THERMO-MECHANICAL MODEL DEVELOPMENT AND
EXPERIMENTAL VALIDATION FOR METALLIC PARTS IN
ADDITIVE MANUFACTURING

A Dissertation in
Mechanical Engineering

by

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Abstract

The objective of this work is to experimentally validate thermal and mechanical finite element models of metallic parts produced using additive manufacturing (AM) processes. AM offers advantages over other manufacturing processes due the fact that it can produce net and near-net shapes directly from a digital drawing file. Parts can be produced on a layer by layer basis by melting wire or powder metal using a laser or an electron beam. The material then cools and solidifies to form a fully dense geometry. Unfortunately the large thermal gradients cause a buildup of residual stress often taking parts out of tolerance or causing failure by cracking or delamination. To successfully reduce distortion and residual stress in metallic AM parts without expensive and time consuming trial and error iterations, an experimentally validated physics based model is needed.

In this work finite element (FE) models for the laser directed energy deposition (LDED), the Electron Beam Directed Manufacture (EBDM) process, and the Laser Powder-Bed Fusion (LPBF) process are developed and validated. In situ distortion and temperature measurements are taken during the LDED processing of both Ti-6Al-4V and Inconel® 625. The in situ experimental results are used in addition to post-process residual stress measurements to validate a thermo-mechanical model for each alloy. The results show that each material builds distortion differently during AM processing, a previously unknown effect that must be accounted for in the model. The thermal boundary conditions in the model are then modified to allow for the modeling of the
EBDM process. The EBDM model is validated against in situ temperature and distortion measurements as well as post-process residual stress measurements taken on a single bead wide Ti-6Al-4V wall build. Further model validation is provided by comparing the predicted mechanical response of a large EBDM aerospace component consisting of several thousand deposition tracks to post-process distortion measurements taken on the actual part. Several distortion mitigation techniques are also investigated using an FE model. The findings are used to reduce the maximum distortion present on the large industrial aerospace component by 91%. Finally, the modeling work for the LDED and the EBDM processes is extended to Laser Powder-Bed Fusion (LPBF) processing of Inconel® 718. The necessary boundary conditions and material properties to include in the models are identified by comparing the model with in situ experimental results.
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Chapter 1

Introduction

Laser Directed Energy Deposition (LDED), Electron Beam Directed Manufacture (EBDM), and Laser Powder Bed Fusion (LPBF) are commonly used additive manufacturing (AM) processes. In each of these processes, net and near net shape components are fabricated in a layer-by-layer manner directly from digital drawing files. The flexibility of the AM process allows different part geometries, on the scale of mm$^3$ to m$^3$, to be created. Each layer is formed by melting individual passes from a wire or powder feedstock material, which undergoes rapid heating, melting, solidification, and cooling during the deposition process. As the part is fabricated, the deposited material undergoes repeated heating and cooling cycles as more passes and layers are added. One of the consequences of the thermal gradients induced in the components by the layer-by-layer deposition of material in AM processes is the build-up of undesirable levels of distortion and residual stress. This unwanted distortion and residual stress can lead to loss of tolerance and failure. Currently these issues are dealt with using an experimental trial and error approach where several samples are manufactured to iteratively reach the desired outcome. In addition to being time consuming and expensive, the trial and error approach yields little understanding pertaining to distortion and residual stress mitigation, as even slight changes to the build plan can result in large changes.
in distortion and stress accumulation. In order to circumvent the trial and error process an experimentally validated predictive model is needed.

1.1 Prior work

1.1.1 Welding

The use of finite element modeling (FEM) of AM to predict distortion and residual stress originates from the prior research performed on multi-pass welding. Multi-pass welding is a similar process to AM, in that a heat source is used to melt material onto a workpiece where it is allowed to cool and solidify. Like in AM, thermal gradients lead to unwanted distortion and residual stress.

Due to the fact that welding and AM processes share many similarities, the numerical models are also comparable. Distortion and residual stress modeling of welding dates back to the 1970’s, when Hibbit and Mercal [7] showed uncoupled 2D thermo-mechanical modeling to be a capable means to predict the mechanical response of simple bead-on-plate welds. Several weld models have been used to predict thermal and mechanical behavior [7–13]. Tekriwal and Mazumder [11] describe a method of modeling weld mechanical response where a 3-dimensional transient heat transfer analysis is first performed and the results are then used in a transient thermo-elasto-plastic analysis to calculate stress and strain, yielding accurate results when compared with experiment. The approach is now commonly applied in analyses for both welding and AM. Lindgren has written detailed summaries on the development of weld model complexity [14], material modeling in welding [15], and improvements in computational efficiency for weld
modeling [16]. In more recent weld model work, Michaleris et al. focused on predicting residual stresses and out-of-plane distortion modes, as first defined by Masubuchi [17], caused by welding [18]. Other modeling work has focused on material phase change during welding [19,20].

The inclusion of transformation strains caused by solid-state phase transformation present in steels has been shown to be critical in weld modeling due to the fact that transformation strains influence bulk distortion and residual stress of workpieces, with lower martensite-start temperature yielding reduced levels of residual stress and distortion [21, 22]. Francis et al. have written a thorough review of welding residual stress present in steel and note that transformation strains present in welded steels can completely erase the strains caused by thermal contraction of the weld region [23]. The offsetting of strain attributable to thermal contraction by transformation strains alters the bulk residual stress distribution throughout a workpiece and has been shown to reduce bending stresses on the bottom of surface of base plates for bead-on-plate welds [24]. Because AM is a similar process to welding, the welding literature suggests that transformation strains also need to be included in AM models.

Generally the goal driving model development in welding has been distortion mitigation. In order to reduce distortion the appropriate distortion mode must be identified as first defined by Masubuchi [17]. The out-of-plane distortion modes include angular, buckling, and longitudinal bending. Angular and buckling distortion are caused by similar mechanisms. Angular distortion is caused by transverse shrinkage in the deposition region, while buckling occurs when residual stress caused by longitudinal shrinkage exceeds the workpiece critical buckling strength. Both angular and buckling
distortion are less common in AM than in welding as the substrates used are generally thicker than weld panels. Of the 3 possible out-of-plane modes, longitudinal bending is of primary concern in AM processes. The longitudinal bending distortion is caused by the contraction of the molten material after heating and deposition. Weld research has shown reducing the heat input [25], balancing the residual stress to minimize the bending moment [25], and creating a temperature difference between parts to be welded (known as transient differential heating) [26] to be effective in reducing longitudinal bending distortion levels.

1.1.2 Directed Energy Deposition Additive Manufacturing

AM process simulation using FEM introduces challenges that do not present themselves during multi-pass welding modeling. The increase in number of passes and processing time, as well the addition of material throughout the deposition results in increased complexity and computational expense. Simulations are typically performed using the same thermo-elasto-plastic analysis technique as in the weld literature.

1.1.2.1 Temperature

The thermal response of an AM workpiece drives the mechanical response, thus before the mechanical response can be accurately modeled an adequate transient thermal model is needed.
Thermal modeling

The addition of material into the AM simulation requires an element activation strategy [27]. Two such strategies are the quiet element technique and the quiet-inactive hybrid element technique. The quiet element technique includes all elements in the analysis from the start, but assigns them material properties such that they do not effect the analysis until contacted by the heat source. The hybrid technique does not include the deposited elements at the start of the deposition, but rather brings them into the analysis on a layer-by-layer basis where they are then set to quiet.

Thermal modeling of AM processes is typically performed using the same transient approach that is found in the welding [26] and laser forming [28] literature. The important boundary conditions to consider are the heat source, which is typically modeled using Goldak’s double ellipsoid [29], radiation, and convection. Convection is typically modeled by accounting for free convection [30, 31], or by using a heightened convection intended to account for both free and forced convection [32]. Ghosh and Choi modeled forced convection from the shielding gas flow using an empirical equation [33]. Michaleris determined that neglecting convection and radiation on the evolving interface between active and quiet elements leads to large errors in predictions [27]. Unknown process parameters such as the absorption coefficient and the heat flux are typically determined either by experimental measurement [34] or by inverse simulation [35]. In addition to driving the mechanical response, the thermal response is also known to affect microstructure and material properties [36].
Thermal model validation

Thermal models are validated using in situ measurements to capture the repeated thermal cycles seen during the AM process. In situ temperature measurements can be easily taken using thermocouples [31,37–39]. Thermocouples have been used specifically to validate transient thermal models as done by Peyre et al. on a 25 layer wall build [38]. While the use of thermocouples limits measurement locations to only the substrate, they are typically adequate for model validation. Other researchers have chosen to use optical pyrometers to collect additional temperature data [31]. In situ temperature measurements can also be used to calibrate unknown process parameters [35].

1.1.2.2 Distortion and residual stress

Distortion in AM parts is caused by the buildup of residual stress as a result of the large thermal gradients induced during the repeated melting and solidifying of material. The build-up of residual stress can be illustrated using the 3-bar analogy, shown in Figure 1.1 [6]. The anterior bars represent the substrate while the interior bar represents the deposition. Figure 1.1 (a) shows the workpiece initially at room temperature. Figure 1.1 (b) shows the deposited material expanding as it is melted by the heat source. The interior bar is put into compression and the anterior bars are in tension. If the deposited material were unbonded as shown in Figure 1.1 (c), the system would return to a zero stress state upon cooling to room temperature. However, as illustrated in Figure 1.1 (d) the deposited material is bonded to the substrate resulting in the interior bar being put into tension and the anterior bars experiencing compression.
Mechanical models are used to predict and study the manner in which distortion accumulates in AM workpieces, which must be understood if distortion is to be mitigated. Experiments must be performed to validate the models. They are commonly performed using in situ measurements, post-process measurements, or both.

**Mechanical response modeling**

Many researchers have modeled distortion and residual stress for AM processes, typically limiting simulations to few deposition lines to insure feasibility of computation [3, 31, 37, 39, 40]. Additional work has focused on predicting stress and distortion in materials that undergo phase transformation [41]. Ghosh and Choi concluded that simulated distortion results are significantly affected by failing to properly take into account the microstructural changes present in the deposited material [33]. Griffith *et al.* showed that the high temperatures reached during the laser deposition of stainless steel 316 can cause the material to anneal, thus reducing the measured residual stress [42]. Song *et al.* accounted for the plastic strain relaxation in a deposited nickel based alloy.

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**Fig. 1.1.** Formation of AM residual stress: (a) room temperature (b) molten deposition material (c) return to room temperature unbonded molten material (d) return to room temperature for bonded material [6].
by setting a threshold temperature that, when surpassed, sets the plastic strain value to zero [43]. Qiao et al. performed micro-hardness measurements to demonstrate that the equivalent plastic strain needs to be dynamically adjusted at high temperatures in thermo-elasto-plastic models to simulate material annealing [44].

**Post-process measurement of the final distortion**

Post-process measurements can be used to confirm that a model has accurately predicted the final distortion and residual stress of an AM workpiece. While these measurement methods say nothing about the manner in which distortion and stress accumulate during the deposition process, they are still frequently used for model validation.

Chiumeti et al. measured post-process longitudinal and transverse distortion of an 10 layer Inconel\textsuperscript{R} 718 wall and used the measurements for model validation [39]. Anca et al. validated a model for the mechanical response of Ti-6Al-4V using post process distortion measurements of a small deposition, reporting a 40\% error [3].

The effects of changes in path planning and dwell time between passes on distortion and residual stress have also been studied using models validated with post-process measurements. For example, the selection of different deposition paths has been shown by Nickel et al. [45] to significantly affect distortion in AISI 1117 C-Mn steel, and has also been found by Mercelis and Kruth [46] to affect residual stress in AISI 316 austenitic stainless steel. In the case of changing dwell time between passes, Jendrzejewski et al. [47] concluded that shorter delays decrease the measured residual stress in deposited Co-based stellite SF6 alloy, and Klingbeil et al. [40] found delays to
decrease distortion in AISI 308 austenitic stainless steel. Fessler et al. [48] concluded that allowing deposition to cool between passes reduces distortion of nickel-base INVAR alloys and austenitic stainless steels.

**In situ measurement of distortion**

For improved model validation, researchers commonly compare calculated results from a model to measurements taken in situ, i.e., continuously during the entire duration of the deposition. This provides greater insight than could be achieved using only post-process measurements. Plati et al [37] compared mechanical results to distortion measured during the laser cladding of 1 powder deposition track onto a stainless steel substrate. Ocelik et al. applied 3D digital image correlation (DIC) techniques to measure the strain and distortion of a substrate during laser cladding. The experiment was used to validate a model depositing up to 10 tracks and it was noted that greater distortion accumulated during the earlier heating passes. Lundback and Lindgren [31] developed a flow stress model and validated it against 40 pass builds. In situ temperature was monitored with thermocouples and in situ distortion was recorded with an optical measurement system allowing for measurements to be taken on multiple points of the substrate. The model was shown to capture the distortion of the first 6 passes but then begins to over predict distortion.

**Measurement of residual stress**

The repeated thermal cycles of the deposition process cause residual stress to accumulate in AM workpieces [48–53]. Residual stress levels are dependent on processes
parameters, deposited geometry, and the number of layers \([30, 47]\). Measurement techniques commonly used for model validation include hole drilling, neutron diffraction, and x-ray diffraction \([54]\).

**Mitigation techniques**

Researchers investigating distortion mitigation techniques in AM have shown that altering the laser scanning pattern can reduce distortion, with shorter deposition passes resulting in lower distortion levels \([32, 45, 55, 56]\). However, shorter scanning patterns add processing time by requiring a greater number of deposition passes to add the same amount of material. This problem becomes more significant for large parts with a greater number of deposition passes. Residual stress may be reduced by preheating the substrate and holding it at a high bulk temperature \([30,47,57]\) or by heating the deposition region immediately prior to deposition (localized preheating) \([58]\). Bulk substrate heating is only feasible for small workpieces as it is not practical to hold a large substrate at a high temperature for a long period of time, while localized preheating requires modifications to the laser or electron beam deposition system.

**1.1.3 Powder-Bed Fusion Additive Manufacturing**

The Laser Powder-Bed Fusion (LPBF) process shares many similarities with the DED process. Both processes involve melting feedstock material to form fully dense geometries on a layer by layer basis and can create net and near-net shape parts. Large thermal gradients arise, as in the DED process, causing unacceptable levels of residual
stress to build up in the part, frequently leading to failure by cracking or delamination from the build plate.

The primary difference between the processes is that DED injects powder or wire feed-stock into the melt pool, whereas in LPBF the powder is pre-placed. The LPBF process operates by first spreading a thin layer of powder (on the order of 10 µm) across a build plate. Next a laser melts the material which then cools and solidifies to form a fully dense geometry. The build plate then lowers, a recoater spreads a new layer of powder, and the process is repeated allowing for parts to be additively constructed. The LPBF process also operates under different processing conditions. The laser speed is typically 2 orders of magnitude greater than that used for DED processing and the laser spot size is an order of magnitude smaller than that used for DED.

1.1.3.1 LPBF modeling

From a modeling standpoint the primary differences between the previously discussed AM process and the LPBF process include the presence of the pre-placed powder layer and a smaller laser spot size (as little as 70 µm). The pre-placed powder may affect the thermal response. A smaller spot size necessitates strict spacial and temporal discretization requirements. To reduce lengthy computation times imposed by the discretization requirements associated with LPBF researchers have attempted removing the build plate from the analysis [59] or using a 2-D model [60]. Others have neglected the powder in thermal analyses [43,59,61], essentially assuming that the powder is a perfect insulator. Dai and Shaw [32,62] assumed that the powder material has the same thermal properties as its matching solid material in a 3-D thermomechanical
model. In other research the conductivity of the powder is scaled by a porosity dependent factor \([63,64]\). Another common strategy \([65–69]\) involves assigning the metallic powder thermal properties based on powder-solid relationships developed by Sih et al. \([70–72]\).

### 1.1.3.2 LPBF model validation

Some researchers provide no model validation \([32,62,64,65]\), while others validate the developed models against measured relative densities \([66]\), deposition track width \([67]\), microstructure and melt pool dimensions \([68,69]\), or post process residual stress \([43]\).

### 1.2 The Need

Thermo-mechanical models of AM processes must be validated using both in situ and post-process measurements. While models are presented in the published literature they are frequently not experimentally validated. Models that are validated are often not compared to in situ measurements. Several needs have been identified:

1. A large number of deposition tracks need to be investigated for model validation:
   
   (a) An increase in the number of passes causes a greater accumulation of heat which can expose shortcoming of the model.

   (b) Larger processing time and a greater number of passes can cause errors, that would otherwise go unnoticed, to accumulate and become distinguishable.

2. Mechanical models should be validated using post-process measurements:

   (a) Information pertaining to part distortion can be gathered in all 3 dimensions.
(b) Post-process measurements can record the residual stress accumulated in the sample, allowing for further model validation.

3. Thermo-mechanical models should be validated using in situ measurements:

(a) In situ measurement methods capture information that would go unreported when using only post-process measurements.

(b) Measurements taken in situ can be used to calibrate unknown process parameters.

(c) Time dependent distortion data reveals the manner in which distortion accumulates throughout the build, thus improving the understanding of the mechanical response.

1.3 Objective of This Research

The objective of this work is to design experiments and develop FE models to experimentally validate thermo-mechanical simulations for AM processes. The models predict temperature history, in situ distortion, post-process distortion, and residual stresses in AM workpieces. The models are capable of simulating depositions with a large number of degrees of freedom to demonstrate their relevance to industrial applications. In situ measurements of the temperature and distortion in addition to post-process distortion and residual stress measurements are used to validate the models for depositions using $10^1$-$10^3$ tracks. An experiment is designed and an FE model is used to investigate the difference in mechanical response of Ti-6Al-4V and Inconel$^{\text{®}}$ 625. Distortion mitigation techniques are investigated and applied to a large industrial
Finally, the effect of the pre-placed powder layer present in LPBF process on the workpiece thermal response is examined using a FE model and in situ measurements.

1.4 Outline and Novelties

The following is an outline of the work completed for developing and experimentally validating thermo-mechanical models for AM processes. Contributions and novelties from the work are listed. The work is presented as follows:

- Chapter 2: Effect of inter-layer dwell time on distortion and residual stress in Additive Manufacturing of Titanium and Nickel Alloys

1. Experiments are performed to compare the mechanical response of an alloy that undergoes solid-state phase transformation (Ti-6Al-4V) to an alloy that does not (Inconel® 625) during LDED processing.

2. Inconel® 625 is shown to build distortion consistently throughout processing independent of dwell time.

3. Post-process measurements of the Ti-6Al-4V builds show that reducing dwell time results in less distortion accumulation in the workpiece.

4. The in situ distortion measurements reveal that Ti-6Al-4V does not accumulate distortion consistently throughout the build with some layers actually reducing the measured distortion.

5. Post-process CMM results show that Inconel® 625 builds more than twice the distortion of Ti-6Al-4V for the same heat input and dwell time.
6. Post-process residual stress measurements show that Ti-6Al-4V builds less residual stress as dwell times are reduced. The opposite is shown to be true for Inconel® 625.

- Chapter 3: Effect of solid-state phase transformation in additive manufacturing process modeling.

1. LDED Model validation is performed by comparing simulated mechanical responses to the experimental results presented in Chapter 2.

2. It is shown that the mechanical response of Inconel® 625 can be simulated using the standard constitutive model which accounts for elastic, plastic, and thermal strain.

3. The model predictions for the Ti-6Al-4V builds simulated using the standard constitutive model result in an over-prediction of residual stress and distortion of more than 500 %.

4. The assumption that transformation strain present in Ti-6Al-4V acts to eliminate other strain components at high temperatures is shown to give close agreement with experiment.

- Chapter 4: Residual stress and distortion modeling of Electron Beam Direct Manufacturing Ti-6Al-4V

1. The modeling approach presented in Chapter 3 is applied to simulate the thermal and mechanical response of EBDM deposited Ti-6Al-4V.
2. An experiment is designed to measure the in situ thermal and mechanical response of electron beam deposited Ti-6Al-4V.

3. The experimental results, including in situ temperature, in situ distortion, and post-process residual stress are used to validate the thermo-mechanical model.

4. The necessary boundary conditions needing to be applied to model the electron beam deposition process are determined by comparing simulated and measured temperatures. The results agree closely with the literature.

- Chapter 5: Thermo-mechanical modeling of Additive Manufacturing large parts

  1. The modeling work performed in Chapter 4, paired with an approach to limit the number of DOF, is extended to large industrial EBDM parts.

  2. A thermo-mechanical model for electron beam deposition is shown to predict the post-process distortion of a large industrial aerospace component.

  3. The layers responsible for the majority of the part deformation are identified.

  4. Tall structures built on the aerospace component are shown to not effect distortion after 9 layers, due to the fact the thermal cycles become localized.

- Chapter 6: Mitigation of distortion of Additive Manufacturing large parts
1. Several distortion mitigation strategies are studied on small models of the EBDM processes. Suggestions are made and tested for use on the large industrial aerospace part presented in Chapter 5.

2. It is shown that applying additional deposition to workpieces yields improved distortion mitigation when compared to applying only heating.

3. The importance of the order of addition of balancing layers is investigated using an FE model.

4. The model predictions are used to eliminate distortion present on a large aerospace component.

- Chapter 7: Experimental Validation for Thermal Modeling of Powder Bed Fusion Processes using in situ Measurements

1. A method to account for the pre-placed powder layer in LPBF processes is presented. The effect of neglecting powder in thermal simulations is investigated.

2. Accounting for powder in the model by using powder-solid relationships from the literature is shown to give good agreement with the in situ experimental measurements.

3. Neglecting powder in thermal analyses results in an over-prediction of temperature accumulation during processing.
Chapter 2

Effect of inter-layer dwell time on Distortion and residual stress in Additive Manufacturing of Titanium and Nickel Alloys

2.1 Abstract

In situ measurements of the accumulation of distortion during Additive Manufacturing (AM) of titanium and nickel base alloys are made as a function of changes in dwell time between the deposition of individual layers. The inclusion of dwell times between individual layers to allow for additional cooling during the deposition process is a common technique utilized in AM processes. Experimental observations made here in Inconel® 625 laser deposited builds show that the accumulation of distortion occurs with a consistent trend over the course of the builds and both distortion and residual stress levels decrease with increasing dwell times from 0 to 40 seconds. On the other hand, changes in dwell time for the Ti-6Al-4V laser deposited builds have a significant impact on the accumulation of distortion, with shorter and no dwell times minimizing the distortion accumulation and even reducing it over a range of build heights. These shorter dwell times also produce builds with significantly lower residual stress and distortion levels.

particularly when no dwell time is applied. Based on these results, the materials to be deposited should be considered when developing appropriate path planning schedules.

2.2 Introduction

Laser and electron beam directed energy deposition are two commonly used additive manufacturing (AM) processes. In each of these processes, net and near net shape components are fabricated in a layer-by-layer manner directly from digital drawing files. The flexibility of the AM process allows different part geometries, on the scale of $\text{mm}^3$ to $\text{m}^3$, to be created. Each layer is formed by melting individual passes from a wire or powder feedstock material, which undergoes rapid heating, melting, solidification, and cooling during the deposition process. As the part is fabricated, the deposited material undergoes repeated heating and cooling cycles as more passes and layers are added. One of the consequences of the thermal gradients induced in the components by the layer-by-layer deposition of material in AM processes is the build-up of undesirable levels of distortion and residual stress.

The build-up of distortion and residual stress in AM processes has a number of similarities with multi-pass welding. Several researchers have investigated techniques to reduce distortion in similar multi-pass welding processes including work by Michaleris and DeBiccari [73] who showed that reducing heat input results in a reduction in workpiece distortion and Masubuchi [17] who determined that constraining the workpiece can limit distortion. More recently Deo and Michaleris [26] studied the effect of heating the weld region immediately prior to welding and found that the technique can be used to achieve zero net distortion. In AM research Klingbeil et al. [40] and Jendrzejewski
et al. [30] each found that bulk substrate preheating can be used to reduce distortion in deposited workpieces.

In AM processes, the effects of changes in path planning and dwell time between passes on distortion and residual stress have also been studied. For example, the selection of different deposition paths has been shown by Nickel et al. [45] to significantly affect distortion in AISI 1117 C-Mn steel, and has also been found by Mercelis and Kruth [46] to affect residual stress in AISI 316 austenitic stainless steel. In the case of changing dwell time between passes, Jendrzejewski et al. [47] concluded that shorter delays decrease the measured residual stress in deposited Co-based stellite SF6 alloy, and Klingbeil et al. [40] found delays to decrease distortion in AISI 308 austenitic stainless steel. Fessler et al. [48] concluded that allowing deposition to cool between passes reduces distortion of nickel-base INVAR alloys and austenitic stainless steels. Costa et al. [36] investigated the effect of adding dwell time between deposition layers on the resulting thermal history and microstructure of laser deposited AISI 420 steel. The study found that shorter dwell times result in higher temperature levels and significantly altered microstructure when compared with longer dwell times.

Whereas these studies have focused on post-process measurements of accumulated distortion, others have used in situ measurements to monitor the temporal accumulation of distortion. Lundbäck and Lindgren [31] measured in situ distortion on single wall builds using an optical measurement system to validate their model of a Gas Tungsten Arc welding process. Plati et al. [37] used a linear variable differential transformer to measure the deflection of the free end of a cantilevered substrate during powder based cladding. Grum and Žnidaršič [51] recorded in-process strain using resistance
measuring rosettes to measure strain accumulation during laser cladding. Ocelik et al. [74] used digital image correlation to measure distortion of single and multi-bead Nanosteel, Eutroloy 16012, and MicroMelt 23 powder laser depositions on C45 steel and 301 stainless steel substrates. The in situ measurements taken in these works offer greater insight into distortion accumulation than common post-process distortion measurements, but do not compare the effects of changing materials or dwell time. The in situ distortion measurements have primarily been conducted for model validation.

Much of this previous work on the accumulation of distortion during the deposition has concentrated on welding and laser cladding processes. Additive manufacturing, on the other hand, involves the deposition of multiple layers and potentially large volumes of material in order to produce discrete shapes and components. In cases where AM processes have been investigated, the impact of changes in the dwell time between multiple passes on the accumulation of distortion and the impact of the selection of different materials has not been investigated. In the work reported here, in situ measurements of distortion during the deposition of thin walls in an α/β titanium (Ti-6Al-4V) and a Ni-Cr-Mo solid solution strengthened nickel (Inconel® 625) alloy are made to investigate the impact of both dwell time and material selection. The investigation of these two materials allows for the comparison of alloy systems that undergo a solid-state allotropic transformation to ones that do not. These measurements show that different distortion responses are present in the different materials, with the titanium alloy displaying no accumulation of distortion with increasing deposition layers when no dwell time is used between each layer. On the other hand, the Ni-Cr-Mo alloy
displays a consistent accumulation of distortion with increasing passes and independent of dwell time.

2.3 Experimental Procedure

2.3.1 Process Parameters

A series of single wall structures are fabricated using a laser-based directed energy deposition process on a matching 12.7 mm thick substrate for two material systems. An α/β Ti alloy (Ti-6Al-4V) and a Ni-Cr-Mo based solid solution strengthened nickel alloy (Inconel® 625) are investigated. Spherical gas atomized Inconel® 625 and Plasma Rotating Electrode Process (PREP®) Ti-6Al-4V powder sieved to a -100/+325 sieve range, which corresponds to powder diameters between 44 μm and 149 μm, are used for each deposition. Laser based depositions for each material are made using a deposition set up which consisted of an IPG Photonics® YLR-12000 laser system which operates at a laser wavelength of 1070 to 1080 nm and a Precitec YC-50 cladding head, through which powder is fed. The cladding head is positioned at a location 10 mm above the build. A 200 μm diameter fiber with a 200 mm collimator and a 200 mm focal length lens is used for the optics. The beam diameter at the part surface is measured to be 4 mm and has a Gaussian beam profile, as measured using a Primes Beam Focus Monitor. All deposits are made at a laser power of 2 kW and a travel speed of 10.6 mm/s. Because of differences in the density of the two materials, the powder feed rates are varied to ensure that the same amount of material is deposited for each pass. In the case of the Ti-6Al-4V powder, the feed rate is set at 8 g/min, while for the Inconel® 625 it is set at
16 g/min. A 9.4 L/min argon gas flow is used for shielding the optics from the deposited powder, while a second gas flow of the same rate is used for powder delivery.

The wall deposited on each substrate is 38.1 mm tall, 101.6 mm long, and 6.7 mm wide, while the substrate plate is 152.4 mm long, 38.1 mm wide, and 12.7 mm thick. The walls are constructed from 3-bead wide depositions built 42 layers high, with a bead thickness of 0.89 mm and a step-over of 2.29 mm. After the deposition of each bead, a period of cooling occurs as the laser travels to the next deposition starting point. After three beads are deposited, the layer is complete and a dwell time is added allowing for additional cooling. Figure 2.1 shows the laser path for odd and even number layers. The deposition direction changes based on layer number, with even numbered layers deposited in the opposite direction of odd numbered layers. All paths within a single layer are deposited in the same direction. Table 1 provides a summary of the changes in dwell time used for each material as well as the time required to deposit a single track and the total cooling time between the layers for each case. In this work, the dwell time is defined as the pause programmed into the build between the completion of one layer and the beginning of the next, not including laser travel time. Dwell times of 0, 20, and 40 seconds are used for each material.

2.3.2 In situ Distortion Measurement

In situ measurements of distortion in the substrate are made using the experimental set up shown in Figure 2.2. In this experimental set up, the substrate material is mounted and clamped on one end, allowing the opposite end to be free to distort during the deposition. In situ distortion measurements are taken with a Keyence
Fig. 2.1. Schematic of the laser path order during the deposition for (a) odd numbered layers and (b) even numbered layers.

![Diagram](image)

Table 2.1. Description of build for all cases.

<table>
<thead>
<tr>
<th>Case number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Ti-6Al-4V</td>
<td>Inconel® 625</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Powder feed rate (g/min)</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>16</td>
<td>16</td>
<td>16</td>
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<tr>
<td>Dwell time (s)</td>
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<td>20</td>
<td>40</td>
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<td>40</td>
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<tr>
<td>Cooling time between layers (s)</td>
<td>4</td>
<td>24</td>
<td>44</td>
<td>4</td>
<td>24</td>
<td>44</td>
</tr>
</tbody>
</table>
LK-031 laser displacement sensor (LDS) and LK-2001 controller. The LDS target has a diameter of 10 μm, a range of 10 mm, a linear accuracy of ± 1 μm, and a resolution of 1 μm. It is positioned near the edge of the bottom of the free end of the substrate to capture the bowing distortion mode of the sample substrate in the z-direction.

![Image of experimental setup](image)

**Fig. 2.2.** Experimental setup used for each case.

The use of in situ methods for monitoring the distortion of the substrate during the deposition process provides a wealth of additional information that would remain uncaptured if only post-process measurements were used. In analyzing the in situ results, it is important to understand the progression that the substrate goes through and to identify important components of the resulting deflection measurements. Figure 2.3 provides a schematic diagram showing the three important stages in the evolution of distortion during each deposition pass and how the location of the LDS impacts
the measurements. For example, Figure 2.3(a) shows the baseline condition in which the substrate is undeformed prior to deposition. Figure 2.3(b) shows the mechanical response of the substrate during deposition, in which the free end of the substrate distorts downward due to the larger thermal expansion of the top surface relative to the bottom surface. This downward deflection results in the LDS measuring a decreasing distortion. At the completion of the deposition layer, the substrate begins to cool, which allows the molten metal to contract, causing the free end of the substrate to distort upward, as shown in Figure 2.3(c). The upward deflection results in the LDS reading an increasing distortion. An example in situ mechanical response measured by the LDS is shown in

Fig. 2.3. Illustration showing distortion of the substrate during the deposition process (a) prior to deposition, (b) during deposition, and (c) during cooling.

Figure 2.4. In this figure, the period prior to the deposition of the first 3 passes, periods between the deposition of each pass, and the post-layer dwell time are identified for the first layer deposited on the substrate. It can be seen that the plate deflects in a downward
direction during each deposition, followed by an upward deflection, even during the short pauses needed to move the laser back to its starting position. It is clear in this figure, though, that the majority of the upward deflection occurs during the programmed dwell time.

Fig. 2.4. Illustration of the mechanical response during individual laser passes. The gray background indicates that the laser power is on. The different areas correspond to measurements taken (a) prior to deposition, (b) during deposition, and (c) during cooling.

2.3.3 In situ Temperature Measurement

In addition to the in situ measurement of the distortion, measurements of the temperature are made at three selected locations on the substrate, as shown in Figure 2.5(b), using 0.25 mm diameter Type-K thermocouples which have an accuracy of ±0.75%. Each thermocouple (TC) is located on the substrate bottom along the deposition axis. The LDS analog voltage signal and thermocouple signals are read by National
Instruments modules 9250 and 9213, respectively. Both modules record data in LabView at a sampling rate of 20 Hz.

2.3.4 Post-process Distortion and Residual Stress Measurement

In addition to the in situ measurements obtained by the laser displacement sensor, the pre and post-process plate profiles are also obtained using a coordinate-measurement machine (CMM). Measurements, with a linear accuracy of ± 4.5 μm, are performed at 10 points along the top of the substrate at the locations identified in Figure 2.6. The changes in plate profile and out-of-plane distortion are then calculated by subtracting the pre-process from the post-process measurements. Figure 2.7 provides a schematic diagram of how the out-of-plane distortion is determined by calculating the distance of the center of the plate (CMM 5 and CMM 6) from a line connecting a point near the clamped end of the substrate (CMM 9 and CMM 10), and a point near the free end of the substrate (CMM 1 and CMM 2).

Post-process residual stress in the substrate is measured using the hole-drilling method defined in ASTM E837. A single residual stress measurement is made at the bottom center of the substrate, which corresponds to the location of Thermocouple 2 shown in Figure 4.3(b). Only one measurement is taken, as the residual stress should be constant along the length of the substrate under the deposition. A detailed investigation of the residual stress distribution in similar AM parts has previously been performed by Denlinger et al. [75]. Micro-Measurements® strain gauges, model EA-06-062RE-120, are bonded to the bottom center of each substrate, and calibrated using the procedure described in manufacturer engineering data sheet U059-07 and technical note 503.
Fig. 2.5. Schematic of the (a) top of the substrate and the (b) bottom of the substrate, showing the LDS measurement location and the thermocouple (TC) locations.

Fig. 2.6. Schematic of all ten CMM points on the top of the substrate.
Incremental drilling is then performed using an RS-200 Milling Guide and a high-speed drill from Micro-Measurements® with a 1.52 mm diameter, carbide-tipped, Type II Class 4A drill bit. Strain measurements are read by a Micro-Measurements® P-3500 Strain Indicator. Bridges are balanced with a Micro-Measurements® Switch and Balance Unit, model SB-1. The hole-drilling method has an accuracy of ± 50 MPa.

2.4 Results

2.4.1 In situ Distortion and Temperature Measurements

In situ measurements of the distortion of 12.7 mm thick Ti-6Al-4V and Inconel 625® substrate plates during laser-based directed energy deposition processes are completed for dwell times ranging from 0 to 40 seconds. Figure 2.8 shows the LDS measurements of the distortion for Ti-6Al-4V in Case 1 through Case 3 and for Inconel® 625 in Case 4 through Case 6 as a function of process time. By capturing the deflection of the substrate in real time using the LDS, the vertical displacement that the substrate experiences with each deposition pass can be captured and monitored. With the increase
of the dwell time between each layer, the rise and fall of the substrate becomes more prominent.

In the majority of the cases analyzed here, both material systems display an accumulation of distortion that is rather consistent over the duration of the build. This behavior is particularly prevalent in the Inconel\textsuperscript{®} 625 samples, in which the distortion accumulates at very similar rates over the duration of each build and displays relatively little dependence on dwell time. The Inconel\textsuperscript{®} 625 samples also generally display much higher levels of distortion than the Ti-6Al-4V samples, even though the same processing conditions are used for both sets of builds.

The most prominent observation from these figures, though, appears in the Ti-6Al-4V depositions with no dwell time (Case 1) and a 20 second dwell time (Case 2). In each of these cases, the Ti-6Al-4V depositions do not display a similar accumulation of distortion over the duration of the build. In fact, the distortion trends measured in the case of no dwell time (Case 1), shown in Figure 2.8(a), for the Ti-6Al-4V sample show a decrease in distortion after the initial passes followed by little to no accumulation over the course of the build. This effect is much less prominent in the case of the 20 second dwell time (Case 2), shown in Figure 2.8(c), and disappears completely when the dwell time is increased to 40 seconds (Case 3). In addition to the in situ distortion measurements, the thermal response during the different depositions was also monitored at three specific locations on the bottom of the substrate. The corresponding thermal histories for each of the in situ distortion measurement cases described above are provided in Figure 2.9. Because of the different locations where the temperature measurements are made, each thermocouple trace displays a different trend and peak temperatures. The
Fig. 2.8. In situ distortion history for all wall builds. *Note: The irregularity in the data is caused by a temporary powder build-up on the LDS. The sudden increase in distortion after layer 42 is caused by the powder being removed from the sensor.
The measured temperatures at each of the three locations display similar trends for each of the materials systems and the three dwell times. In general, the measured temperatures rise rather rapidly during the initial deposition and plateau around the point in time where approximately one third of the build height is completed. At this point, the temperatures measured at the three locations begin to slowly decrease through the remaining portions of the build as the distance between the thermocouple location and the newly deposited material is increasing with time.

Even though similar trends are observed, the magnitude of the temperatures experienced in the different materials and dwell times varies. Table 2 shows the maximum temperature recorded by each thermocouple for all cases. It is observed that the thermocouple measurements on the Titanium substrates generally show higher temperatures than those on the Inconel\textsuperscript{®} substrates. The difference in the magnitude of the temperatures seen in the two materials is likely due to the fact that Inconel\textsuperscript{®} 625 has a higher thermal conductivity (ranging from 9.9 to 22.8 W/m/°C) than does Ti-6Al-4V (ranging from 6.6 to 17.5 W/m/°C) between temperatures from 20°C to 870°C, according to Special Metals [2] and Boyer [1], respectively.

The highest temperatures are observed with no dwell time for both the Ti-6Al-4V and the Inconel\textsuperscript{®} 625 materials, with temperatures in the Ti-6Al-4V build approaching
Fig. 2.9. In situ thermal history of each thermocouple for all wall builds.
a peak of 612°C. As the dwell time is increased, the peak temperatures decrease, with the 40 second dwell time, shown in Figure 2.9(e), reaching a level of only 455°C. The decrease in the temperature magnitude with increasing dwell time is expected, as longer dwell times permit greater heat loss to the environment through convection and radiation.

<table>
<thead>
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<td>Material</td>
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<td>Inconel® 625</td>
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</tr>
<tr>
<td>Dwell time (s)</td>
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<td>20</td>
<td>40</td>
<td>0</td>
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<tr>
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<td>460</td>
<td>405</td>
<td>515</td>
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<td>404</td>
</tr>
<tr>
<td>Thermocouple 2 (°C)</td>
<td>612</td>
<td>545</td>
<td>455</td>
<td>523</td>
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<td>Thermocouple 3 (°C)</td>
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<td>368</td>
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<td>400</td>
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</table>

### 2.4.2 Post-process Distortion and Residual Stress Measurements

Post-process measurements of the out-of-plane distortion experienced by each substrate plate have also been made. A larger out-of-plane distortion value indicates that the sample has experienced greater bowing distortion as a result of the deposition process. These out-of-plane distortion measurements are based on the CMM results obtained prior to and following the deposition on each substrate. Figure 2.10 provides a summary of these results. It is clear from this figure that the Ti-6Al-4V builds experience much lower levels of distortion than the Inconel® 625 builds, but the two materials also display different trends as a function of dwell time. In the Ti-6Al-4V builds, out-of-plane
distortion increases with dwell time, while the increase in dwell time appears to have a minimal impact on the distortion in the Inconel\textsuperscript{R} 625 builds. In addition to the

![Graph showing residual stress measurements](image)

**Fig. 2.10.** Final out-of-plane distortion calculated using the CMM results.

out-of-plane distortion measurements, residual stress measurements were made on the substrate for each Ti-6Al-4V and Inconel\textsuperscript{R} 625 build. The results of these measurements, shown in Figure 2.11, represent the magnitude of the measured stress in the longitudinal direction and correspond to the condition in which the surface stress is fully relieved. As observed in Figure 2.10 for the out-of-plane distortion, the measured residual stresses exhibit significantly different behavior for the Ti-6Al-4V and the Inconel\textsuperscript{R} 625 builds, with the Ti-6Al-4V builds displaying residual stress levels nearly an order of magnitude lower than than the Inconel\textsuperscript{R} 625 builds. With increasing dwell times, though, the two
material systems also display contradictory behavior, with the Ti-6Al-4V builds showing increasing residual stress levels with increasing dwell times and the Inconel® 625 builds displaying decreasing residual stress levels with increasing dwell times.

![Fig. 2.11. Residual stress measurements taken using the hole-drilling method.](image)

2.5 Discussion

2.5.1 Effect of Material and Dwell Time on in situ Distortion Measurements

The in situ distortion measurements allow for the distortion due to each deposition layer to be captured and show that the accumulation of distortion depends on both the dwell time and the material used. In order to evaluate the total distortion attributable
to the deposition of individual layers, the following relationship is used:

\[
%\text{distortion} = 100 \frac{\delta_j}{\delta_{\text{final}}} \tag{2.1}
\]

where, \(\delta_j\) is the distortion measured by the LDS after cooling following the deposition of layer \(j\), and \(\delta_{\text{final}}\) is the final distortion measured by the LDS.

Figure 2.12(a) and Figure 2.12(b) illustrate the accumulation of the total distortion attributable to the completion of individual layers for the Ti-6Al-4V and Inconel\textsuperscript{®} 625 builds, respectively. After layer 42, the build is complete and the total distortion will reach 100% for each case. While all of the Inconel\textsuperscript{®} 625 builds shown in Figure 2.12(b) show distortion accumulating at a near constant rate, regardless of the dwell time, over the course of the deposition, the same is not true for the Ti-6Al-4V builds. As shown in Figure 2.12(a), lowering or eliminating the dwell time impacts the resulting distortion for titanium, especially after the second layer of deposition, by not allowing for the distortion to develop for as long of a time period. Up until the second layer, the titanium builds undergo a majority of their distortion, regardless of the dwell time. For example, after the second layer, Case 1 with no dwell time experiences 55.4% of the total distortion, Case 2 with a 20 second dwell time experiences 57.2%, and Case 3 with a 40 second dwell time experiences 47.8% of the total distortion. After the completion of the second deposition layer in the Ti-6Al-4V builds, though, the distortion behavior changes as a function of dwell time. This behavior is especially true when no dwell time between layers is employed. In Case 1, with no dwell time, the distortion shows a significant decrease from a peak of approximately 54% at the completion of the
second layer through the completion of layer 16, with the overall distortion decreasing to a level of approximately 13%. In absolute terms, this corresponds to a decrease in measured distortion from 0.630 mm to 0.115 mm, representing an 81.7% decrease. After the completion of layer 16, the distortion begins to accumulate again, but does not reach the same magnitudes as observed for the 20 second and 40 second dwell time conditions. There are two likely causes for the observed phenomenon, the first being that at the high temperatures (around 600 °C) recrystallization results in a dynamic reduction of plastic strain and thus the annealing of residual stresses in the test sample. This effect has been studied by Qiao et al. [44] on SS304L stainless steel butt welds as well as by Elmer et al. [76] on Ti-6Al-4V samples. Another possibility is that Ti-6Al-4V undergoes a stress relaxation at temperatures reached during the AM process, which manifests itself through a significant reduction in accumulated distortion. This possibility was first proposed by Denlinger et al. [75] in a study on electron beam deposited Ti-6Al-4V.

When dwell times of 20 seconds and 40 seconds are utilized, the overall trends vary. With the addition of the 20 second dwell time, the distortion levels between layers 2 and 16 display more of a plateau in the level of distortion than the substantial decrease observed with no dwell time. In this 20 second dwell time condition, the overall distortion levels only fall from a level of approximately 57% at the completion of the second layer to a level of approximately 54% at the completion of layer 16, after which the distortion increases at a more consistent rate. This decrease in distortion is not present when a 40 second dwell time is used, with the distortion increasing at a consistent rate similar to all conditions observed with the Inconel® 625 builds in Figure 2.12(b). At the completion of layer 16 in this 40 second dwell time, the distortion has accumulated to approximately
73% of the total. These results indicate that decreasing dwell time significantly reduces distortion accumulation in Ti-6Al-4V.

2.5.2 Effect of Material and Dwell Time on Post-process Distortion and Residual Stress

The results obtained from the in situ distortion measurements are supported by the post-process CMM measurements made on each build substrate. These post-process measurements also highlight the impact of the dwell time and allow for the overall distortion of the different builds to be directly compared. As shown with the in situ measurements, the changes in the dwell time between deposition layers have different impacts on the distortion in Ti-6Al-4V and Inconel® 625 builds. Whereas the final distortion measured in the Ti-6Al-4V builds increases with increasing dwell time, the Inconel® 625 builds show little change. For example, an increase from no dwell time to 20 and 40 second dwell times in the Ti-6Al-4V builds leads to a 35.1% and a 54% increase in distortion, respectively. On the other hand, the Inconel® 625 samples display a 2.2% and a 4.4% decrease in distortion with dwell times of 20 and 40 seconds, respectively, when compared to no dwell time.

In addition to the impact on the post-process distortion measurements, the increases in dwell time also have a similar impact on the residual stress measurements in the substrate. In the case of the Ti-6Al-4V builds, the residual stress increases substantially with an increase in dwell time, with the residual stress increasing by 79.6% and 122.4% over the baseline no dwell condition for the 20 second and 40 second dwell times, respectively. On the other hand, the Inconel® 625 samples display much higher
Fig. 2.12. The percent of the total distortion attributed to the completion of individual deposition layers for (a) Ti-6Al-4V and (b) Inconel® 625.
residual stress levels overall, but they significantly decrease with increasing dwell times. For these builds, the measured residual stresses for the 20 second and 40 second dwell time conditions decrease by 12.4% and 23.5%, respectively, with respect to the baseline no dwell condition.

In general, these results represent deviations from commonly accepted means for reducing distortion and residual stress and amplify the differences which arise with the use of different material systems. Whereas the introduction of dwells during deposition to reduce distortion and residual stress are commonly used, these results show that this decision should be guided by the material being deposited. Even though Ti-6Al-4V and Inconel\textsuperscript{R} 625 are commonly used for AM deposition, they display widely different trends in the accumulation of distortion and residual stress in even simple builds, and the inclusion of a dwell time can have an adverse impact on the properties of the build.

These differences in behavior may be attributable to the differences in their structure at the temperatures experienced during laser-based deposition processes. For example, it is observed that Inconel\textsuperscript{R} 625 is a solid solution strengthened nickel base alloy that does not undergo any solid state allotropic transformations and maintains an fcc structure throughout its solid state temperature range. On the other hand, Ti-6Al-4V is a two phase ($\alpha + \beta$) titanium alloy that undergoes an allotropic phase transformation, where it transforms from a bcc to an hcp structure. On heating and cooling, the presence of these two phases across the solid state stability range introduces a number of different complexities, including a difference in the thermal expansion properties of the two phases which may create an annealing of the residual stress, as described by Elmer et al. [76]. These conditions are also impacted by the differences in the thermo-physical properties of
the different alloys, as evidenced by the differences in peak temperatures measured during the individual laser depositions. Such changes in the temperature of the substrate also play a role and should be considered when developing deposition processes for different alloys.

2.6 Conclusions

Understanding the manner in which distortion accumulates in a workpiece during AM is important for designing techniques to mitigate stress and distortion. A series of in situ and post-process distortion measurements have been performed on Ti-6Al-4V and Inconel® 625 laser deposited builds fabricated using the same heat input and with different inter-layer dwell times. These measurements have allowed the accumulation of distortion during individual passes to be captured and for the evolution of the distortion accumulation to be monitored over the entire build. Concurrent temperature measurements have also allowed the impact of changes in the temperature to be tracked and correlated with the observed distortion behavior. The results obtained in this work show that adding dwell time to allow for additional cooling during the deposition process results in reduced distortion and residual stress in Inconel® 625. The opposite is true of Ti-6Al-4V, where decreasing dwell time results in significantly lower residual stress and distortion levels. The conclusions can be summarized as follows:

1. When depositing Inconel® 625, the addition of longer dwell times results in the consistent accumulation of distortion with increasing layers. Similar behavior is also observed in the Ti-6Al-4V builds only at dwell times of 40 seconds.
2. On the other hand, the elimination of dwell time during the laser deposition of Ti-6Al-4V results in a decrease in distortion and much lower overall distortion levels in the builds. Decreasing distortion trends are most prevalent between layers 2 and 16, during which the overall measured distortion decreases from 0.630 mm to 0.115 mm, representing an 81.7% decrease in the accumulated distortion.

3. Concurrent in situ temperature measurements show that higher peak temperatures are obtained with no dwell time included in the deposition path planning, with the highest peak temperatures observed in the Ti-6Al-4V samples.

4. Post-process dimensional inspections show that the distortion exhibited by the Inconel® 625 builds is more than twice that of the Ti-6Al-4V builds for the same heat input and dwell time.

5. The corresponding post-process residual stress measurements show lower residual stresses in the Ti-6Al-4V builds, with the residual stress reaching a minimum when no dwell time is implemented. On the other hand, increasing dwell time results in a decrease in the measured residual stress of the Inconel® 625 builds.

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Chapter 3

**Effect of solid-state phase transformation in additive manufacturing process modeling**

### 3.1 Abstract

A method for modeling the effect of transformation strain at high temperatures during additive manufacturing processes is experimentally validated for Ti-6Al-4V samples subject to different inter-layer dwell times. The predicted mechanical responses are compared to those of Inconel® 625 samples, which experience no allotropic phase transformation, deposited under identical process conditions. The thermal response of workpieces in additive manufacturing is known to be strongly dependent on dwell time. In this work the dwell times used vary from 0 to 40 s. Based on past research on ferretic steels and the additive manufacturing of titanium alloys it is assumed that the effect of solid-state phase transformation in Ti-6Al-4V acts to oppose all other strain components, effectively rapidly eliminating all strain at temperatures above 690°C. The model predicts that Inconel® 625 exhibits increasing distortion with decreasing dwell times but that Ti-6Al-4V displays the opposite behavior, with distortion dramatically decreasing with lowering dwell time. These predictions are accurate when compared with experimental in situ and post-process measurements.

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1. The work presented in this chapter has been submitted for Publication: Denlinger, E R., Michaleris P., "Effect of solid-state phase transformation in additive manufacturing process modeling," Materials and Design (2015). In Review.
3.2 Introduction

Laser directed energy deposition is a frequently used additive manufacturing (AM) process. The system allows for parts to be built up by adding metal layer-by-layer rather than subtracting material through the application of typical machining techniques. This leads to increased flexibility in manufacturing which can in turn reduce costs. The process is also useful for the repair and modification of existing parts. Unfortunately AM processes result in large thermal gradients in the workpiece, causing the emergence of residual stresses and distortion. Finite element modeling (FEM) can be used to predict distortion and residual stress levels, allowing for optimization of the build plan prior to part manufacture and thus avoiding costly trial and error iterations. Residual stresses are caused by either plasticity due to contraction of the weld region or by solid-state phase transformations present in the material [25]. To accurately predict distortion and residual stress present in a workpiece, a model should account for both.

Modeling of AM has primarily focused on predicting thermal response [38,77–81], predicting distortion and residual stress, [3, 31, 39, 82, 83], and developing distortion mitigation techniques [40, 84]. Several researchers have performed thorough model validations using in situ experimental measurement techniques [31,37–39,75].

The use of FEM to predict distortion and residual stress in AM originates from the prior research performed on multi-pass welding. The inclusion of transformation strains caused by solid-state phase transformation present in steels has been shown to be critical in weld modeling due to the fact that transformation strains influence bulk distortion and residual stress of workpieces, with lower martensite-start temperature yielding reduced
levels of residual stress and distortion [21, 22]. Francis et al. have written a thorough review of welding residual stress present in steel and note that transformation strains present in welded steels can completely erase the strains caused by thermal contraction of the weld region [23]. The offsetting of strain attributable to thermal contraction by transformation strains alters the bulk residual stress distribution throughout a workpiece and has been shown to reduce bending stresses on the bottom of surface of base plates for bead-on-plate welds [24]. Because AM is a similar process to welding, the welding literature suggests that transformation strains also need to be included in AM models.

While the effect of transformation strains in steel is now well understood due to the prevalent use of the material in welding, the effect of transformation strains in Ti-6Al-4V, a material commonly used in AM, has not been extensively studied. Unlike steel which transforms from a FCC structure to a BCC structure upon cooling, 2 phase (α + β) Ti-6Al-4V undergoes an allotropic transformation from a BCC to a HCP structure. However, similarly to steel, the difference in coefficient of thermal expansion, volume, and hardness of the 2 Ti-6Al-4V phases alters the stress stress state of the material [76]. Denlinger et al. experimentally investigated the effect of the phase transformation on in situ distortion and residual stress in laser deposited Ti-6Al-4V [85], by comparing the mechanical responses of Ti-6Al-4V workpieces to the response of Inconel® 625, a material that does not undergo an allotropic phase change. Inter-layer dwell times were varied as previous work by Costa et al. concluded that microstructure was highly dependent on dwell times [36]. Shorter dwell times in the Ti-6Al-4V builds, which cause higher in-process temperatures to be reached and thus greater transformation to the β phase, were shown to result in dramatically reduced levels of distortion and residual
stress, a trend not seen in the Inconel® 625 builds. The results indicate that the transformations strains oppose the strain attributed to the contraction of the molten material, similar to what has been observed in steel. Currently no model presented in the literature has been validated to capture this effect.

This work serves to validate a method for introducing the effect of transformation strains present in Ti-6Al-4V into FE models of direct energy deposition, a full description of which is included in [75]. First, a transient thermal model is validated against in situ temperature measurements taken during the deposition of 42 layer high, 3 pass wide, Ti-6Al-4V and Inconel® 625 wall builds with varying inter-layer dwell times. Ti-6Al-4V is chosen due to the fact that it undergoes a solid-state phase transformation and thus will be subject to transformation strains. Inconel® 625 should be possible to accurately model without the inclusion of transformation strains, as it does not undergo a solid-state phase transformation. Next, the constitutive model neglecting transformation strains is used to model the mechanical responses of the materials and compared with experimental in situ distortion measurements and post-process residual stress measurements. The Ti-6Al-4V builds are then simulated while accounting for the transformation strains to investigate if improved correlation with experiment is achieved.

3.3 Lagrangian Modeling Approach

The thermal history is first calculated by performing a 3-dimensional transient thermal analysis. The thermal results are then input into a 3-dimensional quasi-static incremental analysis which simulates the mechanical response. The thermal analysis can be performed independently of the mechanical analysis because the plastic strain energy
is small compared to the laser source energy, making the analyses weakly coupled [28].

A detailed description of the model is available in reference [75].

### 3.3.1 Transient Thermal Analysis

The governing transient heat transfer energy balance in the entire volume of the material is given as:

\[
\rho C_p \frac{dT}{dt} = -\nabla \cdot q(r, t) + Q(r, t)
\]  

(3.1)

where \(\rho\) is the material density, \(C_p\) is the specific heat capacity, \(T\) is the temperature, \(t\) is the time, \(Q\) is the internal heat generation rate, \(r\) is the relative reference coordinate, and \(q\) is the heat flux. The Fourier heat flux constitutive relation is given by:

\[
q = -k\nabla T
\]

(3.2)

dependent on temperature dependent thermal conductivity \(k\).

The temperature dependent material properties for Ti-6Al-4V and Inconel\textsuperscript{®} 625 are listed in Table 3.1. The properties between those listed are obtained by linear interpolation over the temperature range. Properties at temperatures above and below those listed are taken as the nearest tabulated value. Density is a constant $4.43 \times 10^{-6}$ g/mm\textsuperscript{3} and $8.44 \times 10^{-6}$ g/mm\textsuperscript{3} for Ti-6Al-4V and Inconel\textsuperscript{®} 625, respectively.

Thermal radiation \(q_{rad}\) is accounted for using the Stefan-Boltzmann law:

\[
q_{rad} = \varepsilon \sigma (T_s^4 - T_\infty^4)
\]

(3.3)
Table 3.1. Temperature dependent thermal properties of Ti-6Al-4V [1] and Inconel\textsuperscript{625} [2]

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$k$ (W/m/°C)</th>
<th>$C$ (J/kg)</th>
<th>$k$ (W/m/°C)</th>
<th>$C$ (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>Inconel\textsuperscript{625}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>565</td>
<td>9.9</td>
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<td>-</td>
<td>-</td>
<td>25.2</td>
<td>645</td>
</tr>
</tbody>
</table>
where $\varepsilon$ is the surface emissivity, $\sigma$ is the Stefan-Boltzmann constant, $T_s$ is the surface temperature of the workpiece, and $T_\infty$ is the ambient temperature. The emissivity $\varepsilon$ of Ti-6Al-4V is set as 0.25, as experimentally determined by Yang [34]. The emissivity $\varepsilon$ of Inconel 625 is taken as 0.28 as found in reference [86]. The reference does state that the value is only an approximation, as the emissivity will vary with surface finish.

Newton’s law of cooling describes the heat loss due to convection $q_{\text{conv}}$:

$$q_{\text{conv}} = h(T_s - T_\infty)$$

(3.4)

here $h$ is taken as 18 W/m$^2$/°C by correlating simulated and experimental results for a metal deposition process.

### 3.3.2 Mechanical Analysis

The governing mechanical $\sigma$ equilibrium equation is written as:

$$\nabla \cdot \sigma = 0$$

(3.5)

The mechanical constitutive law is:

$$\sigma = C \epsilon_e$$

(3.6)

Total strain $\epsilon$, assuming small deformation thermo-elasto-plasticity, is decomposed as:

$$\epsilon = \epsilon_e + \epsilon_p + \epsilon_T + \epsilon_t$$

(3.7)
where \( \mathbf{C} \) is the fourth order material stiffness tensor, and \( \mathbf{e}_e, \mathbf{e}_p, \mathbf{e}_T, \) and \( \mathbf{e}_t \) are the elastic, plastic, thermal, and transformation strain, respectively.

For an incremental formulation, Equation (4.7) is re-written as:

\[
\mathbf{n}\sigma = \mathbf{n}^{-1}\sigma + \Delta \sigma
\]  
(3.8)

where \( \mathbf{n}^{-1}\sigma \) and \( \mathbf{n}\sigma \) are the stress and the previous and current increment, and \( \Delta \sigma \) is the stress increment computed as:

\[
\Delta \sigma = \Delta \mathbf{C}(\mathbf{n}^{-1}\mathbf{e}_e - \mathbf{n}^{-1}\mathbf{e}_p - \mathbf{n}^{-1}\mathbf{e}_T - \mathbf{n}^{-1}\mathbf{e}_t) + \mathbf{C}(\Delta \mathbf{e} - \Delta \mathbf{e}_p - \Delta \mathbf{e}_T - \Delta \mathbf{e}_t)
\]  
(3.9)

where left superscripts denote the time increment where a quantity is computed and \( \Delta \mathbf{e} \) is the total strain increment corresponding to the current displacement increment.

Stress relaxation present in Ti-6Al-4V at high temperatures is accounted for when the norm of temperature of an element’s Gauss points exceeds 690 °C. At this occurrence the following relation is enforced:

\[
\mathbf{n}^{-1}\mathbf{e}_t = -\left(\mathbf{n}^{-1}\mathbf{e}_e + \mathbf{n}^{-1}\mathbf{e}_p + \mathbf{n}^{-1}\mathbf{e}_T\right)
\]  
(3.10)

meaning that an assumption is being made that the transformation strains will completely negate the strain components that are attributable to the thermal contraction of the deposition region, a phenomenon that has been observed in the welding of steel, and an approximation previously shown to yield accurate distortion and residual stress results [75].
The temperature dependent mechanical properties for Ti-6Al-4V and Inconel 625, listed in Table 3.2, include the elastic modulus $E$, yield strength $\sigma_y$, and the coefficient of thermal expansion $\alpha$.

<table>
<thead>
<tr>
<th>$T$ ($^\circ$C)</th>
<th>$E$ (GPa)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\alpha$ ($\mu m/m^\circ C$)</th>
<th>$E$ (GPa)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\alpha$</th>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.2</td>
</tr>
</tbody>
</table>

3.4 Experimental Validation

The modeling approach laid out in Section 2 is applied to experimental results for Ti-6Al-4V and Inconel$^\text{®}$ 625 wall builds from reference [85] in an attempt to capture the mechanical responses reported. A brief overview of the experiment is provided here.

Single wall structures were fabricated on a 12.7 mm substrate of matching material using a laser-directed energy deposition process. The 2 materials chosen to compare were
Ti-6Al-4V and Inconel® 625 with the reasoning that Ti-6Al-4V undergoes an allotropic phase transformation and Inconel® does not. The experimental setup is shown in Figure 3.1. The setup allows 1 end of the substrate to be clamped and allows the free end of a cantilevered substrate to distort during the deposition. A laser displacement sensor (LDS) with a linear accuracy of $\pm 1\ \mu m$ was placed to record the in situ bowing distortion in the z-direction. Thermocouples with an accuracy of $\pm 0.75\%$ recorded the in situ thermal response of the workpiece. Figure 3.2(a) shows the locations of the thermocouples and LDS used for model validation.

![Experimental setup](image)

**Fig. 3.1.** Experimental setup

The depositions are performed at a laser power of 2 kW and a laser scan speed of 10.6 mm/s. The laser beam spot size was measured to be 4 mm in diameter at the
part surface. The laser penetration depth was found to be 1.1 mm by sectioning as recommended by Goldak [29].

Figure 3.2(b) shows the dimensions of each build. Each deposited 42-layer high, 3-bead wide, wall was 38.1 mm tall, 101.6 mm long, and 6.7 mm wide. The substrates used measured 152.4 mm long, 38.1 mm wide, and 12.7 mm thick. After the deposition of each layer a dwell time was added to allow for a period of cooling. Dwell times of 0, 20, and 40 seconds were used for each material to expose the parts to differing amounts of cooling time. Table 3.3 summarizes the cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Material</th>
<th>Dwell time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inconel® 625</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Inconel® 625</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
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<td>40</td>
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<td>Ti-6Al-4V</td>
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</tr>
<tr>
<td>5</td>
<td>Ti-6Al-4V</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>Ti-6Al-4V</td>
<td>40</td>
</tr>
</tbody>
</table>

In addition to the in situ distortion and temperature measurements, residual stress measurement were taken post-process. The hole drilling method was used to take a stress measurement on the substrate of each sample. Figure 3.2(a) shows the measurement location. The hole drilling method has and accuracy of ± 50 MPa.
(a) Locations of the thermocouple 1 (A), thermocouple 2 (B), LDS (C), and blind hole drilling (B) measurements.

(b) Dimensions of the substrate and wall.

Fig. 3.2. Illustration of the sample dimensions and measurement locations
3.5 Numerical Implementation

Figure 3.3 displays the 3-dimensional finite element mesh, generated in Patran 2012 by MSC, used for both the thermal and mechanical analyses. The same mesh is used for all cases. The mesh contains 52,472 Hex-8 elements and 62,231 nodes. Hex-8 elements were chosen because they have been shown to yield more accurate results than tetrahedral elements when plastic deformation occurs [87]. The elements for the deposited material are allotted as 2 per laser spot size radius and 1 per deposition thickness, making the elements $1 \times 1 \times 0.87$ mm$^3$ in volume. The mesh is coarsened as it moves away from the deposition. The fixture clamp is included in the model to capture heat loss through conduction.

Fig. 3.3. Finite element mesh used for all simulations.
3.5.1 Solution Method

The thermal and mechanical analyses are performed using the code CUBIC by Pan Computing LLC. The analyses are performed in a series of time steps with the current time step taking the solution at the previous time step as an initial condition. At each time step the discrete equilibrium equations are solved using the Newton-Raphson method.

3.5.2 Boundary Conditions

Convection and radiation are applied to all free surfaces of the mesh. The model substrate is mechanically constrained as cantilevered to represent the experimental conditions. The laser heat source is modeled using the Goldak double ellipsoid model as follows:

\[
Q = \frac{6\sqrt{3}P\eta}{abc\sqrt{\pi}} e^{-\left[\frac{3x^2}{a^2} + \frac{3y^2}{b^2} + \frac{3(z+vt)^2}{c^2}\right]}
\]

(3.11)

where \(P\) is the incident laser power, \(\eta\) is the absorption efficiency; \(x, y,\) and \(z\) are the local coordinates; \(a, b,\) and \(c\) are the transverse, depth, and longitudinal dimension of the ellipsoid respectively, \(v\) is the scan speed, and \(t\) is the time.

3.5.3 Material Deposition Modeling

A hybrid quiet-inactive element approach is used to simulate the addition of deposited material. The method was proposed by Michaleris in reference [27]. At the start of the simulation all deposited elements are inactive, i.e., not part of the analysis.
The elements are switched to quiet, meaning that they are given properties such that they do not affect the analysis, on a layer-by-layer basis. A quiet element is made active when the following condition is met at any Gauss point of the element:

\[
\frac{6\sqrt{3}}{abc\pi}\sqrt{\pi} e^{-\left[\frac{3x^2}{a^2} + \frac{3y^2}{b^2} + \frac{3(z+v) t^2}{c^2}\right]} \geq 0.05
\]  

Both surface radiation and convection are applied as a boundary condition to the evolving interface between quiet and active elements, which is identified at each time increment.

### 3.6 Modeling results and comparison to the experiment

#### 3.6.1 Thermal History

The thermal response of the workpiece is calculated by the model and compared to the experimental measurements. Figure 4.7 shows the experimental results, as measured by thermocouples 1 and 2, compared to calculated results at nodes corresponding to the thermocouple locations for each case. Reference [85] notes that because thermocouple 1 and 2 are at different locations on the substrate, they record different thermal histories. Thermocouple 1, at the free end of the substrate, shows a lower peak temperature than thermocouple 2, at the middle of the substrate. Samples with the same dwell times display similar thermal histories independent of the sample material. For each case the sample temperatures increase for roughly the first third of the build and then begin to decrease due to the heat source moving vertically away from the thermocouple locations. Longer dwell times results in lower peak temperatures with the 40 s dwell cases.
3 and Case 6) exceeding 400 °C and the 0 s dwell cases (Case 1 and Case 4) exceeding 600 °C.

The results of the transient thermal analyses are in close agreement with the experimental results as can be seen in Figure 3.4. The average percent error for the entire simulation of each case can be calculated as

$$\text{\% Error} = \frac{100 \sum_{i=1}^{n} \frac{(x_{\text{exp},i}) - (x_{\text{sim},i})}{x_{\text{exp},i}}}{n}$$

(3.13)

where $n$ is the total number of simulated time increments, $i$ is the current time increment, $x_{\text{sim}}$ is the simulated value, and $x_{\text{exp}}$ is the experimentally measured value. The largest error at any thermocouple is found to be 12.0 %. Table 3.4 shows the percent error at each thermocouple for all cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Material</th>
<th>Dwell time (s)</th>
<th>% Error TC 1</th>
<th>% Error TC 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inconel® 625</td>
<td>0</td>
<td>7.45</td>
<td>12.0</td>
</tr>
<tr>
<td>2</td>
<td>Inconel® 625</td>
<td>20</td>
<td>6.37</td>
<td>3.24</td>
</tr>
<tr>
<td>3</td>
<td>Inconel® 625</td>
<td>40</td>
<td>9.65</td>
<td>4.46</td>
</tr>
<tr>
<td>4</td>
<td>Ti-6Al-4V</td>
<td>0</td>
<td>2.20</td>
<td>1.48</td>
</tr>
<tr>
<td>5</td>
<td>Ti-6Al-4V</td>
<td>20</td>
<td>7.12</td>
<td>3.85</td>
</tr>
<tr>
<td>6</td>
<td>Ti-6Al-4V</td>
<td>40</td>
<td>9.14</td>
<td>3.14</td>
</tr>
</tbody>
</table>
Fig. 3.4. In situ thermal history of each thermocouple for all wall builds. (a) Case 1, (b) Case 4, (c) Case 2, (d) Case 5, (e) Case 3, (f) Case 6.
3.6.2 Distortion History

In situ distortion is calculated and compared to the experimental measurements of the distortion of 12.7 mm thick Inconel® and Ti-6Al4V substrates during laser deposition with dwell times ranging from 0 to 40 s. Figure 3.5 shows the final calculated distortion of Inconel® wall deposited with no dwell time (Case 1). The distortion is caused by the shrinking of the deposited material upon cooling. The LDS monitors the in situ distortion of the free end of the substrate. The simulated in situ distortion is recorded at a node corresponding to the LDS measurement location in the model.

Fig. 3.5. Mechanical model showing the final distorted shape of the workpiece (4 x magnification), Case 1.

3.6.2.1 Inconel® 625

Figure 3.6 presents the measured and simulated distortion for the Inconel® 625 builds with 0 (Case 1), 20 (Case 2), and 40 s (Case 3) dwell times. Periods of decreasing distortion are caused by the thermal expansion of the top of the substrate due to
heating from the laser. An increase in distortion is attributable to the shrinking of the deposited material upon cooling. Each of the Inconel $\textsuperscript{R} 625$ experimental builds displays a consistent accumulation of distortion over the duration of the build which occurs independent of dwell time. The simulated distortion of the model captures these trends and is in close agreement with the experiment throughout the deposition for each case.

3.6.2.2 Ti-6Al-4V

Figure 3.7 shows the calculated distortion time trace for the Ti-6Al-4V samples, both with and without the inclusion of transformation strain, compared with the experimental measurements for dwell times of 0 (Case 4), 20 (Case 5), and 40 s (Case 6). The simulations that employ the constitutive model with no transformation strain show distortion accumulating consistently throughout the deposition, similar to the trend seen in the Inconel $\textsuperscript{R}$ builds, but with significantly higher levels of distortion. This prediction is at odds with the experimental measurements which show significantly lower distortion levels in the Ti-6Al-4V builds when compared with the Inconel $\textsuperscript{R}$ builds. The fact that the model predicts a consistent accumulation of distortion with respect to time is also not in agreement with the measurements, which in the Case of the 0 and 20 s dwell cases show a reduction of distortion accumulation following the initial deposited layers. This reduction in distortion levels occurs due to the fact that the relaxation of residual stress, caused by the transformation strain negating the strain attributed to the contraction of the deposited material, manifests itself as a reduction in accumulated distortion [75].
Fig. 3.6. Calculated in situ distortion results for the Inconel® 625 samples compared with the experimental data.
In addition to not capturing the trend of distortion accumulation, the constitutive model neglecting transformation strain also significantly over predicts the final distortion of the workpiece. For example in Case 4 (0 s dwell), the calculated distortion of 6.39 mm over predicts the measured final distortion of 1.14 mm by 461%. This percent error is similar to that reported in reference [75] when a constitutive model without stress relaxation was used to model the distortion of Ti-6Al-4V deposited by an electron beam.

Unlike the constitutive model that neglects transformation strain, the simulations using the constitutive model that includes the transformation strain term show that in the 0 s dwell case, Ti-6Al-4V does not consistently accumulate distortion over the duration of the build. The model actually registers a reduction of distortion after the initial layers, shown in Figure 3.8, followed by little accumulation over the rest of the build. This effect is lessened when adding a 20 s dwell time and is not present at all when a 40 s dwell is added, showing consistent distortion accumulation throughout the build. These predictions, showing a strong dependence of distortion on dwell time, are in close agreement with the experimental observations.

3.6.3 Residual Stress

Residual stress values are calculated for all time-steps during the simulation. The final calculated value for residual stress at the location of the blind hole drilling measurement is compared to the experimental measurement for the Inconel® 625 and Ti-6Al-4V substrates with dwell times ranging from 0 to 40 s.
Fig. 3.7. Calculated in situ distortion results for the Ti-6Al-4V samples compared with the experimental data.
(a) Calculated in situ distortion results for Case 4 (Ti-6Al-4V 0 s dwell time, compared with experiment for the first 15 deposition layers.

(b) Calculated in situ distortion results for Case 5 (Ti-6Al-4V 20 s dwell time, compared with experiment for the first 15 deposition layers.

Fig. 3.8. Calculated in situ distortion results for the Ti-6Al-4V samples compared with the experimental data showing the loss in accumulated distortion.
3.6.3.1 **Inconel® 625**

The residual stress predictions from the model are compared with the results from the blind hole drilling experiments in Figure 3.9. The model predicts that residual stress in the Inconel® 625 exhibits nearly no dependence on dwell time, showing a post-process residual stress of just over 500 MPa. The experiment shows decreasing residual stress with increasing dwell time, a trend not captured by the model likely due to the fact that temperature dependent precipitation hardening present in Inconel® 625 is not included in the model.

![Graph showing calculated residual stress results for Inconel® 625 samples compared with experimental data.](image)

**Fig. 3.9.** Calculated residual stress results for the Inconel® 625 samples compared with the experimental data.
3.6.3.2 Ti-6Al-4V

Figure 3.10 shows the residual stress results for the model when including transformation strain and neglecting transformation strain compared with the experimental measurements. The experimental measurements show that residual stress in the Ti-6Al-4V increases with increasing dwell time. While the model neglecting transformation strain captures this trend, it over predicts the residual stress magnitude by over 500% in all cases. When transformation strain is incorporated into the model the residual stress results are under predicted in each case, however the the calculations are all within 22% of the measurement error bars and thus represent a significant reduction in the percent error when compared with the experiment. The under prediction of the residual stress is an expected consequence of implementing the relaxation of residual stresses instantaneously. In reality the stress relaxation likely takes place over time. Further experimental work would need to be performed to incorporate a dynamic stress relaxation into the model and improve correlation with the experiments.

3.7 Conclusions

Being able to predict the mechanical response of AM workpieces is important if expensive trial and error iterations are to be avoided. A model has been validated to adequately account for the transformation strain present in Ti-6Al-4V, a common alloy used in direct metal deposition. The model was validated by varying inter-layer dwell times, which are known to impact sample temperature and thus phase transformation effects, for Ti-6Al-4V and Inconel® 625 wall builds. The in situ temperature,
Fig. 3.10. Calculated residual stress results for the Ti-6Al-4V samples compared with the experimental data.

in situ distortion, and post-process residual stress predictions were compared with experimental measurements. The Inconel\textsuperscript{®} 625 builds can be adequately modeled without the inclusion of transformation strain, as the alloy does not undergo a solid-state phase transformation. However, for Ti-6Al-4V which does undergo solid-state phase transformation, neglecting the effect of transformation strain incurs errors of over 500\% when predicting post-process residual stress and distortion, and also does not capture the distortion accumulation trends observed during the in situ measurements. Modeling that the transformation strain caused by solid-state phase transformation in Ti-6Al-4V acts to oppose all other strain components and effectively eliminates all strain at temperatures above 690\degree C, leads to close agreement with experimental measurements. This modeling effort highlights the need for future work focusing on further development of a high-temperature constitutive model for Ti-6Al-4V. High
temperature Gleeble testing could provide insight into the dynamic nature of the strain relaxation noted in Ti-6Al-4V. Better understanding of the relationship between temperature, microstructure, and material properties is needed to develop a more physically realistic constitutive model.

3.8 Acknowledgements

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Chapter 4

Residual stress and distortion modeling
of Electron Beam Direct Manufacturing Ti-6Al-4V

4.1 Abstract

In this work, a finite element model is developed for predicting the thermo-mechanical response of Ti-6Al-4V during electron beam deposition. A three dimensional thermo-elasto-plastic analysis is performed to model distortion and residual stress in the workpiece and experimental in situ temperature and distortion measurements are performed during the deposition of a single bead wide, 16 layer high wall build for model validation. Post-process Blind Hole Drilling residual stress measurements are also performed. Both the in situ distortion and post-process residual stress measurements suggest that stress relaxation occurs during the deposition of Ti-6Al-4V. A method of accounting for such stress relaxation in thermo-elasto-plastic simulations is proposed where both stress and plastic strain is reset to zero, when the temperature exceeds a prescribed stress relaxation temperature. Inverse simulation is used to determine the values of the absorption efficiency and the emissivity of electron beam deposited wire-fed Ti-6Al-4V, as well as the appropriate stress relaxation temperature.

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1 The work presented in this chapter has been Published: Denlinger, Erik R., Jarred C. Heigel, and Panagiotis Michaleris. "Residual stress and distortion modeling of electron beam direct manufacturing Ti-6Al-4V." Journal of Engineering Manufacture (2014): 0954405414539494.
4.2 Introduction

Additive Manufacturing (AM) has seen increased attention in recent years due to the ability of the process to produce near-net shape parts directly from CAD files without the retooling cost associated with casting or forging. The electron beam deposition process is of particular interest to the aerospace industry due to its ability to deposit large amounts of feedstock material at rapid rates. The process involves melting metal wire onto a substrate, and allowing for it to cool and form a fully dense geometry on a layer by layer basis. Unfortunately, large thermal gradients during the deposition process result in undesirable distortion and residual stress. Modifications to the build plan may reduce distortion and residual stress. To optimize the build plan without the expensive trial and error iterations an accurate predictive model is needed.

Numerical modeling for the prediction of temperature, distortion, and residual stress caused by the additive manufacturing process is similar to that of multi-pass welding. Weld modeling has been an active area of research for nearly 4 decades. Several weld models have been used to predict thermal and mechanical behavior [7–13]. Other modeling work has focused on material phase change during welding [19, 20]. Lindgren has written detailed summaries on the development of weld model complexity [14], material modeling in welding [15], and improvements in computational efficiency for weld modeling [16]. In more recent weld model work, Michaleris et al. focused on predicting distortion modes caused by welding, as well as residual stress [18].

AM modeling adds significant computational cost when compared to multi-pass welding due to the increased amount of deposited material, passes, and process time. The
addition of material into the simulation requires an element activation strategy [27]. Like weld models, AM models require accurate process parameters to yield acceptable results. The parameters of particular interest are absorption efficiency and surface emissivity, as these parameters determine the energy entering and exiting the system during the process. Yang et al. experimentally determined the absorption efficiency and surface emissivity of deposited Ti-6Al-4V using a laser assisted machining process [34]. Shen and Chou modeled the efficiency of a powder based electron beam system as 0.90 and assumed the emissivity of Ti-6Al-4V to be a constant 0.70, resulting in close agreement with experimental results [79]. No available literature provides values of efficiency and emissivity for a wire-fed electron beam system. Over the past decade significant work has been performed to model AM [3,31,32,39].

Some researchers have focused on material phase change caused by AM such as predicting the resulting microstructure of deposited Ti-6Al-4V [38,78,88–92]. Additional work has focused on predicting stress and distortion in materials that undergo phase transformation [41]. Ghosh and Choi concluded that simulated distortion results are significantly affected by failing to properly take into account the microstructural changes present in the deposited material [33]. Griffith et al. showed that the high temperatures reached during the laser deposition of stainless steel 316 can cause the material to anneal, thus reducing the measured residual stress [42]. Song et al. accounted for the plastic strain relaxation in a deposited nickel based alloy by setting a threshold temperature that, when surpassed, sets the plastic strain value to zero [43]. Qiao et al. performed micro-hardness measurements to demonstrate that the equivalent plastic strain needs to be dynamically adjusted at high temperatures in thermo-elasto-plastic models to
simulate material annealing [44]. An approach for managing the stress relaxation in AM deposited Ti-6Al-4V has not yet been presented.

The objective of the present work is to develop a finite element model for predicting the in situ thermo-mechanical response of a Ti-6Al-4V workpiece deposited using a wire-fed electron beam system. Workpiece distortion and residual stress is modeled using a three dimensional thermo-elasto-plastic analysis. The model is validated using experimental in situ temperature and distortion measurements performed during deposition of a 16 layer high, single bead wide, wall build as well as post-process residual stress measurements taken using Blind Hole Drilling. Both the in situ distortion and post-process residual stress measurements suggest that stress relaxation occurs in Ti-6Al-4V during deposition. The thermo-elasto-plastic model presented accounts for the observed stress relaxation by resetting both stress and plastic strain to zero when the temperature exceeds a prescribed stress relaxation temperature. The absorption efficiency, emissivity, and stress relaxation temperature are determined by applying inverse simulation.

4.3 Electron Beam Deposition Simulation

The thermal and mechanical histories are determined by performing a three dimensional transient thermal analysis and a three dimensional quasi-static incremental analysis, respectively. The thermal and mechanical analyses are performed independently and are weakly coupled, meaning that the mechanical response has no effect on the thermal history of the workpiece [28].
4.3.1 Thermal Analysis

The governing heat transfer energy balance is written as:

\[ \rho C_p \frac{dT}{dt} = -\nabla \cdot q(r, t) + Q(r, t) \]  

(4.1)

where \( \rho \) is the material density, \( C_p \) is the specific heat capacity, \( T \) is the temperature, \( t \) is the time, \( Q \) is the heat source, \( r \) is the relative reference coordinate, and \( q \) is the heat flux vector, calculated as:

\[ q = -k \nabla T \]  

(4.2)

Table 4.1 lists the values of all temperature dependent material properties (thermal conductivity \( k \) and specific heat capacity \( C_p \)) for Ti-6Al-4V. Material properties at temperatures between those listed are obtained by linear interpolation over the temperature range. The material thermal properties above 800\(^\circ\)C are assumed to be constant. The density is a constant \( 4.43 \times 10^{-6} \text{ g/mm}^3 \). The electron beam heat source is modeled using the Goldak double ellipsoid model as follows:

\[ Q = \frac{6\sqrt{3}P\eta f_s}{abc\pi\sqrt{\pi}} e^{-\left[\frac{3\pi}{2a^2} + \frac{3\pi}{2b^2} + \frac{3\pi}{4c^2}\right] t^2} \]  

(4.3)

where \( P \) is the power, \( \eta \) is the absorption efficiency, \( f_s \) is the process scaling factor; \( x \), \( y \), and \( z \) are the local coordinates; \( a \), \( b \), and \( c \) are the transverse, melt pool depth, and longitudinal dimensions of the ellipsoid respectively, \( v_w \) is the heat source travel speed, and \( t \) is the time. The heat source is circular, making \( a = c = 6.35 \text{ mm} \), and \( f_s = 1 \).
Table 4.1. Temperature dependent thermal properties of Ti-6Al-4V [1]

<table>
<thead>
<tr>
<th>$T$ ($^\circ$C)</th>
<th>$k$ (W/m/$^\circ$C)</th>
<th>$C$ (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.6</td>
<td>565</td>
</tr>
<tr>
<td>20</td>
<td>6.6</td>
<td>565</td>
</tr>
<tr>
<td>93</td>
<td>7.3</td>
<td>565</td>
</tr>
<tr>
<td>205</td>
<td>9.1</td>
<td>574</td>
</tr>
<tr>
<td>315</td>
<td>10.6</td>
<td>603</td>
</tr>
<tr>
<td>425</td>
<td>12.6</td>
<td>649</td>
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<td>760</td>
<td>17.5</td>
<td>858</td>
</tr>
<tr>
<td>870</td>
<td>17.5</td>
<td>959</td>
</tr>
</tbody>
</table>

The penetration depth $b$ of the heat source is found to be 3.81 mm by cross-sectioning a deposited sample, as recommend by Goldak in reference [29]. Velocity is a constant 12.7 mm/s. Power is entered as the average power of each individual pass during the deposition. The efficiency $\eta$ is initially unknown and is calibrated based on experimental results.

Radiation is applied to all free surfaces, including those of the newly deposited material, using the Stefan-Boltzmann law:

$$ q_{rad} = \varepsilon \sigma (T_s^4 - T_\infty^4) $$

where $\varepsilon$ is the surface emissivity, $\sigma$ is the Stefan-Boltzmann constant, $T_s$ is the surface temperature of the workpiece, and $T_\infty$ is the ambient temperature (25°C). The efficiency $\varepsilon$ is unknown, and determined by correlating computed and experimentally measured
temperatures. Due to the deposition taking place in a vacuum there is no surface convection.

When the electron beam is on, the magnitude of the time increments is calculated such that the beam travels a distance equal to its radius as follows:

\[ t^i = t^{i-1} + \frac{r_i}{v_w} \]  \hspace{1cm} (4.5)

where, \( t \) is the time, \( i \) is the current increment number, and \( r \) is the modeled electron beam radius. When the electron beam is off, time steps are allowed to increase to magnitudes as large as 10 seconds.

### 4.3.2 Mechanical Analysis

A thermal history dependent quasi-static mechanical analysis is performed to obtain the mechanical response of the workpiece during deposition. The results of the thermal analysis are imported as a thermal load into the mechanical analysis. The governing stress equilibrium equation is:

\[ \nabla \cdot \sigma = 0 \]  \hspace{1cm} (4.6)

where \( \sigma \) is the stress. The mechanical constitutive law is:

\[ \sigma = C \epsilon_e \]  \hspace{1cm} (4.7)

\[ \epsilon = \epsilon_e + \epsilon_p + \epsilon_T \]  \hspace{1cm} (4.8)
where $C$ is the fourth order material stiffness tensor, and $\epsilon^e$, $\epsilon^p$, and $\epsilon^T$ are the total elastic strain, plastic strain, and thermal strain, respectively. The thermal strain is computed as:

\begin{align*}
\epsilon^T &= \epsilon^T_j j \\
\epsilon^T &= \alpha (T - T_{\text{ref}}) \\
\dot{j} &= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}^T
\end{align*}

where $\alpha$ is the thermal expansion coefficient and $T_{\text{ref}}$ is reference temperature. The plastic strain is computed by enforcing the von Mises yield criterion and the Prandtl-Reuss flow rule:

\begin{align*}
f &= \sigma_m - \sigma_Y(\epsilon^q, T) \leq 0 \\
\dot{\epsilon}^p &= \dot{\epsilon}^a \qquad (4.13) \\
a &= \left( \frac{\partial f}{\partial \sigma} \right)^T \qquad (4.14)
\end{align*}

where $f$ is the yield function, $\sigma_m$ is Mises' stress, $\sigma_Y$ yield stress, $\epsilon^q$ is the equivalent plastic strain, and $a$ is the flow vector.

For an incremental formulation, Equation (4.7) is re-written as:

\begin{equation}
\sigma^n = \sigma^{n-1} + \Delta \sigma
\end{equation}
where \( n^{-1} \sigma \) and \( n \sigma \) are the stress and the previous and current increment, and \( \Delta \sigma \) is the stress increment computed as:

\[
\Delta \sigma = \Delta C(n^{-1} e - n^{-1} \epsilon_p - n^{-1} \epsilon_T) + C(\Delta \epsilon - \Delta \epsilon_p - \Delta \epsilon_T)
\] (4.16)

where left superscripts denote the time increment where a quantity is computed and \( \Delta \epsilon \) is the total strain increment corresponding to the current displacement increment.

Incorporation of annealing in the constitutive system involves re-setting the plastic strain \( \epsilon_p \) to zero when the annealing temperature is reached [43]. Alternatively, Qiao et al. proposed a gradual (dynamic) reduction of the equivalent plastic strain \( \epsilon_q \) based on the time duration that the material is exposed at high temperatures [44]. As discussed in further detail in sections 4.6.2 and 4.6.3 a relaxation mechanism is present during the deposition of Ti-6Al-4V manifested by the reduction of distortion recorded by \textit{in situ} distortion measurements and reduced residual stress as measured by post-process Blind Hole Drilling. Implementation of such annealing models did not result into significant improvement in the correlation of computed results with \textit{in situ} displacement and post-process residual stress measurements. The deviation was in the order of 500%.

A stress relaxation is proposed in this work where instantaneous annealing and creep occurs when a stress relaxation temperature \( T_{\text{relax}} \) is reached. The relaxation is implemented by re-setting the stress \( n^{-1} \sigma \), elastic strain \( n^{-1} \epsilon_e \), thermal strain \( n^{-1} \epsilon_T \), plastic strain \( n^{-1} \epsilon_p \), and equivalent plastic strain \( n^{-1} \epsilon_q \) at the previous time increment to zero when the norm of the temperature at all Gauss points of an element
exceeds $T_{\text{relax}}$. The material response corresponding to this relaxation model for various relaxation temperatures are presented in section 4.6.

Table 4.2 displays the temperature dependent mechanical properties of Ti-6Al-4V, including the elastic modulus $E$, the yield strength $\sigma_y$, and the coefficient of thermal expansion $\alpha$. Material properties between the temperatures listed are linearly interpolated. Mechanical properties are assumed constant above 800°C. The Poison’s ratio $\nu$ is assumed to be a constant value of 0.34. The model assumes perfect plasticity, i.e., no material hardening occurs.

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$E$ (GPa)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\alpha$ ($\mu$m /m°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>105.00</td>
<td>777.15</td>
<td>8.60</td>
</tr>
<tr>
<td>20</td>
<td>103.95</td>
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<td>78.63</td>
<td>552.15</td>
<td>9.70</td>
</tr>
<tr>
<td>800</td>
<td>62.80</td>
<td>417.15</td>
<td>9.70</td>
</tr>
</tbody>
</table>

4.4 Calibration and Validation

4.4.1 Deposition Process

Electron beam freeform fabrication is used to deposit Ti-6Al-4V in a vacuum chamber. A Ti-6Al-4V plate 254 mm long, 101.6 mm wide, and 12.7 mm thick is
used as a substrate. The substrate is clamped at one end and cantilevered, allowing the unconstrained end to deflect freely, while monitored by a laser displacement sensor (LDS). Figure 4.1 shows the test fixture and constrained substrate after deposition.

The AM system used is the Sciaky VX-300, which welds in the range of $10^{-4}$ to $10^{-5}$ Torr. The work envelope is approximately $5.8 \times 1.2 \times 1.2$ m$^3$ in volume. The deposition process begins by preheating the portion of the substrate where metal will be deposited. Figure 4.2 shows the preheating path. The electron beam power is set to 4.4 kW and scan speeds range between 35 mm/s and 42 mm/s.

After heating the top surface of the substrate a one-bead wide deposition is built 16 layers high and 203.2 mm long. The wire feed rate is set to 50.8 mm/s. The scan speed is a constant 12.7 mm/s. The electron beam operates at a power varying between
Fig. 4.2. Scan pattern of the preheat performed on the top of the substrate.

8 and 10 kW. The power is fluctuated to control the melt pool size. Figure 4.1 shows the resulting deposited geometry. The sloped wall is approximated as being 47.625 mm high near the clamped end of the substrate and 28.6 mm high near the free end for the succeeding finite element modeling work. Table 4.3 shows, for each deposition layer, the average power and start time, as well as the beginning and ending z-coordinates.

4.4.2 In situ Distortion and Temperature

In situ distortion measurements are taken using a Micro-Epsilon LDS, model LLT 28x0-100 and Micro-Epsilon controller scan CONTROL 28x0. The LDS is positioned to measure the longitudinal bowing distortion mode in the z-direction at the free end of the substrate, as shown in Figure 4.3(b). The LDS targets a point approximately 6.3 mm from the free end of the substrate. A National Instruments 9250 module reads the LDS analog voltage signal.

Figure 4.3 shows the locations of the 4 thermocouples (0.25 mm diameter) used to monitor in situ temperature. Thermocouples 1-3 measure the temperature on the bottom of the substrate, parallel to the axis of deposition, and are located 63.5 mm from
Table 4.3. Varying electron beam process and path parameters

<table>
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<tr>
<th>Pass number</th>
<th>Power (kW)</th>
<th>Start time (s)</th>
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<td>2.9766</td>
<td>1.7875</td>
</tr>
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<td>216.02</td>
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<td>3.5750</td>
</tr>
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<td>8.9297</td>
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</tr>
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<td>7.1500</td>
</tr>
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<td>8.6</td>
<td>535.33</td>
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<td>8.9375</td>
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<td>8.6</td>
<td>624.49</td>
<td>17.8594</td>
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<td>7</td>
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<td>12.5125</td>
</tr>
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<td>8.4</td>
<td>2188.92</td>
<td>47.6250</td>
<td>28.6000</td>
</tr>
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</table>
the clamped end of the substrate, at the center of the substrate, and 6.35 mm from the free end of the substrate, respectively. Thermocouple 4 measures temperature along the axis of deposition, on the top edge, at the free end of the substrate. The thermocouples are placed to allow for temperature readings at various substrate locations, without interfering with the LDS. A National Instruments 9213 module reads the thermocouple analog voltage signals. Both National Instruments modules record data into LabView at a frequency of 20 Hz.

Fig. 4.3. Schematic showing the LDS measurement location and the thermocouple (T/C) locations on the (a) top of the substrate and (b) the bottom of the substrate.

4.4.3 Residual Stress

Post-process residual stress is measured using the hole-drilling method. Eight residual stress measurements are taken, seven measurements on the bottom of the substrate along the axis of deposition (see Figure 4.4(a)), and one measurement on the deposited material (see Figure 4.4(b)). The majority of the measurements are taken
on the substrate, as it provides a large smooth surface appropriate for applying strain gauges and placing the milling guide. Micro-Measurements\textsuperscript{\textregistered} strain gauges, model EA-06-062RE-120, are bonded to the bottom center of the substrate. Gauges are calibrated using the procedure described in manufacturer engineering data sheet U059-07 and technical note 503. The ASTM E837 drilling process is followed. Incremental drilling is done using RS-200 Milling Guide and high speed drill from Micro-Measurements\textsuperscript{\textregistered}. A 15.2 mm diameter, carbide-tipped, Type II Class 4A drill bit was used. Strain measurements are read by a Micro-Measurements\textsuperscript{\textregistered} P-3500 Strain Indicator. Bridges are balanced with a Micro-Measurements\textsuperscript{\textregistered} Switch and Balance Unit, model SB-1.

4.5 Numerical Implementation

Figure 4.5 displays the 3-D finite element mesh used for both the thermal and mechanical analysis. The mesh contains 6848 Hex-8 elements and 9405 nodes. The mesh is generated using Patran 2012 by MSC. The mesh allows one element per deposition thickness and one element per heat source radius. A mesh convergence study was performed using two and then four elements per heat source radius resulting in 1.47\% and 7.17\% peak change respectively in the computed temperatures at the nodes corresponding to thermocouples and a 1.02\% and 2.85\% average change respectively at the nodes corresponding to thermocouples. However, the mesh with one element per heat source radius is used in this work because computational efficiency is critical in modeling electron beam deposition. The thermal and mechanical analyses are performed using the code CUBIC by Pan Computing LLC [27]. The quiet element activation approach is used in this work, where elements representing the metal deposition regions are present from
Fig. 4.4. Schematic showing the locations of the residual stress measurements (a) on the bottom of the substrate and (b) on the build portion.
the start of the analysis. However, they are assigned properties so that they do not affect the analysis. For the heat transfer analysis, the thermal conductivity $k$ is set to a lower value to minimize conduction into the quiet elements, and the specific heat $C_p$ is set to a lower value to adjust energy transfer to the quiet elements [27]:

$$k_{\text{quiet}} = s_k k$$

$$C_p^{\text{quiet}} = s_{C_p} C_p$$

(4.17) (4.18)

where, $k_{\text{quiet}}$ and $C_p^{\text{quiet}}$ are the thermal conductivity and the specific heat used for quiet elements, and $s_k = 10^{-6}$ and $s_{C_p} = 10^{-2}$ are the respective scaling factors. When the heat source contacts any Gauss point of an element, the element is activated by switching $s_k$ and $s_{C_p}$ to 1. The initial temperature of the element is also reset to the ambient temperature to minimize erroneous heating of the element due to the activation. For each time increment, the continuously evolving surface between active and quiet elements is identified, and surface radiation is applied as a boundary condition.

For the mechanical analysis, the quiet elements are assigned a lower elastic modulus:

$$E_{\text{quiet}} = s_E E$$

(4.19)

where, $s_E = 10^{-4}$. 
Figure 4.6 illustrates the mechanical constraints applied to the model. Three corner nodes on the clamped end of the substrate are constrained in all translational directions to model clamping the end of the substrate.

The numerical model is calibrated, using inverse simulation, as in reference [35], to determine the unknown values of efficiency $\eta$ in Equation (4.3), surface emissivity $\varepsilon$ in Equation (4.4), and stress relaxation temperature $T_{\text{relax}}$.

Efficiencies are varied from 0.90 [79] to 0.95 [93], based on the available literature. The emissivity is varied from 0.44 to 0.69 [94]. The percent error is calculated as:

$$\% \text{ Error} = \frac{100 \sum_{i=1}^{n} \left| \frac{x_{\text{exp}} \ i}{x_{\text{sim}} \ i} \right|}{n}$$

(4.20)

where $n$ is the total number of simulated time increments, $i$ is the current time increment, $x_{\text{sim}}$ is the simulated value, and $x_{\text{exp}}$ is the experimentally measured value. The
Fig. 4.6. Mechanical constraints used to fix the clamped end of the substrate.

The results from the calibrated thermal model are imported as a thermal load into the mechanical simulation. The mechanical model is calibrated by adjusting the relaxation temperature \( T_{relax} \). Temperatures between 600 °C (below the beginning of the alpha-beta phase transition [76]) and 980°C (approaching the beta-transus of Ti-6Al-4V) are tested. The error is calculated using Equation (4.20).

4.6 Results and Discussion

4.6.1 Thermal History

Table 4.4 shows the results of the simulated cases used to calibrate the thermal model. The efficiency \( \eta \) and emissivity \( \varepsilon \) values are found to be 0.90 and 0.54, respectively, as this combination results in the lowest percent error (7.7%) compared with experiment. The calibrated value for efficiency agrees with that used for the electron beam deposition
in reference [79]. The calibrated emissivity value falls within the range experimentally established by Coppa [94].

Figure 4.7 displays the thermal histories experimentally measured by the 4 thermocouples compared with the simulation results for the calibrated process efficiency of $\eta = 0.90$ and emissivity of $\varepsilon = 0.54$ at nodes corresponding to the locations of the thermocouples. A process time of 0 minutes corresponds to the start of the preheat. The ambient temperature is approximately $25^\circ$C. Thermocouple 2 (located at the middle of the bottom of the substrate) records the highest temperature. Thermocouple 3 (located on the bottom of the substrate near the clamped end) records the lowest temperature, as it is nearest to the clamp which absorbs heat through conduction. Although the process efficiency $\eta$ and emissivity $\varepsilon$ are calibrated to match the thermocouple measurements, a good correlation (7.7% error) is achieved for the entire duration of the process. In addition, thermocouples 1, 3, and 4, which are not used for calibration, also display close correlation with simulated results.

### 4.6.2 Distortion History

Table 4.5 lists the results of the simulated cases run to investigate the effect of changing the relaxation temperature on the in situ distortion. A final error is also computed by comparing the post-process experimental distortion with the post-process simulation distortion. A stress relaxation temperature of $690^\circ$C provides results in closest agreement with the experimental post-process distortion.

Figure 4.8 illustrates the computed model distortion at the LDS point for selected relaxation temperatures. As seen in the figure, no relaxation (case 26) results in a
Table 4.4. Cases examined for thermal model calibration

<table>
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<tr>
<th>Case</th>
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<th>ε</th>
<th>% Error</th>
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</tr>
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<td>0.64</td>
<td>9.5</td>
</tr>
<tr>
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<td>0.69</td>
<td>11.1</td>
</tr>
<tr>
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<td>0.49</td>
<td>10.5</td>
</tr>
<tr>
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<td>0.54</td>
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<td>0.925</td>
<td>0.59</td>
<td>8.9</td>
</tr>
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</tr>
<tr>
<td>18</td>
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<td>0.69</td>
<td>11.5</td>
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</table>

Table 4.5. Cases examined for mechanical model calibration

<table>
<thead>
<tr>
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<th>$T_{relax}$ (°C)</th>
<th>% Error</th>
<th>Final % Error</th>
</tr>
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<td>600</td>
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<td>45.3</td>
</tr>
<tr>
<td>20</td>
<td>650</td>
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<td>23.2</td>
</tr>
<tr>
<td>21</td>
<td>670</td>
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<td>12.9</td>
</tr>
<tr>
<td>22</td>
<td>680</td>
<td>4.7</td>
<td>8.0</td>
</tr>
<tr>
<td>23</td>
<td>690</td>
<td>7.4</td>
<td>2.2</td>
</tr>
<tr>
<td>24</td>
<td>700</td>
<td>14.3</td>
<td>4.4</td>
</tr>
<tr>
<td>25</td>
<td>980</td>
<td>144.3</td>
<td>157.5</td>
</tr>
<tr>
<td>26</td>
<td>None</td>
<td>431.0</td>
<td>520.3</td>
</tr>
</tbody>
</table>
(a) Thermocouple 1
(b) Thermocouple 2
(c) Thermocouple 3
(d) Thermocouple 4

Fig. 4.7. Computed thermal history with $\eta = 0.90$ $\varepsilon = 0.54$ compared with the experimental measurements.
nearly linear increase of distortion with time up to 20 minutes. The increase rate becomes lower afterwards. The distortion trend is quite different from that of the LDS measurement which shows a decrease of distortion after 7 minutes of processing. Also, the final distortion is over predicted by 520.3%. A high relaxation temperature (case 25, 980°C) exhibits a similar distortion trend to that of with no relaxation, but with lower magnitudes. The distortion is still over predicted by 157.5%. Case 23 with $T_{\text{relax}}$ of 690°C exhibits the same distortion behavior as the LDS measurement. The distortion decreases after 7 minutes of processing and starts to increase again at 20 minutes of processing time. The error compared to the LDS measurement is 7.4% over the entire process duration. An even lower relaxation temperature (case 19, 600°C) exhibits a decrease of distortion after 5 minutes and an increase again after 25 minutes. This case under predicts the final distortion, by 45.3%. These results illustrate the importance of accounting for stress relaxation in the model when predicting in situ and post-process distortion.

Figure 4.9 shows the post-process distorted shape of the workpiece as predicted by the calibrated model. The maximum distortion occurs at the free end of the substrate. Figure 4.10 displays a close up of the in situ measured distortion and the computed results for the calibrated relaxation temperature of 690°C. It is noted that although the relaxation temperature is calibrated to match the in situ measured distortion, a very good correlation (7.4% error) is achieved for the entire duration of the process.
Fig. 4.8. Computed distortion histories at various stress relaxation temperatures compared with the experimental measurement.

Fig. 4.9. Final post-process distortion (1x magnification) and longitudinal ($\sigma_{yy}$) residual stress (MPa).
4.6.3 Residual Stress

Figure 4.11 displays the computed post-process longitudinal component of the residual stress at nodes on the bottom of the substrate along the axis of deposition as well as experimental values at corresponding locations. Computed results for no stress relaxation (case 26) and stress relaxation temperature of $T_{\mathrm{relax}} = 690 \, ^\circ\mathrm{C}$ (case 23) are shown in the figure. The results with no relaxation are over predicted by more than 500%, and reach values as high as 1040 MPa. The computed residual stress also exhibits a shift to negative values close to the edges which is not present in the measured residual stress. The computed results with a stress relaxation temperature of $T_{\mathrm{relax}} = 690 \, ^\circ\mathrm{C}$ do not show this shift to negative values and are in close agreement (within 25%) with the measured residual stress. The maximum stress predicted using stress relaxation is 41.3 MPa. Figure 4.9 also illustrates the computed longitudinal residual stress on
the entire part. A high stress concentration is seen along the interface between the substrate and the build, and lower values (less than 250 MPa) elsewhere. The stress concentration located at the interface is artificially high in the model due to the sudden transition from the substrate to the deposition. On the actual workpiece the transition is filleted, meaning the stress concentration will not occur. The measured residual stress on the deposition is 20.0 MPa, which is in good agreement with the prediction from the model of 3.15 MPa considering that the measurement error of Blind Hole Drilling can be ±50MPa [54]. The residual stress results on the deposition are lower than those on the substrate. This is because the deposited material reaches a temperature exceeding the stress relaxation temperature more frequently, leading to a reduction in residual stress. These results show that, in addition to causing errors in the simulated distortion, failure to account for stress relaxation present in Ti-6Al-4V during electron beam deposition results in erroneous residual stress predictions.

![Computed residual stress results with and without stress relaxation, compared with the experimental measurements along the axis of deposition.](image)

**Fig. 4.11.** Computed residual stress results with and without stress relaxation, compared with the experimental measurements along the axis of deposition.
4.7 Conclusions

A finite element model is developed for predicting the thermo-mechanical response of Ti-6Al-4V during electron beam deposition. A three dimensional thermo-elasto-plastic analysis is performed and experimental in situ temperature and distortion measurements are performed during deposition of a single bead wide, 16 layer high wall build for model validation. Post-process Blind Hole Drilling residual stress measurements are also performed.

Both the in situ distortion and post process residual stress measurements suggest that stress relaxation occurs during the deposition of Ti-6Al-4V. A method of accounting for such stress relaxation in thermo-elasto-plastic simulations is proposed where both stress and plastic strain is reset to zero, when the temperature exceeds a prescribed stress relaxation temperature.

Inverse simulation is performed to determine the values of the absorption efficiency and the emissivity of electron beam deposited wire-fed Ti-6Al-4V, as well as the appropriate stress relaxation temperature. An efficiency of 0.90 and an emissivity of 0.54 result into the best correlation between measured and computed temperature history. Both values are in agreement to those published for laser assisted machining and powder based additive manufacturing. A stress relaxation value of 690°C is found to provide the best correlation between in situ measured and computed distortion. The results show that failure to implement stress relaxation in the constitutive model leads to errors in the residual stress and the in situ distortion predications of over 500% when compared with the experimental measurements.
Suggested future work includes sectioning and microstructural analysis of the test piece and establishing a correlation with the computed results. In addition, the possibility of a gradual stress relaxation model should be investigated to compare with the instantaneous model used in this work. This would require further testing to establish the suitable rate of relaxation.

4.8 Acknowledgements

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Chapter 5

Thermo-mechanical modeling of Additive Manufacturing large parts

5.1 Abstract

A finite element modeling strategy is developed to allow for the prediction of distortion accumulation in additive manufacturing large parts (on the order of meters). A 3D thermo-elasto-plastic analysis is performed using a hybrid quiet inactive element activation strategy combined with adaptive coarsening. At the beginning for the simulation, before material deposition commences, elements corresponding to deposition material are removed from the analysis, then elements are introduced in the model layer by layer in a quiet state with material properties rendering them irrelevant. As the moving energy source is applied on the part, elements are switched to active by restoring the actual material properties when the energy source is applied on them. A layer by layer coarsening strategy merging elements in lower layers of the build is also implemented such that while elements are added on the top of build, elements are merged below maintaining a low number of degrees of freedom in the model for the entire simulation. The effectiveness of the modeling strategy is demonstrated and experimentally validated.

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on a large electron beam deposited Ti-6Al-4V part consisting of 107 deposition layers. The simulation and experiment show good agreement with a maximum error of 29 %.

5.2 Introduction

Electron beam directed energy deposition is a commonly used additive manufacturing (AM) process. The process is of particular interest to the aerospace industry, due to its ability to rapidly deposit bulk material at rates as high as 2500 cm$^3$/hr [93]. The electron beam process constructs the part on a layer by layer basis, directly from a digital drawing file. Wire feedstock material is rapidly heated until melting and then allowed to solidify and cool, forming a fully dense geometry. The repeated thermal cycling causes large thermal gradients which, in turn, result in undesirable levels of distortion in the workpiece. Targeted alterations to the build plan may be used to reduce distortion levels. To avoid expensive trial and error iterations, an accurate predictive model is needed to optimize the build plan. Electron beam deposited parts are typically large, involving the deposition of several hundred kilograms of material, resulting in computationally expensive finite element models. Thus, for large parts, a modeling strategy is required to reduce computation time in order to make the simulations feasible.

Thermal and mechanical modeling of AM is similar to that of multi-pass welding, which has been an active research area for nearly 40 years. Numerous weld models capable of predicting thermal and mechanical behavior are reported in references [7–13, 95–97]. State of the art summaries by Lindgren et al. detail the development of
weld model complexity [14], weld material modeling [15], and weld model computational efficiency [16].

Modeling AM processes adds significant computational expense when compared to multi-pass weld simulations. This is due to an increase in the process time and the number of passes, as well as the addition of the deposited material. The deposition of material can add a large number of elements into the simulation and requires an element activation strategy [27]. Modeling of AM has primarily focused on predicting thermal response [38, 77–81], predicting distortion and residual stress, [3, 31, 39, 82, 83], and developing distortion mitigation techniques [40, 84].

All aforementioned models focus on the simulation of small parts with a simple deposited geometry to insure feasibility of computation. The capability to model large parts is of use in industry applications where parts are commonly large and geometries complex. The use of 2D models is one approach used to minimize computational expense. This strategy can result in an increasing loss of accuracy with added weld passes and would not be suitable for complex geometries [98]. Another approach is to perform a steady-state analysis rather than a transient analysis. In the area of welding research, Zhang and Michaleris developed a finite element model using an Eulerian approach which decreased computation time by a factor of 2. The approach, however, resulted in a loss of accuracy when compared with a Lagrangian approach [99]. For AM processes, Ding et al. presents an Eulerian simulation capable of simulating the deposition of four 0.5 m long beads in under 20% of the computation time required to do the same simulation using a transient approach [100]. The use of an Eulerian reference frame allows for a
nonuniform mesh along the deposition line, resulting in reduced computation time, but prohibits the modeling of more complex geometries.

Adaptive meshing can also be used to reduce computation time [101,102]. Work has been done on adaptive meshing outside the field of additive manufacturing, especially in computational fluid dynamics where singularities need to be resolved. Most techniques first use an error estimator [103] or error indicator [104] to determine which regions of the mesh need to be refined or coarsened. Then, the necessary changes are made to the mesh and the solution is transferred to new grid points. Many different techniques exist. Berger and Oliger used adaptive subgrid generation to refine meshes for finite difference solutions of 1 and 2 dimensional hyperbolic partial differential equations [101]. They showed that adaptive meshing can yield more accurate results than fine static meshing with shorter computation times. Berger and Colella further developed this method in [105]. Bell et al. extended this work to 3 dimensions [106]. Bank, et al. developed algorithms for refining quadrilateral meshes which can be generalized to hexahedral meshes [107]. Shepherd et al. developed algorithms for coarsening structured and unstructured hexahedral meshes [108].

In the field of welding, Prasad and Narayanan developed a triangular adaptive mesh method for 2 dimensional thermal models [109]. Runnemalm and Hyun presented a fully automatic 3 dimensional thermo-mechanical adaptive technique for weld modeling with hexahedral elements [110], but were not able to assess the accuracy or the efficiency of their technique due to the high physical memory demands of a benchmark fine mesh. Unfortunately there is currently no model presented in the available literature capable
of simulating the deposition of many layers of complex geometry for the manufacture of a large (on the order of meters) workpiece.

Previous work performed by the authors presents a method for calibrating and validating a model for simulating the thermal and mechanical response of an electron beam deposited titanium workpiece [111]. A single bead wide 16 layer deposition was manufactured while monitoring substrate temperatures using in situ thermocouple measurements and in situ substrate distortion using a laser displacement sensor (LDS). A finite element model was used to identify the absorption efficiency and the emissivity, as well as the stress relaxation temperature for Ti-6Al-4V. The model was further validated by comparing measured and predicted residual stress values along the substrate. The values for absorption efficiency, emissivity, and stress relaxation temperature found in reference [111] are applied in this work with no further calibration.

The objective of this work is to develop a finite element modeling strategy to allow for the prediction of the distortion accumulation in large AM parts. A thermo-mechanical finite element analysis is performed using a hybrid quiet inactive element activation strategy combined with adaptive coarsening. At the beginning for the simulation, before material deposition commences, elements corresponding to deposition material are removed from the analysis, then elements are introduced in the model layer by layer in a quiet state with material properties rendering them irrelevant. As the moving energy source is applied on the part, elements are switched to active by restoring the actual material properties when the energy source is applied on them. A layer by layer coarsening strategy merging elements in lower layers of the build is also implemented such that while elements are added on the top, elements are also merged on the bottom
maintaining a low number of degrees of freedom in model for the entire simulation. The algorithm requires less overhead than a fully automatic scheme, but requires no manual mesh modification as in reference [112]. The effectiveness of the approach is demonstrated by modeling the manufacturing of a large Ti-6Al-4V part consisting of 107 total layers and several thousand weld passes. Model validation is performed using experimental measurements. The large part is built by Sciaky Inc., and the distortion measurements are taken by Neomek Inc.

5.3 Electron Beam Deposition Simulation

The thermal and mechanical histories are determined by performing a 3D transient thermal analysis and a 3D quasi-static incremental analysis, respectively. The thermal analysis is performed independently of the mechanical analysis, as the mechanical response has no effect on the thermal history of the workpiece [28].

5.3.1 Thermal Analysis

The governing heat transfer energy balance is written as:

$$\rho C_p \frac{dT}{dt} = -\nabla \cdot \mathbf{q}(\mathbf{r}, t) + Q(\mathbf{r}, t)$$

(5.1)

where $\rho$ is the material density, $C_p$ is the specific heat capacity, $T$ is the temperature, $t$ is the time, $Q$ is the heat source, $\mathbf{r}$ is the relative reference coordinate, and $\mathbf{q}$ is the heat flux vector, calculated as:

$$\mathbf{q} = -k \nabla T$$

(5.2)
Table 5.1 lists the values of all temperature dependent material properties (thermal conductivity $k$ and specific heat capacity $C_p$) for Ti-6Al-4V. The material properties at temperatures between those listed are obtained by linear interpolation over the temperature range. The material thermal properties above $800^\circ$C are assumed to be constant, as accurate properties above this temperature are difficult to determine. The density of Ti-6Al-4V is $4.43 \times 10^{-6}$ g/mm$^3$.

Table 5.1. Temperature dependent thermal properties of Ti-6Al-4V [1]

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$k$ (W/m/°C)</th>
<th>$C_p$ (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.6</td>
<td>565</td>
</tr>
<tr>
<td>20</td>
<td>6.6</td>
<td>565</td>
</tr>
<tr>
<td>93</td>
<td>7.3</td>
<td>565</td>
</tr>
<tr>
<td>205</td>
<td>9.1</td>
<td>574</td>
</tr>
<tr>
<td>315</td>
<td>10.6</td>
<td>603</td>
</tr>
<tr>
<td>425</td>
<td>12.6</td>
<td>649</td>
</tr>
<tr>
<td>540</td>
<td>14.6</td>
<td>699</td>
</tr>
<tr>
<td>650</td>
<td>17.5</td>
<td>770</td>
</tr>
<tr>
<td>760</td>
<td>17.5</td>
<td>858</td>
</tr>
<tr>
<td>870</td>
<td>17.5</td>
<td>959</td>
</tr>
</tbody>
</table>

Radiation is accounted for by using the Stefan-Boltzmann law:

$$ q_{rad} = \varepsilon \sigma (T_s^4 - T_\infty^4) $$

(5.3)
where $\varepsilon$ is the surface emissivity, $\sigma$ is the Stefan-Boltzmann constant, $T_s$ is the surface temperature of the workpiece, and $T_\infty$ is the ambient temperature. The emissivity $\varepsilon$ is set as 0.54 as determined in reference [111] by applying inverse simulation.

5.3.2 Mechanical Analysis

The thermal history dependent quasi-static mechanical analysis is performed to obtain the mechanical response of the workpiece during the deposition. The results of the thermal analysis are loaded into the mechanical analysis. The governing stress equilibrium equation is:

$$\nabla \cdot \sigma = 0 \quad (5.4)$$

where $\sigma$ is the stress.

The mechanical constitutive law is:

$$\sigma = C \epsilon_e \quad (5.5)$$

$$\epsilon = \epsilon_e + \epsilon_p + \epsilon_T \quad (5.6)$$

where $C$ is the fourth order material stiffness tensor, and $\epsilon_e$, $\epsilon_p$, and $\epsilon_T$ are the total elastic strain, plastic strain, and thermal strain, respectively. Plastic strain relaxation is implemented, as described in reference [111], such that when the norm of the temperature at all Gauss points of an element exceeds a relaxation temperature $T_{relax} = 690^\circ C$, $\epsilon_e$, $\epsilon_p$, and $\epsilon_T$ in Equation (4.8) are set to zero.
Table 5.2 displays the temperature dependent mechanical properties (elastic modulus $E$, yield strength $\sigma_y$, and the thermal expansion coefficient $\alpha$) of Ti-6Al-4V. The material properties between the temperatures listed are found by interpolating over the temperature range. The properties are assumed constant above 800°C. Poison’s ratio $\nu$ is assumed to be a constant value of 0.34. The model assumes perfect plasticity, i.e. no material hardening occurs as in reference [111]. This assumption is made due to Ti-6Al-4V experiencing very little hardening and due to the presence of large plastic strains.

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$E$ (GPa)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\alpha$ ($\mu$m /m °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>105.00</td>
<td>777.15</td>
<td>8.60</td>
</tr>
<tr>
<td>20</td>
<td>103.95</td>
<td>768.15</td>
<td>8.64</td>
</tr>
<tr>
<td>250</td>
<td>91.81</td>
<td>664.65</td>
<td>9.20</td>
</tr>
<tr>
<td>500</td>
<td>78.63</td>
<td>552.15</td>
<td>9.70</td>
</tr>
<tr>
<td>800</td>
<td>62.80</td>
<td>417.15</td>
<td>9.70</td>
</tr>
</tbody>
</table>

5.4 Mesh Coarsening Algorithm

5.4.1 Merging of Elements Layer by Layer

To reduce computational expense, a coarsening algorithm is developed for large models. Layers are merged two layers below active deposition, as shown in Figure 5.1,
such that there is always at least one fine layer below the deposition. The coarsening algorithm is combined with a hybrid quiet inactive element activation method that adds elements for the new layer. Nodes are removed and DOFs are deleted in the merged layers. For mechanical analysis, Gauss point variables such as plastic strain and hardening need to be interpolated.

![Diagram of mesh coarsening algorithm](image)

Fig. 5.1. The mesh coarsening algorithm merges two layers of elements and deletes an entire plane of nodes. This is combined with a hybrid quiet inactive element activation method.

### 5.4.2 Interpolation of Gauss Point Values

In order to interpolate the solution values to the Gauss points of the new element, the global coordinates of the Gauss points must be transformed to the local coordinates of each of the two old elements. Transforming local coordinates to global coordinates, known as isoparametric mapping, can be accomplished using the shape functions, as shown in Equation (5.7) for linear hexahedral elements [113]:

\[
\phi = \frac{1}{8} \sum_{i=1}^{8} \Phi_i \prod_{k=1}^{3} \left( 1 + \xi_k \Xi_{k_i} \right)
\]

(5.7)
where $\phi$ is the variable being interpolated, $\Phi$ are the values of $\phi$ at the element’s nodes, $\xi$ are the local coordinates of the point where the quantity is being interpolated, and $\Xi$ are the local coordinates of the element’s nodes. Subscript $i$ ranges from 1 to 8 for each node of the element, and $k$ ranges from 1 to 3 for each spatial dimension.

For prismatic elements, a simple linear interpolation could be used to transform global coordinates to local coordinates. However, for a general element, a more sophisticated method is needed. In this case, the Newton-Raphson method is used. This is an inverse isoparametric mapping, first described by Lee and Bathe for 2 dimensional elements [114]. Here, the procedure is described in detail for 3 dimensional hexahedrons.

By substituting global coordinates $x$ for $\phi$ and global node coordinates $X$ for $\Phi$ into Equation (5.7), the residual $R$ can be formulated for $j = 1$ to 3 as follows:

$$R_j = x_j - \frac{1}{8} \sum_{i=1}^{8} X_{ij} \prod_{k=1}^{3} \left(1 + \xi_k \Xi_{ki}\right)$$

(5.8)

Then the Jacobian $J$ can be formulated as shown in Equation (5.9):

$$J_{mj} = \frac{\partial R_j}{\partial \xi_m} = -\frac{1}{8} \sum_{i=1}^{8} X_{ij} \Xi_{mi} \prod_{k=1}^{3} \frac{1 + \xi_k \Xi_{ki}}{1 + \xi_m \Xi_{mi}}$$

(5.9)

Beginning with $\xi^0 = 0$, the Newton-Raphson scheme is iteratively applied as follows:

$$\xi^{n+1} = \xi^n - \left[J^n\right]^{-1} R^n$$

(5.10)
updating the residual and the Jacobian at each step for \( n = 0, 1, 2, \ldots \) until the residual is sufficiently small. Note that \( J \) is only a \( 3 \times 3 \) matrix, thus solving Equation (5.10) is computationally inexpensive. For the models tested here, this method always converges.

Because the solution values are known at the Gauss points of the new elements rather than at the nodes, a modified version of Equation (5.7) must be used:

\[
\phi = \frac{1}{8} \sum_{i=1}^{8} \gamma_i \prod_{k=1}^{3} \left( 1 + \sqrt{3} \xi_k \Xi_{ki} \right)
\]  

(5.11)

where \( \gamma \) contains the values of \( \phi \) at the Gauss points and the \( \sqrt{3} \) accounts for the fact that the Gauss points for linear hexahedral elements are located at local coordinates of \( \pm \frac{1}{\sqrt{3}} \). By substituting the local coordinates \( \xi \) obtained from the Newton-Raphson method into Equation (5.11), the six stress components and other solution values can be interpolated to the new elements.

5.4.3 Verification of Layer by Layer Coarsening Algorithm

A small scale model is used to verify the coarsening algorithm. Thermo-mechanical modeling of electron beam deposition is performed on the small model using mesh density and processing conditions similar to that of modeling a large part. Results from a model without coarsening are used as a baseline. The four successively coarser meshes used in each stage of the simulation are shown in Figure 5.2. The numbers of nodes and elements in each mesh, including the full wall height, are shown in Table 5.3.
Fig. 5.2. Mesh 1 has a uniform density in the $z$ direction. Element edges which are subsequently eliminated by coarsening are highlighted in yellow.
Table 5.3. Number of nodes and elements in each mesh.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Number of nodes</th>
<th>Number of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2261</td>
<td>1632</td>
</tr>
<tr>
<td>2</td>
<td>1479</td>
<td>912</td>
</tr>
<tr>
<td>3</td>
<td>1428</td>
<td>880</td>
</tr>
<tr>
<td>4</td>
<td>1377</td>
<td>848</td>
</tr>
</tbody>
</table>

5.4.4 Verification Results

Displacement magnitude results at the end of the simulation are shown in Figure 5.3 and Figure 5.4 respectively for the uniform fine baseline mesh and the coarse mesh. Displacements at node 1 at the free end of the substrate and node 2 between the fifth and sixth layers are plotted versus time in Figure 5.5 for both cases. Note that node 1 is active for the duration of the simulation, but node 2 does not become active until approximately 82 s. Displacement results at the instant shown in Figure 5.3 along line AA are extracted and plotted in Figure 5.6 for both cases. The largest percent error for the results shown is 24.4% at $y = 44.45$ mm in Figure 5.6. This location corresponds to the end of the deposition track meaning that bending of the substrate ends at this point. The finer mesh is more able to capture this transition. The simulation wall times for both cases are compared in Table 5.4. There is no improvement in the first stage because at this point, both meshes are the same.
Table 5.4. Quasistatic mechanical simulation wall times for the small model.

<table>
<thead>
<tr>
<th>Simulation stage</th>
<th>Uniformly fine baseline mesh</th>
<th>Coarse mesh</th>
<th>Percent reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.4 s</td>
<td>17.4 s</td>
<td>0 %</td>
</tr>
<tr>
<td>2</td>
<td>23.2 s</td>
<td>15.2 s</td>
<td>34.7 %</td>
</tr>
<tr>
<td>3</td>
<td>23.5 s</td>
<td>9.69 s</td>
<td>58.8 %</td>
</tr>
<tr>
<td>4</td>
<td>23.9 s</td>
<td>8.72 s</td>
<td>63.5 %</td>
</tr>
</tbody>
</table>

Fig. 5.3. Displacement magnitude results (mm) at the end of simulation for the uniformly fine baseline case of the small model.

Fig. 5.4. Displacement magnitude results (mm) at the end of simulation for the coarse case of the small model.
Fig. 5.5. Displacement magnitude at two different nodes versus time for both cases of the small model. The locations of these nodes are shown in Figure 5.3.
Fig. 5.6. Displacement versus $y$ location at end of simulation along line AA for both cases of the small model.

5.5 Validation on a Large Part

Electron beam freeform fabrication is used to deposit Ti-6Al-4V wire feedstock material in a vacuum chamber, to form the workpiece shown in Figure 5.7. A Ti-6Al-4V plate 3810 mm long, 457 mm wide, and 25.4 mm thick is used as a substrate. The substrate is placed on the fixture and held in place by 40 evenly spaced clamps. The clamps have a spring constant of 22.5 N/mm, allowing for some distortion of the substrate.

The AM system used is the Sciaky VX-300, which welds in the range of $10^{-4}$ to $10^{-5}$ Torr. The work envelope is approximately $5.8 \times 1.2 \times 1.2$ m$^3$ in volume. The electron beam power is varied from 8 kW to 10 kW in order to control the melt pool size. The scan speed of the electron beam is 12.7 mm/s. The 12.7 mm diameter wire has a feed rate set to 50.8 mm/s. Previous work by the authors found the absorption
efficiency \( \eta \) to be 0.90 [111]. The largest builds are deposited 80 layers high, with a total deposition layer count of 107. A post-process 3D scan of the part is performed by Neomek Inc. using a Surphaser Laser with an estimated scanning accuracy of \( \pm 0.5 \) mm, which allows for the distortion levels to be quantified.

Fig. 5.7. Large workpiece, deposited on 3810 mm long substrate, for model validation, figure provided by Sciaky, Inc.

5.6 Numerical Implementation

Due to the size and complexity of the large part, a 3 stage modeling approach is implemented to simulate the entire deposition process. Stage 1 models the first layer of deposition, stage 2 models the second through ninth layers of deposition, and stage 3 models all succeeding layers. This approach allows for mesh coarsening to be implemented between each stage, thus reducing the computational expense. All meshes are generated using Patran 2012 by MSC. Anti-symmetry is also used to model only half of the deposition. The thermal and mechanical analysis are performed using CUBIC by Pan Computing LLC. The elements are activated using a hybrid quiet inactive approach.
for all simulations [27]. The inactive elements are initially not part of the simulation, but rather, are added on a layer by layer basis requiring the addition of an equation to the linear system in conjunction with a renumbering of the equations. The initially inactive elements are brought into the simulation as quiet elements, meaning that they are given material properties such that they do not affect the thermal or mechanical model. When the heat source contacts a quiet element it is activated by switching the element’s material properties to their actual value.

The electron beam heat source is modeled using the Goldak double ellipsoid model [29] as follows:

$$Q = 6\sqrt{3}P \eta f e^{-\left[ \frac{3x^2}{a^2} + \frac{3y^2}{b^2} + \frac{3(z+v)t^2}{c^2} \right]}$$

(5.12)

where $P$ is the power, $\eta$ is the absorption efficiency, $f$ is the process scaling factor; $x$, $y$, and $z$ are the local coordinates; $a$, $b$, and $c$ are the transverse, melt pool depth, and longitudinal dimensions of the ellipsoid respectively, $v_w$ is the heat source travel speed, and $t$ is the time. The front quadrant and rear quadrant of the heat source are modeled separately. The heat source radius $a$ is 6.35 mm, and the melt pool depth $b$ is 3.8 mm, for each quadrant. The length $c$ measures 12.7 mm and 25.4 mm for the front and rear quadrant, respectively. The process scaling factor $f$ equals 0.6 for the front quadrant and 1.4 for the rear quadrant. The elongation of the heat source allows for larger time steps and thus, reduced computation time. Velocity is a constant 12.7 mm/s. Power is entered as the power used during deposition, which varies between 8 and 10 kW. The
absorption efficiency $\eta$ is set at 0.90. Details on the determination of these parameters are discussed in reference [111].

All free surfaces are subject to radiative heat loss, including those located at the interface of active and quiet elements. Figure 5.8(a) shows the mechanical constraints applied to the model. Rigid constraints are placed to prevent the symmetry face from moving in the x direction. A constrained degree of freedom is placed on a single node at the symmetry face to prevent rigid-body motion in the y direction. Spring constraints are used to simulate the clamps of the fixture. The spring constant is set to a large value for negative deflections, simulating the rigid fixture, and is set to 11.25 N/mm for positive deflections, simulating the clamps present on the fixture. Table 5.5 lists the nonlinear spring constant values. Figure 5.8(b) shows a magnified isometric view of a portion of the mesh.

<table>
<thead>
<tr>
<th>z-deflection (mm)</th>
<th>Spring Constant (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>500</td>
</tr>
<tr>
<td>-4</td>
<td>170</td>
</tr>
<tr>
<td>-2</td>
<td>100</td>
</tr>
<tr>
<td>&gt;= 0</td>
<td>11.25</td>
</tr>
</tbody>
</table>

The first modeling stage models the preheat (i.e. initial heating passes performed before any deposition to make the substrate surface uniform) and the first layer of
Fig. 5.8. Illustration of the mesh used for layers 1-9.
deposition on each side of the substrate. Stage 1 accounts for 79.3 hours of experimental process time, representing periods of material deposition and cooling. Mesh coarsening cannot be performed before the deposition of layer 1 is completed.

The substrate for stage 1 is 4 elements thick, with the top and bottom layer having a thickness of 3.175 mm (equal to the thickness of each deposition layer), and the middle layers having a thickness of 9.925 mm. The mesh for the deposited layers measures 3.175 mm thick, allowing for 1 element per deposition thickness. The mesh is not uniform. Deposited elements are dimensioned to provide 1 element per heat source radius [111], resulting in an element size of roughly $6.35 \text{ mm} \times 6.35 \text{ mm}$. The mesh is coarsened outward from the deposition in order to reduce the number of required elements. The model contains 154,104 hexagonal elements and 179,939 nodes.

The second modeling stage models layers 2 through 9 of the deposition, and simulates 121.0 hours of process time. Mesh coarsening can be utilized after the completion of layer 1 and before the start of the deposition of layer 2. Once the deposition of layer 9 is completed, the majority of the material comprising the part has been deposited.

The substrate elements are coarsened in the z-direction for stage 2. This results in a 50% reduction in the number of elements located in the substrate. The model contains 120,174 hexagonal elements and 145,883 nodes. Note that due to the implementation of the mesh coarsening algorithm, the number of required elements for this model has decreased when compared with its predecessor, despite the increase in the amount of the deposited material.
The third modeling stage begins after the completion of the deposition of layer 9. The majority of the material has now been deposited. The remaining deposition completes the tallest structures present on the part, and accounts for 431.1 hours of process time. The deposition passes become shorter, and the heating caused by the electron beam becomes more localized.

Stage 3 simulates layers 10 through completion of the deposition. All elements that are not part of the tall structures have been coarsened and. The mesh is now comprised of 104,025 hexagonal elements and 120,941 nodes. The model is run independently of the results of the previous 2 simulations in order to examine if the additional deposition layers have a significant impact on the overall substrate distortion. Some small structures have been omitted from the model as they will have a negligible effect on distortion compared with the larger structures.

5.7 Results and Discussion

Table 5.6 displays the CPU run time for each of the 3 models run using 16, 3.1 GHz, cores. The thermal models for stages 1, 2, and 3 run in 13.8, 16.9, and 12.7 hours, respectively. The mechanical models run in 25.33, 32.2, and 13.2 hours, respectively. Stage 2 is the most computationally expensive because, while the model does contain fewer elements than that of stage 1, stage 2 simulates significantly more process time.

Figure 5.9 shows the distortion results of the anti-symmetric model after completion of 9 layers of deposition on each side of the substrate. Figure 5.9(a) displays the distortion results while the substrate is clamped in the fixture, showing a maximum distortion of 54.8 mm. Figure 5.9(b) shows that significant distortion is accumulated
once the fixture clamps are released, resulting in a maximum distortion of 186 mm. The increased distortion occurs because the force applied by the clamps is no longer acting on the substrate.

Results of the simulation of the distortion accumulated from layer 10 to the completion of the deposition show that distortion attributed to these layers is 0.025 mm. These layers cause negligible distortion of the substrate, showing that nearly all substrate distortion is attributable to the first 9 layers of deposition. This result is expected, as a majority of the feedstock material is deposited during the first 9 layers of the deposition. The succeeding layers are deposited using shorter deposition passes, resulting in more localized thermal gradients.

Figure 5.10 shows the final simulation results for the large part, which are compared to the 3D scan distortion results taken by Neomek Inc. Figure 5.11 shows the coordinate system that is used to compare the 3D scan distortion results to the computed distortion results, as well as the location of the data points being compared. The distortion is compared along the x direction due to the fact that it is the coordinate direction which distortion varies along most greatly. The face with the maximum distortion was chosen for comparison.
Fig. 5.9. Mechanical results (mm) after 9 layers of deposition (a) while clamped in the fixture (b) after release of the clamps. Significant distortion can be seen after the release of the clamps (2× magnification).
Fig. 5.10. Displacement magnitude (mm) results after the model has been rotated to the same orientation as the scan results. (2 × magnification)

Fig. 5.11. Top view of the coordinate system used for the simulated and experimental results, figure provided by Neomek, Inc.
Figure 5.12 displays a comparison between the simulated results and the actual experimental results. The experimental scan results and simulation results plotted correspond to distortions in the x-z plane at y = 457 mm. The simulated and experimental results show good agreement, with a maximum error of 29%.

![Distortion Graph](image)

Fig. 5.12. Experimental and simulated distortion results in the x-z plane at y = 457 mm.

### 5.8 Conclusions

A finite element modeling strategy has been developed to allow for the prediction of distortion accumulation in large workpieces in additive manufacturing. The strategy involves performing a 3D Lagrangian thermo-elasto-plastic analysis using a combined hybrid quiet inactive element activation strategy with adaptive coarsening. The
effectiveness of the modeling strategy is demonstrated and experimentally validated on a large electron beam deposited Ti-6Al-4V part consisting of 107 deposition layers.

The proposed modeling strategy is particularly suited for models with a high number of elements, as the computational overhead to introduce new elements and to coarsen existing elements is negligible compared to the computational savings of the reduced total number of degrees of freedom in the model. Unlike an Eulerian approach, the presented modeling strategy is capable of modeling complex geometries which are likely to appear in industry applications.

Model validation using experimental measurements shows that the proposed strategy can accurately predict the distortion of additive manufacturing large parts (maximum error of 29%). It can also enable determining which deposition passes are responsible for the majority of the substrate distortion which is important in designing build plans that mitigate distortion.

Possible future work could include implementing an octree-based element refining/coarsening strategy for structured or general unstructured hexahedral meshes. This could include an automatic algorithm that refines in the vicinity of the electron beam or laser path, or an error estimator/indicator method. Such an approach could significantly further reduce required computation time.

5.9 Acknowledgements

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Chapter 6

Mitigation of Distortion in Large Additive Manufacturing Parts

6.1 Abstract

Distortion mitigation techniques for large parts constructed by Additive Manufacturing processes are investigated. Unwanted distortion accumulated during deposition is a common problem encountered in AM processes. The proposed strategies include depositing equal material on each side of a substrate to balance the bending moment about the neutral axis of the workpiece and applying heat to straighten the substrate. Simple finite element models are used to predict the effectiveness of the mitigation strategies in order to reduce computation time and to avoid costly experiments. The strategy of adding sacrificial material is shown to be most effective and is then applied to the manufacture of a large electron beam deposited part consisting of several thousand deposition passes. The deposition strategy is shown to reduce the maximum longitudinal bending distortion in the large AM part by 91%. It is shown that after the distortion mode of concern is identified, simple finite element models can be used to study distortion accumulation trends relevant to the large part. Experimental observations made here, as well as finite element model results, suggest that the order

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in which the balancing material is added significantly affects the success of the proposed distortion mitigation strategy.

6.2 Introduction

Additive manufacturing (AM) processes allow for the construction of metallic parts directly from a digital geometry file without the retooling cost associated with casting and forging. Wire or powder is melted onto a substrate by a laser or electron beam and allowed to cool and solidify to form a fully dense geometry built up on a layer by layer basis. The large thermal gradients caused by the deposition process often lead to significant workpiece distortion, especially in large builds, taking the part out of tolerance. In order to combat the issue of process induced distortion, and in order to make AM processes useful in industry applications, techniques to mitigate distortion in large deposited parts must be developed.

In order to reduce distortion the appropriate distortion mode must be identified as first defined by Masubuchi [17]. The out-of-plane distortion modes, include angular, buckling, and longitudinal bending distortion. Angular and buckling distortion are caused by similar mechanisms. Angular distortion is caused by transverse shrinkage in the deposition region, while buckling occurs when residual stress caused by longitudinal shrinkage exceeds the workpiece critical buckling strength. Both angular and buckling distortion are less common in AM than in welding as the substrates used are generally thicker than weld panels. Of the 3 possible out-of-plane modes, longitudinal bending is of primary concern in AM processes. The longitudinal bending distortion is caused by the contraction of the molten material after heating and deposition. The progression
of substrate distortion during deposition is illustrated in Figure 6.1. When the molten material is applied the thermal expansion of the top of the substrate causes bending and plastic deformation, shown in Figure 6.1(a). The final longitudinal bending of the substrate, displayed in Figure 6.1(b), is caused by the cooling and contraction of the deposited material. When depositing large parts the problem of longitudinal bending can be exacerbated by the use of longer longitudinal deposition passes commonly used as a means to reduce processing time [115]. Several methods to reduce distortion incurred during welding and AM processes have been previously investigated.

Distortion mitigation techniques used in AM originate from research performed on a similar process, multi-pass welding. In welding research, finite element modeling (FEM) is commonly used to assess the effectiveness of distortion mitigation strategies while avoiding costly trial and error iterations. FEM development in welding dates back several decades and has focused on predicting both thermal and mechanical response of welded panels [7,9–13]. Weld research has shown reducing the heat input [25], balancing the residual stress to minimize the bending moment [25], and creating a temperature difference between parts to be welded (known as transient differential heating) [26] to be effective in reducing longitudinal bending distortion levels. AM differs from multi-pass welding in that AM involves the addition of large volumes of material, resulting in a larger number of deposition passes and longer processing times.

Researchers investigating distortion mitigation techniques in AM have shown that altering the laser scanning pattern can reduce distortion, with shorter deposition passes resulting in lower distortion levels [32,45,55,56]. However, shorter scanning patterns add processing time by requiring a greater number of deposition passes to add the same
Fig. 6.1. The progression of the substrate distortion throughout the deposition process. The undeformed and deformed substrate is illustrated by a dashed and solid line respectively.
amount of material. This problem becomes more significant for large parts with a greater
number of deposition passes. Residual stress may be reduced by preheating the substrate
and holding it at a high bulk temperature [30,47,57] or by heating the deposition region
immediately prior to deposition (localized preheating) [58]. Bulk substrate heating is
only feasible for small workpieces as it is not practical to hold a large substrate at a high
temperature for a long period of time, while localized preheating requires modifications
to the laser or electron beam deposition system. Industry applications are frequently
focused on large workpieces, but all of the distortion mitigation techniques thus far
investigated have only been shown to be effective on models of small parts.

This work investigates three new distortion mitigation techniques, useful for
reducing the longitudinal bending distortion mode in large AM parts. The first strategy
involves applying heat to the workpiece substrate in an attempt to straighten it after
deposition. The subsequent techniques involve depositing equal material on each side
of a substrate in order to balance the bending moment about the neutral axis of the
workpiece. The added deposition passes used to balance the bending moment are referred
to as balancing passes, with the deposition passes needed to construct the actual part
geometry referred to as build passes. The balancing passes deposit sacrificial material
that are machined away post process. Because electron beam deposition typically adds
significant extra material that is machined away post-process, the additional sacrificial
material will will not require a prohibitive amount of additional machining time. The
second strategy examines the effect of depositing the build passes consecutively after the
completion of all balancing passes is investigated. The third investigates the possibility of
depositing a balancing layer after each build layer is considered. The effectiveness of each
technique is investigated using a small FE model consisting of fewer than 4000 elements. The techniques found to be most successful on the small model are then applied to the manufacture of a large Ti-6Al-4V electron beam deposited part.

6.3 Evaluation of Distortion Mitigation Techniques

The feasibility of applying heat to straighten a substrate or adding additional deposited material to balance the bending moment acting on the substrate was first investigated using differing deposition strategies on small models. Significant computation time was saved by using small finite element models to predict the effectiveness of different distortion mitigation strategies on large parts when compared with simulating the actual large workpieces.

A brief overview of the model is first provided. Deposition cases are outlined which allow for the investigation of the aforementioned mitigation techniques. The deposition strategies that successfully achieve significant distortion mitigation on the small models are applied to the manufacture of an actual large part.

6.3.1 Electron Beam Deposition Simulation

A 3D thermo-elasto-plastic analysis was performed to predict the effectiveness of the presented distortion mitigation techniques. The results of the thermal simulation were imported as a load file into the mechanical analysis, which does not affect the thermal analysis due to the fact that the two are weakly coupled. Electron beam deposition was modeled, as the process is commonly used to deposit large parts due
to its ability to quickly deposit large amounts of bulk material. A detailed validation of both the thermal and the mechanical model model is provided in reference [111].

6.3.1.1 Thermal Analysis

The governing heat transfer energy balance is written as:

$$\rho C_p \frac{dT}{dt} = -\nabla \cdot \mathbf{q}(\mathbf{r}, t) + Q(\mathbf{r}, t)$$

(6.1)

This equation depends on the material density $\rho$, the specific heat capacity $C_p$ of the material, the temperature $T$, the heat source $Q$, the time $t$, the relative reference coordinate vector $\mathbf{r}$, and the heat flux vector $\mathbf{q}$ which is dependent on the thermal conductivity $k$:

$$\mathbf{q} = -k\nabla T$$

(6.2)

Heat loss attributable to radiation is calculated as:

$$q_{rad} = \varepsilon \sigma (T_s^4 - T_\infty^4)$$

(6.3)

where $\varepsilon$ is the surface emissivity, $\sigma$ is the Stefan-Boltzmann constant, $T_s$ is the surface temperature of the workpiece, and $T_\infty$ is the ambient temperature. A constant emissivity of 0.54 was used in the simulations [111]. Convection was not present, as the electron beam deposition process takes place in a vacuum chamber.
The temperature dependent thermal material properties for Ti-6Al-4V are listed in Table 6.1. Properties at temperatures between those listed were determined by linear interpolation over the temperature range and values at temperatures above those listed were assumed to be the nearest tabulated value. Density $\rho$ was set as a constant $4.43 \times 10^{-6}$ g/mm$^3$.

Table 6.1. Temperature dependent thermal properties of Ti-6Al-4V [1].

<table>
<thead>
<tr>
<th>$T$ ($^\circ$C)</th>
<th>$k$ (W/m/$^\circ$C)</th>
<th>$C$ (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.6</td>
<td>565</td>
</tr>
<tr>
<td>20</td>
<td>6.6</td>
<td>565</td>
</tr>
<tr>
<td>93</td>
<td>7.3</td>
<td>565</td>
</tr>
<tr>
<td>205</td>
<td>9.1</td>
<td>574</td>
</tr>
<tr>
<td>315</td>
<td>10.6</td>
<td>603</td>
</tr>
<tr>
<td>425</td>
<td>12.6</td>
<td>649</td>
</tr>
<tr>
<td>540</td>
<td>14.6</td>
<td>699</td>
</tr>
<tr>
<td>650</td>
<td>17.5</td>
<td>770</td>
</tr>
<tr>
<td>760</td>
<td>17.5</td>
<td>858</td>
</tr>
<tr>
<td>870</td>
<td>17.5</td>
<td>959</td>
</tr>
</tbody>
</table>

6.3.1.2 Elasto-Plastic Mechanical Analysis

Once the thermal simulation was complete, the results were imported into the mechanical analysis. A quasi-static incremental analysis was performed. The stress equilibrium equation is:
\[ \nabla \cdot \sigma = 0 \]  
(6.4)  

where \( \sigma \) is the stress.  

The mechanical constitutive law is:

\[ \sigma = C \epsilon_e \]  
(6.5)  

\[ \epsilon = \epsilon_e + \epsilon_p + \epsilon_T \]  
(6.6)  

where \( C \) is the fourth order material stiffness tensor and \( \epsilon \) is the total strain, which due to the fact that displacement gradients will be much smaller than unity, can be decomposed into: \( \epsilon_e \), \( \epsilon_p \), and \( \epsilon_T \), which represent elastic, plastic, and thermal strain, respectively. Stress relaxation present in Ti-6Al-4V was accounted for in the constitutive model by specifying a relaxation temperature \( T_{relax} \) of 690 °C. When the norm of the temperature of the Gauss points of an element surpasses \( T_{relax} \) the stress is set to zero [111].  

Table 6.2 lists the temperature dependent mechanical properties for Ti-6Al-4V used in the model, including the elastic modulus \( E \), the yield strength \( \sigma_y \), and the coefficient of thermal expansion \( \alpha \). A constant value of 0.34 was used for Poisson’s ratio.
Table 6.2. Temperature dependent mechanical properties of Ti-6Al-4V [1,3].

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$E$ (GPa)</th>
<th>$\sigma$ (MPa)</th>
<th>$\alpha$ ($\mu$m/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>105.00</td>
<td>777.15</td>
<td>8.60</td>
</tr>
<tr>
<td>20</td>
<td>103.95</td>
<td>768.15</td>
<td>8.64</td>
</tr>
<tr>
<td>250</td>
<td>91.81</td>
<td>664.65</td>
<td>9.20</td>
</tr>
<tr>
<td>500</td>
<td>78.63</td>
<td>552.15</td>
<td>9.70</td>
</tr>
<tr>
<td>800</td>
<td>62.80</td>
<td>417.15</td>
<td>9.70</td>
</tr>
</tbody>
</table>

6.3.2 Numerical Model

A 203.2 mm long × 28.6 mm tall × 12.7 mm wide wall was constructed on the topside of the substrate and a 203.2 mm × 6.35 mm × 12.7 mm wall was built up on the backside of the substrate, as large and complex industry parts commonly require material to be placed on both sides of the substrate. Also, the part investigated in Section 4 possesses 9 layers and 2 layers on the top and bottom of the substrate, respectively. The depositions were made upon a 254 mm long × 101.6 mm wide × 12.7 mm thick substrate using unidirectional longitudinal passes. The part dimensions, which are arbitrary, were chosen to be sufficiently small to allow for the model to be discretized into fewer than 4000 elements, thus keeping simulation times feasible. A cooling time of 1200 s occurred between the deposition of each layer. On large parts long cooling times are typical because each deposition layer can be time consuming, allowing the deposited material to cool significantly as the layer is completed. All meshes were generated using Patran 2012 by MSC. The thermal and mechanical analyses were performed using CUBIC by Pan.
Computing LLC. The electron beam heat source was modeled using the Goldak double ellipsoid model as follows:

\[ Q = \frac{6\sqrt{3}P\eta}{abc\pi\sqrt{\pi}} e^{-\left[\frac{3x^2}{a^2} + \frac{3y^2}{b^2} + \frac{3(z+vt)}{c^2}\right]} \]  

(6.7)

where \( P \) is the electron beam power; \( x, y, \) and \( z \) are the local coordinates; \( a, b, \) and \( c \) are the transverse, melt pool depth, and longitudinal dimensions of the ellipsoid respectively, \( v_w \) is the heat source travel speed, and \( t \) is the time. The laser spot size is 6.35 mm and penetrates to a depth of 3.81 mm [111], making \( a \) and \( c \) equivalent to 6.35 mm, and \( b \) equal to 3.81 mm. The absorption efficiency \( \eta \) for Ti-6Al-4V deposited by using the electron beam system is 0.9 [111].

The addition of deposited material was simulated using the quiet element approach where all elements begin as part of the simulation, however elements belonging to the deposited material are given material properties such that they do not affect the analysis until contacted by the heat source. Surface radiation was applied to all free surfaces, including those on the evolving surface between active and quiet elements. The heat source has a power \( P \) of 8 kW and moves at a speed \( v_w \) of 12.7 mm/s. After the completion of the build, the parts cooled to room temperature. The model was mechanically constrained as cantilevered allowing distortion to be monitored at a node on the free end of the substrate, as shown in Figure 6.2, to compare the different deposition strategies.
Fig. 6.2. Mesh showing the node observed to monitor distortion.
6.3.3 Deposition Strategies

Four cases were studied to determine the effectiveness of several mitigation strategies. Straightening the substrate by applying only heat and depositing additional material to balance the bending moment were compared to a baseline case. For actual builds, in the cases where additional material is deposited, the extra material is sacrificial and would be machined away post-process. A detailed description of each case is provided and the cases are summarized in Table 6.3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>baseline</td>
</tr>
<tr>
<td>2</td>
<td>topside heating</td>
</tr>
<tr>
<td>3</td>
<td>sequential balancing layers</td>
</tr>
<tr>
<td>4</td>
<td>alternating balancing layers</td>
</tr>
</tbody>
</table>

A 3-step convergence study was performed on the baseline case comparing the discretization of the mesh using 1, 2, and 4 elements per heat source radius. Using 2 elements and 4 elements per heat source radius resulted in an increase in distortion of 2.8 % and 3.1 % when compared to 1 element per heat source radius, respectively. Thus, 1 element per heat source radius was chosen.
Case 1: Baseline with topside deposition only

Figure 6.3 shows the baseline case, Case 1. Nine layers and 2 layers were deposited on the topside and backside of the substrate respectively, as labeled in Figure 6.3. The mesh was comprised of 3264 elements and 4574 nodes. No attempt was made to reduce distortion accumulated during the deposition of material.

Case 2: Topside heating after each topside layer

Figure 6.4 shows Case 2 which applied topside heating in an attempt to lessen distortion of the part without adding additional material. The mesh was comprised of 3264 elements and 4574 nodes. After the deposition of each unbalanced deposition layer on the topside of the substrate 2 heating passes were performed each 203.2 mm long. The longitudinal bending distortion caused by the thermal expansion of the top of the substrate was intended to counter the distortion caused by the contraction of the molten material.
Case 3: Backside deposition after topside layers deposited

Case 3, whose deposition pattern is seen in Figure 6.5, used consecutive balancing layers on the backside of the substrate to balance the build layers. The first 2 build layers on the topside of the substrate and the 2 build layers on the backside of the substrate were deposited in alternating fashion. Then the remaining 7 build layers on the topside of the substrate were deposited sequentially followed by 7 consecutive balancing layers on the backside of the substrate. The mesh has 3712 elements and 5267 nodes.

Case 4: Backside deposition after each topside layer

Case 4, shown in Figure 6.6, examined the possibility of mitigating distortion by depositing a balancing layer on the backside of the substrate after each build layer on the topside of the substrate in order to continually balance the bending moment about the neutral axis of the workpiece throughout the build.

The end result was a workpiece with a 9 layer high wall on each side of the substrate. When the balancing layers cool after deposition, the bending moment caused
by the contraction of the molten material of the balancing layers equals the bending moment caused by the contraction of the molten material making up the build layers and thus may help to straighten the substrate. The mesh consists of 3712 elements and 5267 nodes.

### 6.3.4 Small Model Results

#### 6.3.4.1 Distortion

Figure 6.7 plots the results of Case 1, the baseline case, to illustrate how distortion is being quantified. In each case the maximum substrate distortion occurred at the free end of the cantilevered substrate. Table 6.4 summarizes the results of the simulations.

For Case 1, the baseline where 9 and 2 layers were deposited on the topside and backside of the substrate respectively, it can be seen in Figure 6.8 that nearly no distortion accumulated after the first 4 deposition layers (around 4900 s) as the bending moment remained balanced about the neutral axis of the substrate. The remaining 7 deposition layers on the topside of the substrate caused a final distortion of 3.32 mm.
Fig. 6.6. Alternating deposition pattern, Case 4.

Fig. 6.7. Final distortion for the baseline case, Case 1.

Table 6.4. Distortion results for the small models.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>Final Distortion (mm)</th>
<th>% Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>baseline</td>
<td>3.32</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>topside heating</td>
<td>2.04</td>
<td>38.9</td>
</tr>
<tr>
<td>3</td>
<td>sequential balancing layers</td>
<td>1.17</td>
<td>64.8</td>
</tr>
<tr>
<td>4</td>
<td>alternating balancing layers</td>
<td>0.06</td>
<td>98.2</td>
</tr>
</tbody>
</table>
Applying topside heating to the model in Case 2 resulted in a final distortion of 2.04 mm, a 38.9% decrease compared with the baseline.

The in situ distortion results from Case 3, applying sequential balancing layers, shows that distortion accumulated in the build until 12000 s when balancing layers began to reduce it. The balancing layers were unable to mitigate all distortion and result in a final distortion of 1.17 mm which represents a 64.8% reduction in distortion relative to the baseline case.

Case 4, using alternating balancing layers, can be seen to have accumulated little distortion throughout the build. The balancing of the bending moment after each deposition layer resulted in a final distortion of 0.06 mm, representing a 98.2% reduction in distortion compared with Case 1, the baseline case.

Fig. 6.8. In situ distortion for Cases 1-4.
The results from the small model suggest that balancing the bending moment about the substrate by adding material is the most capable method of those explored for reducing distortion in AM workpieces. The method yielded superior results to applying only heating.

The distortion mitigation was achieved when the deposited balancing layers cool from their molten state and shrink to form a fully dense deposition, essentially canceling out the distortion caused by the build layers as illustrated in Figure 6.9. The evolution of substrate distortion is responsible for the peaks and valleys seen in the in situ distortion results in Figure 6.8.

The observed accumulation of distortion in the models indicates that the sequence in which the balancing layers are added is important. The balancing layers in Case 3, which were deposited sequentially, resulted in significant distortion mitigation, however, the balancing layers in Case 4, which were deposited in alternating fashion, eliminated nearly all distortion attributed to the deposition of the build layers. The alternating balancing layers in Case 4 eliminated significantly more distortion than their sequentially deposited counterparts in Case 3. This result suggests that depositing the build layers and balancing layers in an alternating manner can be used to eliminate virtually all longitudinal bending distortion.

### 6.3.4.2 Residual stress

Distortion built up during electron beam processing is driven by the accumulation of residual stress due to large thermal gradients during the deposition process. An understanding of how the alternating balancing layers in Case 4 eliminate more distortion
Fig. 6.9. Evolution of substrate distortion during deposition and solidification leading to reduced distortion of the workpiece.
than the consecutive balancing layers in Case 3 can be gained by examining the residual stress results from the model. The residual stress results are compared along the cross-section in Figure 6.10. The residual stress distribution in Case 1, the baseline case, is shown in Figure 6.11. After the deposition of layer 4 the residual stresses were balanced about the center of the substrate, as seen in Figure 6.11(a). Figure 6.11(b) shows that after the remaining 7 layers were deposited the contraction of the additional molten Ti-6Al-4V on the top of the substrate pulled the top of the substrate into compression, leading to a high level of upward bowing distortion. In Case 2, which applied only heating to counter substrate distortion, the thermal expansion of the substrate was not adequate to overcome the stress generated by the contraction of the molten metal upon cooling, making it the least effective of the investigated strategies.

Fig. 6.10. Cross-section, located at x = 101.6 mm, at which residual stress results are analyzed.
Fig. 6.11. Residual stress (MPa) results for the baseline Case 1.

(a) residual stress distribution after 4 layers are deposited, 2 on the top and 2 on the bottom

(b) post-process residual stress distribution
Figure 6.12 displays the residual stress distribution in the workpiece when applying the consecutive balancing layers in Case 3. Prior to the addition of the balancing layers the residual stress distribution was identical to that plotted in Figure 6.11(b). The sacrificial layers added entered into tension upon cooling, exerting a compressive force on the bottom side of the substrate as shown in Figure 6.12(a). The compressive residual stress acted to eliminate a portion of the upward bowing of the substrate. Figure 6.12(b) illustrates that when all sacrificial material has been deposited and permitted to cool the added contraction of the molten material was unable to counter the residual stress above the neutral axis of the substrate, meaning there will be an upward bow of the substrate.

Unlike the addition of consecutive sacrificial layers, alternating sacrificial layers were able to balance the residual stress about the neutral axis of the substrate throughout the build. Figure 6.13(a) shows that after the deposition of 12 total layers the residual stress was nearly uniform through the thickness of the substrate, i.e, there was little bowing distortion. As the additional layers were added the balance of residual stress was maintained throughout the deposition, ending with the substrate in uniform compression through the thickness in its post-process state as illustrated in Figure 6.13(b).

6.4 Mitigation Techniques Applied on a Large Part

To further investigate the effectiveness of the distortion mitigation strategies presented, a large electron beam deposited build was constructed twice to study if the most successful strategies from the small models result in similar distortion reduction on a larger scale.
Fig. 6.12. Residual stress (MPa) results when using consecutive balancing layers, Case 3.
Fig. 6.13. Residual stress (MPa) results when using alternating balancing layers, Case 4.
6.4.1 Experimental Procedure

An electron beam freeform fabrication system was used to deposit 9.5 mm diameter Ti-6Al-4V wire feedstock material, at a rate of 0.85 mm/s, in a vacuum chamber, to form the workpiece shown in Figure 6.14. The AM system used was the Sciaky VX-300, which welds in a vacuum in the range of $10^{-4}$ to $10^{-5}$ Torr. The work envelope was approximately $5.8 \times 1.2 \times 1.2$ m$^3$ in volume. The electron beam power was varied from 8 kW to 10 kW in order to control the melt pool size. The largest builds were deposited 80 layers high, with a total deposition layer count of 107. A Ti-6Al-4V plate 3810 mm long, 457 mm wide, and 25.4 mm thick was used as a substrate. Two parts, Part A and Part B, were deposited on each substrate during the build. Two builds were performed, Build 1 and Build 2, allowing for a total of 4 parts to be manufactured. Post-process scan results taken by Neomek Inc quantified the distortion using a Surphaser Laser with an accuracy of ± 0.5 mm.

![Fig. 6.14. Large deposited part.](image_url)

Figure 6.15 shows the test fixture used. The substrate was placed on the fixture and held in place by 40 evenly spaced clamps. The clamps have a spring constant of 22.5
N/mm, allowing for in situ distortion of the substrate. The fixture can rotate to allow for deposition on both sides of the substrate.
6.4.2 Deposition Cases

Due to the effectiveness of the distortion mitigation strategies applied on Cases 3 and 4 these same strategies were implemented on a large part. The new Cases, L2 and L3, were deposited using scan patterns which deposit balancing layers sequentially and alternatingly, respectively and compared to a new baseline deposition Case L1. These cases are summarized in Table 6.5

Case L1

Figure 6.16 shows Case L1 which was applied to Build 1 and excluded the use of balancing layers. The deposition strategy is the same as that applied in Case 1 and was applied to both parts deposited on the substrate of Build 1 making Part A and Part B identical. Case L1 is the baseline case used to determine the effectiveness of the
Table 6.5. Case descriptions for the large part.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>baseline</td>
</tr>
<tr>
<td>L2</td>
<td>sequential balancing layers</td>
</tr>
<tr>
<td>L3</td>
<td>alternating balancing layers</td>
</tr>
</tbody>
</table>

distortion mitigation strategies. The layers on Case L1 which were responsible for the majority of the longitudinal bending distortion of the workpiece have been identified in previous finite element modeling work by the authors [115] and are labeled in Figure 6.16.

**Case L2 and Case L3**

Case L2 employed a strategy that is analogous to that of Case 3 and applies sequential build layers, followed by sequential balancing layers. Figure 6.17(b) shows the additional layers added to Part B of Build 2. After all build layers were added to the part, it was turned over to allow for the addition of the balancing layers. This strategy required fewer rotations of the part and thus reduces processing time.

The deposition strategy used for Case L3 identical to Case 4, using alternating build layers and balancing layers. The strategy was applied to Part A of Build 2 as shown in Figure 6.17(a). After the completion of each build layer the part was physically turned over to allow for the deposition of a balancing layer, and then returned to its initial position to resume the deposition of the next build layer.
Fig. 6.16. Schematic of the large build illustrating both parts deposited on Build 1, Case L1.

Fig. 6.17. Schematic of the large build illustrating both parts deposited on Build 2, Cases L2 and L3.
After the completion of the deposition of the 2 builds, post-process scan results were taken by Neomek, Inc to allow for the workpiece distortion levels to be quantified.

6.5 Results and Discussion

The distortion results are plotted in Figure 6.18. Case L1, the unbalanced deposition applied to Build 1, accumulated a maximum of 37.2 mm of distortion on Part A and 32.6 mm on Part B. Case L2 and Case L3 were implemented on Part B and Part A of Build 2, respectively. Case L2 resulted in distortion levels as high as 10.2 mm on Part B. Case L2 has a maximum distortion 69% smaller than Case L1. The Case L3 deposition pattern resulted in a maximum substrate distortion of 3.3 mm on Part A, representing an 91% decrease in distortion compared with the baseline case. Table 6.6 summarizes the results.

![Figure 6.18](image)

Fig. 6.18. Comparison of distortion results before and after application of the distortion mitigation techniques on the targeted portion of the substrate.
Table 6.6. Distortion results for the large part.

<table>
<thead>
<tr>
<th>Case</th>
<th>Deposition Description</th>
<th>% Decrease in Max Distortion</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>unbalanced</td>
<td>-</td>
</tr>
<tr>
<td>L2</td>
<td>sequential</td>
<td>69</td>
</tr>
<tr>
<td>L3</td>
<td>alternating</td>
<td>91</td>
</tr>
</tbody>
</table>

The application of the distortion mitigation strategies on the large part yielded similar results as are found from the small models. The small model intended to represent a large workpiece, predicted a percent distortion mitigation of 65% when adding sequential balancing layers whereas the manufactured large part using sequential balancing layers saw a 69% reduction in distortion. The use of alternating balancing layers on the small model yielded a distortion 98% less than the baseline case compared with an 91% decrease seen on the large part compared with its baseline case.

The results also confirm that the use of balancing layers can be used to significantly mitigate distortion of large workpieces. Depositing the balancing layers sequentially after the deposition has been completed can be done to save processing time, however the greatest distortion mitigation was achieved when depositing a balancing layer after each build layer.

6.6 Conclusions

Several distortion mitigation strategies for AM parts have been presented to allow for a significant reduction in the longitudinal bending of a workpiece. The
approaches involve using heating to straighten a substrate or depositing additional material to balance the bending moment about the neutral axis of the workpiece. The distortion mitigation strategies are well suited for large parts, as they do not require any modifications to the AM system and do not require the impractical heating of the entire substrate to limit thermal gradients.

The effectiveness of the strategies is demonstrated on a small FE model. This allows for the lengthy computation time associated with simulating large parts to be avoided. The small models show that adding additional material is a superior mitigation technique compared with applying only heating, thus the strategy is applied to the manufacture of large parts.

The distortion results of the large parts indicate that applying a balancing layer after each build layer yields favorable results when compared with depositing all balancing layers sequentially after the completion of the build layers and is capable of eliminating nearly all bending distortion. This consequence is in agreement with the prediction from the small models. The percent reduction in distortion on the large parts is found to be in close agreement with that calculated in the simulations. This shows that if the proper distortion mode is identified, a small model can be used to study distortion accumulation trends when applying different mitigation strategies and used to make decisions pertaining to a larger part.

It is suggested that when designing a build plan an effort should be made to deposit roughly equal volume of material on each side of the substrate in order to minimize substrate distortion.
Acknowledgements

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Chapter 7

Experimental Validation for Thermal Modeling of Powder Bed Fusion Processes using in situ Measurements

7.1 Abstract

A model for predicting the thermal response of Inconel® 718 during Laser Powder-Bed Fusion processing (LPBF) is developed. The approach includes the pre-placed powder layer in the analysis by initially assigning powder properties to the top layer of elements before restoring the solid properties as the heat source traverses the layer. Different linear heat inputs are examined by varying both laser power and scan speed. The effectiveness of the model is demonstrated by comparing the predicted temperatures to in situ experimental thermocouple data gathered during LPBF processing. The simulated temperatures accurately capture the measured peak temperatures (within 11% error) and temperature trends. The effect of neglecting the pre-placed powder layer in the simulations is also investigated demonstrating that conduction into the powder material should be accounted for in LPBF analyses. The simulation neglecting the powder predicts temperatures more than 30% higher than the simulation including the powder.
7.2 Introduction

The Laser Powder Bed Fusion (LPBF) Additive Manufacturing (AM) process allows for the rapid production of net shape metallic parts directly from a digital drawing file. The LPBF process operates by first spreading a thin layer of powder (on the order of 10 µm) across a build plate. Next a laser melts the material which then cools and solidifies to form a fully dense geometry. The build plate then lowers, a recoater spreads a new layer of powder, and the process is repeated allowing for parts to be additively constructed on a layer-by-layer basis. During the process large thermal gradients arise causing unacceptable levels of residual stress to build up in the part, frequently leading to failure by cracking or delamination from the build plate. Current efforts to deal with this issue utilize a costly trial and error approach where parts are manufactured several times until an acceptable final product results. Finite element model (FEM) predictions can be used to circumvent the costly trial and error process assuming that they accurately capture the necessary process physics.

In situ experimental measurements have been used to thoroughly validate FE models of AM processes as they provide more insight into the process than could be realized using only post-process measurements. Lundbäck and Lindgren performed in situ distortion and temperature measurements during single wall depositions using a Gas Tungsten Arc Welding process and used the results for FEM validation [31]. Ocelik et al. used digital image correlation (DIC) to validate a predictive FE model by measuring in process strain during laser cladding of single and multi-bead Nanosteel, Eutroloy 16012, and MicroMelt 23 on C45 steel and 301 stainless steel substrates [74]. Peyre
et al. used thermocouples and thermal imaging to validate numerical predictions of the direct metal deposition thermal response [38]. Plati et al. used thermocouples to monitor in situ temperatures during laser cladding and validated a 3-D FE model using the results [37]. Heigel et al. developed a thermo-mechanical FE model of the Laser Engineered Net Shape (LENS) processing of Ti-6Al-4V which includes the effect of forced convection from the inert gas jets [116]. In situ temperature measurements performed using thermocouples were used to validate the model and to illustrate that the numerical predictions neglecting forced convection were insufficient. Gouge et al. implemented forced convection measurements in a thermal FE model to accurately capture the thermal response of laser clad Inconel® 625 [117]. The results were validated against in situ measurements taken with thermocouples. While each of these studies offers an in depth validation of numerical models of AM processes, none focuses on the LPBF process.

From a modeling standpoint the primary differences between the previously discussed AM process and the LPBF process include the presence of the pre-placed powder layer and a smaller laser spot size (as little as 70 µm). The pre-placed powder may affect the thermal response. A smaller spot size necessitates strict spacial and temporal discretization requirements. To reduce lengthy computation times imposed by the discretization requirements associated with LPBF researchers have attempted removing the build plate from the analysis [59] or using a 2-D model [60]. Others have neglected the powder in thermal analyses [43,59,61], essentially assuming that the powder is a perfect insulator. The effect of this assumption is unknown.

Several other approaches to modeling the powder layer have been presented that are not intended to reduce computational expense. Dai and Shaw [32,62] assumed that
the powder material has the same thermal properties as its matching solid material in a 3-D thermomechanical model. In other research the conductivity of the powder is scaled by a porosity dependent factor [63,64]. Another common strategy [65–69] involves assigning the metallic powder thermal properties based on powder-solid relationships developed by Sih et al. [70–72]. Some of these works provide no model validation [32, 62, 64, 65], while others validate the developed models against measured relative densities [66], deposition track width [67], microstructure and melt pool dimensions [68, 69], or post process residual stress [43]. None of the aforementioned models have been validated against in situ temperature measurements.

The objective of this work is to develop a thermal model of the LPBF process and validate the predicted thermal response against in situ temperature measurements. A thermal finite element analysis is performed using a modified version of the quiet element approach. The modified approach permits the metallic powder material to have a thermal conductivity and emissivity based off of the solid-powder material property relationships developed by Sih et al. [72]. Prior to processing all deposition elements are given powder properties, however when the heat source is applied to the elements their solid properties are restored. The necessary number of degrees of freedom (DOF) in the model imposed by the required spacial discretization requirement is reduced by applying condensation and recovery [118]. The effectiveness of the approach is demonstrated by simulating the LPBF processing of a single layer 100 deposition track Inconel® 718 build. Experimental model validation is performed by comparing the model to the measured thermal response of actual LPBF processed builds of the same geometry. The model
results are also compared to numerical cases where the powder elements are removed from the analysis.

7.3 Modeling Approach

The temperature history is calculated using a 3D Lagrangian transient thermal analysis accounting for both solid and powder properties of the metallic material.

7.3.1 Transient thermal analysis

The governing transient heat transfer energy balance in the entire volume of the material is given as:

\[ Q(x, t) - \nabla \cdot q(x, t) - \rho C_p \frac{dT}{dt} = 0 \]  \hspace{1cm} (7.1)

where \( \rho \) is the material density, \( C_p \) is the specific heat capacity, \( T \) is the temperature, \( t \) is the time, \( Q \) is the internal heat generation rate, \( x \) is the relative reference coordinate, and \( q \) is the heat flux.

The initial condition for Equation (7.1) is:

\[ T(x, t_0) = T_\infty \]  \hspace{1cm} (7.2)

where \( T_\infty \) is the ambient air temperature. The Fourier heat flux constitutive relation is given by:

\[ q = -k \nabla T \]  \hspace{1cm} (7.3)
which depends on temperature dependent thermal conductivity $k$.

Thermal radiation $q_{rad}$ is accounted for using the Stefan-Boltzmann law:

$$q_{rad} = \varepsilon\sigma(T_s^4 - T_{\infty}^4) \tag{7.4}$$

where $\varepsilon$ is the surface emissivity, $\sigma$ is the Stefan-Boltzmann constant, and $T_s$ is the surface temperature of the workpiece. The Stefan-Boltzmann equation can be linearized and put into heat transfer coefficient form:

$$q_{rad} = h_{rad}(T_s - T_{\infty}) \tag{7.5}$$

where the heat transfer coefficient for radiation $h_{rad}$ is calculated as:

$$h_{rad} = \varepsilon\sigma(T_s + T_{\infty})\left(\frac{T_s^2}{T_s^2 + T_{\infty}^2}\right) \tag{7.6}$$

Newton’s law of cooling describes the heat loss due to convection $q_{conv}$:

$$q_{conv} = h(T_s - T_{\infty}) \tag{7.7}$$

where $h$ is the convective heat transfer coefficient.

### 7.3.2 Powder-bed properties

The properties of the metallic Inconel® 718 powder are assigned based on powder-solid relationships developed by Sih et al [72]. The conductivity $k_p$ of the Inconel® 718 powder consisting of spherical particles can be calculated as follows:
\[
k_p = k_f \left(1 - \sqrt{1 - \phi} \right) \left(1 + \phi \frac{k_r}{k_f}\right) + \sqrt{1 - \phi} \left(1 - \frac{f}{k} \left(1 - \frac{f}{k} \right) \ln \frac{k_s}{k_f} - 1 \right) + \frac{k_r}{k_f}\right) \right]^{(7.8)}
\]

where \(k_f\) is the thermal conductivity of the argon gas surrounding the particles, \(\phi\) is the porosity of the powder bed, \(k_s\) is the conductivity of the solid, and \(k_r\) is heat transfer attributed to the radiation amongst the individual powder particles.

\[
k_r = \frac{4}{3} \sigma T^3 D_p \tag{7.9}
\]

where \(D_p\) is the average diameter of the powder particles.

The emission of radiation from the heated porous powder surface is caused by emission from the individual particles as well as from cavities present in the powder bed.

The emissivity \(\varepsilon_p\) of the Inconel\textsuperscript{10} 718 powder bed can be calculated as:

\[
\varepsilon_p = A_H \varepsilon_H + (1 - A_H) \varepsilon_s \tag{7.10}
\]

where \(A_H\) is the porous area fraction of the powder surface:

\[
A_H = \frac{0.908\phi^2}{1.908\phi^2 - 2\phi + 1} \tag{7.11}
\]

and \(\varepsilon_H\) is the emissivity of the powder surface vacancies:
\[ \varepsilon_H = \frac{\varepsilon_s \left[ 2 + 3.082 \left( \frac{1-\phi}{\phi} \right)^2 \right]}{\varepsilon_s \left[ 1 + 3.082 \left( \frac{1-\phi}{\phi} \right)^2 \right] + 1} \] (7.12)

7.4 Experimental validation

7.4.1 Process parameters

A pair of single layer depositions were fabricated using a LPBF process on a matching substrate. A Nickel-Chromium alloy Inconel\textsuperscript{®} 718 was chosen as it is commonly used for AM applications due to high corrosion resistance and good weldability. The powder had a particle diameter cutoff size of 44 \( \mu \text{m} \) and was initially spread to a thickness of 152.4 \( \mu \text{m} \) with a porosity of 52 \%. The laser depositions were performed using a Nd:YAG fiber laser operating at a wavelength of 1070 nm. The beam diameter at the part surface was 485 \( \mu \text{m} \) with a tophat intensity distribution. The depositions were performed in an inert atmosphere. In order to test the robustness of the model 2 different sets of process parameters were used, each utilizing a unique laser power and scan speed, which are listed in Table 7.1. Deposition A applied a laser power of 230 W and a scan speed of 55.03 mm/s and Deposition B applied a laser power of 115 W and a scan speed of 50.80 mm/s.

The single layer deposited geometry measures 27.6 mm by 10.7 mm in area and is melted by 100 laser passes with a hatch spacing of 100 \( \mu \text{m} \). The direction of the laser scan alternates with even numbered deposition tracks deposited in the opposite direction.

\textsuperscript{1}The work presented in this section was performed at United Technologies Research Center by Vijay Jagdale, GV Srinivasan, and Tahanay El-Wardany.
of odd numbered tracks. The substrate used was 101.6 mm by 101.6 mm with a thickness of 925 \( \mu \text{m} \). Because the objective of this work is to validate the ability of the thermal model to account for effects of the powder layer the relatively thin substrate is desirable as it minimizes the effect of conduction into the substrate thus allowing for greater heat transfer into the powder.

<table>
<thead>
<tr>
<th>Deposition</th>
<th>Power ( P ) [W]</th>
<th>Scan speed ( v ) [mm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>230</td>
<td>55.03</td>
</tr>
<tr>
<td>B</td>
<td>115</td>
<td>50.80</td>
</tr>
</tbody>
</table>

### 7.4.2 In situ measurements

In situ measurements of the substrate temperature were made using the experimental setup shown in Figure 7.1. A schematic of the measurement locations is shown in Figure 7.2. In the experimental setup, the Inconel\( ^{\text{R}} \) 718 substrate was mounted and clamped on all four sides by an insulating material. The fixture resembles a picture frame, leaving a 76.2 mm\(^2\) area exposed on both the top and bottom of the substrate. While the entire area on top was covered with powder the bottom area was monitored to record in situ temperature. In situ temperature measurements were taken at two locations on the bottom of the substrate using K-type thermocouples with an
accuracy of ± 2.2%. All data was recorded at a frequency of 5 Hz using a Vishay system 7000 DAQ.

![Experimental setup](image)

**Fig. 7.1.** Experimental setup

### 7.5 Numerical implementation

#### 7.5.1 Solution method

The thermal analyses are performed in CUBES® (version 262) by Pan Computing LLC. The analyses are done in a series of time steps with the current time step taking the solution at the previous time step as an initial condition. The temporal discretization requirement for laser based AM processes is known from previous work [111, 116, 117]:

\[
    t^j = t^{j-1} + \frac{r}{v}
\]

(7.13)

where \( j \) is the current time step, \( r \) is the laser radius, and \( v \) is the heat source scan speed. The Newton-Raphson method solves the discrete equations at each time step.
(a) Deposition A: 230 W and 55.03 mm/s

(b) Deposition B: 115 W and 50.80 mm/s

Fig. 7.2. Bottom view of the workpiece illustrating of the sample dimensions and measurement locations (all dimensions in mm).
The as-used temperature dependent thermal properties for Inconel\textsuperscript{®} 718 are listed in Table 7.2. Properties falling between the tabulated temperatures are found through linear interpolation. Properties at temperatures above those that are listed are taken as the nearest tabulated value. The latent heat of fusion $H$ is accounted for over the solidus $T_s$ to liquidus $T_L$ temperature range.

Table 7.2. Temperature dependent thermal properties of solid Inconel\textsuperscript{®} 718 [4, 5]

<table>
<thead>
<tr>
<th>$T$ [°C]</th>
<th>$k$ [W/m/°C]</th>
<th>$C$ [J/kg]</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>p</td>
<td>s</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>11.4</td>
<td>427</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>11.4</td>
<td>427</td>
<td>-</td>
</tr>
<tr>
<td>100</td>
<td>12.5</td>
<td>441</td>
<td>-</td>
</tr>
<tr>
<td>300</td>
<td>14.0</td>
<td>481</td>
<td>-</td>
</tr>
<tr>
<td>500</td>
<td>15.5</td>
<td>521</td>
<td>-</td>
</tr>
<tr>
<td>538</td>
<td>-</td>
<td>-</td>
<td>0.28</td>
</tr>
<tr>
<td>649</td>
<td>-</td>
<td>-</td>
<td>0.42</td>
</tr>
<tr>
<td>700</td>
<td>21.5</td>
<td>601</td>
<td>-</td>
</tr>
<tr>
<td>760</td>
<td>-</td>
<td>-</td>
<td>0.58</td>
</tr>
<tr>
<td>1350</td>
<td>31.3</td>
<td>691</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.3 lists the as-used constant material properties and processing conditions for the thermal analyses. The conditions were chosen in accordance with the experimental conditions used for model validation.
7.5.2 The finite element mesh

Figure 7.3 shows the finite element mesh used in the analyses. The mesh contains 29,030 Hex-8 elements and 39,120 nodes. Each node has 1 DOF (temperature). The mesh is discretized to allow for 2 elements per laser radius in the x-y plane of the deposition region and 1 element per deposition thickness in the z direction. The spacial discretization was chosen based on known requirements from the literature [111,116,117]. The mesh is coarsened as it moves away from the region that interacts with the heat source.

![Finite element mesh](image)

**Fig. 7.3.** Finite element mesh.

A nonconforming mesh is used by implementing a h-Adaptive scheme. A dependent node occurs when an element’s node is not shared by an adjacent element. The dependent (condensed) nodes are accounted for in the analysis by applying condensation and recovery [118]. As each time-step \( t \) the incremental temperature \( \delta T \) is calculated as follows:
\[ \delta T = - \left[ \frac{dR}{dT} (T^I) \right]^{-1} R(T^I) \]  \hfill (7.14)

where \( R \) is the residual.

The independent (retained) temperatures \( T_r \) and dependent (condensed) temperatures \( T_c \) can be grouped allowing for (7.14) to be written as:

\[
\begin{bmatrix}
\delta T_r \\
\delta T_c
\end{bmatrix} = \begin{bmatrix}
\frac{dR}{dT} (T^I_r) & \frac{dR}{dT} (T^I_c) \\
\frac{dR}{dT} (T^I_r) & \frac{dR}{dT} (T^I_c)
\end{bmatrix}^{-1} \begin{bmatrix}
R(T^I_r) \\
R(T^I_c)
\end{bmatrix} \quad (7.15)
\]

The constraint equations have the following form:

\[ T_c = C_r \sum_{k=1}^{n} T_k \]  \hfill (7.16)

where \( C_r \) is a constraint coefficient and \( n \) is the number of nodes that \( T_c \) is dependent upon. If an element’s node lies on an adjacent element’s edge \( C_r \) and \( n \) equals \( \frac{1}{2} \) and 2, respectively. If an element’s node lies on an adjacent element’s face \( C_r \) and \( n \) equals \( \frac{1}{4} \) and 4, respectively. By enforcing the constraints the system can be solved by including only the retained degrees of freedom:

\[
\begin{bmatrix}
\delta T_r
\end{bmatrix} = \begin{bmatrix}
\frac{dR}{dT} (T^I_r) - \frac{dR}{dT} (T^I_r)C_r + C^T_r \frac{dR}{dT} (T^I_c)C_r \\
C_r \frac{dR}{dT} (T^I_c)C_r
\end{bmatrix}^{-1} \begin{bmatrix}
T_r - C^T_r T_c
\end{bmatrix} \quad (7.17)
\]
The solution of the condensed nodes is then determined by enforcing the constraint equations.

### 7.5.3 Boundary Conditions

The laser heat source $Q$ is modeled using the Goldak double ellipsoid model [29] defined as follows:

$$Q = \frac{6\sqrt{3}P\eta}{abc\pi} e^{-\left[\frac{3x^2}{a^2} + \frac{3y^2}{b^2} + \frac{3(z+vt)^2}{c^2}\right]}$$  \hspace{1cm} (7.18)

where $\eta$ is the absorbtion efficiency; $x$, $y$, and $z$ are the local coordinates; $a$, $b$, and $c$ are the transverse, depth, and longitudinal dimension of the ellipsoid respectively. Convection and radiation are applied to all free surfaces that were not insolated during the experiment.

### 7.5.4 Material deposition modeling

The quiet element approach is commonly used to simulate material deposition for the modeling of powder or wire fed AM processes [27]. The strategy involves initially assigning scaling factors to the material properties of the deposition elements effectively eliminating any effect they have on the analysis. When the deposition elements are contacted by the heat source their solid properties are restored. The approach is not physically realistic for the LPBF process due to the fact that the powder material is pre-placed. Here the deposition elements are initially given a thermal conductivity and emissivity as found from the powder-solid material property relations described by Sih
et al. [72], presented in Section 2. Elements are switched from having powder properties to having solid properties when the following condition is met at any of the Gauss points of an element [27]:

$$\frac{6\sqrt{3}}{abc\pi\sqrt{\pi}} e^{-\left[\frac{3x^2}{a^2} + \frac{3y^2}{b^2} + \frac{3(z+v)^2}{c^2}\right]} \geq .05 \quad (7.19)$$

### 7.5.5 Investigated analysis cases

Three numerical cases are investigated and are summarized in Table 7.4. Case 1 initially assigns powder properties to the deposition elements as calculated in Section 2 and uses a power of 230 W and a scan speed of 55.03 mm/s in order to be validated against the experimental results of Deposition A. Case 2 assigns powder properties as calculated in Section 2 and uses the processing parameters from Deposition B, a power of 115 W and a laser scan speed of 50.80 mm/s. The results of Case 2 are validated using the thermocouple measurements from Deposition B. Case 3 simulates the powder as a perfect insulator and uses a power of 230 W and a scan speed of 55.03 mm/s, the same parameters used in Case 1. Case 3, which excludes the powder layer, is compared to Case 1 which includes the pre-place powder layer to study the importance of including the powder effects on the heat transfer in the model.
7.6 Results and discussion

7.6.1 Simulation validation

In order to determine the effectiveness of the numerical model the thermal results predicted by the simulation are compared with the experimentally measured in situ thermal response presented in Section 3. The thermal results from Case 1 (P = 230 W, \( v = 55.03 \) mm/s, pre-placed power included) and Case 2 (P = 115 W, \( v = 50.80 \) mm/s, pre-placed power included) are compared to the corresponding experimental cases Deposition A and Deposition B, respectively.

Figure 7.4 shows the predicted temperatures for Case 1 (Figure 7.4(a)) and Case 2 (Figure 7.4(b)) compared with the experimentally measured in situ temperatures. The oscillations in the predicted temperatures are caused by the laser traversing the substrate.

The model for Case 1 predicts that at the location of thermocouple 1 (the center of the bottom side of the substrate) the temperature will rapidly rise at the start of processing, reaching a peak of 778 °C around 31 s after the deposition begins. The peak occurs at this time because the heat source is located directly above the thermocouple 1 location. As the heat source moves past the center of the substrate in the Y direction the temperature begins to decrease as the meltpool becomes further removed from the measurement location. The model prediction at the location of thermocouple 2, which is located on the bottom of the substrate near the end of the laser scan path, shows a steady rise in temperature throughout the build and peaks at 711 °C after 63 s of processing. These predictions are in close agreement with the experimental thermocouple measurements which capture similar peak temperatures and trends.
Fig. 7.4. Comparison between the simulated and the experimentally measured in situ temperatures.
The simulation results for Case 2 show that at the location of thermocouple 1 (the center of the bottom side of the substrate) the temperature reaches a maximum of 510 °C as the heat source approaches. Case 2 has a linear heat input 46 % less than that of Case 1, resulting in the predicted thermocouple 1 peak temperature for Case 2 reaching a value more than 200 °C cooler than in Case 1. This closely agrees with the experimental results. The predicted thermocouple 2 (located 1.8 mm from the deposition area on the bottom of the substrate) in situ temperature reaches a peak temperature of 205 °C. The prediction is also in close agreement with the experiment. Table 7.5 shows the percent errors for the simulated peak temperatures, calculated as:

\[
\%Error = 100 \frac{T_{\text{sim}} - T_{\text{exp}}}{T_{\text{exp}} - T_{\infty}}
\]  

(7.20)

where \(T_{\text{sim}}\) and \(T_{\text{exp}}\) are the simulated and experimental temperatures, respectively.

7.6.2 Effect of neglecting pre-place powder

Now that the thermal model has been validated against the in situ experimental results it can be used to examine the effect of neglected the pre-placed powder layer from the analysis. Excluding the powder from the analysis is a common practice [43, 59, 61] though the effect of assuming the powder to be an insulator is unknown. Neglecting the powder from the analysis provides a benefit as the mesh will require fewer elements, resulting a computational time savings. To study the validity of the assumption the thermal results of numerical Case 1, which has been validated, are compared with the
results of Case 3 which neglects the pre-placed powder layer. Figure 7.5 shows the 4 nodes used for the comparison.

Figure 7.6 shows the thermal history during the simulation at each of the 4 nodes used for the comparison of Case 1 and Case 3. At each node the temperature for the simulation neglecting the pre-placed powder (Case 3) exhibits higher temperatures than the simulation which includes the pre-placed powder (Case 1). This occurs because in Case 3 the powder effectively acts as an insulator.

Figure 7.6(b) shows the in situ results after melting caused by the heat source occurs at Node 2, located at the deposition and substrate interface, for both Cases. Both the numerical model including (Case 1) and excluding (Case 3) the pre-place powder are in close agreement. The maximum discrepancy calculated at any time step is a 4.4 % over prediction of temperature by the simulation neglecting the powder (Case 3) compared with the results when including the powder (Case 1). However as the distance of the node used for comparison from the deposition region is increased, the error becomes large. At Node 4, located 22.5 mm from the deposition region, the maximum difference in temperature prediction is 31.4 %. Table 7.6 records the maximum percent by which Case 3 (no powder) over predicts the temperature calculated in Case 1 (including powder) at each node.

The results suggest that estimating that the powder behaves as a perfect insulator, in order to reduce the required number of elements in the FE mesh, incurs large temperature prediction errors. Whereas only a single layer example is presented here, LPBF builds typically consist of many layers, making the assumption that the powder
(a) Top view of the mesh

(b) Isometric view of the sectioned mesh

(c) Magnified view showing the 4 nodes used for comparison between the models.

Fig. 7.5. Illustration of the node locations used for the comparison of the powder models.
Fig. 7.6. In situ temperature results comparing the thermal response when using various powder bed models.
does not affect the analysis even less accurate. Adding more powder will increase the influence of conduction losses into the powder, thus further increasing errors.

7.7 Conclusions

A finite element model has been developed to allow for the prediction of the temperature history of LPBF manufactured parts. The approach involves performing a 3D Lagrangian heat transfer analysis where the elements representing the powder material are given material properties based off of solid-powder material property relationships found in the literature. The elements are switched to having solid properties when contacted by the heat source to simulate the LPBF process. Condensation and recovery was applied to reduce the computational expense. The predicted temperature results are compared with experimentally measured temperatures during the LPBF process. The effect of neglecting the pre-placed powder from the analyses was also investigated. The conclusions can be summarized as follows:

1. Including powder properties calculated from powder-solid relationships found in the literature in the numerical model results in simulated temperatures in close agreement (maximum 10 % error) with the experimentally measured thermal history.

2. In addition to closely predicting the peak temperature measured during the deposition process the model also captures the trends shown in the in situ measurements.
3. Assuming that the powder layer is an insulator in order to eliminate elements from the FE mesh results in temperatures being over predicted by more than 30% when compared with the validated model. This issue which will be exacerbated by the edition of more layers.

Future work should focus on developing a methodology for replacing the effect of conduction losses into the powder with a heat flux boundary condition. This approach could allow for the powder elements to be eliminated from the analysis to save computation time without incurring temperature prediction errors.
Table 7.3. As-used constant material properties and processing conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient temperature $T$ [°C]</td>
<td>22</td>
</tr>
<tr>
<td>Argon gas conductivity $k$ [W/m/°C]</td>
<td>0.016 [119]</td>
</tr>
<tr>
<td>Convection coefficient $h$ [W/m²/°C]</td>
<td>15 [58]</td>
</tr>
<tr>
<td>Density $\rho$ [kg/m³]</td>
<td>8146 [4]</td>
</tr>
<tr>
<td>Laser absorptivity $\eta$</td>
<td>0.45 [120]</td>
</tr>
<tr>
<td>Latent heat $H$ [J/kg]</td>
<td>227,000 [121]</td>
</tr>
<tr>
<td>Liquidus temperature $T_L$ [°C]</td>
<td>1337 [121]</td>
</tr>
<tr>
<td>Solidus temperature $T_S$ [°C]</td>
<td>1255 [121]</td>
</tr>
<tr>
<td>Powder particle diameter $D_p$ [µm]</td>
<td>44</td>
</tr>
</tbody>
</table>

Table 7.4. Cases investigated in the numerical analyses.

<table>
<thead>
<tr>
<th>Case</th>
<th>Power $P$ [W]</th>
<th>Scan speed $v$ [mm/s]</th>
<th>Powder included in analysis?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>230</td>
<td>55.03</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>115</td>
<td>50.80</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>230</td>
<td>55.03</td>
<td>No</td>
</tr>
</tbody>
</table>
Table 7.5. Errors of the predicted maximum temperatures.

<table>
<thead>
<tr>
<th>Case</th>
<th>T sim TC 1 [°C]</th>
<th>T exp TC 1 [°C]</th>
<th>Error TC 1 [%]</th>
<th>T sim TC 2 [°C]</th>
<th>T exp TC 2 [°C]</th>
<th>Error TC 2 [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>778</td>
<td>775</td>
<td>0.4</td>
<td>711</td>
<td>796</td>
<td>11.0</td>
</tr>
<tr>
<td></td>
<td>510</td>
<td>508</td>
<td>0.4</td>
<td>205</td>
<td>218</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 7.6. Percent by which the model neglecting powder (Case 3) over predicts the temperature predicted by the model including powder (Case 1) at each comparison node.

<table>
<thead>
<tr>
<th>Node</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.4</td>
</tr>
<tr>
<td>2</td>
<td>4.4</td>
</tr>
<tr>
<td>3</td>
<td>12.2</td>
</tr>
<tr>
<td>4</td>
<td>31.4</td>
</tr>
</tbody>
</table>
Chapter 8

Conclusions and Recommendations for Future Work

Thermo-mechanical models of AM processes must be developed in order to circumvent the costly trial and error approach currently being widely applied to produce parts. In order to have confidence in the FE models their predictions must be validated against experimental measurements. Typically the validation is performed against post-process experimental results such as the final distortion or residual stress. However, greater insight into the physics of the process and more thorough model validation can be gained through the use of in situ measurements.

This work has primarily focused on the development of thermo-mechanical models for both the LDED, EBDM, and LPBF processes as well as the design of both post-process and in situ experiments to validate the simulations.

Chapter 2 presented experimental in situ distortion results in addition to post-process distortion and residual stress measurements for the LDED processing of Inconel® 625 and Ti-6Al-4V. The goal of the work was to improve the understanding of how distortion accumulates during AM processing. It was shown that the materials accumulate distortion very differently during manufacture. Increasing inter-layer dwell times resulted in greater distortion and residual stress present in the Ti-6Al-4V builds. The opposite trend was seen in the Inconel® 625 depositions.
Chapter 3 focused on developing a thermo-mechanical model of the LDED process capable of capturing the thermal and mechanical responses of Ti-6Al-4V and Inconel® 625 measured in Chapter 2. The primary finding was that the standard material constitutive model accounting for elastic, plastic, and thermal strain was capable of closely capturing the mechanical response of Inconel® 625 but was unable to capture the response of Ti-6Al-4V. The model over-predicted the distortion and residual stress present in the Ti-6Al-4V samples by over 500%. A new modified constitutive model was developed to account for the transformation strain present in Ti-6Al-4V, yielding close agreement with experiment.

Chapter 4 applied the validated LDED model from Chapter 3 to model the in situ temperature and distortion of EBDM processing of Ti-6Al-4V. In situ experiments were designed and performed to capture the thermal and mechanical response during the deposition. Blind hole drilling was performed to quantify the post-process residual stress. The experimental results were used to validate the model.

Chapter 5 applied the EBDM model studied in Chapter 4 to model the post-process distortion of an actual EBDM industrial aerospace component consisting of thousands of deposition tracks. The model was used to identify the deposition layers most responsible for the accumulated distortion. The post-process distortion predicted the measured distortion within 29% maximum error.

Chapter 6 utilized EBDM FE models to investigate various distortion mitigation techniques. Findings from the FE models were used to guide distortion mitigation on the large aerospace component from Chapter 5. The maximum distortion was reduced by 91%.
Chapter 7 extends the LDED and EBDM modeling work to the LPBF process. The effect of the pre-placed powder layer is incorporated into the analysis by modifying the quiet element approach. Elements representing the powder were assigned properties based on powder-solid material property relationships found in the literature. The model results were shown to be in close agreement with experiment. Assigning the powder to be an insulator in the analysis was found to result in elevated temperatures.

Future work should focus on the further development of LPBF process models. Due to the number of laser passes and the strict spacial discretization requirement imposed by the small laser spot size it is infeasible to model part-scale builds using a moving source simulation. Instead, moving source simulations should be used to gain an understanding of the plastic strain accumulated within the individual layers during processing. Once the plastic strain is known and understood it could be incorporated into the model on a layer-by-layer basis thus dramatically reducing computation time.
References


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