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Abstract

Bargaining models of multilateral exchange have to contend with the possibility that a part of a proposed multilateral deal for a set of parties may still be a feasible and consensual deal for some of the parties. The first two chapters are based on embedding this possibility in two extensive form bargaining games for coalitional environments known in the literature. The third chapter explores in a simple credit relationship between a creditor and an entrepreneur with a sequential investment project, whether the strategy to commit not to refinance a project in the event of default of any value to the creditor. It is also concerned with the extent to which competition on the entrepreneurial side of the market and the informational environment may make such commitment sequentially rational. The final chapter studies an exchange environment with weak protection of property rights so that power is the basis of exchange. With property rights and income distribution constantly in flux, the question is whether the imperative to maintain stability can act as a constraint on classical allocative efficiency.

In Chapter 1, a new feature pertaining to proposer's ability to implement offers is introduced in the extensive form bargaining mechanism studied in Okada (1996). This mechanism is used to analyze two classes of coalitional games with transferable utility. One class is that of strictly supermodular games; the other has the property that per capita value is increasing as a coalition adds to its members. The new feature in the mechanism is that the proposer has a choice to implement his proposal with any subset of responders who have accepted it. It is shown that for all sufficiently high discount factors δ , there exists an efficient subgame perfect equilibrium in pure stationary strategies (SSPE) whose limiting outcome is the core-

constrained Nash Bargaining Solution. For strictly supermodular games, Core constraints are binding on Nash Bargaining Solution while for the other class they are not. Also, all efficient SSPE are payoff-equivalent in the limit as $\delta \to 1$.

In Chapter 2, a new feature pertaining to proposer's ability to implement offers is introduced in the extensive form bargaining mechanism studied in Chatterjee, Dutta, Ray and Sengupta (1996). This mechanism is used to analyze two classes of coalitional games with transferable utility. One class is that of strictly supermodular games; the other has the property that per capita value is increasing as a coalition adds to its members. The new feature in the mechanism is that the proposer has a choice to implement his proposal with any subset of responders who have accepted it. It is shown that for all sufficiently high discount factors δ , there exists an efficient subgame perfect equilibrium in pure stationary strategies (SSPE) whose limiting outcome is the core-constrained Nash Bargaining Solution. For strictly supermodular games, Core constraints are binding on Nash Bargaining Solution while for the other class they are not.

In Chapter 3 which is coauthored with my advisor Kalyan Chatterjee, a simple contracting environment with a creditor who has wealth and a entrepreneur who has a two-period investment project is studied. After observing the partial completion of the project at the end of first period, the creditor may decide whether to refinance it or liquidate it. Contracting is subject to moral hazard and limited liability each period. The creditor prefers a contract that commits him not to refinance if and only if the extent of moral hazard problem is sufficiently high. Such a commitment may not be sequentially rational, however. The role of competition and hidden information in enforcing commitment is studied. When credit supply is scarce and the creditor lacks commitment, competition from other potential entrepreneurs may not be a credible deterrent against refinancing because of the wedge between the liquidation value of the output and its value as an input for second period. Sufficient conditions are developed under which hidden information makes commitment sequentially rational.

In Chapter 4, an equilibrium existence theorem is established for a stochastic game model with discounted payoffs in which at each date every player has opportunities to redefine the prevailing state. A state in the model is a property rights allocation over an asset and a distribution of income from the asset among all the players. Property rights may be held by coalitions which may differ in terms of their power and their productivity with the asset. A transition from one state to another is feasible if the proposed owning coalition is at least as powerful as the current owning coalition and everyone in the proposed owning coalition consents to the change. State transitions in the stochastic game are endogenous because they are constrained by players' threat positions which are endogenous. A stylized example illustrates that when power is the basis of exchange, stability may be a constraint on classical allocative efficiency.

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Chapter 1

Efficient Coalitional Bargaining with Noncontingent Offers
Part I: Random Proposer Protocol

1.1 Introduction

Bargaining models of multilateral exchange must contend with the possibility that a part of a proposed multilateral deal for a set of parties may still be a feasible and consensual deal for some of the parties. This possibility is nonexistent in a bilateral deal because a deal by its definition needs a minimum of two parties to consent.

To facilitate collective decision making, institutions have evolved that are sparse in terms of the criteria needed to conclude multilateral deals. The criteria is set in terms of the number of parties who consent to the deal. Two arrangements at the extreme ends of such a criteria are dictatorship which requires consent of just one party and veto power to everyone (unanimity) that requires consent of every party. In between such extremes are majority rules. A veto power given to parties in a bilateral deal is equivalent to individual consent. In a multilateral deal, however, a veto power given to any party is more than individual consent. It is the power to encroach over the consent of other parties, irrespective of their number. It is also the power to encroach over voluntary and mutually beneficial deals that may be struck by other parties.

Often there are no institutionalized negotiation rules, for instance, parliamentary voting based on some kind of majority rule, that are legally enforceable on the parties. Sovereign debt renegotiation is a prominent example. In such cases, there are parties whose consent is more important than the consent of others in concluding any deal. Such relative importance is determined by whatever is the relevant notion of power for the environment. For our purposes, it is the ability to create value as captured in a coalitional game.

As documented in Hornbeck (2010) and Alfaro (2014), Argentina, after defaulting in 2002, on its legally incurred sovereign debt, went in for negotiations with its private creditors for debt restructuring. The negotiation process for sovereign debt restructuring is not legally enforceable. After failing to agree on the terms, Argentina made a unilateral take-it-or-leave-it offer to settle in the 2005 Bond Exchange. 76% of the creditors accepted the offer. The 2010 Bond Exchange took the acceptance to 91.3%. This has created two coalitions of bondholders: the exchange bondholders who have consented to the restructured deal and the holdouts who have not and are litigating in an attempt to get their full face value. In 2012, US Court of Appeals for Second Circuit, interpreting *pari passu* clause, prohibited Argentina from paying one class of creditors while others receive nothing, effectively giving a huge leverage to holdouts. In 2014, US Supreme Court declined to hear Argentina?s appeal against the ruling[1]. In theoretical terms, the court has ruled in favor of an extreme criteria among a spectrum of criteria possible to conclude a deal. It has effectively given veto power in the hands of every creditor and the ruling is being widely seen as an impediment to restructuring deals.

Our objective, in this paper, is not to study the specific setting of sovereign debt renegotiation, but rather to take existing models of multilateral bargaining and relax any enforceable institutional requirement like veto power or majority rule for conclusion of deals. Thus the relative importance of consent of various parties will be determined endogenously by their ability to create value and their threat positions. Some parties will endogenously get veto

[1]http://www.nytimes.com/2014/06/20/business/economy/ruling-on-argentina-gives-investors-an-upper-hand.html?$r_r = 0$

power, others will not. The upshot is that relative to existing models, efficiency is robust to minor procedural details of who makes the offer.

Our environment is an n-person coalitional game with transferable utility for which a well developed solution concept is the Core. The stability requirements imposed on allocations in the definition of the Core is the primary reason for its theoretical appeal. The Core emerges exactly as the set of stationary equilibrium outcomes of the n-person bargaining model (without discounting) of Perry and Reny (1994) [2]. Moldovanu and Winter (1995) also find the Core emerging as the set of payoffs of their order-independent equilbria of a family of undiscounted bargaining games. [3]

The n-person pure bargaining game (where the only possible outcomes are complete cooperation of all players or complete breakdown of cooperation) has been studied independently for which a well developed solution concept is the Nash Bargaining Solution. It was axiomatically derived for bilateral case by Nash Jr (1950) and later shown by Binmore et al. (1986) to be the limiting (as $\delta \to 1$) equilibrium outcome of two-person bargaining model of Rubinstein (1982). A similar limiting result was shown by Krishna and Serrano (1996) for the n-person case.

The central question of our concern is: What payoff outcomes can we expect in n-person coalitional games when parties do not have recourse to legally enforceable institutions, for instance, majority rule or universal veto power, for concluding multilateral deals in negotiations. In this paper, we only give a limited answer to this question. We find there is a class of n-person coalitional games which may be analyzed essentially as n-person pure bargaining games. For this class, the Nash Bargaining Solution remains a limiting equilibrium outcome. More interestingly, there is a class of n-person coalitional games which cannot be analyzed as their associated n-person pure bargaining games but the pure bargaining games play a critical role for payoff outcomes. Thus for this class, both Core and Nash Bargaining

[2] The environment in Perry and Reny (1994) is a totally balanced TU game.

[3] The environment in Moldovanu and Winter (1995) is a strictly superadditive NTU game that has nonempty core for each of its component games.

Solution are important for a limiting equilibrium outcome. Some of these findings have been noticed by Chatterjee et al. (1993) in a bargaining mechanism where every responder has veto power over the proposed offer but their results are limited by possible inefficiencies that may arise if the initial proposer is not chosen carefully. In this paper, we take a variant of the their model studied by Okada (1996) and relax any requirement (based on number of parties giving consent which include unanimity and majority rules) to conclude a deal that may not be enforceable. In a companion paper Chaturvedi (2013b), we embed this feature in the mechanism studied by Chatterjee. et. al. (1993).

We focus on studying two settings that have been studied before. One class, **S**, is that of strictly supermodular games [4]. It has the property that players are complements for coalition formation. The other class of games, **G**, have the property that per capita value is increasing as a coalition adds to its members. For either class, an efficient outcome has immediate formation of grand coalition. The class of games **G** and **S** are unrelated in that neither is a subset of the other. However both have nonempty cores [5]. An example of a strictly supermodular setting is a production partnership game [6]. The problem to be studied in this environment is to determine the coalitional structure i.e. which coalitions form and for each such coalition formed, how is the surplus that accrues to that coalition shared among its members.

The literature on noncooperative analysis of coalitional games was pioneered by Selten (1980) and Harsanyi (1974). Since then, the literature can be classified along several dimensions. Selten (1980), Moldovanu and Winter (1994) and Compte and Jehiel (2010) study bargaining protocols that terminate as soon as the first coalition is formed. Chatterjee et. al.(1993), Okada (1996) and Moldovanu and Winter (1995) study bargaining protocols that allow the

[4] They are traditionally called strictly convex games.

[5] The necessity of nonempty core for existence of efficient stationary equilibria is a result that holds in a variety of mechanisms studied and holds in the mechanism studied here as well.

[6] Each player owns some factors of production (like land or labor). Players have access to a (convex) production technology that displays increasing marginal productivity. Players only make participation decisions. A coalition S by cooperating can pool their factors of production and generate a value that is just the production output. See Rosenmüller (1981) and Example 18.A.A.6 in Mas-Colell et al. (1995).

possibility of multiple agreements.

Previous literature on bargaining mechanisms for games in **S** that deals with impatient players either does not get efficiency with probability one (Okada (1996)) or gets it only for a particular choice of the player who makes the first proposal (Chatterjee et. al.(1993)) or only by imposing that bargaining terminates after the first coalition forms (Compte and Jehiel (2010)). A common feature of this literature is that it gives veto power to all responders towards whom the proposal is directed. This embodies a constraint on the proposer?s ability to implement offers- he cannot implement his offer if some responder rejects it. Put in a different way, the offers made by the proposer are *contingent offers* - their implementation is contingent on acceptance by everyone to which the offer is directed. Indeed, the reason for inefficiency in these models is that when you give veto power to every responder, it increases their demands and so it gets costly for some players to propose to the grand coalition. They would rather propose to a smaller coalition and satisfy their veto-demands than propose to the grand coalition and satisfy everyone?s veto-demands.

We now describe our mechanism recursively. Suppose the state of the game is such that players in S are still negotiating while the rest have left the game with some agreements reached in some fashion. At this point, a player in S is randomly chosen to be the proposer with probability $1/|S|$ [7]. The proposer makes a proposal which is a coalition $T \subset S$ and a distribution x_T of surplus of that coalition. Responders in T then move sequentially in some order saying Yes or No. After everyone has responded, the proposer decides whether to partially implement his offer with all, some or none of the responders who have accepted the offer. When a proposer decides to implement his offer with T_I (which if not empty necessarily includes the proposer), he gives each responder j in T_I what he offered to him i.e. x_j and he gets the residual of the surplus. The state then changes to one in which $S \setminus T_I$ is the set of players still negotiating in the game [8]. There is discounting when a new proposer

[7] Chatterjee et. al.(1993) mechanism differs at this point in that it has a fixed player chosen.

[8] Compte and Jehiel (2010) do not allow the rest of the players to continue bargaining once a coalition has formed.

is chosen. Players are expected utility maximizers. The notion of equilibrium is stationary subgame perfect equilibrium (SSPE).

Our mechanism embodies a proposer's ability to make noncontingent offers- even if some responder in his proposed coalition has rejected it, he has a choice to implement it with a subset of responders who have accepted it. In strictly supermodular environments, this ability to walk away with a subcoalition makes the proposer more powerful in that it potentially gives him access to a threat. It turns out this is enough for getting efficiency with probability 1. We will elaborate it further after stating our results.

The main result in this paper is to show for all games in $\mathbf{G} \cup \mathbf{S}$, for all sufficiently high discount factors, there exists an efficient pure strategy SSPE whose limiting outcome is the core-constrained Nash Bargaining Solution [9]. For games in $\mathbf{G}$, the Core does not act as a binding constraint on the Nash Bargaining Solution. For games in $\mathbf{S}$, the Core is a binding constraint on the Nash Bargaining Solution. We give a constructive proof describing a recursive algorithm for computing the proposals made by the players to the grand coalition in this SSPE. Also, efficient SSPE are payoff-equivalent in the limit as $\delta \to 1$. This limit value is the core-constrained Nash Bargaining Solution. To summarize, efficiency for strictly supermodular games in our mechanism is not sensitive to the choice of initial proposer, obtains in pure strategies and without artificially imposing any behavior that is unnatural to the environment.

The ideas behind our constructive existence proof are properties of strictly supermodular settings. These are the algorithmic characterization of the core-constrained Nash Bargaining Solution for supermodular games shown by Dutta and Ray (1989), a further monotonicity result about such allocation shown in Dutta (1990) and a result due to Compte and Jehiel (2010) about nested structure of coalitions for which the core constraints are binding at any core allocation. In the equilibrium we construct, the set of coalitions for which the core constraints bind at the core-constrained Nash Bargaining Solution are precisely those that

[9] Compte and Jehiel (2010) call it the Coalitional Nash Bargaining Solution.

constitute credible coalitional threats. First we partition the players by using the result of Compte and Jehiel (2010) at the core-constrained Nash Bargaining Solution. Our description of equilibrium proposals is a result of two recursive algorithms. The first algorithm inductively describes what will be threat points of players in the equilibrium. In doing this, we 'anchor' the discounted continuation value of our equilibrium in the core-constrained Nash Bargaining Solution. A responder who is not part of a coalitional threat that a proposer uses must be willing to lower his demand relative to what he would have demanded if he had veto power over the offer. This responder can therefore be held *hostage* and suffer a *disadvantage* at the hands of this proposer. An interesting feature of the equilibrium is that the proposer is forced to concede this responder more than what he demands. This is the cost he has to pay in order to maintain the credibility of his coalitional threat. Care needs to be exercised at this point in deciding which players are held *hostage* by which players. We do this in a natural way suggested by the hierarchical [10] nature of partition of players. The purpose of the second algorithm is to describe the proposer's advantage and the *hostage's disadvantage*. This is done by inductively using the equilibrium condition and the feasibility condition.

The idea of the proof for uniqueness of the limit allocation in an asymptotically efficient SSPE starts with the observation that limit value of any such equilibrium is a core allocation. The result of Compte and Jehiel (2010) then tells us that credible coalitional threats must have a nested structure. This generates a partition of players. Next we argue that all players in any block of partition get the same payoff in the limit. Since the blocks of partition are also credible coalitional threats, the limit allocation must have a hierarchical structure. The limit allocation and the coalitional threats must be such that every player's payoff is simultaneously maximized in equilibrium. These simultaneous maximization problems lead to hierarchical maximization problem of precisely the same sort that characterizes the core-

[10]The interpretation of the partition as hierarchical follows from Dutta and Ray (1989). They show that the core-constrained Nash Bargaining Solution has the property that players higher in hierarchy get more than those lower in hierarchy.

constrained Nash Bargaining Solution of supermodular games as showed in Dutta and Ray (1989). Each step's problem must take as a constraint the solution to previous steps. This unravels the optimal coalitional threats as well as the limit allocation inductively.

The plan of the paper is as follows. After introducing the model in Section 2, we give a statement of our results in Section 3. The proof of the existence result is exposited as follows. First we discuss the equilibrium construction in a simple 3-player example and contrast the efficiency implication with the other mechanisms that have been studied. The candidate equilibrium is described in Sections 4.1, 4.2, 4.3 and 4.4 both for a restricted model in which only one coalition is permitted to form as well as the model without this restriction. The reason we exhibit the equilibrium for a restricted model first is that we only have to deal with the game with all players in it. There are no subgames with a smaller population. Also, the acceptance-rejection strategies are simple for the restricted model. Optimality of the strategies is discussed in Section 4.5. A monotonicity property of core-constrained Nash Bargaining Solution for strictly supermodular games shown in Dutta (1990) then assures us that the strategies so constructed can be supported as an SSPE in our model for all sufficiently high discount factors. The uniqueness of limit allocation for any asymptotically efficient equilibrium is discussed in Section 5. We conclude in Section 6.

1.2 The Model

1.2.1 The Coalitional Game

Let $N = \{1, \ldots, n\}$ be the set of all players. Let (N, v) be a coalitional game with transferable utility. Any coalition, $S \subset N$ has a nonnegative worth, $v(S) \geq 0$. We will denote the set of all coalitions of N by $\mathscr{C}$. When a coalition agrees to a payoff allocation, it can fully commit to it and there are no enforcement problems in implementing that agreement. We now describe some coalitional environments that have been studied in the literature.

(N, v) is strictly superadditive if

$$\forall S, T \subset N, \quad S \cap T = \emptyset, \quad v(S \cup T) > v(S) + v(T)$$

(N, v) has increasing returns per capita as a coalition adds to its members if

$$\forall S, T \subset N, \quad S \supset T, \quad \frac{v(S)}{|S|} > \frac{v(T)}{|T|}$$

(N, v) is strictly convex if

$$\forall i \in N, \quad \forall S, T \subset N \setminus i, \quad S \supset T, \quad v(S \cup \{i\}) - v(S) > v(T \cup \{i\}) - v(T)$$

Let **G** denote the class of games that has increasing returns per capita. Let **S** denote the class of strictly supermodular games. For both these environments, the economy splitting up into coalitions is an inefficient coalitional structure. The only efficient structure is the formation of the grand coalition. Also supermodular environments are superadditive as well. The following definition will be useful to us.

Definition 1. Suppose $b^\star$ is a core allocation for a game (N, v). Then we say S is a binding coalition with respect to $b^\star$ if $b^\star(S) = v(S)$.

1.2.2 The Bargaining Mechanism

In any period $t = 1, 2, \ldots$, let S be the set of active players still in the game. A player from S is randomly chosen to be the proposer. Draws are independent and each player has an equal chance $1/|S|$ of being chosen. A proposer makes an offer (T, x_T) where $i \in T \subset S$ and $\sum_{i \in T} x_i = v(T)$. Players in $T \setminus i$ then respond sequentially according to some given order ϕ. Suppose $T_A \subset T$ accept the offer. Then player i can choose whether to implement his offer (T, x_T) with a coalition $T_I \subset T_A$, $i \in T_I$ or discard the offer. If i implements (T, x_T) with some T_I , then coalition T_I exits the game. Every player j in $T_I \setminus i$ gets x_j and i gets the residual $v(T_I) - \sum_{j \in T_I \setminus i} x_j$. The game continues with the set of active players

$S \setminus T_I$. If i discards the offer (i.e. chooses not to partially implement his offer), the game continues with the set of active players unchanged at S. The offers made to each prospective coalition partner are *noncontingent* in the sense that they are not contingent on acceptance by everyone in the proposed coalition. Preferences are linear in the share and intertemporal preferences are just discounted utility preferences with a common discount factor δ. Players are expected utility maximizers.

For any SSPE σ of the extensive form game $G(N, v)$ described above, let $u(S, \sigma) \in \mathbb{R}^S$ be expected payoff at a chance node when S is the set of players still in the game. Let $u^\star(S, \sigma) = \lim_{\delta \to 1} u(S, \sigma)$. Let $b(S, \sigma) = \delta u(S, \sigma)$.

We?ll refer to the environment and the mechanism described above as the unrestricted model. This is the object of our study and our results pertain to the unrestricted model. However, for expositional purposes, we find it convenient to work with a version of the mechanism where the bargaining terminates as soon as one coalition forms. We refer to this version as the restricted model. As noted in the last paragraph of the introduction, we exposit the strategies for the restricted model only. However, we do point out, what will be the corresponding strategies in the unrestricted model. Again when we discuss optimality of strategies, we carry out the proof for the restricted model. But we do point out what ensures perfection in the unrestricted model.

1.3 Results

For stating our results, we will need the following definition.

Definition 2. Core-constrained Nash Bargaining Solution is the allocation that maximizes the Nash product among all allocations in the core. For any (N, v) with a nonempty core, this is uniquely defined.

$$\max_{x\in\mathbb{R}^n} \prod_{i\in N} x_i$$
$$\text{subject to } x(N) = v(N)$$
$$\forall S \subsetneq N,\ x(S) \geq v(S)$$

Proposition 1. For all games in $\mathbf{G} \cup \mathbf{S}$ and for all sufficiently high discount factors, there exists an efficient pure strategy SSPE whose limiting outcome is the core-constrained Nash Bargaining Solution.

Proposition 2. For all games in $\mathbf{G} \cup \mathbf{S}$, all *efficient* SSPE are payoff-equivalent in the limit as $\delta \to 1$. This limit value is the core-constrained Nash Bargaining Solution.

1.4 Proof of Proposition 1

For strictly supermodular games $\mathbf{S}$, the equilibrium construction is rather involved. So for ease of exposition, we will first consider a 3-player example that will illustrate the equilibrium construction and contrast it with the other mechanisms that have been studied.

Example 1. Consider the following 3-player strictly supermodular coalitional game.

S	v(S)	S	v(S)	S	v(S)
{1}	0	{2}	0	{3}	0
{1,2}	0.7	{1,3}	0.2	{2,3}	0.2
{1, 2, 3}	1	∅	0		

Remark. 1. The core-constrained NBS in this game is the allocation $u^\star = (0.35, 0.35, 0.3)$.

2. The set of coalitions for which the core constraints are binding, $\mathscr{S}_b = \{\{1,2\},\{1,2,3\}\}$.

3. For the Chatterjee et. al.(1993) mechanism, if Player 1(or 2) is the initial proposer, he proposes to $\{1,2\}$ in the unique SSPE and the limiting outcome is $(0.35, 0.35, 0)$.

4. For the Okada mechanism (1996), a pure strategy SSPE does not exist.

We claim that for $\delta \geq 0.7$, the following proposal strategies are supported as an SSPE.

Proposal Strategies. Player i makes an offer $u(\delta, N, i)$ to the grand coalition N. The offers are:

$$u(\delta, N, 1) = (b_1 + a_1, b_2, b_3 - h_3)$$
$$u(\delta, N, 2) = (b_1, b_2 + a_2, b_3 - h_3)$$
$$u(\delta, N, 3) = (b_1, b_2, b_3 + a_3)$$

where

$$b_1 = 0.35 \qquad b_2 = 1.5 \qquad b_3 = \delta - 0.7$$
$$a_1 = \big(\frac{3}{\delta} - 3\big)b_1 \qquad a_2 = a_1 \qquad h_3 = a_1 - (1 - \delta)$$
$$a_3 = \big(\frac{3}{\delta} - 3\big)b_3 + 2h_3$$

Implementation Strategies. For equilibrium proposals

(1) Player 1.

• P1 threatens to partially implement the equilibrium offer with $\{1, 2\}$ if P3 rejects. He's indifferent between doing so and discarding the offer (i.e. pass and let the game continue).

• P1 strictly prefers to discard the equilibrium offer if P2 rejects.

(2) Player 2.

• P2 threatens to partially implement the equilibrium offer with $\{1, 2\}$ if P3 rejects. He's indifferent between doing so and discarding the offer.

• P2 strictly prefers to discard the equilibrium offer if P1 rejects.

(3) Player 3 discards the equilibrium offer whenever anyone rejects.

Acceptance-Rejection Strategies. Any responder j's strategy depends on whether conditional on his rejection, the proposer i discards the offer or partially implements his proposal. Suppose i discards it. Then for the equilibrium proposal, j accepts if it gives him at least $b_j(N,\sigma)$ and rejects otherwise while for off-equilibrium proposals, he accepts if he gets more than $b_j(N,\sigma)$ and rejects otherwise. Suppose i partially implements it. Then for the equilibrium proposal, j accepts if it gives him at least $\delta v(j)$ and rejects otherwise while for off-equilibrium proposals, he accepts if he gets more than $\delta v(j)$ and rejects otherwise.

We illustrate the argument that the equilibrium offer is accepted in the subgame when P1 makes a proposal. The arguments are order independent i.e. do not depend on the order in which responders move. Consider P2 irrespective of his place in the order. Suppose P2 rejects. Then P1 will discard the offer and P2 gets b_2. Since $u_2(\delta,N,1) = b_2$, P2 is indifferent in which case he accepts as per his ARS. Consider P3. No matter whether he responds before or after P2, P2 has accepted or will accept the offer by the preceding argument. So if P3 rejects, P1 will certainly implement with $\{1,2\}$ and he'll be left with a payoff of 0. If P3 accepts, the grand coalition will form and he will get $u_3(\delta,N,1)$. Since $u_3(\delta,N,1) = b_3 - h_3 > 0$ for sufficiently high δ, P3 accepts.

The optimality of Acceptance-Rejection Strategies is clear and the optimality of Implementation Strategies can be checked.

Optimality of Proposal Strategy. Again we show the arguments for $G(\delta,N,1)$. Classify deviations as

(1) (N,d) where $d_2 = u_2(\delta,N,1) = b_2$, $0 < d_3 < u_3(\delta,N,1) = b_3 - h_3$. This is the case where P1 tries to gain unilaterally at the expense of P3. Consider the order $\phi = 23$. Consider P2. If he rejects, then P3 also rejects because the continuation game is more profitable to P3. So P2 gets b_2. If he accepts, P3 also is forced to accept and P2 gets b_2. So P2 is indifferent in which case he rejects as per his ARS. This prompts a rejection by P3 as well. So the deviation is unanimously rejected.

Consider the order $\phi = 32$. If P3 rejects, then P2 finds himself indifferent between accepting and rejecting. So P2 rejects as per his ARS. P3 certainly prefers rejecting, so he rejects as well.

(2) (N, d) where $d_2 > u_2(\delta, N, 1)$, $0 < d_3 < u_3(\delta, N, 1) = b_3 - h_3$. This is the case where P1 tries to bribe P2 in order to gain at the expense of P3. P3 is certain that if he rejects, P1 will not implement d with $\{1,2\}$ because $v(\{1,2\}) - d_2 < v(\{1,2\}) - u_2(\delta, N, 1) = \delta u_1(\sigma)$. Now P3 certainly prefers rejecting to accepting as $d_3 < u_3(\delta, N, 1) < \delta u_3(\sigma)$. So P3 rejects. Now P2 knows that P3 reasoning as above will reject and P1 will not implement d with $\{1,2\}$ and hence P1's sweetened offer to him is just a *mirage* which is never going to materialize. So P2 is also indifferent and hence he rejects as per his ARS.

(3) $(\{1,2\}, d)$ where $d_2 = u_2(\delta, N, 1) + \epsilon$ for $\epsilon > 0$. The payoff for P1 from such an offer is

$$\begin{aligned} d_1 &= v(\{1,2\}) - d_2 \\ &= 1 - (u_2(\delta, N, i) + \epsilon) \\ &= 1 - (b_2 + \epsilon) \\ &= b_1 - \epsilon \\ &< b_1 + a_1 = u_1(\delta, N, 1) \end{aligned}$$

Thus P1 cannot gain by this deviation.

(4) $(\{1,3\}, d)$ where $d_3 = b_3 + \epsilon$ for $\epsilon > 0$. The payoff for P1 from such an offer is

$$\begin{aligned} d_1 &= v(\{1,3\}) - d_3 \\ &= 0.2 - (b_2 + \epsilon) \\ &< b_1 - \epsilon \\ &< b_1 + a_1 = u_1(\delta, N, 1) \end{aligned}$$

Thus P1 cannot gain by this deviation.

Now we will deal with any strictly supermodular game.

1.4.1 Partition of players

Given a strictly supermodular (N, v), let $\{N_1, \ldots, N_L\}$ be the partition of the set of players N induced by the core-constrained Nash Bargaining Solution. We know this partition can be provided because of the following known result.

Claim D. Compte and Jehiel (2010). For (N, v) strictly supermodular, the set of binding coalitions $\mathscr{S}$ with respect to a core allocation $u^\star$ is nested. That is $\mathscr{S}$ is of the form $\{S_1, S_1 \cup S_2, \ldots, S_1 \cup \ldots \cup S_L\}$. This naturally induces a partition $\{S_1, S_2, \ldots, S_L\}$ of players.

We remark here that the partition generated from this result may differ from that generated by the algorithm described in Dutta and Ray (1989) to characterize the core-constrained Nash Bargaining Solution. Also note that for games in $\mathbf{G}$, there is no partition of players i.e. $L = 1$. From the construction described in what follows, the strategies for games in $\mathbf{G}$ can be read out accordingly keeping this in mind.

1.4.2 Proposal Strategy

is described in terms of base payoffs, proposer's advantage and hostage's disadvantage.

Consider the game $G(N, v)$ with the full set N as the population of players. The proposal strategy is described as additions and subtractions from the discounted value of the game, $b(N, \sigma) \in \mathbb{R}^n_+$. Player i can get more than $b_i(N, \sigma)$ only as a proposer and we call that addition as proposer's advantage. Player i can get less than $b_i(N, \sigma)$ only as a responder. When this happens, we say that the proposer has taken i as a *hostage* and i's loss relative to $b_i(N, \sigma)$ as *hostage's disadvantage.*

We now describe the proposal strategies. Whenever $i \in N$ makes a proposal, he offers

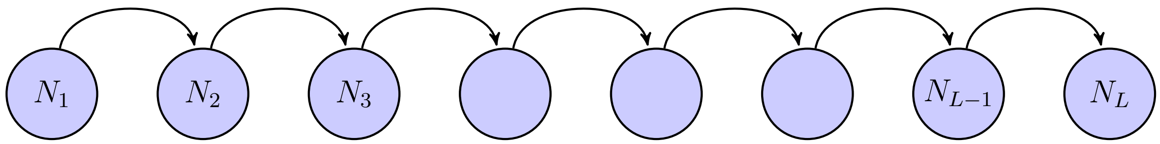

Figure 1.1: Equilibrium Structure of "Hostages"

$(N, u(\delta, N, i))$ where for $i \in N_k$, for $j \in N_{k+1}$ and for $l \in N \setminus (N_{k+1} \cup \{i\})$

$$u_i(\delta, N, i) = b_i(N, \sigma) + a_i(N, \sigma)$$
$$u_j(\delta, N, i) = b_j(N, \sigma) - h_j(N, \sigma)$$
$$u_l(\delta, N, i) = b_l(N, \sigma)$$

Figure 1 depicts the structure of *hostages* in the equilibrium that we construct. For any $k < L$, any player in N_k takes all the players in N_{k+1} as his hostages. It's fruitful now to define what we mean by a threat point of a player as a proposer.

Definition 3. For a strategy profile σ of the bargaining game $G(N, v)$, let $(S, u(\delta, S, i))$ be i's offer in in the subgame $G(S, v)$. Then coalition $T \subsetneq S$ and the restricted offer to T, $u_T(\delta, S, i)$ is a credible threat for $i \in S$ with respect to his offer if
(i) i gives his coalition partners in T their discounted expected continuation value. i.e. $\forall j \in T \setminus i$, $u_j(\delta, S, i) = b_j(S, \sigma)$.
(ii) It is locally optimal for i to implement T at his implementation node. i.e. $v(T) - \sum_{j \in T \setminus i} u_j(\delta, S, i) \geq b_i(S, \sigma)$.
(iii) i threatens to implement his offer $u(\delta, S, i)$ with T at his implementation node.

Our description of equilibrium proposals is a result of two recursive algorithms. The first algorithm inductively describes what will be threat points of players in the equilibrium. In doing this, we 'anchor' the discounted continuation value of our equilibrium in the core-constrained Nash Bargaining Solution. A responder who is not part of a coalitional threat that a proposer uses must be willing to budge from his discounted continuation payoff. This responser can therefore be held *hostage* and suffer a *disadvantage* at the hands of this proposer. Care needs to be exercised at this point in deciding which players are held *hostages* by which players. We do this in a natural way suggested by the hierarchical nature of partition of players. The purpose of the second algorithm is to describe the proposer's advantage and the hostage's disadvantage. This is done by inductively using the equilibrium condition and the feasibility condition.

RECURSIVE ALGORITHM TO COMPUTE CREDIBLE THREATS

We now describe a simple recursive algorithm for computing credible threats. The computation is based on three features. First, the set of credible coalitional threats is precisely $\{N_1, N_1 \cup N_2, \ldots, N_1 \cup \ldots \cup N_L\}$. Owing to the way we have partitioned the set of players, this means the set of credible coalitional threats is precisely the set of coalitions for which the core constraints are binding at the core-constrained Nash Bargaining Solution. Thus the only players who do not have a coalitional threat are those in the last block of partition, N_L. The second feature can be described as symmetry. Players in the same block of partition (because of the first feature, this means they have the same coalitional threat) have the same discounted value of equilibrium. Lastly, a player who is part of a coalitional threat is indifferent between implementing this threat and not implementing it.

Step 1. For $i \in N_1$, his coalitional threat is N_1. The symmetry and indifference feature immediately give $b_i(N, \sigma)$.

$$v(N_1) - (|N_1| - 1)b_i(N, \sigma) = b_i(N, \sigma)$$

which gives

$$b_i(N, \sigma) = \frac{v(N_1)}{|N_1|}$$

Suppose the credible threats have been computed for $i \in N_1, \ldots, i \in N_{k-1}$.

Step k. For $i \in N_k$, his coalitional threat is $\cup_{a=1}^{k-1} N_a$. Note for $j < k$, $b_j(N, \sigma)$ has already been computed at Step j. The symmetry and indifference feature immediately give

$$v(\cup_{a=1}^{k} N_a) - \sum_{j<k} b_j(N, \sigma) - (|N_k| - 1)b_i(N, \sigma) = b_i(N, \sigma)$$

which gives

$$b_i(N,\sigma) = \frac{v(\cup_{a=1}^{k} N_a) - v(\cup_{a=1}^{k-1} N_a)}{|N_k|}$$

Step L. For $i \in N_L$, set

$$b_i(N,\sigma) = \frac{\delta v(N) - v(\cup_{a=1}^{L-1} N_a)}{|N_L|}$$

The full construction of proposals will justify the above assignment.

The equilibrium we construct has the feature that for $L \geq 2$, for $k < L$, for $i \in N_k$, if player i is the proposer, then players in N_{k+1} are *hostages* of i.

RECURSIVE ALGORITHM TO COMPUTE PROPOSER'S ADVANTAGE AND HOSTAGE'S DISADVANTAGE

In what follows, for a player $i \in N_k$, a_i will denote his proposer's advantage when he is the proposer and h_i will denote his disadvantage when he is held a hostage (this happens when some player in N_{k-1} is a proposer). The computation has the feature that players in the same block of partition have the same disadvantage.

Step 1. For $i \in N_1$, set his proposer advantage as

$$a_i(N,\sigma) = \Big(\frac{|N|}{\delta} - |N|\Big) b_i(N,\sigma)$$

This is derived by writing the equilibrium condition. If $L = 1$, then there's nothing more to describe since there are no hostages in this case. Stop. If $L \geq 2$, move to the next Step.

Step k for $k \leq L$. This is the inductive step in the Algorithm. There are two substeps.

Step k.1. For $i \in N_k$, his disadvantage as a hostage, $h_i(N,\sigma)$ is determined from the feasibility condition of the proposal made by $j \in N_{k-1}$ and by noting that $b(N) = \delta v(N)$:

$$b_j + a_j + (|N_{k-1}| - 1) b_j + |N_k| (b_i - h_i) + \sum_{m \in N \setminus (N_k \cup N_{k-1})} b_m = v(N)$$

$$h_i(N,\sigma) = \frac{a_j(N,\sigma) - (1-\delta) v(N)}{|N_k|}$$

Step $k.2$. For $i \in N_k$, his proposer's advantage, a_i is determined from equilibrium condition

$$\begin{aligned} a_i(N,\sigma) &= \Big(\frac{|N|}{\delta} - (|N| - |N_{k-1}|)\Big) b_i - |N_{k-1}| (b_i - h_i) \\ &= \Big(\frac{|N|}{\delta} - |N|\Big) b_i(N,\sigma) + |N_{k-1}| h_i(N,\sigma) \end{aligned}$$

where $h_i(N,\sigma)$ has already been computed in Step $k.1$.

The construction carried out above fully describes the proposal strategies in the restricted version of the model where only one coalition may form. This is because there are no subgames with a smaller population. For the model as we have described, in any subgame $G(S,v)$ with the set of players being S, the proposal strategies are computed as outlined above with the algorithms being carried out over (S,v). The reduced game (S,v) is also strictly supermodular and hence the same procedure carries over.

Remark. 1. For any player, his proposer's advantage and his hostage's disadvantage is continuously and monotonically decreasing in δ and vanishes in the limit as $\delta \to 1$. For a player $j_k \in N_k$, it is easy to arrive at formulas for

$$\begin{aligned} \frac{\mathrm{d}a_{j_1}}{\mathrm{d}\delta} &= -\frac{|N|}{\delta^2}\frac{v(N_1)}{|N_1|} < 0 \\ \frac{\mathrm{d}h_{j_2}}{\mathrm{d}\delta} &= -\frac{|N|}{|N_2|}\Big(\frac{v(N_1)}{\delta^2\,|N_1|} - \frac{v(N)}{|N|}\Big) < 0 \end{aligned}$$

For k ¿ 2

$$\frac{\mathrm{d}h_{j_k}}{\mathrm{d}\delta} = -\frac{|N|\sum_{a=1}^{k-1}|N_a|}{|N_{k-1}|\,|N_k|}\Big(\frac{v(\cup_{a=1}^{k-1}N_a)}{\delta^2\sum_{a=1}^{k-1}|N_a|} - \frac{v(N)}{|N|}\Big) < 0$$

The inequalities follow because the expressions in the parentheses are always positive for strictly super modular games by Claim 4.2 in Appendix A. For $k \geq 2$, from Step $k.2$ of Algorithm

$$\frac{\mathrm{d}a_{j_k}}{\mathrm{d}\delta} < 0$$

2. For any $i \in N$, the limiting payoff vector as $\delta \to 1$ when i is the proposer is core-constrained Nash Bargaining Solution. We know this from the algorithmic characterization of the core-constrained Nash Bargaining Solution for supermodular games shown by Dutta and Ray (1989).

1.4.3 Implementation Strategy

Consider the game $G(N, v)$ with the full set N as the population of players. Fist consider the equilibrium-offers. Suppose $i \in N_k$. If $k = L$, the last block of our partition of players, then i always discards his offer whenever someone rejects. If $k < L$, then i's strategy is as follows.

(a) If every player $j \in N \setminus i$ accepts the offer, then i implements the offer.

(b) If the set of players who have accepted or proposed the offer is exactly $\cup_{a=1}^{k} N_a$, then i implements his offer with this coalition.

(c) If some player in $\cup_{a=1}^{k} N_a$ rejects, then i discards the offer.

(d) For any other set of acceptances, i implements that coalition which gives him the maximum payoff provided this payoff is at least as great as the payoff he gets by discarding the offer.

For off-equilibrium offers, i implements that coalition which gives him the maximum payoff provided this payoff is at least as great as the payoff he gets by discarding the offer.

This describes the implementation strategies of players in the restricted version of the model where only one coalition may form. For the unrestricted version as we have described, for any subgame $G(S, v)$ with S as the population of players, the corresponding partition is $(S_a)_a$. Players follow the implementation strategies as described above with this change.

1.4.4 Acceptance-Rejection Strategy

Consider j's response node in the game $G(N, v)$ where Q is the set of players who have accepted or proposed the standing offer (S, x_S) made by i so far. Since the responders move in a pre-determined order ϕ, anyone of them while contemplating accepting or rejecting the offer has to consider the effect of his decision on the decisions of the responders following him. The ARSs of responders are defined inductively, first for the last responder according to ϕ, then for the penultimate responder and so on backwards in order.

Let $Y_j \subset S \setminus (Q \cup \{j\})$ be the set of responders who will accept the proposal (S, x_S) according to their respective ARSs if j rejects it. For the last responder in S, $Y_j = \emptyset$. The ARS of j is conditioned on whether i would implement his offer with a coalition $S_I \subset Q \cup Y_j$ or discard his offer.

For equilibrium offers,

(a) If i discards his offer conditional on a rejection by j, then j accepts the offer if it gives him at least $b_j(N, \sigma)$ and rejects otherwise.

(b) If i implements his offer with a coalition conditional on a rejection by j, then j accepts the offer if it gives him at least $\delta v(j)$ and rejects otherwise.

For off-equilibrium offers,

(a) If i discards his offer conditional on a rejection by j, then j accepts the offer if it gives him more than $b_j(N, \sigma)$ and rejects otherwise.

(b) If i implements his offer with a coalition conditional on a rejection by j, then j accepts the offer if it gives him more than $\delta v(j)$ and rejects otherwise.

Note for the equilibrium proposal, responders resolve any indifference by accepting while for any off-equilibrium proposal, they resolve it by rejecting. This describes the acceptance-rejection strategies of players in the restricted version of the model where only one coalition may form. For the unrestricted version as we have described, for any subgame $G(S, v)$ with S as the population of players, the corresponding partition is $(S_a)_a$. Players follow the acceptance-rejection strategies as described above with this change keeping in view that when a proposer threatens to form a subcoalition $T \subsetneq S$, the responders while making their decision look forward to their payoff in the continuation game $G(S \setminus T, v)$. We omit writing it here.

1.4.5 Optimality

Again for ease of exposition, we only show that the strategies described constitute an SSPE for the restricted model. The proof can be found in Appendix A. A remark is in order as to how the equilibrium we have constructed achieves efficiency no matter who proposes as opposed to Chatterjee et. al.(1993). In their mechanism if any player who was not in the last block of partition was chosen as a proposer, efficiency was not obtained. In the equilibrium we have constructed, each such player has a credible coalitional threat using which he can relax the demands of players who are not part of this coalitional threat. Thus each such player strictly prefers to make an offer to the grand coalition. The equilibrium is sustained by the following expectations. Any deviation by a proposer in which he tries to unilaterally gain at the expense of players outside his coalitional threat (outsiders) will be met by a rejection by players inside his coalitional threat (insiders). A rejection by insiders is a best response since they are indifferent between rejecting and accepting. Any deviation by a proposer in which he tries to gain at the expense of outsiders by bribing insiders will be met by a rejection by outsiders since the act of bribing insiders renders the threat incredible. Outsiders call it a bluff and anticipating/seeing this response, the insiders reject it as well since they realize the sweetened offer to them is just a mirage which is never going to materialize.

For the unrestricted model as we have described, the corresponding strategies described constitute an SSPE as well. This follows because the limit allocation, the core-constrained Nash Bargaining Solution satisfies a monotonicity property shown in Dutta (1990). For stating this property, let $u^\star(N, v)$ denote the core-constrained Nash Bargaining Solution for the coalitional game (N, v). For a vector $y \in R^N$, let y_S denote the projection of y along the axes of players in S.

Dutta (1990). Suppose (N, v) is strictly supermodular. For any $S \subsetneq N$, $u^{\star}_S(N, v) > u^{\star}(S, v)$ where the strict inequality is for all coordinates.

The result above ensures that in any subgame $G(S, v)$, for all sufficiently high δ, every responder agrees to the proposal no matter who proposes. Thus in every subgame $G(S, v)$, the entire coalition S is formed immediately. Thus we have shown Proposition 1.

We now show with the help of an example that the equilibrium we constructed for strictly supermodular games may fail to be an equilibrium if the coalitional game is supermodular.

Example 2. Consider the following 3-player strictly supermodular coalitional game.

S	v(S)	S	v(S)	S	v(S)
{1}	0	{2}	0	{3}	0
{1,2}	1.8	{1,3}	1.6	{2,3}	0.1
{1, 2, 3}	2.4	∅	0		

Remark. 1. The core-constrained NBS in this game is the allocation $u^{\star} = (1, 0.8, 0.6)$.

2. The set of coalitions for which the core constraints are binding at cNBS, $\mathscr{S}_b = \{\{1, 2\}, \{1, 3\}\}$.

Candidate Equilibrium for Example 2.

Proposal Strategies. Player i makes an offer $u(\delta, N, i)$ to the grand coalition N. The offers are:

$$u(\delta, N, 1) = (b_1 + a_1, b_2 - h_2, b_3)$$

$$u(\delta, N, 2) = (b_1, b_2 + a_2, b_3 - h_3)$$

$$u(\delta, N, 3) = (b_1, b_2, b_3 + a_3)$$

where

$$
\begin{aligned}
&b_1 = 1 && b_2 = 0.8 && b_3 = 2.4\delta - 1.8 \\
&a_1 = \big(\frac{3}{\delta} - 3\big) && a_3 = 2.4(1-\delta) && h_2 = a_1 - a_3 \\
&a_2 = \big(\frac{3}{\delta} - 3\big)b_2 + h_2 && h_3 = a_2 - a_3 &&
\end{aligned}
$$

In this candidate SSPE, $b1 + b3 < v(\{1, 3\})$. So P3 strictly prefers to partially implement his offer with $\{1, 3\}$ rather than discarding the offer. Thus $\{1, 3\}$ is a credible threat for P3. A deviation (N, d) in which $d_1 = b_1 + \epsilon$, $d_2 = b_2 - 2\epsilon$ where $\epsilon > 0$ is profitable to P3 because $v(N) - (b_1 + b_2 - \epsilon) > b_3 + a_3$. Thus the proposed strategies do not constitute an equilibrium. The problem is that a deviation in which P3 bribes P1 in taking further advantage of P2 is not deterred because this 'act of bribing' does not render the threat incredible. This is due to the strict preference of P3 in implementing his offer with $\{1, 3\}$ over discarding the offer.

1.5 Proof of Proposition 2

Let us define some sets pertaining to the equilibrium points of the game.

$$
\begin{aligned}
E &= \{\sigma : \sigma \text{ is an SSPE of } G(N, v) \text{ for all sufficiently high } \delta\} \\
E_e &= \{\sigma \in E : \text{ everyone makes an offer to } N\} \\
E_c &= \{\sigma \in E : u^\star(N, \sigma) \in \text{ core } (N, v)\}
\end{aligned}
$$

The following result says that for strictly superadditive coalitional environments, no SSPE can exhibit delay in any subgame. The proof can be found in Appendix B.

Lemma 1. Suppose (N, v) is strictly superadditive. For an SSPE $\sigma \in E$, for every $S \subset N$, every player $i \in S$ must make an acceptable offer in every subgame $G(S, v)$.

The proof of Lemma 1 does not involve any new ideas. It is reminiscent of Okada (1996) who gets this result for the mechanism he studies. It will be an input to our next observation which constitutes the first key step in our proof.

Lemma 2. Suppose (N, v) is strictly superadditive. Let σ be an asymptotically efficient SSPE. Then it must have its limit value $u^\star(N, \sigma)$ in the core. Formally, $E_e \subset E_c$.

Corollary. The set of credible threats in $\sigma \in E_e$ is a subset of the set of binding coalitions with respect to $u^\star(N, \sigma)$.

The idea of proof of Lemma 4 is as follows. For any SSPE σ, define $\Delta(\delta) = v(N) - \sum_{j \in N} b_j(N, \sigma)$. In an asymptotically efficient SSPE, $\lim_{\delta \to 1} \Delta(\delta) = 0$. Our method of proof is to show that $b(N, \sigma) \in C(\Delta)$ where $C(\Delta)$ is the core of a game derived from (N, v) in which the value of all coalitions has been reduced by the same small amount . A limit argument will then give the result. The proof can be found in Appendix B.

The idea of the proof of Proposition 2 is to utilize the nested structure of credible coalitional threats in an asymptotically efficient SSPE. This feature of equilibrium is due to Corollary to Lemma 4 and Claim D of Compte and Jehiel (2010) which was reproduced while describing the proposal strategies in Section 1. This generates a partition of players. Next we argue that all players in any block of partition get the same payoff in the limit. Since the blocks of partition are also credible coalitional threats, the limit allocation must have a hierarchical structure. The limit allocation and the coalitional threats must be such that every player's payoff is simultaneously maximized in equilibrium. These simultaneous maximization problems lead to hierarchical maximization problem of precisely the same sort that characterizes the core-constrained Nash Bargaining Solution of supermodular games as showed in Dutta and Ray (1989). Each step's problem must take as a constraint the solution to previous steps. This unravels the optimal coalitional threats as well as the limit allocation inductively. See Appendix B for the formal proof.

1.6 Concluding Remarks

In the classical environment of coalitional game with transferable utility, the efficiency implications of relaxing unanimity (as a requirement for agreement on an offer) in a bargaining model was examined. An asymptotically efficient equilibrium in pure strategies was displayed for strictly supermodular coalitional games, a result that does not obtain in the models of Chatterjee et al. (1993) and Okada (1996) where every responder has veto power and obtains only in a restricted setting (where only one coalition may form) in Compte and Jehiel (2010). The limiting outcome of this equilibrium is found to be the core-constrained Nash Bargaining Solution. Stationary subgame perfect equilibria that are efficient for all sufficiently high are shown to be payoff equivalent in the limit. That limit is the limit of the efficient equilibrium displayed i.e. the core-constrained Nash Bargaining Solution. The efficiency implications of noncontingent offers are underscored by similar result we find when we embed this feature in Chatterjee et al. (1993) mechanism where the rule governing the selection of proposers is taken to be a fixed protocol and the first rejector becomes the new proposer. Such an exercise is undertaken in Chaturvedi (2013b). We have found it difficult to establish uniqueness of the limiting allocation in the class of SSPE. Investigating this and the efficiency implications of the mechanism for strictly superadditive games with nonempty cores are avenues for future work.

Appendix A

Claim 2.1. Suppose (N, v) is strictly supermodular. Let $u^\star$ be the core-constrained Nash Bargaining Solution for (N, v). Then for every $S \subset N$ such that $S \neq \cup_{a=1}^{k} N_a$ for some $k = 1, \ldots, L$, we have $\sum_{j \in S} u_j^\star > v(S)$.

Proof. This is because $u^\star$ is in the core of (N, v) and by definition the set of all coalitions that are binding at $u^\star$ is precisely $\{N_1 N_1 \cup N_2, \ldots, N_1 \cup \ldots \cup N_L\}$. Q.E.D.

Claim 2.2. Suppose (N, v) is strictly supermodular. Then $\forall k = 1, \dots, L-1$

$$\frac{v(\cup_{a=1}^{k} N_a)}{\sum_{a=1}^{k} |N_a|} \geq \frac{v(N)}{|N|}$$

Proof. Step 1. Let $N_0 = \emptyset$. Given (N, v), for $k = 1, \dots, L(v)$, define inductively the restricted game $(N \setminus \cup_{a=0}^{k-1} N_a, v_k)$ by $v_k(S) = v(\cup_{a=0}^{k-1} N_a \cup S) - v(\cup_{a=0}^{k-1} N_a)$. In this step, we show a property of supermodular games that may be called the 'principle of cascading averages'. It says that the average values of restricted (in the sense defined above) games of a supermodular game are ordered in a decreasing fashion. We show the following statement is true:

$$\forall k \in \{1, \dots, L-1\}, \quad \frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} \geq \frac{v(N) - v(\cup_{a=0}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|}$$

Suppose to the contrary that for some k

$$\frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < \frac{v(N) - v(\cup_{a=0}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|} \tag{1.1}$$

Since $k \neq L$, $N_k \subsetneq N \setminus \cup_{a=0}^{k-1} N_a$ is a maximizer at Step k of our algorithm for generating partitions. So

$$\frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} \leq \frac{v(\cup_{a=0}^{k} N_a) - v(\cup_{a=0}^{k-1} N_a)}{|N_k|} \tag{1.2}$$

Adding (2.1) and (2.2),

$$\frac{\left|N \setminus \cup_{a=0}^{k} N_a\right| + |N_k|}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|}[v(N) - v(\cup_{a=0}^{k-1} N_a)] < [v(N) - v(\cup_{a=0}^{k-1} N_a)]$$

By strict supermodularity of (N, v), $v(N) - v(\cup_{a=0}^{k-1} N_a) > 0$. So we have

$$1 = \frac{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < 1$$

a contradiction.

Step 2. By Step 1, we get $\forall k \in \{1, \dots, L-1\}$

$$\begin{aligned}\frac{v(N)}{|N|} &\geq \frac{v(N) - v(\cup_{a=1}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|} \\ \frac{\left|N \setminus \cup_{a=0}^{k} N_a\right|}{|N|} &\geq \frac{v(N) - v(\cup_{a=1}^{k} N_a)}{v(N)} \\ \frac{v(\cup_{a=1}^{k} N_a)}{\sum_{a=1}^{k} |N_a|} &\geq \frac{v(N)}{|N|}\end{aligned}$$

Q.E.D.

Proof of Proposition 1 (ctd.) *Optimality of Implementation Strategy*

For off-equilibrium proposals, the optimality is clear. The lemma below implies optimality for equilibrium proposals.

Claim 2.3. Consider the implementation node of $i \in N_k$

(1) Suppose $k < L$ and $\cup_{a=1}^{k} N_a$ is the set of players who have accepted or proposed i's equilibrium offer. Then i is indifferent between partially implementing the proposal with $\cup_{a=1}^{k} N_a$ and discarding it. Moreover i cannot gain by implementing a subcoalition among $\cup_{a=1}^{k} N_a$. Hence it is optimal to implement $\cup_{a=1}^{k} N_a$.

(2) i cannot gain by implementing his equilibrium offer with a subcoalition that excludes players from $\cup_{a=1}^{k} N_a$.

Proof. . Let

$$\mathscr{T} = \operatorname*{argmax}_{T \subset N_A,\, i \in T} \left[v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i) \right]$$

We want to prove the following statements. (1) (a) $\cup_{a=1}^{k} N_a \in \mathscr{T}$ and (b)

$$v(\cup_{a=1}^{k} N_a) - \sum_{j \in T \setminus i} u_j(\delta, N, i) = b_i(N, \sigma)$$

(2) Suppose $T \subset N_A$ is such that (a) $i \in T$ and (b) $\cup_{a=1}^{k} N_a \setminus T \neq \emptyset$. Then for all sufficiently high δ, it is optimal for i to discard the equilibrium offer rather than partially implement it

with T. That is

$$v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i) \leq b_i(N, \sigma)$$

1(b) is true by virtue of our construction of equilibrium proposals. As regards 1(a), take $T \subset N_A = \cup_{a=1}^{k} N_a$. Let $u^\star$ be the core-constrained Nash Bargaining Solution algorithmically given by Dutta and Ray (1989).

$$\begin{aligned}
v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j(\delta, N, i) &= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j^\star \\
&= u_i^\star && \textbf{Dutta and Ray (1989)} \\
&\geq v(T) - \sum_{j \in T \setminus i} u_j^\star && b^\star \textbf{ is a core allocation} \\
&= v(T) - \sum_{j \in T \setminus i} b_j \\
&= v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i)
\end{aligned}$$

For (2), any such T is not a binding coalition with respect to $u^\star$. Thus $v(T) - \sum_{j \in T \setminus i} u_j^\star < u_i^\star$. By construction, $v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i) \to v(T) - \sum_{j \in T \setminus i} u_j^\star$ and $b_i(N, \sigma) \to u_i^\star$ as $\delta \to 1$. Q.E.D.

Optimality of Acceptance-Rejection Strategy is clear.

Optimality of Proposal Strategy

We first show that the equilibrium offer is accepted no matter who proposes. Suppose $i \in N_k$. Consider the last responder in $\cup_{a=1}^{k} N_a$ who finds himself in a situation where every other responder in $\cup_{a=1}^{k} N_a$ has already accepted the proposal. By rejecting, the most j can get is $b_j(N, \sigma)$. If j accepts, then by part 1 of Claim 4.3, $\cup_{a=1}^{k} N_a$ is certainly a candidate for i to consider implementing. Part 2 of Claim 4.3 assures that j is part of any coalition that i will choose to implement. In any case, j is certain to get $u_j(\delta, N, i) = b_j$. Thus j is indifferent between accepting and rejecting the proposal which means accepting is optimal for him. By induction on the number of responders in $\cup_{a=1}^{k} N_a \setminus j$ who have not yet responded to the proposal, it follows for any responder $j \in \cup_{a=1}^{k} N_a$ that if every other responder in $\cup_{a=1}^{k} N_a \setminus j$ who has already responded to the proposal has accepted it, then it is optimal for j to accept it.

Consider a responder $j \in N \setminus \cup_{a=1}^{k} N_a$. If all players in $\cup_{a=1}^{k} N_a$ who have already responded before him have accepted the proposal, then j knows by the arguments of the preceding paragraph that the other players in $\cup_{a=1}^{k} N_a$ who will follow him will accept the proposal as well. Since $\cup_{a=1}^{k} N_a$ is certainly one of the candidates that meets the requirements of implementation, j knows that some coalition will form and the game would end. If j accepts, two situations may arise. Either j is part of the coalition that i forms or he isn?t. In the former case, he gets $u_j(\delta, N, i)$ while in the latter $\delta v(j)$. If he rejects, some coalition forms and the game ends. So he gets $\delta v(j)$. Thus accepting weakly dominates rejecting the proposal for j.

Deviations from equilibrium offer. Suppose $i \in N_k$.

Case 1. $k < L$. Classify deviations as:

(1) (N, d) where $\forall j \in \cup_{a=1}^{k} N_a \setminus i, d_j = u_j(\delta, N, i), \forall j \in N \setminus \cup_{a=1}^{k} N_a, d_j \leq u_j(\delta, N, i)$ and $\exists j \in N \setminus \cup_{a=1}^{k} N_a$ such that $d_j < u_j(\delta, N, i)$. This is a deviation where the proposer i tries to further gain unilaterally at the expense of his "hostages" i.e. players in N_{k+1} or other players not in $\cup_{a=1}^{k} N_a$. A unanimous rejection of this off-equilibrium offer is obtained in what follows.

Consider the last responder j in $\cup_{a=1}^{k} N_a$ who finds himself in a situation where every other responder in $\cup_{a=1}^{k} N_a$ has already rejected the proposal. If j is the only responder in $\cup_{a=1}^{k} N_a \setminus i$, then by part 2 of Claim 4.3, if j rejects, i will certainly pass and he gets $b_j(N, \sigma) = u_j(\delta, N, i) = d_j$ which is what he gets by accepting. So j is indifferent and it is optimal for him to reject. If j is not the only responder in $\cup_{a=1}^{k} N_a \setminus i$, then consider j's response node where $j \in \cup_{a=1}^{k} N_a$, players in $\cup_{a=1}^{k} N_a \setminus j$ have already responded and $(\cup_{a=1}^{k} N_a \setminus j) \cap (Q \setminus i) = \emptyset$. By Claim 4.3, j knows that i is certainly going to pass at his implementation node no matter what his response. So he is indifferent and hence it is optimal for him to reject. The argument is easily extended now by induction on the number of responders in $\cup_{a=1}^{k} N_a \setminus j$ who have not yet responded to the proposal to say that for any responder $j \in \cup_{a=1}^{k} N_a$, if every other responder in $\cup_{a=1}^{k} N_a \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it.

Since all responders in $\cup_{a=1}^{k} N_a$ reject this off-equilibrium proposal, any responder $j \in N_{k+1}$ knows that i's threat to implement $\cup_{a=1}^{k} N_a$ is an empty threat. j certainly prefers rejecting the proposal as $d_j < u_j(\delta, N, i) < b_j(N, \sigma)$. By a similar argument, everyone else in $N \setminus \cup_{a=1}^{k+1} N_a$ rejects it as well.

(2) (N, d) where $\forall j \in \cup_{a=1}^{k} N_a \setminus i, d_j \geq u_j(\delta, N, i), \exists j \in \cup_{a=1}^{k} N_a \setminus i, d_j > u_j(\delta, N, i), \forall j \in N \setminus \cup_{a=1}^{k} N_a, d_j \leq u_j(\delta, N, i)$ and $\exists j \in N \setminus \cup_{a=1}^{k} N_a$ such that $d_j < u_j(\delta, N, i)$. This is a deviation where the proposer i tries to further gain at the expense of his "hostages" i.e. players in N_{k+1} or other players not in $\cup_{a=1}^{k} N_a$ and offers to share the gains with some players in $\cup_{a=1}^{k} N_a$. A unanimous rejection of this off-equilibrium offer is obtained in what follows.

Consider the last responder $j \in N \setminus \cup_{a=1}^{k} N_a$ who finds himself in a situation where every other responder in $N \setminus \cup_{a=1}^{k} N_a$ has already rejected the proposal. Now j is certain that if he rejects, i would not implement $\cup_{a=1}^{k} N_a$ and would prefer to pass. This is because

$$
\begin{aligned}
v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} d_j &=< v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j(\delta, N, i) \\
&= b_i(N, \sigma)
\end{aligned}
$$

If j accepts then either $\cup_{a=1}^{k} N_a \cup \{j\}$ is a coalition that is implemented by i in which case j gets d_j or i chooses to pass in which case j gets $b_j(N, \sigma)$. If he rejects, he gets $b_j(N, \sigma)$.

Since $b_j(N,\sigma) \geq u_j(\delta, N, i) \geq d_j$ with a strict inequality for some j, it is optimal for j to reject the proposal. Argue by induction on the number of responders in $N \setminus \cup_{a=1}^{k} N_a$ who have not yet responded to the proposal to say that for any responder $j \in N \setminus \cup_{a=1}^{k} N_a$, if every other responder in $(N \setminus \cup_{a=1}^{k} N_a) \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it.

Since all responders in $N \setminus \cup_{a=1}^{k} N_a$ reject this off-equilibrium proposal, any responder j in $\cup_{a=1}^{k} N_a$ knows that i's sweetened offer is just a "mirage" because even if everyone in $\cup_{a=1}^{k} N_a$ accepts, i will pass. So j rejects the proposal as well.

(3) (S,d) where $S = \cup_{a=1}^{k} N_a$ and $\forall j \in S \setminus i, d_j = u_j(\delta, N, i) + \epsilon$ for $\epsilon > 0$. The payoff for i from such an offer is

$$\begin{aligned}
d_i &= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} d_j \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j(\delta, N, i) - \Big(\sum_{a=1}^{k} |N_a| - 1\Big)\epsilon \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} b_j - \Big(\sum_{a=1}^{k} |N_a| - 1\Big)\epsilon \\
&= b_i - \Big(\sum_{a=1}^{k} |N_a| - 1\Big)\epsilon \\
&< b_i + a_i = u_i(\delta, N, i)
\end{aligned}$$

Thus i cannot gain by this deviation.

(4) (S,d) where S does not include i's credible threat $\cup_{a=1}^{k} N_a$. The payoff for i from such an offer is arbitrarily close to $v(S) - \sum_{j \in S \setminus i} u_j^\star$ while the payoff from the equilibrium strategy is arbitrarily close to $u_i^\star$ as δ gets high enough. By Claim 4.1 in Appendix A, i cannot gain by this deviation.

Case 2. $k = L$. Consider any deviation (N,d) where $i \in N \setminus S, \forall j \in S, d_j < u_j(\delta, N, i)$ and $\forall j \in N \setminus S, d_j \geq u_j(\delta, N, i)$. This is a deviation where the proposer i tries to gain at the expense of players in S possibly by sweetening the offer for some players in $N \setminus S$. A unanimous rejection of this off-equilibrium offer is obtained in what follows.

Consider the last responder j in S who finds himself in a situation where every other responder in S has already rejected the proposal. If j is the only responder in S, then by rejecting j gets $b_j(N,\sigma) = u_j(\delta, N, i)$. If he accepts, then either j is a part of a coalition which i implements in which case he gets d_j or i passes in which case he gets $b_j(N,\sigma) = u_j(\delta, N, i)$. Since $d_j < u_j(\delta, N, i)$, j weakly prefers to reject. If j is not the only responder in S, then by part (2) of Claim 4.3, j knows that i is certainly going to pass at his implementation node if he rejects. He certainly prefers rejecting because $b_j(N,\sigma) = u_j(\delta, N, i) > d_j$. So it is optimal for him to reject. The argument is easily extended now by induction on the number of responders in $S \setminus j$ who have not yet responded to the proposal to say that for any responder $j \in S$, if every other responder in $S \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it. Q.E.D.

Appendix B

Proof of Lemma 1 Step 1. Suppose $\sigma \in E$ involves a player $i \in S$ making an unacceptable offer in some subgame $G(S, v)$. We construct a profitable deviation (S, d) for i. Let d be defined by $\forall j \in S \setminus i$, $d_j = b_j(S,\sigma) + \epsilon$ where $0 < \epsilon < (v(S) - \sum_{j \in S} b_j(S,\sigma))/(|S| - 1)$. Then d will be unanimously accepted since any responder does worse by rejecting. i gets $d_i = v(S) - \sum_{j \in S \setminus i}(b_j(S,\sigma) + \epsilon)$. By making an unacceptable offer, i's payoff at his proposal node in $G(S, v)$ is $b_i(S,\sigma)$. It is then easy to see by the choice of ϵ that $d_i > b_i(S,\sigma)$ i.e. $v(S) > \sum_{j \in S} b_j(S,\sigma) + (|S| - 1)\epsilon$.

Step 2. We now show that we can always choose such an ϵ. In other words, $v(S) - \sum_{j \in S} b_j(S,\sigma) > 0$. Let $u_j(\delta, S, i)$ be the payoff to j in a unanimously acceptable offer

$(S, u(\delta, S, i))$ made by i. If $j \notin S$ or i makes an unacceptable offer, $u_j(\delta, S, i) = 0$. Now

$$\begin{aligned}
\sum_{j \in S} b_j(S, \sigma) &= \sum_{j \in S} \frac{\delta}{|S|} \Big[\sum_{k \in S} u_j(\delta, S, k) \Big] \\
&= \frac{\delta}{|S|} \Big[\sum_{j \in S} u_j(\delta, S, i) + \sum_{j \in S} \sum_{k \in S \setminus i} u_j(\delta, S, k) \Big] \\
&= \frac{\delta}{|S|} \sum_{k \in S \setminus i} \sum_{j \in S} u_j(\delta, S, k)
\end{aligned}$$

Step 3. Now $\forall k \in S \setminus i$

$$\sum_{j \in S} u_j(\delta, S, k) = \begin{cases} 0 & \text{if } i \text{ makes an unacceptable offer} \\ v(T) & \text{if } i \text{ makes an acceptable offer to } T \subset S \\ v(S) & \text{if } i \text{ makes an acceptable offer to } S \end{cases}$$

By strict superadditivity,

$$\forall k \in S \setminus i, \quad \sum_{j \in S} u_j(\delta, S, k) \leq v(S)$$

$$\begin{aligned}
\sum_{j \in S} b_j(S, \sigma) &\leq \frac{\delta}{|S|} (|S| - 1) v(S) \\
&< v(S)
\end{aligned}$$

Thus we have shown a profitable deviation by i. This contradicts the premise that $\sigma \in E$.

Q.E.D.

Proof of Lemma 2 Fix an asymptotically efficient SSPE $\sigma \in E_e$. By definition of $\Delta(\delta)$,

$$\sum_{j \in N} b_j(N, \sigma) = v(N) - \Delta(\delta)$$

Fix $S \subsetneq N$. There are two cases.

Case 1. S is a credible threat for some $i \in S$ in σ. Then by definition of credibility, $v(S) \geq \sum_{j \in S} b_j(N, \sigma)$. Suppose $\eta(\delta) = v(S) - \sum_{j \in S} b_j(N, \sigma) > 0$. We construct a profitable deviation for i. Let $u(\delta, N, i)$ be i's acceptable (by Lemma 3) offer to N in σ. Since S is a credible threat for i, $\exists j \in N \setminus S$ such that $u_j(\delta, N, i) = b_j - h_j$ where $h_j > 0$.

Construct a feasible deviation (N, d) where

$$\begin{aligned} \forall k \in S, \quad d_k &= u_k(\delta, N, i) + \frac{\epsilon}{|S|} \qquad\qquad 0 < \epsilon < \eta(\delta) \\ d_j &= u_j(\delta, N, i) - \epsilon \\ \forall k \in N \setminus \{j \cup S\}, \quad d_k &= u_k(\delta, N, i) \end{aligned}$$

For this deviation, the credibility of i's threat to implement S is preserved. The requirement for this is

$$\begin{aligned} v(S) - \sum_{j \in S \setminus i} d_j &> b_i(N, \sigma) \\ \leftrightarrow v(S) &> \sum_{j \in S} b_j(N, \sigma) + \frac{|S| - 1}{|S|}\epsilon \\ \leftrightarrow \eta(\delta) &> \frac{|S| - 1}{|S|}\epsilon \end{aligned}$$

which is true. So the deviation (N, d) in which i by bribing $S \setminus i$ enlists their support in further taking advantage of j is not deterred as the credibility of i's threat to implement S is unaffected by this deviation.

Thus in an asymptotically efficient SSPE

$$\sum_{j \in S} b_j(N, \sigma) = v(S) > v(S) - \Delta(\delta)$$

Case 2. S is not a credible threat for any $i \in S$ in $\sigma(\delta)$. Then by definition of credibility, $\sum_{j \in S} b_j(N, \sigma) \geq v(S) > v(S) - \Delta(\delta)$.

We have thus proved $b(N, \sigma) \in C(\Delta)$. Q.E.D.

Proof of Proposition 2 (ctd.) Let $\mathscr{S}$ be the set of credible threats in σ. Since $\sigma \in E_e$, by the corollary to Lemma 4 and Claim D of Compte and Jehiel (2010), we know that wlog

$\mathscr{S}$ has the structure $\{S_1, S_1 \cup S_2, \ldots \cup_{a=1}^{L-1} S_a$. Let $S_L = N \setminus \cup_{a=1}^{L-1} S_a$. By definition of credible threats, $b(N,\sigma)$ which we will hereafter write b must satisfy the system of equations E:

$$\sum_{j \in S_1} b_j = v(S_1)$$
$$\sum_{j \in S_1} b_j + \sum_{j \in S_2} b_j = v(S_1 \cup S_2)$$
$$\ldots$$
$$\sum_{j \in S_1} b_j + \ldots + \sum_{j \in S_{L-1}} b_j = v(\cup_{a=1}^{L-1} S_a)$$
$$\sum_{j \in S_1} u_j^\star + \ldots + \sum_{j \in S_L} u_j^\star = v(N)$$

Now we show that $u^\star(N,\sigma)$ must satisfy

$$\forall a = 1, \ldots, L \quad \forall \{i,j\} \subset S_a \quad u_i^\star = u_j^\star$$

so that

$$\forall j \in S_1 \quad u_j^\star = b_j = \frac{v(S_1)}{|S_1|}$$
$$\forall j \in S_2 \quad u_j^\star = b_j = \frac{v(S_1 \cup S_2) - v(S_1)}{|S_2|}$$
$$\ldots$$
$$\forall j \in S_{L-1} \quad u_j^\star = b_j = \frac{v(\cup_{a=1}^{L-2} S_a \cup S_{L-1}) - v(\cup_{a=1}^{L-2} S_a)}{|S_{L-1}|}$$
$$\forall j \in S_L \quad u_j^\star = \frac{v(N) - v(\cup_{a=1}^{L-1} S_a)}{|S_L|}$$

The proof is by induction on k, the index of the finite nested sequence of coalitions $\{S_k\}_{k=1}^{L-1}$ that can act as credible threats. It is then easily extended to the case $k = L$.

For k=1. Step 1. Suppose $\{i,j\} \subset S_1$ and S_1 is a credible threat for i. Then we claim S_1 is also a credible threat for j. Now take a candidate SSPE $\sigma \in E_e$ such that S_1 is a credible threat for i but not for j. Since

$$(v(S_1) - \sum_{k \in S_1 \setminus i} b_k(N,\sigma) = b_i(N,\sigma)) \leftrightarrow (v(S_1) - \sum_{k \in S_1 \setminus j} b_k(N,\sigma) = b_j(N,\sigma))$$

criterion (ii) of the definition of a credible threat is met. Now j cannot give any player $k \in S_1$ less than $b_k(N, \sigma)$ unless he has access to another credible threat that does not include k in it. But we have already seen that any other credible threat in the game must include S_1 and hence k. Thus criterion (i) of the definition is also met. So it must be that j's strategy in σ is to pass at his implementation node. We construct a deviation for j, σ'_j which differs from σ_j at two decision nodes:

(a) at j's proposal node.

$$\begin{aligned} \forall k \in S_2 \quad & u'_k(\delta, N, j) = u_k(\delta, N, j) - \epsilon \qquad\qquad 0 < \epsilon < b_k(N, \sigma) - b_k(N \setminus S_1, \sigma) \\ \forall k \in N \setminus S_2 \quad & u'_k(\delta, N, j) = u_k(\delta, N, j) + \frac{|S_2|\,\epsilon}{|N \setminus S_2|} \end{aligned}$$

(b) at j's implementation node. j threatens to implement his offer $u'(\delta, N, j)$ with S_1.

In accordance with the ARSs that responders must follow in σ, j's offer $u'(\delta, N, j)$ will be unanimously accepted and hence $u'_j(\delta, N, j) > u_j(\delta, N, j)$. This implies $u_j(\sigma'_j, \sigma_{-j}) > u_j(\sigma)$. This contradicts that σ was an equilibrium point. This proves that S_1 is also a credible threat for j.

Since S_1 acts as a credible threat for both i and j, each of hem derives the same total advantage from other players. So from the feasibility equation for i and j's offers, $a_i = a_j$. That is, i and j get the same proposer advantage.

Step 2. For every other credible threat T, $S_1 \subset T$. So

$$\forall k \in N \setminus i \quad u_i(\delta, N, k) = b_i(N, \sigma)$$

$$\forall k \in N \setminus j \quad u_j(\delta, N, k) = b_j(N, \sigma)$$

so that

$$b_i(N,\sigma) = \frac{\delta}{|N|}\big[a_i + |N|\, b_i(N,\sigma)\big]$$
$$(1-\delta)b_i(N,\sigma) = \frac{\delta}{|N|}a_i$$

Similarly

$$(1-\delta)b_j(N,\sigma) = \frac{\delta}{|N|}a_j$$

Now $a_i = a_j$ implies $b_i(N,\sigma) = b_j(N,\sigma)$.

Inductive Hypothesis. Let $1 \leq k < L-1$. Suppose $\forall m \leq k$

$$\forall i \in S_m \quad b_i(N,\sigma) = b_j(N,\sigma)$$

Suppose $\{i,j\} \subset S_{k+1}$ and S_{k+1} is a credible threat for i. Then it follows by the same reasoning as in the base step $k=1$ that S_{k+1} is also a credible threat for j.

Step 1. So from the feasibility equation for i and j's offers, $a_i = a_j$. That is, i and j get the same proposer advantage.

Step 2. For $a \leq k$, consider any player i_a in N_a, say i_a is deriving a total advantage of $H_i + H_j$, where H_i is the advantage i_a derives from i and H_j is the advantage i_a derives from j. Then i_a can derive the same total advantage by treating player i and j symmetrically i.e. by deriving an advantage of $h_i^{i_a} = h_j^{i_a} = \frac{H_i+H_j}{2}$ from each.

Step 3. For every other bigger credible threat T, $S_{k+1} \subset T$. So

$$\forall p \in N \setminus \cup_{a=1}^{k} S_a \quad u_i(\delta, N, p) = b_i(N,\sigma)$$
$$\forall p \in N \setminus \cup_{a=1}^{k} S_a \quad u_j(\delta, N, p) = b_j(N,\sigma)$$

so that from the equilibrium conditions

$$b_i = \frac{\delta}{|N|}\big[\sum_{l \in \cup_{a=1}^{k} S_a} (b_i - h_i^l) + b_i + a_i + (\big|N \setminus \cup_{a=1}^{k} S_a\big| - 1)b_i\big]$$
$$b_i = \frac{\delta}{|N|}\big[\,|N|\, b_i + a_i - \sum_{l \in \cup_{a=1}^{k} S_a} h_i^l\big]$$

Similarly

$$b_j = \frac{\delta}{|N|}\big[\,|N|\,b_j + a_j - \sum_{l \in \cup_{a=1}^{k} S_a} h_j^l\big]$$

Now $a_i = a_j$ and $\forall l \in S_k$, $h_i^l = h_j^l$ implies $b_i = b_j$.

Now since σ is asymptotically efficient, $(u^\star, (S_t)_{t=1}^{L-1})$ must solve

1. the system of equations E.

2. for every i, max $v(N) - \sum_{j \in N \setminus i} u_j^\star$. By efficiency, this is the same as for every i, max $u_i^\star$.

In the presence of (1), (2) leads to a series of hierarchical maximization problems that lead to the determination of optimal threats $(S_t)_{t=1}^{L-1}$ that must coincide with the partition induced by our algorithm and hence $u^\star$ must be the core-constrained Nash Bargaining Solution.

Q.E.D.

Chapter 2

Efficient Coalitional Bargaining with Noncontingent Offers Part II: Sequential Proposer Protocol

2.1 Introduction

This paper should be seen as a companion paper to Chaturvedi (2013b) and therefore borrows much of the motivation from there. We refrain from repeating the introduction. For coalitional games, two alternative bargaining models have been studied in the literature that deals with impatient players; each differing in the way a new proposer is chosen. Chatterjee et al. (1993) study a variant with fixed order in which players move while Okada (1996) studies a random proposer variant. Both these variants embody a constraint on the proposer's ability to implement offers- he cannot implement his offer if some responder rejects it. Put in a different way, the offers made by the proposer are 'contingent offers'- their implementation is contingent on acceptance by everyone to which the offer is directed.

We focus on studying two settings that have been studied before. One class, **S**, is that of strictly supermodular games. It has the property that players are complements for coalition formation. The other class of games, **G**, have the property that per capita value is increasing as a coalition adds to its members. For either class, an efficient outcome has immediate formation of grand coalition. The class of games **G** and **S** are unrelated in that neither is a subset of the other. However both have nonempty cores [1].

We now describe our mechanism recursively. The mechanism is defined relative to a order ϕ which is a collection of orderings ϕ_S of players in S, one for each coalition. An ordering ϕ_S should be seen as arranging the players in S on a circle in clockwise direction with a pointer pointed at some player in S. Suppose the state of the game is such that players in S are still negotiating while the rest have left the game with some agreements reached in some fashion. The player that has the pointer in S, say i, is chosen to be the proposer. The proposer makes an offer (T, x_T) where $i \in T \subset S$ and $\sum_{j \in T} x_j = v(T)$. Responders in $T \setminus i$ then move sequentially according to order ϕ_T moving along the circle in clockwise direction choosing from Yes or No. After everyone has responded, the proposer decides whether to partially implement his offer with all, some or none of the responders who have accepted the offer. Let j be the first rejector, if any. When a proposer decides to implement his offer (T, x_T) with T_I (which if not empty necessarily includes the proposer), he gives each responder j in T_I what he offered to him i.e. x_j and he gets the residual of the surplus. The state then changes to one in which $S \setminus T_I$ is the set of players still negotiating in the game with the pointer at j. If i discards the offer (i.e. chooses not to partially implement his offer), the game continues with the set of active players unchanged at S but the pointer at the player next to i in ordering ϕ_S. There is discounting when a new proposer is chosen. Players are expected utility maximizers. The notion of equilibrium is stationary subgame perfect equilibrium (SSPE).

Our mechanism embodies a proposer's ability to make noncontingent offers- even if some responder in his proposed coalition has rejected it, he has a choice to implement it with a subset of responders who have accepted it. In strictly supermodular environments, this ability to walk away with a subcoalition makes the proposer more powerful in that it potentially gives him access to a threat. It turns out this is enough for getting efficiency irrespective of

[1] The necessity of nonempty core for existence of efficient stationary equilibria is a result that holds in a variety of mechanisms studied and holds in the mechanism studied here as well.

the order. We will elaborate it further after stating our results.

We show for all games in $\mathbf{G} \cup \mathbf{S}$ and for all sufficiently high discount factors, there exists an order-independent efficient subgame perfect equilibrium in pure stationary strategies whose limiting outcome is the core-constrained Nash Bargaining Solution. For games in $\mathbf{G}$, the Core does not act as a binding constraint on the Nash Bargaining Solution. For games in $\mathbf{S}$, the Core is a binding constraint on the Nash Bargaining Solution. We give a constructive proof describing a recursive algorithm for computing the proposals made by the players to the grand coalition in this SSPE. To summarize, efficiency for strictly supermodular games in our mechanism is not sensitive to the choice of initial proposer, obtains in pure strategies and without artificially imposing any behavior that is unnatural to the environment.

The ideas behind our constructive existence proof are properties of strictly supermodular settings. These are the algorithmic characterization of the core-constrained Nash Bargaining Solution for supermodular games shown by Dutta and Ray (1989), a further monotonicity result about such allocation shown in Dutta (1990) and a result due to Compte and Jehiel (2010) about nested structure of coalitions for which the core constraints are binding at any core allocation. In the equilibrium we construct, the set of coalitions for which the core constraints bind at the core-constrained Nash Bargaining Solution are precisely those that constitute credible coalitional threats. First we partition the players by using the result of Compte and Jehiel (2010) at the core-constrained Nash Bargaining Solution. Our description of equilibrium proposals is a result of two recursive algorithms. The first algorithm inductively describes what will be threat points of players in the equilibrium. In doing this, we 'anchor' the discounted continuation value of our equilibrium in the core-constrained Nash Bargaining Solution. A responder who is not part of a coalitional threat that a proposer uses must be willing to lower his demand relative to what he would have demanded if he had veto power over the offer. This responder can therefore be held *hostage* and suffer a *disadvantage* at the hands of this proposer. An interesting feature of the equilibrium is that the proposer is forced to concede this responder more than what he demands. This is the cost he has to pay in order to maintain the credibility of his coalitional threat. Care needs to be exercised at this point in deciding which players are held *hostage* by which players. We do this in a natural way suggested by the hierarchical nature of partition of players. The purpose of the second algorithm is to describe the proposer's advantage and the *hostage's disadvantage*. This is done by inductively using the equilibrium condition and the feasibility condition.

The plan of the paper is as follows. After introducing the model in Section 2, we give a statement of our results in Section 3. The proof of the existence result is exposited as follows. First we discuss the equilibrium construction in a simple 3-player example and contrast the efficiency implication with the other mechanisms that have been studied. The candidate equilibrium is described in Sections 4.1, 4.2, 4.3 and 4.4 both for a restricted model in which only one coalition is permitted to form as well as the model without this restriction. The reason we exhibit the equilibrium for a restricted model first is that we only have to deal with the game with all players in it. There are no subgames with a smaller population. Also, the acceptance-rejection strategies are simple for the restricted model. Optimality of the strategies is discussed in Section 4.5. A monotonicity property of core-constrained Nash Bargaining Solution for strictly supermodular games shown in Dutta (1990) then assures us that the strategies so constructed can be supported as an SSPE in our model for all sufficiently high discount factors. Some further results are discussed in Section 5.

2.2 The Model

2.2.1 The Coalitional Game

Let $N = \{1, \ldots, n\}$ be the set of all players. Let (N, v) be a coalitional game with transferable utility. Any coalition, $S \subset N$ has a nonnegative worth, $v(S) \geq 0$. We will denote the set of all coalitions of N by $\mathscr{C}$. When a coalition agrees to a payoff allocation, it can fully commit to it and there are no enforcement problems in implementing that agreement. We now describe some coalitional environments that have been studied in the literature.

(N, v) is strictly superadditive if

$$\forall S, T \subset N, \quad S \cap T = \emptyset, \quad v(S \cup T) > v(S) + v(T)$$

(N, v) has increasing returns per capita as a coalition adds to its members if

$$\forall S, T \subset N, \quad S \supset T, \quad \frac{v(S)}{|S|} > \frac{v(T)}{|T|}$$

(N, v) is strictly supermodular if

$$\forall i \in N, \quad \forall S, T \subset N \setminus i, \quad S \supset T, \quad v(S \cup \{i\}) - v(S) > v(T \cup \{i\}) - v(T)$$

Let **G** denote the class of games that has increasing returns per capita. Let **S** denote the class of strictly supermodular games. For both these environments, the economy splitting up into coalitions is an inefficient coalitional structure. The only efficient structure is the formation of the grand coalition. Also supermodular environments are superadditive as well. The following definition will be useful to us.

Definition 4. Suppose $b^\star$ is a core allocation for a game (N, v). Then we say S is a binding coalition with respect to $b^\star$ if $b^\star(S) = v(S)$.

2.2.2 The Bargaining Mechanism

The mechanism is defined relative to a order ϕ which is a collection of orderings ϕ_S of players in S, one for each coalition. An ordering ϕ_S should be seen as arranging the players in S on a circle in clockwise direction with a pointer pointed at some player in S. Suppose the state of the game is such that players in S are still negotiating while the rest have left the game with some agreements reached in some fashion. The player that has the pointer in S, say i, is chosen to be the proposer. The proposer makes an offer (T, x_T) where $i \in T \subset S$

and $\sum_{j \in T} x_j = v(T)$. Responders in $T \setminus i$ then move sequentially according to order ϕ_T moving along the circle in clockwise direction choosing from Yes or No. After everyone has responded, the proposer decides whether to partially implement his offer with all, some or none of the responders who have accepted the offer. Let j be the first rejector, if any. When a proposer decides to implement his offer (T, x_T) with T_I (which if not empty necessarily includes the proposer), he gives each responder j in T_I what he offered to him i.e. x_j and he gets the residual of the surplus. The state then changes to one in which $S \setminus T_I$ is the set of players still negotiating in the game with the pointer at j. If i discards the offer (i.e. chooses not to partially implement his offer), the game continues with the set of active players unchanged at S but the pointer at the player next to i in ordering ϕ_S. There is discounting when a new proposer is chosen. Players are expected utility maximizers. The notion of equilibrium is stationary subgame perfect equilibrium (SSPE).

For any SSPE σ of the extensive form game $G(N, v)$ described above, let $u(S, \sigma, i) \in \mathbb{R}^S$ be payoff vector of players in S at a node when S is the set of players still in the game and i is the proposer. Let $u^\star(S, \sigma, i) = \lim_{\delta \to 1} u(S, \sigma, i)$.

We'll refer to the environment and the mechanism described above as the unrestricted model. This is the object of our study and our results pertain to the unrestricted model. However, for expositional purposes, we find it convenient to work with a version of the mechanism where the bargaining terminates as soon as one coalition forms. We refer to this version as the restricted model. As noted in the last paragraph of the introduction, we exposit the strategies for the restricted model only. However, we do point out, what will be the corresponding strategies in the unrestricted model. Again when we discuss optimality of strategies, we carry out the proof for the restricted model. But we do point out what ensures perfection in the unrestricted model.

2.3 Results

For stating our result, we will need the following definition.

Definition 5. Core-constrained Nash Bargaining Solution is the allocation that maximizes the Nash product among all allocations in the core. For any (N, v) with a nonempty core, this is uniquely defined.

$$\max_{x \in \mathbb{R}^n} \prod_{i \in N} x_i$$

$$\text{subject to } x(N) = v(N)$$

$$\forall S \subsetneq N,\ x(S) \geq v(S)$$

Proposition 3. For all games in $\mathbf{G} \cup \mathbf{S}$ and for all sufficiently high discount factors, there exists an order-independent efficient subgame perfect equilibrium in pure stationary strategies whose limiting outcome is the core-constrained Nash Bargaining Solution.

2.4 Proof of Proposition 1

For strictly supermodular games **S**, the equilibrium construction is rather involved. So for ease of exposition, we will first consider a 3-player example that will illustrate the equilibrium construction and contrast it with the other mechanisms that have been studied.

Example 1. Consider the following 3-player strictly supermodular coalitional game.

S	v(S)	S	v(S)	S	v(S)
{1}	0	{2}	0	{3}	0
{1,2}	0.7	{1,3}	0.2	{2,3}	0.2
{1, 2, 3}	1	∅	0		

Remark. 1. The core-constrained NBS in this game is the allocation $u^\star = (0.35, 0.35, 0.3)$.

2. The set of coalitions for which the core constraints are binding, $\mathscr{S}_b = \{\{1,2\},\{1,2,3\}\}$.

3. For the Chatterjee et. al.(1993) mechanism, if Player 1(or 2) is the initial proposer, he proposes to $\{1,2\}$ in the unique SSPE and the limiting outcome is $(0.35, 0.35, 0)$.

It may be verified that for all orders ϕ, for sufficiently high δ, the following proposal strategies are supported as an SSPE.

Proposal Strategies. Player i makes an offer $u(\delta, N, i)$ to the grand coalition N. The offers are:

$$\begin{aligned}
u(\delta, N, 1) &= \Big(\frac{1}{\delta}\frac{0.7}{1+\delta}, \frac{0.7}{1+\delta}, 1 - \frac{0.7}{\delta}\Big) \\
u(\delta, N, 2) &= \Big(\frac{0.7}{1+\delta}, \frac{1}{\delta}\frac{0.7}{1+\delta}, 1 - \frac{0.7}{\delta}\Big) \\
u(\delta, N, 3) &= \Big(\frac{0.7}{1+\delta}, \frac{0.7}{1+\delta}, 1 - \frac{1.4}{\delta}\Big)
\end{aligned}$$

Now we will deal with any strictly supermodular game.

2.4.1 Partition of players

Given a strictly supermodular (N, v), let $\{N_1, \ldots, N_L\}$ be the partition of the set of players N induced by the core-constrained Nash Bargaining Solution. We know this partition can be provided because of the following known result.

Claim D. Compte and Jehiel (2010). For (N, v) strictly supermodular, the set of binding coalitions $\mathscr{S}$ with respect to a core allocation $u^\star$ is nested. That is $\mathscr{S}$ is of the form $\{S_1, S_1 \cup S_2, \ldots, S_1 \cup \ldots \cup S_L\}$. This naturally induces a partition $\{S_1, S_2, \ldots, S_L\}$ of players.

We remark here that the partition generated from this result may differ from that generated by the algorithm described in Dutta and Ray (1989) to characterize the core-constrained Nash Bargaining Solution. Also note that for games in $\mathbf{G}$, there is no partition of players i.e. $L = 1$. From the construction described in what follows, the strategies for games in $\mathbf{G}$ can be read out accordingly keeping this in mind.

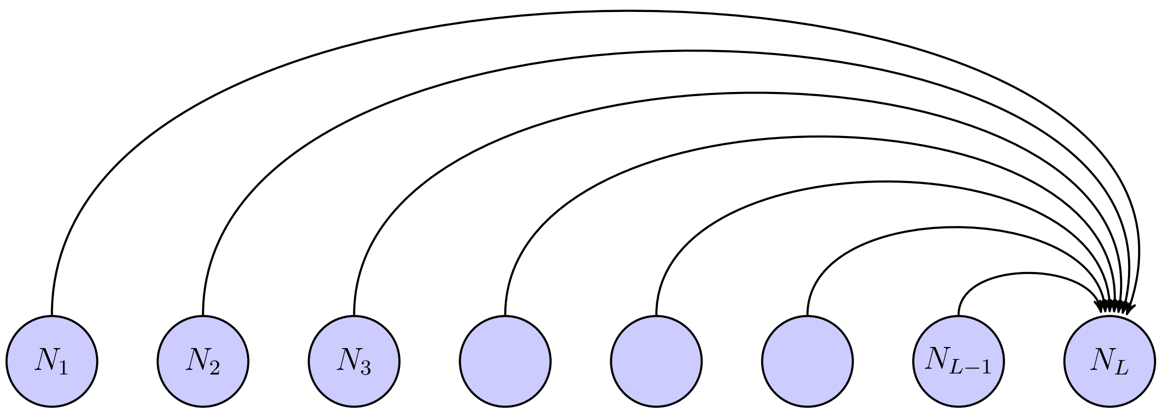

Figure 2.1: Equilibrium Structure of "Hostages"

2.4.2 Proposal Strategy

is described in terms of base payoffs, proposer's advantage and hostage's disadvantage.

Consider the game $G(N, v)$ with the full set N as the population of players. The proposal strategy is described as additions and subtractions from a vector, $b(N, \sigma) \in \mathbb{R}^n_+$. Player i can get more than $b_i(N, \sigma)$ only as a proposer and we call that addition as proposer's advantage. Player i can get less than $b_i(N, \sigma)$ only as a responder. When this happens, we say that the proposer has taken i as a *hostage* and i's loss relative to $b_i(N, \sigma)$ as *hostage's disadvantage*.

We now describe the proposal strategies. Whenever $i \in N$ makes a proposal, he offers $(N, u(\delta, N, i))$ where for $a = 1, \ldots, L$, for $i \in N_a$, for $j \in N \setminus (N_L \cup i)$ and for $l \in N_L \setminus i$

$$u_i(\delta, N, i) = b_i(N, \sigma) + a_i(N, \sigma)$$

$$u_j(\delta, N, i) = b_j(N, \sigma)$$

$$u_l(\delta, N, i) = b_l(N, \sigma) - h_i(N, \sigma)$$

Figure 1 depicts the structure of *hostages* in the equilibrium that we construct. For any $k < L$, any player in N_k takes all the players in N_L as his hostages. It's fruitful now to define what we mean by a threat point of a player as a proposer.

Definition 6. For a strategy profile σ of the bargaining game $G(N, v)$, let $(S, u(\delta, S, i))$ be i's offer in in the subgame $G(S, v)$. Then coalition $T \subsetneq S$ and the restricted offer to T, $u_T(\delta, S, i)$ is a credible threat for $i \in S$ with respect to his offer if

(i) i gives his coalition partners in T their discounted continuation value. i.e. $\forall j \in T \setminus i$, $u_j(\delta, S, i) = b_j(S, \sigma)$.

(ii) It is locally optimal for i to implement T at his implementation node. i.e. $v(T) - \sum_{j \in T \setminus i} u_j(\delta, S, i) \geq b_i(S, \sigma)$.

(iii) i threatens to implement his offer $u(\delta, S, i)$ with T at his implementation node.

Our description of equilibrium proposals is a result of two recursive algorithms. The first algorithm inductively describes what will be threat points of players in the equilibrium. In doing this, we 'anchor' the discounted continuation value of our equilibrium in the core-constrained Nash Bargaining Solution. A responder who is not part of a coalitional threat that a proposer uses must be willing to budge from his discounted continuation payoff. This responser can therefore be held *hostage* and suffer a *disadvantage* at the hands of this proposer. Care needs to be exercised at this point in deciding which players are held *hostages* by which players. In the equilibrium we will construct, any proposer who has a threat takes all the players in N_L as his 'hostages'. In the second step, a player i's proposer's advantage is described in terms of $b_i(N, \sigma)$. In the third step, the hostage's disadvantage is described by using feasibility condition of the offers.

RECURSIVE ALGORITHM TO COMPUTE CREDIBLE THREATS

We now describe a simple recursive algorithm for computing credible threats. The computation is based on three features. First, the set of credible coalitional threats is precisely $\{N_1, N_1 \cup N_2, \ldots, N_1 \cup \ldots \cup N_{L-1}\}$. Owing to the way we have partitioned the set of players, this means the set of credible coalitional threats is precisely the set of coalitions for which the core constraints are binding at the core-constrained Nash Bargaining Solution. Thus the only players who do not have a coalitional threat are those in the last block of partition, N_L. The second feature can be described as symmetry. Players in the same block of partition (because of the first feature, this means they have the same coalitional threat) have the same $b_i(N,\sigma)$. Lastly, a player who is part of a coalitional threat is indifferent between implementing this threat and not implementing it.

Step 1. For $i \in N_1$, his coalitional threat is N_1. The symmetry and indifference feature immediately give $b_i(N,\sigma)$.

$$v(N_1) - (|N_1| - 1)b_i(N,\sigma) = \delta b_i(N,\sigma)$$

which gives

$$b_i(N,\sigma) = \frac{v(N_1)}{|N_1| - 1 + \delta}$$

Suppose the credible threats have been computed for $i \in N_1, \ldots, i \in N_{k-1}$.

Step k. For $k < L$, $i \in N_k$, his coalitional threat is $\cup_{a=1}^{k-1} N_a$. The symmetry and indifference feature immediately give $b_i(N,\sigma)$.

$$v(\cup_{a=1}^{k} N_a) - \sum_{a<k} \sum_{j \in N_a} b_j(N,\sigma) - (|N_k| - 1)b_i(N,\sigma) = \delta b_i(N,\sigma)$$

which gives

$$b_i(N,\sigma) = \frac{1}{|N_k| - 1 + \delta}\Big[v(\cup_{a=1}^{k} N_a) - \sum_{a<k} \sum_{j \in N_a} b_j(N,\sigma)\Big]$$

Step L. For $i \in N_L$, set

$$b_i(N,\sigma) = \frac{\delta}{1 + \delta(|N_L| - 1)}\Big[v(N) - \sum_{a<k} \sum_{j \in N_a} b_j(N,\sigma)\Big]$$

The full construction of proposals will justify the above assignment.

PROPOSER'S ADVANTAGE

For $k = 1, \ldots, L$, for any $i \in N_k$, the principle behind computing $a_i(N,\sigma)$ is that for any $j \in \cup_{a \geq k} N_a$, conditional on the event in $G(N,v,j)$ that i rejects and $N \setminus i$ accept the equilibrium offer, the game played next period is $G(N, v, i)$ i.e. i gets to be the proposer.

$$a_i(N, \sigma) = \frac{1-\delta}{\delta} b_i(N, \sigma)$$

Since for each $a = 1, \ldots, L$, for $\{i, j\} \subset N_a$, $b_i = b_j$, we have $a_i = a_j$. Thus we only have L distinct proposer advantages $a^1(N, \sigma) > a^2(N, \sigma) > \ldots > a^L(N, \sigma)$ where for sufficiently high δ, the ordering is due to the corresponding ordering of $\{b^k(N, \sigma)\}_{k=1}^{L}$.

HOSTAGE'S DISADVANTAGE The disadvantage $h_i(N, \sigma)$ of players in N_L at the hands of player i is calculated from the feasibility of i?s equilibrium offer.

$$h_i(N, \sigma) = \sum_{a=1}^{L} \sum_{i \in N_a} b_i(N, \sigma) - v(N) + a_i(N, \sigma)$$

Since for each $a = 1, \ldots, L$, for $\{i, j\} \subset N_a$, $a_i = a_j$, we have $h_i = h_j$. Thus we only have L distinct proposer advantages $h^1(N, \sigma) > h^2(N, \sigma) > \ldots > h^L(N, \sigma)$ where for sufficiently high δ, the ordering is due to the corresponding ordering of $\{a^k(N, \sigma)\}_{k=1}^{L}$.

The construction carried out above fully describes the proposal strategies in the restricted version of the model where only one coalition may form. This is because there are no subgames with a smaller population. For the model as we have described, in any subgame $G(S, v)$ with the set of players being S, the proposal strategies are computed as outlined above with the algorithms being carried out over (S, v). The reduced game (S, v) is also strictly supermodular and hence the same procedure carries over.

Remark. 1. For any player, his proposer's advantage and his hostage's disadvantage is continuously and monotonically decreasing in δ and vanishes in the limit as $\delta \to 1$.

2. For any $i \in N$, the limiting payoff vector as $\delta \to 1$ when i is the proposer is core-constrained Nash Bargaining Solution. We know this from the algorithmic characterization of the core-constrained Nash Bargaining Solution for supermodular games shown by Dutta and Ray (1989).

2.4.3 Implementation Strategy

Consider the game $G(N, v)$ with the full set N as the population of players. Fist consider the equilibrium-offers. Suppose $i \in N_k$.

$\underline{k = L}$. L is the last block of our partition of players, then i strategy is as follows

(a) If every player $j \in N \setminus i$ accepts the offer, then i implements the offer.

(b) If any player $j \in N \setminus i$ rejects the offer, then i discards the offer.

$\underline{k < L}$ Then i's strategy is as follows.

(a) If every player $j \in N \setminus i$ accepts the offer, then i implements the offer.

(b) If every player $j \in \cup_{a=1}^{k} N_a \setminus i$, then i implements his offer with this coalition.

(c) If some player in $\cup_{a=1}^{k} N_a$ rejects, then i discards the offer.

(d) For any other set of acceptances, i implements that coalition which gives him the maximum payoff provided this payoff is at least as great as the payoff he gets by discarding the offer.

For off-equilibrium offers, i implements that coalition which gives him the maximum payoff provided this payoff is at least as great as the payoff he gets by discarding the offer.

This describes the implementation strategies of players in the restricted version of the model where only one coalition may form. For the unrestricted version as we have described, for any subgame $G(S, v)$ with S as the population of players, the corresponding partition is $(S_a)_a$. Players follow the implementation strategies as described above with this change.

2.4.4 Acceptance-Rejection Strategy

Consider j's response node in the game $G(N, v)$ where Q is the set of players who have accepted or proposed the standing offer (S, x_S) made by i so far. Since the responders move in a pre-determined order ϕ_S, anyone of them while contemplating accepting or rejecting the offer has to consider the effect of his decision on the decisions of the responders following him. The ARSs of responders are defined inductively, first for the last responder according to ϕ, then for the penultimate responder and so on backwards in order.

Let $Y_j \subset S \backslash (Q \cup \{j\})$ be the set of responders who will accept the proposal (S, x_S) according to their respective ARSs if j rejects it. For the last responder in S, $Y_j = \emptyset$. The ARS of j is conditioned on whether i would implement his offer with a coalition $S_I \subset Q \cup Y_j$ or discard his offer.

For equilibrium offers,

$\underline{j \notin N_L}$

(a) If i discards his offer conditional on a rejection by j, then j accepts the offer if it gives him at least $b_j(N, \sigma)$ and rejects otherwise.

(b) If i implements his offer with a coalition conditional on a rejection by j, then j accepts the offer if it gives him at least $\delta v(j)$ and rejects otherwise.

$\underline{j \in N_L}$

(a) If i discards his offer conditional on a rejection by j and the game next period is $G(N, v, l)$, then j accepts the offer if it gives him at least $\delta u_j(\delta, N, l)$ and rejects otherwise.

(b) If i implements his offer with a coalition conditional on a rejection by j, then j accepts the offer if it gives him at least $\delta v(j)$ and rejects otherwise.

For off-equilibrium offers,

$\underline{j \notin N_L}$

(a) If i discards his offer conditional on a rejection by j, then j accepts the offer if it gives him more than $b_j(N, \sigma)$ and rejects otherwise.

(b) If i implements his offer with a coalition conditional on a rejection by j, then j accepts the offer if it gives him more than $\delta v(j)$ and rejects otherwise.

$\underline{j \in N_L}$

(a) If i discards his offer conditional on a rejection by j and the game next period is $G(N, v, l)$, then j accepts the offer if it gives him more than $\delta u_j(\delta, N, l)$ and rejects otherwise.

(b) If i implements his offer with a coalition conditional on a rejection by j, then j accepts the offer if it gives him more than $\delta v(j)$ and rejects otherwise.

Note for the equilibrium proposal, responders resolve any indifference by accepting while for any off-equilibrium proposal, they resolve it by rejecting. This describes the acceptance-rejection strategies of players in the restricted version of the model where only one coalition may form. For the unrestricted version as we have described, for any subgame $G(S, v)$ with S as the population of players, the corresponding partition is $(S_a)_a$. Players follow the acceptance-rejection strategies as described above with this change keeping in view that when a proposer threatens to form a subcoalition $T \subsetneq S$, the responders while making their decision look forward to their payoff in the continuation game $G(S \setminus T, v)$. We omit writing it here.

2.4.5 Optimality

Again for ease of exposition, we only show that the strategies described constitute an SSPE for the restricted model. The proof can be found in Appendix A. A remark is in order as to how the equilibrium we have constructed achieves efficiency no matter who proposes as opposed to Chatterjee et. al.(1993). In their mechanism if any player who was not in the last block of partition was chosen as a proposer, efficiency was not obtained. In the equilibrium we have constructed, each such player has a credible coalitional threat using which he can relax the demands of players who are not part of this coalitional threat. Thus each such player strictly prefers to make an offer to the grand coalition. The equilibrium is sustained by the following expectations. Any deviation by a proposer in which he tries to unilaterally gain at the expense of players outside his coalitional threat (outsiders) will be met by a rejection by players inside his coalitional threat (insiders). A rejection by insiders is a best response since they are indifferent between rejecting and accepting. Any deviation by a proposer in which he tries to gain at the expense of outsiders by bribing insiders will be met by a rejection by outsiders since the act of bribing insiders renders the threat incredible. Outsiders call it a bluff and anticipating/seeing this response, the insiders reject it as well since they realize the sweetened offer to them is just a mirage which is never going to materialize.

For the unrestricted model as we have described, the corresponding strategies described constitute an SSPE as well. This follows because the limit allocation, the core-constrained Nash Bargaining Solution satisfies a monotonicity property shown in Dutta (1990). For stating this property, let $u^\star(N,v)$ denote the core-constrained Nash Bargaining Solution for the coalitional game (N,v). For a vector $y \in R^N$, let y_S denote the projection of y along the axes of players in S.

Dutta (1990). Suppose (N,v) is strictly supermodular. For any $S \subsetneq N$, $u^\star_S(N,v) > u^\star(S,v)$ where the strict inequality is for all coordinates.

The result above ensures that in any subgame $G(S,v)$, for all sufficiently high δ, every responder agrees to the proposal no matter who proposes. Thus in every subgame $G(S,v)$, the entire coalition S is formed immediately. Thus we have shown Proposition 1.

2.5 Further Results

Let us define some sets pertaining to the equilibrium points of the game $G_\phi(N,v)$.

$$E^\phi = \{\sigma : \sigma \text{ is an SSPE of } G_\phi(N,v) \text{ for all sufficiently high } \delta\}$$
$$E^\phi_e = \{\sigma \in E^\phi : \text{ everyone makes an offer to } N\}$$
$$E^\phi_c = \{\sigma \in E^\phi : u^\star(N,\sigma) \in \text{ core } (N,v)\}$$

The following result says that for strictly superadditive coalitional environments, no SSPE can exhibit delay in any subgame. The proof can be found in Appendix B.

Lemma 3. Suppose (N,v) is strictly superadditive. For an SSPE $\sigma \in E^\phi$, for every $S \subset N$, every player $i \in S$ must make an acceptable offer in every subgame $G(S,v)$.

The proof of Lemma 1 does not involve any new ideas. It is reminiscent of Okada (1996) who gets this result for the mechanism he studies. It will be an input to our next observation.

Lemma 4. Suppose (N, v) is strictly superadditive. Suppose for some order ϕ, σ is an asymptotically efficient SSPE as $\delta to 1$. Then it must have its limit value $u^\star(N, \sigma)$ in the core. Formally, $E_e^\phi \subset E_c^\phi$.

Corollary. The set of credible threats in $\sigma \in E_e^\phi$ is a subset of the set of binding coalitions with respect to $u^\star(N, \sigma)$.

The idea of proof of Lemma 4 is as follows. For any symptotically efficient SSPE σ of $G_\phi(N, v)$, we show that $u(N, \sigma, i) \in C(N, v)$ where $C(N, v)$ is the core of a game derived from (N, v). A limit argument will then give the result. The proof can be found in Appendix B.

2.6 Concluding Remarks

In the classical environment of coalitional game with transferable utility, the efficiency implications of relaxing unanimity (as a requirement for agreement on an offer) in a bargaining model was examined. An asymptotically efficient equilibrium in pure strategies was displayed for strictly supermodular coalitional games, a result that does not obtain in the models of Chatterjee et al. (1993). The limiting outcome of this equilibrium is found to be the core-constrained Nash Bargaining Solution. The efficiency implications of noncontingent offers are underscored by similar result we find when we embed this feature in Okada (1996) mechanism proposers are selected randomly. Such an exercise is undertaken in Chaturvedi (2013b). We have found it difficult to establish uniqueness of the limiting allocation in the class of SSPE. Investigating this and the efficiency implications of the mechanism for strictly superadditive games with nonempty cores are avenues for future work.

Appendix A

Claim 4.1. Suppose (N, v) is strictly supermodular. Let $u^\star$ be the core-constrained Nash Bargaining Solution for (N, v). Then for every $S \subset N$ such that $S \neq \cup_{a=1}^{k} N_a$ for some $k = 1, \ldots, L$, we have $\sum_{j \in S} u_j^\star > v(S)$.

Proof. This is because $u^\star$ is in the core of (N, v) and by definition the set of all coalitions that are binding at $u^\star$ is precisely $\{N_1 N_1 \cup N_2, \ldots, N_1 \cup \ldots \cup N_L\}$. Q.E.D.

Claim 4.2. Suppose (N, v) is strictly supermodular. Then $\forall k = 1, \ldots, L-1$

$$\frac{v(\cup_{a=1}^{k} N_a)}{\sum_{a=1}^{k} |N_a|} \geq \frac{v(N)}{|N|}$$

Proof. Step 1. Let $N_0 = \emptyset$. Given (N, v), for $k = 1, \ldots, L(v)$, define inductively the restricted game $(N \setminus \cup_{a=0}^{k-1} N_a, v_k)$ by $v_k(S) = v(\cup_{a=0}^{k-1} N_a \cup S) - v(\cup_{a=0}^{k-1} N_a)$. In this step, we show a property of supermodular games that may be called the 'principle of cascading averages'. It says that the average values of restricted (in the sense defined above) games of a supermodular game are ordered in a decreasing fashion. We show the following statement is true:

$$\forall k \in \{1, \ldots, L-1\}, \quad \frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} \geq \frac{v(N) - v(\cup_{a=0}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|}$$

Suppose to the contrary that for some k

$$\frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < \frac{v(N) - v(\cup_{a=0}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|} \tag{2.1}$$

Since $k \neq L$, $N_k \subsetneq N \setminus \cup_{a=0}^{k-1} N_a$ is a maximizer at Step k of our algorithm for generating partitions. So

$$\frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} \leq \frac{v(\cup_{a=0}^{k} N_a) - v(\cup_{a=0}^{k-1} N_a)}{|N_k|} \tag{2.2}$$

Adding (2.1) and (2.2),

$$\frac{\left|N \setminus \cup_{a=0}^{k} N_a\right| + |N_k|}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|}[v(N) - v(\cup_{a=0}^{k-1} N_a)] < [v(N) - v(\cup_{a=0}^{k-1} N_a)]$$

By strict supermodularity of (N, v), $v(N) - v(\cup_{a=0}^{k-1} N_a) > 0$. So we have

$$1 = \frac{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < 1$$

a contradiction.

Step 2. By Step 1, we get $\forall k \in \{1, \ldots, L-1\}$

$$\begin{aligned}
\frac{v(N)}{|N|} &\geq \frac{v(N) - v(\cup_{a=1}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|} \\
\frac{\left|N \setminus \cup_{a=0}^{k} N_a\right|}{|N|} &\geq \frac{v(N) - v(\cup_{a=1}^{k} N_a)}{v(N)} \\
\frac{v(\cup_{a=1}^{k} N_a)}{\sum_{a=1}^{k} |N_a|} &\geq \frac{v(N)}{|N|}
\end{aligned}$$

Q.E.D.

Proof of Proposition 1 (ctd.) *Optimality of Implementation Strategy*

For off-equilibrium proposals, the optimality is clear. The lemma below implies optimality for equilibrium proposals.

Claim 4.3. Consider the implementation node of $i \in N_k$

(1) Suppose $k < L$ and $\cup_{a=1}^{k} N_a$ is the set of players who have accepted or proposed i's equilibrium offer. Then i is indifferent between partially implementing the proposal with $\cup_{a=1}^{k} N_a$ and discarding it. Moreover i cannot gain by implementing a subcoalition among $\cup_{a=1}^{k} N_a$. Hence it is optimal to implement $\cup_{a=1}^{k} N_a$.

(2) i cannot gain by implementing his equilibrium offer with a subcoalition that excludes players from $\cup_{a=1}^{k} N_a$.

Proof. . Let

$$\mathscr{T} = \underset{T \subset N_A,\, i \in T}{\operatorname{argmax}} \left[v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i)\right]$$

We want to prove the following statements.

(1) When $k < L$ and $N_A = \cup_{a=1}^k N_a$ (a) $\cup_{a=1}^k N_a \in \mathscr{T}$ and (b)

$$v(\cup_{a=1}^k N_a) - \sum_{j \in \cup_{a=1}^k N_a \setminus i} u_j(\delta, N, i) = b_i(N, \sigma)$$

(2) Suppose $T \subset N_A$ is such that (a) $i \in T$ and (b) $\cup_{a=1}^k N_a \setminus T \neq \emptyset$. Then for all sufficiently high δ, it is optimal for i to discard the equilibrium offer rather than implement it with T. That is

$$v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i) \leq u_i(\delta, N, l)$$

where l is the proposer next period if i discards the equilibrium offer.

1(b) is true by virtue of our construction of equilibrium proposals. As regards 1(a), take $T \subsetneq \cup_{a=1}^k N_a$. Let $u^\star$ be the core-constrained Nash Bargaining Solution algorithmically given by Dutta and Ray (1989). Then we know T is not a binding coalition with respect to $u^\star$. Thus $u_i^\star > v(T) - \sum_{j \in T \setminus i} u_j^\star$. Also by construction, $v(\cup_{a=1}^k N_a) - \sum_{j \in \cup_{a=1}^k N_a \setminus i} u_j(\delta, N, i) = \delta b_i \to u_i^\star$ as $\delta \to 1$; while $v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i) \to v(T) - \sum_{j \in T \setminus i} u_j^\star$ as $\delta \to 1$. Thus we conclude that for sufficiently high δ, $v(\cup_{a=1}^k N_a) - \sum_{j \in \cup_{a=1}^k N_a \setminus i} u_j(\delta, N, i) > v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i)$.

For (2), any such T is not a binding coalition with respect to $u^\star$. Thus $v(T) - \sum_{j \in T \setminus i} u_j^\star < u_i^\star$. By construction, $v(T) - \sum_{j \in T \setminus i} u_j(\delta, N, i) \to v(T) - \sum_{j \in T \setminus i} u_j^\star$ and $u_i(\delta, N, l) \to u_i^\star$ as $\delta \to 1$. Q.E.D.

Optimality of Acceptance-Rejection Strategy is clear.

Optimality of Proposal Strategy

We first show that the equilibrium offer is accepted no matter who proposes. Suppose $i \in N_k$. Consider the last responder in $\cup_{a=1}^k N_a$ who finds himself in a situation where every other responder in $\cup_{a=1}^k N_a$ has already accepted the proposal. By rejecting, the most j can get is $b_j(N, \sigma)$. If j accepts, then by part 1 of Claim 4.3, $\cup_{a=1}^k N_a$ is certainly a candidate for i to consider implementing. Part 2 of Claim 4.3 assures that j is part of any coalition that i will choose to implement. In any case, j is certain to get $u_j(\delta, N, i) = b_j$. Thus j is indifferent between accepting and rejecting the proposal which means accepting is optimal for him. By induction on the number of responders in $\cup_{a=1}^{k} N_a \setminus j$ who have not yet responded to the proposal, it follows for any responder $j \in \cup_{a=1}^{k} N_a$ that if every other responder in $\cup_{a=1}^{k} N_a \setminus j$ who has already responded to the proposal has accepted it, then it is optimal for j to accept it.

Consider a responder $j \in N \setminus \cup_{a=1}^{k} N_a$. If all players in $\cup_{a=1}^{k} N_a$ who have already responded before him have accepted the proposal, then j knows by the arguments of the preceding paragraph that the other players in $\cup_{a=1}^{k} N_a$ who will follow him will accept the proposal as well. Since $\cup_{a=1}^{k} N_a$ is certainly one of the candidates that meets the requirements of implementation, j knows that some coalition will form and the game would end. If j accepts, two situations may arise. Either j is part of the coalition that i forms or he isn?t. In the former case, he gets $u_j(\delta, N, i)$ while in the latter $\delta v(j)$. If he rejects, some coalition forms and the game ends. So he gets $\delta v(j)$. Thus accepting weakly dominates rejecting the proposal for j.

Deviations from equilibrium offer. Suppose $i \in N_k$.

Case 1. $k < L$. Classify deviations as:

(1) (N, d) where $\forall j \in \cup_{a=1}^{k} N_a \setminus i, d_j = u_j(\delta, N, i), \forall j \in N \setminus \cup_{a=1}^{k} N_a, d_j \leq u_j(\delta, N, i)$ and $\exists j \in N \setminus \cup_{a=1}^{k} N_a$ such that $d_j < u_j(\delta, N, i)$. This is a deviation where the proposer i tries to further gain unilaterally at the expense of his "hostages" i.e. players in N_L or other players not in $\cup_{a=1}^{k} N_a$. A unanimous rejection of this off-equilibrium offer is obtained in what follows.

Consider the last responder j in $\cup_{a=1}^{k} N_a$ who finds himself in a situation where every other responder in $\cup_{a=1}^{k} N_a$ has already rejected the proposal. If j is the only responder in $\cup_{a=1}^{k} N_a \setminus$

i, then by part 2 of Claim 4.3, if j rejects, i will certainly pass and he gets $b_j(N, \sigma) = u_j(\delta, N, i) = d_j$ which is what he gets by accepting. So j is indifferent and it is optimal for him to reject. The argument is easily extended now by induction on the number of responders in $\cup_{a=1}^{k} N_a \setminus j$ who have not yet responded to the proposal to say that for any responder $j \in \cup_{a=1}^{k} N_a$, if every other responder in $\cup_{a=1}^{k} N_a \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it.

Since all responders in $\cup_{a=1}^{k} N_a$ reject this off-equilibrium proposal, any responder $j \in N \setminus \cup_{a=1}^{k} N_a$ knows that i's threat to implement $\cup_{a=1}^{k} N_a$ is an empty threat. Whether j accepts or rejects, i will discard the offer but if j rejects, he might get to be the proposer if he is the first rejector which may be better for him relative to the current offer. Thus it is weakly better for him to reject as well.

(2) (N, d) where $\forall j \in \cup_{a=1}^{k} N_a \setminus i, d_j \geq u_j(\delta, N, i), \exists j \in \cup_{a=1}^{k} N_a \setminus i, d_j > u_j(\delta, N, i), \forall j \in N \setminus \cup_{a=1}^{k} N_a, d_j \leq u_j(\delta, N, i)$ and $\exists j \in N \setminus \cup_{a=1}^{k} N_a$ such that $d_j < u_j(\delta, N, i)$. This is a deviation where the proposer i tries to further gain at the expense of his "hostages" i.e. players in N_L or other players not in $\cup_{a=1}^{k} N_a$ and offers to share the gains with some players in $\cup_{a=1}^{k} N_a$.

If there is a player j in i's coalitional threat that i does not "bribe" in the sense above, then the argument works as in (1). j being indifferent, rejects the offer and i cannot gain. Suppose i bribes every player in his coalitional threat. That is, $\forall j \in \cup_{a=1}^{k} N_a \setminus i, d_j > u_j(\delta, N, i)$. Consider the last responder $j \in N \setminus \cup_{a=1}^{k} N_a$ who finds himself in a situation where every other responder in $N \setminus \cup_{a=1}^{k} N_a$ has already rejected the proposal. Now j is certain that if he rejects, i would not implement $\cup_{a=1}^{k} N_a$ and would prefer to pass. This is because

$$\begin{aligned} v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} d_j &=< v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j(\delta, N, i) \\ &= \delta b_i(N, \sigma) \end{aligned}$$

If he rejects, he gets $u_j(\delta, N, l)$ for some $l \in N \setminus \cup_{a=1}^{k} N_a$. Since $u_j(\delta, N, l) \geq u_j(\delta, N, i) \geq$

d_j with a strict inequality for some j, it is optimal for j to reject the proposal. Argue by induction on the number of responders in $N \setminus \cup_{a=1}^{k} N_a$ who have not yet responded to the proposal to say that for any responder $j \in N \setminus \cup_{a=1}^{k} N_a$, if every other responder in $(N \setminus \cup_{a=1}^{k} N_a) \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it.

Since all responders in $N \setminus \cup_{a=1}^{k} N_a$ reject this off-equilibrium proposal while all responders in $\cup_{a=1}^{k} N_a$ accept it, i cannot gain from this deviation.

(3) (S, d) where $S = \cup_{a=1}^{k} N_a$ and $\forall j \in S \setminus i, d_j = u_j(\delta, N, i) + \epsilon$ for $\epsilon > 0$. The payoff for i from such an offer is

$$\begin{aligned}
d_i &= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} d_j \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} u_j(\delta, N, i) - \Big(\sum_{a=1}^{k} |N_a| - 1 \Big) \epsilon \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \setminus i} b_j - \Big(\sum_{a=1}^{k} |N_a| - 1 \Big) \epsilon \\
&= \delta b_i - \Big(\sum_{a=1}^{k} |N_a| - 1 \Big) \epsilon \\
&< b_i + a_i = u_i(\delta, N, i)
\end{aligned}$$

Thus i cannot gain by this deviation.

(4) (S, d) where S does not include some players from i's credible threat $\cup_{a=1}^{k} N_a$ and $\forall j \in S \setminus i, d_j = u_j(\delta, N, i)$. The payoff for i from such an offer is arbitrarily close to $v(S) - \sum_{j \in S \setminus i} u_j^\star$ while the payoff from the equilibrium strategy is arbitrarily close to $u_i^\star$ as δ gets high enough. By Claim 4.1 in Appendix A, i cannot gain by this deviation.

Case 2. $k = L$. Consider any deviation (N, d) where $i \in N \setminus S, \forall j \in S, d_j < u_j(\delta, N, i)$ and $\forall j \in N \setminus S, d_j \geq u_j(\delta, N, i)$. This is a deviation where the proposer i tries to gain at the expense of players in S possibly by sweetening the offer for some players in $N \setminus S$. A unanimous rejection of this off-equilibrium offer is obtained in what follows.

Consider the last responder j in S who finds himself in a situation where every other responder in S has already rejected the proposal. If j is the only responder in S, then by rejecting j gets $b_j(N, \sigma) = u_j(\delta, N, i) > d_j$. This is because by part (2) of Claim 4.3, j knows that i is certainly going to pass at his implementation node if he rejects. So j prefers to reject. The argument is easily extended now by induction on the number of responders in $S \setminus j$ who have not yet responded to the proposal to say that for any responder $j \in S$, if every other responder in $S \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it. Q.E.D.

Appendix B

Proof of Lemma 1 Step 1. Suppose $\sigma \in E^{\phi}$ involves a player $i \in S$ making an unacceptable offer in some subgame $G(S, v)$ following which $G(S, v, l)$ is played. We construct a profitable deviation (S, d) for i. Let d be defined by $\forall j \in S \setminus i$, $d_j = \delta u_j(S, \sigma, l) + \epsilon$ where $0 < \epsilon < (v(S) - \sum_{j \in S} \delta u_j(S, \sigma, l))/(|S| - 1)$. Then d will be unanimously accepted since any responder does worse by rejecting. i gets $d_i = v(S) - \sum_{j \in S \setminus i}(\delta u_j(S, \sigma) + \epsilon)$. By making an unacceptable offer, i's payoff at his proposal node in $G(S, v)$ is $\delta u_i(S, \sigma, l)$. It is then easy to see by the choice of ϵ that $d_i > \delta u_i(S, \sigma, l)$ i.e. $v(S) > \sum_{j \in S} \delta u_j(S, \sigma) + (|S| - 1)\epsilon$.

Step 2. We now show that we can always choose such an ϵ. In other words, $v(S) - \sum_{j \in S} \delta u_j(S, \sigma, l) > 0$. By strict superadditivity,

$$\sum_{j \in S} u_j(S, \sigma, l) \leq v(S) \sum_{j \in S} \delta u_j(S, \sigma, l) < v(S)$$

Thus we have shown a profitable deviation by i. This contradicts the premise that $\sigma \in E^{\phi}$. Q.E.D.

Proof of Lemma 2 Fix an asymptotically efficient SSPE $\sigma \in E^{\phi}_e$. Then if i is the initial proposer in $G_{\phi}(N, v)$

$$\sum_{j \in N} u_j(N, \sigma, i) = v(N)$$

Fix $S \subsetneq N$. There are two cases.

Case 1. S is a credible threat for some $i \in S$ in σ. Then by definition of credibility, $v(S) \geq \sum_{j \in S \setminus i} u_j(N, \sigma, i) \geq \delta u_i(N, \sigma, l)$ where $G_\phi(N, v, l)$ is the game played next period if i discards the offer. Suppose the inequality is strict i.e. let $\eta(\delta) = v(S) - \sum_{j \in S \setminus i} u_j(N, \sigma, i) - \delta u_i(N, \sigma, l) > 0$. We construct a profitable deviation for i. Now $u(N, \sigma, i)$ is i's acceptable (by Lemma 3) offer to N in σ. Since S is a credible threat for i, $\exists j \in N \setminus S$ such that $u_j(N, \sigma, i) = \delta u_j(N, \sigma, j) - h_j$ where $h_j > 0$.

Construct a feasible deviation (N, d) where

$$\begin{aligned} \forall k \in S, \quad d_k &= u_k(N, \sigma, i) + \frac{\epsilon}{|S|} \qquad\qquad 0 < \epsilon < \eta(\delta) \\ d_j &= u_j(N, \sigma, i) - \epsilon \\ \forall k \in N \setminus \{j \cup S\}, \quad d_k &= u_k(N, \sigma, i) \end{aligned}$$

For this deviation, the credibility of i's threat to implement S is preserved. The requirement for this is

$$\begin{aligned} v(S) - \sum_{j \in S \setminus i} d_j &> \delta u_i(N, \sigma, l) \\ \leftrightarrow v(S) &> \sum_{j \in S \setminus i} u_j(N, \sigma, i) + \delta u_j(N, \sigma, l) + \frac{|S| - 1}{|S|}\epsilon \\ \leftrightarrow \eta(\delta) &> \frac{|S| - 1}{|S|}\epsilon \end{aligned}$$

which is true. So the deviation (N, d) in which i by bribing $S \setminus i$ enlists their support in further taking advantage of j is not deterred as the credibility of i's threat to implement S is unaffected by this deviation.

Thus in an asymptotically efficient SSPE

$$\sum_{j \in S} u_j(N, \sigma) > \sum_{j \in S \setminus i} u_j(N, \sigma) + \delta u_j(N, \sigma, l) = v(S)$$

Case 2. S is not a credible threat for any $i \in S$ in σ. Then by definition of credibility, $\delta u_i(N, \sigma, l) > v(S) - \sum_{j \in S \setminus i} u_j(N, \sigma, i)$. By efficiency, $u_i(N, \sigma, i) \geq \delta u_i(N, \sigma, l)$ so that

$\sum_{j \in S} u_j(N, \sigma) > v(S)$.

We have thus proved $u(N, \sigma, i) \in C(N, v)$. A limit argument as $\delta \to 1$ thus proves that $u^\star(N, \sigma) \in C(N, v)$. Q.E.D.

Chapter 3

Creditor Commitment in a Sequential Investment Model

3.1 Introduction

In a creditor-entrepreneur relationship, is the strategy to commit not to refinance when faced with default of any value to the creditor. If so, what determines this value. We argue that certain sequential production technologies- technologies that need sequential investment input and that produce output in earlier rounds that is valuable as an input for later rounds have a role. We also argue that this limits the ability of competition on the entrepreneurial side of the market to enforce this commitment for the creditor. When hidden information is incorporated as a basic feature of the environment besides lack of commitment, there are information structures under which the creditor does not refinance after a default, an outcome the same as that which obtains with complete information and commitment.

The standard one period model of investment with moral hazard is well known and illustrates why there might be credit rationing due to the wedge between a firm's value and pledgeable income [1]. The baseline model that we use is borrowed from Holmstrom and Tirole (1997) although it has many precursors like Hölmstrom (1979) , Grossman and Hart (1983) and

[1] Pledgeable Income is the maximum income that can credibly (i.e. net of agency rents) be promised to investors

Innes (1990) [2]. We use a two period version of that model to allow for default and create possibilities for refinancing.

The model has an entrepreneur who has access to a two period investment project that needs fixed capital investment each period and generates binary returns that are independent across periods. Each period, the entrepreneur has a binary effort choice. Effort is hidden but returns are observable. When returns are high, the entrepreneur can pay his dues and remains solvent. When returns are low, although nonzero, he cannot and therefore there is default. The project is only partially complete at the end of first period which is modeled as: the first period output is more valuable as an input for second period. There is limited liability each period with liability being limited to current income. Parties are risk neutral.

In the first part, we study the creditor's preference over two simple contracts that differ in their degree of commitment only in the event of default. The "commitment contract" binds the creditor to commit not to refinance when he observes a default while a "lack of commitment" contract does not bind him. The creditor's preference over these commitment scenarios is not unambiguous and depends on the extent of the moral hazard problem. The higher the amount of private benefit that the entrepreneur may potentially enjoy, the more likely it is that the creditor prefers to commit not to refinance. It is instructive to look at the cost and benefit that determine this preference. The commitment to terminate financing when faced with default implies that incentives to work in the first period can be provided with lower rewards. This is the benefit of commitment. The cost, however, is that the creditor must forego the future surplus that could be exploited by refinancing. When the entrepreneur's private benefit is sufficiently high, the creditor cares a lot about incentives as opposed to exploiting the future surplus. When the private benefit is low, however, getting a share of future pie is more tempting at the cost of lower share of first period pie.

The commitment may not be easy to enforce. So it is natural to inquire into features of the environment that may help the creditor enforce this commitment. When creditor

[2] A recent exposition is in Chapter 1 of Holmström and Tirole (2011). See also Chapter 3 in Tirole (2010).

lacks commitment, one may hope that markets may work as enforcement mechanism for commitment by providing more choices or outside options. In the second part, we test the ability of a competitive pool of entrepreneurs to act as a commitment device when the creditor cannot himself commit against refinancing and when credit supply is scarce. We point out that this is limited by the amount of output produced by the sequential investment project. Against a backdrop of depressed liquidation values, the output produced is more valuable in the hands of entrepreneur who can use it as inside equity towards the next round of investment. Other potential entrepreneurs must now match this project in terms of inside equity in order to secure financing. The higher the output produced, the more difficult it is for outside competitors to match him. To the extent that they may not be able to match him, the competition fails to act as a commitment device of the creditor. But when an outside competitor does match the project in terms of inside equity pledged, competition does act as a credible deterrent against refinancing and its benign effect on first period incentives is exactly the same as if the creditor had made the commitment.

Adverse selection or hidden information is considered an important feature in modeling credit markets. In the third part, we enrich the no-commitment moral hazard model with hidden information and study its role in enforcing commitment. The entrepreneur can be of two types- high and low ability. Ability affects the output distribution only under high effort. It turns out that in both periods, the creditor prefers to implement a high effort from a high ability type and a low effort from a low ability type entrepreneur. The optimal contract that implements this is a pooling contract. We display sufficient conditions on informativeness of output under which hidden information makes commitment sequentially rational. Although the contracting problem is now involved because both moral hazard and adverse selection are present, the main idea is that an informational environment sufficiently precise allows the creditor to statistically infer that the entrepreneur is likely to be of low ability when he observes a default.

In related literature, Dewatripont and Maskin (1995) look at how the institutional structure on the creditor side, specifically, the degree of centralization impacts creditor's commitment. Using a stylized model, they argue centralization is more prone to lack of commitment and decentralization may help creditors make the commitment not to refinance. The impact of commitment problems on financial contracting has been an active area of research. Hart and Moore (1994) identify a source of credit rationing that is quite different from that based on managerial ability to divert funds- one based on lack of commitment on the part of an entrepreneur not to withdraw his human capital i.e. his ability to quit the project and the consequent costly substitution of human capital for the creditor. If the entrepreneur were able to make such a commitment, claims on project returns that allow a creditor to break even are enough to secure financing. But due to lack of commitment, such claims are no longer credible. The possibility of the entrepreneur to quit at any date puts a constraint on the size of the claim that he can credibly promise. Hart and Moore (1994) derive the constraint as the equilibrium payoff the creditor gets in a structured renegotiation game following a quit that takes into account the prospect that he may quit again in the future. Neher (1999) explores the role of sequential investments in mitigating the commitment problem of the sort pointed out by Hart and Moore (1994). Under a view that lifecycle of a project involves the gradual conversion of "inalienable" human capital into "alienable" physical capital, sequential investments help because they allow the creditor to use the collateral created during early rounds of investments and the threat to withhold subsequent investment to guard against any attempt by the entrepreneur to diminish the claim of creditor owing to lack of commitment.

There is a different literature, comprising among many others, Myerson (2012) (see references therein) that rationalizes credit cycles in economies with moral hazard in financial intermediation. Myerson (2012) in particular builds on earlier insights of Becker and Stigler (1974) and Shapiro and Stiglitz (1984) that enforcement in environments with repeated shirking opportunities is efficiently provided by compensation contracts that promise large end of the career (backloaded) rewards for agents with consistent good performance records. This literature relies on compensation contracts as instruments for enforcement. In contrast, we investigate the extent to which elements of the underlying environment, namely competition on entrepreneurial side of the market and hidden information help in enforcement.

3.2 Model

3.2.1 Environment

There is a creditor **C** who has wealth but no idea and a debtor **D** who has an idea but no wealth. In effect, **D** has access to profitable investment project. Unless both agree to write a financial contract, the gains from trade cannot be realized.

The investment project that lasts for $T = 2$ periods is modeled as a stochastic process (q_1, q_2) of independent random variables q_t denoting output in period t. Assume physical assets of the technology collapse at the end of period 2. Over its horizon, the technology needs a fixed capital investment (I, I) and a labor/effort input of (e_1, e_2), $e_i \in \{e_L, e_H\}$ where $e_H > e_L$. The project's physical assets in combination with **D**'s human capital yields a stochastic flow of returns (q_1, q_2), $q_i \in \{q_L, q_H\}$ where $q_H > q_L$. Project returns are independent across periods. The probability law of each random variable q_t is determined by the effort choice e_t made by D in period t. In each period t, conditional on capital investment, higher effort makes higher return more likely in the first order stochastic dominance sense [3].

$$\forall t = 1, 2 \quad \pi_1 = Pr(q_t = q_H | e_t = e_H, I) > Pr(q_t = q_H | e_t = e_L, I) = \pi_0$$

Both parties are risk neutral. A choice of low effort by **D** is construed as his ability to divert an amount $B > 0$ from I as his private benefit while a choice of high effort means no such diversion.

[3] Note there is no richness in the way output relates to capital input.

Assumption 1. The one-period project has positive NPV if the entrepreneur exerts high effort and negative NPV if he exerts low effort.

$$\pi_1 q_H + (1 - \pi_1) q_L - I > 0 > \pi_0 q_H + (1 - \pi_0) q_L - I$$

Of course, even if the project has positive NPV, it may not still be financed because the pledgeable income may be less than the total income of the project.

There is partial project completion at the end of first period. Assumption 2 below gives formal expression to this by saying that there is a wedge between liquidation value and input value of the first period output. The low output produced in the event of default is worth more in the hands of the entrepreneur than in the hands of the creditor at the end of the first period. At the end of second period, it has the same value in the either hands. Implicit here is the limited liability of **D**. The maximum cover that **C** has in the event of default is the liquidation value of the output.

Assumption 2. The output q_1 at the end of first period has a liquidation value of αq_1 while its value as a second period input is q_1. For simplicity, fix $\alpha = 0$.

3.2.2 Contracting Process and Feasible Contracts

The creditor **C** is viewed as the principal who has all the bargaining power at any time. So attention can be restricted to take-it-or-leave-it contract offers made by the creditor. At the beginning of $t = 1$, **C** offers either of the following two contracts:

(1) A "Lack of Commitment Contract" is the provision of initial investment and continued second period investment whenever outstanding debt is settled. It encodes the repayments $(p(H), p(L), p(HH), p(HL))$. $p(H)$ is the repayment after a high output history $q_1 = q_H$ in the first period. $p(HH)$ is the repayment in *Debt Settlement* HH after the output history $(q_1 = q_H, q_2 = q_H)$. Similarly $p(L)$ and $p(HL)$.

However when a default is incurred, **C** cannot commit to pull out. If he doesn't, he offers a second period contract that encodes the repayments $(p(LH), p(LL))$. $p(LL)$ is the promise of repayment in *Debt Settlement LL* after the output history $(q_1 = q_L, q_2 = q_L)$. Similarly $p(LH)$.

(2) A "Commitment Contract" differs from a Lack of Commitment Contract only in the event of default. **C** can commit not to refinance and the relationship terminates.

D then chooses his effort level e_1 following which the first period output q_1 realizes. If the first period output is high $(q_1 = q_H)$, then *Debt Settlement H* takes place. Following this, in period 2, **D** chooses e_2 and output q_2 realizes. After this, *Debt Settlement* Hq_2 takes place.

If, however, the first period output is low $(q_1 = q_L)$ so that **D** is forced to default, then with lack of commitment, **C** has the choice to either offer a one-period *refinancing contract* or liquidate the project. With commitment, **C** must liquidate. If refinanced, **D** then chooses e_2 and output q_2 realizes. After this, *Debt Settlement* Lq_2 takes place.

3.2.3 Payoffs

Table 3.1 below shows the payoffs of the creditor and the entrepreneur after various histories.

Table 3.1: Payoffs after all histories except those following a default in first period

	Debt Settlement H	*Debt Settlement* Hq_2	*Debt Settlement L*
C	$p(H) - I$	$p(Hq_2) - I$	$-I$
D	$q_H - p(H)$	$q_2 - p(Hq_2)$	q_L

Note the cash flows in *Debt Settlement L*. The low output q_L is not enough to settle the debt I and hence there has been a default. The only recourse the creditor has in the event of default is liquidation of output q_L which yield him nothing by force of Assumption 2. Payoffs in *Debt Settlement* Lq_2 will depend on the commitment possibilities of the creditor and will be written as we discuss them.

3.3 Value of Creditor Commitment

We say that the creditor lacks commitment when there is a possibility of refinancing in the event of default. **C** is committed to continue investment in the next period if **D** never defaults on his repayments. However he cannot commit not to refinance after a low output at the end of period-1 when **D** has defaulted. In contrast, the creditor has commitment when there is no possibility of refinancing. **C** can commit not to refinance in the event of default i.e. after a low output at the end of period-1. Our first result pertains to what drives the value of commitment to the creditor.

Proposition 4. The creditor's preference over commitment scenarios is determined by the degree to which financial contracting is plagued by the moral hazard problem. The creditor prefers a commitment contract if and only if the size of private benefit B is sufficiently high.

Proof. Step 1. Contracting Problem under Lack of Commitment

Second period. The one period continuation project is identical at either the solvency node after $q_1 = q_H$ or the default node after $q_1 = q_L$ and is a standard one period model of investment with moral hazard. The only difference is the investor outlay after a default node is less than that after the solvency node by the amount q_L. The solution to this problem is well known. The repayments must satisfy three constraints.

The moral hazard constraint and limited liability constraints [4] imply that **D** must earn a

[4]Liability in the second period in any state is limited to the second period income in that state. In particular, liability does not extend to wealth in the second period low output state. See the remarks immediately following the proof.

positive agency rent which is minimized by setting

$$p(HL) = p(LL) = q_L$$
$$p(HH) = p(LH) = q_H - \frac{B}{\Delta\pi}$$

With this **C**'s break-even condition for the continuation project at the solvency node is that the expected pledgeable income has to exceed the investor outlay I.

$$\pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L \geq I$$

At the default node, a similar break-even condition must hold with the same expected pledgeable income but a reduced investor outlay $I - q_L$.

D's expected continuation payoff at either the solvency node or default node is $\frac{\pi_1 B}{\Delta\pi}$.

Initial (period-1) Contracting Node.

Table 3.2: Lack of Commitment: Payoffs at initial contracting node

q_1	D's payoffs	C's payoffs
q_H	$q_H - p(H) + \frac{\pi_1 B}{\Delta\pi}$	$p(H) - I + \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - I$
q_L	$\frac{\pi_1 B}{\Delta\pi}$	$-I + \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - (I - q_L)$

Again, we are interested in the conditions under which the project goes ahead. The break-even condition for **C** is

$$\pi_1 p(H) + \underbrace{\pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L}_{\text{Pledgeable Income from Continuation Project}} \geq I + \underbrace{\pi_1 I + (1-\pi_1)(I - q_L)}_{\text{Expected Investment in Continuation Project}}$$

The moral hazard constraint is

$$\pi_1(q_H - p(H) + \frac{\pi_1 B}{\Delta\pi}) + (1-\pi_1)\frac{\pi_1 B}{\Delta\pi} \geq \pi_0(q_H - p(H) + \frac{\pi_1 B}{\Delta\pi}) + (1-\pi_0)\frac{\pi_1 B}{\Delta\pi} + B \quad \text{(MH)}$$

The limited liability condition is

$$q_H - p(H) \geq 0 \quad \text{(LLH)}$$

Now (LLH) and (MH) give the following upper bounds on $p(H)$ respectively

$$p(H) \le q_H$$
$$p(H) \le q_H - \frac{B}{\Delta\pi}$$

Note the debtor's rent is minimized by setting the repayment $p(H)$ equal to the least upper bound which is given by (MH). So

$$p(H) = q_H - \frac{B}{\Delta\pi}$$

Step 2. Contracting Problem under Commitment

Solvency Node after $q_1 = q_H$. **C** provides the entire capital input. This is again a standard one period investment problem with moral hazard whose solution is well known.

$$p(HL) = q_L$$
$$p(HH) = q_H - \frac{B}{\Delta\pi}$$

Initial (period-1) Contracting Node.

Table 3.3: Commitment: Payoffs at initial contracting node

q_1	D's payoffs	C's payoffs
q_H	$q_H - p(H) + \frac{\pi_1 B}{\Delta\pi}$	$p(H) - I + \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - I$
q_L	0	$-I$

The break-even condition for **C** is

$$\pi_1 p(H) + \underbrace{\pi_1\Big[\pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L\Big]}_{\text{Pledgeable Income from Continuation Project}} \ge I + \underbrace{\pi_1 I}_{\text{Expected Investment in Continuation Project}}$$

The incentive compatibility condition is

$$\pi_1(q_H - p(H) + \frac{\pi_1 B}{\Delta\pi}) \ge \pi_0(q_H - p(H) + \frac{\pi_1 B}{\Delta\pi}) + B \qquad \text{(MH)}$$

The limited liability condition is

$$q_H - p(H) \geq 0 \qquad \text{(LLH)}$$

Now (LLH) and (MH) give the following upper bounds on $p(H)$ respectively

$$p(H) \leq q_H$$
$$p(H) \leq q_H - \frac{B}{\Delta\pi} + \frac{\pi_1 B}{\Delta\pi}$$

The debtor's rent is minimized by setting the repayment $p(H)$ equal to the least upper bound which is given by (MH) so that

$$p(H) = q_H - \frac{B}{\Delta\pi} + \frac{\pi_1 B}{\Delta\pi}$$

Note that the the first period repayment $p(H)$ is higher in the case of commitment than in the case of lack of commitment as the expression $\frac{\pi_1 B}{\Delta\pi} > 0$. In other words, commitment of the creditor not to refinance improves first period incentives because incentives can be provided with lower rewards.

Step 3. Costs and benefits of creditor's commitment. Note that the creditor's continuation payoff after a high output (no default node) is the same in both cases. Also the first period sunk cost of investment is the same in the event of default. The commitment scenarios affect payoffs in two places- one, the first period payoffs in Debt Settlement H and two, the continuation payoffs in the event of a default (after a low first period output).

Table 3.4: Creditor's payoffs

	Period-1 payoffs in Debt Settlement H	Continuation payoffs after Default
Commitment	$q_H - \frac{B}{\Delta\pi} + \frac{\pi_1 B}{\Delta\pi} - I$	0
No Commitment	$q_H - \frac{B}{\Delta\pi} - I$	$\pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - (I - q_L)$

As already pointed out in the section on commitment, the benefit of commitment is that it improves incentives in the first period. The commitment to terminate next period of investment means incentives can be preserved using lower rewards. This translates to higher repayments to creditor in the commitment case. The cost, however, is that he must forego the future surplus that could be exploited by refinancing after a default. Denoting the size of benefit by **B** and the size of cost by **C**, we can quantitatively write this

$$\mathbf{B} = \frac{\pi_1 B}{\Delta\pi}$$
$$\mathbf{C} = \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - (I - q_L)$$

Now the creditor ex-ante prefers commitment to no commitment if and only if $\mathbf{B} \geq \mathbf{C}$, that is, if and only if the size of the private benefit is sufficiently high. In other words, the degree to which financial contracting is plagued by the moral hazard problem determines creditor's ex-ante preference over commitment and no commitment. Of course, the ex-post preference is clear. Sequential rationality implies that the creditor ignore the first period sunk cost of investment and contract over future surplus. By refinancing the project, there is scope for old debt recovery. For instance, the proportion of old debt expected to be recovered by refinancing is

$$\frac{\pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - (I - q_L)}{I}$$

Q.E.D.

Remark. The formulation of limited liability constraint in the second period merits some attention. This is because after the high output state, the entrepreneur is potentially richer than he is after the default state. Suppose we work with a formulation where the liability extends to wealth. He takes a wealth of $q_H - p(H)$ into the second period after the solvency node where $p(H)$ is his repayment in the Debt Settlement H. The limited liability constraint in the second period now reads

$$\underbrace{q_H - p(H)}_{\text{Wealth brought into period 2}} + \underbrace{q_L - p(HL)}_{\text{Period 2 income}} \geq 0$$

The optimal $p(HL)$ is obtained when this constraint is binding. Note that $p(HL) = p(LL) + q_H - p(H)$. Now the moral hazard constraint is

$$\underbrace{q_H - p(HH)}_{\text{Income in state HH}} - \underbrace{q_L - p(HL)}_{\text{Income in state HL}} \geq \frac{B}{\Delta\pi}$$

The need to preserve incentives means that the optimal $p(HH)$ also has to be scaled up by $q_H - p(H)$. Thus we have

$$\underbrace{(p(HL), p(HH))}_{\text{Optimal Repayments after Solvency Node}} = (q_H - p(H), q_H - p(H)) + \underbrace{\left(q_L, q_H - \frac{B}{\Delta\pi}\right)}_{\text{Optimal Repayments after Default Node}}$$

This means that **D**'s continuation payoff at the solvency node is $\frac{\pi_1 B}{\Delta\pi} - (q_H - p(H))$ while after the default node, it is $\frac{\pi_1 B}{\Delta\pi}$. Now coming to the initial contracting node

Table 3.5: Entrepreneur's payoffs at initial node

q_1	D's payoffs
q_H	$q_H - p(H) + \frac{\pi_1 B}{\Delta\pi} - (q_H - p(H)) = \frac{\pi_1 B}{\Delta\pi}$
q_L	$\frac{\pi_1 B}{\Delta\pi}$

With a non-contingent payoff, the moral hazard constraint is necessarily violated. Thus if the liability extends to total wealth rather than current income, first period incentives to work are compromised. This conclusion extends to the commitment case as well. Thus when the creditor prefers to implement high effort in both periods as is the case here, he optimally restricts liability to current income in the second period.

3.4 Competition and Commitment

When the creditor lacks the commitment not to refinance the continuation project in the event of default, one may think that competition from other potential entrepreneurs who also seek financing may help the creditor make that commitment. In this section, we wish to show that the sequential nature of investment projects that produce some output with time may make this commitment difficult. Due to depressed liquidation values of which

Assumption 2 is a stark expression, the low project output is worth more in the hands of entrepreneur and of no value in the hands of the creditor. The entrepreneur can use that output as inside equity for next round of investment. The potential entrepreneurs seeking investment may not be able to match this inside equity. The preceding discussion must be understood in the context of limited supply of credit. [5]

3.4.1 The Model with Competition and Lack of Commitment

We augment the model to introduce competition on the entrepreneurial side in the event of default. **C** is committed to continue the relationship with **D** when output is high. However in case of low output, **C** faces a choice between refinancing the same project (in which case **D** uses the low output as inside equity for the second period), financing a second debtor $\hat{\mathbf{D}}$ who has a new project that lasts one period that is technologically the same as the continuation project of **D**. The only difference is $\hat{\mathbf{D}}$ has an inside equity of $\hat{A}$. This means that the investor outlay for $\hat{\mathbf{D}}$'s project is $I - \hat{A}$ while that for **D**'s project is $I - q_L$.

Proposition 5. When credit supply is scarce, the ability of competition on entrepreneurial side of the market to enforce commitment is limited by the amount of output produced in the event of default. The creditor now has to contend with prospect of entrepreneur using that output as inside equity towards refinancing.

Remark. We have chosen to make this point in the form of a proposition above only as a matter of presentation since it follows immediately from Assumption 2. In Case 1, we show when competition may not enforce creditor commitment. In Case 2, we show when it does.

Case 1. $\hat{A} < q_L$. In this case, **C** prefers to refinance the continuation project of **D** because the threat to finance $\hat{\mathbf{D}}$'s project is not credible. The reason is that the sequential production process of **D**, of which one period has elapsed, allows the generation of some output. Due

[5] If credit supply is not a constraint, potential entrepreneurs can always get it if they meet all the required condtions.

to Assumption 2, there is a wedge between the liquidation value and the inside equity value of that output. This makes the second period financing more attractive to **C**. $\hat{\mathbf{D}}$'s project, being technologically the same as the continuation project of **D**, must now better it in terms of inside equity in order to be credible. This case reduces to the section on contracting problem under lack of commitment in proof of Proposition 4.

Case 2. $\hat{A} > q_L$. When other potential entrepreneurs have sufficient inside equity, competition from them indeed serves as a credible threat and helps the creditor enforce the commitment.

Default Node after $q_1 = q_L$. At this node, **C** prefers to finance $\hat{\mathbf{D}}$'s project because the optimal repayments are the same as what they would be if he chose to refinance **D** but **C**'s investment is less because $\hat{\mathbf{D}}$'s inside equity is higher than **D**'s.

$$\hat{p}(LL) = q_L$$
$$\hat{p}(LH) = q_H - \frac{B}{\Delta\pi}$$

so that

$$\mathbf{D}\text{ 's continuation payoff} = q_L$$
$$\mathbf{C}\text{ 's continuation payoff} = \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - (I - \hat{A})$$

Solvency Node after $q_1 = q_H$. This is exactly the same as in Section 3.

$$\hat{p}(HL) = q_L$$
$$\hat{p}(HH) = q_H - \frac{B}{\Delta\pi}$$

so that

$$\mathbf{D}\text{ 's continuation payoff} = \frac{\pi_1 B}{\Delta\pi}$$
$$\mathbf{C}\text{ 's continuation payoff} = \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - I$$

Initial (period-1) Contracting Node

Table 3.6: Initial Payoffs

q_1	D's payoffs	C's payoffs
q_H	$q_H - \hat{p}(H) + \frac{\pi_1 B}{\Delta\pi}$	$\hat{p}(H) - I + \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - I$
q_L	0	$-I + \pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L - (I - \hat{A})$

The break-even condition for the creditor is

$$\pi_1 \hat{p}(H) + \underbrace{\pi_1\Big[\pi_1(q_H - \frac{B}{\Delta\pi}) + (1-\pi_1)q_L\Big]}_{\text{Pledgeable Income from Competitor's Project}} \geq I + \underbrace{\pi_1 I + (1-\pi_1)(I-\hat{A})}_{\text{Expected Investment in Competitor's Project}}$$

The incentives constraint is

$$\pi_1(q_H - \hat{p}(H) + \frac{\pi_1 B}{\Delta\pi}) \geq \pi_0(q_H - \hat{p}(H) + \frac{\pi_1 B}{\Delta\pi}) + B \tag{MH}$$

Now (LLH) and (MH) give the following upper bounds on $p(H)$ respectively

$$\hat{p}(H) \leq q_H$$
$$\hat{p}(H) \leq q_H - \frac{B}{\Delta\pi} + \frac{\pi_1 B}{\Delta\pi}$$

The least upper bound is given by (MH). So the optimal first period repayment is

$$\hat{p}(H) = q_H - \frac{B}{\Delta\pi} + \frac{\pi_1 B}{\Delta\pi}$$

Note the first period repayment here is exactly the same as in the lack of commitment case when there was no outside option for **C**.

3.5 Hidden Information and Commitment

Adverse selection is considered an important feature in modeling credit markets. We now wish to study whether hidden information helps in enforcing creditor commitment.

3.5.1 The Model with Lack of Commitment and Hidden Information

We augment the model by letting entrepreneur have an ability parameter which is his private information. The entrepreneur is drawn from an ability distribution with two-point support $\{\theta_H, \theta_L\}$ with ν being the probability of type θ_H which is common knowledge. The output distribution now depends both on the hidden ability and hidden effort. Thus we have a model with both hidden Information and moral hazard.

Table 3.7: Output configuration for ability-effort combinations

$P(q_H \mid \theta, e)$	e_H	e_L
θ_H	μ_H	μ_L
θ_L	μ_M	μ_L

where $0 < \mu_L < \mu_M < \mu_H < 1$. Thus the production technology is sensitive to ability when and only when both ability types choose high effort [6].

We will impose some restriction on parameters (μ_H, μ_L). These restrictions are tantamount to whether the break-even constraint of the creditor is met in different circumstances. For describing these restrictions, let

$$A_1 = \mu_H q_H + (1-\mu_H) q_L - I - \frac{\mu_H B}{\mu_H - \mu_L}$$
$$A_2 = \mu_L q_H + (1-\mu_L) q_L - (I - q_L)$$

A_1 is an increasing and continuous function of μ_H. $A_1(\mu_H = 0) = q_L - I < 0$ and $A_1(\mu_H = 1) = q_H - I - \frac{B}{1-\mu_L} > 0$. By Intermediate Value Theorem, there is a $\underline{\mu}_H$ such that for every $\mu_H \in (\underline{\mu}_H, 1)$, $A_1 > 0$. Similarly there is a $\bar{\mu}_L$ such that for every $\mu_L \in (0, \bar{\mu}_L)$, $A_2 < 0$.

The parameter restrictions we impose are $(\mu_H, \mu_L) \in (\underline{\mu}_H, 1) \times (0, \bar{\mu}_L)$. These restrictions will have the consequence that for a one period project, if the creditor knows that the entrepreneur is of high ability and chooses high effort, he is willing to finance him. While

[6] The specification is tractable

if the creditor knows the entrepreneur is of low ability, then he is unwilling to finance him even with lack of commitment.

The output distribution must follow the probability law $P(q|\theta) = \sum_e P(e|\theta)P(q|\theta, e)$. The principal, of course, is contractually implementing for every θ, a distribution $\{P(e|\theta)\}_e$. After observing the output, the creditor updates his beliefs by Bayes Rule $P(\theta|q) = \frac{P(q|\theta)P(\theta)}{\sum_q P(q|\theta)P(\theta)}$.

Remark. 1. In moral hazard models with risk neutrality and limited liability of the sort we have been studying, the optimal contract always hits the limited liability constraint in the low output state. Thus we restrict attention to what repayment the contract specifies in high output state.

2. Since we are now studying a model that has both hidden Information and moral hazard, the truth telling constraints are complicated because when a debtor deviates by misreporting his type, he is free to choose any effort level. The contract design has to take into account what action each type will choose when they deviate.

3.5.2 Contracting Problem

Solvency Node after $q_1 = q_H$. The payoff environment for each type is as follows.

Table 3.8: Payoff environment in second period for high ability entrepreneur

q_2	$P(q_2\|\theta_H, e_H)$	$P(q_2\|\theta_H, e_L)$	Payoff of Type θ_H
q_H	μ_H	μ_L	$q_H - p(HH\|\theta_H)$
q_L	$1 - \mu_H$	$1 - \mu_L$	0

Table 3.9: Payoff environment in second period for low ability entrepreneur

q_2	$P(q_2\|\theta_L, e_H)$	$P(q_2\|\theta_L, e_L)$	Payoff of Type θ_L
q_H	μ_M	μ_L	$q_H - p(HH\|\theta_L)$
q_L	$1 - \mu_M$	$1 - \mu_L$	0

A detailed analysis of the contracting problem is done in the Appendix. Based on that analysis, we examine what the creditor prefers to implement. Thus the creditor prefers to

Table 3.10: Effort choices in second period and contracts that implement them

Choices	Implementing Contracts
$e(\theta_H) = e(\theta_L) = e_H$	$p(HH\|\theta_H) = p(HH\|\theta_L) = q_H - \frac{B}{\mu_M - \mu_L}$
$e(\theta_H) = e_H, e(\theta_L) = e_L$	(i) $p(HH\|\theta_H) = p(HH\|\theta_L) = q_H - \frac{B}{\mu_H - \mu_L}$
	(ii) $p(HH\|\theta_H) = p(HH\|\theta_L) = q_H - \frac{B}{\mu_M - \mu_L}$

implement $e(\theta_H) = e_H, e(\theta_L) = e_L$ by a pooling contract

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H - \frac{B}{\mu_H - \mu_L}$$

We now give an intuitive idea as to why the optimal contract is a pooling contract by illustrating it for the case when the creditor wants to implement high effort from high ability type and low effort from low ability type. First note that type only affects the distribution over output. Second, limited liability always binds when output is low no matter what the type. Thus the only instruments undetermined are the type-contingent repayments in case of high output. To fix ideas, suppose when a high ability debtor misreports his type, he prefers high effort to low effort. When low ability debtor misreports, he prefers low effort to high effort. Thus no matter what the type, when they deviate by misreporting, they don't change the output distribution relative to when they report truthfully. Then the truth telling constraint for a high ability type means that his rewards (repayments) in case of high output can be no less (more) than the rewards (repayments) for low ability type. Similarly, the truth telling constraint for a low ability type means that his rewards (repayments) in case of high output can be no less (more) than the rewards (repayments) for high ability type. Thus the truth telling constraints force the optimal contract to be a pooling contract.

Let us now come to the question of why the creditor wants to implement high effort from high ability type and low effort from low ability type rather than high effort from both types. The creditor, by implementing a low effort from low ability type, is able to relax the upper bound on repayments compared to the case when he implements a high effort from low ability type. Since the upper bound on repayments imposed by moral hazard constraint of a high ability type is higher than that imposed by a similar constraint of low ability type, the creditor does better when he does not have to give incentives to work to a low ability type.

We briefly comment on why other choices of effort profile like $e(\theta_H) = e_L, e(\theta_L) = e_H$ and $e(\theta_H) = e(\theta_L) = e_L$ and other options like shutting off trade with low ability type are dominated choices for the creditor. The highest upper bound on repayments from any type is obtained from the moral hazard constraint of a high ability debtor. In other words, a high ability type can be made to work at the least cost. By choosing not to implement high effort from low ability type, he can offer a pooling contract that has repayments hitting that upper bound. Thus other choices of effort profiles are necessarily dominated. A contract that shuts off trade with low ability type must hit the limited liability constraint in the high output state. But this compromises incentives to work for a high ability type debtor.

Record the continuation payoff of each type.

Table 3.11: Continuation Payoffs

Type	Continuation Payoff
θ_H	$\frac{\mu_H B}{\mu_H - \mu_L}$
θ_L	$\frac{\mu_L B}{\mu_H - \mu_L} + B = \frac{\mu_H B}{\mu_H - \mu_L}$

Initial Node. The payoff environment now includes the continuation payoff of each type in the high output state.

Table 3.12: Payoff environment in first period for high ability entrepreneur

q_1	$P(q_1 \vert \theta_H, e_H)$	$P(q_1 \vert \theta_H, e_L)$	Payoff of Type θ_H
q_H	μ_H	μ_L	$q_H - p(HH \vert \theta_H) + \frac{\mu_H B}{\mu_H - \mu_L}$
q_L	$1 - \mu_H$	$1 - \mu_L$	0

The analysis closely parallels the one we did after the solvency node and hence omitted. We

Table 3.13: Payoff environment in first period node for high ability entrepreneur

q_1	$P(q_1\|\theta_L, e_H)$	$P(q_1\|\theta_L, e_L)$	Payoff of Type θ_L
q_H	μ_M	μ_L	$q_H - p(HH\|\theta_L) + \frac{\mu_H B}{\mu_H - \mu_L}$
q_L	$1-\mu_M$	$1-\mu_L$	0

merely record that

Table 3.14: Effort choices in first period and contracts that implement them

Choices	Implementing Contracts
$e(\theta_H) = e(\theta_L) = e_H$	$p(HH\|\theta_H) = p(HH\|\theta_L) = q_H + \frac{\mu_H B}{\mu_H - \mu_L} - \frac{B}{\mu_M - \mu_L}$
$e(\theta_H) = e_H, e(\theta_L) = e_L$	(i) $p(HH\|\theta_H) = p(HH\|\theta_L) = q_H + \frac{\mu_H B}{\mu_H - \mu_L} - \frac{B}{\mu_H - \mu_L}$
	(ii) $p(HH\|\theta_H) = p(HH\|\theta_L) = q_H + \frac{\mu_H B}{\mu_H - \mu_L} - \frac{B}{\mu_M - \mu_L}$

Thus the creditor prefers to implement $e(\theta_H) = e_H, e(\theta_L) = e_L$ by a pooling contract

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H + \frac{\mu_H B}{\mu_H - \mu_L} - \frac{B}{\mu_H - \mu_L}$$

The intuitive explanations of why choices of other effort profiles are dominated carry over from earlier. So we refrain from repeating them. Now that we know the effort profile that the creditor wants to implement in the first period, we can write down his posterior beliefs.

$$P(q_L|\theta_H) = P(q_L|\theta_H, e_H) = 1 - \mu_H$$

$$P(q_L|\theta_L) = P(q_L|\theta_L, e_L) = 1 - \mu_L$$

$$P(\theta_H|q_L) = \frac{P(q_L|\theta_H)P(\theta_H)}{P(q_L|\theta_H)P(\theta_H) + P(q_L|\theta_L)P(\theta_L)} = \frac{\nu(1-\mu_H)}{\nu(1-\mu_H) + (1-\nu)(1-\mu_L)}$$

$$P(\theta_H|q_H) = \frac{P(q_H|\theta_H)P(\theta_H)}{P(q_H|\theta_H)P(\theta_H) + P(q_H|\theta_L)P(\theta_L)} = \frac{\nu\mu_H}{\nu\mu_H + (1-\nu)\mu_L}$$

Proposition 6. Let $\mathbf{P}(t)$ be the parameter space with $\mu_H - \mu_L > t$ where $t \in (0, 1)$. For sufficiently high t, creditor commitment is sequentially rational.

Proof. We will show that the break-even constraint of the creditor is violated after a default while it is satisfied at the solvency node. Formally these constraints are written as

$$\frac{\nu(1-\mu_H)}{\nu(1-\mu_H)+(1-\nu)(1-\mu_L)}A_1+\frac{(1-\nu)(1-\mu_L)}{\nu(1-\mu_H)+(1-\nu)(1-\mu_L)}A_2>0$$
$$\frac{\nu\mu_H}{\nu\mu_H+(1-\nu)\mu_L}A_1+\frac{(1-\nu)\mu_L}{\nu\mu_H+(1-\nu)\mu_L}A_2<0$$

These inequalities are equivalent to

$$\frac{1-\mu_L}{1-\mu_H}>\frac{\nu A_1}{(1-\nu)|A_2|}>\frac{\mu_L}{\mu_H}$$

Now as $\mu_H \to 1$ and $\mu_L \to 0$ (i.e. $t \to 1$), the above inequalities take the form

$$\infty \geq \frac{\nu[q_H-I-B]}{(1-\nu)|2q_L-I|} \geq 0$$

which is satisfied as the fraction in the middle is a finite positive number. Q.E.D.

Appendix

(I) Suppose the creditor wants to implement $e(\theta_H)=e(\theta_L)=e_H$. He must maximize the repayments $(p(HH|\theta_H), p(HH|\theta_L))$ subject to the following incentive and limited liability constraints.

$$q_H-p(HH|\theta_H)\geq\frac{B}{\mu_H-\mu_L} \qquad \text{MH}(\theta_H)$$
$$q_H-p(HH|\theta_L)\geq\frac{B}{\mu_M-\mu_L} \qquad \text{MH}(\theta_L)$$
$$\mu_H[q_H-p(HH|\theta_H)]\geq\max\left[\mu_H[q_H-p(HH|\theta_L)],\mu_L[q_H-p(HH|\theta_L)]+B\right] \qquad \text{IC}(\theta_H)$$
$$\mu_M[q_H-p(HH|\theta_L)]\geq\max\left[\mu_M[q_H-p(HH|\theta_H)],\mu_L[q_H-p(HH|\theta_H)]+B\right] \qquad \text{IC}(\theta_L)$$
$$p(HH|\theta_H)\leq q_H \qquad \text{LLH}(\theta_H)$$
$$p(HH|\theta_L)\leq q_H \qquad \text{LLH}(\theta_L)$$

Depending on which action a type prefers when he misreports his type, there are four cases. In what follows, each case is formally defined by (1) and (2). (1) simplifies the right side of IC(θ_H) while (2) simplifies the right side of IC(θ_L).

Case 1. When θ_H deviates by misreporting his type, he prefers e_H to e_L. When θ_L deviates by misreporting his type, he prefers e_H to e_L. That is

$$q_H - p(HH|\theta_L) \geq \frac{B}{\mu_H - \mu_L} \tag{1}$$

$$q_H - p(HH|\theta_H) \geq \frac{B}{\mu_M - \mu_L} \tag{2}$$

$$p(HH|\theta_L) \geq p(HH|\theta_H) \tag{IC(θ_H)}$$

$$p(HH|\theta_H) \geq p(HH|\theta_L) \tag{IC(θ_L)}$$

IC(θ_H) and IC(θ_L) imply that the optimal contract is a pooling contract and the upper bounds on repayments given by (1), (2), MH(θ_H) and MH(θ_L) imply that it is given by

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H - \frac{B}{\mu_M - \mu_L}$$

Case 2. When θ_H deviates by misreporting his type, he prefers e_H to e_L. When θ_L deviates by misreporting his type, he prefers e_L to e_H. That is

$$q_H - p(HH|\theta_L) \geq \frac{B}{\mu_H - \mu_L} \tag{1}$$

$$q_H - p(HH|\theta_H) \leq \frac{B}{\mu_M - \mu_L} \tag{2}$$

$$p(HH|\theta_L) \geq p(HH|\theta_H) \tag{IC(θ_H)}$$

$$\mu_M[q_H - p(HH|\theta_L)] \geq \mu_L[q_H - p(HH|\theta_H)] + B \tag{IC(θ_L)}$$

MH(θ_H) and (2) imply that

$$p(HH|\theta_H) \in \left[q_H - \frac{B}{\mu_M - \mu_L}, q_H - \frac{B}{\mu_H - \mu_L}\right] \tag{3}$$

Note (1) is redundant in face of MH(θ_L). MH(θ_L) implies

$$p(HH|\theta_L) \leq q_H - \frac{B}{\mu_M - \mu_L}$$

Now MH(θ_L) must bind at the optimum. If it does not, then

$$p(HH|\theta_H) \leq p(HH|\theta_L) < q_H - \frac{B}{\mu_M - \mu_L}$$

where the weak inequality is due to IC(θ_H). But this contradicts (3). Thus MH(θ_L) binds at the optimum. But then IC(θ_H) and (3) force the optimal contract to be a pooling contract which is the same as in Case 1.

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H - \frac{B}{\mu_M - \mu_L}$$

IC(θ_L) is satisfied with this contract.

Case 3. When θ_H deviates by misreporting his type, he prefers e_L to e_H. When θ_L deviates by misreporting his type, he prefers e_H to e_L. That is

$$q_H - p(HH|\theta_L) \leq \frac{B}{\mu_H - \mu_L} \qquad (1)$$

$$q_H - p(HH|\theta_H) \geq \frac{B}{\mu_M - \mu_L} \qquad (2)$$

$$\mu_H[q_H - p(HH|\theta_H)] \geq \mu_L[q_H - p(HH|\theta_L)] + B \qquad \text{IC}(\theta_H)$$

$$p(HH|\theta_H) \geq p(HH|\theta_L)B \qquad \text{IC}(\theta_L)$$

MH(θ_L) and (1) imply that

$$p(HH|\theta_L) \in \left[q_H - \frac{B}{\mu_H - \mu_L}, q_H - \frac{B}{\mu_M - \mu_L}\right] \qquad (3)$$

But since $\mu_H > \mu_M$ this interval is an empty set, no contract exists.

Case 4. When θ_H deviates by misreporting his type, he prefers e_L to e_H. When θ_L deviates by misreporting his type, he prefers e_L to e_H. MH(θ_L) and (1) are the same as in Case 3 and so no contract exists in this case as well.

(II) Suppose the creditor wants to implement $e(\theta_H) = e_H$ and $e(\theta_L) = e_L$. He must maximize the repayments $(p(HH|\theta_H), p(HH|\theta_L))$ subject to the following incentive and limited liability constraints.

$$
\begin{aligned}
q_H - p(HH|\theta_H) &\geq \frac{B}{\mu_H - \mu_L} && \text{MH}(\theta_H) \\
\mu_H[q_H - p(HH|\theta_H)] &\geq \max\left[\mu_H[q_H - p(HH|\theta_L)], \mu_L[q_H - p(HH|\theta_L)] + B\right] && \text{IC}(\theta_H) \\
\mu_L[q_H - p(HH|\theta_L)] + B &\geq \max\left[\mu_M[q_H - p(HH|\theta_H)], \mu_L[q_H - p(HH|\theta_H)] + B\right] && \text{IC}(\theta_L) \\
p(HH|\theta_H) &\leq q_H && \text{LLH}(\theta_H) \\
p(HH|\theta_L) &\leq q_H && \text{LLH}(\theta_L)
\end{aligned}
$$

Case 1. When θ_H deviates by misreporting his type, he prefers e_H to e_L. When θ_L deviates by misreporting his type, he prefers e_H to e_L. That is

$$
\begin{aligned}
q_H - p(HH|\theta_L) &\geq \frac{B}{\mu_H - \mu_L} && (1) \\
q_H - p(HH|\theta_H) &\geq \frac{B}{\mu_M - \mu_L} && (2) \\
p(HH|\theta_L) &\geq p(HH|\theta_H) && \text{IC}(\theta_H) \\
\mu_L[q_H - p(HH|\theta_L)] + B &\geq \mu_M[q_H - p(HH|\theta_H)] && \text{IC}(\theta_L)
\end{aligned}
$$

In presence of (2), MH(θ_H) is redundant. Now a contract with both (1) and (2) binding violates IC(θ_L). A contract with (1) and IC(θ_L) binding violates (2). A contract with (2) and IC(θ_L) binding is a pooling contract given by

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H - \frac{B}{\mu_M - \mu_L}$$

that satisfies (1) and IC(θ_H).

Case 2. When θ_H deviates by misreporting his type, he prefers e_H to e_L. When θ_L deviates by misreporting his type, he prefers e_L to e_H. That is

$$
\begin{aligned}
q_H - p(HH|\theta_L) &\geq \frac{B}{\mu_H - \mu_L} && (1) \\
q_H - p(HH|\theta_H) &\leq \frac{B}{\mu_M - \mu_L} && (2) \\
p(HH|\theta_L) &\geq p(HH|\theta_H) && \text{IC}(\theta_H) \\
p(HH|\theta_H) &\geq p(HH|\theta_L) && \text{IC}(\theta_L)
\end{aligned}
$$

MH(θ_H) and (2) imply that

$$p(HH|\theta_H) \in \left[q_H - \frac{B}{\mu_M - \mu_L}, q_H - \frac{B}{\mu_H - \mu_L}\right] \tag{3}$$

By (1)

$$p(HH|\theta_L) \leq q_H - \frac{B}{\mu_H - \mu_L}$$

IC(θ_H) and IC(θ_L) force the optimal contract to be a pooling contract

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H - \frac{B}{\mu_H - \mu_L}$$

Case 3. When θ_H deviates by misreporting his type, he prefers e_L to e_H. When θ_L deviates by misreporting his type, he prefers e_H to e_L. That is

$$q_H - p(HH|\theta_L) \leq \frac{B}{\mu_H - \mu_L} \tag{1}$$
$$q_H - p(HH|\theta_H) \geq \frac{B}{\mu_M - \mu_L} \tag{2}$$
$$\mu_H[q_H - p(HH|\theta_H)] \geq \mu_L[q_H - p(HH|\theta_L)] + B \tag{IC(θ_H)}$$
$$\mu_L[q_H - p(HH|\theta_L)] + B \geq \mu_M[q_H - p(HH|\theta_H)] \tag{IC(θ_L)}$$

In presence of (2), MH(θ_H) is redundant. A contract with (2) and IC(θ_L) binding violates (1). A contract with (2) and IC(θ_H) binding violates (1). A contract with (1) and IC(θ_H) binding violates (2). A contract with (1) and IC(θ_L) binding violates (2). A contract with (1) and (2) binding violates IC(θ_L). Thus no optimal contract exists in this case.

Case 4. When θ_H deviates by misreporting his type, he prefers e_L to e_H. When θ_L deviates by misreporting his type, he prefers e_L to e_H. That is

$$q_H - p(HH|\theta_L) \leq \frac{B}{\mu_H - \mu_L} \tag{1}$$
$$q_H - p(HH|\theta_H) \leq \frac{B}{\mu_M - \mu_L} \tag{2}$$
$$\mu_H[q_H - p(HH|\theta_H)] \geq \mu_L[q_H - p(HH|\theta_L)] + B \tag{IC(θ_H)}$$
$$p(HH|\theta_H) \geq p(HH|\theta_L) \tag{IC(θ_L)}$$

(2) and MH(θ_H) imply

$$p(HH|\theta_H) \in \left[q_H - \frac{B}{\mu_M - \mu_L}, q_H - \frac{B}{\mu_H - \mu_L}\right] \tag{3}$$

(1) and (3) imply

$$p(HH|\theta_L) \geq p(HH|\theta_H) \tag{4}$$

IC(θ_L) and (4) imply

$$p(HH|\theta_H) = p(HH|\theta_L) = q_H - \frac{B}{\mu_H - \mu_L}$$

This satisfies IC(θ_H).

Chapter 4

Stable Property Rights

4.1 Introduction

The appropriative struggle for control of productive resources sits right beside the economic activity of production and exchange through market institutions. The following quotes are suggestive.

"Throughout the history of mankind, it has been quite common that economic agents, individually or collectively, use power to seize control of assets held by others."

Piccione and Rubinstein (2007)

"All aspects of human life are responses ... to the interaction of two great life strategy options: on the one hand, production and exchange, on the other hand, appropriation and defense against appropriation."

Hirshleifer (1994)

The acts of appropriation can take many forms- international war, local and national struggles, bank robbery and theft and strikes and lockouts. Milder forms include rent seeking activities like lobbying for licenses and monopoly privileges. Legal systems often moderate such struggles by providing for liability rules where an agent's entitlements may be accorded only a weak protection and some other agent may take it against his free will by paying legally stipulated damages. In many societies, there are instances of government acting under provisions of *eminent domain* to confiscate the private land of landowners which can seldom be legitimized by public goods provision (which is the primary justification for clauses of *eminent domain*).

The classical efficiency of exchange in a free market economy is predicated on enforcement of property rights. The first welfare theorem postulates voluntary exchange as a premise for allocative efficiency of exchange. A strand of literature in law and economics starting with Calabresi and Melamed (1972) and Kaplow and Shavell (1996) look at the economic analysis of property and liability rules. Recently, Bar-Gill and Persico (2012) study an exchange economy with a single durable asset protected by liability rules in which exchange happens through a bargaining game that has the structure that in each period, there is a bilateral spot interaction of the current owner and a potential taker that determines the next period's owner. The player who is not the owner at the end of each period exits the game and a different taker appears next period. They find under some conditions, there exists an efficient equilibrium of the economy in which property is accorded only a liability rule protection.

The central concern of this paper is whether the need to maintain a stable balance of power is a constraint on classical allocative efficiency. Put differently, the most productive coalition may not have the power to defend the asset against appropriation. So stability considerations may imply the property rights be held by a less productive coalition. We develop a dynamic model with a single durable asset in which agents are unconstrained in their ability to enter a Coasian bargain at each date. Joint ownership of asset is feasible. Coalitions are heterogeneous both in their productivity with the asset and their vulnerability to appropriation. We do away with any liability rules and instead use the construct of a power function to define the opportunities for appropriation. The model is written as a stochastic game with discounted payoffs in which at each date every player has opportunities to redefine the prevailing state. A state in the model is a property rights allocation over an asset and a distribution of income from the asset among all the players. A transition from one state to another is feasible if the proposed owning coalition is at least as powerful as the current owning coalition and everyone in the proposed owning coalition consents to the change. State transitions in the stochastic game are endogenous because they are constrained by players' threat positions which are endogenous.

There is a growing literature on resource allocation in environments with weak protection of property rights. Jordan (2006) invents a class of coalitional games of allocation by force which he calls pillage games and studies the core and stable sets of these games. Jordan (2009) studies how efficient allocation of resources may be achieved in an environment of pillage where any reallocation may change the distribution of power as well. Garfinkel and Skaperdas (2007) provide an overview of a different class of models that deal with conflict. Game theoretic literature that studies process models of coalition formation in the spirit of dynamic stochastic games with recontracting opportunities at every date include Konishi and Ray (2003) without side payments and Gomes and Jehiel (2005) with side payments. Ray and Vohra (2013) is a recent survey.

4.2 Model

4.2.1 Environment

There is a single indivisible and durable asset. There is a set $N = \{1, \dots, n\}$ of n agents who live forever. The asset may be owned by a coalition of agents. Various groups of agents may have varying productivity with the asset which is described by a function F that for a coalition S gives its productivity $F(S)$, in units of lifetime payoffs.

Coalitions also have power both with and without the ownership of the asset. A power function π gives the power $\pi(S)$ of a coalition S that is defined by

$$\pi(S) = \begin{cases} \alpha + |S| & \text{if } S \text{ owns the asset} \\ |S| & \text{if } S \text{ does not own the asset} \end{cases}$$

The question is how does the distribution of coalitional power determine the allocation of the asset. To answer this, we will model the agents interacting through a dynamic process and look at equilibrium point allocations.

4.2.2 State of the System

There is a set of states $\mathbb{Z}$. A state $z \in \mathbb{Z}$ specifies a coalition S that jointly owns the asset and a distribution $u = (u_1, \ldots, u_n) \in \mathbb{R}^n_+$ of lifetime payoff $F(S)$ that S gets from the asset.

$$\mathbb{Z} = \{(S, u) \in 2^N \setminus \phi \times \mathbb{R}^n_+ : \sum_{i \in N} u_i = F(S)\}$$

4.2.3 Feasible Transitions between States

Fix a status quo state (S, u) and a suggested change of state (T, w). The relation $\rightsquigarrow$ is introduced on pairs of states with a view of defining which transitions satisfy the power calculus of the transition. Such calculus depends on the relative power of two coalitions- the currently prevailing owning coalition and the suggested owning coalition.

$(S, u) \rightsquigarrow (T, w)$ denotes that the transition from the state (S, u) to the state (T, w) satisfies the power calculus. It is helpful for the exposition to define various types of transition that have a bearing on power calculus.

- D-transition: $T \subset S$. This is a proposal to downscale the owning coalition.
- E-transition: $S \subset T$. This is a proposal to expand the owning coalition.
- PO-transition: $S \setminus T \neq \emptyset$ and $T \setminus S \neq \emptyset$. This is a proposal to partially overthrow the owning coalition.
- CO-transition: $S \cap T = \emptyset$. This is a proposal to completely overthrow the owning coalition.

Definition 7. $(S, u) \rightsquigarrow (T, w)$. The definition of when does the transition from the state (S, u) to the state (T, w) satisfies the power calculus is given in a tabular form

Type of Transition	$(S, u) \rightsquigarrow (T, w)$ if
D-transition	$\pi(T) \geq \pi(S \setminus T)$
E-transition	None
PO-transition	$\pi(T) \geq \pi(S \setminus T)$
CO-transition	$\pi(T) \geq \alpha + \pi(S)$

Definition 8. Feasible Transition. The transition from the state (S, u) to the state (T, w) is feasible if the following two conditions are met

1. $(S, u) \rightsquigarrow (T, w)$
2. Everyone in T approves of the transition.

4.2.4 Dynamic Game

We model the dynamic process through which the agents interact as a dynamic stochastic game. Time is discrete and runs from $t = 1, 2, \ldots$. The game is described recursively. Suppose the state of the game at the start of period t is $z = (S, u)$. Denote the subgame at this node as $\mathscr{G}_t(S, u)$.

Choices in $\mathscr{G}_t(S, u)$:

A proposer $i \in N$ is randomly chosen with probability $p_i \in (0, 1)$ who then has the choice to make a transition offer $(T, w) \in \mathbb{Z}$ [1]. Responders $j \in T \setminus i$ then respond sequentially in some order choosing accept or reject.

Consequences in $\mathscr{G}_t(S, u)$:

If the transition offer is infeasible, the game moves to $\mathscr{G}_{t+1}(S, u)$. Otherwise, the game moves

[1] We want to allow any player to be able to propose the prevailing state. So we do not impose the requirement $i \in T$.

to $\mathscr{G}_{t+1}(T, w)$.

Payoffs: Note the game is such that period t actions determine period $t+1$ state. The state encodes all the payoff relevant information. For a given strategy profile σ, the symbol $V_t^i(S_t, u_t)$ denotes player i's continuation payoffs at the node $\mathscr{G}_t(S_t, u_t)$ at the start of period t when the state is (S_t, u_t). Deterministic sequences of states starting at time t

$$(S_t, u_t), (S_{t+1}, u_{t+1}), \ldots$$

are evaluated by player i as

$$(1-\delta)\sum_{\tau=t}^{\infty} \delta^{t-\tau} u_\tau^i$$

where the discount factor $\delta \in (0, 1)$.

4.3 Equilibrium Existence for Finite State Space

In this section we derive an existence theorem when the state space is finite. Let $|\mathbb{Z}| = m$. Let $p_i \in (0, 1)$ be the probability with which player i is chosen to be the proposer at any state. Let $p = \{p_1, \ldots, p_n\}$. We refer to the dynamic game as $\mathscr{G}(\pi, F, p, \delta)$.

We will consider strategies in which players may use mixed action at proposal node but use pure strategies at the response node.

Definition 9. A Markov Strategy for i, denoted $\sigma_i = (\sigma_i, \sigma_i^{AR})$ consists of
(1) A Proposal Strategy for i which is a function $\sigma_i : \mathbb{Z} \to \Delta(\mathbb{Z})$ such that $\sigma[z, i](z')$ is the probability with which state z' is proposed by i when chosen as a proposer at the state z.

(2) An Acceptance Rejection Strategy for i at state z which is a function $\sigma_i^{AR}[z] : \mathbb{Z} \to \{A, R\}$.

A Markov Perfect Equilibrium (MPE) σ is a subgame perfect equilibrium in which every player is playing a Markov Strategy. For every player $i \in N$, let $V^i(\sigma) : \mathbb{Z} \to \mathbb{R}$ be a value function for player i in σ. Since lifetime productivity of any coalition is bounded and the evaluation map of deterministic sequences of states is bounded, $V^i(\sigma)$ is a bounded function. Let $V(\sigma) = (V^1(\sigma), \ldots, V^n(\sigma))$ be the profile of value functions.

Acceptance Rejection Strategies Consider $\mathscr{G}_t(z)$ where $z = (S, u)$. We will now describe the Acceptance Rejection Strategy of responder j at his response node $[i, (T, w), Q]$ where i is the proposer who has the made the transition offer $z = (T, w)$ and Q is the set of players who have accepted it. $i \in Q$. ARSs of responders is defined inductively, first for the last responder according to the order ϕ, then for the penultimate responder and so on backwards in order. Let F_j be the set of responders in T who follow j in the order ϕ. For the last responder in T, $F_j = \emptyset$.

(1) If every responder $k \in F_j$ would accept i's offer, then j accepts if $V^j(T, w) \geq V^j(S, u)$ and rejects otherwise.
(2) If some responder $k \in F_j$ would reject i's offer, then j is indifferent between accepting and rejecting.

We will restrict attention to equilibria in which j follows weakly undominated strategy: Accept if $V^j(T, w) \geq V^j(S, u)$ and Reject otherwise.

The first result characterizes an MPE conditional on its existence.

Proposition 7. Let σ be a behavioral strategy profile with the associated profile of value functions $V(\sigma) = (V^1(\sigma), \ldots, V^n(\sigma))$. Then σ is a Markov Perfect Equilibrium if and only if the following conditions hold:

(1) For every state z and every player i, the support of i's mixed action satisfies

$$\text{supp } \sigma[z,i] \subset \underset{z' \in \mathbb{Z}}{\text{argmax}} \Big\{ V[z,i](z', \sigma(-[z,i])) : z \rightsquigarrow z' = (T,w) \text{ and} \\ \forall j \in T \quad V[z,j]((z', \sigma(-[z,i])) \geq V[z,j](\sigma) \Big\}$$

(2) For every state z and every player i,

$$V[z,i](\sigma) = (1-\delta) u_i(z) + \delta \sum_{j \in N} p_j \sum_{z' \in \mathbb{Z}} \sigma[z,j](z')\, V[z',i](\sigma)$$

Proof. Given $\sigma(-[z,i])$, player i at state z faces an infinite horizon Markov decision problem. The first part of the characterization is the formal expression of optimality of $\sigma[z,i]$. The second part of the characterization expresses the familiar "optimal strategy satisfies Bellman Equation" condition. Q.E.D.

The second result is the existence theorem for the stochastic game. The proof follows Nash's template of setting up a set-valued map for strategy iteration and applying Kakutani's fixed point theorem to it. Since our game is dynamic, we need to derive how farsighted players evaluate any σ at any state from their per period payoff functions. This is done in Step 1 by applying linear algebra to the payoff accounting equations (similar to Bellman equations). Showing the continuity of choice correspondence in σ takes some effort and this is done in Step 2. First the constraints that express consent are reduced to linear constraints using one-shot deviation principle. The difficulty of showing continuity lies in the dependence of the coefficient matrix of the linear constraints in σ. We rely on results in the stability theory of linear programming. Intuitively, at a σ that places positive probability on an inefficient state for the candidate owning coalition, it is feasible to simultaneously improve the payoffs for everyone in this coalition. This is sufficient for continuity at such a point. However, at a σ where this is not possible, the constraints that express consent (which hold as equality at such a point) are full rank. Again, this suffices for continuity at such a point. Step 3 now completes the proof by setting up the set-valued map (that expresses optimization at every decision node) to which Kakutani's fixed point theorem is applied.

Proposition 8. The dynamic game $\mathscr{G}(\pi, F, p, \delta)$ has a Markov Perfect Equilibrium.

Proof. Let $[z, i]$ denote player i when he is the proposer at state z and $\sigma[z, i] \in \Delta(\mathbb{Z})$ denote his local mixed choice. A behavioral proposal strategy σ_i for player i is a list of his mixed choices at every state. A profile of behavioral proposal strategies $\sigma = (\sigma_i)_{i \in N}$. The set of all behavioral proposal strategies is $[\Delta(\mathbb{Z})]^{mn}$. We will construct a correspondence $F : [\Delta(\mathbb{Z})]^{mn} \rightrightarrows [\Delta(\mathbb{Z})]^{mn}$ with the property that existence of MPE is equivalent to existence of fixed point of F. We will then apply Kakutani Fixed Point Theorem to F to show it has a fixed point.

Step 1. Given σ, let $V[z, i](\sigma)$ denote payoff to agent $[z, i]$ under σ. The value vector for player i is denoted $V^i(\sigma) = (V[z, i](\sigma))_{z \in \mathbb{Z}}$ and it lives in $\mathbb{R}^m$. Let $V^i(\sigma)$ be given by the following system of m linear equations

$$\forall z \in \mathbb{Z}, \quad V[z, i](\sigma) = (1 - \delta) u_i(z) + \delta \sum_{j \in N} p_j \sum_{z' \in \mathbb{Z}} \sigma[z, j](z') \, V[z', i](\sigma)$$

Let $u_i = (u_i(z))_{z \in \mathbb{Z}}$ and then the system of m equations may be written as

$$[I_{m \times m} - \delta \mathbb{P}^{\sigma}_{m \times m}] V^i(\sigma) = (1 - \delta) u_i$$

where $\mathbb{P}^{\sigma}$ is the transition matrix induced by σ on $\mathbb{Z}$ given by $\mathbb{P}^{\sigma}(z, z') = \sum_{j \in N} p_j \sigma[z, j](z')$.

We now observe that the matrix $[I - \delta \mathbb{P}^{\sigma}]$ is strictly diagonally dominant. This is because the diagonal and the off-diagonal entries

$$\begin{aligned} [I - \delta \mathbb{P}^{\sigma}]_{zz} &= 1 - \delta \sum_{j \in N} p_j \sigma[z, j](z) && \in [1 - \delta, 1] \\ [I - \delta \mathbb{P}^{\sigma}]_{zz'} &= -\delta \sum_{j \in N} p_j \sigma[z, j](z') && \in [-\delta, 0] \end{aligned}$$

are such that the property of strict diagonal dominance

$$\forall z \in \mathbb{Z}, \quad \big| [I - \delta \mathbb{P}^{\sigma}]_{zz} \big| > \sum_{z' \neq z} \big| [I - \delta \mathbb{P}^{\sigma}]_{zz'} \big|$$

is equivalent to $1 > \delta$ which is true. By Levy Desplanques Theorem, the matrix $[I - \delta \mathbb{P}^{\sigma}]$ is nonsingular and hence invertible. Thus $V^i(\sigma)$ is uniquely solved from the linear system and is continuous in σ.

Step 2. Let $C[z,i](\sigma)$ be the choice correspondence of agent $[z,i]$ for parameter σ.

$$C[z,i](\sigma) = \Delta\Big\{z' = (T,w) \in \mathbb{Z} : z \rightsquigarrow z' \text{ and } \forall j \in T \quad V[z,j](z', \sigma(-[z,i])) \geq V[z,j](\sigma)\Big\}$$

In this step, we establish the continuity of the choice correspondence by deducing it from the continuity of an auxiliary correspondence. Let $\mathbb{Z}_T = \{z' \in \mathbb{Z} : z' = (T,w) \text{ for some } w \text{ and } z \rightsquigarrow z'\}$ be the set of states in which the owning coalition is T and the power calculus of transition from z is satisfied. Define the auxiliary correspondence to be the bounded polyhedron-valued map

$$C_T[z,i](\sigma) = \Big\{\beta[z,i] \in \Delta(\mathbb{Z}_T) : \forall j \in T \quad V[z,j](\beta[z,i], \sigma(-[z,i])) \geq V[z,j](\sigma)\Big\}$$

describes agent $[z,i]$'s local mixed choices with support in $\mathbb{Z}_T$ that ensure simultaneous payoff improvement of players in T. Upper semicontinuity is immediate by the continuity of $V^i(\sigma)$. For lower semicontinuity, we need to look closer at the structure of the choice correspondence.The payoffs that occur in the constraints are the payoffs in a discounted ($\delta < 1$) dynamic program with additive and uniformly bounded rewards where agent $[z,i]$ may potentially move at several times depending on other's choices each time playing the same choice prescribed by $\beta[z,i]$. By the one-shot deviation principle of dynamic programming, this constraint set is equivalent to one in which agent $[z,i]$ is restricted to using one-shot deviations from $\sigma[z,i]$ in order to generate these payoff improvements. This also makes the constraint set linear in $\beta[z,i]$ as we shall show now.

$$V[z,j](\sigma) = (1-\delta)u_j(z) + \delta \sum_{k \in N} p_k \sum_{x \in \mathbb{Z}} \sigma[z,k](x) V[x,j](\sigma)$$

$$
\begin{aligned}
&= \underbrace{(1-\delta)u_j(z) + \delta \sum_{k\in N\setminus i} p_k \sum_{x\in\mathbb{Z}} \sigma[z,k](x)V[x,j](\sigma)}_{K(\sigma)} + \delta p_i \sum_{x\in\mathbb{Z}} \sigma[z,i](x)V[x,j](\sigma) \\
&= K(\sigma) + \delta p_i \sum_{x\in\mathbb{Z}} \sigma[z,i](x)\big[(1-\delta)u_j(x) + \delta \sum_{k\in N} p_k \sum_{y\in\mathbb{Z}} \sigma[x,k](y)V[y,j](\sigma)\big] \\
&= K(\sigma) + \delta p_i(1-\delta)\langle \sigma[z,i], u_j\rangle + \delta^2 p_i \sum_{x\in\mathbb{Z}} \sigma[z,i](x) \sum_{k\in N} p_k \sum_{y\in\mathbb{Z}} \sigma[x,k](y)V[y,j](\sigma)
\end{aligned}
$$

where $K(\sigma)$ is an expression that is independent of $\sigma[z,i]$.

Let $\sigma^1[z,i](y) := \sum_{x\in\mathbb{Z}} \sigma[z,i](x) \sum_{k\in N} p_k \sigma[x,k](y) = \langle \sigma[z,i], \mathbb{P}^\sigma(.,y)\rangle$ be the one step transition probability from $[z,i]$ to y. Then the one-step distribution over states regarding $\sigma[z,i]$ as the initial distribution is $\sigma[z,i]\mathbb{P}^\sigma$. Using this, we can simplify the payoff expression as

$$
\begin{aligned}
V[z,j](\sigma) &= K(\sigma) + \delta p_i(1-\delta)\langle \sigma[z,i], u_j\rangle + \delta^2 p_i \langle \sigma^1[z,i], V^j(\sigma)\rangle \\
&= K(\sigma) + \delta p_i\big[\langle \sigma[z,i], (1-\delta)u_j\rangle + \langle \sigma[z,i], \delta\mathbb{P}^\sigma V^j(\sigma)\rangle\big] \\
&= K(\sigma) + \delta p_i\big[\langle \sigma[z,i], (1-\delta)u_j + \delta\mathbb{P}^\sigma V^j(\sigma)\rangle\big] \\
&= K(\sigma) + \delta p_i \langle \sigma[z,i], V^j(\sigma)\rangle
\end{aligned}
$$

The constraint set $C_T[z,i](\sigma)$ may now be seen to be linear.

$$
\begin{bmatrix} (V^1(\sigma))_{z\in\mathbb{Z}} \\ (V^2(\sigma))_{z\in\mathbb{Z}} \\ \cdots \\ (V^t(\sigma))_{z\in\mathbb{Z}} \end{bmatrix}_{|T|\times m} \beta[z,i] \geq \begin{bmatrix} \langle V^1(\sigma), \sigma[z,i]\rangle \\ \langle V^2(\sigma), \sigma[z,i]\rangle \\ \cdots \\ \langle V^t(\sigma), \sigma[z,i]\rangle \end{bmatrix}_{|T|\times 1}
$$

$$
\begin{aligned}
\forall z' \in \mathbb{Z} \quad & \beta[z,i](z') \geq 0 \\
\forall z' \in \mathbb{Z}\setminus\mathbb{Z}_T \quad & \beta[z,i](z') = 0 \\
\sum_{z'\in\mathbb{Z}} & \beta[z,i](z') = 1
\end{aligned}
$$

Note the constraints are of two types- one that demand unanimity and the other that restrict $\beta[z,i]$ to be a probability distribution over $\mathbb{Z}_T$. The difficulty of ascertaining lower semicontinuity of the constraint set in the parameter σ lies in dependence of the coefficient matrix in the unanimity constraints on σ [2].

[2] Indeed if the coefficient matrix were fixed along the perturbations, we have continuity by Lemma 2 of Hart and Mas-Colell (2010)

It is pertinent to distinguish between two kinds of points in the domain space. For $\lambda \neq 0$, let $\mathbb{Z}_T^{\star\sigma}(\lambda) = \text{argmax}_{z\in\mathbb{Z}_T} \sum_{j\in T} \lambda_j V[z,j](\sigma)$ be the set of efficient states with T as the owning coalition and the pareto weight is λ and $\mathbb{Z}_T^{\star\sigma} = \cup_{\lambda\neq 0}\mathbb{Z}_T^{\star\sigma}(\lambda)$ be the set of all efficient states with T as the owning coalition.

Case 1. Suppose σ is such that $\sigma[z,i]$ does not put probability 1 on $\mathbb{Z}_T^{\star\sigma}$. We rely on sufficient conditions under which lower semicontinuity obtains as stated in Theorem 4.32 of Rockafellar and Wets (2009). What those conditions require is that there exist $\beta[z,i]$ such that the unanimity constraints hold as strict inequalities. Formally the condition is

$$\exists \beta[z,i] \quad \text{st} \qquad \begin{bmatrix} (-V^1(\sigma))_{z\in\mathbb{Z}_T} \\ (-V^2(\sigma))_{z\in\mathbb{Z}_T} \\ \cdots \\ (-V^t(\sigma))_{z\in\mathbb{Z}_T} \end{bmatrix}_{|T|\times|\mathbb{Z}_T|} \beta[z,i] < \begin{bmatrix} -\langle V^1(\sigma), \sigma[z,i]\rangle \\ -\langle V^2(\sigma), \sigma[z,i]\rangle \\ \cdots \\ -\langle V^t(\sigma), \sigma[z,i]\rangle \end{bmatrix}_{|T|\times 1}$$

$$-I_{|\mathbb{Z}_T|\times|\mathbb{Z}_T|}\beta[z,i] \leq 0_{|\mathbb{Z}_T|\times 1}$$

$$[-1,\ldots,-1]_{1\times|\mathbb{Z}_T|}\beta[z,i] \leq -1$$

$$[1,\ldots,1]_{1\times|\mathbb{Z}_T|}\beta[z,i] \leq 1$$

Regarding this as the primal, we make use of Motzkin's Transposition Theorem (see p94 in Schrijver (1998)) for equivalent conditions on the dual. Let $p = |\mathbb{Z}_T|$. Motzkin's Transposition Theorem states

$$\exists \beta[z,i] \quad \text{st} \quad A\beta[z,i] < a \text{ and } B\beta[z,i] \leq b$$

$$\text{iff}$$

$$\forall \lambda = (\lambda_1,\ldots,\lambda_j,\ldots,\lambda_t) \geq 0 \text{ and } \mu = (\mu_1,\ldots,\mu_p,\mu_{p+1},\mu_{p+2}) \geq 0$$

$$(i) \quad \lambda A + \mu B = 0 \implies \lambda a + \mu b \geq 0$$

$$(ii) \quad \lambda A + \mu B = 0,\ \lambda \neq 0 \implies \lambda a + \mu b > 0$$

The condition $\lambda A + \mu B = 0$ is written as

$$\forall z \in \mathbb{Z} \quad -\sum_{j\in T} \lambda_j V[z,j](\sigma) - \mu_z - \mu_{p+1} + \mu_{p+2} = 0$$

which implies

$$\forall z \in \mathbb{Z} \quad \mu_{p+2} - \mu_{p+1} = \sum_{j \in T} \lambda_j V[z, j](\sigma) + \mu_z$$
$$\geq \sum_{j \in T} \lambda_j V[z, j](\sigma) \qquad\qquad \mu_z \geq 0$$

This implies

$$\mu_{p+2} - \mu_{p+1} \geq \sum_{j \in T} \lambda_j V[z^\star, j](\sigma) \qquad\qquad \text{for } z^\star \in \mathbb{Z}_T^{\star\sigma}(\lambda)$$
$$> \Big\langle \sum_{j \in T} \lambda_j V^j(\sigma), \sigma[z, i] \Big\rangle$$

where the strict inequality is because a weighted sum cannot exceed the greatest of the terms being averaged and must be strictly less than this greatest term whenever a positive weight is put on a term that is not the greatest. This implies

$$-\Big\langle \sum_{j \in T} \lambda_j V^j(\sigma), \sigma[z, i] \Big\rangle - \mu_{p+1} + \mu_{p+2} > 0$$

Thus we have shown $\lambda a + \mu b > 0$.

Case 2. Suppose σ is such that $\sigma[z, i]$ puts probability 1 on $\mathbb{Z}_T^{\star\sigma}$. At such a point, the value of the correspondence is $C_T[z, i](\sigma) = \Delta(\mathbb{Z}_T^{\star\sigma})$. At any $\beta[z, i] \in \Delta(\mathbb{Z}_T^{\star\sigma})$, the unanimity constraints hold with equality. We verify the sufficient condition developed by Dantzig et al. (1967) (cf. Wets (1985)) for continuity of $C_T[z, i](\sigma)$ at such a point. We will show that the coefficient matrix $A(\sigma)$ for unanimity constraints is full rank. This ensures that for a sequence $\sigma^n \to \sigma$

$$\limsup_{n \to \infty} \text{ rank } A(\sigma^n) \leq \text{ rank } A(\sigma)$$

We verify that the rows of $A(\sigma)$ are linearly independent and hence the matrix is full rank

$|T|$.

$$\begin{aligned}
\sum_{j\in T}\lambda_j V^j(\sigma) &= 0 && \lambda_j\in\mathbb{R}\\
\implies \sum_{j\in T}\lambda_j[I-\delta\mathbb{P}^\sigma]^{-1}(1-\delta)u_j &= 0 && \text{Step 1}\\
\implies [I-\delta\mathbb{P}^\sigma]^{-1}\big((1-\delta)\sum_{j\in T}\lambda_j u_j\big) &= 0\\
\implies \sum_{j\in T}\lambda_j u_j &= 0
\end{aligned}$$

The last implication is because the matrix $[I-\delta\mathbb{P}^\sigma]^{-1}$ is nonsingular and hence full rank. So its nullspace is $\{0\}$. We also have $\delta<1$. We now show that the vectors $(u_j)_{j\in N}$ are linearly independent. This would imply that any subset thereof, in particular $(u_j)_{j\in T}$ is linearly independent. Hence $\lambda_j=0$ for every $j\in T$. This would conclude the verification that $A(\sigma)$ is full rank.

For $k\in N$, let $z_k=(N,F(N)e_k)\in\mathbb{Z}$ where e_k is the k-th unit vector in $\mathbb{R}^n$. We have $u_j(z_k)=F(N)$ if $j=k$ and 0 if $j\neq k$. Suppose $\sum_{j\in N}\lambda_j u_j=0$. This implies for every $k\in N$, $\sum_{j\in N}\lambda_j u_j(z_k)=0$ and consequently $\lambda_k=0$. Thus $(u_j)_{j\in N}$ are linearly independent.

We have shown that the auxiliary correspondence $C_T[z,i](\sigma)$ is continuous at every point σ. We now construct our original choice correspondence $C[z,i](\sigma)$ from the auxiliary correspondence. The idea is to view the auxiliary correspondence as expressing conditional choice (conditional on the choice of owning coalition T) and original choice correspondence as expressing unconditional choice of $[z,i]$. Let $\rho\in\Delta(T:z\rightsquigarrow(T,w))$ denote a randomization over coalitions such that the transition from state z to a state with T as the owning coalition satisfies the power calculus. Then $C_\rho[z,i](\sigma)=\prod_T\rho_T C_T[z,i](\sigma)$ is continuous as a cartesian product of continuous correspondences and $C[z,i](\sigma)=\cup_\rho C_\rho[z,i](\sigma)$ is continuous as a union of continuous correspondences. (See Theorem 7.3.12, 7.3.14 and 7.3.8 in Klein and Thompson (1984)).

Step 3. Define $[z,i]$- th coordinate map of $F : [\Delta(\mathbb{Z})]^{mn} \Rightarrow [\Delta(\mathbb{Z})]^{mn}$ by

$$F[z,i](\sigma) = \underset{\beta[z,i]\in C[z,i](\sigma)}{\operatorname{argmax}} V[z,i](\beta[z,i], \sigma(-[z,i]))$$

By an argument that mimics Lemma 3.1 in Myerson (2013), $F[z,i](\sigma)$ is nonempty and convex. By continuity of $V[z,i]$ on $\Delta(\mathbb{Z})]^{mn}$, continuity of the choice correspondence $C[z,i](\sigma)$ and Theorem of the Maximum, we have that $F[z,i](\sigma)$ is upper semicontinuous. Thus we have F is a nonempty, convex-valued and upper semicontinuous correspondence defined on a nonempty compact convex set $\Delta(\mathbb{Z})]^{mn}$. By Kakutani Fixed point Theorem, F has a fixed point. By Proposition 7 and the definition of F, there is a bijection between the the set of fixed points of F and the set of MPE of $\mathscr{G}(\rightsquigarrow, F, p, \delta)$. In particular, the game has an MPE. Q.E.D.

4.4 Long Run Equilibrium Behavior

Any MPE σ of the dynamic game $\mathscr{G}(\pi, F, p, \delta)$ has an associated transition matrix $\mathbb{P}^\sigma$ that describes the transition probability from one state to another. The long run behavior of an MPE σ is described by the stable sets of its transition matrix $\mathbb{P}^\sigma$. It is defined by two requirements [3]. First, starting from any state that is in a stable set, the Markov process remains in the stable set forever. Second, no nonempty and proper subset of a stable set is stable. These two properties are formally expressed in the definition below.

Definition 10. A set $E \subset \mathbb{Z}$ is called a stable set of $\mathscr{G}(\pi, F, p, \delta)$ if there is an MPE σ such that

1. E is closed under $\mathbb{P}^\sigma$.

$$\forall z \in E, \quad \sum_{z'\in E} \mathbb{P}^\sigma(z, z') = 1$$

[3] See Gomes and Jehiel (2005)

2. E is a communication class [4] under $\mathbb{P}^\sigma$. In other words, any pair of states in E communicate. Formally, for any pair of states $z, z' \in E$, there exists a sequence $(z_0, z_1, \ldots, z_k, \ldots, z_m)$ with $z = z_0$ and $z_m = z'$ such that for every $k = 1, \ldots, m$ $z_k \in E(\sigma)$ and $\mathbb{P}^\sigma(z_{k-1}, z_k) > 0$.

Since any stable set is associated with some MPE σ, the expression "...is a stable set of σ" is meaningful.

Definition 11. A state z is called a stable state or an absorbing state of $\mathscr{G}(\pi, F, p, \delta)$ if there is an MPE σ such that $\{z\}$ is a stable set of σ.

4.5 Example

We present a simple example that exposits the idea that when power is the basis of exchange, stability may be a constraint on allocative efficiency.

Example. There are three players, $N = \{1, 2, 3\}$. There is a symmetric submodular productivity environment given by

$$F(S) = \begin{cases} F & \text{if } |S| = 3 \\ G & \text{if } |S| = 2 \\ 0 & \text{if } |S| = 1 \end{cases}, \qquad F > G, \qquad G > \frac{3}{4}F$$

There are six possible states. i.e. $\mathbb{Z} = \{a, b, c, d, e, f\}$ where

$$a = \Big(12, \Big(\frac{G}{2}, \frac{G}{2}, 0\Big)\Big), \quad b = \Big(23, \Big(0, \frac{G}{2}, \frac{G}{2}\Big)\Big), \quad c = \Big(13, \Big(\frac{G}{2}, 0, \frac{G}{2}\Big)\Big)$$
$$d = \Big(123, \Big(\frac{G}{2}, \frac{G}{2}, F - G\Big)\Big), \quad e = \Big(123, \Big(F - G, \frac{G}{2}, \frac{G}{2}\Big)\Big), \quad f = \Big(123, \Big(\frac{G}{2}, F - G, \frac{G}{2}\Big)\Big)$$

Every transition satisfies the power calculus. The recognition probabilities are equal $p_1 = p_2 = p_3 = 1/3$. We claim that the following strategies σ constitute an equilibrium. The transition matrix $\mathbb{P}^\sigma$ induced by σ is also listed.

[4]See Bremaud (1999)

1	a	b	c	d	e	f
a	1	0	0	0	0	0
b	1/2	0	1/2	0	0	0
c	0	0	1	0	0	0
d	0	0	0	1	0	0
e	1/2	0	1/2	0	0	0
f	0	0	0	0	0	1

2	a	b	c	d	e	f
a	1	0	0	0	0	0
b	0	1	0	0	0	0
c	1/2	1/2	0	0	0	0
d	0	0	0	1	0	0
e	0	0	0	0	1	0
f	1/2	1/2	0	0	0	0

3	a	b	c	d	e	f
a	0	1/2	1/2	0	0	0
b	0	1	0	0	0	0
c	0	0	1	0	0	0
d	0	1/2	1/2	0	0	0
e	0	0	0	0	1	0
f	0	0	0	0	0	1

$\mathbb{P}^\sigma$	a	b	c	d	e	f
a	2/3	1/6	1/6	0	0	0
b	1/6	2/3	1/6	0	0	0
c	1/6	1/6	2/3	0	0	0
d	0	1/6	1/6	2/3	0	0
e	1/6	0	1/6	0	2/3	0
f	1/6	1/6	0	0	0	2/3

The first step is to compute the value functions $(V^1(z), V^2(z), V^3(z))$ of σ. For this, we need the following matrix

$I - \mathbb{P}^\sigma$	a	b	c	d	e	f
a	$1-2\delta/3$	$-\delta/6$	$-\delta/6$	0	0	0
b	$-\delta/6$	$1-2\delta/3$	$-\delta/6$	0	0	0
c	$-\delta/6$	$-\delta/6$	$1-2\delta/3$	0	0	0
d	0	$-\delta/6$	$-\delta/6$	$1-2\delta/3$	0	0
e	$-\delta/6$	0	$-\delta/6$	0	$1-2\delta/3$	0
f	$-\delta/6$	$-\delta/6$	0	0	0	$1-2\delta/3$

$V^i(z) = (V[a,i], V[b,i], V[c,i], V[d,i], V[e,i], V[f,i])$ is computed as the solution to the following system of equations

$$[I - \mathbb{P}^\sigma]V^i(z) = (1-\delta)u_i(z)$$

The value functions so computed are displayed below

$$
\begin{aligned}
V[a,1] &= V[c,1] = V[d,1] = V[f,1] = \frac{3-2\delta}{3(2-\delta)}G \\
V[b,1] &= \frac{\delta}{3(2-\delta)}G \\
V[e,1] &= \frac{3(1-\delta)}{3-2\delta)}F + \Big[\frac{\delta}{3(2-\delta)} - \frac{3(1-\delta)}{3-2\delta)}\Big]G \\
V[a,2] &= V[b,2] = V[d,2] = V[e,2] = \frac{3-2\delta}{3(2-\delta)}G \\
V[c,2] &= \frac{\delta}{3(2-\delta)}G \\
V[f,2] &= \frac{3(1-\delta)}{3-2\delta)}F + \Big[\frac{\delta}{3(2-\delta)} - \frac{3(1-\delta)}{3-2\delta)}\Big]G \\
V[b,3] &= V[c,3] = V[e,3] = V[f,3] = \frac{3-2\delta}{3(2-\delta)}G \\
V[a,3] &= \frac{\delta}{3(2-\delta)}G \\
V[d,3] &= \frac{3(1-\delta)}{3-2\delta)}F + \Big[\frac{\delta}{3(2-\delta)} - \frac{3(1-\delta)}{3-2\delta)}\Big]G
\end{aligned}
$$

We verify σ is unimprovable for player 1. By symmetry, it is unimprovable for player 2 and 3. We claim the following order relations among payoffs.

$$V[a,1] > V[b,1]$$

$$V[a,1] > V[e,1]$$

The first inequality is immediate from $1 > \delta$. The second follows from the assumption that $G > \frac{3}{4}F$ and $1 > \delta$. The verification that σ is unimprovable for player 1 is now immediate.

Discussion. The state space can be partitioned into inefficient states $\{a, b, c\}$ and efficient states $\{d, e, f\}$. Figure 4.1 shows the transition graph of the markov process induced by the equilibrium σ. The markov process has four communication classes viz. $\{a, b, c\}, \{d\}, \{e\}, \{f\}$ and one closed set $\{a, b, c\}$. Thus the set of inefficient states $\{a, b, c\}$ is a stable set. Note that the markov process is not irreducible as it has more than one communication class.

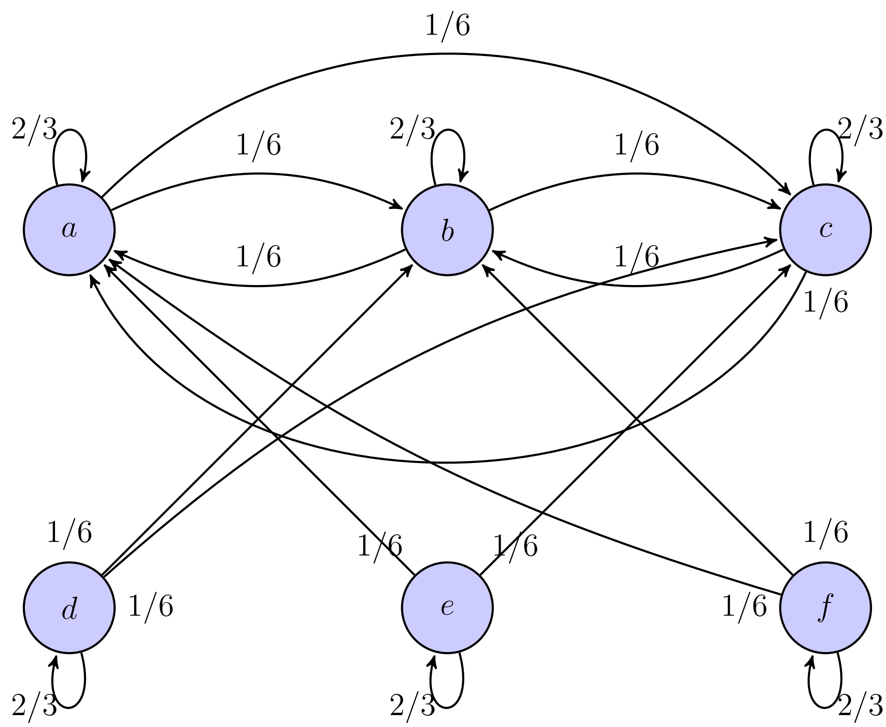

Figure 4.1: Transition Graph of Markov Process induced by σ

It is instructive to look at the equilibrium expectations that stabilize the inefficient allocation of property rights. At an inefficient state like a, while players 1 and 2 have no incentives to move to an efficient state, player 3 has an incentive to offer the efficient state d. Such an offer is feasible for player 3. However, the efficient state d is unstable and itself vulnerable to acts of appropriation in which player 3 can join ranks with either player 1 or player 2 to strip the other of his property rights. Since player 3 cannot commit against such future acts of appropriation, a move to an efficient state d is dominated by a move to inefficient states of either b or c in which player 3 appropriates now rather than later.

4.6 Concluding Remarks

A stochastic game was introduced as a model of persistent power struggle for property rights. An equilibrium existence theorem was proved for the finite state space version of the game. A simple three player example illustrates the scope of the paper that when power is the basis of exchange, there is a tension between efficiency and stability. One would like to see whether an existence theorem can be established for the stochastic game with infinite state space by a limiting argument. A characterization of productivity environments for which there is tension between stability and efficiency is also desirable.

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