MEASUREMENT OF ATMOSPHERIC MUON NEUTRINO DISAPPEARANCE WITH ICECUBE-DEEPCORE

A Dissertation in
Physics
by
Matthew Gregory Dunkman

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The dissertation of Matthew Gregory Dunkman was reviewed and approved* by the following:

Tyce DeYoung  
Professor of Physics  
Dissertation Co-Advisor, Co-Chair of Committee

Douglas Cowen  
Professor of Physics  
Dissertation Co-Advisor, Co-Chair of Committee

Irina Mocioiu  
Professor of Physics

Stephane Coutu  
Professor of Physics  
Professor of Astronomy and Astrophysics

Derek Fox  
Professor of Astronomy and Astrophysics

Nitin Samarth  
Professor of Physics  
Head of the Department of Physics

*Signatures are on file in the Graduate School.
Abstract

The measurement of muon neutrino disappearance presented here was designed to make use of neutrinos of all flavors, exploiting $\nu_e$ events and more poorly reconstructed cascade-like neutrino events to better constrain systematic uncertainties associated with neutrino fluxes and interactions. In accordance with IceCube Collaboration policy, the analysis was developed “blind,” with reference to Monte Carlo simulations and down-going events where no oscillation signal could be visible. Discrepancies observed in the data when they were examined after the analysis was finalized led to the conclusion that the cascade-like and intermediate data samples were affected by unexpected systematic effects, so that their inclusion in the analysis produced substantial bias. This analysis thus relied exclusively on the track-like data sample, while investigations into the nature of these systematic effects continue in preparation for an IceCube journal article. The best fit and 68% confidence interval from this analysis on the two oscillation parameters which drive muon neutrino disappearance are

$$\sin^2 \theta_{23} = 0.53^{+0.07}_{-0.11},$$
$$\Delta m^2_{32}/10^{-3} \text{eV}^2 = 2.35^{+0.14}_{-0.17},$$

which are compatible and competitive with measurements made by dedicated beam-line neutrino experiments.
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6.10 Top: comparison between data (black points) and simulation with and without oscillations (blue and red respectively) as a function of reconstructed L/E. The events displayed are from the most track-like sample (C), using best fit parameters from the full sample shown in Table 6.1. Bottom: ratio of data and oscillated simulation to the unoscillated simulation. Here the data agrees well with the best fit MC estimates at all values of L/E, and with the unoscillated expectation in the down-going region where oscillations are not expected to occur. Error bars on the data are statistical only.

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6.3 Fit results for track-like sample
Dedication

For my wife Michela, and our daughter Hannah.
Chapter 1  Neutrino Theory

In the Standard Model of particle physics [1], everything in the universe is composed of combinations of twelve fundamental particles shown as the quarks and leptons in Figure 1.1. The additional column of gauge bosons work as messengers, transferring energy and momentum between the quarks and leptons. The strong force uses the gluon to bind together the quarks to form protons, neutrons, and all sorts of other particles. All electromagnetic force is propagated by photons, whether it is light from the sun warming our skin or the sustaining push on electrons as an electronic signal travels down a wire. The remaining force that is relevant to individual sub-atomic particles is the weak interaction, which, through the W and Z bosons, is responsible for radioactive decays and all sorts of changes in particle type.

These weak interactions fuel the sun, but electrically charged particles will typically interact several thousand times through the electromagnetic force for each weak interaction. The result of this dominance of electromagnetic interactions on well known particles is a large mismatch between the precision with which weak and electromagnetic interaction processes are understood. Neutrinos are the only electrically neutral fundamental particle; therefore, they are an ideal probe of the weak interaction.

1.1 Neutrino eigenstates

In the Standard Model, lepton number is a conserved quantity in all interactions. As there are three flavors of charged leptons, only three neutrinos can couple with the $W^\pm$ boson as shown in Figure 1.2. Therefore, neutrinos produced in weak
Figure 1.1: The particles in the Standard Model. All matter is composed of combinations of leptons and quarks, with the gauge bosons acting as carriers the fundamental forces. This image was downloaded from https://commons.wikimedia.org/wiki/File:Standard_Model_of_Elementary_Particles.svg, using the version modified 19:54, 6 August 2013.

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Leptons</th>
<th>Gauge Bosons</th>
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<tr>
<td>u</td>
<td>e</td>
<td>e⁻</td>
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<tr>
<td>c</td>
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<td>t</td>
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<td>d</td>
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<td>s</td>
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<td>ν₅⁻</td>
</tr>
<tr>
<td>b</td>
<td>ν₇</td>
<td>ν₇⁻</td>
</tr>
</tbody>
</table>

**QUARKS**
- u: up, mass ≈ 2.3 MeV/c², charge 2/3, spin 1/2
- c: charm, mass ≈ 1.275 GeV/c², charge 2/3, spin 1/2
- t: top, mass ≈ 173.07 GeV/c², charge 2/3, spin 1/2
- d: down, mass ≈ 4.8 MeV/c², charge -1/3, spin 1/2
- s: strange, mass ≈ 95 MeV/c², charge -1/3, spin 1/2
- b: bottom, mass ≈ 4.18 GeV/c², charge -1/3, spin 1/2

**LEPTONS**
- e: electron, mass 0.511 MeV/c², charge -1, spin 1/2
- μ: muon, mass 105.7 MeV/c², charge -1, spin 1/2
- τ: tau, mass 1.777 GeV/c², charge -1, spin 1/2
- νₑ: electron neutrino, mass < 2.2 eV/c², charge 0, spin 1/2
- ν₅: muon neutrino, mass < 0.17 MeV/c², charge 0, spin 1/2
- ν₇: tau neutrino, mass < 15.5 MeV/c², charge 0, spin 1/2

**GAUGE BOSONS**
- g: gluon, mass 0 GeV/c², charge 0, spin 1
- W⁻: W⁻ boson, mass 91.2 GeV/c², charge ±1, spin 1
- H: Higgs boson, mass 126 GeV/c², charge 0, spin 0

Figure 1.2: The three possible neutrino interactions involving the W⁻ boson. Antineutrinos undergo conjugate interactions with the W⁺ boson.
interactions are in a pure flavor eigenstate, $\nu_e$, $\nu_\mu$, $\nu_\tau$. As the $W^\pm$ transfers both electric charge as well as momentum, these neutrino interactions are referred to as charged current (CC). Weak interactions can also be mediated by the slightly heavier and neutral $Z^0$ boson, with these processes denoted as neutral current (NC) interactions. Precision measurements of the $Z^0$ resonance shown in Figure 1.3 confirmed that only three neutrinos couple to the $Z^0$ boson [3]; however, these neutrinos can be in any superposition of flavor states.

Figure 1.3: Measurement of the hadronic production cross-section near the $Z$ resonance strongly shows that only three families of neutrinos can be produced in $Z$ decays. [3]
1.2 Neutrino interactions with nucleons

As with all probes of nuclear structure, the result of a neutrino-nucleon interaction varies greatly as the energy of the neutrino increases. Below 1 GeV, neutrinos will typically elastically scatter off the entire nucleon, rather than individual quarks. In the process, as shown in Figure 1.4a, enough energy may be deposited to free one or more individual nucleons from the target nucleus. If the neutrino interacts through the charged current, with the final state including a charged lepton as well as the target nucleon, then this process is referred to as quasielastic scattering. If the target nucleus is large enough, the nucleon struck by the neutrino will likely undergo multiple strong interactions with the remainder of the nucleus, dispersing the energy through final state interactions.

As the energy of the incident neutrino increases, so too does the energy transferred to the target nucleons. Several excited baryon states exist between 1-3 GeV, and neutrino interactions can provide the required energy to boost a proton or neutron into one of these resonant states. The excited nuclei quickly decay back to the ground state through emission of a pion or heavier meson. These interactions are referred to as resonance processes. In the same energy range, but away from the resonant peaks, a single pion can be produced that coherently carries a large portion of the transferred energy and momentum. As before, these emitted mesons may undergo final state interactions within the nucleus before escaping or being absorbed by the surrounding nucleons.

Just as X-ray photons are able to resolve molecular structure, while radio frequency photons cannot, at even higher energies the neutrino will probe deeper

Figure 1.4: Feynman diagrams of quasielastic (a), coherent (b), and deep inelastic (c) neutrino-nucleon scattering.
into the nucleon, interacting with individual partons (quarks and gluons). This final interaction method is referred to as deep inelastic scattering (DIS), and is the primary process above 10 GeV.

### 1.2.1 Deep inelastic scattering

At these energies, nucleons are best described using sea and valence quarks and are fundamentally complicated and probabilistic [4]. As a result, deep inelastic scattering processes are described completely in terms of three dimensionless kinematic invariants, the inelasticity $y$, the four-momentum transfer $Q^2$, and the Bjorken scaling variable $x$. In this context, these parameters can be reconstructed from observables that are (at least in principle) measurable experimentally,

\begin{align}
y &= \frac{E_{\text{had}}}{E_{\nu}} \quad (1.1) \\
Q^2 &= -m_\ell^2 + 2E_\ell(E_\ell - p_\ell \cos \theta_\ell) \quad (1.2) \\
x &= \frac{Q^2}{2M_N E_{\text{had}}} = \frac{Q^2}{2M_N E_{\nu} y}, \quad (1.3)
\end{align}

where $E_{\text{had}}$ is the collective energy of the outgoing hadronic shower; $E_{\nu}$ is the incident neutrino energy; $M_N$ is the mass of the target nucleon; and $E_\ell$, $m_\ell$, $p_\ell$, and $\cos \theta_\ell$ are the energy, mass, momentum and scattering angle of the outgoing lepton in the laboratory frame, respectively. In the case of neutral current scattering, the outgoing lepton is a neutrino which cannot be observed and all information about the interaction must be inferred from the hadronic component. Using these variables and three additional nuclear structure functions ($F_{1,2,3}$), the inclusive cross section for DIS can be written as

\begin{align}
\frac{d^2\sigma}{dxdy} &= A \left[ b F_1(x, Q^2) + c F_2(x, Q^2) + dx F_3(x, Q^2) \right] \quad (1.4) \\
A &= \frac{G_F^2 M_N E_{\nu}}{\pi(1 + Q^2/M_W^2)^2} \quad (1.5) \\
b &= \left( \frac{y^2}{2} \right) 2x \quad (1.6) \\
c &= 1 - y - \frac{M_N xy}{2E_{\nu}} \quad (1.7)
\end{align}
\[ d = \pm y \left( 1 - \frac{y}{2} \right), \] (1.8)

where \( G_F \) is the Fermi weak coupling constant; \( M_{W,Z} \) is the mass of the \( W^\pm (Z^0) \) boson in the case of CC (NC) scattering; and the \( +(-) \) sign in (1.8) is used for neutrino (anti-neutrino) interactions. The structure functions are generally expressed in the context of the parton model, in which the quark composition is defined through parton distribution functions (PDFs). The measurement of this inclusive neutrino cross section has been performed by several experiments as shown in Figure 1.5, which is found to be linear with respect to neutrino energy above 10 GeV [4].

**Table XIV** Attributes of neutrino experiments that have recently studied DIS, including CHORUS (Kayis-Topaksu et al., 2008a; Onegut et al., 2006), MINOS (Adams et al., 2010), NOMAD (Wu et al., 2008), and NuTeV (Mason et al., 2007a; Tzanov et al., 2006a; Zeller et al., 2002).

**Figure 1.5:** Measurement of the inclusive neutrino and antineutrino CC cross sections \( \nu_\mu N \rightarrow \mu^- X \) and \( \bar{\nu}_\mu N \rightarrow \mu^+ X \) divided by neutrino energy, plotted as a function of neutrino energy, from [4, Fig. 28].
1.2.2 Kinematic suppression

There is a very large difference between the mass of a neutrino and each of the charged leptons. Thus even highly relativistic neutrinos may not possess enough energy to produce their charged partner when interacting with a nucleon in its rest frame. This is especially relevant for CC $\nu_\tau$ interactions, where the interaction cross section is severely suppressed below 100 GeV, as shown in Figure 1.6. As a charged lepton is not produced during neutral current interactions, these do not experience such suppression at low energies.

![Figure 1.6: Comparison of total charged current $\nu_\mu$ (solid) and $\nu_\tau$ (dashed) cross section divided by neutrino energy as a function of neutrino energy from [4] Fig. 10.](image)

Figure 1.6: Comparison of total charged current $\nu_\mu$ (solid) and $\nu_\tau$ (dashed) cross section divided by neutrino energy as a function of neutrino energy from [4] Fig. 10.
1.3 Neutrino oscillations

It is now well established that neutrinos and undergo flavor oscillations [1]. These oscillations require that neutrinos have mass and that their mass eigenstates are distinct from their flavor eigenstates. Global fits of the oscillation parameters measured experimentally (such as those provided by [5–8]) show that the mass splittings are significantly different with $\Delta m_{21}^2 \ll (|\Delta m_{32}^2| \simeq |\Delta m_{31}^2|)$, where $\Delta m_{ij}^2 = m_j^2 - m_i^2$ as shown in Figure 1.7. We can therefore decompose the state of an arbitrary neutrino $|\nu\rangle$ into either the flavor (interaction) basis or mass (propagation) basis, which equally span the space as follows:

$$|\nu\rangle = \sum_\alpha c_\alpha |\alpha\rangle, \quad \langle \alpha | \beta \rangle = \delta_{\alpha \beta}$$

(1.9)

$$|\nu\rangle = \sum_{j=1,2,3} c_j |j\rangle, \quad \langle j | k \rangle = \delta_{jk},$$

(1.10)

for flavor eigenstates $\alpha \in (\nu_e, \nu_\mu, \nu_\tau)$ and mass eigenstates $j \in (1, 2, 3)$. To reduce the number of summations in the formalism, I introduce the following vector forms:

Figure 1.7: Flavor composition of neutrino mass eigenstates from global fit of [8]. The separation of mass states illustrates the large separation between $\nu_{1,2}$ and $\nu_3$ and is not to scale, as the real separation would require several additional pages. The relative position of the $\nu_3$ is also not known with respect to the other mass eigenstates, shown here is the “normal hierarchy”, in which $\nu_3$ is the most massive, but the alternative ordering, referred to as the “inverted hierarchy,” in which $\nu_3$ is the lightest neutrino is also experimentally allowed.
for the neutrino state projected into a given basis:

\[
|\nu_\alpha\rangle = \sum_\alpha c_\alpha |\alpha\rangle = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad \text{and} \quad |\nu_j\rangle = \sum_j c_j |j\rangle = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},
\]

which are acted on by matrices, here denoted in boldface e.g. \( \mathbf{A} \). The mapping between flavor and mass bases is provided by a unitary mixing matrix

\[
|\nu_\alpha\rangle = \mathbf{U} |\nu_j\rangle, \quad \mathbf{U}^\dagger |\nu_\alpha\rangle = |\nu_j\rangle,
\]

which can be defined as the set of projections between bases

\[
|\alpha\rangle = \left( \sum_j |j\rangle \langle j| \right) |\alpha\rangle = \sum_j |j\rangle \langle j| \alpha \rangle = \sum_j |j\rangle U_{j\alpha},
\]

\[
|j\rangle = \left( \sum_\alpha |\alpha\rangle \langle \alpha| \right) |j\rangle = \sum_\alpha |\alpha\rangle \langle \alpha| j \rangle = \sum_\alpha |\alpha\rangle U^*_{\alpha j}.
\]

Following the PMNS convention [1] and ignoring the Majorana phases, we can expand Equation 1.11 as the product of rotation matrices and a single complex phase

\[
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{1.14}
\]

with \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \), and \( \delta = \delta_{CP} \) as the CP-violating phase. The values of these parameters from [8] are provided in Table 1.1.

We can solve Schrödinger’s equation to find that the state of an arbitrary neutrino after some time is

\[
|\nu\rangle_t = e^{-iHt} |\nu\rangle, \tag{1.15}
\]

where \( H \) is the neutrino’s Hamiltonian. The mass states previously discussed are thus defined as the eigenstates of the vacuum Hamiltonian, that is

\[
H |\nu\rangle = \sum_j E_j |j\rangle. \tag{1.16}
\]
Table 1.1: Values of neutrino oscillation parameters from [8] under the assumption of a normal mass hierarchy as illustrated in Figure 1.7. Here bfp refers to the best fit point.

<table>
<thead>
<tr>
<th></th>
<th>bfp ±1σ</th>
<th>3σ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{12}/^\circ$</td>
<td>$33.48^{+0.78}_{-0.75}$</td>
<td>$31.29 \rightarrow 35.91$</td>
</tr>
<tr>
<td>$\theta_{23}/^\circ$</td>
<td>$42.3^{+3.0}_{-1.6}$</td>
<td>$38.2 \rightarrow 53.3$</td>
</tr>
<tr>
<td>$\theta_{13}/^\circ$</td>
<td>$8.50^{+0.20}_{-0.21}$</td>
<td>$7.85 \rightarrow 9.10$</td>
</tr>
<tr>
<td>$\delta_{CP}/^\circ$</td>
<td>$306^{+39}_{-70}$</td>
<td>$0 \rightarrow 360$</td>
</tr>
<tr>
<td>$\Delta m^2_{21}/10^{-5}$ eV$^2$</td>
<td>$7.50^{+0.19}_{-0.17}$</td>
<td>$7.02 \rightarrow 8.09$</td>
</tr>
<tr>
<td>$\Delta m^2_{31}/10^{-3}$ eV$^2$</td>
<td>$2.457^{+0.047}_{-0.047}$</td>
<td>$2.317 \rightarrow 2.607$</td>
</tr>
</tbody>
</table>

In the relativistic limit, where $E_j \gg m_j$, the Hamiltonian can be written as

$$H |\nu\rangle = \sum_j \left( \frac{m_j^2}{2E_\nu} \right) |j\rangle = \left( \frac{m^2}{2E_\nu} \right) |\nu_j\rangle,$$  \hspace{1cm} (1.17)

with

$$m^2 |\nu_j\rangle = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$  \hspace{1cm} (1.18)

We note that the Hamiltonian can also be written in the interaction (flavor) basis as

$$H |\nu\rangle = U \left( \frac{m^2}{2E_\nu} \right) U^\dagger |\nu_\alpha\rangle.$$  \hspace{1cm} (1.19)

The previous results require that the neutrinos be relativistic, which is easily met for neutrinos whose rest mass is less than an electron volt. Given the neutrino’s very small interaction cross section, we can assume that a typical neutrino will have a mean free path on the scale of a few light years. Therefore, we assume the velocity of the neutrino is constant from production until it reaches our detector and exchange propagation distance $L = ct$ with the time of propagation in Equation 1.15. Over that distance, the neutrino’s flavor state will be modified according to

$$|\nu\rangle_L = e^{-i \int_0^L dx H} |\nu\rangle = e^{-i \int_0^L dx H} \sum_j |j\rangle.$$  \hspace{1cm} (1.20)
\[ \sum_j e^{-im_j^2 L/2E_\nu} |j\rangle. \] (1.21)

This extends naturally to the amplitude of a transition from flavor state \( \alpha \rightarrow \beta \) over the distance \( L \):

\[
\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta |E_\nu, L) = \langle \beta | e^{-i \int_0^L dx H} |\alpha\rangle \\
= \langle \beta | e^{-i \int_0^L dx H} \sum_j |j\rangle \langle j|\alpha\rangle \\
= \sum_j \langle \beta|j\rangle e^{-im_j^2 L/2E_\nu} U_{j\alpha} \\
= \sum_j U_{\beta j}^* e^{-im_j^2 L/2E_\nu} U_{j\alpha}. \] (1.22)

The probability of flavor transition is given by the modulus squared of the amplitude,

\[
\mathcal{P}(\nu_\alpha \rightarrow \nu_\beta |E_\nu, L) = |\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta |E_\nu, L)|^2 \\
= \left( \sum_k U_{k\beta} e^{im_k^2 L/2E_\nu} U_{ak}^* \right) \left( \sum_j U_{\beta j}^* e^{-im_j^2 L/2E_\nu} U_{ja} \right) \\
= \sum_{jk} U_{k\beta} U_{\beta j}^* e^{-i\Delta m_{jk}^2 L/2E_\nu} U_{ja} U_{ak}^*. \] (1.24)

### 1.3.1 Two-neutrino approximation

Given the large separation between eigenstates, and the knowledge that the third mass eigenstate is almost entirely composed of muon and tau neutrinos, we can describe the oscillations of atmospheric muon neutrinos to leading order with a two-flavor approximation, in this case neglecting \( \nu_e \) and \( \nu_1 \). We have previously presented the standard three-flavor case, but we can reduce Equations 1.14 and 1.18 to include fewer neutrinos. The simplified forms are

\[
\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta_{23} & \sin \theta_{23} \\ -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}, \] (1.26)

and

\[
\mathbf{m}^2 |\nu_j\rangle = \begin{pmatrix} m_2^2 & 0 \\ 0 & m_3^2 \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}. \] (1.27)
If we plug these two-flavor forms into Equation 1.25 and churn through some trigonometry, we get

\[ P(\nu_\mu \to \nu_\tau | E_\nu, L) = \sin^2(2\theta_{23}) \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E_\nu} \right). \quad (1.28) \]

Using the global fit values for \( \Delta m_{32}^2 \) and \( \theta_{23} \) from [8] and the relevant unit conversions, to one significant digit, this two flavor probability further simplifies to

\[ P(\nu_\mu \to \nu_\tau | E_\nu, L) \simeq (1.000) \times \sin^2 \left( 0.003 \frac{L}{E_\nu} \right). \quad (1.29) \]

with \( L \) in kilometers and \( E_\nu \) in GeV. For neutrinos traversing the earth’s diameter, which is \( 1.3 \times 10^4 \) km, the first two oscillation maxima will occur for neutrinos of 25 GeV and 8 GeV respectively. These maxima signify that, at these energies, essentially all muon-flavor neutrinos produced on one side of the earth will have transitioned into a pure tau-flavor state by the time they arrive at the other side of the planet.

### 1.3.2 Modifications due to matter

If the neutrino travels through the earth, then we must modify the Hamiltonian to include weak interaction potentials through charged and neutral currents. These modifications are required because the earth contains a large number of stable electrons, but does not contain any stable muons or tau leptons. In the simplified two-flavor case, these modifications will have no net impact on the \( \nu_\mu \to \nu_\tau \) oscillations described in the previous section, but a full treatment of neutrino oscillations allows for \( \nu_\mu \to \nu_e \) and can be represented as

\[ H |\nu\rangle = U \left( \frac{m_2^2}{2E_\nu} \right) U^\dagger |\nu_\alpha\rangle + V |\nu_\alpha\rangle, \quad (1.30) \]

in which

\[ V |\nu_\alpha\rangle = \begin{pmatrix} V_{ee} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad \text{with } V_{ee} = A_{ee} \rho_e = \sqrt{2}G_F \rho_e. \quad (1.31) \]

Here we note that both the charged current interaction potential with nucleons

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and the neutral current interaction potential are proportional to the identity matrix. Thus they will only contribute to the complex phase of the neutrino state, and have no impact on the oscillation probabilities; therefore we have not included those terms. If we were to include sterile neutrinos in our model these potentials would no longer be proportional to the identity matrix and we would be required to include them. As their inclusion would not alter our approach, they will not be discussed further.

As the vacuum and matter terms of Equation [1.30] are Hermitian matrices, that is $A^\dagger = A$, they can be diagonalized by a unitary matrix, and all their eigenvalues are real. So just as in the vacuum case, we will use the eigenstates of the Hamiltonian to define effective masses for the neutrinos in matter,

$$H |\nu\rangle = U_m \left( \frac{m_\text{e}^2}{2E_\nu} \right) U_m^\dagger |\nu_\alpha\rangle .$$  \hfill (1.32)

Following the same procedure as with vacuum, the probability amplitude of a transition from flavor state $\alpha \rightarrow \beta$ over the distance $L$ in matter can be computed, however we must note that the effective mass eigenstates and mixing matrix are now functions of the electron density. If that density is not constant or adiabatically varying, then we must divide the integration into $N$ adiabatic regions, that is

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta |E_\nu, L) = \prod_{k=1}^{N} \mathcal{A}_k(\nu_\alpha \rightarrow \nu_\beta |E_\nu, L_k)$$  \hfill (1.33)

for $L = \sum_{k=1}^{N} L_k$. We will leave the Hamiltonian in terms of vacuum eigenstates to avoid myriad subscripts, noting that matrix exponentiation is often numerically calculated by computing the eigenvalues and unitary matrices described in Equation [1.32] Therefore, in a region of reasonably constant density, the transition amplitude is given by

$$\mathcal{A}_k(\nu_\alpha \rightarrow \nu_\beta |E_\nu, L_k) = \langle \beta | \exp \left\{-i \int_{x_{k-1}}^{x_k} dx \left[ U \left( \frac{m^2}{2E_\nu} \right) U^\dagger + A \rho_e(x) \right] \right\} |\alpha\rangle$$  \hfill (1.34)

$$= \langle \beta | \exp \left\{-i \left[ U \left( \frac{m^2 L_k}{2E_\nu} \right) U^\dagger + A \int_{x_{k-1}}^{x_k} dx \rho_e(x) \right] \right\} |\alpha\rangle ,$$  \hfill (1.35)

which can be computed numerically. The overall oscillation probability in matter
is calculated as in Equation 1.24 with the insertion of Equation 1.33:

\[ P(\nu_\alpha \rightarrow \nu_\beta | E_\nu, L) = \left| \prod_{k=1}^{N} A_k(\nu_\alpha \rightarrow \nu_\beta | E_\nu, L_k) \right|^2. \]  

(1.36)

A visual comparison of the oscillation probabilities from the two-flavor approximation (Equation 1.28) to three flavor oscillation probabilities through a vacuum (Equation 1.25) and through matter (Equation 1.36) is provided in Figure 1.8.

### 1.4 Atmospheric neutrinos

The earth’s upper atmosphere is constantly bombarded by high energy nuclei referred to as cosmic rays. Accurately predicting the flux of atmospheric neutrinos requires extensive simulation including propagation and interaction of cosmic ray primaries and secondaries through the earth’s atmosphere and magnetic field. The practical result of these simulations is a set of tables providing the number of neutrinos of a given flavor arriving at a specified detector location as a function of energy and arrival direction [11]. Using first order approximations, we can gain an intuitive picture of the overall shape of the neutrino flux and cosmic ray flux from which it is derived. These approximations also motivate a set of flux-related systematic variables that are used in the analysis described in Chapter 5.

#### 1.4.1 Atmospheric primary flux

The flux of cosmic ray primary nuclei has been measured extensively [1]. Between 1 GeV and 100 TeV, these data can be parametrized to first order with a series of modified power laws [13,14]

\[ \Phi_N(E) = a \left( E + b \exp\left\{-c\sqrt{E}\right\} \right)^{-\gamma}. \]  

(1.37)

These approximate spectra are fit to nucleon-specific flux measurements, including two separate fits to describe the Helium spectra: one for data below 100 GeV/nucleon taken by magnetic spectrometers, and another for higher energy data taken by calorimeters of various kinds. The fit parameters for each nucleon in this model are provided in Table 1.2.
Figure 1.8: Probability for a muon flavor neutrino of energy $E_{\nu}$ to be in flavor state $\beta$ after traveling through the entire earth, $L \approx 12700$ km, under the two flavor approximation (top), three flavor vacuum approximation (center), and full three flavor with matter effects (bottom). The oscillation parameters used in this figure are from Table 1.1 and the matter densities are from the PREM [9]. The energy axis runs from 5 GeV to 250 GeV on a logarithmic scale, above which there are no further oscillations in any of these models. Muon neutrino disappearance analyses attempt to measure the shape of the dark blue curve. The enhancement of electron flavor neutrinos (light orange line) due to the earth’s matter is clearly visible below 10 GeV when comparing the lower two figures. Above 20 GeV, however, the differences between any of the three figures is minimal.
Table 1.2: Parameters for the five leading nucleon fluxes in the fit of Equation 1.37 as in [14].

<table>
<thead>
<tr>
<th>parameter/component</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\gamma$</th>
</tr>
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<tbody>
<tr>
<td>Hydrogen (A=1)</td>
<td>14800</td>
<td>±600</td>
<td>2.15</td>
<td>0.21</td>
</tr>
<tr>
<td>He (A=4, low)</td>
<td>750</td>
<td>±100</td>
<td>1.50</td>
<td>0.30</td>
</tr>
<tr>
<td>He (A=4, high)</td>
<td>600</td>
<td>±30</td>
<td>1.25</td>
<td>0.14</td>
</tr>
<tr>
<td>CNO (A=14)</td>
<td>33</td>
<td>±5</td>
<td>0.97</td>
<td>0.01</td>
</tr>
<tr>
<td>Mg-Si (A=25)</td>
<td>34</td>
<td>±6</td>
<td>2.14</td>
<td>0.01</td>
</tr>
<tr>
<td>Iron (A=56)</td>
<td>4.5</td>
<td>±0.5</td>
<td>3.07</td>
<td>0.41</td>
</tr>
</tbody>
</table>

### 1.4.2 Neutrino production in the upper atmosphere

As a nucleus enters the upper atmosphere, it interacts strongly in the atmosphere, producing a collection of relativistic hadrons, among other things. The lightest hadron is the pion, and it is produced most abundantly. Charged pions decay almost exclusively into muon neutrinos and muons, which subsequently decay into electrons and electron neutrinos as shown in Figure 1.9. As electron neutrinos are not produced in the initial decay of pions, the yield of high energy electron neutrinos from a given cosmic ray interaction is significantly smaller than that of muon neutrinos. There is an additional decrease in the yield of high energy electron neutrinos due to the relatively long lifetime of the muon itself. If the muon reaches the earth before it decays, then it will interact and lose the majority of its energy before decaying.

In the most simple model [14], the flux of atmospheric neutrinos is a modification to the primary nucleon flux accounting for the relative abundance of pions and kaons produced in the air shower, that is

$$
\Phi_{\nu}(E, \theta) = \Phi_N(E) \times (w_\pi + w_K)
$$

$$
w_\pi = \frac{A_\pi}{1 + B_{\pi\nu}E \cos \theta^*/\epsilon_\pi}
$$

$$
w_K = \frac{A_K}{1 + B_{K\nu}E \cos \theta^*/\epsilon_K},
$$

where the numerators $A_{\pi,K}$ are related to the production of pions and kaons, and the denominators $B_{\pi\nu,K\nu}$ correspond to the attenuation lengths and decay kinematics.
Here $\cos \theta^*$ provides the line-of-sight angle from production to detection and $\epsilon_{\pi,K}$ is the characteristic decay energy of mesons produced in the atmosphere. Because the mass of pions is so similar to that of muons, approximately 75% of the pion’s energy is transferred to the charged lepton, with the remaining energy given to the neutrino. Because of this, for neutrino energies above 100 GeV the atmospheric neutrinos are produced primarily through kaon decay, which divides its energy equally between the muon and neutrino.

Further corrections to the flux are applied to account for the geometry of neutrino production in the atmosphere to the location and physical extent of detectors on (or under) the surface of the earth. These corrections take into account geomagnetic effects as well as variations in atmospheric density, doing so through extensive computer simulation and neutrino tracking. The results of one such flux model [11] are shown in Figure 1.10. In this figure, we note that the muon neutrino flux extends to higher energies than the electron neutrino flux. This is due to the fact that the primary source of electron neutrinos is muon decay, and higher energy muons such as those produced by kaon decay are less likely to decay in flight before interacting with the dense matter of the earth’s crust. The electron neutrino angular flux is also much more peaked near the horizon where the atmosphere is thickest, which provides the muons more opportunity to decay.

Figure 1.9: Charged pions produced in the upper atmosphere will decay into muons which subsequently can decay into electrons. In each of these decay processes, a neutrino is created to conserve lepton number.
1.4.3 **Flux of atmospheric neutrinos at an underground detector**

The neutrinos produced in the atmosphere are exclusively muon and electron flavor. However, from their point of production in the upper atmosphere to their detection these neutrinos can travel between 20 km and 13000 km, depending on whether they were produced in the atmosphere above the detector or on the opposite side of the earth. As previously discussed, neutrinos undergo flavor oscillations during propagation. Given measured values for these oscillation parameters, the observable flux of neutrinos at a detector will be modified, sometimes extensively, by these oscillations.

In particular, at energies around 25 GeV muon neutrinos produced on the opposite side of the earth will be almost exclusively in the tau flavor state when they arrive at a detector. Due to the small interaction cross section of neutrinos relative to charged particles, there are still moderately large errors on the oscillation parameters $\theta_{23}$ and $\Delta m^2_{31}$, which are the dominant terms in atmospheric oscillations as shown in Table 1.1. The precision on these parameters is currently driven by dedicated neutrino beams that operate on a narrow energy band. Atmospheric neutrinos are measured over several decades of energy and the goal of this dissertation is to use them to improve upon the precision of the oscillation measurements.

Figure 1.10: Theoretical atmospheric neutrino fluxes at the South Pole from [11].
Chapter 2
The IceCube Neutrino Observatory

The IceCube Neutrino Observatory [16, 19] is an ice Cherenkov detector located at the South Pole. It consists primarily of 5160 light sensors, known as DOMs, distributed in a 3D array through a cubic kilometer of ice between 1.5 and 2.5 km below the glacial surface, which are used to detect neutrino interactions in the ice via Cherenkov emission from charged particles produced in the interaction. Included in the center of this array is a region of higher instrumentation density, referred to as DeepCore, which allows for the detection of neutrinos down to a few GeV [20]. This “in-ice array” is complemented by a surface array of light sensors frozen in ice-filled tanks called IceTop, which is used to detect cosmic ray air showers [21, 22]. These elements are shown in Figure 2.1. For this analysis, we consider only the DeepCore region, which comprises a cylinder of approximately 125 m radius and 350 m in height, to be our fiducial volume.

In the previous chapter, we discussed how neutrinos—being stable, colorless, electrically neutral particles—generally do not interact with matter. Before fully describing the IceCube detector, we will briefly discuss the ways in which other particles interact in the glacial ice at the South Pole in which the IceCube Neutrino Observatory is situated.
Figure 2.1: The IceCube Neutrino Observatory consists of an in-ice array and surface array; on-site data processing and storage take place in the IceCube Lab on the surface of the South Pole glacier. DeepCore is a sub-array in the bottom center of the in-ice array, which is more densely instrumented than the remainder of the detector.
2.1 Interactions of charged particles with matter

The strength of the strong interaction falls off extremely rapidly with distance; therefore, electromagnetic interactions account for all relevant processes after the initial weak interaction between the neutrino and nucleon. As the energy of a charged particle increases additional electromagnetic interaction mechanisms become available to it. At very low energies, the charged particle interacts with the Coulomb field of the medium as a whole. As its energy increases, these interactions begin to excite individual electrons, liberating them and ionizing the medium. Further increases to the energy of the charged particle allow it to resolve nuclei and electromagnetic interactions with these nucleons become dominant.

2.1.1 Cherenkov radiation

As a charged particle moves through a dense medium, it polarizes the local electromagnetic field. This polarization is mediated by photons, and thus its response is limited to the speed of light in that material provided by the group velocity of photons. If the charged particle moves faster than this, then the field’s response piles up resulting in coherent emission of photons, known as Cherenkov radiation, at an angle $\theta_c$ with respect to the charged particle such that

$$\cos \theta_c = \frac{1}{\beta n} = \frac{v_{\text{phase}}}{v_{\text{particle}}},$$

(2.1)

where $\beta$ is the speed of the particle relative to the speed of light in vacuum and $n$ is the index of refraction of the medium, which relates the phase velocity of light in the medium to the speed of light in vacuum and is in general dependent on wavelength. The constraint of $\cos \theta_c \leq 1$ implies that once the velocity of a charged particle dips below some $\beta_{\text{threshold}}$, Cherenkov light can no longer be produced. Equivalently,

$$E_{\text{threshold}} = mc^2 \frac{n}{\sqrt{n^2 - 1}},$$

(2.2)

where $mc^2$ is the rest mass of the particle. The threshold energies for a sample of particles are provided in Table 2.1. The number of photons emitted per particle per
Table 2.1: Cherenkov threshold energy for various particles commonly produced in neutrino interactions, calculated in ice \((n = 1.31)\) using Equation 2.2. As both the phase and group indices of refraction are weakly dependent on the wavelength of light, these thresholds are approximate. Units are MeV/c^2.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Rest mass</th>
<th>(E_{\text{threshold}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^\pm)</td>
<td>0.511</td>
<td>0.791</td>
</tr>
<tr>
<td>(\mu^\pm)</td>
<td>105.7</td>
<td>161</td>
</tr>
<tr>
<td>(\tau^\pm)</td>
<td>1776.8</td>
<td>2750</td>
</tr>
<tr>
<td>(\pi^\pm)</td>
<td>139.6</td>
<td>215</td>
</tr>
<tr>
<td>(K^\pm)</td>
<td>493.7</td>
<td>765</td>
</tr>
<tr>
<td>(p^\pm)</td>
<td>938.3</td>
<td>1450</td>
</tr>
</tbody>
</table>

unit wavelength \(\lambda\) and length \(x\) can be estimated using the Frank-Tamm formula

\[
\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha}{\lambda^2} \sin^2 \theta_c
\]

(2.3)

where \(\alpha\) is the fine structure constant \([1]\).

### 2.1.2 Muons in ice

Even at very low energies, muons are capable of exciting individual electrons from nearby atoms. This process has been studied in detail over several decades of particle energy as shown in Figure 2.2. The average energy loss per unit distance can be inverted and integrated to obtain the average energy as a function of muon length, which is shown in Figure 2.3. Given the DeepCore fiducial region mentioned at the start of this chapter and the criterion that events be fully contained in this fiducial volume as discussed in Chapter 4, this analysis is only sensitive to a narrow energy range of approximately 1 GeV to 100 GeV; below this range, muons do not produce enough light to be detected, while at higher energies, the muon is no longer contained. Throughout this energy range, in which the muons are considered Minimum Ionizing Particles (MIPs), the ionization rate is approximately constant with respect to energy. A muon will lose approximately 220 MeV/m due to ionization as it travels through ice, which is four orders of magnitude greater than the energy lost to the polarization which yields Cherenkov light. However,
Figure 2.2: The average rate of energy loss per meter by muons in water ice is shown as a function of the initial muon energy [1,2]. As the initial muon energy decreases toward the muon rest mass on the left side of the plot, the rate of energy loss increases dramatically. Between 0.3 GeV and 100 GeV, the muon is considered a minimum ionizing particle. At higher energies, the total rate of energy loss is dominated by stochastic interactions with nuclei such as bremsstrahlung and pair-production.

Due to their much lower rest mass, electrons produced in neutrino interactions cannot be considered MIPs. While they still lose some energy through ionization, the ionized electrons are absorbed by the detector media much more rapidly than optical Cherenkov photons. Therefore, only densely instrumented calorimetric detectors can observe this ionization; sparsely instrumented detectors, such as IceCube, can only effectively measure Cherenkov radiation. It is not until the muon energies exceed 700 GeV that bremsstrahlung and pair-production of electrons and positrons become the dominant mode of energy loss; however, muons of this energy cannot be contained in the small fiducial volume of DeepCore and are not relevant for neutrino oscillation studies.

2.1.3 Electromagnetic cascades

Due to their much lower rest mass, electrons produced in neutrino interactions cannot be considered MIPs. While they still lose some energy through ionization,
Figure 2.3: The average energy of a muon that travels a given length through ice. By inverting, integrating, and interpolating the total losses from Figure 2.2, we obtain the gray line shown. The light green band encloses the energies of muons that travel between 7 m and 350 m, which correspond to the DOM-to-DOM separation and the total spatial extent of our fiducial region respectively. In blue is the linear approximation used in the reconstruction described in Section 3.2.4.

The dominant interaction mechanism is bremsstrahlung. Through bremsstrahlung, an electron emits a high energy photon that, in turn, produces an electron-positron pair when it further interacts with matter. This defines one iteration of a cascading process that is repeated until the average particle reaches some critical energy $E_c$, below which ionization is the dominant energy loss mechanism.

The collection of electrons, positrons, and photons is referred to as an electromagnetic cascade. In these cascades, we will define the average distance over which the energy of an electron is reduced by a factor of $e$ as the radiation length $X_0$. This can be practically achieved by assuming that for every radiation length, one full iteration of the process occurs. In this model, each electron-positron pair produced in a given iteration equally shares the energy lost by their parent electron, and once a particle in the cascade reaches $E_c$, it is not included in subsequent generations. With these assumptions, the number of particles in a given cascade will reach its
maximum after traveling
\[ X_{\text{max}} = X_0 \ln \frac{E}{E_c} \]  
(2.4)

where \(E\) is the energy of the electron that initiates the cascade; for electrons in ice, the scaling factors are given by: \(X_0 \sim 40\,\text{cm}\) and \(E_c \sim 80\,\text{MeV}\). Because each particle above threshold will produce Cherenkov light, the luminosity of a cascade is directly proportional to the number of charged particles. Therefore, cascades initiated by electrons with energies relevant to this analysis (roughly 1–100 GeV) will reach their maximum brightness within a few meters of their initiation.

2.1.4 Hadronic cascades

In Section 1.2, we mentioned that the final state of DIS between neutrinos and nucleons contains a hadronic shower. As these hadrons propagate through the ice, they interact with the surrounding medium generating daughter hadrons and leptons resulting in a hadronic cascade. While the overall behavior of hadronic and electromagnetic cascades may be similar, hadronic cascades contain particles significantly heavier than electrons, which drop below the Cherenkov threshold much more rapidly. The net result of this is that hadronic cascades yield significantly fewer Cherenkov photons than electromagnetic cascades of the same initial energy, and possess much more variability on an event-by-event basis.

One of the hadrons that can be produced in these cascades is the neutral pion, which will decay into a pair of photons. These photons subsequently initiate their own electromagnetic cascades within the overall cascade. As the total energy of the cascade increases, so too does the number of neutral pions produced. Therefore, the Cherenkov light yield of hadronic cascades can be parametrized as a function \(F_{\text{em}}\), of the fraction of the total energy \(E\) that is transferred into the electromagnetic portion of the cascade,

\[ F_{\text{em}} = 1 - (E/E_0)^{-m}, \]  
(2.5)

where \(E_0\) and \(m\) are obtained through Monte Carlo studies [28,29]. Using this, we can compute the electromagnetic equivalent energy that would be required to produce the Cherenkov light from a hadronic cascade of energy \(E_{\text{had}}\),

\[ E_{\text{em}} = (F_{\text{em}} + f_0(1 - F_{\text{em}})) \times E_{\text{had}}, \]  
(2.6)
Table 2.2: Fit parameters for the hadronic cascade scaling factors in Equations 2.6 and 2.5 from [29].

<table>
<thead>
<tr>
<th>$E_0$</th>
<th>$m$</th>
<th>$f_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19 GeV</td>
<td>0.16</td>
<td>0.31</td>
</tr>
</tbody>
</table>

where $f_0$ is also obtained through Monte Carlo studies. Sample values for these hadronic scaling parameters are provided in Table 2.2

2.2 Ice at the South Pole

The ice of the Antarctic glacier was formed as layers of snow were very slowly compacted under their own weight. As a result, the glacial ice possesses a layered structure with an air bubble content that decreases as a function of depth due to the pressure exerted by the higher layers. At a depth exceeding 1500 m, these air bubbles transition from the gas phase to a solid air hydrate clathrate phase which is optically identical to the bulk ice; thereby light can pass between the ice and air hydrate crystals with almost no scattering [23]. Beyond this depth, both the scattering and absorptivity of the ice follow variations in the concentration of dust impurities, which track large scale changes in the global climate. These dust particles are typically micron-sized; as a result, Cherenkov photons are in the regime of Mie scattering, in which scattering has only a minor impact on the direction of the photon.

As photons propagate from their point of emission to a receiving sensor, they are affected by both scattering and absorption in the ice. These optical properties are studied using in-situ measurements acquired while the detector operated in “flasher mode”, in which LEDs mounted within a given DOM are turned on for a short period of time. The data from an ensemble of LED flashes are fitted to provide a global model of the scattering and absorption parameters for the instrumented volume of glacial ice [24]. The average distance traveled by a photon before it is absorbed is $1/a$, where $a$ is the absorption coefficient. The geometric scattering coefficient $b$ is used to define an effective scattering coefficient $b_e = b (1 - \langle \cos \theta \rangle)$, where $\theta$ represents the deflection angle at a given scatter point. The average distance between successive scatters is given by $1/b$, whereas $1/b_e$ is related to the
distance scale over which a given photon’s directional information is lost due to repeated scattering.

Figure 2.4: (a) Scattering and absorption coefficients from the South Pole ice (SPICE) model [24] at various depths for photons with \( \lambda = 405 \text{ nm} \). The large peak around 2000 m is referred to as the dust layer. (b) DOMs on the DeepCore strings were deployed in two groups, with a few DOMs above the dust layer to veto vertical muons and the remainder deployed in the clearest ice below the dust layer.
The depth dependence of these parameters is shown in Figure 2.4a with typical effective scattering and absorption lengths of 50 m and 200 m respectively in the deepest ice. These fits are validated using additional data from the “dust logger” device that ran during detector deployment [27]. In both the flasher and dust logger data, the region of depth between 1950-2100 m has significantly shorter scattering and absorption lengths. The reason for this is an unusually high accumulation of dust deposited 71,000 years ago during the last glacial period.

2.3 IceCube layout and design

IceCube consists of 78 vertical strings in a triangular lattice with characteristic spacing of 125 m. Each string contains 60 light sensors, vertically separated by 17 m. A region containing eight additional strings located within 100 m of the central string of IceCube is referred to as DeepCore, with string-to-string separations between 40-75 m, as shown in Figure 2.5. The additional DeepCore strings are also instrumented more densely, with 10 sensors above the dust layer and 50 sensors below the dust layer, as shown in Figure 2.4b.

![Figure 2.5](image)

Figure 2.5: The footprint of the in-ice array shows that the typical inter-string separation is 125 m, and highlights the DeepCore in-fill region which reduces that separation to 40-75 m.
These light sensors, called Digital Optical Modules (DOMs), consist of a 10-inch Hamamatsu R7081-02 photomultiplier tube [25] and associated electronics [26] enclosed in a pressurized glass sphere as shown in Figure 2.6a. These DOMs efficiently record Cherenkov light with wavelengths between 350-550 nm as shown in Figure 2.6b. In addition to having a denser spatial configuration, the DOMs on DeepCore strings also feature newer Hamamatsu R7081-MOD PMTs with a relative quantum efficiency 1.35 times greater than the standard IceCube PMTs. The DOMs digitize the recorded waveforms from their PMTs, generating a time-stamped “hit” when the signal rises over a threshold of about 0.25 photoelectrons (PEs). The signal is passed through both ATWD (Analog Transient Waveform Digitizer) and fADC (fast Analog to Digital Converter) chips which digitize the incoming analog waveform at a rate of 300 and 25 MHz respectively. The ATWD can record for 430 ns (128 samples) with a charge resolution of \( \sim 30\% \) for single PEs and a timing resolution of \( \sim 2 \) ns. The fADC system is designed to capture long, late pulses, recording for 6400 ns (256 samples). If the nearest, or next-to-nearest, DOM also

![Figure 2.6: The Digital Optical Module (DOM) shown schematically in (a) contains a 10" PMT and the associated digitizing hardware in a glass pressure vessel. In (b) the DOM’s sensitivity to Cherenkov radiation emitted by one meter of simulated bare muon track, i.e., with no bremsstrahlung or other stochastic emission, convolved with the module’s optical acceptance is shown. The integral under this curve is 2450 photons.](image)

29
records a hit within ±1000 ns, then the DOM is considered to be in “hard” local coincidence (HLC) and transmits the full digitized readout to the surface; otherwise, it only sends a brief digitized summary “soft” local coincidence (SLC). The digitized hits are collected and triggers are defined in electronics at the surface. After the trigger criteria have been satisfied, all isolated and non-isolated hits are recorded into an event. The trigger relevant for this analysis (SMT3) requires three HLC hits within a 2500 ns window inside the DeepCore volume.

2.4 DeepCore event topologies

Two broad classes of events are recorded in DeepCore, namely track-like and cascade-like events. We define an event to be track-like if it contains one Cherenkov-emitting particle that travels a distance of at least the string-to-string spacing. For the minimum ionizing muon described in Section 2.1.2, this corresponds to an energy of approximately 15 GeV. The Cherenkov light of muons below this energy will be engulfed in that of the hadronic cascade initiated at the interaction vertex. As the radiation lengths for both electromagnetic and hadronic cascades are significantly shorter than this string-to-string distance, under this definition, only neutrino interactions that produce muons are considered track-like; all others are described as cascade-like.

These two categories include within them all neutrino-induced events, as well as two distinct varieties of background events described in the following sections: noise and atmospheric muons. Before event selection is applied, these backgrounds describe the data reasonably well, as shown in Figure 2.7. Strict topological, veto, and event containment cuts eliminate the vast majority of atmospheric muons that trigger the IceCube detector as well as neutrino-induced events with energies above a few hundred GeV. A full description of event reconstruction is provided in Section 3.2 and the process of selecting neutrino-induced events is in Chapter 4.

2.4.1 Neutrino-induced events

As mentioned in Section 1.2, neutrinos which have enough energy to consistently trigger DeepCore will typically undergo deep inelastic scattering with the ice. In these interactions, the energy transferred to the quark is sufficient to overcome
its binding energy and multiple relativistic mesons are generated as the atomic nucleus attempts to return to its ground state. These liberated mesons will produce a hadronic cascade as described in Section 2.1.4. This cascade is visible until the average particle energy in the cascade drops below the Cherenkov limit provided in Equation 2.2. These particle cascades are typically ellipsoidal with the major axis extending from the initial interaction vertex to a point a few meters away. Through the eyes of the IceCube detector, these hadronic cascades are considered to be point-like particles due to the sparsity of instrumentation.

If the neutrino interacts through the charged current, then a charged lepton is produced at the interaction vertex in addition to the hadronic cascade initiated by the struck nucleon. Electrons will initiate their own electromagnetic cascades, which are indistinguishable from the accompanying hadronic cascade due to scattering and detector granularity. Muons of sufficient energy will escape the hadronic cascade and be visible as tracks. Tau leptons will decay within picoseconds of their creation and decay primarily into hadrons, which will initiate a secondary hadronic cascade.

Figure 2.7: After applying the in-situ trigger and filter, the data are described by the combination of atmospheric muons (blue) and dark noise (red). The noise model “Vuvuzela” shown here includes contributions from thermal, radioactive decay, and scintillation as described in Section 3.1.3. A 20% deficit between simulation and data is observed for the lowest number of DOMs hit, which is reduced by improving the fit to noise model [31].
Table 2.3: Possible experimental signatures of neutrino interactions in DeepCore. Dashed lines represent neutrinos, orange lines are muons, red lines are particles originated in a hadronic shower, and blue lines are particles originated in an electromagnetic shower.

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Secondary particles</th>
<th>Detector signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC $\nu_\mu$</td>
<td>$\mu$ track and hadronic cascade</td>
<td>Track with cascade</td>
</tr>
<tr>
<td>CC $\nu_\tau$</td>
<td>$\tau$ decays into $\mu$ ($\sim 17%$ b.r.)</td>
<td></td>
</tr>
<tr>
<td>$\tau$ decays into $e$ / hadrons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC $\nu_e$</td>
<td>Hadronic and EM cascades</td>
<td>Cascade</td>
</tr>
<tr>
<td>NC $\nu_\alpha$</td>
<td>Hadronic cascade</td>
<td></td>
</tr>
</tbody>
</table>

Their other decay modes produce an electron or muon with the requisite neutrinos to conserve flavor. The electrons will start an electromagnetic cascade, and the muons will likely not have sufficient energy to escape the envelope of the hadronic cascade. All of these neutrino-induced event topologies are summarized in Table 2.3.

2.4.2 Dark noise events

Even in total darkness, DOMs are known to record current. Two processes are known to contribute, a thermal term and radioactive decays in the PMT and pressure sphere glass [31]. Each of these mechanisms results from Poisson processes which are uncorrelated to one another with respect to time. However, radioactive decays can also induce a small amount of scintillation light in the glass, thereby causing a burst of light correlated in time to the initial decay. These correlated hits combined with the 1 $\mu$s local coincidence window enhance the number of SMT3 triggers due to dark noise. The result is a dark noise trigger rate on the order of 5 Hz, which is several orders of magnitude above the rate of neutrino events. The
rejection of these events is described in Chapter 4.

2.4.3 Atmospheric muons

The vast majority of events observed in IceCube involve muons produced in cosmic ray air showers penetrating the detector. The majority of these muons leave clean down-going tracks through the entire detector, and can be easily categorized. However, even unusual muons at the one in a million level would still trigger the detector at the same rate as the most common neutrino. Additionally, the geometry of the IceCube detector leaves wide highways down which a muon can travel, and the thick dust layer further reduces the ability of outer strings to observe atmospheric muons incident at specific angles. As the muons travel through the ice, they will lose energy, and so a substantial portion of these hard-to-reject muons will even decay within the DeepCore fiducial volume.

After a few levels of cuts, the remaining atmospheric muons look very similar to track-like neutrino events. If you trace the reconstructed “neutrino” path backwards from the vertex, the majority of these “sneaky” atmospheric muons will pass by the outer layers of the detector inside the dust layer and point directly toward the center of DeepCore. This directional information can therefore be incorporated into another level of cuts, and the final sample of atmospheric muons appears to consist of fully contained cascade-like events. This class of events was first described in the measurement of the atmospheric $\nu_e$ flux by IceCube [55]. The selection bias on these “sneaky” muons is so strong that they are best viewed as a separate class of events with properties distinct from more typical muons.

These remaining events triggered by atmospheric muons, but not appearing to be muon-like in any fashion, create an irreducible background for this analysis. They are so rare that it becomes computationally impossible to simulate enough events to generate a smooth template. In this analysis, we estimate the rate of these background events using a side band of detector data. This categorization relies on the number of DOMs that measure charge along any path from the core of the event to the edge of the IceCube detector in events which pass all other selection criteria. If there are zero or one DOM observing charge, then the event is considered to be neutrino-like; but if two or more DOMs observe charge, then the event is categorized as an atmospheric muon. Whether or not a given DOM
observes a single photoelectron from a distant passing track is a fundamentally random process, and so this is an unbiased metric.

### 2.4.4 Summary

All events are classified on a continuum from very cascade-like to very track-like. Those events triggered by dark noise, while cascade-like, are typically low in charge, have minimal directionality, and just barely satisfy the trigger timing criterion. At low analysis levels, atmospheric muons are clearly down-going tracks, but as these are progressively removed the remaining events become more and more cascade-like. The final level muons still have some information about their directionality, and typically point back through the dust layer to the edge of the IceCube volume. As the flux of muon neutrinos is almost an order of magnitude higher than that of electron neutrinos above 10 GeV, the vast majority of neutrino-induced events will be the result of muon neutrino interactions. The direction of these events will closely follow their parent neutrino flux, which is a factor of two greater at the horizon than in the vertical direction [11].

### 2.5 Previous DeepCore measurements

We acknowledge here a series of previous measurements which have motivated several of the decisions made in this analysis. In 2013, IceCube published their first statistically significant measurement of neutrino-induced cascades using the first year of DeepCore data [55]. These events were reconstructed to have energies between approximately 80 GeV and 6 TeV. Of the 1029 events observed in 281 days of data taking, $496 \pm 66\text{(stat)} \pm 88\text{(syst)}$ were estimated to be cascades caused by charged current electron neutrinos and neutral current neutrino interactions. These data are shown as the hollow black triangles in Figure 2.8.

In addition to measuring the energy spectrum of atmospheric electron neutrinos, IceCube has also published two previous analyses observing and then measuring muon neutrino disappearance with DeepCore. The first observation [56] used the same year of partial detector data as the cascade measurement, and used two separate data samples with different energy distributions to reject the null oscillation hypothesis at a significance greater than 5$\sigma$. The second measurement of muon
neutrino disappearance with IceCube used a “golden” sample of 5174 events from three years of complete detector data from May 2011 to April 2014. We compare the result of this three year measurement to other leading experimental measurements of muon neutrino disappearance in Figure 2.9.

The analysis presented in this dissertation uses the same three years of data as [57], but is built around an event reconstruction (described in Section 3.2.4) that has much higher acceptance of low energy and cascade-like events than the one used to obtain the golden sample. Based on the previous success of DeepCore to measure the flux of atmospheric electron neutrinos, we plan to keep these cascade-like events, in contrast to the exclusively track-like sample used for previous oscillation

![Figure 2.8](image_url)

Figure 2.8: The atmospheric neutrino flux measured by IceCube across multiple channels is shown with markers, and predictions from theoretical models [10][12] with lines. The open triangles show the \( \nu_e \) measurement with DeepCore [55], with the filled red triangles representing a higher energy follow-up [36]. The black circles and blue band come from the through-going upward \( \nu_\mu \) analyses [34,35].
measurements in IceCube. This cascade-like sample is populated by lower energy muon neutrinos and electron neutrinos, which experience minimal oscillations over baselines relevant for atmospheric neutrinos. Extensive work was also performed for this analysis to keep a full-sky sample, rather than the purely up-going sample used in the previous measurement. Including the combination of these down-going events and cascade-like events at all angles, increases our ability to constrain systematic uncertainties, especially those related to the neutrino flux.

As much work on event simulation and parametrization of systematic uncertainties was done for the previous IceCube oscillation measurement, we use the same analysis software and the same Monte Carlo simulation. Slight modifications to this analysis software were necessary to include the additional cascade channel and extend the zenith range.

Figure 2.9: The 90% confidence contours in the \( \sin^2 \theta_{23} - \Delta m_{32}^2 \) plane for IceCube [57] and three other experiments [58–60]. The log-likelihood profiles for the individual oscillation parameters are also shown (right and top) for the current IceCube result.
Chapter 3  
Event Simulation and Reconstruction

The previous chapter described the physical by-products of neutrino interactions within the IceCube detector. This chapter will focus on how those interactions are modeled and how we reconstruct the neutrino itself from detector data.

3.1 Simulation chain

The IceCube collaboration uses a large suite of software to simulate events in our detector. A brief overview of the main software projects is provided here.

3.1.1 Atmospheric muon and neutrino interaction

For atmospheric muon interactions, the CORSIKA [39] software package is used. This package simulates extensive air showers generated through interactions of various cosmic ray primary particle provided by models given by Hoerandel [40] and Beatty et al. [41]. These air showers include a substantial number of muons, which are propagated toward the IceCube detector.

Atmospheric neutrino interactions are handled by a combination of software packages. For energies between 1 GeV and 200 GeV, the range relevant for this analysis, GENIE [44] is used. Neutrino interactions produced by GENIE are weighted in accordance with the energy dependence of various cross sections and kinematic constraints. The particles produced by GENIE, such as hadrons and leptons, are passed to the GEANT4 [43] software package for propagation through
At higher energies, the number of particles produced in neutrino interactions increases significantly, and the event-to-event variations decrease. For these events, IceCube uses the I3NeutrinoGenerator module, which implements high-energy approximations to the neutrino cross section. This module is based upon the earlier ANIS generator [42], the output of which is a lepton and a single collinear hadronic cascade object. The higher energy aggregate particles generated by I3NeutrinoGenerator are propagated using the MMC (Muon Monte Carlo) tool [45].

3.1.2 Photon propagation and detection

As particles are propagated through ice, they will interact with the medium and emit photons. Each of these photons is then propagated through a specific ice model as described in Section 2.2. This photon propagation is handled using the CLSim [46] package, which utilizes OpenCL to run these processes in parallel. In addition to using a specific ice model, CLSim also is aware of the location of individual DOMs within the detector, and the final output of the propagation process is a list of photons that arrive at the surface of each DOM. These photons are then converted to “hits”, which are used by the later simulation steps.

3.1.3 Noise generation

In addition to observing photons produced by neutrinos and atmospheric muons whizzing through the IceCube detector, each DOM will also record some amount of dark noise. The physics behind this dark noise was described in Section 2.4.2. The Vuvuzela noise generator, described in [31], accounts for all of these physical processes and is used at this point to supplement the physics hits on each DOM in the detector.

3.1.4 DOM simulation

The combined series of physics and noise hits is then passed into the PMTResponseSimulator module, which converts these hits into photoelectron-pulses and generates pre-pulses, late pulses, and after-pulses for each existing hit with probabilities 0.7%, 3.5%, and 5.93% respectively. In pre-pulses, photoelectrons bypass the
first dynode producing a small early signal. After-pulses are generated when the residual gas inside the PMT is ionized, and these ions then drift into the dynode chain. The full ensemble of pulses is then passed to the DOMLauncher module, which is responsible for modeling the behavior of the discriminator, coincidence logic, and digitizer circuitry, thereby producing simulated I3DOMLaunches. These I3DOMLaunches contain the full digital waveform from the ATWD and fADC channels that is sent to the surface from the DOMs as in the real detector.

3.1.5 Triggering of data

At this point, simulated data and raw detector data are functionally identical; therefore, all further processing steps apply equally to both simulated and measured data. Once triggered, the IceCube detector records all I3DOMLaunches within 10 $\mu$s of the triggering launches. All triggers with overlapping time windows are merged, and the union of these trigger windows forms an event [20].

3.1.6 Pulse extraction

As data taken at the South Pole must be transmitted over satellite to be analyzed, the waveform information stored in the I3DOMLaunches must be compressed due to bandwidth limitations. This compression is done using the WaveDeform module, which uses a non-negative linear least squares algorithm to generate a collection of I3RecoPulses for each DOM. These I3RecoPulses are a discrete representation of the arrival time, total charge, and duration of each bunch of photoelectrons; however they also contain many sub-threshold pulses to replicate the fine-grained structure of the I3DOMLaunch.

3.2 Event reconstruction

The primary goal of the reconstruction process is to undo each step in the simulation process to accurately determine as much information as possible about the original neutrino or atmospheric muon that interacted in the detector.
3.2.1 Hit cleaning methods

The primary objective of hit cleaning is to identify clusters of I3RecoPulses which are causally connected; through this process, isolated hits caused by noise are excluded and events with multiple interactions can be separated.

The simplest of these cleanings utilizes a static time window around a given trigger at time $t_0$, which has the effect of reducing the trigger readout window. This is strongly motivated for low energy events in DeepCore, in which light from the average neutrino interaction is fully dissipated after a few microseconds. Therefore, the readout window is reduced from $[t_0 - 10\,\mu s, t_0 + 10\,\mu s]$ to $[t_0 - 5\,\mu s, t_0 + 4\,\mu s]$, which excludes a region dominated by dark noise.

Within this reduced time window, there will still be a non-negligible amount of hits due to dark noise. However, simply rejecting all hits that do not satisfy the HLC criteria described in Section 2.3 severely limits our ability to tag incoming “sneaky” muons—those muons which due to their direction and pattern of energy loss leave very few hits in the veto region. In order to reject isolated noise hits, but keep isolated “physics hits,” we employ the SeededRT method. This approach starts with a subset of DOMs containing hits that satisfy the HLC condition, and then checks for other hits that are within a radius $R = 150\,\text{m}$ and a time window $T = 1\,\mu s$ of any hit within the existing cluster. Any hits that satisfy this “RT” condition are included in the cluster, and the process is then repeated until no new hits are found.

3.2.1.1 Direct hit selection

In addition to looking for simple clusters of hits, we can also use the geometry of Cherenkov emission to gain additional hit cleaning power. The leading edge of the Cherenkov wavefront forms a cone with its apex at the current position of the relativistic particle. If we project this cone onto a single string of the detector, then it will form a hyperbola in depth as a function of time as shown in Figure 3.1. As detector noise will, on the whole, not form a similar hyperbola, we can use this template to identify events containing Cherenkov light that has minimally scattered. The bootstrapping method of identifying these minimally scattered, or direct, photons is described exhaustively in [30].
Recalculating the neutrinos in ice

threshold effects (e.g. modifying the absorption coefficient can reduce the number of direct photons expected so that events are missed). The analysis presented in this work is built around the idea of selecting these direct photons, and using an event only when enough of them are found.

Consider a Cherenkov emitter with infinite range crossing the DeepCore volume. The light wavefront expands in the shape of a cone, which eventually meets a string of DOMs. When the cone passes by a string it projects a conic section in space and time: a hyperbola. A sketch of this is shown in Fig. 6.4.

Figure 6.4: Formation of hyperbolic patterns. Top: diagrams depicting the differences in arrival time of photons to DOMs as a function of the orientation of the Cherenkov emitter (red). Dark yellow means early signals, bright yellow means late. Bottom: hyperbolas formed by the intersection of Cherenkov light with the detector’s strings for the two geometric configurations on top and a distance between [0, 50] m.

Assuming that the string is perfectly vertical the configuration has rotational symmetry. It is then possible to remove one dimension from the problem and describe the situation in the plane perpendicular to the vector connecting the cone’s axis and the string at the point where they are the closest. This point is referred to as the point of closest approach in short. Doing so allows uniquely defining the hyperbola by means of four quantities. A sketch that illustrates the definition of the quantities is shown in Fig. 6.5. The top view in panel (a) shows the point of closest approach, while the projection in the plane just described is contained in panel (b).

In [142] the ANTARES Collaboration published the idea of using hyperbolic patterns.

3.2.2 Simple vertex reconstructions

Many of the cuts described in the following chapter rely on knowledge of the interaction vertex, that is, the position and time at which the first Cherenkov light is emitted. The simplest vertex guess comes from the location and time of the first HLC hit in the cleaned pulse series of the event, $(\vec{x}_{1\text{st}}, t_{1\text{st}})$. 

Figure 3.1: Formation of Cherenkov hyperbolas [30]. Top: diagrams depicting the differences in arrival time of photons to DOMs as a function of the orientation of the Cherenkov emitter (red). Dark yellow means early signals, bright yellow means late. Bottom: hyperbolas formed by the projection of Cherenkov light with a vertical string for the two configurations on top with the emitter and the string between [0, 50] m.
3.2.3 Simple track reconstructions

The linefit reconstruction algorithm is run on data that has passed the SeededRT cleaning. It is an extremely simple and fast approach that ignores the details of Cherenkov emission and scattering, and does not account for any noise pulses that survive the cleaning process. For an ensemble of \( N \) pulses, let the position and time of the \( i \)th pulse be given by \( \vec{x}_i \) and \( t_i \) respectively. Then a reconstructed muon track having a velocity \( \vec{v} \) passing through the point \( x_0 \) at time \( t_0 \) can be found analytically with the solution to the least-squares optimization problem:

\[
\min_{t_0, \vec{x}_0, \vec{v}} \sum_{i=1}^{N} \rho_i(t_0, \vec{x}_0, \vec{v})^2
\]

where

\[
\rho_i(t_0, \vec{x}_0, \vec{v}) = \sqrt{|| \vec{v} \cdot (t_i - t_0) ||^2 - ||(\vec{x}_i - \vec{x}_0)||^2}
\]

The linefit result is improved using three additional pulse cleaning methods described in [50], which reduce the impact of scattering and noise.

A secondary track reconstruction, the Single-Photo-Electron Fit (SPE fit), includes terms accounting for basic scattering and absorption as well as the geometry of Cherenkov emission of photons from a muon. It uses an analytic function described in [51] to estimate the number of photoelectrons arriving at a DOM from a muon. This function is modified with the addition of a noise term and the inclusion of a Gaussian convolution to account for timing resolutions from PMT jitter due to the variability of photoelectron transit times through the dynode chain.

3.2.4 The hybrid reconstruction

The reconstruction method used in this analysis assumes that the light produced by particles created in the initial interaction can be modeled as a combination of an ionizing muon and an electromagnetic cascade, where this cascade is used to represent the hadrons produced in the original interaction. Due to the large string-to-string distances inherent to IceCube events, the geometric differences between electromagnetic and hadronic cascades are considered minor, whereas the differences in light yield are significant and included as an energy-dependent scaling factor applied a posteriori.
Table 3.1: Description of reconstruction parameters and boundaries used in the fit. Note that the vertex time is relative to the closest trigger time, \( t_0 \).

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex Time</td>
<td>( t \in [t_0 - 1000 \text{ ns}, t_0 + 500 \text{ ns}] )</td>
</tr>
<tr>
<td>Vertex Position X</td>
<td>( x \in [-155 \text{ m}, 245 \text{ m}] )</td>
</tr>
<tr>
<td>Vertex Position Y</td>
<td>( y \in [-235 \text{ m}, 165 \text{ m}] )</td>
</tr>
<tr>
<td>Vertex Position Z</td>
<td>( z \in [-550 \text{ m}, -150 \text{ m}] )</td>
</tr>
<tr>
<td>Zenith Direction</td>
<td>( \cos \theta \in [-1, 1] )</td>
</tr>
<tr>
<td>Azimuth Direction</td>
<td>( \varphi \in [0, 2\pi] )</td>
</tr>
<tr>
<td>Muon Length</td>
<td>( L_\mu \in [0, 1 \text{ km}] )</td>
</tr>
<tr>
<td>Cascade Energy</td>
<td>( E_c \in [0, 1 \text{ TeV}] )</td>
</tr>
</tbody>
</table>

The combination of the muon and cascade, which share a common starting point and propagate in the same direction, is called an event hypothesis. Given these assumptions, the entire event hypothesis can be summarized by 8 physical parameters, which are listed in Table 3.1. We also run the reconstruction in cascade-mode, in which the muon length is fixed to 0 m. By comparing the likelihood values of the full hybrid reconstruction to the cascade reconstruction, we can judge the importance of the muon track to the overall fit and partially separate events containing muons from hadronic or electromagnetic cascades. A third likelihood value is obtained for the so-called noise hypothesis, which is computed for each event under the assumption that \( E_c = 0 \text{ GeV} \) and \( L_\mu = 0 \text{ m} \).

In order to estimate the number of photons incident on a given DOM from a given source particle, we employ photonics tables. These tables are generated at a series of depths in our ice model, and at each depth, a given source is simulated at a series of zenith angles and a fixed energy. We assume azimuthal symmetry and a linear scaling of energy to light yield to keep the disk size and lookup times manageable. The ensemble of tables at all depths and zenith directions is then stacked together and a set of splines is fitted to provide the light amplitude and arrival time distributions for a source and observer DOM at any two given points in the given ice model. This process is described in detail in \[52,53\].

As these tables are produced by simulating photon propagation through the
ice, these tables naturally take into account scattering and absorption which vary greatly at different depths. A separate set of tables is produced for the cascades and ionizing muons as they have different Cherenkov profiles. Furthermore, the muon tables are produced using 15 meter muon track segments. These segments can be stitched together end-to-end to produce ionizing muons that are longer, or equivalently have higher energies.

### 3.2.5 The LogLikelihood function

In a given event, suppose a DOM observes a series of photoelectrons recording a total charge $q$. Let the amount of expected charge on that DOM from a given hypothesis be

$$\mu = \mu_{\text{track}} + \mu_{\text{cascade}} + \mu_{\text{noise}}$$  \hspace{1cm} (3.3)

where $\mu_{\text{track}}$ and $\mu_{\text{cascade}}$ are computed from the photonics tables previously described, and $\mu_{\text{noise}}$ is a small amount of charge calculated by multiplying the Poisson noise rate of this DOM by the duration of the event. Additional event information is contained in the arrival time of the charge, so the event can be divided into $N_t$ time bins such that

$$q = \sum_{i=0}^{N_t} q_i, \quad \mu = \sum_{i=0}^{N_t} \mu_i.$$  \hspace{1cm} (3.4)

The likelihood of a hypothesis that yields some expected charge $\mu_i$ in that time bin given an observed charge $q_i$, can be computed using Poisson statistics:

$$L_{\text{DOM},i} = L_{\text{DOM}}(\mu_i|q_i) = \frac{\mu_i^{q_i} e^{-\mu_i}}{q_i!} = \frac{q_i^{\mu_i} e^{-\mu_i}}{\Gamma(q_i + 1)}$$  \hspace{1cm} (3.5)

where the Gamma function allows for non-integer values of $q_i$, which are a byproduct of the pulse extraction process described previously. The total event likelihood is therefore computed by taking the joint likelihood over all time bins and DOMs

$$\mathcal{L} = \prod_{\text{DOMs}} \prod_{i=0}^{N_t} L_{\text{DOM}}(\mu_i|q_i)$$  \hspace{1cm} (3.6)

Taking the natural logarithm of the likelihood, we arrive at:

$$LLH = \ln(\mathcal{L}) = \sum_{\text{DOMs}} \left[ \sum_{i=0}^{N_t} [q_i \ln \mu_i - \mu_i - \ln \Gamma(q_i + 1)]_{\text{DOM}} \right]$$  \hspace{1cm} (3.7)
Figure 3.2: The LogLikelihood function for a given event is not smooth and is filled with small scale features. With all parameters not shown fixed to their true values we scan, from top to bottom, $x$, $y$, and $z$. The red star denotes the true value and negative LogLikelihood for the true hypothesis, smaller y-values are correspond to more likely estimates of a given parameter.
3.2.6 Finding the maximum likelihood

In order to find the physical parameters $\vec{\theta}$ that best reproduce the data, we want to maximize the $LLH(\vec{\theta})$ function, or equivalently minimize $-LLH(\vec{\theta})$. For very well behaved likelihood functions, an analytic solution can be found, but in practice one often needs to use numerical techniques. The majority of these techniques start with a single test point and therefore can be very sensitive to initial conditions and local maxima. As shown in Figure 3.2, the likelihood space of events in DeepCore is full of local features. As a result, we chose to use MultiNest \cite{47,49} in place of a standard minimizer.

3.2.6.1 The MultiNest algorithm

MultiNest is an implementation of the nested sampling algorithm, which was designed to efficiently compute the Bayesian evidence $Z$, which effectively integrates the likelihood space similar to Markov Chain Monte Carlo (MCMC). A distinct advantage over vanilla MCMC is that nested sampling uses a large number of starting points and over several iterations, the points cluster into progressively tighter regions of favorable likelihood. By allowing for multiple clusters, all local features in the likelihood space can be simultaneously explored and sequentially excluded until only a single cluster including the global maximum remains active. This ability to explore multiple modes simultaneously is illustrated in Figure 3.3.

To begin, $N_{\text{live}}$ trial hypotheses are drawn randomly according to the priors defined by the user. The likelihood of each hypothesis is then evaluated and then run through the X-means clustering algorithm. The resulting 8-dimensional ellipsoid approximates the iso-likelihood contour with all points inside assumed to have higher likelihood than those outside. The hypothesis with lowest likelihood is then discarded and a new trial hypothesis is drawn from inside the ellipsoidal volume, and is kept only if it has a higher likelihood than the previously rejected hypothesis. By iterating the clustering algorithm and forcing all new hypotheses to have better likelihood than the previous steps, the ensemble of hypotheses will converge to the global maximum. In order to avoid falling prey to local maxima, support for multiple clusters is naturally built in. Once every point inside a given cluster is found to have lower likelihood than another cluster it is excluded from future iterations. Furthermore, by recursively shrinking the allowed parameter space,
the sampling efficiency can be kept high. In practice, I have observed sampling efficiencies around 10% in our 8-dimensional highly degenerate space.

3.2.6.2 Implementation of MultiNest in IceCube

In addition to providing MultiNest with Bayesian priors for each physics parameter we want to fit, the algorithm itself has a series of internal configuration options that dictate its behavior. The option settings used by this analysis are provided in Table 3.2 for completeness. MultiNest is designed to explore the entire likelihood space and perform parameter inference through its Bayesian posterior distributions; however, the robustness of these estimates is constrained by the sampling density. As our likelihood evaluation depends directly on lengthy queries to look-up tables, the achievable sample density is severely limited with respect to the ideals outlined in literature.

In practice, time limits of 45 minutes/event and 5 minutes/event are imposed for the 8- and 7-dimensional reconstructions respectively. We have observed that around 5% of events hit this time limit. The convergence criterion within MultiNest is measured with respect to the change in the integrated volume of the allowed parameter space between iterations, so this failure to converge will inflate the estimated width of the posterior distributions of each parameter compared to the true width based on the LLH space. To reduce the impacts of undersampling and

Figure 3.3: Example of MultiNest’s ability to explore complicated likelihood spaces. (a) The two-dimensional likelihood function; (b) dots denoting the points with the lowest likelihood at successive iterations of the MultiNest algorithm. Different colors denote points assigned to different isolated modes as the algorithm progresses.
Table 3.2: Description and value of parameters passed to the MultiNest algorithm in my processing. A full description of these parameters is provided in [49].

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS</td>
<td>False</td>
<td>Sample with “Importance Nested Sampling”</td>
</tr>
<tr>
<td>mmodal</td>
<td>True</td>
<td>Allow mode separation</td>
</tr>
<tr>
<td>nlive</td>
<td>150</td>
<td>Number of live points</td>
</tr>
<tr>
<td>cef</td>
<td>False</td>
<td>Sample in “constant efficiency” mode</td>
</tr>
<tr>
<td>tol</td>
<td>1.1</td>
<td>Evidence tolerance factor (used for convergence)</td>
</tr>
<tr>
<td>efr</td>
<td>1.0</td>
<td>Sampling efficiency (scaling of ellipsoids)</td>
</tr>
<tr>
<td>ndims</td>
<td>8</td>
<td>Number of dimensions</td>
</tr>
<tr>
<td>nPar</td>
<td>8</td>
<td>Number of parameters (including bystanders)</td>
</tr>
<tr>
<td>nCdims</td>
<td>3</td>
<td>Minimum cluster dimensionality (used for splitting modes)</td>
</tr>
<tr>
<td>maxModes</td>
<td>25</td>
<td>Maximum number of modes during processing</td>
</tr>
<tr>
<td>pWrap[(i, \phi)]</td>
<td>True</td>
<td>Azimuth is a periodic parameter</td>
</tr>
</tbody>
</table>

lack of convergence, we use the maximum a posteriori estimate, that is the single event hypothesis which yielded the maximum likelihood value during our scan. To further mitigate these impacts, we reject any event which is not significantly more likely than the noise hypothesis, described previously.

One known cause of this undersampling is the large size of the priors relative to the number of live points used for evaluation. These prior bounds, shown earlier in Table 3.1, were chosen to be universal for all events contained in the DeepCore fiducial volume; however, typical events in this analysis are significantly smaller than that. Future work will establish dynamic event-by-event priors that enclose only as much detector volume as needed for a given event. These smaller priors will increase the sampling density achievable by the MultiNest algorithm, while keeping the number of live points and computation time manageable.

3.2.7 Reconstruction performance

In order to measure neutrino oscillations, as in Equation 1.36, the two most important parameters to reconstruct are the interacting neutrino’s path length through the earth and energy. This path length is geometrically represented to first order by the cosine of the neutrino’s zenith angle. Here we compare the results of
our hybrid reconstruction on simulated neutrino events in which we know the true values injected into the simulation.

These comparisons are performed on events that survive the complete event selection, which will be described in Chapter [4]. When making these comparisons, there are two metrics that must be considered. First we want to know if the reconstruction is unbiased, which is accomplished by looking at the median value of the ensemble distribution of the difference between each reconstructed value and its corresponding true value. The second metric, the median resolution, describes the typical size of these deviations, which here is considered to be the width of the central 50% of those ensemble distributions.

3.2.7.1 Zenith resolution

Our hybrid reconstruction is shown to be a nearly unbiased estimator of the true zenith angle across the relevant range of true neutrino energy for charged current $\nu_\mu$ and $\nu_e$ events in Figure [3.4]. As expected, the median resolution becomes smaller at higher energies and is better at all energies for $\nu_\mu$ events, which are more track-like than $\nu_e$ events.

A clear bias is visible in Figure [3.5] in which we look at the same events with respect to the cosine of the true zenith angle. This is not unexpected, as there are strong phase-space limitations near the vertical poles, and indeed, we see that events which are truly vertical are typically reconstructed as slightly more inclined angles. The median zenith resolution, however, is not impacted by the true zenith angle, which is shown by the consistent width of the vertical error bars across all injected angles.
Figure 3.4: The reconstructed zenith bias and median resolution are shown for 5 GeV slices of true neutrino energy. The position of the horizontal bar in each slice represents the bias, and the vertical error bars represent the median resolution containing the central 50% of the distribution. This is shown for both CC $\nu_\mu$ (solid black) and CC $\nu_e$ (dashed red) simulated events.

Figure 3.5: The reconstructed zenith bias and median resolution are shown for slices of the cosine of the true neutrino zenith angle. The position of the horizontal bar in each slice represents the bias, and the vertical error bars represent the median resolution containing the central 50% of the distribution. This is shown for both CC $\nu_\mu$ (solid black) and CC $\nu_e$ (dashed red) simulated events.
3.2.7.2 Energy resolution

Rather than looking at the raw difference between the reconstructed neutrino energy and the true neutrino energy from simulation, we instead consider the fractional energy resolution. By dividing this raw difference by the true neutrino energy, we measure the relative reconstruction performance across a wide energy range. When comparing the fractional energy resolution to the cosine of the true zenith angle, the distribution is independent of the true zenith angle as seen in Figure 3.6.

In Figure 3.7, we see that the energy is overestimated for all events below 10 GeV, and consistently underestimated for $\nu_\mu$ events above 30 GeV. The overestimation at low energies is likely due to the trigger threshold imposing a selection effect; most events at these energies will not trigger the detector, and those which do are the ones with higher-than-expected light detected. At higher energies, muons created in CC $\nu_\mu$ events will travel greater distances through the ice. Unfortunately, the photonics tables describing the expected light from a contained muon do not exist for the most up-to-date ice model used in the simulation. This mismatch between the ice model used in simulation and reconstruction can account for some

![Figure 3.6](image_url)

Figure 3.6: The reconstructed neutrino energy bias and median resolution is shown for slices of the cosine of the true neutrino zenith angle. The position of the horizontal bar in each slice represents the bias, and the vertical error bars represent the median resolution containing the central 50% of the distribution. This is shown for both CC $\nu_\mu$ (solid black) and CC $\nu_e$ (dashed red) simulated events.
of the bias at higher energies; other effects include the fixed value of $dE/dx$ and the increasing possibility of bremsstrahlung. Up to date photonics tables for cascades do exist, so there is much less bias on higher energy CC $\nu_e$ events, although the assumption of a purely hadronic cascade injects some bias in the reconstructed energy.

![Graph](image)

Figure 3.7: The reconstructed neutrino energy bias and median resolution are shown for 5 GeV slices of true neutrino energy. The position of the horizontal bar in each slice represents the bias, and the vertical error bars represent the median resolution containing the central 50% of the distribution. This is shown for both CC $\nu_\mu$ (solid black) and CC $\nu_e$ (dashed red) simulated events.
Chapter 4  
Event Selection

In order to identify neutrino interactions among the overwhelming background of atmospheric muons, an iterative process of cuts was applied to the data sample. These cuts fall into one of three categories: triggers, vetoes, or quality. Triggering cuts are designed to reject events caused by detector noise, and keep events caused by Cherenkov radiation from relativistic charged particles traveling through the detector. Veto cuts are designed to reject events caused by muons produced in cosmic ray interactions in the atmosphere, and to keep events caused by neutrinos. And quality cuts specify that we are only interested in neutrino-induced events that start and stop within our fiducial volume and can be effectively reconstructed.

4.1 Executive summary

In all of the upcoming summary tables, the final column is used to denote the relative order in which cuts are applied. Here I summarize the ordering of cuts and provide the number of events surviving each analysis level.

Level 1
The in-situ trigger is applied creating both physics and noise events.

Level 2
The online filter is applied rejecting 99% of physics events caused by atmospheric muons.

Level 3
Common cuts developed by the IceCube collaboration to reject noise events
and physics events caused by atmospheric muons.

Level 4

Straight cuts specific to this analysis using simple-to-calculate variables designed to further increase the purity of neutrino-induced physics events. Preliminary containment cuts are applied which reduce the overall neutrino rate by a factor of 2 due to volumetric effects.

Level 5

A multivariate cut to reject physics events caused by sneaky atmospheric muons. This reduces the overall rate of events to a level at which a computationally expensive reconstruction can be applied.

Level 6

Strict starting and stopping containment criteria are applied to the reconstructed events. This has the two-fold benefit of reducing the errors on reconstructed neutrino energy and direction, as well as preferentially rejecting atmospheric muon events. These cuts also separate the data sample into neutrino-like and atmospheric muon-like events as described in Section 4.3.6.

Level 7–ABC

Events are binned for the analysis; this includes classification by event topology into three samples described in Section 4.5 from the very cascade-like Sample A to the very track-like Sample C. Hard cuts on the reconstructed energy are also applied, keeping only the oscillation region with $\log_{10}(E_\nu/\text{GeV}) \in [0.75, 1.75]$.

The impact of these cuts is visible in the rate of events as shown in Table 4.1 and Figure 4.1. The distributions of reconstructed energy and the cosine of the reconstructed zenith direction using the hybrid reconstruction, described Section 3.2.4, are provided at Levels 3, 4, 5, and 6 in Figures 4.2, 4.3, 4.4, and 4.5 respectively. These plots show the impact of each collection of cuts on the two primary variables used in the analysis described in Chapter 5.
Table 4.1: Counts (in thousands) of observed and expected events in three years of data with statistical errors when relevant. Statistical errors in the observed data columns are computed using a 10% subset of total observed data. After all other cuts are applied, the data are divided into neutrino-like and atmospheric muon-like regions as described in Section 4.3.6. Simulation estimates are drawn from GENIE and CORSIKA respectively as described in Section 3.1.

<table>
<thead>
<tr>
<th>Level</th>
<th>Neutrino-like Data</th>
<th>Neutrino-like Simulation</th>
<th>Atmospheric muon-like Data</th>
<th>Atmospheric muon-like Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3</td>
<td>333</td>
<td>110000</td>
<td>113000</td>
<td></td>
</tr>
<tr>
<td>L4</td>
<td>119</td>
<td>3410</td>
<td>3850</td>
<td></td>
</tr>
<tr>
<td>L5</td>
<td>104</td>
<td>262 ± 3</td>
<td>196 ± 9</td>
<td></td>
</tr>
<tr>
<td>L6</td>
<td>67.6 ± 1.1</td>
<td>64.9 ± 0.1</td>
<td>10.2 ± 0.4</td>
<td>17.7 ± 0.3</td>
</tr>
<tr>
<td>L7-A</td>
<td>21.3 ± 0.6</td>
<td>20.8 ± 0.1</td>
<td>2.6 ± 0.3</td>
<td>5.5 ± 1.6</td>
</tr>
<tr>
<td>L7-B</td>
<td>19.6 ± 0.6</td>
<td>20.3 ± 0.1</td>
<td>3.3 ± 0.3</td>
<td>4.8 ± 1.4</td>
</tr>
<tr>
<td>L7-C</td>
<td>17.7 ± 0.6</td>
<td>16.2 ± 0.1</td>
<td>3.3 ± 0.3</td>
<td>7.4 ± 1.7</td>
</tr>
</tbody>
</table>
Figure 4.1: Graphical representation of Table 4.1 showing the reduction of atmospheric muon background events (red) compared to neutrino-induced events (shaded blue region). The data is divided at L6 into neutrino-like (dark gray) and atmospheric muon-like (light gray) sub-sets. The atmospheric muon-like data is predicted up to a factor of order unity and its normalization is allowed to float in the fit; in that regard, the observed differences between the estimated rates for the two background samples are reasonable.
Figure 4.2: Reconstructed energy (a) and direction (b) after the Level 3 cuts have been applied. The data (black points) is compared to the total expectation (hollow blue boxes) in each of the top figures. The ratio of data to this total expectation is shown in each lower subfigure. The expectation is composed of simulated neutrino-induced events (shaded blue region) and simulated atmospheric muon events (red crosses). Vertical lines with arrows indicate the region used in the final analysis. At this early analysis stage, the atmospheric muons dominate the expectation and their data points are frequently completely hidden by the total expectation. The large disagreement at low reconstructed energies is caused by under-simulation of pure noise events.
Figure 4.3: Reconstructed energy (a) and direction (b) after the Level 4 cuts have been applied. Colors and lines as in Figure 4.2. As the L4 cuts are designed to reject noise triggers, the discrepancy at small reconstructed energies has been effectively eliminated. Preliminary containment cuts have decreased the overall rate of neutrino-induced events by a factor of two.
Figure 4.4: Reconstructed energy (a) and direction (b) after the Level 5 cuts have been applied. Colors and lines as in Figure 4.2. The multivariate Level 5 cut removes the vast majority of down-going atmospheric muons from the right side of (b), with minimal impact to the total neutrino expectation. At this point, we have a neutrino-dominated sample throughout the up-going and horizontal regions, that is $\cos(\theta) \leq 0$. However, as the primary muon neutrino disappearance occurs around $\log_{10}(20 \text{ GeV}) = 1.3$, further background rejection will enhance our ability to constrain the atmospheric neutrino mixing parameters.
Figure 4.5: Reconstructed energy (a) and direction (b) after the Level 6 cuts have been applied. Colors and lines as in Figure 4.2. Here, the atmospheric muon estimate (red crosses) is provided using a side-band of data as described in Section 4.3.6 rather than simulation. A final set of containment cuts on the reconstructed starting and stopping positions of the event is primarily visible at high energies, having only minor impact near the expected oscillation maxima around $\log_{10}(20 \text{ GeV}) = 1.3$. Neutrino-induced events are now the dominant factor across all angles and energies. Statistically significant deviations between the data (black points) and the total expectation (blue boxes) are visible throughout the up-going region ($\cos(\theta) < 0$). This is not entirely unexpected, as the normalization and oscillation parameters implicit in the neutrino expectation have not been fitted to the data.
4.2 Triggering algorithms

The full set of trigger-type cuts applied in this analysis is listed in Table 4.2. We will focus the following section on describing the cuts and variables that are unique to this analysis, that is, those labeled as L4 in the final column of the table.

Table 4.2: Summary of trigger-type cuts

<table>
<thead>
<tr>
<th>Category</th>
<th>Test or variable</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-situ trigger</td>
<td>$N_{\text{HLC DOMs}}^{\text{fid}}$ in 2.5 $\mu$s</td>
<td>$\geq 3$</td>
</tr>
<tr>
<td>Event is large enough</td>
<td>$\max(\sum Q)$ in 300 ns</td>
<td>$&gt; 2$ PE</td>
</tr>
<tr>
<td></td>
<td>$\max(N_{\text{pulses}})$ in 300 ns</td>
<td>$&gt; 2$</td>
</tr>
<tr>
<td></td>
<td>$\max(\sum Q_{\text{clustered}})$ around trigger</td>
<td>$\geq 7$ PE</td>
</tr>
<tr>
<td></td>
<td>$N_{\text{DOMs}}$ in seeded R-T search</td>
<td>$\geq 8$ PE</td>
</tr>
<tr>
<td></td>
<td>$\sigma_z$</td>
<td>$\geq 7$ m</td>
</tr>
<tr>
<td>Event is physical</td>
<td>$\max(N_{\text{pulses}})$ with directionality</td>
<td>$\geq 3$</td>
</tr>
<tr>
<td></td>
<td>$N_{\text{DOMs}}$ with direct light</td>
<td>$\geq 3$</td>
</tr>
<tr>
<td></td>
<td>$(\Delta s)^2$</td>
<td>$\leq (0 \text{ m})^2$</td>
</tr>
</tbody>
</table>

4.2.1 In-situ trigger

As mentioned earlier, the most basic trigger is applied during the data-taking process. It requires only three HLC DOMs in the DeepCore fiducial volume during a 2.5 $\mu$s window. If the trigger condition is satisfied, then the detector readout is recorded for $t_0 \pm 10$ $\mu$s and is called an event. While the online trigger (SMT3) is extremely efficient at catching neutrino-induced events, it also captures a significant number of events in which there is only detector noise. As such, several other offline triggers must be applied to reject these noise events and keep only physics events.

4.2.2 Clustering triggers

One of the simplest methods to remove noise events uses the amount of charge deposited in a narrow time window around the trigger time, $[t_0 - 250 \text{ ns}, t_0 + 500 \text{ ns}]$. Using the assumption that detector noise is uncorrelated on short time-scales, we select the largest cluster of pulses using the SeededRT approach. This method
iteratively groups pulses within a given spherical radius and time of each other, starting from the “seeds” of the HLC pulses. Here we used the values of $R = 150 \, \text{m}$ and $T = 1000 \, \text{ns}$. The largest cluster must contain 7 PEs, as shown in Figure 4.6.

A slightly more generic version of this cut passes a time-window of 300\,ns through the entire event. The window observing the maximum charge is returned, and we require that this window contain at least two pulses, and 2\,PE of charge.

A further method to remove events caused by detector noise takes advantage of the geometry of Cherenkov light emission. For this analysis, we require that at least three DOMs in our detector record direct light from a single Cherenkov source as described in Section 3.2.1.1. This is shown in Figure 4.7.

Events are also examined for hints of directionality by using pairs of pulses with what is referred to within IceCube as the “NoiseEngine” algorithm. Allowing each recorded pulse as a reference, each other pulse is placed in one of 48 directional bins covering $4\pi$ steradians. In each of these spatial bins, pairs of pulses with an apparent velocity between $(0.1, 1.0) \, \text{m/ns}$ are flagged as physics candidates. If no spatial bin with respect to any reference pulse contains three or more candidate pairs, then the event is rejected.

The final collection of triggering methods relies on how the event evolves as a function of space and time. For each pulse on a DOM in the detector, we know $(x, y, z, t, q)$, that is the position of the DOM, the time the photoelectrons started to arrive, and number of photoelectrons recorded. We can therefore sort all the pulses in a given event by their starting times, and divide this sorted list into four quartiles, such that each quartile has equal charge. The average position and time of each quartile is computed using the observed charge, $q$, to weight each pulse. For example,

$$x_{Qi} = \frac{\sum_{j=N_{i-1}}^{N_i} q_j x_j}{\sum_{j=N_{i-1}}^{N_i} q_j} \quad \text{with} \quad N_0 = 0 \quad (4.1)$$

for the $i^{th}$ quartile containing $N_i - N_{i-1}$ pulses. Within IceCube, these average positions are frequently referred to as the center of gravity, or COG, of a given event or sub-event. The progression of these average positions for each quartile is used to describe the extent of the event, and we use both the separation

$$\Delta r = \sqrt{(x_{Q4} - x_{Q1})^2 + (y_{Q4} - y_{Q1})^2 + (z_{Q4} - z_{Q1})^2}, \quad (4.2)$$
and the space-time invariant interval

$$(\Delta s)^2 = (\Delta r)^2 - (ct_{Q4} - ct_{Q1})^2$$  (4.3)

for various triggering and veto cuts as indicated in Tables 4.2, 4.3, and 4.4. In a similar fashion, we use the undivided set of pulses to calculate both the charge-weighted average position and standard deviation in each dimension ($\hat{x}, \hat{y}, \hat{z}, \hat{t}$), and $(\sigma_x, \sigma_y, \sigma_z, \sigma_t)$ respectively. These variables are used to remove noise events by requiring:

- At least 8 DOMs must observe charge after applying a seeded R-T cleaning to remove isolated noise hits, shown in Figure 4.8.
- The event should not be dominated by a single DOM, i.e. $\sigma_z \geq 7 \text{ m}$, which is the DOM-DOM separation, shown in Figure 4.9.
- The space-time interval between the $Q_1$ and $Q_4$ should be time-like, i.e. $(\Delta s)^2 \leq (0 \text{ m})^2$, shown in Figure 4.10.
Figure 4.6: Charge of the largest cluster of pulses within [-250 ns, 500 ns] of the trigger time. The total neutrino expectation (shaded blue) combines with the estimated atmospheric muon background (red) to form the total expectation (hollow blue boxes). Observed data (black) is shown, along with a ratio of the (observed data)/(total expectation) in the lower subfigure. The peak at low Q is due to noise events, including a few events from the neutrino simulations where noise promotes the event above the trigger threshold.
Figure 4.7: Number of DOMs observing direct Cherenkov light as described in Section 3.2.1.1. Colors and lines as in Figure 4.6. In this figure, the total expectation completely overlays the atmospheric muon background, so the red lines are not visible. No events are visible below the cut because this cut had already been applied to all events before the data were plotted.
Figure 4.8: The number of DOMs that observe charge in a single spatial cluster. Colors and lines as in Figure 4.6. A trigger-type cut is placed at 8 DOMs, requiring that the event is sufficiently large to aid reconstructions. The data/expectation ratio shown at bottom has very poor agreement at this phase of analysis. This disagreement is a result of the atmospheric muon simulation using an old version of our noise model; this disagreement is reduced at higher cut levels after noise triggers have been removed, as shown in Figure 4.26.
Figure 4.9: The standard deviation of the average depth of pulses in an event. Colors and lines as in Figure 4.6. A trigger-type cut is placed at 7 m, requiring that the charge is distributed between several DOMs. An additional veto-type cut is placed at 100 m, where the expectation from neutrino-induced events is falling off much more rapidly than that of atmospheric muons.
Figure 4.10: The space-time interval \((\Delta s)^2\) between the average positions of the first and fourth quartiles of the event computed with Equation 4.3. Colors and lines as in Figure 4.6. Positive values correspond to space-like events; that is, events in which the pattern of charge deposition cannot be causally connected to a single particle. Events with positive values are therefore likely caused by uncorrelated noise and rejected with a trigger-type cut. An additional veto-type cut is placed at \(-(400 \text{ m})^2\), where the expectation from neutrino-induced events is falling off much more rapidly than that of atmospheric muons.
4.3 Veto algorithms

The full ensemble of veto-type cuts is provided in Table 4.3 with the final column listing the relative order in which these cuts are applied. In the following section, we describe these cuts focusing on those that are unique to this analysis.

Table 4.3: Summary of veto-type cuts. Here $Q^{V(F)}$ refers to charge in the online veto (fiducial) region described in Section 4.3.1 and $Q^{EV}$ refers to charge in the extended veto region described in Section 4.3.2.

<table>
<thead>
<tr>
<th>Category</th>
<th>Test or variable</th>
<th>Requirement</th>
<th>L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-situ filter</td>
<td>$N^V_{HLC}$ in online filter window</td>
<td>$\leq 1$</td>
<td>L2</td>
</tr>
<tr>
<td>Charge deposition</td>
<td>$\sum(Q^V)$ in online filter window</td>
<td>$\leq 7$ PE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum(Q^V)$ in 2 $\mu$s window before trigger</td>
<td>$&lt; 12$ PE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum(Q^V)/\sum(Q^F)$</td>
<td>$\leq 1.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum(Q)$ in first 600 ns / $\sum(Q)$</td>
<td>$&gt; 0.4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum(Q^{EV})$ in online filter window</td>
<td>$\leq 5$ PE</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>$\sum(Q)$ in causal veto region</td>
<td>$\leq 7$ PE</td>
<td>L4</td>
</tr>
<tr>
<td>Topological</td>
<td>$\sigma_z$</td>
<td>$\leq 100$ m</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>$\sigma_t$</td>
<td>$\leq 1$ $\mu$s</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>$(\Delta s)^2$</td>
<td>$\geq -(400$ m$)^2$</td>
<td>L4</td>
</tr>
<tr>
<td>Other</td>
<td>Multivariate BDT score</td>
<td>$\geq 0.1$</td>
<td>L5</td>
</tr>
<tr>
<td></td>
<td>$N_{DOMs}$ with from blind search</td>
<td>$\leq 1$</td>
<td>L6</td>
</tr>
</tbody>
</table>

4.3.1 Online filter

The online filter calculates the average vertex position $\hat{r}$ and time $\hat{t}$ of the pulses in the fiducial region as defined in [20]. Then the speed of a hypothetical particle is calculated between each pulse in the veto region and this fiducial average as

$$v_i = \frac{r_i - \hat{r}}{(t_i - \hat{t})}$$

(4.4)

for a pulse starting at time $t_i$ on a DOM at position $r_i$.

This speed is defined to be positive if the hit occurred before $\hat{t}$ and negative if it appeared afterward. Causally related hits in the veto region are generally expected
to have a speed close to that of the incoming atmospheric muon, which is very close to the speed of light in vacuum, \( c \approx 0.3 \text{ m/ns} \). A pulse is marked as causal if this speed from Equation 4.4 falls within a narrow window with respect to the speed of light,

\[
\frac{5}{6} c \leq v_i \leq \frac{8}{6} c.
\]

(4.5)

Smaller speeds occur for hits caused by light which has scattered and thus arrive late at the DOM in the veto region. Speeds larger in magnitude than the speed of light are in principle acausal, but since the vertex time \( \hat{t} \) represents the average time of the event in the fiducial region, the causal window must be extended significantly. The specific values chosen for Equation 4.5 were chosen from a simulation study, as

![Figure 4.11](image.png)

Figure 4.11: Event-by-event probability of observing a given veto hit speed as calculated in Equation 4.4 for simulated atmospheric muons (black) and atmospheric neutrinos (red) from [31]. A particle entering the DeepCore fiducial volume will most likely leave some hits in the veto region with apparent speeds close to the speed of light in vacuum (dashed blue line). The solid blue box denotes the veto window defined in Equation 4.5. Extending the window to higher speeds does not increase the power of the veto significantly because the vast majority of atmospheric muon events are already removed with the existing window, while the chance of accidentally vetoing neutrino events due to noise hits increases.
shown in Figure 4.11. During online processing, an event is rejected if more than 1 DOM have an HLC pulse within this window. Further cuts are applied offline limiting the total charge in this window to no more than 7 PE, as well as requiring the ratio of charge in the veto volume to that in the fiducial volume be less than 1.5.

4.3.2 Offline filter

In this analysis, we define the fiducial volume as the bottom portion of all strings within a cylindrical radius $\rho \leq 150$ m of string 36. The online filter, described in the previous section, uses a larger fiducial volume enclosing all strings except the three outer layers. This extended online fiducial volume is sparsely instrumented when compared to the central region, and its inclusion degrades the accuracy of our reconstructions. We therefore run the online filter algorithm a second time, with this reduced fiducial volume and enlarged veto region. An event is rejected by this offline filter if more than 5 PE are observed within the veto window defined in Equation 4.5, including all pulses not in our reduced fiducial volume. This veto charge is shown in Figure 4.12.

4.3.3 Topological cuts

In addition to dedicated veto algorithms, low energy neutrinos and high energy atmospheric muons from cosmic ray air showers have very different event lengths and charge deposition patterns. A few variables described in the triggering section are also used as here, namely $\sigma_z$, $(\Delta s)^2$, and $\sigma_t$, which are shown in Figures 4.9, 4.10, and 4.13 respectively.

Furthermore, neutrinos in our energy range always include a cascade at the point of interaction. These cascades will deposit a relatively large portion of their total energy in the first few hundred nanoseconds of the event, while an atmospheric muon will deposit its energy over much longer timescales. We can therefore use the ratio of charge observed shortly after the trigger to the total amount of charge in the event to reject atmospheric muons. We use three versions of this charge ratio as in [33].
Figure 4.12: Total charge observed in the extended offline veto region. Colors and lines as in Figure 4.6. Disagreement between the atmospheric muon simulation (red crosses) and the observed data (black points) is significant above charges of 10 PE, but this region is rejected due to the presence of a clear incoming atmospheric muon track.
Figure 4.13: The standard deviation of the average time of pulses in an event. Colors and lines as in Figure 4.6. A relativistic particle will travel through the fiducial volume in approximately 1.5 $\mu$s. If charge is continuously deposited through this transit time, then the standard deviation in time for a through-going particle should be approximately 1 $\mu$s. Therefore, any event with a larger value for $\sigma_t$ is not fully contained in our fiducial volume and is rejected. Again, we see that the disagreement between the data (black) and the simulated atmospheric muons (red crosses) is largely in the region rejected by this cut.
\[ QR6 = \sum_{t_0}^{t_0+600\text{ns}} (q_i) / \sum Q \]  
\[ QR3 = \sum_{t_0}^{t_0+300\text{ns}} (q_i) / \sum Q \]  
\[ C2QR3 = \sum_{t_2}^{t_2+300\text{ns}} (q_i) / \sum Q, \]  

where \( t_0 \) is the time of the trigger, and \( t_2 \) is the time of the second pulse after the trigger. As the trigger requires three pulses within a time window, there is a non-zero chance that the first and second pulse will be caused by noise before the neutrino interaction; the third charge ratio provided by Equation 4.8 is robust with regard to this case.

### 4.3.4 Causal track veto

A complementary approach to the online filter algorithm uses the pulse closest in time to the trigger as a reference, rather than the average fiducial position as in [32]. We iterate through every other pulse in the event and check if it exists in a region of phase space defined by

\[ \Delta r/c < 2.5 \mu s \]  
\[ \Delta r/c < -\frac{2}{3} \Delta t + \frac{1}{3} \mu s \]  
\[ \Delta t - 0.15 \mu s < \Delta r/c < \Delta t + 1.85 \mu s, \]

where \( \Delta r \) and \( \Delta t \) are the distance and time interval to the reference pulse. This is illustrated as the red region in Figure 4.14. Its width accounts for a reasonable amount of scattering in the ice combined with the typical time scale of a GeV-scale neutrino event in IceCube (1 \( \mu \)s). In order to keep events in which the trigger marks the true start of the event, only hits that cannot be caused by an outgoing particle are tagged as veto; this criterion is Equation 4.10. As random noise can also populate the veto region, we reject an event only if the total charge of all the veto hits is above a threshold of 7PE as shown in Figure 4.15.
Figure 4.14: The two shaded regions show where hits that are causally correlated with the given trigger, which occurs at $\Delta r/c = 0 = \Delta t$ can be found. The blue region contains hits that are created by an outgoing particle, and the red region contains hits from an incoming particle and is defined by the inequalities of Equations 4.9, 4.10, 4.11.
Figure 4.15: The total causal charge found in the red region of Figure 4.14. Colors and lines as in Figure 4.6. As before, the largest discrepancies between data (black) and simulated atmospheric muons (red, here completely hidden by the total prediction in blue lines) exist in the region rejected by this cut.
4.3.5 Multivariate veto

The previous cuts were all designed to reject as many atmospheric muons as possible, while keeping the maximum number of neutrino-induced events. Even after the cuts, some of these variables still have shape differences between the background and signal samples. Fortunately, machine learning algorithms are able to combine small differences over a large collection of variables to create a classification that can robustly separate two samples. A Boosted Decision Tree (BDT) was trained using the TMVA package \[61\] with eleven variables detailed in Table 4.4. The linefit and SPE reconstructions were described in Section 3.2.3. Distributions of the BDT input variables are shown in Figures 4.18–4.28. The configuration settings in Table 4.5 are included for completeness.

For a multivariate cut to be more effective than straight cuts, the input variables should be correlated, and the correlation coefficients should be different for the background and signal samples. Both of these criteria are met, as shown in Figure 4.16. The output metric of the BDT, shown in Figure 4.17, is a score between $-1$ (background-like) and $+1$ (neutrino-like). A cut keeping only events with a score greater than $+0.1$ was applied.

![Correlation Matrix (background)](image1)

![Correlation Matrix (signal)](image2)

Figure 4.16: Correlation matrices from the BDT training for (a) atmospheric muon background events from CORSIKA simulation and (b) charged-current muon neutrino interactions from GENIE simulation. Differences between the two correlation matrices allow multivariate cuts to be more effective than sequential univariate straight cuts. The order of the rows of variables is identical to the ordering of variables listed in Table 4.4.
Table 4.4: Variables included in the multivariate BDT, the names and ordering of variables in this table match those shown in Figure 4.16. Distributions of each of these variables are shown over the following pages.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linefit_speed</td>
<td>4.18</td>
<td>Particle speed [m/ns] from a simple reconstruction</td>
</tr>
<tr>
<td>linefit_zenith</td>
<td>4.19</td>
<td>Zenith angle from a simple reconstruction</td>
</tr>
<tr>
<td>spe11_zenith</td>
<td>4.20</td>
<td>Best zenith angle from an 11-iteration likelihood reconstruction</td>
</tr>
<tr>
<td>C2QR3</td>
<td>4.21</td>
<td>Cleaned charge ratio from Equation 4.8</td>
</tr>
<tr>
<td>QR3</td>
<td>4.22</td>
<td>Charge ratio from Equation 4.7</td>
</tr>
<tr>
<td>z_spread</td>
<td>4.23</td>
<td>Standard deviation with respect to average depth of the event</td>
</tr>
<tr>
<td>separation</td>
<td>4.24</td>
<td>$\Delta r_{Q1-Q4}$ from Equation 4.2</td>
</tr>
<tr>
<td>total_charge</td>
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<td>$\sum (Q)$</td>
</tr>
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<td>num_hit_doms</td>
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<td>Number of DOMs in seeded R-T search</td>
</tr>
<tr>
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<td>$z_{Q1}$ from Equation 4.1</td>
</tr>
<tr>
<td>cog_q1_rho</td>
<td>4.28</td>
<td>$\rho_{Q1}$ from Equation 4.1</td>
</tr>
</tbody>
</table>

Table 4.5: Configuration settings used in training the L5 BDT. A full description of the options and settings is provided in the TMVA Users Manual [61]. These settings are included for completeness; only those which were modified are listed, in most cases the TMVA defaults were used.

<table>
<thead>
<tr>
<th>TMVA option name</th>
<th>Modified</th>
<th>Value</th>
</tr>
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<tbody>
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<td>False</td>
</tr>
<tr>
<td>H</td>
<td>Yes</td>
<td>False</td>
</tr>
<tr>
<td>NTrees</td>
<td>Yes</td>
<td>400</td>
</tr>
<tr>
<td>BoostType</td>
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</tr>
<tr>
<td>SeparationType</td>
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<td>“giniindex”</td>
</tr>
<tr>
<td>nEventsMin</td>
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<td>400</td>
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<tr>
<td>nCuts</td>
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<td>PruneMethod</td>
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<td>“nopruning”</td>
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<tr>
<td>MaxDepth</td>
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<td>3</td>
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</tbody>
</table>

TMVA Release 4.1.3 [262403]
ROOT Release 5.34/03 [336387]
AnalysisType Classification

78
Figure 4.17: The output metric of the Boosted Decision Tree is a “score” in the range $[-1, 1]$. Background events (red crosses) score lower than neutrino-induced signal events (shaded region). A subset of the data (black dots) is in good agreement with the total expectation (hollow blue boxes), which transitions smoothly from the background dominated region to the signal dominated region. The cut keeping only events with a score greater than 0.1 is shown as a vertical black line.
Figure 4.18: Speed in [m/ns] of the reconstructed particle using the linefit algorithm described in [50]. Very clear atmospheric muon tracks with no scattering will produce a speed close to $c = 0.3\, \text{m/ns}$. The distributions of data (black) and atmospheric muons (red), which are hidden by the total expectation (blue boxes), are peaked closer to $0.2\, \text{m/ns}$ as a result of scattered light. The neutrino prediction (shaded blue region) is peaked at much lower speeds because the typical energy of these neutrino events is lower than that of the atmospheric muons as shown in Figure 4.3, the result of which is that scattering will have a larger impact.
Figure 4.19: Zenith angle of the reconstructed particle using the linefit algorithm described in [50]. Both the data (black points) and the total expectation (hollow blue boxes), which is dominated by atmospheric muons (red crosses barely visible on the right of the plot) are strongly peaked near a value of 1 radian. The location and width of this peak correspond to muons that enter the detector and pass the outer layers of veto strings in the dust layer shown in Figure 2.4.
Figure 4.20: Zenith angle of the most likely of 11 iterations of the SPE likelihood reconstruction described in [51]. As in Figure 4.19, the location and width of the peak at 1 radian correspond to atmospheric muons that enter the detector and pass the outer layers of veto strings in the dust layer shown in Figure 2.4.
Figure 4.21: “Cleaned” charge ratio, from Equation 4.8. A value close to 1 occurs if nearly 100% of the charge is deposited in the 300 ns following the third pulse after the trigger. The third pulse is used because the trigger requires three pulses to occur within 1 µs, and here is a non-zero chance that at least one of the first two triggering pulses are caused by detector dark noise. Low energy neutrinos (shaded blue region) typically have a larger fraction of their total charge deposited in this 300 ns window than atmospheric muons (red crosses) that constitute the majority of data (black points) at this analysis level.
Figure 4.22: Charge ratio from Equation 4.7. A value close to 1 occurs if nearly 100% of the charge is deposited in the 300 ns following the trigger. In contrast to Figure 4.21 there are significantly more events with values close to zero, especially in the expectation from neutrino induced events (shaded blue region) this is due to dark noise contributing to the trigger.
Figure 4.23: The standard deviation of the average depth of pulses in an event, as in Figure 4.9. The straight veto-type cut at 100 m applied at L4 is intentionally soft, as the difference between neutrino expectation and the atmospheric muons and data is better handled by the BDT.
Figure 4.24: The spatial separation of the average positions of the first and fourth quartiles of charge, as described in Equation 4.2. While the distributions of neutrino events (shaded blue) and data (black points) are clearly different, application of a straight cut around 75 m would remove a large portion of the neutrino signal.
Figure 4.25: The total charge observed in the cleaned pulse series used for event reconstruction. Atmospheric muon events tend to be slightly brighter than those from neutrinos. Lines and colors as in Figure 4.17.
Figure 4.26: The number of DOMs that observe charge in a single spatial cluster, as in Figure 4.8. As mentioned previously, after noise events are rejected in the L4 cuts, the agreement between data and the total expectation has improved in shape.
Figure 4.27: The average depth of the first quarter of charge in the event, as in Figure 4.32. Even after the straight containment cuts (black vertical lines) are applied at L4, the atmospheric muons (red crosses) have many more events beginning at the top of the fiducial region than the total neutrino expectation (shaded blue).
Figure 4.28: The average radial position of the first quartile of charge in the event, as in Figure 4.34. Even after the straight containment cut (black vertical line) are applied at L4, the atmospheric muons (red crosses) have many more events beginning at the edge of the fiducial region than the total neutrino expectation (shaded blue).
4.3.6 Corridor cut

Thus far, all of the veto techniques have looked for various clusters of charge, either in the fiducial volume or in the veto region. The IceCube detector, unfortunately, was configured with a regular triangular lattice. This layout creates large highways down which dim atmospheric muons can travel largely without being observed as shown in Figure 4.29.

In order to tag these “sneaky” muons, we perform a brute-force search for isolated pulses along these blind directions. This method was used in a previous DeepCore oscillation analysis [57], and is described in full detail in [30] pp. 87ff. The number of DOMs hit in these corridors is used to tag events as neutrino-like or atmospheric muon-like as shown in Figure 4.30. These “sneaky” muons make up the bulk of the background remaining at the final level of the analysis. However, they are very difficult to simulate, so we use a tagged data sample in place of a Monte Carlo simulated sample in the fitting procedure described in Chapter 5.

Figure 4.29: The blue dot in the center of DeepCore marks the location of the first strong signal observed in a hypothetical event. Dashed arrows illustrate the pre-defined blind corridors for this location.
Figure 4.30: The distribution of the number of DOMs above threshold for blind directions is shown at L5, after the multivariate BDT cut. Colors and lines are as in Figure 4.6. The cut at 2 DOMs (black vertical line) divides the event sample into two regions, neutrino-like to the left of the cut, and muon-like to the right. The muon-like data is used to model the background in the final analysis.
4.4 Containment cuts

In addition to selecting higher quality neutrino candidates for reconstruction and analysis, these cuts also remove a large number of background atmospheric muons that interact toward the top and outside of the fiducial volume. As summarized in Table 4.6, the cuts are applied in two batches: once using very simple vertex estimates and a second time using the starting and stopping position of the reconstruction described in Section 3.2.4.

Table 4.6: Summary of containment cuts. Here the detector origin is considered to be at (46.29, -24.88, -350.) with respect to the nominal IceCube coordinates, which is on String 36 just above DOM 51.

<table>
<thead>
<tr>
<th>Object to be contained</th>
<th>Variable</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position of first HLC DOM</td>
<td>$z_{1st}$</td>
<td>$[-125\text{ m}, 150\text{ m}]$ L4</td>
</tr>
<tr>
<td></td>
<td>$\rho_{1st}$</td>
<td>$\leq 150\text{ m}$ L4</td>
</tr>
<tr>
<td>Average position of Q1</td>
<td>$z_{Q1}$</td>
<td>$[-125\text{ m}, 200\text{ m}]$ L4</td>
</tr>
<tr>
<td></td>
<td>$\rho_{Q1}$</td>
<td>$\leq 150\text{ m}$ L4</td>
</tr>
<tr>
<td>Reconstructed starting position</td>
<td>$z$</td>
<td>$\geq -125\text{ m}$ L6</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>$\leq 100\text{ m}$ L6</td>
</tr>
<tr>
<td></td>
<td>$r \cdot (z/</td>
<td>z</td>
</tr>
<tr>
<td>Reconstructed stopping position</td>
<td>$z_{stop}$</td>
<td>$[-150\text{ m}, 150\text{ m}]$ L6</td>
</tr>
<tr>
<td></td>
<td>$\rho_{stop}$</td>
<td>$\leq 150\text{ m}$ L6</td>
</tr>
</tbody>
</table>

4.4.1 Preliminary containment

We use two first guess vertex estimators to apply containment cuts at Level 4. Cuts are applied to the depth of the first HLC pulse, and on the average depth of the first charge quartile, shown in Figures 4.31 and 4.32 respectively. Cutting slightly harder on the depth of the first HLC pulse reduces the number of rejected low energy up-going neutrinos that start high in the fiducial volume. The radial cuts applied to each of these vertex estimates, shown in Figures 4.33 and 4.34, are applied 150 m from the central string of DeepCore, which encloses all of the strings inside the blue dashed region of Figure 4.29.
Figure 4.31: Depth of the first HLC DOM in the event. Atmospheric muons tend to be seen first near the top of the fiducial volume. Lines and colors as in Figure 4.6.
Figure 4.32: Depth of the average position of the first quarter of charge deposited in the event. Lines and colors as in Figure 4.6. The large pile-up of events at a depth of 0 m corresponds to events that contain fewer than four DOMs hit, and for which the event cannot be split into quartiles.
Figure 4.33: Cylindrical radius of the first HLC DOM in the event, with respect to the central string of DeepCore. The lack of events with radii between 150 m and 200 m is a result of no strings existing in that range. Lines and colors as in Figure 4.6.
Figure 4.34: Cylindrical radius of the average position of the first quartile of charge deposited in the event, with respect to the central string of DeepCore. By using the average position, the gap visible in the previous plot is reduced. Lines and colors as in Figure 4.6.
4.4.2 Final containment

After all previous cuts have been applied, the event rate is low enough that we can run the computationally expensive reconstruction described in the previous chapter. This reconstruction provides excellent estimates of the starting and stopping points of the event, and further containment can be applied to both improve the quality of the energy reconstruction and to reject atmospheric muons which entered the detector through the dust layer. The final set of containment cuts is shown schematically in Figure 4.35.

As with the previous containment cuts, the starting depth and cylindrical radius are restricted to a narrow volume around the central string of the detector as seen in Figures 4.36 and 4.37. However, rather than apply a cut such as $z \leq +125 \text{ m}$, we have instead applied a cut on the spherical radius in the upper hemisphere, as shown in Figure 4.38. This spherical cap was chosen after studying the reconstructed starting positions of the remaining simulated atmospheric muon events. Events are also required to stop within the fiducial volume to avoid energy biases from the 50 m gap between strings seen in Figure 4.33. These stopping containment cuts are applied on the depth and cylindrical radius, shown in Figures 4.39 and 4.40 respectively.

The events surviving these cuts agree well with expectations up to a small normalization factor consistent with rate uncertainties due to both the atmospheric neutrino normalization and the (unsimulated) possibility of a few neutrino events being accidentally vetoed due to temporally coincident atmospheric muons observed in the veto region.
Figure 4.35: Schematic view of containment cuts using the reconstructed starting and stopping points. Events must start within the red dashed region, which has a spherical cap chosen to exclude the upper edges of the fiducial volume where the majority of remaining atmospheric muons are reconstructed as starting.
Figure 4.36: The depth of the neutrino interaction reconstructed as described in Section 3.2.4. Here $z = 0 \text{ m}$ refers to the center of our fiducial volume as defined in Table 4.6. The data (black points) agrees with the total expectation (hollow boxes) up to a small normalization offset across all depths. We reject events, mostly neutrino-induced events (shaded blue region), that interact below our fiducial volume (vertical black line at -125 m). In the top half of our fiducial volume, $z > 0 \text{ m}$, this total expectation is dominated by atmospheric muons (red crosses), which are rejected with a subsequent cut on the spherical radius in Figure 4.38.
Figure 4.37: The cylindrical radius from the center of our fiducial volume of the neutrino interaction reconstructed as described in Section 3.2.4. Colors and lines as in Figure 4.36. The cut here is tighter than previous cylindrical cuts shown in Figures 4.33 and 4.34.
Figure 4.38: The spherical radius from the center of our fiducial volume of the neutrino interaction reconstructed as described in Section 3.2.4. Colors and lines as in Figure 4.36. To illustrate the position of the cut, we multiply the always-positive value of the radius by $+1 (-1)$ if the interaction occurs in the top (bottom) half of our fiducial volume. The cut at $+125\, \text{m}$ rejects the majority of the remaining atmospheric muon expectation (red crosses).
Figure 4.39: The depth of the stopping point of the muon reconstructed as described in Section 3.2.4. Colors and lines as in Figure 4.36. If the event is cascade-like, the muon length will be approximately zero, and the stopping point will be identical to the starting point shown in Figure 4.36. Cuts are applied to reject events that leave the densely instrumented fiducial volume to improve the energy resolution.
Figure 4.40: The cylindrical radius of the stopping point of the muon reconstructed as described in Section 3.2.4. Colors and lines as in Figure 4.36. If the event is cascade-like, the muon length will be approximately zero, and the stopping point will be identical to the starting point shown in Figure 4.37. Cuts are applied to reject events that leave the densely instrumented fiducial volume to improve the energy resolution.
4.5 Event classification

As mentioned previously, by fixing the length of the muon to 0 m, we can reconstruct events that are cascade-like. Therefore, we can run a simple likelihood ratio test to determine how cascade-like or track-like a given event is. This style of determination is referred to as Particle Identification, or PID, for historical reasons. Using a null hypothesis of pure cascade and an alternative hypothesis of cascade+track, we can construct a test statistic, shown in Figure 4.41 from the LogLikelihood difference:

\[ \Delta_{LLH} = (LLH_{\text{track}} - LLH_{\text{cascade}}) \]  

(4.12)

Wilks’ theorem states that the distribution of this test statistic under certain conditions will asymptotically converge to a \( \chi^2 \), that is

\[ \Delta_{LLH} \sim \frac{\chi^2(\Delta_{dof})}{2}, \]  

(4.13)

with the alternative hypothesis containing one more degree of freedom (the muon length) than the null hypothesis [1], Chapter 38. However, looking at the likelihood ratios on simulated data, it is clear that Wilks’ theorem does not apply. As shown in Figure 4.42, the majority of cascade-like events prefer the simpler 7-dimensional fit, i.e. \( \Delta_{LLH} < 0 \). Such values of \( \Delta_{LLH} \) should not occur, and are believed to arise from cases where the minimizer failed to converge to the true minimum in the 8D space. These events likely have more convergence issues with the additional degree of freedom, but can still be used in the analysis provided their values are not too negative. Additionally, as the 7-dimensional fit fixes the final parameter to one of its boundary values and the first and second derivatives of the likelihood function are not well behaved at this boundary, we should not expect Wilks’ theorem to hold explicitly.

For these reasons, although we can still use the likelihood ratio as a PID test statistic, we cannot assign p-values from the \( \chi^2 \) distribution. Event classification was performed empirically using simulation dividing the sample into three regions of approximately equal statistics, which we denote by A, B, and C as listed in Table 4.7 and shown in Figure 4.42. By using this partitioning, the sample of up-going events from the most track-like region (C) contains a similar subset of
Table 4.7: Composition of the event classification regions divided by true interaction type. Oscillation parameters are taken from a recent global fit \cite{6}. The contribution from atmospheric muons is estimated from data as described in Section 4.3.6. We see that moving from Sample A to C, the relative number of CC $\nu_e$ decreases as the relative abundance of CC $\nu_\mu$ increases.

<table>
<thead>
<tr>
<th>True Interaction</th>
<th>$\Delta_{LLH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A $\in [-3, 0)$</td>
</tr>
<tr>
<td>CC $\nu_\mu$</td>
<td>50%</td>
</tr>
<tr>
<td>CC $\nu_e$</td>
<td>31%</td>
</tr>
<tr>
<td>CC $\nu_\tau$</td>
<td>5%</td>
</tr>
<tr>
<td>NC $\nu$</td>
<td>4%</td>
</tr>
<tr>
<td>Atmospheric $\mu$</td>
<td>10%</td>
</tr>
</tbody>
</table>

This division into separate PID samples should improve the result both because neutrino backgrounds to the CC $\nu_\mu$ sample are primarily categorized as cascade-like and because higher values of $\Delta_{LLH}$ are associated with better track reconstructions and therefore improved resolutions.
Figure 4.41: The tail distribution, i.e. $1 - CDF$, of the test statistic $\Delta_{LLH}$ is shown for several classifications of events after the common Level3 cuts have been applied. Each of these event classifications is shown using a different color, with: simulated atmospheric muons (red), 1% sample of data (black), simulated CC $\nu_\mu$ (green), simulated CC $\nu_\tau$ (magenta), simulated CC $\nu_e$ (blue). The dashed red horizontal line corresponds to 10% of the initial sample, and a vertical red line is drawn where the CC $\nu_e$ distribution crosses below this threshold. This vertical line represents a potential cut value that would reject 90% of the cascade-like events. A dashed black line is drawn where this potential cut crosses the CC $\nu_\mu$ distribution, and it is seen that 40% of those events would survive this theoretical cut.
Figure 4.42: The track/cascade likelihood ratio used as a PID statistic. A value of $\Delta_{LLH} = 0$ is obtained if the best likelihood value obtained in the track-like fit is equal to that obtained in the cascade-like fit. Events are divided into three samples as described in Table 4.7, with Sample C being the most track-like. Colors and lines as in Figure 4.36.
Chapter 5  
Analysis Method

We perform a binned likelihood analysis, dividing the data along three dimensions relevant for atmospheric neutrino oscillations as outlined in Table 5.1. The number of observed events in a given bin is denoted by $x_{ijk}$ for bin $i$ in energy, $j$ in zenith, and $k$ in PID, and is compared to the expected number of events in that bin, $n_{ijk}$, such that

$$n_{ijk} = n_{\mu,ijk} + n_{\nu,ijk}$$

(5.1)

letting $n_{\mu,ijk}$ be the number of atmospheric muons in a given bin derived from a side band of the data as described in Section 4.3.6 up to a normalization term, and $n_{\nu,ijk}$ be the number of neutrinos in the data sample. The baseline expectation for each bin is shown in Figure 5.1. These baseline templates are modified by various systematic effects as well as the values of neutrino oscillation parameters, as we describe in the following section.

Table 5.1: The data are binned according to three observables which are representative of the variables responsible for neutrino oscillations as described in Section 1.3. The flavor of the interacting neutrino is approximated by the PID variable, $\Delta_{LLH}$, which is separated into three regions such that $A$ is the most cascade-like, and $C$ is the most track-like as shown in Table 4.7.

<table>
<thead>
<tr>
<th>Oscillation Variable</th>
<th>Observable</th>
<th># bins</th>
<th>Range</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutrino Energy</td>
<td>$\log_{10}(E_{\nu}/\text{GeV})$</td>
<td>8</td>
<td>[0.75, 1.75]</td>
<td>$i$</td>
</tr>
<tr>
<td>Neutrino Path Length</td>
<td>$\cos(\theta_{\nu})$</td>
<td>16</td>
<td>$[-1, 1]$</td>
<td>$j$</td>
</tr>
<tr>
<td>Neutrino Flavor</td>
<td>PID</td>
<td>3</td>
<td>$A, B, or C$</td>
<td>$k$</td>
</tr>
</tbody>
</table>
Figure 5.1: Baseline templates for $n^{\nu}_{ijk}$ (left) and $n^{\nu'}_{ijk}$ (right). The top row contains the most cascade-like sample (A), and the bottom row the most track-like sample (C) containing the strongest oscillation signal, which is a reduction in rate of events at 10–20 GeV for $\cos \theta_z \sim [-0.5, -1.0]$.

### 5.1 Handling of systematic uncertainties

As the atmospheric muon background is derived directly from data, apart from a normalization term on the atmospheric muon rate, all systematic uncertainties must be accounted for in the estimation of neutrino events. Following the example
Table 5.2: Description of nuisance parameters included in the fit. Because the atmospheric muon background is derived from a side-band of data, its normalization is allowed to vary with a flat prior constraining it to positive values. To compensate for this variation in the normalization of $n^\mu_{ijk}$, the overall neutrino normalization is also treated with a flat prior, only prohibiting negative values.

<table>
<thead>
<tr>
<th>Nuisance parameter</th>
<th>Prior</th>
<th>Included in</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric $\mu$ normalization</td>
<td>Flat, must be $\geq 0$</td>
<td>$n^\mu_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>Overall neutrino flux normalization</td>
<td>Flat, must be $\geq 0$</td>
<td>$F_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>$\nu_e/\nu_\mu$ flux deviation</td>
<td>$\mu = 1.00, \sigma = 0.02$</td>
<td>$F_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>spectral index deviation</td>
<td>$\mu = 0.00, \sigma = 0.05$</td>
<td>$F_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>$K/\pi$ rescaling</td>
<td>$\mu = 1.00, \sigma = 0.10$</td>
<td>$F_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>$\sin^2(2\theta_{13})$</td>
<td>$\mu = 0.093, \sigma = 0.008$</td>
<td>$F_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>DOM optical efficiency</td>
<td>$\mu = 1.00, \sigma = 0.10$</td>
<td>$G_{ijk}$</td>
<td></td>
</tr>
<tr>
<td>Hole ice $\alpha$ [1/cm]</td>
<td>$\mu = 0.02, \sigma = 0.01$</td>
<td>$H_{ijk}$</td>
<td></td>
</tr>
</tbody>
</table>

Of [30,57], we separate the neutrino related systematics into three classes,

$$n^\nu_{ijk} = F_{ijk}(\theta_{23}, \Delta m^2_{32}; q_w) \times G_{ijk}(q_{\text{DOM eff}}) \times H_{ijk}(q_{\text{Hole ice}})$$ \hspace{1cm} (5.2)

where $F_{ijk}$ provides the flux of atmospheric neutrinos at the detector modified by oscillation parameters and the set of flux related systematics $q_w$, and $G_{ijk}$ and $H_{ijk}$ are linear interpolations between event counts in bin $ijk$ derived from varying the DOM’s optical efficiency and the scattering coefficient in the re-frozen ice columns respectively. The full set of nuisance parameters, $q$, are listed with their priors in Table 5.2.

### 5.1.1 Constructing $F_{ijk}$

The baseline atmospheric neutrino flux is taken from [10], and is dependent on the simulated neutrino energy and zenith angle,

$$\Phi_{\nu_\alpha} = \Phi_{\nu_\alpha}(E_\nu, \theta_\nu),$$ \hspace{1cm} (5.3)

where $\alpha$ denotes the flavor of the neutrino or anti-neutrino. This baseline flux is modified by two systematic effects: a term which adjusts the relative abundance of kaons and pions in the atmospheric air shower, and a term which alters the spectral...

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index. The combination of these terms yields a flux with a few knobs to turn

$$\Phi'_{\nu_\alpha}(E_\nu, \theta_\nu; R_{K/\pi}, \delta \gamma) = \Phi_{\nu_\alpha}(E_\nu, \theta_\nu) \times f_{K/\pi}(E_\nu, \theta_\nu; R_{K/\pi}) \times f_{\delta \gamma}(E_\nu; \delta \gamma) \quad (5.4)$$

with $f_{K/\pi}$ given by

$$f_{K/\pi} = f_{K/\pi}(E_\nu, \theta_\nu; R_{K/\pi}) = \frac{w_{\pi \nu} + R_{K/\pi} w_{K \nu}}{w_{\pi \nu} + w_{K \nu}}. \quad (5.5)$$

Here $w_{\pi \nu}$, $w_{K \nu}$ are the fluxes of neutrinos from pion and kaon decay respectively as in Equation [1.38] and $R_{K/\pi}$ is the scaling factor used to adjust relative abundances. In the energy ranges for this analysis this is a sub-percent scale effect and is weakest around the horizon where the flux is highest. The spectral index term

$$f_{\delta \gamma}(E_\nu; \delta \gamma) = \left( \frac{E_\nu}{E_0} \right)^{\delta \gamma}, \quad (5.6)$$

where $\log_{10}(E_0) = 1.25$ GeV, adjusts the relative abundance of high energy to low energy neutrinos, with positive values of $\delta \gamma$ yielding more high energy neutrinos. The impact of this function is illustrated in Figure [5.2] and typical variations result in a change to the energy distribution on the order of a few percent. There is almost no impact on the angular distribution.

We also add a normalization factor, $N_{\nu_\alpha}$, to this modified neutrino flux that is nominally equal to unity. This allows us to adjust the relative abundance of electron neutrinos to muon neutrinos at production, which is known to 2% in these energy ranges [10].

As oscillations are only measurable in charged current interactions, we divide the overall number of events $F_{ijk}$ into neutral current and charged current subsamples before combining the flux, oscillation probabilities, and interaction cross sections as follows:

$$F_{ijk} = \left( F_{\nu}^{NC} + \sum_{\alpha} F_{\nu_{\alpha}}^{CC} \right)_{ijk} \quad (5.7)$$

$$F_{\nu}^{NC} = N_{\nu_\mu} \left( N_{\nu_e/\nu_\mu} \Phi'_{\nu_e} + \Phi'_{\nu_\mu} \right) \sigma_{\nu}^{NC} \quad (5.8)$$

$$F_{\nu_{\alpha}}^{CC} = N_{\nu_\mu} \left( N_{\nu_e/\nu_\mu} \Phi'_{\nu_e} P_{\nu_e \rightarrow \nu_\alpha} + \Phi'_{\nu_\mu} P_{\nu_\mu \rightarrow \nu_\alpha} \right) \sigma_{\nu_{\alpha}}^{CC} \quad (5.9)$$

with $P_{\nu_{\beta} \rightarrow \nu_\alpha}$ being the oscillation probability from Equation [1.36].
We assume that uncertainties on the neutrino cross section with ice will produce either energy dependent shifts, or simply overall scaling factors to the interaction rate. In the first case these cross-section uncertainties are degenerate with the uncertainty on the atmospheric flux spectral index; in the latter, they are degenerate with the existing normalizations. A quick test of this hypothesis was performed by scaling up and down the reconstructed energy of the hadrons produced in the neutrino-nucleon interaction to roughly approximate uncertainties in the deep inelastic scattering mechanism. As shown in Figure 5.3, these variations produce the same sort of shift in energy of the final sample as the spectral index shown previously.

Figure 5.2: Impact of the spectral index factor $f_{\delta \gamma}$ on energy (left) and angular (right) distributions. Modifying the spectral index has minimal impact on the angular distribution (right), but decreasing its value does shift events from the track-like sample (C) into the two cascade-like samples. This effect is visible on a few percent scale, but follows intuition, as only events with relatively high energy muons are classified as track-like and decreasing the spectral index reduces the relative abundance of high energy neutrinos, which is visible in the left figure.
Figure 5.3: Rough estimate of cross section uncertainties from scaling up and down the reconstructed energy of the hadronic cascade to account for different amounts of pion production. The distortions on the energy (left) and angular (right) distributions are very similar to those created by modifying the spectral index (Figure 5.2).
5.1.2 Constructing $G_{ijk}$

The second class of systematics considered is the overall optical efficiency of the IceCube DOMs. In situ measurements of the DOMs’ efficiency are performed using the flasher boards on neighboring modules [24], and have an overall uncertainty of ±10%. As increasing the overall efficiency of the DOMs will have a direct impact on whether low energy events reach the trigger threshold, several full sets of simulation were produced with discrete efficiencies. After each set is binned in energy, zenith, and PID value, we use a linear interpolation of event counts per bin to allow the DOM efficiency to take any value. The impact of varying the DOM efficiency by ±10% from its nominal value is shown in Figure 5.4. We assume variations in the DOM efficiency to be orthogonal to other systematics, and higher order effects from covariance are neglected beyond their basic treatment inside of MIGRAD.

![Figure 5.4: Impact of the overall optical efficiency of IceCube DOMs on the energy (left) and angular (right) distributions. Increasing the optical efficiency allows more light from low energy muons to be collected, which is visible in the relative increase in Sample C relative to Samples A and B in the first two energy bins. The jagged nature of the angular dependence is a limitation of existing simulation statistics, and is not thought to be physical, but currently unavoidable.](image-url)

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5.1.3 Constructing $H_{ijk}$

Another source of uncertainty is the detector medium itself, specifically the scattering coefficient of the re-frozen bore hole ice surrounding the DOMs. The amount of scattering in the hole ice has a direct impact on the angular acceptance of the DOMs as seen in Figure 5.5. As with the DOM efficiency, we produced several sets of simulation and use a linear interpolation to determine the number of events in a given bin of energy, zenith, and PID value for a given scattering coefficient. Modifications to the scattering coefficient have a large impact on the zenith distribution of the final sample, but have only minor impact on the energy spectrum as shown in Figure 5.6.

![Figure 5.5: Modifications to the angular acceptance of simulated DOMs for various hole ice scattering coefficients ($\alpha$). The case of no scattering in the hole ice is represented by the blue line, which is derived from average lab measurements of the angular acceptance of IceCube DOMs. The black line indicates our best prior understanding of the hole ice. As the effective scattering length, $\alpha^{-1}$, in the hole ice decreases, the ability to observe light traveling directly up the column of refrozen ice (zenith angle of 0°) likewise decreases.](image-url)

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Data analysis for $\nu^{}\mu$ disappearance

Borehole ice

To deploy the DOMs, columns of ice have to be melted. The water freezes back in a few weeks, forming ice with very different properties than the one that surrounds them. The borehole ice is approximated as a medium filled with bubbles, and described by the effective scattering $\alpha$. Simulations show that the change in scattering has the same effect as modifying the angular acceptance of the DOMs. Figure 7.17 shows the effect, together with the variations that this introduces on the final $\nu^{}\mu$ sample. The most noticeable change is a distortion of the zenith angle distribution, coming from energies below 20 GeV. The impact decreases rapidly with energy.

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7.3 Fitting the oscillation parameters

The method to measure neutrino oscillations followed in this work is to compare the data with simulation templates. The parameters in the simulation are varied and the set that is most likely to explain the data is taken. The simulation is done by reproducing the interaction and detection steps, as described in Chapter 5, and then passing the events through the same analysis as the data. The resulting events are used to fill a two-dimensional histogram with the reconstructed energy and zenith angle on the axes. The performance of the reconstructions varies depending on the true parameters of the neutrino, and reproducing them in full in the simulation is the most straightforward way of correctly accounting for them.

7.3.1 Statistical method

The method used to determine the oscillation parameters that the data favor is the binned maximum likelihood in the presence of nuisance parameters [152]. The two observables that modify the effect are the neutrino energy and the travel distance, while the two physical parameters of interest are the mixing angle $\theta_{23}$ and the mass difference $|\Delta m^2_{32}|$. The two-neutrino approximation, shown in Eq. 3.30, is used to calculate transition probabilities.
Figure 5.6: Impact of the scattering coefficient of the re-frozen ice surrounding the DOMs on the energy (left) and angular (right) distributions. Variations in the hole ice scattering are simulated by modifying each DOMs angular acceptance. As such, it comes as no surprise that this systematic has a large impact on the number of events accepted as a function of reconstructed zenith angle. Increasing the scattering coefficient, which decreases the effective scattering length, significantly reduces the number of vertically up-going events, as light from these particles will scatter away from the DOMs. Conversely, the number of downward going events increases with more scattering, as it now becomes more likely for photons to scatter up into the PMT, which is situated on the bottom half of each DOM. There is minimal impact on the energy distribution of events, as the overall sensitivity of the DOMs is not affected.

5.2 Fitting procedure

Oscillation parameters are fitted using a binned likelihood following Poisson statistics to compare the number of expected events $n_{ijk}$ to the number of observed events $x_{ijk}$ in a bin, with $ijk$ as described previously in Table 5.1. A raster scan is performed on the oscillation parameters $\theta_{23}$ and $\Delta m^2_{32}$, finding the maximum likelihood at each point of the scan by fitting the nuisance parameters from Table 5.2 using the MIGRAD package from Minuit2. Those parameters that have gaussian
priors are included as penalty terms in the binned LogLikelihood,

\[-LLH(n, q|x) = \sum_{ijk} (n_{ijk} - x_{ijk} \ln n_{ijk}) + \sum_{l} \frac{(q_l - \bar{q}_l)^2}{2\sigma_{q_l}^2}, \quad (5.10)\]

for \( q_l \in \{N_{\nu_e/\nu_\mu}, R_{K/\pi}, \delta\gamma, q_{\text{DOM eff}}, q_{\text{Hole ice}}, \sin^2(2\theta_{13})\} \). After the raster scan is complete, a final minimization is performed allowing all oscillation and nuisance parameters to vary in order to find the best fit of the oscillation parameters. The likelihood value obtained for each point in the scan is then compared to the likelihood obtained in the final minimization. These likelihood ratios are used to draw 68% and 90% confidence limits using Wilks’ theorem with two degrees of freedom, one for each of the oscillation parameters fit, which then define our sensitivity to these parameters.

### 5.2.1 Goodness of fit

The calculation of a reduced \( \chi^2 \) (i.e., \( \chi^2 \) divided by the number of degrees of freedom in the fitted data) as a measure of goodness of fit is predicated on the assumption that the (possibly composite) hypothesis is perfectly specified. That is to say, although there may be free parameters in the hypothesis, for any given combination of parameters the expectation is assumed to be known with perfect precision.

This is not the situation in this analysis, for two related reasons. Most importantly, a significant fraction of the observed data are believed to be residual backgrounds from cosmic ray muons, particularly in the down-going portion of the data. Because Monte Carlo simulations of those muons are insufficiently accurate and available in insufficient numbers, a data-driven estimate of the characteristics of the background was used instead. This estimate was derived from events which would have been accepted into the final data set except for a small amount of activity (usually one or two photoelectrons) detected in specific areas of the veto region during the event. While this approach has been shown to be more accurate than a Monte Carlo estimate [30], the template of the muon distribution thus obtained is only an estimate of the true underlying shape of the distribution, and is subject to the usual sampling errors associated with a single realization from an ensemble.

The same issue also affects the neutrino Monte Carlo sets used to generate the
expectations for that portion of the data, but to a much lesser degree. Where
the tagged background muon sample used to generate the expectation for that
distribution contained approximately the same number of events as are fitted to be
present in the real data set, 30 years of simulated IceCube live time is available in
the neutrino Monte Carlo sets – approximately 10.8 times the actual amount of
data recorded.

As a result, there is an intrinsic statistical uncertainty in the expectations to
which the data are fitted, particularly in regions where muon contamination is
high. A correct calculation of goodness of fit must take these uncertainties into
account. This problem has been studied in another context related to IceCube by
D. Chirkin \cite{62}, and we adapt his approach to the current situation.

We define two hypotheses: that the observed data and the expected number of
counts are described by the same process (i.e., have the same mean $\mu^i$ in each bin
$i$), or that they are unrelated and have separate means $\mu^i_d$ and $\mu^i_s$ respectively.\footnote{Using the notation described earlier in this chapter would only further complicate the following equations. We include the conversion here for clarity: $\mu^i_d = x_{ijk}$, $\mu^i_s = n_{ijk}$.}

Here we follow the notation of \cite{62} with ‘s’ denoting simulation, but in fact $\mu_s$ has
two components, $\mu_s = \mu_\mu + \mu_\nu$ for the muons and the neutrinos respectively. This
is discussed further below. The former hypothesis is nested within the latter, as
it can be formed by setting $\mu^i_d = \mu^i_s = \mu^i$ in each bin, reducing the number of fit
parameters. We can compare the probabilities of these two hypotheses by forming
the likelihood ratio

\[
\Lambda = \frac{P(\text{same process})}{P(\text{independent processes})} = \prod_i \left( \frac{\mu^i_s}{s_i/n^i_s} \right)^{s_i} \cdot \prod_i \left( \frac{\mu^i_d}{d_i/n^i_d} \right)^{d_i},
\]

(5.11)

where $d_i$ is the number of data observed in bin $i$, $n^i_d$ is the number of measurements
(here always 1), $n^i_s$ is the number of independent simulations used to determine
$\mu^i_s$, and $s_i$ is the total number of simulated data observed in bin $i$ across the $n^i_s$
simulations. These variables are related by $\mu^i_s = s_i/n^i_s$, and similarly for the data-
related variables. The reader is directed to \cite{62} for the derivation of Equation 5.11.

For the data, only one three-year-long measurement was made for each bin, so
$n^i_d = 1$ in all cases. For the expectation, the statistical precision varies between
bins: for a bin in which only tagged muons were predicted (no neutrinos), we would
likewise have $n^i_s = n^i_\mu \approx 1$, but a bin in which only neutrinos were expected would
have $n_i^s = n_i^\nu \approx 10.8$ corresponding to the amount of neutrino Monte Carlo available. It is not obvious how to generalize the derivation of Equation 5.11 to multiple components of the expectation with differing uncertainties, so we define an “effective” $n_i^s$ for each bin to reproduce the usual relationship between the uncertainty on a single measurement and the uncertainty on the mean: $\sigma_{\text{mean}} = \sigma_{\text{sample}}/\sqrt{n_{\text{eff}}}$. By comparing the estimated statistical uncertainty in each bin to that which would be expected from a single measurement, where $\sigma = \sqrt{s} = \sqrt{\mu}$, we obtain

$$n_{\text{eff}}^i = \left(\frac{\sigma_{\text{sample}}}{\sigma_{\text{mean}}}\right)^2 = \frac{\mu_i^s}{\mu_i^s + \mu_i^\nu/n_i^\nu}. \quad (5.12)$$

In Equation 5.12, we have assumed the uncertainties combine in quadrature, $\sigma^2 = \sigma_\mu^2 + \sigma_\nu^2$.

We can then calculate the likelihood ratios for the sample to compare the hypothesis that the data and best-fit expectations are consistent (are realizations of the same underlying process) to the hypothesis that they are inconsistent. Because the latter have additional free parameters (the second mean $\mu$ associated with each bin), some improvement in the fit is expected even if the data and expectations are consistent. Wilks’ theorem suggests that if this is the case, $-2 \ln \Lambda$ should be distributed as a $\chi^2$ distribution with number of degrees of freedom equal to the number of additional fit parameters (here, the number of bins). If the data and expectations are not consistent, then the fit quality will improve by a greater amount when the additional meaningful parameters are added. We have not proved rigorously that the conditions for Wilks’ theorem are satisfied in this instance, but we expect it should be at least approximately correct.

5.3 Monte Carlo expectations

In accordance with IceCube Collaboration policy, the analysis was developed “blind,” with reference to Monte Carlo simulations and down-going events where no oscillation signal could be visible. Therefore, before we discuss the measured results from our three year data sample, we will discuss our expected sensitivity from Monte Carlo studies. In these, we generate a pseudo-data set from the simulated templates with a given injected value of each oscillation and nuisance parameter. Figure 5.7 illustrates the raster scan approach for one of these data challenges, in
which the injected values are recovered through the fitting procedure.

Figure 5.7: The output of a raster scan of the oscillation parameters. In the top left, the LogLikelihood value is shown the best fit of nuisance parameters for fixed values of $\theta_{23}$ and $\Delta m^2_{31}$. The black dot marks both the injected and best fit values of the oscillation parameters. The other 8 figures show the best fit value of a given nuisance parameter at each point of the scan normalized as a % deviation with respect to the injected value.

**5.3.1 Tests of potential bias**

To validate the overall performance of the fitter, we injected oscillation parameters that spanned the $3\sigma$ and $5\sigma$ ranges for $\theta_{23}$ and $\Delta m^2_{32}$ respectively for both normal and inverted hierarchies. Of the 121 points scanned in each hierarchy, no significant bias was seen, with deviations on the 0.01% scale. The injected and best fit points in the normal hierarchy are shown in Figure 5.8.

In this test, the simulated neutrino and background data templates were used directly as the pseudo-data. A more realistic test for bias must include statistical fluctuations on the raw pseudo-data. We therefore show in Figure 5.9 the results
Figure 5.8: Fit errors for 121 simulated pseudo-data sets created for different sets of oscillation parameters, in the presence of muon background and systematic uncertainties. The blue inverted triangles mark the injected parameters, while the red upright triangles connected to them by lines indicate the parameters fitted from the corresponding pseudo-data set. As expected, there is no obvious bias and characteristic error scales for both oscillation parameters. In these tests, the pseudo-data sets were generated and fitted without being modified by statistical fluctuations.
of fits to 81 sets of pseudo-data generated for different combinations of oscillation parameters, with muon backgrounds drawn from the tagged muon sample (which is assumed to be a perfect model of the background for this study). For each set of parameters, Poisson variations are generated around the expected numbers of counts in each bin, and the full fit procedure is applied to the resulting pseudo-data. One can see that, as expected, there is a typical error in the fitted parameters, with a characteristic range of sizes but no obvious bias, which would show up as preferred direction of the errors.

One can investigate whether the observed residuals from the fit data might inject a bias into the result, for example due to the sampling error in the background estimate. As an extension of the bias study, we follow the suggestion of [63], to apply the observed residuals to Monte Carlo pseudo-data generated for a variety of possible oscillation parameters. Because we have only one set of residuals to the real data, by construction, we must first consider what level of agreement is expected between injected and fitted parameters for single realizations of the pseudo-data, in the absence of systematic bias.

To alleviate the concern that systematic errors persisting in the real data sample may cause a bias toward maximal or beyond-maximal mixing, we select one of these 81 trials from Figure 5.9 with non-maximal parameters injected, where the error in the fit had a typical magnitude and pulled the fit toward maximal mixing rather than to lower mixing. The pseudo-data selected for this purpose was the set injected at $\sin^2(\theta_{23}) = 0.464$, $\Delta m^2_{32} = 2.43$.

The residuals from the fit to this pseudo-data set were extracted and were applied as an offset to the expectations for each set of pseudo-data used to generate Figure 5.9. Each set was then re-fitted, and the results are shown in Figure 5.10. In contrast to Figure 5.9, there is a clear pattern to the fit errors arising from the perfect correlation of the residuals. In all cases, the fits are pulled toward maximal mixing; whatever downward fluctuations in the pseudo-data caused the original fit to prefer a higher level of $\nu_\mu$ disappearance have qualitatively the same impact on every other set, although the magnitude varies. It is worth reiterating that there is no real bias in these residuals – they were generated from pure Poisson fluctuations around a chosen model. The correlations seen in Figure 5.10 are solely due to the

\[\text{In the interest of efficiency for these tests, a three-flavor vacuum oscillation model is used, with matter effects turned off. This should not affect the result.}\]
fact that the random fluctuations have been “frozen in” for each subsequent test. Interestingly, the fitter is able to correct almost perfectly for the error in mass splitting when re-fitting the residuals applied to the original pseudo-data set, but

Figure 5.9: Fit errors for 81 simulated pseudo-data sets created with Poisson fluctuations around expected bin counts for different sets of oscillation parameters, in the presence of muon background and systematic uncertainties. The blue inverted triangles mark the injected parameters, while the red upright triangles connected to them by lines indicate the parameters fitted from the corresponding pseudo-data set. As expected, there is no obvious bias and characteristic error scales for both oscillation parameters.
not for the mixing angle error. This is in accord with the predicted higher accuracy on $\Delta m^2_{32}$ compared to the mixing angle.

It should be noted that these frozen residuals are only Poisson distributed about the pseudo-data for a single point in phase space. As we vary the oscillation

Figure 5.10: Fit errors when 81 sets of pseudo-data are generated with identical Poisson residuals (see text for details). There is a universal trend toward greater muon neutrino disappearance, with some trends in mass splitting appearing depending on the injected value. These trends are strictly artifacts of the identical residuals used in each pseudo-data set; no real bias is present in this Monte Carlo study.
parameters, we can substantially change the number of expected events in a given bin, such that the frozen residuals may be larger than expected for Poisson fluctuations for that set of parameters. As a result the likelihood differences will not exactly follow the $\chi^2$ distribution from Wilks’ theorem, but can be used nevertheless as a rough indication of goodness of fit.

We now turn back to the goodness of fit metric. In the absence of statistical fluctuations, the best fit template should be in perfect agreement with the pseudo-data. In this case, we should almost perfectly recover the injected values for all systematic and nuisance parameters, and furthermore, the reduced $\chi^2$ metric should be as valid as the likelihood ratio from Equation 5.11. However, if we apply these frozen residuals to the pseudo-data, we should neither expect to perfectly recover the injected parameters, nor expect the reduced $\chi^2$ goodness of fit metric to be valid. These expectations are confirmed in Table 5.3. During the second phase of this test in which we apply the frozen residuals, we run the fitting procedure a second time including the raster scan over a wide range of plausible oscillation parameters. In this case, the reduced $\chi^2$ is extremely poor, and at first glance the p-value from the likelihood ratio is also poor. As we previously explained, in this case Wilks’ theorem is not explicitly valid, so the mapping between a well defined likelihood ratio and a p-value should be taken with a grain of salt. As such, a p-value of 3.1% from this likelihood ratio can be considered plausible. Because this second test is so similar to the actual process with measured data, we will not include the reduced $\chi^2$ in any further results.
Table 5.3: Values for the nuisance and oscillation parameters obtained by fitting one set of pseudo-data with and without residuals from statistical fluctuations applied. This test was performed only using the most track-like events using three flavor oscillations in a vacuum for simplicity. Column three shows that when the fit is performed on pseudo-data without statistical fluctuations, the injected values are recovered to three significant digits, with the exception of the mixing angle which is recovered to one significant digit. Small changes to the mixing angle near maximal mixing have negligible effect on the data, as seen in the $\chi^2$ and $\Lambda$ values. In this case, either goodness of fit metric produce p-values of 100%. When these unbiased statistical fluctuations are present, there is reasonable agreement between injected and fit values, only the likelihood ratio from Equation 5.11 produces a reasonable p-value.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Injected value</th>
<th>Best fit value with and without fluctuations</th>
<th>Best fit value with frozen fluctuations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\mu}$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.77</td>
</tr>
<tr>
<td>$N_{\nu_{\mu}}$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>$N_{\nu_{e}/\nu_{\mu}}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>$\delta_{\gamma}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>$R_{K/\pi}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>$q_{\text{DOM eff}}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>$q_{\text{Hole ice}}$</td>
<td>0.020</td>
<td>0.020</td>
<td>0.028</td>
</tr>
<tr>
<td>$\sin^2(2\theta_{13})$</td>
<td>0.093</td>
<td>0.093</td>
<td>0.093</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.515</td>
<td>0.504</td>
<td>0.516</td>
</tr>
<tr>
<td>$\Delta m_{32}^2/10^{-3}\text{eV}^2$</td>
<td>2.50</td>
<td>2.50</td>
<td>2.48</td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td></td>
<td>$5.12 \times 10^{-5}$/119</td>
<td>199/119</td>
</tr>
<tr>
<td>P-value</td>
<td></td>
<td>$1.000$</td>
<td>$5.60 \times 10^{-6}$</td>
</tr>
<tr>
<td>$-2 \log(\Lambda)$/dof</td>
<td></td>
<td>$5.36 \times 10^{-3}$/128</td>
<td>159/128</td>
</tr>
<tr>
<td>P-value</td>
<td></td>
<td>$1.000$</td>
<td>0.031</td>
</tr>
</tbody>
</table>
5.3.2 Expected sensitivity

As we mentioned at the end of Section 5.2 our measured sensitivity to the oscillation parameters $\theta_{23}$ and $\Delta m_{32}^2$ are defined using differences between the LogLikelihoods from Equation 5.10 evaluated for discrete values of these oscillation parameters in the raster scan process. Wilks’ theorem (Equation 4.13) with two degrees of freedom is used to convert these differences in LogLikelihoods into a $\chi^2$ from which 68% and 90% confidence limits can be defined. It was noted in [30] that Wilks’ theorem is not explicitly fulfilled for this measurement, and this method yields confidence regions which are larger than they should be in certain regions. As the alternative approaches to correctly address these issues, such as the one proposed by Feldman and Cousins [64], are computationally expensive and the method used here over-estimates the errors, we use Wilks’ theorem to obtain the result in this analysis and accept some over-coverage.

At the start of this section, we showed the output from one of our Monte Carlo data challenges in Figure 5.7. Following the method in the previous paragraph, confidence limits are drawn from the likelihood distribution and are shown in Figure 5.11. As the true value of the oscillation parameters $\theta_{23}$ and $\Delta m_{32}^2$ are unknown, two additional confidence regions are drawn in Figures 5.12 and 5.13 for widely spread, but equally allowed, oscillation parameter values. Included on each of these plots are the closest matching injected and best fit points from Figure 5.10. In each case, the observed difference between the injected and fitted parameters is contained within the 68% confidence region. The relatively large pull in the two non-maximal cases is due to the fact that the measurement fundamentally is sensitive to the magnitude of the disappearance effect, rather than the value of the mixing angle. As one approaches maximal mixing, there is a wide range of angles which lead to comparable amounts of disappearance, which is accounted for in the width of the confidence limits. The pull from unbiased statistical fluctuations on pseudo-data, even those which increase the magnitude of the disappearance, is entirely consistent with the expected sensitivity shown here.
Figure 5.11: Comparison of the error when fitting pseudo-data generated with the pseudo-data residuals, frozen as in Figure 5.10, to measurement contours predicted based on an ensemble of pseudo-data generated for those oscillation parameters. The black dot indicates the injected parameters, the orange cross is the statistical expectation for the best fit point, with the dashed and solid lines indicating the 68% and 90% confidence regions respectively. The red dot connected to the triangle by a line indicates the point from Figure 5.10 closest to the injected value and result of its fit, and is within the 68% confidence region.
Figure 5.12: Comparison of the result of fitting pseudo-data generated with the pseudo-data residuals to the expected range of fit error, as in Figure 5.11 but for different injected oscillation parameters. The red dot connected to the triangle by a line indicates the point from Figure 5.10 closest to the injected value and result of its fit, and is within the 68% confidence region.
Figure 5.13: Comparison of the result of fitting pseudo-data generated with the pseudo-data residuals to the expected range of fit error, as in Figure 5.11 but for different injected oscillation parameters. The red dot connected to the triangle by a line indicates the point from Figure 5.10 closest to the injected value and result of its fit, and is within the 68% confidence region.
Chapter 6  |  Analysis Results

At the May 2015 IceCube collaboration meeting, permission was given to unblind data taken between May 2011 and April 2014 for the purpose of this dissertation. The official analysis was kept blind, as a new parametrization of detector dark noise was under development and IceCube working group leaders decided that it must be included in any future publications and results. Given the extended period of time required for noise model completion, detector simulation, and event reconstruction, the inclusion of this new parametrization was determined to be beyond the scope of this dissertation. Keeping with the IceCube blindness policy, the measured oscillation parameters and any potential problems observed after unblinding this analysis were not to be disclosed to the collaboration members or community outside the dissertation committee.

6.1 Blind sample results

We opened the box, unblinding the full three year sample, and observed a total of 59380 events in the data, corresponding to 21048 events in Sample A, 20953 events in Sample B, and 17379 events in the most track-like Sample C. These event counts are consistent with the 10% subset that was used in Table 4.1 in which the combined expectation from neutrino-like simulation and atmospheric muon-like data exceeded the data by approximately 15%. As the overall normalization terms $N_\mu$ and $N_{\nu_\mu}$ are allowed to vary without a prior, this offset is not problematic, and indeed we observe that the best fit value for the neutrino normalization is 27% lower than the baseline expectation. Note that this normalization term incorporates the detector efficiency and veto efficiency effects as well as the uncertainty in the physical neutrino flux.
Table 6.1: Values for the nuisance and oscillation parameters obtained through a fit of the full event sample. The measured oscillation parameters include $1\sigma$ confidence intervals, which are calculated from their profile likelihoods.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal value</th>
<th>Best fit</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_\mu$</td>
<td>1.00</td>
<td>1.12</td>
<td>+12. %</td>
</tr>
<tr>
<td>$N_{\nu_e}$</td>
<td>1.00</td>
<td>0.73</td>
<td>-27. %</td>
</tr>
<tr>
<td>$N_{\nu_{\mu}}$</td>
<td>1.00 $\pm$ 0.02</td>
<td>1.03</td>
<td>+ 1.7 $\sigma$</td>
</tr>
<tr>
<td>$\delta\gamma$</td>
<td>0.00 $\pm$ 0.05</td>
<td>0.05</td>
<td>+ 0.93$\sigma$</td>
</tr>
<tr>
<td>$R_{K/\pi}$</td>
<td>1.00 $\pm$ 0.10</td>
<td>1.04</td>
<td>+ 0.41$\sigma$</td>
</tr>
<tr>
<td>$q_{DOM,eff}$</td>
<td>1.00 $\pm$ 0.10</td>
<td>1.09</td>
<td>+ 0.95$\sigma$</td>
</tr>
<tr>
<td>$q_{Hole,ice}$</td>
<td>0.020 $\pm$ 0.010</td>
<td>0.026</td>
<td>+ 0.64$\sigma$</td>
</tr>
<tr>
<td>$\sin^2(2\theta_{13})$</td>
<td>0.093 $\pm$ 0.008</td>
<td>0.094</td>
<td>+ 0.15$\sigma$</td>
</tr>
<tr>
<td>$\sin^2\theta_{23}$</td>
<td>0.52$^{+0.04}_{-0.06}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_{32}^2/10^{-3}\text{eV}^2$</td>
<td>2.38$^{+0.10}_{-0.11}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-2\log(\Lambda)/\text{dof}$</td>
<td>525/384</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-value</td>
<td>$2.05 \times 10^{-6}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The full set of parameters and their best fit values are shown in Table 6.1, in which we see that this fit to the full data sample is not particularly good. The confidence limits on the oscillation parameters for this measurement are shown in Figure 6.1. These results show stronger constraints on the mixing angle than had been observed in any of the Monte Carlo data challenges performed prior to unblinding the data. Given the strong pulls on several of the nuisance parameters, and the poor goodness of fit we believe that the limits drawn are beyond the actual sensitivity of this analysis.

Because this analysis is actually sensitive only to the amplitude of oscillation induced disappearance rather than the value of the mixing angle, we also performed a fit in which this amplitude is varied directly. The simplest method for performing this second fit is to use the two flavor approximation from Equation 1.28, varying the entire term $\sin^2(2\theta_{23})$ and allowing this amplitude to take any value it chooses, included those greater than unity. As an example, we look at the result of this “two-flavor test” performed on pseudo-data from one of the Monte Carlo data challenges in the absence of statistical fluctuations. Figure 6.2 shows that the best fit value is maximal (a value of 1.0 on the x-axis). In contrast, the same test
performed on the data sample prefers that more than 100% of the $\nu_\mu$ flux disappear at a confidence level beyond 90%, as shown in Figure 6.3, and obviously unphysical result.

Given the failure of the two-flavor test, and the extremely poor goodness of fit p-value, we now investigate how the goodness of fit metric varies between the three PID samples. The contributions to the full fit are broken down by sample although they are fitted jointly. The likelihood ratios, numbers of additional degrees of freedom, and integrals of $\chi^2$ probability distributions beyond the observed values are shown in Table 6.2. When considering the full sample, the likelihood improves to a greater degree than is consistent with data matching expectations; improvements by that amount or more are expected in only two trials per million.

![Figure 6.1: Measured 90% (solid) and 68% (dashed) confidence limits on oscillation parameters $\sin^2(\theta_{23})$ and $\Delta m^2_{32}$ using the full event sample. The subplot on the right (top) shows the profile likelihood of the mass splitting (mixing angle) from which we compute the 1$\sigma$ confidence interval listed in Table 6.1.](image)
Figure 6.2: Expected 90\% (solid) and 68\% (dashed) confidence limits on two flavor parametrization of the oscillation parameters, $\sin^2(2\theta_{23})$ and $\Delta m^2_{32}$, using a Monte Carlo pseudo-data set approximating the full event sample. The best fit is consistent with maximal mixing, which was injected.
Figure 6.3: Measured 90% (solid) and 68% (dashed) confidence limits on two flavor parametrization of the oscillation parameters, $\sin^2(2\theta_{23})$ and $\Delta m^2_{32}$, using the full event sample. The best fit is inconsistent with maximal mixing at a confidence level greater than 90%.
Table 6.2: Likelihood ratios, numbers of additional parameters, reduced log likelihood ratios, and cumulative probabilities of observing comparable or greater improvement in fits when adding spurious parameters for comparing the hypotheses of correct and incorrect models (see Section 5.2.1 for details). The first set of values refers to the joint fit to the full data sample, with the contributions from the three subsamples listed separately for comparison, with Sample A being the most cascade-like and Sample C the most track-like. The overall fit is extremely poor.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Sample A</th>
<th>Sample B</th>
<th>Sample C</th>
<th>All Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2 \ln \Lambda$</td>
<td>205.9</td>
<td>156.0</td>
<td>163.6</td>
<td>525.4</td>
</tr>
<tr>
<td>Add’l params.</td>
<td>128</td>
<td>128</td>
<td>128</td>
<td>384</td>
</tr>
<tr>
<td>Reduced LLR</td>
<td>1.61</td>
<td>1.22</td>
<td>1.28</td>
<td>1.37</td>
</tr>
<tr>
<td>$\chi^2$ probability</td>
<td>$1.53 \times 10^{-5}$</td>
<td>$4.70%$</td>
<td>$1.85%$</td>
<td>$2.05 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

The fit problems in the full sample are driven by the cascades, particularly Sample A. To gain a qualitative understanding of the fit quality, we compare the number of data events per bin in Sample A to the expected number of events in that bin from the best fit with and without neutrino oscillations. This comparison is shown in Figure 6.4. In each of these expectations, the non-oscillation nuisance parameters take the values from the best fit. In the lowest three energy bins (top three subfigures) there is good agreement between the observed data and the best fit. Over the next two energy bins, discrepancies are visible near the horizon, where due to reconstruction resolution neutrino oscillations have a significant impact. In these bins, which correspond to reconstructed energies between 13.3 GeV and 23.7 GeV, fewer events are observed than expected with the best fit. Because the best fit is already at maximal mixing, these bins contribute significantly to the pull toward greater-than-maximal mixing observed in the two-flavor test of Figure 6.3. At higher energies, particularly in the second to last energy bin, there is an excess of down-going data. This excess occurs in a region where the oscillation parameters have no impact, that is the predictions with and without oscillations overlap. While this surplus certainly contributes to the low value of the goodness of fit metric, it should have no impact on the observed preference to greater than maximal mixing.

In the other cascade-like sample (B), shown in Figure 6.6, we see better agreement in the high energy down-going region, which corresponds to the much better p-value seen in Table 6.2. However, we see an even more pronounced deficit of data near the
Figure 6.4: Comparison of the most cascade-like data (Sample A) as black data points to expected events using the best fit parameters in Table 6.1 with and without oscillations, blue and red lines respectively. Events are displayed in their analysis binning, moving from the lowest energy bin (top) to highest energy bin (bottom). Two distinct mismatches between the observed data and the expected number of events are visible, which contribute to the low probability that these events are drawn from the same parent distribution. These mismatches are described in detail in the text.
Figure 6.5: Comparison of the track-like data (Sample C) as black data points to expected events using the best fit parameters in Table 6.1 with and without oscillations, blue and red lines respectively. Events are displayed in their analysis binning, moving from the lowest energy bin (top) to highest energy bin (bottom).
Figure 6.6: Comparison of the intermediate cascade-like data (Sample B) as black data points to expected events using the best fit parameters in Table 6.1 with and without oscillations, blue and red lines respectively. Events are displayed in their analysis binning, moving from the lowest energy bin (top) to highest energy bin (bottom).
horizon in lower energy bins which also drives the mixing angle beyond maximal.

In the track-like sample, shown in Figure 6.5, there are still some disagreements, most notably the sharp peak visible in the expectations just above $\cos(\theta_{\text{reco}}) = 0.5$ in two of the plots that does not exist in the observed data. This peak is generated by the background estimate from the data-sample and is visible in the lower left plot in Figure 5.1. Given this excess of high energy downgoing events is visible in both the most track-like and cascade-like samples as a fundamental shape mismatch, we must conclude that the data-driven background templates used are not a complete estimate of the atmospheric muons that sneak into the fiducial volume of this analysis. However, the disagreement in the track-like sample is largely restricted to a region in which neutrino oscillations have no impact. As such, this mismatch should decrease any quality of fit metric, but not significantly impact the sensitivity of our analysis.

### 6.2 Track-like sample results

The inclusion of cascade-like events was originally justified as a control sample for the neutrino flux, in which oscillations would have minimal impact. However, upon unblinding, we saw significant differences between the observed cascade-like data and our predicted templates. Due to these differences, especially those in the oscillation region, the best fit to data preferred oscillations remove greater than 100% of the $\nu_{\mu}$ flux. Because the vast majority of the muon neutrino disappearance signal is contained in the most track-like sample, and bolstered by the observation that the differences in the track-like sample were largely restricted to regions of phase space where oscillations did not have significant impact, we decided to perform a second fit to the data using only the track-like events in Sample C.

The results of this track-like only fit are shown in Table 6.3. In this case, the goodness of fit metric is acceptable, with a 10.1% chance of observing comparable fit improvement through random draws of data and expectations from the same parent template. Additionally, the nuisance parameters are pulled significantly less when fitting only the track-like sample; here only one parameter, the scattering length in the refrozen ice, is more than half of a standard deviation from its nominal value. This stands in contrast to the fit to the full sample, in which half of the parameters failed this metric.
Table 6.3: Values for the nuisance and oscillation parameters obtained through a fit of the track-like sample. The measured oscillation parameters include 1σ error bands.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal value</th>
<th>Best fit</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_\mu^\tau$</td>
<td>1.00</td>
<td>1.05</td>
<td>+ 5.2 %</td>
</tr>
<tr>
<td>$N_\nu_\mu$</td>
<td>1.00</td>
<td>0.85</td>
<td>−15.0 %</td>
</tr>
<tr>
<td>$N_{\nu_e/\nu_\mu}$</td>
<td>1.00 ± 0.02</td>
<td>1.01</td>
<td>+ 0.28 σ</td>
</tr>
<tr>
<td>$\delta\gamma$</td>
<td>0.00 ± 0.05</td>
<td>0.01</td>
<td>+ 0.21 σ</td>
</tr>
<tr>
<td>$R_{K/\pi}$</td>
<td>1.00 ± 0.10</td>
<td>1.01</td>
<td>+ 0.12 σ</td>
</tr>
<tr>
<td>$q_{\text{DOM eff}}$</td>
<td>1.00 ± 0.10</td>
<td>1.00</td>
<td>−0.02 σ</td>
</tr>
<tr>
<td>$q_{\text{Hole ice}}$</td>
<td>0.020 ± 0.010</td>
<td>0.027</td>
<td>+ 0.73 σ</td>
</tr>
<tr>
<td>$\sin^2(2\theta_{13})$</td>
<td>0.093 ± 0.008</td>
<td>0.094</td>
<td>+ 0.15 σ</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.53$^{+0.07}_{-0.11}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_{32}^2/10^{-3}\text{eV}^2$</td>
<td>2.35$^{+0.14}_{-0.17}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-2 \log(\Lambda)/\text{dof}$</td>
<td>149/128</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-value</td>
<td>0.101</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As with the fit to the full sample, we performed the two-flavor test to determine if the observed data is consistent with maximal mixing; the result of this test is shown in Figure 6.7. Similarly to what we observed with the full fit, the track-like data prefers greater than maximal mixing. However, in stark contrast to the previous fit, here the best fit is compatible with maximal mixing at a confidence level better than 68%.

We note that the best fit values for oscillation parameters from each of our two fits are entirely consistent. Fitting the track-like (all) events, we obtain

$$\sin^2 \theta_{23} = 0.53^{+0.07}_{-0.11} \quad (0.52^{+0.04}_{-0.06})$$
$$\Delta m_{32}^2/10^{-3}\text{eV}^2 = 2.35^{+0.14}_{-0.17} \quad (2.38^{+0.10}_{-0.14}).$$

This is not unexpected, as the fits were performed on the raw mixing angle $\theta_{23}$, thereby preventing the fitter from pushing into the beyond maximal mixing, which would have biased the result of the full sample. The primary concern with the full fit from the previous section was not the best fit value itself, but rather that the size of the confidence interval on the mixing angle was beyond the actual sensitivity.
of this analysis. Over the previous section, I confirmed that this was indeed the case.

Because the best fit point has not moved substantially, we expect that the distribution of events in the best fit will not change significantly. Looking at the events per bin as before, in Figure 6.8, we do not see any significant differences in comparison to Figure 6.5.

The improvement in fit is much more visible when we project the data into an \( L/E \) plot using the reconstructed zenith direction to estimate the length that the neutrino traveled from its production, which was assumed to be 19 km above the surface of the earth. The reconstructed \( L/E \) distributions show two peaks, the left of which corresponds to down-going events with the other peak near 300 km/GeV.

![Figure 6.7: Measured 90% (solid) and 68% (dashed) confidence limits on two flavor parametrization of the oscillation parameters, \( \sin^2(2\theta_{23}) \) and \( \Delta m^2_{32} \), fitting only the most track-like events. The best fit, while greater than maximal, is consistent with maximal mixing at a confidence level less than 68%.](image)

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Figure 6.8: Comparison of observed data (black) to expected events using the best fit parameters with and without oscillations (blue and red respectively) for the track-like sample when the fit was performed including only this sample. Events are displayed in their analysis binning, moving from the lowest energy bin (top) to highest energy bin (bottom).
corresponding to up-going events. Despite the neutrino flux being maximal near the horizon, the rapid change in neutrino path length between $10^0$ above the horizon and $10^0$ below the horizon creates a valley between the two peaks. This depletion of central values in $L/E$ is further enhanced by the limited energy range of this analysis. Looking first at the full sample in Figure 6.9 we see on the rising edge

Figure 6.9: Top: comparison between data (black points) and simulation with and without oscillations (blue and red respectively) as a function of reconstructed $L/E$. The events displayed are from the most track-like sample (C), using best fit parameters from the full sample shown in Table 6.1. Bottom: ratio of data and oscillated simulation to the unoscillated simulation. There are noticeable excesses in each of the simulated estimates (solid blue and dashed red lines) on the left edge of the peak at 1.5 km/GeV as well as the peak at 300 km/GeV. Error bars on the data (black) are statistical only.
of each of the peaks an over-prediction from the Monte Carlo with respect to the data that persists over several bins. When we look at the distribution of $L/E$ from the fit to just the track-like sample, Figure 6.10, there is much better agreement between the observed data and the fit results. This acts as further confirmation of the previous conclusions with regard to the goodness of fit of each sample.

Figure 6.10: Top: comparison between data (black points) and simulation with and without oscillations (blue and red respectively) as a function of reconstructed $L/E$. The events displayed are from the most track-like sample (C), using best fit parameters from the full sample shown in Table 6.3. Bottom: ratio of data and oscillated simulation to the unoscillated simulation. Here the data agrees well with the best fit MC estimates at all values of $L/E$, and with the unoscillated expectation in the down-going region where oscillations are not expected to occur. Error bars on the data are statistical only.
6.2.1 Measurement and validation of oscillation parameters

We have verified that the confidence limits produced by the full cascade and track sample exceed the sensitivity of this measurement due to mismatch between the observed data and Monte Carlo based predictions in the cascade-like events. Furthermore, we have shown that this disagreement does not bias the best fit values of the neutrino oscillation parameters $\theta_{23}$ and $\Delta m_{32}^2$. But that the disagreement does impact our estimated confidence interval on the mixing angle, significantly over-estimating the precision of our measurement.

When fitting only the track-like sample, the confidence limits are much more in agreement with what we expected to see from the ensemble of data challenges

![Figure 6.11: Measured 90% (solid) and 68% (dashed) confidence limits on oscillation parameters $\sin^2(\theta_{23})$ and $\Delta m_{32}^2$ from fitting only the track-like data sample. The best fit point is shown as an orange cross, and profile likelihoods are drawn by integrating over the other dimension.](image-url)
performed prior to unblinding. The 90% and 68% confidence limits are shown in Figure 6.11. The best fit oscillation parameters show little deviation, as expected because the bulk of the oscillation signal is in the track-like sample.

Although the fit to the track sample alone is statistically acceptable, this does not inherently prove that the fit is unbiased. To address the concern that systematic errors observed in the cascade sample persist in the track sample and may cause a bias toward maximal or beyond-maximal mixing, we again follow the suggestion of [63] to apply the observed residuals to Monte Carlo pseudo-data generated for a variety of possible oscillation parameters. Using the results of the Monte Carlo only bias study presented in Section 5.3.1 as a basis of comparison, we take the actually measured residuals and apply them to the expectations for each bin at different points in the oscillation parameter space. Once again, the fit procedure is applied to each set of pseudo-data and the results are shown in Figure 6.12. The results appear very similar to those obtained from truly random residuals, shown in Figure 5.10; if anything, there may be less bias in the mass splitting apparent with the real residuals than with the Monte Carlo-derived ones. The magnitude of the impact of the frozen residuals on the mixing angle fit appears quite similar to that from the Monte Carlo study. There is no evidence of a systematic error, at least not with magnitude larger than expected from statistical effects.

Finally, one can check whether the fit errors observed for different assumed true oscillation parameters in Figure 6.12 are consistent with the expected statistical precision if those parameters are true, by comparing the fit errors to the predicted contours. Because calculation of expected contours requires ensembles of pseudo-data rather than individual sets, we do this for three widely separated points rather than all 81 shown in Figure 6.12. In each case, the observed difference between the injected and fitted parameters is well within the 68% confidence limit contours predicted for the true values simulated. Of particular note, these contours are created only using the most track-like sample, while the contours presented at the end of Chapter 5, in Figures 5.11–5.13 were generated for the full sample. This is most noticeable in the profile likelihood for the mixing angle in the two pseudo-datasets with non-maximal mixing injected.

The relatively large pull observed in Figure 6.13 is due to the fact that the measurement fundamentally is sensitive to the magnitude of the disappearance rather than the mixing angle per se, and as one approaches maximal mixing there
is a wide range of angles which lead to comparable amounts of disappearance. This effect is accounted for in the expected contours and the observed pull is entirely consistent with those expectations.

Figure 6.12: Fit errors when the residuals observed in the real data are used to generate pseudo-data at 81 sets of oscillation parameters. The results are similar to those from the unbiased Monte Carlo study shown in Figure 5.10; there is a universal trend toward maximal mixing, but consistent with the expectation when applying identical residuals to each pseudo-data set.
Figure 6.13: Comparison of the error when fitting pseudo-data generated with the residuals from the real data to the measurement contours predicted based on an ensemble of pseudo-data generated for those oscillation parameters. The black dot indicates the injected parameters, the orange cross is the statistical expectation for the best fit point, with the dashed and solid lines indicating the 68% and 90% ranges of the ensemble of fits, respectively. The red dot connected to the injected point by a line indicates the result of the fit to the single pseudo-data set based on the residuals from the real data, and is well within the 68% expected contour.
Figure 6.14: Comparison of the result of fitting pseudo-data generated with the real data residuals to the expected range of fit error, as in Figure 6.13, but for different injected oscillation parameters. The result is again consistent with expectations due to statistical fluctuations.
Figure 6.15: Comparison of the result of fitting pseudo-data generated with the real data residuals to the expected range of fit error, as in Figure 6.13, but for different injected oscillation parameters. The result is again consistent with expectations due to statistical fluctuations.
### 6.2.2 Comparison of results

The best fit parameters from the track-only sample are

\[
\sin^2 \theta_{23} = 0.53^{+0.07}_{-0.11}, \\
\Delta m_{32}^2 / 10^{-3} \text{eV}^2 = 2.35^{+0.14}_{-0.17}.
\]

In Figure 6.16, we compare the two dimensional 90% confidence limits from Figure 6.11 to other experimental measurements of \(\sin^2(\theta_{23})\) and \(\Delta m_{32}^2\). A large improvement has been made with respect to the previous IceCube measurement \[57\] shown in solid blue.

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**Figure 6.16:** Measured 90% (solid) confidence limits from the track-like subsample (orange) are compared with the limits set by MINOS \[58\], T2K \[59\], and Super-Kamiokande \[60\]. The best fit point is shown as an orange cross, and profile likelihoods are drawn by integrating over the other dimension.
6.3 Conclusion

We have investigated the goodness of fit of both full data set and the track sample alone. A formal calculation of goodness of fit which takes into account the estimated uncertainties in the model predictions confirms that the fit to the full data set is unacceptable, and that the discrepancies are driven by the cascade-like data; in terms of the probabilities in the tails of a Gaussian, the hypothesis that the data are consistent with the model is rejected at over $4\sigma$. By contrast, there is a 10.1% probability of observing residuals comparable to those seen in the fit to the track sample alone if the model is correct, which we consider an acceptable goodness of fit.

We have also investigated whether the residuals seen in the track-only fit, if due to a systematic error in the model, would produce a bias in the resulting measurement. We find no evidence that the residuals have a greater impact on the fit than would be expected for pure Poisson residuals. The fit errors obtained when these residuals are applied to pseudo-data for several assumed sets of oscillation parameters are well within the 68% confidence level contours.

Investigations into the nature of the problem(s) with the cascade samples continue within the IceCube Collaboration, in preparation for an eventual journal article on these results. While it is not logically possible to disprove the hypothesis that the issues affecting the cascade sample may also produce some subtle bias in the track sample until the nature of the problem is understood, we find no evidence to suggest that the conclusions presented here are incorrect or the results unreliable.

Despite the small differences between the background atmospheric muon template generated from the data side band and the atmospheric muons that sneak into the final event sample, by fitting the only track-like data sample, we have obtained a measurement of the oscillation parameters that drive atmospheric muon neutrino disappearance that is competitive with dedicated beam experiments. This work is being actively continued within the IceCube collaboration with the goal of including the cascade-like events through obtaining a better background estimate and better parameterizing the systematic effects that were found problematic in this analysis when the full event sample that I developed was included.
Bibliography


[63] D. Fox, personal communication.
Vita
Matthew Gregory Dunkman

Education

2015 Ph.D. Pennsylvania State University; University Park, PA

2009 Bachelor of Science Michigan Technological University; Houghton, MI

Experience

2010–2015 Research Assistant Pennsylvania State University
Measurement of atmospheric neutrino oscillation parameters $\theta_{23}$ and $\Delta m^{2}_{31}$ through development of a new energy and angular reconstruction for low-energy (sub TeV) neutrino interactions in the IceCube detector and overhaul of the oscillation event selection. Maintained the Penn State IceCube group’s software on the local HPCC.

2009–2011 Teaching Assistant Pennsylvania State University
Instructed introductory physics labs and recitations.

2008–2009 Undergraduate Research Assistant Michigan Tech University
Studied reconstruction ability of fluorescence detectors on distance events with the Pierre Auger Observatory. Used cosmic ray simulation software AIRES to look for imprint of atmospheric tau leptons on the surface array.

June–August 2006 Summer Intern Fermilab; Batavia, IL
Found correlation between calibration measurements of cathode strips in the muon tracking end-caps of the Compact Muon Solenoid and data taken during test runs.

2006–2009 Undergraduate Teaching Assistant Michigan Tech University
Instructed introductory physics labs and one junior-level optics lab. Received AAPT Outstanding Teaching assistant award in 2008.