stability, which includes a recommendation for the MMF board of directors to impose fees and gate payments in times of heavy redemption activity.\(^{15}\) And Cochrane (2014) calls for an narrow bank sector funded 100\% by equity.

\(^{15}\) See SEC (2013).
There are two downsides of directly regulating contracts. First, each type of financial institution serves a different type of investor and, therefore, requires a different contract. As a result, the regulation needs to be specific to the type of institution. That is, we need one particular regulation for commercial banks, one for mutual funds, one for structured investment vehicles, etc. Which results in a complex regulatory system doomed to feature loopholes and regulatory arbitrage possibilities. The second downside is that, even if we are willing to write complex regulations to every type of financial institution, it is not clear what regulations we should impose. Even a glimpse through the Diamond-Dybvig literature shows that the optimal contract depends on several details of the environment. When you consider models other than Diamond-Dybvig, the possible regulations grow exponentially. Besides, more often than not, regulations have a welfare cost. In which case, the optimal regulation also depends on how the policy-maker evaluates welfare.

In this paper I suggest a different approach to enhance financial stability. Instead of focusing on the particular contract financial intermediaries are offering, I focus on the market where the underlying assets are traded. I show that, if we reduce trade frictions, we enhance financial stability with no need to regulate institutions. Of course, much research still needs to be done to understand how robust this result is to alternative specifications. However, I believe it offers a much more promising path to financial stability than an overly complex regulatory framework.

There are possible extensions that I do not explore in this paper. To start with, over-the-counter trading relates to two frictions; trade delay due to search for trade partners, and bargaining due to bilateral trade. But in this paper I only explore the trade delay friction. A natural extension is to explicitly model the bargaining process. This extension is challenging because involves bargaining under private information.

Each group in my model has a fixed group of investors. In the real world, investors change their group with some frequency and groups are always searching for new investors. An extension that allows for these possibilities would help to understand group
formation and the effect of financial crisis on the size distribution of groups. The challenge of this extension is computational, since the state of a group grows exponentially with the number of investors.

My model completely abstracts from the real side of the economy. An interesting extension would be to build a channel through which panics have real effects. One way of establishing this channel could be to have a real side of the economy producing financial assets from loans to firms and consumers—a form of securitization through which assets are created. In this case, a drop in asset prices would reduce incentives for lending.

3.A. Appendix 3.A: inefficiency results

3.A.1. Proof of claim 3.1

Proof. I will use the one deviation principle to show the result. First, assume a investor deviates in a period \( t = 0 \) by announcing \( \theta_H \). If the investor is of type \( \theta_L \), the total gain of deviation is

\[
-0.0129 - \beta \frac{1 - \beta^5}{1 - \beta} 0.0023.
\]

The first term is the gain of the deviation. And the second is the expected gain of deviation implied by staying 5 consecutive periods announcing \( \theta_H \). Since both are negative, this deviation is not profitable. Now, assume the investor is of type \( \theta_H \). In this case his gain of deviation is

\[
0.0071 - \beta \frac{1 - \beta^5}{1 - \beta} 0.0023 < 0 \iff 0.0071 < \beta \frac{1 - \beta^5}{1 - \beta} 0.0023 \leq \lim_{\beta \to 1} \beta \frac{1 - \beta^5}{1 - \beta} 0.0023 = 0.0117.
\]

Thus, there exist \( \bar{\beta}_0 \) such that, if \( \beta \geq \bar{\beta}_0 \), the gain of deviation is strictly negative.

For the last, let us check a deviation in periods \( t > 0 \). If the investors are supposed to
announce truthfully forever, then a deviation is welfare decreasing since truth-telling is a strictly dominant strategy of the stage game. Assume someone deviate in period $t = 0$ and investors are suppose to announce $\theta_H$ for five consecutive periods—the punishment stage described in bullet three. If the investor is of type $\theta_L$, the total gain of deviation is

$$0.0016 - \beta^{5-s}0.0023,$$

where $s$ is the number of consecutive periods all investors have announced $\theta_H$ before. The gain from deviation is bounded above by

$$0.0016 - \beta^{5}0.0023 < 0 \iff 0.0016 < \beta^{5}0.0023 \leq \lim_{\beta \to 1} \beta^{5}0.0023 = 0.0023.$$

Thus, there exist $\bar{\beta}_1$ such that, if $\beta \geq \bar{\beta}_1$, the gain of deviation is strictly negative. If the investor is of type $\theta_H$, the total gain of deviation is

$$-0.0239 - \beta^{5-s}0.0023$$

which is always negative.

Label $\bar{\beta}$ the maximum between $\bar{\beta}_0$ and $\bar{\beta}_1$. Then, we can conclude that, if $\beta \in [\bar{\beta}, 1)$, the gain of any deviation is strict negative and the proposed strategy profile constitutes a perfect Bayesian equilibrium of the investors-game.

3.B. Appendix 3.B: efficiency results

3.B.1. Proof of proposition 3.1

Proof. Consider the problem

$$\sup \{ \mathbb{E} \sum_t \beta^t [U(A_t, \theta_t) + p_t(A_{t-1} - A_t)]; A = \{A_t\}_t \in \mathcal{A} \}, \quad (3.31)$$

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where $\mathcal{A}$ denotes the set of random sequences, $A = \{A_t(h^t)\}_t$, satisfying, for all $h^t \in H^t$, $A_0 = \bar{A}$ and $(1 - i_t)[A_{t-1}(h^{t-1}) - A_t(h^t)] = 0$.

Let me start showing that conditions (ii) and (iii) are sufficient for $A \in \mathcal{A}$ to solve problem (3.31). Consider a sequence $A \in \mathcal{A}$ satisfying (ii) and (iii) and another arbitrary sequence $\bar{A} \in \mathcal{A}$. The difference in the objective function of problem (3.31) implied by $A$ and $\bar{A}$ is

$$D = \lim_{K \to \infty} \mathbb{E}\left\{\sum_{k=1}^{K-1} \beta^k \{\sum_{d=0}^{d_k-1} \beta^d [U(A_{tk}; \theta_{tk+d}) - U(\bar{A}_{tk}; \theta_{tk+d})]ight.$$  
$$- [p_{tk} - \beta^{d_k} p_{tk+d_k}] [A_{tk} - \bar{A}_{tk}]\} - \beta^{tk} p_{tk} [A_{tk} - \bar{A}_{tk}] \right\}$$

$$\geq \lim_{K \to \infty} \mathbb{E}\left\{\sum_{k=1}^{K-1} \beta^k \{\sum_{d=0}^{d_k-1} \beta^d U'(A_{tk}; \theta_{tk+d}) [A_{tk} - \bar{A}_{tk}]ight.$$  
$$- [p_{tk} - \beta^{d_k} p_{tk+d_k}] [A_{tk} - \bar{A}_{tk}]\} - \beta^{tk} p_{tk} [A_{tk} - \bar{A}_{tk}] \right\}$$

$$= \lim_{K \to \infty} \mathbb{E}\left\{\sum_{k=1}^{K-1} \beta^k \mathbb{E}\left\{\sum_{d=0}^{d_k-1} \beta^d U'(A_{tk}; \theta_{tk+d}) - [p_{tk} - \beta^{d_k} p_{tk+d_k}] |h^t_k\} [A_{tk} - \bar{A}_{tk}]ight.$$  
$$- \beta^{tk} p_{tk} [A_{tk} - \bar{A}_{tk}] \right\}.$$

Condition (ii) implies that

$$\mathbb{E}\left\{\sum_{d=0}^{d_k-1} \beta^d U'(A_{tk}; \theta_{tk+d}) - [p_{tk} - \beta^{d_k} p_{tk+d_k}] |h^t_k\right\}$$

equals zero almost surely. Hence,

$$D \geq - \lim_{K \to \infty} \mathbb{E}\{\beta^{tk} p_{tk} [A_{tk} - \bar{A}_{tk}]\} \geq - \lim_{K \to \infty} \mathbb{E}\{\beta^{tk} p_{tk} A_{tk}\}.$$ 

The last term of the above equation equals zero due to condition (iii). Therefore, we can conclude that conditions (ii) and (iii) are sufficient for a sequence $A \in \mathcal{A}$ to solve problem (3.31). And we have by construction that if $A$ solves problem (3.31) then a policy $\chi$ satisfying condition (i) solves problem (3.14).
3.B.2. Existence and uniqueness of a constrained Pareto efficient allocation

Lemma 3.2 There exists a unique constrained Pareto efficient allocation.

Proof. Note that $W$ is a continuous map and $\mathcal{F}$ is a compact space when equipped with the norm $|| \cdot ||_{\beta}$, which defined as

$$||a||_{\beta} = \sum t \beta^t \sup \{ |a_t(\theta^t, i^t)| ; \theta^t, i^t \in \Theta^t \times \{0, 1\}^t \}.$$ 

Therefore, by the Weierstrass theorem, there exists an allocation $a$ that maximizes $W$ among all outcomes in $\mathcal{F}$. The uniqueness result comes from the strict concavity of $u(\cdot; \theta)$, for each $\theta \in \Theta$, and the convexity of the set $\mathcal{F}$.

3.B.3. Proof of proposition 3.2

Proof. Consider the problem

$$\max \left\{ \mathbb{E} \sum t \beta^t U(A_t; \bar{\theta}_t) ; A \in \mathcal{A} \right\},$$  \hspace{1cm} (3.32)

where $A = \{A_t\}_t$ is a sequence of maps $A_t : \Theta^t \times \{0, 1\}^t \rightarrow [0, NB]$ and $\mathcal{A}$ is the set of sequences $A = \{A_t\}_t$ satisfying

$$A_t(\theta^t, i^{t-1}, 0) = A_{t-1}(\theta^{t-1}, i^{t-1}),$$
$$\sum \theta^t \sum i^t p(\theta^t, i^t) A_t(\theta^t, i^t) = \bar{A}, \text{ and}$$
$$A_0(\theta_0) = \bar{A}.$$ 

It is easy to see that an outcome $a$ is constrained optimal if, and only if, there exists $A = \{A_t\}_t \in \mathcal{A}$ such that: $A_t(\theta^t, i^t) = \sum a_n^t(\theta^t, i^t)$, $a_t(\theta^t, i^t)$ solves (3.15) for $A = A_t(\theta^t, i^t)$, and $A$ achieves the maximum of problem (3.32). Hence, I just need to show that the existence of the Lagrange multipliers satisfying equation (3.25) is necessary and
sufficient for an \( A \in \mathcal{A} \) to solve problem (3.32). This result can be derived from theorem 1, section 8.3, and theorem 1, section 8.4, in Luenberger (1969).

3.B.4. Proof of proposition 3.3

**Proof.** Let \( \lambda = \{ \lambda_t \} \) denote the Lagrange multipliers in proposition (3.2). And consider the deterministic price sequence \( p = \{ p_t \} \) defined by

\[
p_t = \lambda_t - \alpha \beta \sum_{d=0}^{\infty} \beta^d \lambda_{t+1-d}
\]

for all \( t \). Let \( \tilde{\mu}_p^* = \{ \chi_p^*, \psi_p^* \} \in \mathcal{M} \) be the balanced team mechanism associated with the price sequence \( p \), and let \( \sigma \) be the truth-telling strategy. From Athey and Segal (2013), \( \sigma \) is a perfect Bayesian equilibrium of the investors-game implied by \( \tilde{\mu}_p^* \) and \( p \). From equation (3.33), it is easy to see that the sequence \( p \) satisfies

\[
\lambda_t = p_t - \sum_{d=1}^{\infty} \beta^d (1 - \alpha)^{d-1} \alpha p_{t+d}.
\]

Equation (3.34), combined with propositions (3.1) and (3.2), implies that the optimal asset policy satisfies \( \chi_{p_t}^{*n}(\theta_t, \iota_t, p_t) = a_{p_t}^{*n}(\theta_t, \iota_t) \). Since \( a^* \) is feasible, the market clearing condition must be satisfied. And for the last, the implied allocation coincides with the constrained Pareto efficient allocation \( a^* \).

3.B.5. Proof of proposition 3.4

**Proof.** For \( \alpha = 1 \), the optimal mechanism reflects the Walrasian demand associated with the investors type and the market cost of the implied demand of assets. As a result, the balanced team mechanism only replicates the allocation of assets that each investor would choose if they were by themselves accessing the market. And, because this is the Walrasian demand associated with a strictly continuous utility function, investors get strictly worse off by misrepresent their types. Since payoffs in the investors-game are continu-
ous in $\alpha$, investors also get strictly worse off by misrepresent their type when $\alpha$ is in a neighbourhood of 1.

3.C. Appendix 3.C: the non-commitment case

Let me extend the notation introduced before to account for the investors decision to go to autarky. Let $\tilde{\Theta} := \Theta \cup \{\text{aut}\}$ where $\theta^n \in \Theta$ denotes that investor $n$ is not in autarky and have announced type $\theta^n$ and $\theta^n = \text{aut}$ denotes that investor $n$ is in autarky. In a given period $t$, let $\theta_t = (\theta^1_t, \ldots, \theta^N_t) \in \tilde{\Theta} := \tilde{\Theta}^N$ denotes the vector. I denote the period $t$ realized variables $h_t = (\theta_t, \iota_t, p_t) \in H := \Theta \times \{0, 1\} \times \mathbb{R}_+$, and the history of realizations $h^t = (\theta^t, \iota^t, p^t) \in H^t$.

An asset policy is a sequence $\chi = \{\chi_t\}_t$, where $\chi_t = (\chi^1_t, \ldots, \chi^N_t) : H^t \to \mathbb{R}_+^N \cup \{\text{aut}\}$. If $\chi_t \in \mathbb{R}_+^N$ then it denotes how much assets to allocate for each investor in the group. If $\chi_t \in \{\text{aut}\}$ then it denotes a suggestion for investors to go to autarky. An asset policy $\chi = \{\chi_t\}$ is feasible if $\sum_n \chi^n_0 = NA$; for all $h^t \in H^t$, either $\iota_t = 1$ or $\sum_n 1_{\theta^n_t \neq \text{aut}} \chi^n_t(h^t) = \sum_n 1_{\theta^n_t \neq \text{aut}} \chi^n_{t-1}(h^{t-1})$; and $\theta^n_t = \text{aut}$ implies $\chi^n_t(h^t) = 0$. The first condition is the same as before. The second condition is that the group can only adjust its aggregate asset holdings once it accesses the centralized market, but it can only make with investors that are not in autarky. The last condition is that the suggested transfer for people in autarky is always zero. Let $\Gamma$ denotes the set of all feasible asset policies.

Note that a asset policy can be decentralized in many different ways. How much assets investor 1 transfer to investor 2 or investor 3 can be set in different ways so it implies the same aggregate transfer. For simplicity, I impose this arrangement to be proportional. For instance, suppose investors 1 and 3 are the investors to receive net transfers of assets. And suppose that investor 1 is going to receive 60% of the net transfer of assets while
investor 3 is going to receive 40%. That is,
\[
\frac{\chi^1_t(h^t) - \chi^1_{t-1}(h^{t-1})}{\sum_{n=1,3} \chi^n_t(h^t) - \chi^n_{t-1}(h^{t-1})} = 0.6 \quad \text{and} \quad \frac{\chi^2_t(h^t) - \chi^2_{t-1}(h^{t-1})}{\sum_{n=1,3} \chi^n_t(h^t) - \chi^n_{t-1}(h^{t-1})} = 0.4.
\]

Then, each investor \( n \neq 1, 3 \) will transfer 60% to investor 1 and 40% to investor 2 of the net amount he is suppose to transfer, \( \chi^n_{t-1}(h^{t-1}) - \chi^n_t(h^t) \).

A numéraire policy is a sequence \( \psi = \{ \psi_t \}_t \), where \( \psi_t = (\psi^1_t, \ldots, \psi^N_t) : H^t \to \mathbb{R}^N \cup \{aut\} \). If \( \psi_t \in \mathbb{R}^N \) then it denotes how much numéraire good to transfer to each investor in the group. If \( \psi_t \in \{aut\} \) then it denotes a suggestion for investors to go to autarky. A transfer policy \( \psi = \{ \psi_t \}_t \) is feasible if \( \sum_n 1_{\psi^n_t \neq aut} \psi^n_t(h^t) = p_t \sum_n 1_{\psi^n_t = aut} [\chi^n_t(h^t) - \chi^n_{t-1}(h^{t-1})] \) for all \( h^t \in H^t \); and \( \theta^n_t = aut \) implies \( \psi^n_t(h^t) = 0. \) That is, the transfers between investors that have not gone to autarky are budget balanced and a investor in autarky does not receive any numéraire transfer.

A pure strategy for an investor \( n \) is a sequence \( s^n = \{ \sigma^n, a^n, y^n \} = \{ \sigma^n_t, a^n_t, y^n_t \}_t \) of measurable functions where, \( \sigma^n_t \) maps \( (h^{t-1}, \theta^n_t, x^t) \) into \( \Theta \) and \( (a^n_t, y^n_t) \) maps \( (h^t, \theta^n_t, x^t) \) into \( \mathbb{R}^{N-1} \times \mathbb{R}^{N-1} \). \( \sigma^n_t \) is the announcement decision, as before. \( (a^n_t, y^n_t) \) is the asset and numéraire transfer to each other \( N - 1 \) investors. The implied investors-game is analogous to the commitment case.

As before, the only aggregate uncertainty in the economy comes from the public sunspots. Therefore, the price in a period \( t \) is a deterministic function of the public signal vector \( x^{pl}_t \). The market clearing condition requires that, for a given realization of the sequence \( x^{pl}_t \), the implied demand of assets equal the total offer of assets at a period \( t \).

Let \( \eta = \{ \eta_t \}_t \) be the sequence of measures over the space \( H^t \) which is generated by \( Q, \alpha, F, p, \mu \in \mathcal{M} \), and \( \{ s^n \}_n \). The aggregate demand for assets in the centralized market in period \( t \) is
\[
D_t(x^{pl}_t) := \int \sum_{n \neq n} a^n_{t_1}(h^t) d\eta_t(h^t | x^{pl}_t). \quad (3.35)
\]

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Markets clear at a period $t$ if $D_t(x^{pt})$ equals $\bar{A}$ for all $x^{pt}$ in the support $S^{pt}$.

**Definition 3.2** Given a feasible mechanism $\mu \in \mathcal{M}$, a symmetric equilibrium is a pair $(s, p)$ such that: (i) $s^{pt} = s$ for all investors form a perfect Bayesian equilibrium of the investors-game implied by $\mu$; and (ii) markets clear at every period.

The balanced team mechanism need to be extended to account for the possibility of some investor deviate and go to autarky. The modification I propose is simple. The mechanism follow the balanced team mechanism until an investor deviate. After a deviation the mechanism recommend autarky to all remaining investors.

**Proposition 3.11** There exist $\bar{\beta}$ such that, for all $\beta \in [\bar{\beta}, 1)$, there exists a price process $p$, a strategy $s = \{\sigma, a, y\}$ and a feasible mechanism $\mu^* = \{\chi^*, \psi^*\} \in \mathcal{M}$ such that: (i) $\mu^* = \mu^*_p$ is the modified balanced team mechanism associated with the price $p$; (ii) $(s, p)$ is a symmetric equilibrium associated with the mechanism $\mu^*$; (iii) $\sigma$ is the truth-telling strategy; (iii) $a$ and $y$ follow the mechanism recommendation; and (iv) the implied asset allocation is constrained Pareto efficient.

**Proof.** The proof of this proposition follows exactly the same steps of the proof of proposition 3.15. The only difference being that we cannot apply Athey and Segal (2013) to show that follow the mechanism transfer recommendation is an equilibrium since it requires commitment. Athey and Segal (2013) have an extension of their result for the environment without commitment which guarantees that cooperation with the mechanism is an equilibrium for $\beta$ high, this result is given by proposition 4 in their paper. The reason we cannot directly apply proposition 4 in Athey and Segal (2013) is because the payoffs of the investors-game, and also the optimal mechanism, varies with $\beta$. On the other hand, if we exam equation (3.34) and (3.25) we see that the equilibrium price and Lagrange multipliers, normalized by $(1 - \beta)$, converge as $\beta$ goes to 1. Therefore, we can apply the
proposition 4 in Athey and Segal (2013) to show that cooperation is an equilibrium of this limit game. And, since autarky makes the investor strictly worse off than cooperation, the result will also hold for $\beta$ high.

**Proposition 3.12** Autarky is an equilibrium of the investors-game implied by any price process, $p$, and feasible mechanism, $\mu$.

**Proof.** If all investors in the group go to autarky, a investor payoff of staying in the group is bounded above by the autarky payoff. Therefore, going to autarky must be a best response.

### 3.D. Appendix 3.D: optimal policy

#### 3.D.1. Proof of proposition 3.6

**Proof.** Suppose by the way of contradiction that a pair $\{\sigma, p\}$ is an equilibrium and $\sigma$ generates a sequence $\tilde{\Pi}_t \neq \Pi_t$ along the equilibrium path. Let $t_u$ denotes the the first period in which $\tilde{\Pi}_t \neq \Pi_t$. Note that from date $t_u$ forward investors know that the group will maximize his utility individually, hence, truth-telling is a dominant strategy. Therefore, $\sigma$ need to be consistent with truth-telling for $t = t_u$ forwards, which contradicts the fact that $\tilde{\Pi}_t = \Pi_t$.


**Proof.** At any period $t$, the distribution of announcements needs to be generated by the distribution of previous types and sunspots realizations. That is, the announcement vector in period $t$ is a function $g_t(\theta^t, x^t)$. If $g_t$ generates the same distribution of announcement
vector \( \Pi_t \), then it needs to satisfy, for all \( \theta_t \in \Theta \),

\[
P[\mathcal{G}_t(\theta^t, x^t) = \theta_t] = \sum_{\tilde{\theta}^t, \tilde{x}^t \in \mathcal{G}^{-1}_t(\theta_t)} P(\tilde{x}^t) \Pi_0(\tilde{\theta}_0) \mathcal{X}_s=1 Q_c(\tilde{\theta}_{s-1}, \tilde{\theta}_s) \]
\[
= \Pi_t(\theta_t) = \sum_{\tilde{\theta}^t \in \Theta} \mathbb{1}_{\tilde{\theta}_t = \theta_t} \Pi_0(\tilde{\theta}_0) \mathcal{X}_s=1 Q_c(\tilde{\theta}_{s-1}, \tilde{\theta}_s). \tag{3.36}
\]

Note that the above equation always holds if \( \mathcal{G}^{-1}_t(\theta_t) = \{ \tilde{\theta}^t, \tilde{x}^t \in \Theta^t \times S^t; \tilde{\theta}_t = \theta_t \} \), that is, under truth-telling. Now suppose this equation holds for some function \( \mathcal{G}_t \) such that \( \mathcal{G}^{-1}_t(\theta_t) \neq \{ \tilde{\theta}^t, \tilde{x}^t \in \Theta^t \times S^t; \tilde{\theta}_t = \theta_t \} \). Consider an initial distribution \( \hat{\Pi}_0 \neq \Pi_0 \) such that \( ||\hat{\Pi}_0 - \Pi_0|| < \varepsilon \) and a transition \( \hat{Q}_c \) such that \( ||\hat{Q}_c - Q_c|| < \varepsilon. \) Since \( \Theta^t \times S^t \) is finite, any change in \( \mathcal{G}_t \) implies a discontinuous jump on the left hand side of equation (3.36). As a result, equation (3.36) cannot be satisfied for a different \( \mathcal{G}_t \) if we take \( \varepsilon \) to be small enough.

Note that

\[
\sum_{\tilde{\theta}^t, \tilde{x}^t \in \mathcal{G}^{-1}_t(\theta_t)} P(\tilde{x}^t) \Pi_0(\tilde{\theta}_0) \mathcal{X}_s=1 Q_c(\tilde{\theta}_{s-1}, \tilde{\theta}_s)
\]

and

\[
\sum_{\tilde{\theta}^t \in \Theta} \mathbb{1}_{\tilde{\theta}_t = \theta_t} \Pi_0(\tilde{\theta}_0) \mathcal{X}_s=1 Q_c(\tilde{\theta}_{s-1}, \tilde{\theta}_s)
\]

are two different weighted sums of the terms \( \Pi_0(\tilde{\theta}_0) \mathcal{X}_s=1 Q_c(\tilde{\theta}_{s-1}, \tilde{\theta}_s) \). Therefore, when the weights are fixed, any perturbation in the terms \( \Pi_0(\tilde{\theta}_0) \mathcal{X}_s=1 Q_c(\tilde{\theta}_{s-1}, \tilde{\theta}_s) \) will, generically, breaks the equality. Which concludes the proof.

3.D.3. **Proof of proposition 3.8**

**Proof.** Since investors access the market-maker at every period they don’t cause externalities to each other. Therefore, the balanced team mechanism allocates the optimal demand associated with the Walrasian demand given the price sequence \( \{ \bar{p}_t \}_t \). Since this demand is optimal, the investor cannot get any better by misrepresenting his type.

3.D.4. **Proof of proposition 3.9**

**Proof.** Analogous to proposition 3.6.
3.D.5. Proof of proposition 3.10

Proof. Analogous to proposition 3.7.
References


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Job Market Paper
"Financial fragility and over-the-counter markets", 2014
Abstract: I propose a model to study whether trade frictions in an over-the-counter market for financial assets exacerbate or attenuate financial fragility. I model the financial sector as a large number of financial institutions, which I label banks. Each bank is a coalition of depositors and depositors are subject to privately observed liquidity shocks. The banks' problem is to maximize the welfare of depositors by implementing the efficient allocation of financial assets among them. I show that when banks use the balanced team mechanism, proposed by Athey and Segal (2013), there is always a truth-telling equilibrium which supports the constrained Pareto efficient allocation. When the frictions in the over-the-counter market are small, this equilibrium is unique. However, I provide numerical examples in which these frictions are severe and the economy has other equilibria. In one equilibrium depositors claim high liquidity needs, asset price falls, the trade volume collapses and, consequently, the equilibrium allocation is not constrained Pareto efficient. I label this equilibrium a bank-run equilibrium and I interpret the existence of bank-runs as a financial fragility. I propose two policies to eliminate bank-run equilibria. The first is a suspension scheme and the second is an opening of trade facilities similar to the ones established by the Federal Reserve Bank during the 2007-08 financial crisis. Both policies can eliminate bank runs when contingent on announcements of liquidity needs in a large number of banks.

Publications
Abstract: This paper characterizes the direct mechanism which implements the constrained optimal outcome in a version of Diamond and Dybvig (1983) with aggregate uncertainty and a continuum of agents. Using this result, numerical examples where the best direct mechanism has a bank-run-equilibrium are easily obtained.