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PRESERVICE SECONDARY MATHEMATICS TEACHERS' LEARNING OF PURPOSEFUL QUESTIONING AND JUDICIOUS TELLING FOR PROMOTING STUDENTS' MATHEMATICAL THINKING

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Abstract

The field of teacher education is currently experiencing a shift towards curricula focused on practices for teaching and professional coursework designed around different pedagogies of practice (e.g., decomposition of, representations of, and approximations of practice). Researchers and research-informed documents have identified a variety of mathematics teaching practices, one of which is facilitating classroom mathematics discourse. Further, researchers have identified some key practices that constitute facilitating classroom mathematics discourse such as assessing questions and advancing questions (Smith, Bill, & Hughes, 2008) and judicious telling (Lobato, Clarke, & Ellis, 2005). In a methods course designed to engage preservice secondary mathematics teachers (PSMTs) in course activities designed around pedagogies of practice and focused on Types of Teacher Talk (TTT) (i.e., assessing questions, advancing questions, and judicious telling), this study sought to examine what PSMTs learn about TTT and in what ways the course activities were connected to their learning. The design of the study was informed by the interpretive research genre and the study employed qualitative data collection and analysis techniques to explore four case studies of the PSMTs' learning of TTT. The results of the study indicate that the PSMTs, over the course of the semester, constructed conceptions of TTT that were oriented towards focusing on and promoting students' mathematical thinking. In addition, while there were similarities among the PSMTs' conceptions of TTT, they constructed these conceptions in individually distinct ways. Study implications include the design of preservice teacher education professional coursework that engages preservice teachers in multiple and extended experiences with a small set of teaching practices.

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Chapter 1

Introduction, Rationale, and Frameworks for the Study

Introduction

> Currently, the field of teacher education is undergoing a major shift – a turn away from a predominant focus on specifying the necessary knowledge for teaching toward specifying teaching practices that entail knowledge and doing … The fundamental aim undergirding this turn is to better support teachers in learning how to use knowledge in action … (McDonald, Kazemi, & Kavanagh, 2013, p.378)

This short excerpt from McDonald et al. (2013) succinctly captures the current landscape in the professional education of teachers in the United States. For the past twenty years, professional coursework in teacher education has focused on developing teachers' knowledge for teaching and field experiences (e.g., student teaching) have focused on equipping teachers with the capacities to enact the practices that constitute teaching. The shift McDonald et al. described is one in which pedagogical approaches towards professional coursework in teacher education coordinate the development of teachers' knowledge with teachers' learning to enact teaching practices. Some scholars have named these teaching practices *core practices,* which are defined as the constituent components of instruction that place students' ideas as the starting point of instruction and are components of instruction that are accessible for novice teachers.

Core practices as a construct is gaining traction in both research in teaching and research in teacher education. Research in teaching has begun to identify and articulate core practices for teaching within disciplines, such as mathematics (National Council of Teachers of Mathematics, 2014), and seeks to articulate sets of core practices across disciplines. In addition, researchers in teacher education have begun to design and study pedagogical approaches towards professional coursework that focuses on core practices. Further, scholars assert that a focus on core practices in teaching and teacher education provides a common language that enables practitioners and researchers to more accurately communicate and examine teaching as well as establishing a bridge between teaching and teacher education with a set of relevant practices for teaching.

As teacher education shifts its focus towards core practices, scholars are raising important concerns learned from previous reform movements in teacher education. Most important of these concerns is that the "move toward core practices in teacher education risks becoming fad-like, resulting in a proliferation of approaches driven more by the trend than by a deep understanding of how people learn to enact ambitious professional practice" (Lampert et al., 2013, p.379). Scholars are urging that researchers not only identify and develop pedagogical approaches towards professional coursework focused on core practices, but also examine teachers' learning about how to enact core practices. Hence, an important question facing teacher educators is: How can we educate preservice teachers in professional coursework designed around current pedagogical approaches to teacher education?

Researchers have begun to design and identify teacher education pedagogical approaches (e.g., pedagogies of practice) and course activities involved in these pedagogical approaches that support preservice teachers' learning of core practices (e.g., Lampert Beasley, Ghousseini, Kazemi, & Franke, 2010). In addition, researchers are beginning to identify teachers' learning opportunities in professional coursework that are provided by course activities designed around these pedagogical approaches. However, there is limited research that examines what teachers learn about core practices and the ways in which specific course activities support teachers' learning. The purpose of this study is to examine, in the context of a methods course in which course activities are designed around pedagogies of practice, what preservice mathematics teachers learn about a core practice and the ways in which course activities support the preservice mathematics teachers' learning. Implications from this study enable teacher educators and researchers in teacher education to better understand the ways teachers learn core practices and the professional coursework activities that promote their learning.

Background and Study Rationale

Preservice secondary mathematics teachers are entering a profession in which significant attention has been given to what it means to know, learn, and teach school mathematics. In the last twenty-five years, a number of policy documents in mathematics education contend that secondary students' knowing and understanding of mathematics involves more than students recalling isolated facts about mathematical ideas (e.g., using SOH-CAH-TOA to "remember" right-angle trigonometry ratios) and executing procedures (e.g., graphing linear functions in the Cartesian Plane using slope-intercept formulas). The *Curriculum and Evaluation Standards for Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989) and *Principles and Standards for School Mathematics* (NCTM, 2000) include standards for mathematical processes (i.e. reasoning, problem solving, connecting, representing, communicating) as well as standards for mathematical content (i.e. algebra, geometry, numbers and operations, function, measurement). More recently, the Common Core State Standards – Mathematics [CCSS-M] (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010) interweave mathematical processes into the eight mathematical practice standards they propose for grades K-12 mathematics. Prospective teachers not only have to know mathematics as described by these Standards documents, they also need to learn how to teach mathematics in a way that supports these kinds of mathematical understandings. As such, while these standards for K – 12 mathematics advocate what it means for K-12 students to "know" mathematics, research in mathematics education has identified effective teaching practices that play a role in knowing mathematics as characterized by these standards. One of those teaching practices is facilitating meaningful classroom mathematics discourse.

Classroom Mathematics Discourse[1] and Students' Learning of Mathematics

Researchers (e.g., Stein, Grover, & Henningsen, 1996; Truxaw & DeFranco, 2008) found that the nature of classroom mathematics discourse around mathematical tasks plays a critical role in shaping the way in which students engage in mathematics. Research (e.g., Goos, 2004; Staples, 2007) and mathematics teaching standards (e.g., NCTM, 1991) suggest that students' engagement in meaningful classroom mathematics

[1] It is important at this point to articulate what I mean by classroom mathematics discourse as there multiple interpretations of discourse and classroom discourse in the literature. In describing classroom mathematics discourse, Ghousseini (2008) carefully explains the differences between Discourse with a capital "D" – "socially accepted association among ways of using language, of thinking, and of acting that can be used to identify oneself as a member of a socially meaningful group or social network (Gee, 1999) – and discourse with a lower case "d" – language in use (Gee, 1999) – in the context of the mathematics and mathematics education communities. When I talk about classroom mathematics discourse, I am referring to discourse with a lower case "d". That is, I am interpreting discourse as the communication of one's thinking about mathematics to another in a conversation through the use of speech, written statements, or gestures.

discourse enhances students' opportunities to learn mathematics as characterized in the standards for K – 12 mathematics (NCTM, 1989; 2000; NGA & CCSSO, 2010). Meaningful classroom mathematics discourse supports students in discussing their approaches to mathematical problems, and affords them the opportunity to connect mathematical ideas among their solutions, identify and test conjectures, and reason about the validity of solutions. Mathematics teachers have a significant role in shaping the communication and interaction among the members of the classroom, which is essential to students learning to do mathematics (i.e. representing, connecting, reasoning, and generalizing) and learning about themselves as doers of mathematics (Franke, Kazemi, & Battey, 2007).

Teachers' Role in Supporting Meaningful Classroom Mathematics Discourse

Ways that teachers support students as they engage in and sustain meaningful mathematics discourse in secondary mathematics classrooms are more understood as a result of the growth in the body of relevant literature (Boaler & Brodie, 2004; Forman et al., 1998; Goos, 2004; Lobato, Clarke, & Ellis, 2005; Staples, 2007; Truxaw & DeFranco, 2008). The most recent research-informed set of guidelines for effective mathematics teaching (NCTM, 2014) proposes that one important way teachers support student engagement in meaningful classroom mathematics discourse is through purposeful questioning. Boaler and Brodie (2004) asserted that the range of questions a teacher uses in his or her instruction is important for "shaping the nature and flow of classroom discussion and the cognitive opportunities offered to students" (p.781). More specifically, researchers (Goos, 2004; Staples, 2007; Truxaw & DeFranco, 2008) found that teachers' continuous eliciting and gathering of evidence of what students understood during a lesson was important in the teacher diagnosing the students' mathematical understanding as well as establishing a common/shared meaning of a mathematical idea among the members of a classroom before moving forward with the lesson. Researchers (Forman et al., 1998; Goos, 2004; Truxaw & DeFranco, 2008) have also found that effective and purposeful questioning can extend students' understanding of mathematics by scaffolding students' engagement in mathematical processes and promoting student reflection and self-monitoring (e.g., question their assumptions, review approaches to identify errors). In addition, this body of research advocates teachers' purposeful questioning as one of eight mathematical teaching practices that supports students engagement in mathematics as characterized by the standards for school mathematics (CCSS-M, 2010; NCTM, 2000).

Another important way teachers may support students' engagement in meaningful classroom mathematics discourse is through "telling as initiating" (Lobato et al., 2005). Lobato et al. (2005) proposed that "telling as initiating" serves to introduce new mathematical ideas into a classroom conversation in order to "stimulate students' mathematical constructions" (p. 110) and assert that this is an action that does not occur in isolation but in conjunction with eliciting the ways in which students are interpreting information in the mathematics classroom. Teachers may engage in this form of telling when they, for example, introduce and then link mathematical terms to student generated ideas, which in turns supports communication of mathematical ideas (Goos, 2004; Lobato et al., 2005). Also, this form of telling may involve highlighting an aspect of a student's contribution to a conversation or introducing a new representation in order to support students' mathematical thinking about a new or different perspective than the one with which they are currently working (Forman et al., 1998; Goos, 2004; Lobato et al., 2005; Staples, 2007).

In addition to research and standards for mathematics teaching, standards for teacher education (CCSSO, 2013; NCTM & National Council of Accreditation of Teacher Education [NCATE], 2012) that impact beginning teachers, of which preservice secondary mathematics teachers are a subset, also attend to the teacher's role in facilitating classroom mathematics discourse. NCTM and the National Council of Accreditation of Teacher Education (NCATE) recently released the NCTM NCATE standards (2012), which included standards for secondary mathematics teacher preparation programs. A key component of the content pedagogy standard emphasizes and explicates the teacher's role in classroom mathematics discourse; specifically, preservice secondary mathematics teachers are expected to:

> Implement techniques related to student engagement and communication including selecting high quality tasks, guiding mathematical discussions, identifying key mathematical ideas, identifying and addressing student misconceptions, and employing a range of questioning strategies. (NCTM & NCATE, 2012, p. 2)

The Interstate Teacher Assessment and Support Consortium's (InTASC) *Model Core Teaching Standards and Learning Progressions for Teachers 1.0* also explicates the teacher's role in classroom discourse in the *performance* portion of the Instructional Strategies Standard: "The teacher asks questions to stimulate discussion that serves different purposes (e.g., probing for learner understanding, helping learners articulate their ideas and thinking process, stimulating curiosity, and helping learner to question)" (CCSSO, 2013, p. 43).

It is clear from this brief review of research in mathematics education as well as policy documents in mathematics education and teacher education that teachers' capacities for facilitating meaningful classroom mathematics discourse, specifically their purposeful questioning, is valued by both the mathematics education community and the teacher education community. The question then becomes: In what ways can we educate preservice mathematics teachers so that they may learn of purposeful questioning and telling as initiating? I now turn to a discussion of teacher education pedagogical approaches in order to provide further background information for this research study.

Pedagogical Approaches in Teacher Education

The teacher education standards described above provide guidelines for the content of teacher education programs concerning the facilitation of classroom discourse and other aspects of teaching, but do not prescribe particular pedagogical approaches for teacher professional coursework (i.e. methods courses). Historically, there have been a number of pedagogical approaches towards the professional coursework in teacher education (Grossman, 2005; Grossman and McDonald, 2008).

According to Grossman and McDonald (2008), in the past fifty years "research on teaching has followed a heady trajectory – moving from investigations of teacher characteristics and behaviors to a focus on teacher cognition" (p. 189). The pedagogical approaches in teacher professional education courses followed a parallel trajectory, although the trajectory has lagged behind the research on teaching (McDonald, et al., 2013). This trajectory is a shift from equipping preservice teachers (PTs)[2] with particular instructional behaviors to supporting the development of their knowledge for teaching. Grossman and McDonald name the pedagogical approaches at the extremes of this trajectory in teacher education *pedagogies of enactment* and *pedagogies of investigation*. Teacher educators who employ pedagogies of enactment focus on developing the skilled practices important to the clinical aspect of teaching and those who employ pedagogies of investigation focus on developing teachers' knowledge, beliefs, and reflection skills.

In the following section, I use the constructs of pedagogies of enactment and pedagogics of investigation to categorize different pedagogical approaches for methods courses in the last fifty years to better illustrate the historical landscape of professional coursework in teacher education. The following section is heavily influenced by Grossman's (2005) rigorous review of pedagogical approaches for methods courses.

Pedagogies of enactment. Two pedagogical approaches in teacher education, systems approach and computer simulations, illustrate ways PTs are equipped with instructional behaviors in pedagogies of enactment.

For an extended period of time – 1960s into the 1980s – the focus in teacher education courses was on developing PTs' instructional behaviors. In the 1960s to 1970s, PTs learned these behaviors in university based laboratory settings in what Peck and Tucker (1973) called a systems approach. The systems approach involved repeated cycles of performing instructional behaviors in which teacher educators (Teacher educators) specified behaviors that were the "objective of the learning experience" (Peck & Tucker, 1973, p. 943). During the cycles, the Teacher educators had PTs perform the

[2] The acronym *PT* refers to ALL preservice teachers. *Mathematics PTs* refers to ALL preservice mathematics teachers. The acronym *PSMT* refers to preservice secondary mathematics teachers and the acronym *PEMT* refers to preservice elementary mathematics teachers.

behaviors, measured the PTs' performance in terms of behavioral objectives, and provided PTs feedback on the observed results. An example of a systems approach methods course activity is microteaching. Grossman (2005) describes microteaching in the following passage:

> Most forms of microteaching focus on discrete skills of teaching and involve videotaping prospective teachers' efforts to enact these skills and then providing them with feedback on their performance. Despite the depiction of microteaching as behavior modification, the model as enacted is quite complex, involving the close observation of master teachers, the formation of concepts related to teaching, practice and feedback, as well as opportunities to learn from experience. (p. 430)

Microteaching illustrates the qualities of a systems approach in that focusing on enacting discrete skills and giving students feedback on their performance are ways that PTs perform instructional behaviors and receive feedback on performed results. One of the criticisms of microteaching is that this approach was not grounded in a clear theoretical framework for teacher learning (Grossman, 2005).

As research in teaching shifted to "teacher-as-decision-maker" (Grossman, 2005), a second pedagogical approach – computer simulations – made use of an emerging technology to continue to focus on instructional behaviors. Computer simulations, which were an extension of the laboratory setting in teacher education, "assisted preservice teachers in the acquisition of such teaching skills as classroom management, feedback to students, questioning, and the use of wait time" (Grossman, 2005, p. 433). The purpose of this approach was to simplify the "unpredictability of classrooms" and focus PTs' attention on specific instructional skills.

Pedagogies of investigation. Three pedagogical approaches in teacher education coursework, video technology and hypermedia, case methods, and inquiry projects, illustrate the ways PTs are encouraged to develop knowledge, beliefs, and a reflective stance towards teaching in pedagogies of investigation.

As research on teaching shifted from behavioral psychology to cognitive psychology, "teacher educators responded by focusing less on training models than on models that attempted to improve teachers' ability to reason and reflect on teaching" (Grossman, 2005, p. 439). Two pedagogical approaches that align with this focus during this period are "video technology and hypermedia" and "case methods" (Grossman, 2005). As Grossman explained, the use of video technology and hypermedia was a result of the microteaching approach. Video afforded replay and provided a common example of teaching that a teacher educator and PTs could collectively analyze. The other pedagogical approach, case methods, focused on PTs' reasoning about teaching and learning. Grossman explained that researchers (e.g., Careter & Unklesbay, 1989; Shulman, 1986) "proposed that the case method could help prospective teachers learn to think pedagogically, to reason through classroom dilemmas, and to explore possible actions" (p. 439).

Recently, a third pedagogical approach, inquiry projects, continued to focus on teachers' thinking and reflection by encouraging the development of a reflective/inquiry stance towards teaching. An example of a methods course activity used in this pedagogical approach is "portfolios in teacher education" (Grossman, 2005). The use of portfolios occurred around the same time as case methods and were "used both to stimulate preservice teachers' reflections on their development, as well as to assess that development, often in a summative way" (Grossman, 2005, p. 443).

Concern with recent approaches. Grossman and McDonald (2008) reacted to the pedagogical approaches of the last twenty years (e.g., case methods, portfolios, practitioner research) with the claim that professional coursework in teacher education has been dominated by pedagogies of investigation, which focus on developing teachers' knowledge for teaching. My own review of mathematics PTs' learning in methods courses and activities in mathematics methods courses (presented in Chapter 2), as would be expected, reflect this focus on the developing teachers' knowledge for teaching. Grossman and McDonald (2008) asserted that this pedagogical approach of focusing on the development of PTs' knowledge places the PTs' learning of the interactive aspects of teaching solely in hands of field placements – a context in which the teacher educator often has little involvement. As a result, a divide exists between the PTs' learning of the conceptual and relational aspects of teaching. Grossman, Hammerness, and McDonald (2009a) identified a second divide, which exists between the objectives of foundational education coursework and disciplinary methods coursework. That is, foundational education coursework typically focuses on general principles and guidelines for teaching and learning, while disciplinary methods courses, in contrast, focus on providing teachers with practical tools. Grossman et al. (2009a) argued that although the methods courses focus on instructional methods, much of the emphasis is *"about* instructional methods and less about learning to enact such practices fluidly" (p. 275). McDonald et al. (2013) echoed both of these divides and pointed out that neither of the professional approaches to coursework in the 1960s and 1970s nor the coursework of the last thirty years has "attended to what Mary Kennedy (1999) calls *the problem of enactment*, or the gap between what novices can consider and what they are able to do" (p. 379).

In response to these concerns, Grossman et al. (2009a) called for pedagogical approaches towards professional coursework in which the curricula are organized around "practices of the profession" rather than around knowledge domains. Grossman et al. suggested, "to reorganize the curriculum around a set of core practices and then help novices develop professional knowledge, and skill, as well as an emerging professional identity around these practices" (p. 277). Important to a curriculum organized around core practices are teacher educators carefully attending to "both the conceptual and practical aspects associated with any given practice" (p. 278), which involves integrating principles of teaching, academic knowledge, and the enactment of teaching. Hence, a pedagogical approach towards coursework organized around core practice(s) does not overlook the development of PTs' knowledge. Rather, this approach couples the development of PTs' knowledge with the equipment of PTs' instructional behaviors.

Pedagogies of practice are one way of conceptualizing such pedagogical approaches towards professional coursework that has gained traction in teacher education literature (Grossman et al., 2009a; Lampert et al., 2010).

Pedagogies of Practice

Mathematics teacher educators who employ pedagogies of practice focus on developing mathematics PTs' understanding of principles behind core practices that constitute teaching mathematics (e.g., equity, learning for mathematical understanding), develop mathematics PTs' abilities to enact the core practices (e.g., setting up mathematical tasks, leading mathematical discussions), and mathematical content knowledge (MCK) for school mathematics (Lampert et al., 2010; Lampert et al., 2013; McDonald et al., 2013). Pedagogies of practice simultaneously engage PTs in course activities that support their development of pedagogical content knowledge (PCK), mathematical content knowledge (MCK) (knowledge constructs proposed by Hill, Ball, & Schilling, 2008), and noticing/analysis of teaching skills, but balances and coordinates this development with necessary skills for teaching mathematics.

Grossman, Compton, Igra, Shahan, and Williamson (2009b) proposed a framework for understanding pedagogies of practice that has influenced the design and study of methods courses using these pedagogical approaches (e.g., Boerst et al., 2011; Ghousseini & Herbst, 2014; Lampert et al., 2013; McDonald et al., 2013). The framework involves three concepts that were identified from analyses of the professional education of students in three highly interactional professions (psychology, education, and clergy). The three concepts in the framework are: a) *representations of practice*, b) *decomposition of practice*, and c) *approximations of practice*. Grossman et al. (2009b) defined these concepts:

> Representations of practice comprise the different ways that practice is represented in professional education and what these various representations make visible to novices. Decomposition of practice involves breaking down practice into its constituent parts for the purposes of teaching and learning. Approximations of practice refer to opportunities for novices to engage in

> practices that are more or less proximal to the practices of a profession. (p. 2058)[3]

Researchers in teacher education (e.g., Boerst, Sleep, Ball, & Bass, 2011; Lampert, Beasley, Ghousseini, Kazemi, & Franke., 2010; Lampert et al., 2013; McDonald, et al., 2013) responded to this framework by systematically designing models/structures for teacher education activities based on decompositions of practice, representation of practices, and approximations of practice. One such model is Lampert et al.'s Cycle of Enactment and Investigation (CEI). In addition, researchers (e.g., Ghousseini, 2008; Ghousseini & Herbst, 2014; Tyminski, Zambak, Drake, & Land, 2014) have studied the opportunities mathematics PTs have in these methods course activities designed around pedagogies of practice to learn about the core practice of facilitating classroom mathematics discourse.

My dissertation study

While researchers (e.g., Boerst et al., 2011; Ghousseini, 2008; Ghousseini & Herbst, 2014) have identified opportunities for mathematics PTs to learn about the components of facilitating meaningful classroom mathematics discourse and have characterized teacher education activities in methods courses grounded on pedagogies of practice (e.g., Boerst et al., 2011; Lampert et al., 2013), literature is limited in *what* mathematics PTs learn about specific components of facilitating mathematics classroom mathematics discourse (e.g., questioning that elicits and/or extends student mathematical thinking, telling as initiating) and how decomposition of practice, representations of practice, and approximations of practices support this particular aspect of mathematics

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[3] From this point on in the dissertation report, when I refer to pedagogies of practice, I am referring to Grossman et al.'s (2009a) conception of pedagogies of practice – decompositions of practice, representations of practice, and approximations of practice.

PTs' learning. Addressing this gap is necessary in order for teacher educators to understand ways that mathematics PTs learn to integrate their knowledge and analyses of teaching with the clinical instructional skills encompassed in facilitating meaningful mathematics discourse. In addition, addressing this gap identifies in what ways different course activities in a secondary mathematics methods course incorporating pedagogies of practice support PSMTs learning about constituent parts of facilitating meaningful mathematics classroom discourse (specific contributions of this study are included at the conclusion of the literature review).

This study builds on the literature about pedagogies of practice by not only identifying opportunities to learn about constituent parts of facilitating meaningful classroom mathematics discourse (Ghousseini, 2008; Ghousseini & Herbst, 2014; Tyminksi, et al. 2014), but also examining the nature of PSMTs' learning and how teacher education activities grounded on pedagogies of practice support that learning. More specifically, this study addresses the following questions:

In a methods course designed to engage PSMTs in iterative learning cycles, framed by pedagogies of practice and focused on a particular decomposition of practice,

1. What conceptions of assessing questions, advancing questions, and judicious telling, did four PSMTs construct?
2. What were the PSMTs' pathways for constructing their conceptions across the semester?
3. In what ways were course activities, designed around decomposition of practice, representations of practice, and approximations of practice, connected to the PSMTs' construction of their conceptions?

Frameworks

The framework for this study consists of two parts. The first part of the framework is designed to explain how I am theorizing about the object of the PSMTs' learning – the Types of Teacher Talk (more thoroughly explained in a subsequent subsection). This part of the framework contains a set of constructs that describe how I analyze *what* the PSMTs learn about in the teacher education activities designed around pedagogies of practice. While the first part of the framework addresses the object of the PSMTs' learning, the second part of the framework is designed to illuminate *how* I am modeling the individual PSMTs' learning and understanding of the Types of Teacher Talk (TTT). This part of the framework is a lens through which I analyze the PSMTs' psychological activities.

Part One of the Framework: Object of Learning

I begin this section defining several constructs – practice, core practice, and decomposition of practice – that are important to the first part of the framework. These constructs were mentioned in the rationale, but not properly defined. After introducing these constructs, I explain my conceptual framework for how I am theorizing about the practices that constitute the core practice of facilitating meaningful classroom mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling).

Practices, core practice, and decomposition of practice. I interpret the word "practice" in "teaching practice" using Lampert's (2010) reference to the following definition: "The word *practice* implies things that people do, constantly and habitually" (p. 5). Lampert used this definition to characterize teaching as a collection of practices.

Lampert and others (e.g., Boerst et al., 2011; Grossman et al., 2009a) asserted that teaching can be decomposed into discrete practices that constitute the professional doings of teachers. Decomposition of practice involves identifying the habitual or ritual constituent parts of teaching (Grossman et al., 2009b; Lampert, 2010). It is important to note that although these contemporary reformers suggest teaching can be decomposed into "practices," reformers in teacher education (e.g., Lampert, 2010) emphasize that teaching involves careful and purposeful integration of different practices. In order to teach PTs practices that constitute teaching, reformers have identified several practices through research, standards, and observations of classrooms; they characterize these practices as high-leverage or core practices (Lampert, 2010). Although the definition and name for the "type" of practice (i.e. high-leverage, core) varies, Grossman et al. (2009a) identified the following common characteristics for high-leverage/core practices (I refer to these practices as core practices from here on):

- Practices that occur with high frequency in teaching;
- Practices that novices can enact in classrooms across different curricula or instructional approaches;
- Practices that novices can actually begin to master;
- Practices that allow novices to learn more about students and about teaching; and
- Practices that are research-based and have the potential to improve student achievement. (p. 277)

In addition to reformers varying in their definitions of core practices, they also vary in their conception of the grain size of core practices (e.g., Grossman et al., 2009a; Lampert, 2010). For example, Grossman et al. suggested “facilitating classroom discussions” as a core practice. However, Grossman et al. also suggested that three component practices – asking questions, monitoring student participation, and representing student ideas – constitute the practice of facilitating discussions. Hence, decomposing teaching may identify core practices such as facilitating discussions, but a core practice can be further decomposed into more particular component core practices[4].

TTT framework. My conceptual framework – TTT – is composed of three constructs from mathematics education literature that organize the way in which I conceive the object of the PSMTs’ learning. Also, the TTT framework was the key decomposition of practice used in the secondary mathematics methods course that serves as the context for this study (see Chapter 3). These three constructs are component core practices that enable a teacher to engage in facilitating meaningful classroom mathematics discourse – a less particular core practice. The three TTT are:

- Assessing Questions: Questions that “can assess what students understand about the problem (e.g., clarify what the student has done and what the student understands)” (Smith, Bill, & Hughes, 2008, p. 136).
- Advancing Questions: Questions that “help students advance towards the mathematical goals of the lesson. Teachers can extend students beyond their current thinking by pressing them to extend what they know to a new situation or think about something they are not currently thinking about” (Smith et al., 2008, p. 136).

[4] From this point in the dissertation report, the *more particular component core practices* will be referred to as *component core practices*.

- Judicious Telling: "The set of teaching actions that serve the function of stimulating students' mathematical constructions via the introduction of new mathematical ideas into a classroom conversation" (Lobato et al., 2005, p. 110).

Assessing questions, advancing questions, and judicious telling align with the different ways teachers shape classroom mathematics discourse. I argue in my rationale, three important ways teachers support students' engagement in meaningful classroom mathematics discourse – eliciting and gathering evidence of students' mathematical understanding, extending students' mathematical thinking, and telling as initiating. Assessing questions are questions that enable teachers to elicit and gather evidence of students' mathematical understanding, advancing question by nature extend students' mathematical thinking, and judicious telling is based on Lobato et al.'s construct of telling as initiating.

Part Two of the Framework: Theoretical Perspective for Teacher Learning

Although learning has been addressed in mathematics teacher education research, researchers have used different views of learning to examine teacher change and development of practice. Sowder's (2007) review of mathematics teacher development studies indicated that, generally, studies used one of two broad perspectives on learning – the psychological perspective or the sociological perspective –to frame the study of teacher change. Those who adhere to the psychological perspective view learning as cognitive development (Goldsmith & Schifter, 1997). For instance, constructivism has been used to study individual teachers' ways of thinking, ways of understanding, and dimensions of knowledge concerning mathematics and mathematics pedagogy (e.g., Doerr & Bower, 2001; Simon, 1995; Stump, 2001).

I use a constructivist view of learning to examine the PSMTs' learning and understanding from a psychological perspective. Constructivism focuses on the individual's learning process, although it does acknowledge that the individual may be interacting with people in contexts such as classrooms. While the origins of constructivism can be traced to Jean Piaget, this study is grounded on von Glasersfeld's interpretation and description of Piaget's theory. For constructivists, the learning process involves a person actively constructing one's knowledge based on one's prior experiences (von Glasersfeld, 1995; Simon, 1995). Simon (1995) elaborated on this notion in the following passage:

> Constructivism derives from a philosophical position that we as human beings have no access to an objective reality, that is, a reality independent of our ways of knowing it. Rather, we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge. Learning is the process by which human beings adapt to their experiential world. (p. 115)

Hence, a person's prior knowledge shapes the way that one encounters and interprets new situations in one's experiential world as well as the way one constructs knowledge about new situations. There are two key constructs for constructivists in encountering new situations – assimilation and accommodation.

Assimilation involves a person perceiving a situation and reducing the situation to conceptual and sensorimotor elements from which they can match with existing conceptual structures (von Glasersfeld, 1995). For radical constructivists, a person's conceptual structures may or may not match objective reality. The conceptual structure exists for the person because it is "viable" (von Glasersfeld, 1995). Simon (1995) explained, "a concept works or is viable to the extent that it does what we need it to do" (p. 115). Hence, viability emphasizes the idea that radical constructivism involves a person's previous experiences as a mediator of new situations. If these new situations are not assimilated, then there is an opportunity for a person to accommodate.

Von Glasersfeld (1995) explained accommodation in the context of an action scheme. Three components constitute the pattern of an action scheme: "(1) Recognition of a certain situation; (2) a specific activity associated with that situation; and (3) the expectation that the activity produces a certain previously experienced result" (p. 65). A person first perceives and recognizes a situation, which "is always a result of assimilation" (p.65). This encounter with the situation triggers an activity for the person that he or she has associated with the recognition pattern for the situation. At this point, the person will attempt to assimilate the result of the activity to an expected result. However, von Glasersfeld explained that if the person does not assimilate, there will be a perturbation that is an unexpected result, and the person revisits the situation. Further, von Glasersfeld stated,

> If the unexpected outcome of the activity was disappointing, one or more of the newly noticed characteristics may effect a change in the recognition pattern and thus in the conditions that will trigger the activity in the future. Alternatively, if the unexpected outcome was pleasant or interesting, a new recognition pattern may be formed to include the new characteristic." (p. 65)

For both cases, refiguring of a recognizing pattern or generation of a new recognizing pattern as a result of a perturbation "would be an act of learning and we would speak of an 'accommodation'" (von Glasersfeld, 1995, p. 66). While radical constructivists focus on the individual in the process of learning, von Glasersfeld points out that Piaget did not disregard the social context in which learning may take place. von Glasersfeld stated, "Piaget has stressed many times that the most frequent cause of accommodation is the interaction, and especially linguistic interaction, with others" (p. 66). Hence, social interactions are a frequent and rich source of perturbations.

In summary, von Glasersfeld's radical constructivist perspective of learning involves two key constructs – assimilation and accommodation. Assimilation is a person's interpretation of their experiential reality through existing conceptual structures. Accommodation is a person adding conditions to or changing conditions in one's recognition pattern of a situation in response to a perturbation, which von Glasersfeld considers to be an act of learning. Since von Glasersfeld considers recognition as a result of assimilation, I assume that recognition patterns are directly related or representative of a person's existing conceptual structures. Hence, I interpret changes in a person's recognition pattern (accommodation) as representative of changes to a person's conceptual structures. Thus, when I speak of accommodation, I am referring to the addition of constructs to or changes in existing constructs in a person's conception of an entity.

Rationale for the Psychological Perspective in this Study

The constructivist view of learning enabled me to address my research questions in three ways. First, the constructivist view positions the PSMTs as individuals who construct their conceptions based on their experiential realities. Hence, I examined *what* conceptions of the TTT the four PSMTs constructed through their experiences in course activities. Second, a key tenet of constructivism is that the act of learning is accommodation, which involves a person adding conditions to or changing conditions in recognition patterns associated with conceptual structures. This tenet afforded me a way of examining and modeling the PSMTs' pathways for constructing conceptions of TTT throughout the semester. Last, the constructivist view emphasizes learning as an activity that occurs in response to a perturbation, which is often a result of interaction with others. This important tenet of constructivism afforded me a way of examining the data to determine the course activities that immediately preceded the changes in the secondary mathematics PTs' conceptions. I explain in Chapter 3 what I consider to be evidence of conceptions, accommodation, and course activities that may have been perturbing events.

Chapter Conclusion

In this chapter, I presented an argument for PSMTs to learn component core practices of facilitating classroom mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling) and an argument for pedagogies of practice as a viable pedagogical approach to methods courses to support their learning of these component core practices. The confluence of these two arguments is the rationale for my dissertation research study. Also, I presented frameworks for the object of PSMTs' learning – assessing questions, advancing questions, and judicious telling – and the way in which I model their learning – constructivist learning perspective.

Preview of Remaining Chapters

This dissertation contains seven chapters, references, and appendices. In Chapter 2, I present a review of two bodies of literature: research on secondary mathematics teachers facilitating classroom mathematics discourse and research on mathematics PTs' learning in mathematics methods courses (which includes a section on mathematics PTs learning to facilitate classroom mathematics discourse).

In Chapter 3, I present the methods and procedures with which I conducted my research study. In Chapters 4 through 6, I present my data and findings that address my three research questions. In Chapter 4, I present my findings pertaining to the PSMTs' conceptions of assessing questions, advancing questions, and judicious telling. In Chapter 5, I present my findings focused on the PSMTs' pathways for constructing their conceptions of assessing questions, advancing questions, and judicious telling. In Chapter 6, I present my findings on the way in which the course activities connected to the PSMTs' accommodating their conceptions of assessing questions, advancing questions, and judicious telling. In addition, Chapter 6 presents a descriptive account of the key course activities. Chapter 7 contains my conclusions, implications for mathematics teacher education, and recommendations for further research.

Chapter 2

Literature Review

This literature review is composed of three main sections. In order to justify the decision to focus on the core practice of facilitating classroom mathematics discourse, I begin with a brief review of research about the role teachers have in classroom mathematics discourse that supports students' learning of mathematics, especially in secondary classrooms. Then, in order to establish a gap in the literature, I review two areas in mathematics teacher education research. First, I review literature concerning mathematics PT learning and *teacher education activities*[5] (e.g., analyses of a narrative case, designing a lesson, rehearsing teaching with peers and a teacher educator) that support mathematics PTs' learning in methods courses. Second, I review the literature on preservice and inservice mathematics teacher education concerning what and how teachers learn about facilitating classroom mathematics discourse.

Teachers Roles in Facilitating Meaningful Classroom Mathematical Discourse

As mentioned in Chapter 1, teachers have a significant role in shaping meaningful mathematics discourse in the classroom. Franke et al. (2007) emphasized this role in the following statement:

> Shaping mathematical discourse is a significant aspect of a teacher's work. How teachers and students talk with one another in the social context of the classroom is critical to what students learn about mathematics and about themselves as doers

[5] *Teacher education activity* refers to an activity or activities in mathematics methods courses that are mechanisms supporting mathematics PTs' learning about aspects of mathematics teaching and learning.

> of mathematics. What it means to do and learn mathematics is enacted through the discursive practices that form in the classroom. (p. 230)

In relation to students' mathematical thinking and learning of mathematics, researchers identified three ways (different from the previously identified assessing questions, advancing questions, and judicious telling in Chapter 1) in which teachers can support what Franke et al. called "discursive practices that form in the classroom" (p. 230) – eliciting student mathematical thinking, focusing attention on student contributions, and scaffolding strategies for mathematical processes.

Teachers elicit students' thinking to generate discussions in which students establish a common ground or shared understanding of mathematical idea(s) in a task (Staples, 2007; Truxaw & DeFranco, 2008). Truxaw and Defranco (2008) found that a teacher, during whole class discussions, asked for students' interpretations about the definition of a prime number. This led to a brief conversation, during which the group was able to establish a shared understanding of the term. Staples (2007) elicited multiple students' approaches to a homework problem in which the students were to arrive at a generalization for a pattern. The students sharing their responses provided an opportunity for the teacher to focus the class on a particular piece of the mathematics, which supported the students in establishing a common ground from which they could collectively build.

Teachers focus their attention on student contributions and position contributions in order to support students' participation and communication in mathematical conversations (Goos, 2004; O'Connor & Michaels, 1996; Staples, 2007). O'Connor & Michaels (1996) found that teachers restating or reformulating a student's verbal contribution not only clarifies the idea in the contribution, but also helps position the student as a participant in the classroom mathematics discourse. Goos (2004) and Staples (2007) found that the students in their studies entered the classrooms with little experience in participating in collaborative exploration of mathematical ideas. The teachers in these respective studies supported participation by asking the students to explain their thinking to each other (Goos, 2004) and by positioning the students to work collectively during whole group discussions (Staples, 2007).

Effective teachers scaffold strategies for conjecturing and reasoning and use reflection questions to engage students in mathematical processes, such as justifying, representing, and connecting (Goos, 2004; Truxaw & DeFranco, 2008). Goos (2004) found that the teacher "scaffolded" her students' reasoning by not only pressing the students for explanations for their conjectures and answers, but also by "leading them through strategic steps or linking ideas to previously or concurrently developed knowledge" (p. 270). Truxaw and DeFranco (2008) found that a teacher's use of metacognitive prompts helped "the students actively monitor their own thinking" (p. 512), which supported students creating conjectures and investigating the mathematical principle involved in the task.

Each of the three discursive practices, respectively, seem to support students in establishing a shared understanding of mathematical ideas in tasks, in participating and communicating mathematically, and engaging in mathematical processes. However, the image of the teacher's role in facilitating meaningful classroom mathematics discourse described in this section and in Chapter 1 (e.g., assessing questions, advancing questions, and judicious telling) is all too uncommon in many mathematics classes in the United

States (Banilower et al., 2012; Hiebert et al., 2005; Stigler & Hiebert, 1999). In addition, while the constructs of assessing questions, advancing questions, and judicious telling exist in the literature, the definitions of these constructs are underdeveloped. To complicate things further, mathematics teachers who aim to engage with their students in classroom mathematics discourse that aligns with this image may face multiple challenges, such as student participation, focusing attention on the mathematics in tasks (Silver & Smith, 1996; Smith et al., 2008), and pursuing students' explanations with subsequent questioning (Franke et al., 2009). Hence, it seems reasonable that teachers, of which PSMTs are a subset, need opportunities to learn to study representations of the ways in which teachers effectively facilitate meaningful classroom mathematics discourse as well as experiences engaging in the component core practices of facilitating classroom mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling) in order to face these challenges.

Mathematics PTs' Learning

My review of the literature on mathematics PTs' learning identified three areas on which the studies focused – analysis and investigation of mathematics teaching and learning, mathematical knowledge and understanding for teaching, and components of mathematics teaching (i.e. core practices). The following review of the literature focuses mainly on secondary mathematics methods courses, except for the section on components of mathematics teaching, which includes elementary methods courses. Each section addresses both findings concerning mathematics PTs' learning and the teacher education activities that supported the mathematics PTs' learning.

PSMTs Learning to Investigate Mathematics Teaching

In one area of pertinent literature, researchers have examined the changes in PSMTs' abilities to analyze and investigate mathematics teaching and learning as represented in videos of secondary school classrooms. In the next section, I provide a description of the way the researchers characterize these investigations accompanied by an overview of the study's focus and teacher education activities in the study. Then I review findings from the studies concerning changes in PSMTs' investigational abilities. The section concludes with ways this study will contribute to the literature.

A number of the research studies reviewed in this section include titles and descriptions of PSMTs "learning" to analyze, reflect, and notice. However, the researchers often use the term "change" within the body of the article. Hence, in my review I will use the term "change" in place of "learning."

Focus and teacher education activities in the studies. The researchers across these studies differ in how they characterize investigations of teaching and learning as well as the "changes" on which they focus. Van Es and Sherin (2002) and Star and Strickland (2008) focus on PSMTs' abilities with "noticing," which van Es and Sherin characterized as being composed of the following three components:

> (a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) pursuing what one knows about the context to reason about classroom interactions. (p. 573)

Van Es and Sherin studied how PSMTs learn to notice and interpret interactions in video-recordings of their own classroom teaching. The PSMTs used a computer software program – the Video Analysis Support Tool (VAST) – to individually analyze the video-recordings on three occasions. Van Es and Sherin conducted the study during the PSMTs' first semester of a two-semester student teaching experience.

Star and Strickland (2008) limited their definition of noticing to first component, part (a), of Van Es and Sherin's (2002) definition. This is because Star and Strickland (2008) felt that part (a) is the "most foundational [component of noticing] (particularly for preservice teachers)" (p. 111). Rather than studying how PSMTs learn to notice, as did Van Es and Sherin, Star and Strickland focused on changes in what the PSMTs noticed as a result of participating in a secondary mathematics methods course. Instead of individually analyzing videos of their own practice, as did the PSMTs in Van Es and Sherin's study, the PSMTs in Star and Strickland's study collectively watched and discussed videos of mathematics classrooms that were published as a part of The Third International Mathematics and Science Study (TIMSS). The teacher educator used a five-dimensional framework to support the PSMTs learning to notice, which focused PSMTs' observations on: classroom environment, classroom management, tasks, mathematical content, and communication.

Instead of noticing, Stockero (2008) used the term *reflection*. Stockero (2008) cites Rodgers (2002) as influencing her definition of reflection, which is, "analyzing classroom events in order to identify often subtle differences in students' mathematical understandings and the ways in which the teacher's actions contributed to them" (pp. 374-375). Stockero's study differed from the other two studies in that she investigated the nature of the changes in PSMTs' reflections as they engaged in sustained reflection on mathematics teaching and learning in a video-case curriculum. Similar to Star and

Strickland, Stockero had PSMTs collectively watch videos and analyze the videos during small and whole group discussions. Stockero's study differed from the other studies in that the video-cases were a part of a video-case curriculum that involved multiple, connected lessons.

Sanatagata et al. (2007) as well as Santagata and Angelici (2010) used the terms "observations," "analysis," and "reflections" to describe the actions PSMTs engage in during the investigation of mathematics teaching and learning. While Santagata and colleagues did not define these terms, they seem to be thinking about these terms in the same way as the other researchers. That is, the PSMTs were looking for connections between events in the classroom, such as student contributions and teacher actions, as well as reasoning about these interactions (Van Es & Sherin, 2002; Stockero, 2008). Santagata et al. (2007) studied what PSMTs may learn through the analysis of video-cases, and Santagata and Angelici (2010) studied changes in two different groups of PSMTs' analysis of video-cases. Similar to Star and Strickland (2008), the PSMTs used frameworks as a lens for analyzing TIMSS videos. Santagata and colleagues supported teachers' analyses of mathematics teaching and learning in TIMSS videos using a three-part framework. The framework focused the PSMTs' attention on different aspects of the lesson while they viewed the video three times. Each time the PSMTs watched the video they focused on one of the following three aspects of the lesson: "(1) Parts of the lesson and learning goals; (2) Students' thinking and learning; (3) Alternative teaching strategies" (Santagata et al., 2007, p. 128). Santagata and Angelici also had PSMTs individually view and analyze TIMSS video-cases. However, Santagata and Angelici conducted a comparative study between the PSMTs' use of two different frameworks –

one emphasizing reflection (Lesson Analysis Framework) and one emphasizing evaluation (Teaching Rating Framework).

In summary, although researchers differ in how they characterized the teacher education activities in which the PSMTs engaged (observing, noticing, analyzing, and reflecting on teaching and learning), most of the characterizations include identifying teacher and student behaviors, interpreting these behaviors, and hypothesizing links between a teacher's behaviors and student behaviors that are indicative of learning. Also, nearly all the researchers studied PSMTs as they engaged in carefully designed video-analysis activities. Last, the researchers that studied the secondary mathematics methods courses varied in how they structured the video analysis activities – individual versus collective analysis, use of framework versus not using a framework, and sustained opportunities across a semester versus intensive interventional activity.

Findings from this set of studies. The researchers' findings from this particular set of studies suggest three commonalities among the positive change in the PSMTs' observations and analyses of mathematics teaching and learning. First, all the researchers found a positive change in the degree of detail of what was included in the PSMTs' observations, analyses, or reflections. Stockero (2008) found a decrease in the PSMTs' low-level reflections corresponded with an increase in relatively higher-level reflections. Star and Strickland (2008) found a significant increase in specific observation skills: "particularly in teachers' ability to notice features of the classroom environment, mathematical content of a lesson, and teacher and student communication during a lesson" (p. 107). Similarly, Santagata et al. (2007) and Santagata and Angelici (2010) found the PSMTs, having experienced the interventional activity, were more critical and elaborated more on what was observed in the lesson. Also, Van Es and Sherin (2002) found that the PSMTs who used the VAST software and made the most progress

> appeared to be more interpretative in their analyses… [PSMTs] were more likely to attempt to explain what students meant when they analyzed student thinking, how a teacher move may have influenced understanding, or how the discourse evolved in the classroom. (p. 590)

Second, most of the researchers in this set of studies found a change in the PSMTs' use of evidence from the videos. This includes a positive change in PSMTs' tendency to ground reflections in evidence from the video (Santagata & Angelici, 2010; Stockero, 2008) and PSMTs "identifying specific evidence to support call-outs [noteworthy events] they made about teaching and learning" (Van Es & Sherin, 2002, p. 590).

Third, there was a change in the coupling of student thinking and corresponding teacher actions. Stockero (2008) found a positive change in PSMTs' "use of pedagogy and student thinking as a bases for analyzing teaching practice" (p. 389), while Santagata et al. (2007) reported that PSMTs, having experienced the interventional activity, "discussed student learning as a result of tasks organized for them by the teacher" (p. 131). Similar to Stockero's findings, Santagata and Angelici (2010) found that the PSMTs "used evidence of student progress to discuss teachers' decisions" (p. 348). Also, Santagata and Angelici found that the PSMTs' reasoning involved in these types of analyses led PSMTs to propose alternative instructional strategies.

In addition to the findings organized around these similar themes, the researchers also reported the following. Stockero (2008) found that early in the course PSMTs used definitive language in their analyses of student thinking, but, as the course progressed, the PSMTs' analyses of student thinking became more tentative and grounded in evidence from the video. Santagata et al. (2007) found a change in the mathematical content of the PSMTs' analyses. That is, on the pre-test, the PSMTs' comments were often about "general didactic choices" (p. 131), but on the post-test the PSMTs "used their knowledge of mathematical concepts to shed light on what they observed, or to argue for the efficacy (or lack thereof) of the teacher's choices" (p. 131). Van Es and Sherin (2002) found that "VAST appears to have supported teachers organizing their analyses around call-outs rather than chronological descriptions of the events in the video" (p. 590). The PSMTs initially organized their analysis of a teaching episode based on the chronological order of the events in the episode. However, having participated in the VAST intervention, the PSMTs organized their analyses around the call-outs and concepts embedded in the call outs.

In summary, opportunities to engage in video case analysis seems to support PSMTs in the detail of their descriptions of mathematics teaching and learning, the use of evidence to support their claims about mathematics teaching, and linking teachers' decisions and instructional moves to students' mathematical thinking.

Ways this study contributed to this area of the research literature. This study contributed to this body of literature in two ways. First, I built on this literature base by not only examining the nature of teacher education activities involving analyses of mathematics teaching but also examining the ways in which the PSMTs' interactions during these activities were connected to PSMTs accommodating their conceptions of mathematics teaching. Second, I extended this literature by examining the ways in which the analyses of mathematics was connected to PSMTs accommodating their conceptions of mathematics teaching that supports students' mathematical thinking and learning.

Changes in PSMTs' Knowledge and Understanding

I begin the following section with a review of what aspects of change in PSMTs' knowledge and understanding researchers have investigated and the activities in the methods courses in which the PSMTs' knowledge and understanding were investigated. Then, I review the findings of the studies and offer ways this study will contribute to the existing literature.

Focus and teacher education activities in the studies. In the context of secondary mathematics methods courses, researchers examined change in PSMTs' mathematical knowledge for teaching, which includes PSMTs' understanding of mathematics and mathematics teaching. Some researchers (Bowers & Doerr, 2001; Davis, 2009; Kinach, 2002) investigated changes in PSMTs' knowledge or understanding of both mathematics and mathematics teaching and learning. Kinach (2002) investigated the transformation process of PSMTs' subject matter knowledge of mathematics (SMK) to pedagogical content knowledge (PCK) as well as the change in PSMTs' instructional explanations from one level of understanding to another (instrumental to relational [Skemp, 1978]). Kinach's (2002) study involved three tasks, purposefully sequenced to focus PSMTs on developing instructional explanations for integer addition and subtraction. Kinach intended the first and second tasks in her methods course to challenge PSMTs' understanding of integer addition and subtraction, while the second and third tasks were intended to transform and sustain PSMTs' SMK and PCK. The tasks involved the PSMTs create instructional explanations for a set of eight integer addition and subtraction problems for a group of middle school students. The tasks were sequenced so that PSMTs had to confront their own conceptual difficulties about adding and subtracting integers while creating instructional explanations.

Davis (2009) also investigated changes in PSMTs' PCK and SMK, but he used the term content knowledge (CK) rather than SMK. Davis studied changes in PSMTs' PCK and CK of exponential functions as the PSMTs interacted with different mathematical textbooks during a lesson planning assignment. Davis's study involved a single activity in a secondary methods course – PSMTs planned two different lessons from two different mathematics textbooks. The PSMTs individually planned a single lesson about exponential functions using a traditional textbook and a problem-centered textbook. In order to prepare the lesson, the PSMTs were given a target unit from each of the textbooks as well as the unit preceding and following the target unit, supplementary textbook materials, front matter to the textbook, and answers to homework problems.

Rather than investigate PSMTs' knowledge, Bowers and Doerr (2001) investigated PSMTs' thinking and understanding of mathematics and mathematics teaching. Bowers and Doerr (2001) sought to "identify and understand the sources and types of change that we observed in participants' views of pedagogy and their understandings of mathematics of change [i.e. rates of change, instantaneous rates of change]" (p. 122). The study was conducted around three instructional sequences in a secondary methods course emphasizing the role of technology in learning mathematics. Bowers and Doerr incorporated the use of a mathematical software program, MathWorld, in the three instructional sequences based on a constructivist approach to learning. The first two instructional sequences involved investigations of motion in MathWorld in which PSMTs explored the Fundamental Theorem of Calculus. Similar to the purpose of the first two tasks in Kinach's (2002) study, Bowers and Doerr designed the first two instructional sequences in the methods course to "perturb the students' thinking about their views of the relation between position and velocity graphs" (p. 120) and "re-think their understanding of the relationship between rates and accumulations" (pp. 120-121). The third instructional sequence involved PSMTs designing, implementing, and reflecting on a "MathWorlds-based lesson sequence to help younger students interpret various concepts of the mathematics of change" (Bowers & Doerr, 2001, p. 120).

While the previous three sets of researchers investigated PSMTs' knowledge and understanding of both mathematics and mathematics teaching, other researchers (Jenkins, 2010; Stump, 2001) narrowed their focus on changes in PSMTs' PCK. In a secondary mathematics course, Stump (2001) investigated what PSMTs learn about students' difficulties with slope and what students learn about various representations for teaching slope. Like Bowers and Doerr (2001), Stump approached her secondary methods course from a constructivist perspective, designing and implementing teacher education activities that built on the PSMTs' prior knowledge. Several teacher education activities in the course were significant to the PSMTs' learning about students' difficulties with slope and representations for teaching slope. In the first activity, PSMTs interviewed two students, who were enrolled in a basic college mathematics course, about their understanding of slope. Then the PSMTs compared the college students' understandings in a written analysis. The other activities supported PSMTs' "repertoires of representations for teaching the concept of slope" (Stump, 2001, p. 211). These activities involved the PSMTs discussing the different types of representations for slope and learning about slope in functional situations as they completed different mathematical tasks from secondary mathematics.

Similar to Stump's (2001) investigation of PCK, Jenkins (2010) also studied PSMTs' knowledge of students' thinking and understanding of mathematics. Jenkins (2010) conducted an exploratory study to "ascertain how the process [structured interviews with students] affects PSTs' knowledge of middle school students' mathematical thinking" (p. 145). Also similar to Stump's study, Jenkins incorporated student interviews in his methods course as teacher education activities, but more extensively than Stump. In pairs, the PSMTs conducted three structured interviews with three different middle school students as the students worked on a pre-determined mathematical task. The structured interview involved three phases. First, the PSMTs constructed solutions to the mathematical tasks they gave the middle school students and anticipated students' mathematical thinking and knowing. Second, the PSMTs interviewed the middle school students and supported the students in thinking aloud. Last, the PSMTs analyzed the notes and student work to construct reports, which included interpreting the students' work through a particular learning theory.

In summary, researchers studying changes in PSMTs' knowledge and understanding of mathematics and mathematics pedagogy all focused on carefully designed teacher education activities in secondary mathematics methods courses, but varied in their conceptualizations of knowledge and understanding. The researchers studying changes in pedagogical content knowledge focused on a particular dimension – students' thinking and understanding of mathematics. These researchers differed in the breadth and depth of the activities they investigated in the secondary methods courses.

Findings from this set of studies. Three sets of the researchers reported changes in PSMTs ' knowledge or understanding of both mathematics and mathematics teaching (Bowers & Doerr, 2001; Davis, 2009; Kinach, 2002). Kinach (2002) reported that in the process of constructing instructional explanations, PSMTs moved from an instrumental understanding to a relational understanding of integer addition and subtraction. More specifically, "As the teacher-education students became more aware through Tasks 2 and 3 of the different types of learning outcomes made possible by the levels of understanding framework, their instructional explanations became less procedural and more relational" (Kinach, 2002, p. 64). Similarly, Bowers and Doerrs (2001) reported that one of the PSMTs' "pedagogical insights", which developed through the different tasks, was the PSMTs recognizing the potential value of conceptual explanations in teaching mathematics. Additionally, Bowers and Doerrs reported that PSMTs thought about tradeoffs in having students explore mathematical idea(s) before or after "mathematical formalisms" are introduced. Also similar to the PSMTs in Kinach's (2002) study, Bowers and Doerr found the PSMTs in their study developed "mathematical insights" important to secondary mathematics. For example, the PSMTs realized a "velocity graph [for a moving object] determines a family of position graphs [for the same moving object]" (Bowers & Doerr, 2001, p. 123) and began identifying the difference between average velocity and instantaneous velocity of a moving object. Davis (2009) found that PSMTs became more proficient with table to equation translations of exponential functions, but the PSMTs still had difficulties with some aspects of exponentials and exponential functions. In terms of PCK, Davis reported the

PSMTs, after interacting with the traditional textbook, became aware of problematic places for students when learning exponential properties.

Researchers (Jenkins, 2010; Stump, 2001) who focused on change in PSMTs' PCK also found PSMTs' knowledge developed as having participated in the aforementioned activities. Stump (2001) found that the interview and analysis assignment supported PSMTs learning about "students' knowledge and skills with graphs and equations" (p. 223) and "limitations in students' understanding of slope as a measure of steepness and as a measure of rate of change" (p. 223). Also, there was some evidence that the PSMTs learned about representations for teaching. At the beginning of the semester, PSMTs' knowledge of representations for teaching slope involved graphs and physical representations, but the PSMTs created lessons during the methods course that "developed the concept of slope through problems involving real-world situations" (Stump, 2001, p. 225). However, the following semester the PSMTs reverted back to the graphs and physical representations when they taught these lessons. Similar to Stump, Jenkins (2010) found PSMTs developed knowledge about students' mathematical thinking. Jenkins (2010) proposed that the anecdotal data in the PSMTs' reports "demonstrate that the process [structured interviews] develops listening skills for accessing students' mathematical thinking and awareness of the variety of ways middle school students make sense of mathematics" (p. 150).

In summary, these studies suggest carefully designed sequences of teacher education activities (Bowers & Doerr, 2001; Kinach, 2008; Stump, 2001) or individual teacher education activities (Davis, 2009; Jenkins, 2010) can support changes in PSMTs' knowledge and understanding of mathematics and mathematics pedagogy. The studies also caution that although PSMTs may exhibit growth in some dimensions of knowledge and understanding, they may still struggle in others (Davis, 2009) or in applying this knowledge in subsequent opportunities to teach (Stump, 2001).

Ways this study contributed to this area of the research literature. This study builds on this body of literature because I examined how teacher education activities are connected to “changes” in PSMTs’ knowledge of mathematics pedagogy. In addition, this study also investigated a particular component of teachers’ knowledge of mathematics teaching – teacher’s role in facilitating meaningful classroom mathematics discourse. This study extends this body of literature by contributing an understanding of PSMTs’ knowledge of mathematics pedagogy and the trajectory of PSMTs’ development of knowledge of mathematics pedagogy in relation to teacher education activities.

Mathematics PTs Learning to Enact Core Practices

In this section I review literature about PEMTs learning to enact core practices in addition to PSMTs’ learning, because of the limited number of studies focusing on PSMTs. I begin the following section by providing an overview of the focus and teacher education activities in the studies about mathematics PTs learning to enact core practices of mathematics teaching. Then I review findings from the studies and explain how this study will contribute to the existing body of literature.

Focus and teacher education activities in the studies. Researchers have investigated mathematics PTs’ learning of several different core practices of mathematics teaching in methods courses. Crespo (2003), Crespo and Sinclair (2008), and Norton and Kastberg (2012) investigated mathematics PTs posing of mathematics problems. In addition, Crespo (2003) and Crespo and Sinclair (2008) investigated factors contributing to the change in this core practice.

Crespo (2003) and Norton and Kastberg (2012) investigated changes in mathematics PTs' abilities to problem pose as mathematics PTs engaged in a specific teacher education activity – a letter exchange program with mathematics students in, respectively, elementary and high school classrooms. The PSMTs in Norton and Kastberg's study exchanged twelve letters with an Algebra II student pen pal. During the methods course, PSMTs shared student responses with peers as well as designed tasks for the Algebra II students with the help of peers and the mathematics teacher educator. Before exchanging letters, PSMTs "used an assessment form to examine and reflect upon the effectiveness of their tasks" (Norton & Kastberg, 2012, p. 115) in order to record the expected level of cognitive demand the task would elicit. When PSMTs received student responses, they inferred the actual level of cognitive demand of the task and supported inferences with evidence from the student responses. Part of the assessment form also required the PSMTs to explain reasons for discrepancies between the expected and actual levels of cognitive demand of the task.

The PEMTs in Crespo's study (2003) exchanged six math letters with an elementary student pen pal. The PEMTs were allowed to select their own mathematical tasks, but the tasks for three letters were selected from a common set of tasks from the methods course. PEMTs' interactions with the students in the letters "was supported in various ways throughout the course with debriefing sessions, reading and writing letters in collaborative groups, and weekly journal reflections" (Crespo, 2003, p. 248).

Crespo and Sinclair (2008) investigated changes in PEMTs posing of problems as the PEMTs participated in two interventions embedded with four tasks. Instead of a letter exchange program, Crespo and Sinclair (2008) used real-world and mathematical situations as a way of engaging PEMTs in problem posing and investigated the role of exploration in PEMTs' problem posing. A series of four tasks "invite[d] students to pose problems – rather than to solve or come up with an answer – from a given situation" (p. 400). The following is the situation from the first task:

> Generate three different problems with the given situation.
>
> Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles. (Crespo & Sinclair, 2008, p.402)

PEMTs were asked to individually generate problems for situations in the first two tasks without being encouraged to "explore" the context of the situation prior to creating the problems. For the third task, half of the PEMTs were encouraged and supported in the exploration of the situation before generating a problem and the other half after they generated the problem. During the fourth task, the PEMTs and the mathematics teacher educator discussed qualities of a good or interesting problem. In addition, the tasks involved situations that provided opportunities to pose problems for different mathematical topics and situations in different mediums (e.g., paper, manipulatives, computer).

Charalambous, Hill, and Ball (2011) investigated a different core practice – mathematical explanations. Charalambous et al. (2011) investigated whether PEMTs in content and methods courses could learn to provide explanations as well as what the learning entailed and what contributed to the learning. Charalambous et al. followed students in a two-term mathematics content/methods course, but most of the study seemed to involve activities from the content course. The content course was "aimed at helping PTs develop flexible understanding of important ideas and processes within the realm of number theory and operations; it also sought to offer them opportunities to practice using representations, providing explanations, and analyzing others' thinking" (Charalambous et al., 2011, p. 446). The PEMTs developed explanations for "rich problems" around which the course was structured. The following is one of the problems:

> Beth has 3 liters of soda. She wants to share the soda equally among 5 friends. What fraction of a liter will each serving be? What fraction of the soda will each serving be?" (Charalambous et al., 2011, p. 446)

The PEMTs solved the problems individually, and then explained their solution methods to peers in small groups, and groups shared solutions methods with the entire class during whole group discussions.

In summary, this set of researchers studied changes in mathematics PTs enacting core practices by examining the PTs' enactment of core practices prior to, during, and following teacher education activities. These teacher education activities occurred in settings of reduced complexity – methods courses – but still engaged the mathematics PTs in the relational aspects of teaching. That is, mathematics PTs communicated with students in school classrooms or performed core practices with peers.

Findings from this set of studies. Researchers (Crespo, 2003; Crespo & Sinclair, 2008; Norton & Kastberg, 2012) investigating mathematics PTs posing of problems determined positive change in mathematics PTs' posing of mathematics problems as well as factors of the teacher education activities associated with the changes. Crespo (2003) found the PEMTs' early problems were "mathematically straightforward" (p. 250), "tended to restrict and narrow the work the pupil needed to do" (p. 252), and were seldom solved by the PT prior to posing the problem for students. As PEMTs participated in the letter exchange program, Crespo (2003) found these characteristics of problem posing did not disappear, but "became much less prominent" (p. 259). PEMTs began to pose problems that asked students for more elaborate responses in which they were to give justifications or explanations. Also, PEMTs were not posing problems blindly, but "gave serious thought to what needed to be clarified and to ways to scaffold the pupil's work without taking away the opportunity to discover and learn something interesting" (Crespo, 2003, p. 262). The factors that seemed to support PEMTs' problem posing was the interaction with real students, in-class exploration of "non-traditional problems," and collaborating with peers in problem posing.

In their study involving letter writing, Norton & Kastberg (2012) qualitatively analyzed the change in two PSMTs' problem posing. Norton & Kastberg reported both PSMTs – Colt and Patti – initially attributed the disparity in the expected and assessed levels of cognitive demand to lack of effort on the behalf of the student. Over the course of the letter exchange, Colt and Patti differed in how they continued to attribute disparity as well as how they modified tasks. Colt "consistently attributed the disparity to effort on the part of the student" (Norton & Kastberg, 2012, p. 126), whereas Patti became more self-critical in attributing the disparity and questioned how she could better facilitate student engagement. These different perspectives led the Colt and Patti to modify tasks in different ways. Colt included "hints to tasks that previously elicited responses that he felt were disappointing" (Norton & Kastberg, 2010, p. 126). Patti inferred students' mathematical thinking given their responses in the letters, then returned to the original task and restructured the task considering the student's prior approaches.

Crespo and Sinclair (2008) report, "We found that engaging them [PEMTs] in prior exploratory work in a mathematical situation improved both the range and quality of problems they posed" (p. 412). The PEMTs posed problems for task one and two required "little cognitive work to be solved" (Crespo & Sinclair, 2008, p. 403), while the problems generated in task three generated more "reasoning" problems (referent to Vacc's scheme) that were "pedagogically richer problems" (p. 406).

In regards to instructional explanations, Charalambous et al. (2011) found that the four PEMTs investigated in the study, to varying degrees, learned to provide effective explanations. In addition, Charalambous et al.:

> develop[ed] tentative hypotheses about four factors associated with PSTs' learning to provide explanations: a strong link between PSTs' performance in providing explanations and their subject-matter knowledge; active and deliberate reflection on practice appears to also contribute to PSTs' growth in providing explanations; developing alternative images of teaching; and developing a *productive disposition* toward providing explanations and *self-confidence* for autonomously engaging in this practice. (p. 460)

In summary, these studies suggest that mathematics PTs can grow in instructional capacities through planned teacher education activities that reduce the setting of complexity, focus on particular core practices, and have mathematics PTs enact the component teaching.

Ways this study contributed to this area of the research literature. Although this study did not focus on changes in the PSMTs' instructional capacities around core practices, it did focus on changes in the PSMTs' conceptions of component core practices (e.g., assessing questions, advancing questions, and judicious telling) of facilitating classroom mathematics discourse. Hence, this study builds on these studies because it examined the nature of PSMTs' learning (i.e. constructing conceptions) of these component core practices as the PSMTs interacted in teacher education activities situated in settings of reduced complexity (e.g., approximations of practice).

Teachers' Learning to Facilitate Meaningful Classroom Mathematical Discourse

Research in preservice teacher education during the last twenty years has begun to build a knowledge base in relation to facilitating classroom mathematics discourse. In this section, I review what has been studied in preservice teacher education and report in greater detail on a set of studies about mathematics methods courses designed around pedagogies of practice. Then, to provide a better view of the landscape of facilitating classroom mathematics discourse in teacher education, I review a set of studies from in-service teacher education.

Studies in Mathematics PT Education

Researchers studying what and how PSMTs learn about facilitating classroom mathematics discourse have focused on PSMTs in their student teaching semester (Blanton, Berenson, & Norwood, 2001; Brendenfur & Frykholm, 2000; Lloyd, 2005) and in university courses prior to student teaching (Blanton, 2002; Boerst et al., 2011;

Ghousseini, 2008; Ghousseini & Herbst, 2014; Lampert et al., 2013; Rahal & Melvin, 1998; Tyminski, Zambak, Drake, & Land, 2014). I review the studies that occurred in university based mathematics methods courses, as they are the most relative to this dissertation study.

Blanton (2002) examined the change in PSMTs' notions of mathematics discourse in undergraduate mathematics courses for PSMTs. Boerst et al. (2011), Lampert et al. (2013), and Tyminski et al. (2014) conducted their studies in elementary mathematics methods courses. Boerst et al. studied the design of teacher education activities - framed by pedagogies of practice - that provided PEMTs an opportunity to learn about leading productive mathematical discussions. Both Lampert et al. and Tyminski et al. also described their elementary methods course activities designed around pedagogies of practice, but focused on, respectively, opportunities PEMTs had to learn component core practices of organizing mathematical discussions[6] and PEMTs' abilities in organizing a mathematical discussion. Rahal and Melvin (1998), Ghousseini (2008), and Ghousseini and Herbst (2014) studied PSMTs in secondary mathematics methods courses. Rahal and Melvin studied the influence of an instructor's modeling of instructional strategies for facilitating mathematics discourse on the PSMTs' understanding of the instructional strategies. Ghousseini (2008) and Ghousseini and Herbst (2014) identified opportunities course activities – designed around pedagogies of practice - provided PSMTs to learn component core practices of leading classroom mathematics discussions[7]. Within this body of research studies, I elaborate on a small number of the studies (Boerst et al., 2011;

[6] I use the phrase *organizing a mathematical discussion* rather than *facilitating classroom mathematics discourse*, because *organizing a mathematical discussion* was the phrase Tyminski et al. (2014) used to characterize the object of the PEMTs' learning .

[7] Again, I use the phrase *leading classroom mathematics discussions* because this was the phrase used by Ghousseini and Herbst (2014) to characterize the object of the PSMTs' learning.

Ghousseini, Ghousseini and Herbst, 2014; Lampert et al., 2013), which all incorporated pedagogies of practice.

Boerst et al. (2011) examined a collaborating group of elementary methods instructors' decomposition of "leading mathematical discussions" and the use of the decomposition in the design of the methods course. The methods course was designed for "novices to learn how to engage in high-leverage teaching practices and simultaneously develop a sense of why the work is done" (p. 2845). Similar to the ideas of component core practices and core practices presented in Chapter 1 (see p. 20), Boerst et al.'s decomposition of leading discussions led to an identified set of practices varying in grain size, in which more particular practices were nested within less particular practices. For example, "questions that elicit student thinking" was a more particular practice that is nested within the more particular practice of "orchestrating discussions." That is, eliciting student thinking is one way that teachers get students to participate in discussions. In addition to decomposing the practice of leading discussions, Boerst et al. reported several examples of teacher education activities that supported PEMTs in learning to lead mathematical discussions as well as assessments for evaluating PEMTs' progress. Some of the teacher education activities included "fishbowl" experiences in which the instructor modeled practices for the PEMTs who played the role of students, the group analysis and rehearsing of different questions that elicit and extend students' mathematical thinking, and the PEMTs individually enacting practices with small groups of students and a whole class of students.

Lampert et al. (2013) focused on the substance and structure of teacher educator and novice teacher interactions (TE/NT) in a particular phase of their CEI – rehearsals.

Although the study did not focus strictly on component core practices of facilitating classroom mathematical discussions, Lampert et al.'s quantitative analyses indicated that a majority of the substance of TE/NT interactions in rehearsals included the substance coded *elicit and respond* – "eliciting, interpreting, responding to student mathematical work or talk" (p. 7). In addition, a majority of interactions were structured around teacher educators giving directive feedback to the rehearsing novice teacher. The qualitative analyses illustrated ways the teacher educators provided feedback to the novice teachers during rehearsals about eliciting and responding to the "faux" students' mathematical thinking. Lampert et al. suggested, "rehearsals created rich opportunities for novices and Teacher educators to learn to navigate the social and intellectually complex demands of teaching" (p. 13).

Ghousseini (2008) investigated what PSMTs had opportunities to learn, specifically in regards to a particular teacher education activity (see Figure 2-1 for the structure of the instructional activity) that focused on PSMTs observing, investigating, rehearsing, and enacting the discourse routines.

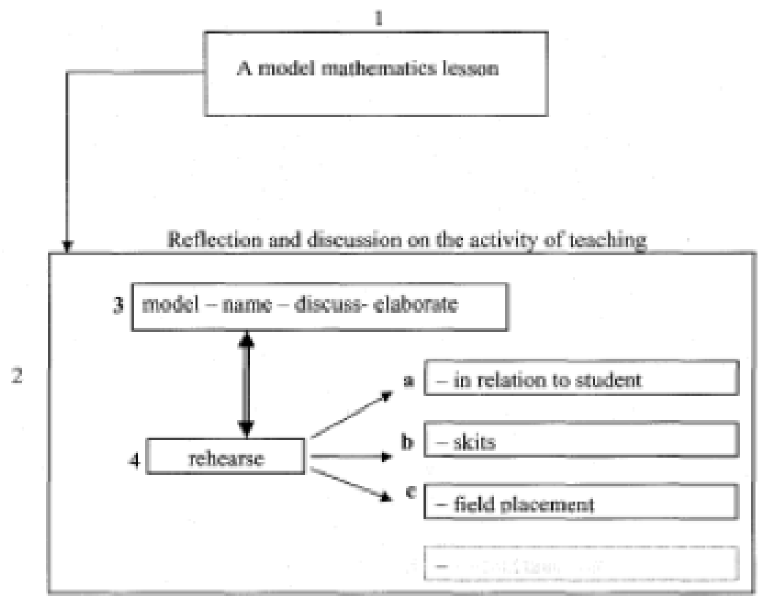


Figure 2-1: The structure of the intervention in Ghousseini (2008)

Ghousseini's (2008) analysis of PSMTs' opportunities to learn to lead mathematical discussions had two perspectives – the mathematics teacher educator's perspective and the PSMTs' perspective. The mathematics teacher educator's perspective indicated four opportunities for learning in the intervention (see Figure 2-1). First, the model mathematics lesson was an opportunity for the PSMTs to learn to engage in mathematical processes such as conjecturing and reasoning. Second, the analysis of the mathematics teacher educator's work in the model lesson was an opportunity to learn about the discourse routines that supported the PSMTs' engagement in the mathematics. Third, the practice of discourse moves in relation to the students was an opportunity for the PSMTs to learn to respond to the student mathematical thinking. Last, the skit activity was another opportunity for the PSMTs to learn to enact discourse moves and attend to student mathematical thinking. The PSMTs' perspective indicated that the model lesson was a significant opportunity for learning about mathematical reasoning, mathematics as a social construction, the nature of mathematical tasks that support reasoning, and the work that teachers do to support mathematical discussions.

Ghousseini and Herbst (2014) examined PSMTs' opportunities to learn about classroom mathematics discussions during two sessions of a secondary mathematics methods course in which the activities were designed around pedagogies of practice. These course activities included the teacher educators modeling practices related to classroom mathematics discussions (representations of practice), a discussion following the teacher educators modeling in which the group collectively labeled the key practices used by the teacher educators to lead the mathematics discussion (decomposing practice), and set of approximations of practices that engaged the PSMTs in enacting practices related to leading classroom mathematics discussions. Ghousseini and Herbst (2014) suggested that opportunities the PSMTs had to learn about classroom mathematics discussions "were distributed across the three pedagogies of practice, with each pedagogy making additional aspects of the practice available for the PSTs' understanding and investigation" (p. 22). Further, "Both pedagogies (representation and decomposition) worked in complementary ways to provide opportunities to learn about leading discussions. The approximations of practice that followed also built on that work by opening opportunities for the PSTs to put into practice the concepts and ideas highlighted in the previous pedagogies and to reflect on their work collectively" (p. 23).

Tyminski et al. (2014) examined the ways in which elementary methods course activities designed around pedagogies of practice develop PEMTs' ability to organize a mathematical discussion. The study focused on a set of course activities that included: (a) PEMTs learning about strategies and questions used to organize discussion through course readings and subsequent discussions to unpack readings (decomposition of practice); (b) PEMTs reading a narrative case and identify examples of the question types used by the teacher to organize the mathematical discussion (representation of practice); and (c) PEMTs' reviewing a set of ten sample student approaches for a mathematical task and organizing a mathematical discussion around the task and the sample students' approaches. Tyminski et al. found that the PEMTs all developed abilities to generate goals for the discussions aimed at supporting students in clarifying and communicating their thinking, to create questions used in order to connect student work, and to sequence student work in accordance to their goal for the discussion. However, Tyminski et al. found that the goals, questions, and sequencing of student work varied in levels of sophistication. For example, in regards to questioning, many of the PEMTs' questions were clarifying and leading questions (two question types from the groups' decomposition of practice) – "there was a comparative dearth of questions providing students opportunity to make mathematical connections" (p. 483).

Studies in Inservice Teacher Education

Studies concerning how inservice teachers learn to facilitate classroom mathematics discourse have been situated in professional development experiences. Researchers have examined changes in the teachers' beliefs and core practices in the classroom as they participated in professional development (Hunter, 2010; Wood, Cobb, & Yackel, 1991) and examined mathematics teachers' experiences in the professional development session (Herbel-Eisenmann, Drake, Cirillo, 2009; Peressini & Peressini, 1998).

Wood et al. (1991) studied changes in one elementary school teacher's beliefs about student learning and teaching as she participated in a teaching experiment with the researchers (who were the authors of the article). The teacher's beliefs changed as she encountered conflicts between her developing, constructivist view of learning and her interactions with students. Resolving these conflicts involved the teacher significantly changing her interactions to solicit their thinking and negotiate mathematical meaning (facilitating classroom mathematics discourse). Hunter (2010) also examined changes in one teacher's beliefs about classroom discourse in mathematics teaching and learning as well as the interconnections between those beliefs with "past experiences, and current, and future expectations for her diverse students" (p. 398). As the teacher participated in a yearlong study group, her facilitation of classroom mathematics discourse in her classroom, among other core practices, supported the diverse students "to actively participate in collective reasoning" (Hunter, 2010, p. 407). Both Wood et al. and Hunter's studies on changes in teacher's beliefs also characterized the changes in the teacher facilitating classroom mathematics discourse and the ways the classroom mathematical course promoted the students' learning of mathematics.

While these two studies mention and briefly describe aspects of the professional development, other studies (Herbel-Essenmann, Drake, & Cirillo, 2009; Peressini and Knuth, 1998) have characterized teachers' experiences in professional development concerned with classroom mathematics discourse. Peressini and Knuth (1998) studied the role and nature of the classroom mathematics discourse experienced by a secondary school mathematics teacher as he participated in an intensive summer mathematics class as well as the role and nature of discourse in the teacher's own mathematics classroom the following school year. The teacher had experiences in the course learning discrete mathematics in which the instructor modeled dialogic conversations and “expected them to think for themselves and to arrive at their own individual understandings” (Peressini & Knuth, 1998, p. 121). Recognizing the ways that the dialogic discourse supported his learning of mathematics, the teacher attempted to incorporate dialogic discourse in his own classroom. However, the teacher struggled in sustaining collective investigations of mathematical ideas and “often spoke to his students in a univocal fashion in order to align them with his thinking” (p. 121). Herbel-Eisenmann et al. extend the work of the previous studies on professional development by examining the way the teachers in the study made sense of a practice involved in facilitating classroom mathematics discourse – revoicing. Herbel-Eissenmann et al. studied a group of middle school teacher-researchers in a study group that actively met over three years. The discussions in the study group, which were grounded in the teachers’ experiences in using revoicing, contributed to the group’s development of a complex scheme that characterized the different forms of revoicing as well as the functions of revoicing. The researchers argue that the study group was a mechanism that helped the teachers critically examine a particular practice involved in facilitating classroom mathematics discourse.

In summary, mathematics teacher education has identified opportunities mathematics PTs and inservice mathematics teachers have for learning to facilitate classroom mathematics discourse. Some of the studies in both areas of mathematics teacher education focused on opportunities PSMTs and inservice had to learn in the methods course and professional development sessions. Also, mathematics teacher education is beginning to examine teacher education activities designed around pedagogies of practice and opportunities these activities provide PSMTs have to learn about the teacher's role in facilitating classroom mathematics discourse. However, few of the studies addressed in great detail the mechanisms in a university-based methods course or PD session that supported learning and even less included specific characterizations of how the teachers learned.

Ways this Study Contributed to This Area of the Research Literature

This study contributes to this body of literature in several ways. First, this study helps mathematics teacher educators more deeply understand the use of teacher education activities designed around decompositions of practice, representations of practice, and approximations of practice. This includes characterizing the nature of such teacher education activities that focus on core practices and the ways in which the teacher education course activities promoted PSMTs' learning of the core practice. Second, this study contributes to the literature on PSMTs' learning (in the context of a secondary methods course) in relation to the teacher's role in facilitating classroom mathematics discourse, specifically learning of assessing questions, advancing questions, and judicious telling. Besides identifying opportunities for learning in teacher education activities designed around pedagogies of practice, this study examined the nature of PSMTs' learning. This includes examining *what* conceptions of the component core practices for facilitating meaningful mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling) the PSMTs' construct and the pathways for their learning of these component core practices.

Chapter 3

Methods and Procedures

I begin this chapter with a description of the design of the study. Then, I explain the context of the study followed by the methods for data collection and data analysis. Last, I conclude with a description of how I established trustworthiness for this study.

Design of the Study

In order to explain the research design for the study, it is important to remind the reader of the constructivist learning perspective used to address the three research questions. As I explained at the end of chapter one, an accommodation of a person's conception of an entity is an act of learning. More specifically, an accommodation involves adding conditions to or changing conditions in a person's recognition pattern, which I interpret as representative of changes to a person's conception of an entity. Also, accommodations are preceded by perturbations, which are often interactions with others. Hence, it was necessary to design the research study so that I could focus on: (a) the PSMTs' conceptions of TTT from their perspectives; (b) changes in PSMTs' conceptions of TTT; and (c) course activities that were connected to the PSMTs' accommodating their conceptions of TTT.

The research design for this study was framed using tenets and features of the interpretive research genre (Borko, Whitcomb, and Byrnes, 2008, p.1025). Borko et al. (2008) explained, "interpretive research seeks to perceive, describe, analyze, and interpret features of a specific situation or context, preserving its complexity and communicating the perspectives of the actual participants" (p. 1025). My research study follows the interpretive research genre in many ways. Specifically, I sought to explore the understandings of four PSMTs' conceptions of assessing questions, advancing questions, and judicious telling from *their* perspective using a constructive learning perspective. This involved explicating specific facets of the PSMTs' conceptions of TTT as well as additions of facets and changes in facets throughout the semester. In addition, I sought to examine features of the context (course activities) in which the four PSMTs constructed conceptions of TTT and to elucidate the complexity of the four PSMTs' pathways for constructing conceptions of TTT (i.e., the sequence in which the PSMTs' added to and changed their conceptions of TTT).

Borko, Whitcomb, and Byrnes (2008) conveyed some of the key features of the interpretative genre in the following statement:

> Participants' voice and discourse are critical to capture, so researchers record interactions in naturalistic settings, conduct interviews, and review written artifacts such as reflective journal … Interactions and activities are documented through field notes, artifacts, audiotapes, and videotapes, thus affording researchers opportunities to identify elements of the educational settings that foster, shape, and/or constrain teacher learning. (p. 1026)

To stay consistent with these features of the interpretive research genre, I employed qualitative data collection and analysis techniques (Erickson, 1986) that focused on four case studies (Merriam, 2009) of PSMTs' learning of TTT.

Merriam (2009) asserted, "A *case study* is an in-depth description and analysis of bounded system" (p. 40). The bounded system that constituted a case in my research study was a PSMT's learning of assessing questions, advancing questions, and judicious telling for mathematics teaching as s/he was enrolled in a secondary mathematics methods course. Merriam's emphasis on *in-depth description* and *analysis* is consistent with tenets and features of the interpretive research genre and qualitative research techniques.

In the following section I first describe the context of the study. I then explain my qualitative data collection methods, followed by methods for analysis. I conclude with a description of how I established trustworthiness for this study.

Context of Study

In this section I report the setting in which this study was conducted, the participants involved in the study, and the course activities in the Fall 2013 semester of the methods course. I begin with a description of Teaching Secondary Mathematics I – the methods course around which this study was conducted. Then, I describe the participants and recruitment of participants in this study as well as my role as a researcher in the study. Last, I give an overview of the design of course activities and the sequence of course activities.

Setting

The study was conducted with a group of PSMTs enrolled in a secondary mathematics methods course at a large Mid-Atlantic university during the Fall 2013 semester. The methods course, Teaching Secondary Mathematics I, is one of three required mathematics education courses in the university's secondary mathematics teacher certification program for undergraduate students. The course is taken concurrently with a second methods course, Teaching Mathematics in Technology-

Intensive Environments, which focuses on teaching and learning secondary mathematics in technology-intensive environments.

The content and selection of teacher education activities during the semester was an iterative dance between a priori-established content and formatively-developed content. In other words, a majority of course content and teacher education activities were based on the instructor's analysis of PSMTs' current thinking and her hypotheses about how she may advance their thinking towards the course's learning goals. While the content and teacher education activities may vary from semester to semester depending on the formative feedback she received from the PSMTs, sources for the content are generally practitioner-based articles/book chapters and teacher education activities normally include analytic reflection assignments and in-class discussions. Further, the instructor designed many of the course activities using the framework of decomposition of practice, representation of practice, and approximation of practice (Grossman et al., 2009b) and organized these course activities into multiple iterations of Cycles of Enactment and Investigation (Lampert et al., 2013). I provide greater detail about assignments and other teacher education activities for the Fall 2013 course in a subsequent section and Appendix A.

Selecting the setting. Teaching Secondary Mathematics I in the Fall 2013 was purposefully selected for this study because of the instructor's decision to redesign several of the course activities using the pedagogies of practice framework (decomposition of practice, representations of practice, and approximations of practice) (Grossman et al., 2009b). In addition, the redesign included restructuring the teacher education activities in modified CEI based on Lampert et al.'s (2013) model. The modified CEI as well as the teacher education activities designed around pedagogies are thoroughly addressed in Appendix A.

Participants

My dissertation study sits within a larger research study designed to investigate a number of research questions. As such, the corpus of data includes data from seventeen PTs, of which four PTs became my case study participants. In this section, I describe the seventeen PSMTs and course instructors who participated in the study. I then explain my choice of four of those 17 PSMTs as the focus of this study. Also, I discuss my role as a researcher in the study.

PSMTs. The PSMTs were all undergraduate students who were prospective middle and secondary school mathematics teachers accepted into the university's secondary education/mathematics option certification program. Most of the PSMTs were beginning their third year at the university, and this Teaching Secondary Mathematics I course was the first methods course for all PSMTs.

Recruitment of PSMTs. At the conclusion of the first day of the methods class, I recruited all eighteen PSMTs to partake in the study. I explained the focus of the study and provided an overview of the type of data that would be collected. I emphasized that the participants' grades in the course would not be impacted by their participation (the course instructor was not present for this presentation, nor would she know who was participating in the study until after grades had been submitted at the end of the semester). The informed consent form provided them with different options as to how they may participate – video during classroom observations and/or; student assignments and/or; interviews or; neither. In addition, I explained that the participants who volunteered for the interviews would be compensated for their time (per interview) with a ten-dollar gift card to a large retail store at the completion of the semester. Seventeen of the eighteen PSMTs signed consent forms volunteering to participate in classroom observations, to participate in interviews, and to share their assignments. One of the eighteen PSMTs did not agree to participate in the study in any capacity.

Four case study PSMTs. I deliberately selected four PSMTs from the seventeen participants to be case studies of PSMTs' learning of The TTT. I selected these four participants using criteria I established through my ongoing analysis of the all the participants during the Fall 2013 semester and my observation notes made during the third interview, as follows:

- The PSMTs were not absent for a considerable number of class lessons.
- The PSMTs' description of the role of the teacher in the first interview were representative of other participants.
- The PSMTs' descriptions of the TTT in the second and third interview were representative of other participants.
- While the PSMTs selected as case studies were representative of other participants, the selected PSMTs exhibited qualitative differences in their descriptions of types of teacher in data sources across the semester.

In the following section I provide a brief description of the four PSMTs I selected as participants for the study.

Nick. Nick was present for all twenty-eight class sessions. During the first interview, Nick expressed that the mathematics instruction in his high school mathematics classes was mostly lecture-based lessons in which examples were presented to students followed by students working on problems similar to the examples. Also in the first interview, he explained mathematics instruction should include students in a pattern of discourse different from a pattern that some would describe as I-R-E (Mehan, 1979), but he did not specifically identify what the teacher and students would be doing in this discourse. During the second interview, he emphasized that the course had changed the way he thought about mathematics instruction and that he would not teach mathematics in the way that his high school teachers taught. Across the data sources, Nick actively commented and responded to questions during conversations, he raised questions during analyses activities, was reflective about engaging in approximations of practice, and his descriptions of the TTT suggested change in his conceptions.

Gretchen. Gretchen was present for all twenty-eight class sessions. During her first interview, Gretchen expressed dissatisfaction with her experience as a student in high school mathematics classes. She explained that the teacher "mostly lectured" (First Interview). She expressed that mathematics instruction should be "student centered" and "project-based," but included descriptions of the teacher's role that one would associate with direct instruction. Throughout the semester, Gretchen was significantly involved in small group conversations and her contributions suggested she was thinking deeply about the big ideas in the course. Her peers were impressed by her teaching abilities in the first rehearsal and made comments about the quality of her use of the TTT (e.g., she was a "Mini-Dr. A in the rehearsal"). My ongoing analysis indicated there was not much change in Gretchen's descriptions of the TTT after mid-semester. In addition, during the second half of the semester, Gretchen's contributions in analyses activities and group conversations were not as thoughtful as her contributions earlier in the semester.

Steve. Steve was present for twenty-seven of the class sessions. During his first interview, Steve's responses to questions were different from a number of the other PSMTs. First, he had significant experience tutoring students in mathematics and had experience teaching mathematics to secondary students during his eighty hours of classroom observation experience (requirement for the secondary certification program). Second, Steve explained that the role of the teacher should involve facilitating classroom discussions in which the teacher monitored students' work and asked open-ended questions. Steve descriptions of types of teacher in early analyses suggested changes in his conceptions of the TTT early in the course. Throughout the semester, Steve's descriptions of teaching and analyses of teaching suggested he was confident in his understanding of secondary mathematics teaching and learning. However, Steve's descriptions also suggested a vision of mathematics instruction in which the teacher was the sole authority of the mathematics in the high school classroom.

Leslie. Leslie was present for all twenty-eight class sessions. During the first interview, Leslie explained that the teacher's role teaching mathematics involved engaging and motivating students to learn mathematics by making it relevant to their lives. However, she did not include any further details as to how a teacher would engage and motivate the students. Leslie expressed difficulties in understanding the TTT early in the semester. In addition, Leslie openly expressed her difficulties enacting instruction during the first two rehearsals. Throughout the semester, Leslie demonstrated a strong desire to improve her understanding and skill to teach mathematics. Last, Leslie was often unbending in communicating her perspectives and interpretations of course readings and frameworks during class discussions.

Instructor and co-instructors. The instructor of record for the course, Dr. A, was an associate professor of mathematics education with over thirteen years of experience as a mathematics teacher educator and over ten years of experience as a high school mathematics teacher. The other two mathematics teacher educators were two doctoral candidates who were teaching assistants for the course. One of the assistants had significant experience as a high school mathematics teacher and both assistants had prior experiences as assistants and/or instructors in mathematics methods courses.

My Role as a researcher. My role as a researcher during the Fall 2013 semester involved collecting data during classroom observations, organizing data that resulted from students' assignments, scheduling and conducting interviews, organizing and securely storing collected data, and distributing the gift cards to the PSMTs at the end of the semester.

My role in the classroom during the semester was mainly as an observer, but included some instances of participating in instruction. My capacity in instruction was limited to the explanation of the teacher education activities (e.g., instructions on how to use StudioCode), handing out and collecting assignments, and handing out materials for the teacher education activities (e.g., markers, poster paper, computers). I also participated in two approximations of practice, playing the role of a typical high school student.

Teaching Secondary Mathematics I Course Activities

In this section, I explain the way the instructor framed the course activities using pedagogies of practice. I more thoroughly address the specific teacher education activities designed around pedagogies of practice and organized in modified Cycles of Enactment and Investigation in Appendix A.

Pedagogies of Practice Framework. The instructor decided to adapt existing course activities and create new course activities grounded on the Grossman et al.'s (2009b) framework for pedagogies of practice – decomposition of practice, representations of practice, and approximations of practice. I explain the way in which the three concepts of the framework were used to frame the course activities.

Decomposition of practice. One of the instructors' learning goals for the PSMTs during the Fall 2013 semester of Teaching Secondary Mathematics I was to develop a capacity to facilitate meaningful classroom mathematics discourse that supports secondary students' mathematical thinking. The instructor introduced two decompositions of practice during the course to support students in learning the component core practices of facilitating meaningful classroom mathematics discourse. Both conceptual tools were used to guide the PSMTs' learning of assessing questions, advancing questions, and judicious telling. The TTT framework involved three talk moves a teacher may use in facilitating meaningful classroom mathematics discourse: assessing questions (Smith et al., 2008); advancing questions (Smith et al., 2008); and judicious telling (Lobato et al., 2005). The second decomposition of practice, the Nine Question Type framework represented in Figure 3-1, was a nine-question typology developed by Boaler and Brodie (2004). The instructors introduced this framework in the second iteration of the modified CEI in order to help the PSMTs better define TTT.

Table 2: Teacher Questions.

Question type	Description	Examples
1. Gathering information, leading students through a method	Requires immediate answer Rehearses known facts/procedures Enables students to state facts/procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3. Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations	Where is this x on the diagram? What does probability mean?
4. Probing, getting students to explain their thinking	Asks student to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5. Generating Discussion	Solicits contributions from other members of class.	Is there another opinion about this? What did you say, Justin?
6. Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life	In what other situations could you apply this? Where else have we used this?
7. Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used	Would this work with other numbers?
8. Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics	What is the lottery? How old do you have to be to play the lottery?

Figure 3-1: Nine Question Type framework (Boaler & Brodie, 2004)

Representations of practice. The instructors of the course selected a variety of representations of practices for the PSMTs to analyze using the decompositions of practice as frameworks to guide their analyses (see Table 3-1). The instructors intended for the decompositions of practice – particularly the TTT Framework – to focus the PSMTs' analyses of the representations of practice on the ways these component core practices were enacted by the teacher and the ways in which the component core practices supported students' mathematical thinking.

Table 3-1: Representations of practice in the methods course

Media Type	**Representation of practice**	**Decomposition of practice used in analysis**
Audio-recording	Dr. A and the Staircase problem	TTT
Narrative Case	Case of Edith Hart	TTT
Audio-recording	PSMTs' first rehearsal	TTT
Narrative Case	Case of Edith Hart	Question Types
Student work samples	Students A-J work samples on the Odd + Odd = Even task.	Criteria for Valid arguments
Narrative Case	Case of Nancy Edwards	Question Types
Video-recording	PSMTs teaching in the Second Rehearsal	TTT and Question Types
Audio-recording	PSMTs teaching students at Field School	TTT and Question Types

***Approximations of practice*.** The instructor designed several different approximations of practice of varying complexity and authenticity over the course of the semester (see Table 3-2). Each of the approximations of practice listed in Table 3-2 involved the PSMTs engaging in instruction and each of these approximations of practice was coupled with a second approximation of practice – planning for the enactment. As the semester progressed the complexity and authenticity of the approximations of practice increased. That is, early in the semester the PSMTs co-taught a group of doctoral students in a methods course, which is less complex and authentic than the final approximation of practice in which each PSMT individually taught an actual high school student.

Table 3-2: Approximations of practice in the methods course

Setting	Objective for PSMTs' teaching	Length of Time	Students
University-based methods course	Support students in addressing a set of	5-8 minutes	3 doctoral students

	tasks from a middle school mathematics unit		
University-based methods course	Support a student in making progress on his/her mathematical argument	3 minutes	1 doctoral student
University-based methods course	Support students in addressing a set of tasks from a middle school mathematics unit	20 minutes	8-10 PSMTs
High School	Support student(s) in addressing a set of three tasks.	25 – 30 minutes	1-2 high school student(s)

Data Collection

In this section, I report the data collection methods for this study. The corpus of data for this study consisted of data collected in three different settings during the Fall 2013 semester: (a) the Teaching Secondary Mathematics I course at the university; (b) the interviews with the PSMTs; and (c) the PSMTs' notebooks and assignments. I first report the data sources and data collection methods for classroom observations, then report the data sources and data collection methods for the three interviews.

Classroom Observations

My report of data sources and data collections for the classroom observations is organized around five data sources – video-recordings, audio-recordings, classroom artifacts, field notes, and student assignments/notebooks – and my data collection methods. While data was collected for all twenty-eight class sessions, only data from classroom observations in Classes 3 – 15 and Classes 23 – 28 were analyzed (more thoroughly addressed in the data analysis section). Table 3-3 presents the data sources collected in Classes 3 – 15 and Classes 23 – 28. It is important to note that all video-recordings, audio-recordings, classroom artifacts and student assignments were saved in electronic form and securely stored on a computer and flash drive.

Table 3-3: Data sources for the research study

Class # and Date	**Data Source**
Class 3 Tuesday 9/03/13	Audio-recordings, Audio-recording of Dr.A, video-recording, photos of whiteboard,
Class 4 Thursday 9/05/13	Audio-recordings, Audio-recording of Dr.A, video-recording, photos of whiteboard,
Class 5: Tuesday 9/10/13	Audio-recordings, Audio-recording of Dr.A, video-recording, photos of whiteboard, StudioCode Timelines
Class 6: Thursday 9/12/13	Audio-recording, StudioCode Timeline, Digital scans of the PSMTs response sheets, Notebooks, Video-recording, Word file
Class 7 Tuesday 9/17/13	Audio-recordings, Audio-recording of Dr. A, Video-recording, Photos of the White Board
Class 7 Thursday 9/19/13	Video-recording, Audio-recording of Dr. A, Audio-recordings, TTLP in Word
Class 9 Tuesday 9/24/13	Video-recordings, Audio-Recordings, Digital Scans of Observation Sheets (Not many of the PSMTs included their names on their observation sheets)
Class 10 Thursday 9/26/13	Audio-recording, StudioCode Timelines
Class 11 10/1/13	Audio-recordings, Video-recording, Audio-recording of Dr. A
Class 12 Thursday 10/3/13	Notebooks, audio-recording, Video-Recording Audio-recording of Instructor, Notebooks
Class 13 Tuesday 10/8/13	Video-recording, photos, audio-recording of Dr. A, Notebooks, Audio-recordings, Notebooks Notebooks, TTLPs in Word
Class 25 Thursday 12/05/13	Audio-recordings, Digital Scans of Student Work
Class 26 Tuesday 12/10/13	StudioCode Timelines, Audio-recordings

Video-recordings. The video-recordings focused on whole-class discussions during the classroom observations. Two cameras were positioned on opposite sides of the rear of the classroom so that the view-finder captured as many of the classroom participants as possible. The researcher zoomed in to focus on and capture publicly displayed inscriptions on the white-boards, projected images or Power Point slide shows, and inscriptions on poster paper. At the end of each week of data collection, the researcher edited the video-recordings from an entire class session so that the collection of video-recordings in storage only included whole-class discussions, Dr. A's whole-class announcements, and rehearsals that took place in the methods courses (Class 9 and Class14). Also, at the end of each week, I saved the video-recordings to a secure computer and erased all video-recordings from the two cameras.

Audio-recordings. Audio-recordings focused on Dr. A and the small-group discussions in all twenty-eight class sessions. The digital audio-recordings (WAV file) of Dr. A were made using an audio-recorder that attached to the lapel of Dr. A's clothing. Dr. A had control of starting and stopping the audio-recorder. She started the audio-recorder at the beginning of the class, stopped the audio-recorder when she had conversations with students and instructors that addressed in-class and out-of-class topics that were not a focus of this study, and began recording when she again addressed a small group of students or the entire class.

Audio-recordings of the small group discussions, each containing two to five PSMTs, were made using digital audio-recorders. The PSMTs were often arranged around a single table and an audio-recorder was placed in the middle of each group. The co-instructors and researchers started the audio-recorders at the beginning of a small group conversation and then stopped the audio-recorders at the conclusion of a small group conversation. Many of the class sessions involved multiple small group conversations that required the co-instructors and researcher to start and stop the audio-recorders appropriately. I saved the files of the audio-recordings to a secure computer and deleted the files from the audio-recorders at the end of each week.

Classroom artifacts. I collected three forms of classroom artifacts during the semester. First, photos were taken of all PSMTs' and instructors' writing on poster paper and white boards. One of the co-instructors and I would take the photo of the artifact at the end of the class or while the PSMTs participated in small group conversations so that the class would not be interrupted. The photos were captured using the video cameras and stored as digital files (JPG files). Second, I made digital scans (PDF files) of any of the in-class handouts, which included assignments, responses to reflection questions, and responses to surveys. Third, I supplied the PSMTs with flash drives on which they saved their StudioCode timelines. At the end of Class 4, Class 5, Class 10, Class 15, and Class 26, the co-instructors and I would collect and upload the content of the flash drives. After uploading digital files from the video-camera and flash drives to a secure computer, the researcher would delete the files from both of these electronic devices at the end of the data collection week.

Field notes. I recorded field notes in Word documents using a lap top computer during and immediately after the class session. I made notes during class sessions that included attendance, information about the grouping of the PSMTs sitting at tables, location of data collection equipment, description of course activities during a class session, and a list of instructional aids used by the course instructors. In addition to the descriptive information of the setting and classroom events, I also recorded observations about instances in class that pertained to the research questions as well as the time of the event and the data collection instrument that would have recorded the instance. After each class, I briefly consulted the co-instructors about their observations of course activities and the PSMTs' engagement in the activities. One of the course instructors shared her course notes with me so that I could include her observations with my field notes.

Students' Assignments and Notebooks

At the end of the semester, I collected electronic files (docx and PDF files) of all PSMTs' assignments (except for the history of mathematics project) and the PSMTs' notebooks. I downloaded all the electronic files from the course instructors' digital folder and uploaded the files to my laptop. While I collected all the student assignments, Table 3-4 displays the assignments that I included in the corpus of data for my research study. The PSMTs' notebooks included the PSMTs' hand written notes, assignments with instructor feedback, responses to in-class writing prompts, and narrative cases that included the PSMTs' markings. Three PSMTs did not submit notebooks – one PSMT disposed of her notes at the end of the course and the other two PSMTs did not submit the notebook after twice being contacted (email and face-to-face) after the third interview. I collected fourteen of the seventeen notebooks on the last day of class, scanned each of the notebooks, uploaded the files to my laptop, and returned the notebooks to the students during the month of January in the Spring 2014 semester.

Table 3-4: Teaching Secondary Mathematics I Fall 2013 assignments

Date Assignment Announced	**Student Assignment**

8/26/13	Reading Journal #1
8/28/13	Reading Journal #2
9/5/13	Course reading (Smith et al., 2008) and reading reaction
9/10/13	Defining Features of TTT
9/12/13	Cal's Dinner Card Problem, Edith Hart Analysis #1 & Instructional Purposes of TTT #1
9/19/13	CMP Problem Set and First Rehearsal TTLP
9/26/13	Course readings (Boaler & Humphreys, 2005; Smith & Stein, 2011), nine question type categorization, Edith Hart Analysis #2, and Instructional Purpose of TTT #2
10/03/13	Nance Edwards Analysis
10/8/13	Second Rehearsal Individual TTLP
10/15/13	Second Rehearsal Reflection
11/12/13	Class 25 Approximation of Practice TTLP and Final Project Paper
12/05/13	Class 25 Approximation of Practice Individual StudioCode Analysis

Interviews

In the following sub-section I begin with an overview of when and with whom I conducted the interviews during the Fall 2013 semester. Then, for each of the three interviews, I describe my data collection methods for the interview and briefly describe the interview protocol.

When the interviews were conducted. I conducted three rounds of interviews with all seventeen participants. The first round of interviews occurred on and between 8/28/13 to 9/04/13. The second round of interviews was conducted in two sets – on and between 9/30/13 to 10/04/13 and 10/17/13 to 10/28/13. The third round of interviews was conducted after the end of the semester on and between 1/13/14 and 1/28/14. All interviews were audio-recorded using a digital audio-recorder located on a desk between the interviewer, the participant, and myself.

First Interview. The first interview was a semi-structured interview organized into two parts. The first part of the interview protocol questioned the PSMTs about their experiences in secondary mathematics, specifically their experiences, if any, with teachers who facilitated classroom mathematics discourse. The following are examples of prompts in the interview protocol:

1. To help me better understand your view of mathematics teaching and learning, it would be helpful for me to know a little bit about your experiences in secondary mathematics. What mathematics classes did you take as a high school student?
2. Do you remember any of your teachers asking you or your classmates to explain how you were thinking about a problem?
   a. What kinds of questions would (s)he ask you to get you and your classmates to explain your thinking? What kinds of explanations were expected?
   b. If someone in your class gave an answer, do you remember if your teachers ever asked others in the class whether they agreed/disagreed with the answer? (First Interview protocol). (First interview protocol)

The second part of the interview asked the participant to talk about the role of teachers and students in mathematics classrooms. The following are examples of interview prompts:

- Imagine your best day of teaching mathematics. Describe what it makes it the best day.
- What is the teacher's role in a mathematics class?
- What is the role of the students in a mathematics class? (First interview protocol)

The first round of interviews typically lasted between twenty-five and thirty-five minutes, the shortest interview lasting twenty minutes and the longest interview lasting fifty-five minutes.

Second Interview. The second round of interviews was conducted at two different times with two different groups of the PSMTs so that I could ask the PSMTs about their experiences in a rehearsal. The first set of interview participants was the eight PSMTs who were teaching pairs in the first rehearsal. The second set of interview participants was the nine PSMTs who were instructed in the Second Rehearsal and NOT in the first rehearsal.

The same interview protocol was used during both sets of interviews. The interview was semi-structured, involved a stimulated recall (Calderhead, 1981) portion, and was organized into three parts. The first part of the interview prompted the PSMTs to talk about their experiences in the Teaching Secondary Mathematics I that they found helpful in thinking about TTT and about their own understanding of TTT in teaching mathematics. The following are examples of interview prompts:

- Talk to me about your defining features for assessing questions, advancing questions, and judicious telling.
- What, if anything, about these three Types of Teacher Talk still is unclear or confusing?
- How have, if at all, course activities impacted your learning of the three Types of Teacher Talk? *[Hand participant outline of course activities]* (Second Interview Protocol)

The second part and third part of the interview, which involved stimulated recall, focused on the ways in which four PSMTs' understanding of TTT was, if at all, connected to their planning for and instruction in the rehearsals. During the second part of the interview, PSMTs were prompted to review the TTT they and their group members wrote in their TTLP. The following is an example question from the protocol, "How did your understanding of the three Types of Teacher Talk contribute to your group's crafting of the questions and judicious telling in your TTLP?" (Second interview protocol). During the third part of the interview, the PSMTs were prompted to review instances in recordings of their rehearsals. In order to prepare for the third part of the interview, I reviewed the StudioCode timelines associated with the rehearsals and selected instances to use in the interview. During the interview, I supplied the participants with a computer and had the participants control StudioCode so they may review the instance and the StudioCode Timeline Memo associated with the instance. The following is an example of a prompt from the third part of the second interview protocol:

- Say to participant: I am providing you with a video and audio clip of your rehearsal group's "teaching episode." As you are watching the clip, I would like you to pause and talk about:
    - The purpose for your questions and/or statements;
    - How did this statement or question support the students' mathematical thinking? Evidence?;
    - How did your understanding of the three Types of Teacher Talk influence your decisions about what to say? (Second interview protocol)

The second interviews were typically forty-five to fifty-five minutes in length.

Third Interview. The third interview was conducted at the beginning of the Spring 2014 semester (January) so that I could collect data on the participants' understanding of TTT and the course activities given some temporal distance from the end of the course in December 2013.

The third interview was semi-structured and the interview protocol was organized into three parts. The first part of the interview involved a writing task in which the participants reviewed outlines of course activities and were prompted to write "about the importance each of the activities had in YOUR [participant] learning about the three Types of Teacher Talk" (Third interview protocol). After the participants finished writing, I read through the documents and asked the participants to talk more about their written explanations of the importance of certain course events. I purposefully selected certain course events based on the ambiguity of the participants' written explanation.

The second part of the interview involved a card sort activity in which the participants categorized the Nine Question Types (9QTs) (Boaler & Brodie, 2004) as assessing questions, advancing questions, and judicious telling. Each question type and its corresponding description from the course reading in which the 9QTs were introduced (Boaler & Humphreys, 2005), were printed on cards (see Appendix B). The participant was prompted to categorize the question type as a type of teacher talk and to talk to me about their reason for their categorization. After the participants categorized all 9QTs, I asked the participants, "Suppose you are working with a student or group of students on the Staircase Problem. *[pause to allow the students to get into that head space]* How would you organize/sequence these questions across a lesson using the Launch-Explore-Summarize format?" (Third interview protocol).

The third part of the interview included a set of questions about the participants' understanding of the TTT and their role in facilitating classroom mathematics discourse. For example, I asked, "How do you distinguish between an assessing question and an advancing question? What is your definition of an assessing question and an advancing question?" (Third interview protocol).

The typical time length of the third interview was sixty to seventy-five minutes. Since participants had been told interviews would typically last no longer than an hour, I felt hurried at times during the third interview, which impacted the third interview in two ways. First, some of the participants were not able to write responses about the importance the final set of course activities during the third modified CEI. Instead, I had the participants to talk to me about the importance these course activities had on their learning of the TTT. Second, not all participants were able to complete the portion of the card sort activity in which they explain how they would incorporate the 9QTs in a lesson using the launch-explore-summarize format.

Analysis Methods

In this section I report my five-stage method of analysis for the corpus of data from this study. First, I explain my procedure for identifying relevant data sources for this study. Then, I report my coding scheme, my procedure for coding different data sources, and products generated from this second stage of analysis. Next, I explain the third stage of analysis that was based on methods of constant comparative analysis. Fourth, I explain the way in which I analyzed data to identify key course activities and accommodations to the PSMTs' conceptions of TTT. I conclude with my fifth stage, which was a cross-case analysis.

Stage 1: Identifying Data Sources

The first stage of my analysis identified data sources relevant to my research questions. Based on my research questions, I used two criteria to identify data sources for this study from the corpus of data collected during the Fall 2013 semester. First, I identified data sources that corresponded with course activities and assignments designed around decomposition of practice, representations of practice, and/or approximations of practice. Second, I identified the data sources corresponding with these course activities and assignments that also involved the TTT. I used these criteria to search the Course Activity Data Log (Description of Classroom Data.docx) and the Homework Assignment Log (HW Assignment Log.docx). Table 3-5 displays an excerpt from the log that includes the date of the lesson, the course activities in the lesson, and the data gathered. Table 3-6 displays two excerpts from the homework assignment log that includes the date the assignment was announced and an excerpt from the instructor's description of the assignment. I used the criteria to search the two catalogs for appropriate activities and assignments, created a chronological list of activities and assignments, and gathered data sources corresponding to the course activities and assignments.

Table 3-5: Example entry in the Course Activity Data Log

Date	**Course Activity**	**Data**
9/12/13	Small group StudioCode analysis of Dr. A and the Staircase Problem	Audio2_130912_Class 6_1-2, Audio3_130912_Class 6_1-2, Audio5_130912_Class 6_1-2, Audio6_130912_Class 6_1-2, DrA Audio StudioCode Analysis: Rehearsal1_Analysis_G1 Rehearsal1_Analysis_G2

		Rehearsal1_Analysis_G2B Rehearsal1_Analysis_G3_Coded926 Rehearsal1_Analysis_G3.TLcodes Rehearsal1_Analysis_G4, PT1_CodingAnalysis.docx
	Students work individually on a reflection assignment – group StudioCode analysis	Class6_Analysis_Ref.pdf
	Launching of defining features of TTT activity with small groups	Class6_Edited_130912_Video030_1-2, DrA Audio
	Small groups share their defining features of TTT	Audio2_130912_Class 6_2-2, Audio3_130912_Class 6_2-2, Audio5_130912_Class 6_2-2, Audio6_130912_Class 6_2-2, DrA Audio
	Whole group reporting of analysis and crafting of defining features of TTT	Class6_Edited_130912_Video030_2-2, DrA Audio

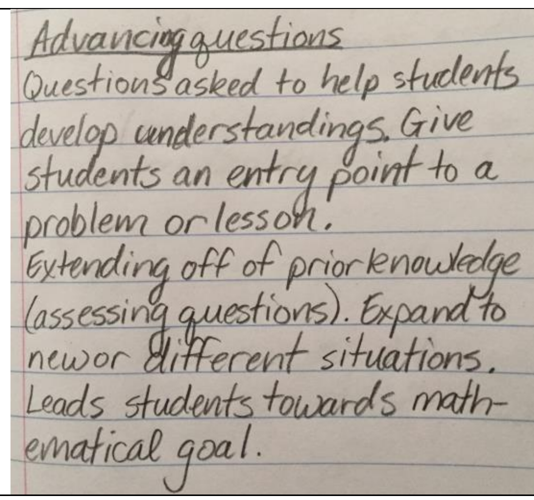
Table 3-6: Example entry in the Course Homework Assignment Log

Date	Homework Assignment
9/5/13	**TTLP Article Reactions & CCSS-M Reactions** 1. Read "Thinking Through the Lesson: Successfully Implementing High-Level Tasks" (uploaded on ANGEL with file name mtms2008-10-132a.pdf). Pay particular attention to the section Creating Questions That Assess and Advance Students' Thinking. Write a working description in your notebooks of a) Assessing Questions; and b) Advancing Questions. 2. Download the file titled CCSSI_MathStandards F13.pdf and save the document in a place where you can easily access the document at home and in class. This document contains the *Common Core State Standards for Mathematics*. Visit the website: http://www.corestandards.org/ Answer a set of questions in your notebook (see assignments folder.)
9/10	**Defining Features of TTT** 1. Download (from ANGEL) and complete the document "Teacher Talk Defining Features." We want you to focus on defining features of these three Types of Teacher Talk. in other words, how do you know it's telling/assessing/advancing when you hear it?

Stage 2: Coding

The second stage of my analysis involved coding the relevant data sources using an a priori coding scheme. Table 3-7 presents my coding scheme and dictionary, which was based on the TTT framework and composed of four codes – Assessing, Advancing, Telling, and Other.

Table 3-7: Coding scheme and dictionary

Code	Rationale	Examples using Nick's Data
Assessing Questions	Data was coded as an Assessing Questions: (a) When object of the PSMTs' and/or Dr.A's verbal statements in a conversation is an Assessing Question(s); (b) when the object of the PT's written assignment or StudioCode Timeline was an Assessing Question; and (c) when the object of the PT's verbal or written response is one of the 9QTs that the PT had categorized as an Assessing Question.	I think there are two instructional goals of Assessing Questions for a teacher whose goal is to make mathematics problematic for students. One instructional goal is to find out what students have come up with through their exploration (Homework assignment after Class 6). a. I asked a few questions in this section and all were assessing. All of these questions gathered information from the student about what he was thinking or doing or why he was doing it. b. Assess what the student was doing and why he was doing it (Class 25 Approximation of Practice StudioCode Timeline).
Advancing Questions	Data was coded as an Advancing Questions: (a) When object of the PSMTs' and/or Dr.A's verbal statements in a conversation is an Assessing Question(s); (b) when the object of the PT's written assignment or StudioCode Timeline was an Advancing Question; and (c) when the object of the PT's verbal or written response is one of the 9QTs that the PT had categorized as an Advancing	 Advancing questions Questions asked to help students develop understandings. Give students an entry point to a problem or lesson. Extending off of prior knowledge (assessing questions). Expand to new or different situations. Leads students towards math-ematical goal.

	Question.	PT1: So advancing questions. PT3: I have questions that kind of help students develop an understanding. It can give them an entry point to a problem or lesson. [Inaudible] at the beginning or during a lesson. PT1: Yeah. PT3: Just help them develop (Class 7 small group discussion).
Judicious Telling	Data was coded as Judicious Telling: (a) When object of the PSMTs' and/or Dr.A's verbal statements in a conversation is judicious telling; (b) when the object of the PT's written assignment or StudioCode Timeline was judicious telling; and (c) when the object of the PT's verbal or written response is one of the 9QTs that the PT had categorized as judicious telling.	This telling statement was like the equivalent of taking your (Interviewer who was Student 3 in the first rehearsal) pencil from you because I was literally pointing like to each number as I went along and just spoon-feeding it to you and straight out just giving you answers (Second Interview).
		Dr. A: Yeah, a mathematical definition. Right? [All the PSMTs are writing at their tables.] So telling, may have a feature of conveying a mathematical definition or putting a mathematical word or vocabulary to an idea (Class 6 Whole Class Discussion).
Other	Data was coded as Other: (a) when the object of the PSMTs' and/or Dr. A's verbal statements in a conversation was about Types of Teacher, but not specifying a specific Type of Teacher Talk; and (b) when the PSMTs self-identified a course activity or assignment that was important to their learning.	Interviewer: What course activities have influenced your understanding of the Types of Teacher Talk? Nick: Definitely uh the staircase problem. When we were were analyzing Dr. A's like questions, because that was awesome seeing. That was a real life example of a teacher working a class through a lesson and we could see advancing, assessment, telling statements. So I know that one really stuck out to me (Second Interview).

		Nick: I know when we first got introduced to them [Types of Teacher Talk] we had like a big list of maybe it was defining features or something like that. And now I guess I'm thinking more of them in terms of their purposes. When you asked me to define the features [early in the second interview] I even gave you more what I thought were purposes than defining features (Second Interview).

Using this coding scheme, I coded all the data sources for a single case in chronological order before moving onto a second case. For example, I coded all the data sources for the case of Leslie beginning with the homework assignment after Class 4 and ending with the third interview. Then, I moved onto a second case, the case of Steve. Once I coded all the data, I collected all the coded instances for each of the four codes in four Collection of Codes documents.

In the next five subsections, I first explain my coding procedures, using the aforementioned coding scheme, which varied based on the four different types of data sources (audio-recordings and video-recordings, StudioCode Timelines, notebooks, and student assignments). Then, I conclude with a subsection that more thoroughly explains the Collection of Codes documents.

Audio-recordings and video-recordings. I used StudioCode (video analysis software) to code audio-recordings of small group conversations, video-recordings of whole class discussions, and audio-recordings of interviews. My StudioCode Code Window (see Figure 3-2) was based on my a priori coding scheme and included labels for all seventeen PSMTs and instructors. I controlled my listening and/or viewing of the recordings using the StudioCode Timeline (see Figure 3-3). In addition, the Timeline also displayed the instances in the recording that I coded using the Code Window. After I coded a recording, I returned to each of the coded instances to: (a) write an overview of the instance in the Memo function of the Timeline (see box in the upper right of Figure 3-3); (b) identify whether one of the four participants were involved in the conversation; and (c) transcribed the instance in the Timeline Memo.

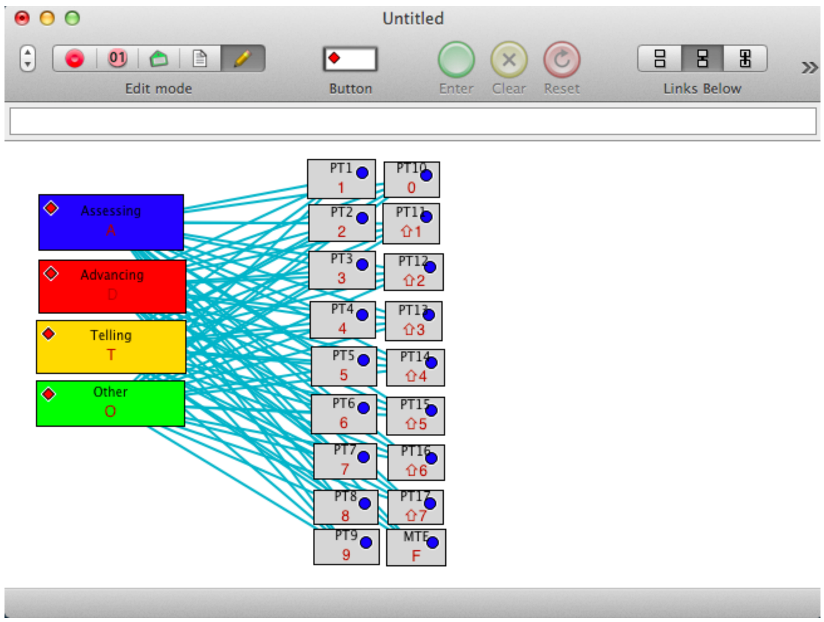


Figure 3-2: Example of a StudioCode CodeWindow

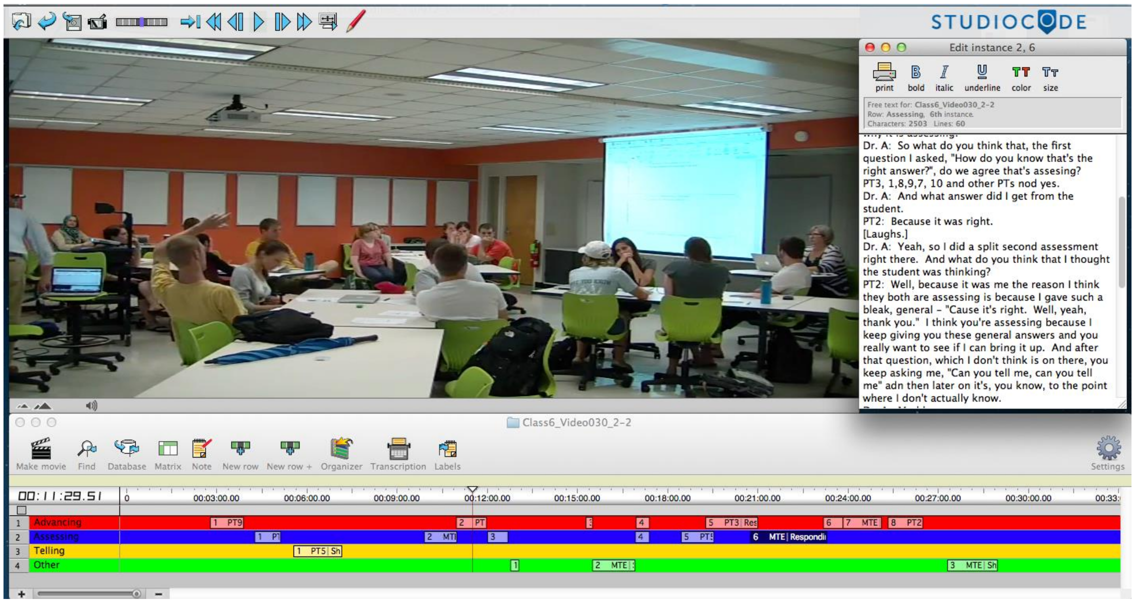


Figure 3-3: Example of a StudioCode Timeline, Recording Window, and Memo Box

After I coded each data source, I exported the Timeline Memos to a Word document and annotated the document to include referents such as the content of StudioCode Timelines the groups were writing, the transcript of an audio-recording or video-recording the group that the group was discussing, and inscriptions on the whiteboard. I would then use the comment feature of Word to code the data a second time. This second pass coded instances in the transcripts in which one of the four participants was speaking or present in the conversation. In the comment box I wrote a low-inference memo that included: lesson number, the course activity, excerpts from the transcript, referents in the conversation (e.g., the transcript of the audio-recording that the group was discussing, inscriptions on the whiteboard).

StudioCode Timelines. The PSMTs wrote StudioCode Timeline Memos during their analyses of Dr. A and the Staircase Problem, the first rehearsal, the Second Rehearsal, and the Class 25 Approximation of Practice. I exported the Timeline Memos to Word documents in order to code the documents and write memos using the comment feature of Word. I wrote low-inference memos that included excerpts from the content of the Timeline Memos and the transcript from the recording to which the Timeline Memo was associated.

Notebook. Digital files were made of the student notebooks and I viewed the files using a PDF reader. I coded data in the notebooks by digitally selecting and “cutting” an image, then “pasting” the image in Word. Similar to my other coding procedures in Word, I used the comment function to code the image in word. I wrote low-inference memos in the comment box that included the date of the notebook entry and excerpts of the content of the notebook entry.

Student Assignments. All student assignments were Word documents. Similar to my other coding procedures in Word, I coded passages in the written documents using the comment feature. I wrote low-inference memos in the comment box that included the name of the assignment and excerpts of the content of the passage.

Collection of Coded Data. Once all the data sources had been coded, I created four documents that were collections of coded data – *Collection of Codes*. Each of the documents corresponded with one of the codes – Assessing Questions, Advancing Questions, Judicious Telling, and Other. I created the documents by returning to the Word documents that corresponded with a data source and contained the coded instances from that data source. Next, I reviewed these documents in chronological order and sorted all the instances coded Assessing Questions into a collection in a Word document (I named this Collection of Assessing Codes). I repeated this process for Advancing Questions, Judicious Telling, and Other. Each of these Collection of Codes included the coded data for one of the respective codes arranged in the chronological order. I conducted the next phase of my analysis using the four Collections of Codes.

Stage 3: Constant Comparative Analysis

The third stage of analysis was based on constant comparative analyses (Strauss & Corbin, 1990) in order to search for patterns and themes among the coded instances for each of my cases. Similar to coding, this stage of my analysis proceeded linearly for each single case. For example, I began with the case of Leslie and first analyzed her Collection of Assessing Codes, then her Collection of Advancing Codes, followed by her Collection of Judicious Telling Codes, and ended with Collection of Other Codes. Proceeding in this manner enabled me to identify clusters of data related to Leslie's conception of Assessing Questions, Advancing Questions, and Judicious Telling.

To better communicate about this stage of my analysis, I refer to the case of Leslie and the Collection of Assessing Codes. My constant comparative analysis of the coded instances involved two main passes of the coded instances in Leslie's Collection of Assessing Codes to search for and identify commonalities, differences, and patterns in the data so that I could model components of Leslie's conception of assessing questions.

During my first pass of the data in Leslie's Collection of Assessing Codes, I began to sort the data into clusters, which were groups of the coded instances that exhibited common feature(s). In addition, I wrote working definitions that characterized the common feature(s) of a cluster and the ways in which the commonality differed from those of other clusters. As these clusters began to form, I interpreted the data in the clusters as indicators of a component of Leslie's conception of assessing questions, which

I call a *facet*. My working definition of the clusters became my way of characterizes the “acets of Leslie’s conception of assessing questions.

On my second pass of the data, which was now organized in clusters associated with facets (see row 1 in Table 3-8), I organized the data into a table and wrote the way in which each datum was an indicator of the facet (see row 2 in Table 3-8). The creation of these tables was important for my analysis in three ways. First, it enabled a structured way of identifying data within a cluster that were and were not indicators of a facet. Second, it led me to further differentiate facets into more particular facets (see Figure 3-4) so that I could more accurately model Leslie’s conception of assessing questions. Third, I was able to more accurately write well-defined definitions for each facet of Leslie’s conception of assessing questions.

Table 3-8: Example of stage 3 analysis table for organizing a cluster of descriptions

<table>
<tr><td colspan="2">Facet 1.5: Assessing questions determine student difficulties
Description: Assessing questions determine what students do not understand or are questions used to elicit students' thinking when the teacher perceives the student to be "stuck."</td></tr>
<tr><td>Data Source</td><td>Data</td></tr>
<tr><td>Class 5 small group discussion about definitions</td><td>In the small group discussion, PT6 shares his definition of Assessing questions, "you also are trying to find those holes which need to be filled. Like those things that you didn't necessarily project as strongly as you should have been. Because sometimes you don't realize that you missed a particular concept until you start asking students what they got from a lesson and then all of a sudden it's obvious that you missed something." Leslie responds, "So, you like find the gaps [essentially.]"</td></tr>
<tr><td>Homework after Class 6</td><td>Leslie wrote the following in her instructional purpose:
"The instructional purpose of Assessing Question is to find any gaps in the students' learning that may need more explanation or further clarification."</td></tr>
<tr><td>Homework after Class 25</td><td>Leslie wrote the following in her purpose for her Assessing Question "can you explain to me one more time what you did when you were putting this formula together":
<blockquote>When he created his own formula for the first part of the problem, I figured that it would be useful in talking through that in order to get a general formula. He was getting stuck with the general formula, so I wanted him to tell me what his thought process was when he came up with the formula he previously used, even though it was based on n=4, which would be difficult to use on n=1 through n=3.</blockquote></td></tr>
<tr><td>Third Interview</td><td>Leslie's response to the interviewer's question about how her understanding influenced her interaction with the student in rehearsal three:
<blockquote>Well, I began with assessing questions to see if he knew what he was doing. um any time that he got stuck I want to say or yeah stuck or he was quiet for a while and really thinking and a decent amount of time passed.</blockquote></td></tr>
</table>

In row 1 and row 2 "find the gaps" indicates that assessing questions determine what the students lack in their understanding of mathematical ideas. In row 3 Leslie's statements "he was getting stuck" and " I wanted him to tell me what his thought process was …" indicate that assessing questions are used when students have difficulties so that the teacher may elicit the students' mathematical thinking. In row 4 Leslie's statement "any time that he got stuck" indicates that assessing questions are used when students have difficulties with a task.

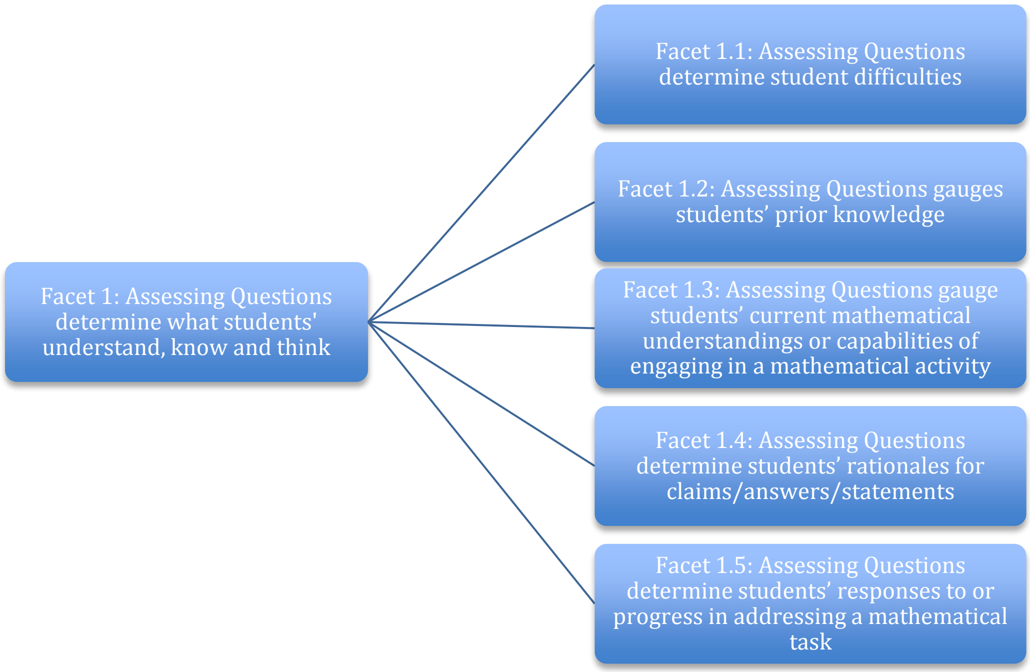


Figure 3-4: Example of more particular facets of PSMTs' conception of TTT

My analyses of the remaining Collection of Events documents proceeded in a similar way for Leslie and the other three participants.

Stage 4: Analysis for Key Course Activities. Key course activities were course activities (e.g., homework assignments, small group discussions, in-class analysis of teaching) that included or immediately preceded data indicating an addition of a facet or change in a facet of the PSMTs' conceptions (accommodation) of assessing questions, advancing questions, and judicious telling. Hence, it was necessary to identify two types of changes in the PSMTs' descriptions of the TTT (I explained in Stage 2 and Stage 3 that I interpreted descriptions as indicators of facets of the PSMTs' conceptions of TTT). One type of change in the PSMTs' descriptions was the emergence of new qualities or functions of the TTT. I interpreted this as the *addition of a facet* to the PSMTs' conception of a TTT. The other type of change was the differentiation of an existing facet into more particular facets (see Figure 3-4). I interpreted this as a *change to a facet* in the PSMTs' conception of a TTT. Since the key course activity included or preceded data indicating an accommodation, a key course activity may have been a perturbing event for the PSMTs.

As with the last section, I use the case of Leslie, specifically the key course activities connected to the facets of her conception of assessing questions, to illustrate the fourth stage of my analysis – the procedure for searching for and identifying key course activities.

The fourth stage of my analysis was based on an analytical model for identifying *critical events* for participants in longitudinal studies (Powell, Francisco, & Maher, 2003). Similar to Stohl Lee's (2005) use of this model, I considered critical events to be course activities in which I found evidence of an accommodation or course activities immediately preceding a data source that included evidence of an accommodation. This stage of my analysis involved analyzing multiple documents associated with Leslie's Assessing Question codes in order to identify and characterize the course activities that preceded the addition of and changes in each facet of Leslie's conception of assessing questions.

I began by creating a list of data sources in which an indicator of a facet first emerged (see Table 3-9). In order to create this list, I reviewed the Excel Tables (see Figure 3-5 for example) to identify the data source in which the initial datum for the assessing questions cluster(s) occurred.

Table 3-9: Example of a list of course activities used in stage 4 of the analysis

Evidence in Class 5
Facet A Assessing questions determine what students' understand, know, and think
Facet A.1 Assessing questions gauges students' prior knowledge
Facet A.4 Assessing questions determine students' rationales for claims/answers/statements

Facet A.5 Assessing questions determine student difficulties
Facet B Assessing questions encourage students' metacognition
Evidence in Class 6
Facet A.3 Assessing questions gauge students' current mathematical understandings or capabilities of engaging in a mathematical activity
Facet D Assessing questions inform teachers how to proceed in a lesson
Evidence in Class 8
Facet A.2 Assessing questions determine students' responses to a mathematical task
Evidence in Class 10 HW
Facet A.2.1 Assessing questions determine students progress in a mathematical task
Facet D.1 Assessing questions set up advancing questions
Evidence in Int 2
Facet C Assessing questions guide students' mathematical thinking
Facet F Assessing questions require teachers to stop and listen
Evidence in Int 3
Facet G Assessing questions encourage student-to-student discussions

I	J	K	L
	Assessing questions encourage students' reflection	Assessing questions guide students' mathematical thinking	Assessing questions inform teachers how to proceed in a lesson
HW Assignment			
Class 5	X		
HW Assignment			
Class 6	X		X
HW Assignment	X		X
Class 7			
Class 8			
Class 9			
Class 10			
HW Assignment			
Class 11			
HW Assignment			
Interview 2		X	X
Class 13			
HW assignment			
Class 14			
Class 15		X	
HW Assignment			
Class 24			
HW Assignment			
Class 25			
HW Assignment			
Class 26		X	X
Final Project		X	X
Interview 3		X	

Figure 3-5: Sample of Excel Table used in stage 4 of the analysis

Next, I used the list to guide multiple passes of the original coded documents and the Collection of Events documents in order to search for and identify data indicating that a course activity was a key course activity and may have been a perturbing event. These passes are reported in detail in the following paragraph.

First, I reviewed the Collection of Other Codes to identify data in which Leslie identified course activities that were important for her learning of the TTT and course activities that I coded as important for her learning of TTT. Next, I examined the original coded documents and the Collection of Leslie's Assessing Question Codes to search for and identify conversations in which Leslie either expressed confusion, responded to a different member of the class introducing a new quality of an assessing question, or was present in a conversation in which a member of the class introduced a new quality of an assessing question. If I was unable to determine evidence in the data source of a perturbation for Leslie, I repeated the analysis in the data source that immediately preceded the one that I had initially analyzed. Last, once I identified data indicating a course activity may have been key course activity, I searched for data corroborating my finding. This involved continuing to analyze the data source within which the indicator of the key course activity was identified, the data sources immediately preceding and following the data source that included the indicator, and the data from the Collection of Other Codes document. I reported in my analysis when I was unable to identify a key course activity for a facet or could not corroborate the initial data indicating a key course activity.

My analyses proceeded in a similar way for the perturbing events connected to the facets of Leslie's conceptions of advancing questions and judicious telling as well as the other participants.

Stage 5: Cross-Case Analysis

The fifth and final stage of my analysis was the cross-case analysis of my four case studies – Leslie, Steve, Gretchen, and Nick. I report my approach for the cross-case analysis in the following sub-section using examples from the portion of my analysis focused on PSMTs' conceptions of assessing questions. My cross-case analysis used a strategy Miles, Huberman, and Saldana (2013) called *stacking comparable cases* that incorporates both *case-oriented strategies* and *variable-oriented strategies*. First, I report the way in which I analyzed the cases for themes among the PSMTs' conceptions of assessing. Then, I report my analysis of the cases concerning the PSMTs' pathways for constructing the conceptions of assessing questions and the key course activities connected to the PSMTs' conceptions of assessing questions.

First, after I spent significant time analyzing each participant's conception of assessing questions in depth, I incorporated a tenet of the *variable oriented strategy* by looking for "themes cut across cases" (Miles et al., 2013, p.103). For example, Table 3-10 displays one theme I identified among the facets of the participants' conceptions of assessing questions. Besides the PSMTs' facets of assessing questions associated with the theme, I also include the perturbing course event associated with the facet and analytical memos characterizing the ways in which the PSMTs' facets associated with the theme were similar or differed. Ultimately, these individual tables and memos enabled me to report *what* qualities and functions of assessing questions were similar among the facets of PSMTs' conceptions. Also, the tables and memos enabled me to report any differences among the ways in which PSMTs' facets addressed the relation of assessing questions to students' mathematical thinking and/or learning.

Table 3-10: Example table for a theme in cross-case analysis

Assessing question Theme 1: Assessing questions determine students' mathematical understanding, thinking, and/or knowing		
Case	Facet	Key Course Activity
Leslie	1	HW after Class 4
Steve	1	HW after Class 4
Gretchen	1	HW after Class 4
Nick	1	HW after Class 4
All four of the PSMTs conceptions of assessing questions included facets that addressed assessing questions determine students' mathematical understanding, thinking, and knowing. All four of the PSMTs further differentiated this facet of their conceptions into more particular facets. However, some of the more particular facets shared similarities while other did not.		

Next, similar to a method described in Miles et al. (2013), I stacked the matrices organized around themes into a *meta-matrix* (see Table 3-11), which enabled me to perform *systematic comparison* of when the PSMTs added and changed facets of their conceptions of assessing questions (accommodated) as well as the key course events connected to these facets. The entries in this meta-matrix were two pieces of information: (a) the PSMT's facet(s) associated with each of the themes; and (b) the key course activities that preceded the PSMT's facets (i.e., course activities that may have caused perturbations). These comparisons resulted in two products. First, I constructed maps illustrating the PSMTs' pathways for constructing the features of assessing questions. Second, I identified similarities and differences about which key course activities were connected to the PSMTs' construction of facets in the themes.

Table 3-11: Example of meta-matrix (Miles et al., 2013) in the cross-case analysis

Assessing question themes.	Case			
	Steve	Gretchen	Leslie	Nick
Theme 1: Determine what students know and think	Facet 1; Reading After course 4 Throughout	Facet 1; Reading after course 4 Throughout	Facet 1; Reading after course 4 Throughout	Facet 1; Reading after course 4 Throughout
Theme 2: Inform teachers' subsequent instruction	Facet 2; Class 5 working def disc	Facet 5; Class 8 Planning Mid-semester. Facet 3.1; class 14 rehearsal	Facet 3, 3.1; Class 5/6 SmGp analysis	Facet 3; Class 5 working def disc
Theme 3: Support Students' Reflection	Facet 3; Class 5 Working def disc	Facet 2; Class 5 Working def disc	Facet 2; Class 5 working def disc	Facet 4; Class 5 working def disc
Theme 4: Role in whole class discussions	Facet 4; Class 5/6 SmGp analysis	Facet 4; Edith Hart Analysis	No facet associated with this theme	Facet 7-7.1; Edith Hart Analyis
Theme 5: Guide Students' mathematical thinking	Facet 7; Class 14 Second Rehearsal	Facet 6, Unknown	Facet 4; Class 9 First Rehearsal	No facet associated with this theme

Trustworthiness

I established trustworthiness of this study by checking evidence analyzed from one data source using data collected from other sources (Bogdan & Biklen, 2006). There are at least four data sources for addressing each of my research questions. As reported in this chapter, the method of analysis for the corpus of data was conducted in a manner that aligned with both parts of the frameworks (TTT framework and constructivist learning perspective) informing this study. Evidence from my analysis of student interviews was checked using data from classroom observations and student work. Evidence from my analysis of student assignments was checked using data from classroom observations and student interviews. Evidence from my analysis of classroom observations was checked using data from student interviews, student assignments, and planning sessions. In addition, I met weekly with two dissertation writing groups that focused on critically and constructively examining multiple aspects of our studies, particularly methods of analysis. After extensive explanations of the research design for this study, members of the two dissertation groups generated feedback that helped verify and clarify my methods of analysis.

Chapter Conclusion

In this chapter I presented the methods and procedures I used to conduct my dissertation research study. In following chapters, I present the data, findings, conclusions, and recommendation that were shaped by these methods and procedures.

Chapter 4

PSMTs' Conceptions of TTT

In this chapter, I present my findings from the case and cross-case analysis of what conceptions of assessing questions, advancing questions, and judicious telling the four PSMTs constructed. I begin with a brief overview of my findings from the case analyses of each of the four PSMTs' conceptions of assessing questions, advancing questions, and judicious telling. Then, I present my findings from the cross-case analysis of the four PSMTs.

The Four PSMTs' Conceptions of TTT

Throughout the semester, the four PSMTs constructed conceptions of TTT that included different facets that addressed both qualities and functions of TTT in mathematics teaching. In addition, some of the PSMTs further differentiated facets of their conceptions into more particular facets. For example, Steve's initial conception of assessing questions included the facet *assessing questions determine what students understand, know, and think*. As the semester progressed, Steve differentiated this facet into three, more particular facets that addressed what about the students' mathematical thinking, understanding, and/knowledge that the assessing questions support teachers in determining – *assessing questions determine students' justifications or approach for an answer to a task, assessing questions gauge students' mathematical understandings, Assessing question determine students' rationales or justifications for their work as they engage in a task* (see Figure 4-1).

Figure 4-1 through Figure 4-12 are diagrams that model Steve's, Leslie's, Gretchen's, and Nick's multi-faceted conceptions of assessing questions, advancing questions, and judicious telling. In the diagrams, the different columns of facets represent different degrees in the particularity of the facets. For example, consider, again, Steve's conception of assessing questions represented in Figure 4-1. The third column contains a set of more particular facets (e.g., *assessing questions determine students' justifications or approach for an answer to a task, assessing questions gauge students' mathematical understandings, assessing questions determine students' rationales or justifications for their work as they engage in a task)* related to a less particular facet in the second column (e.g., *assessing questions determine what students understand, know, and think*).

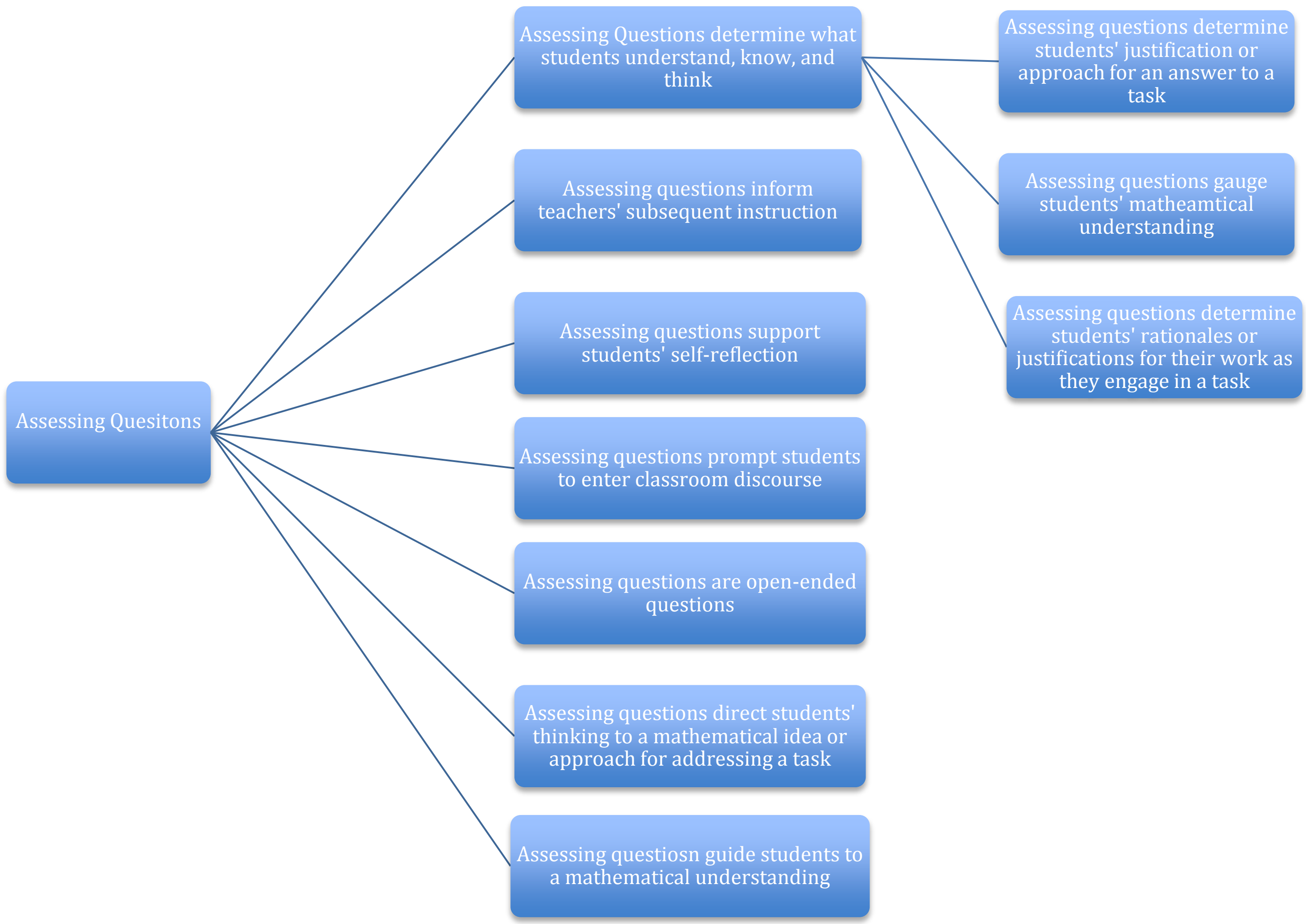


Figure 4-1: Steve's conception of assessing questions

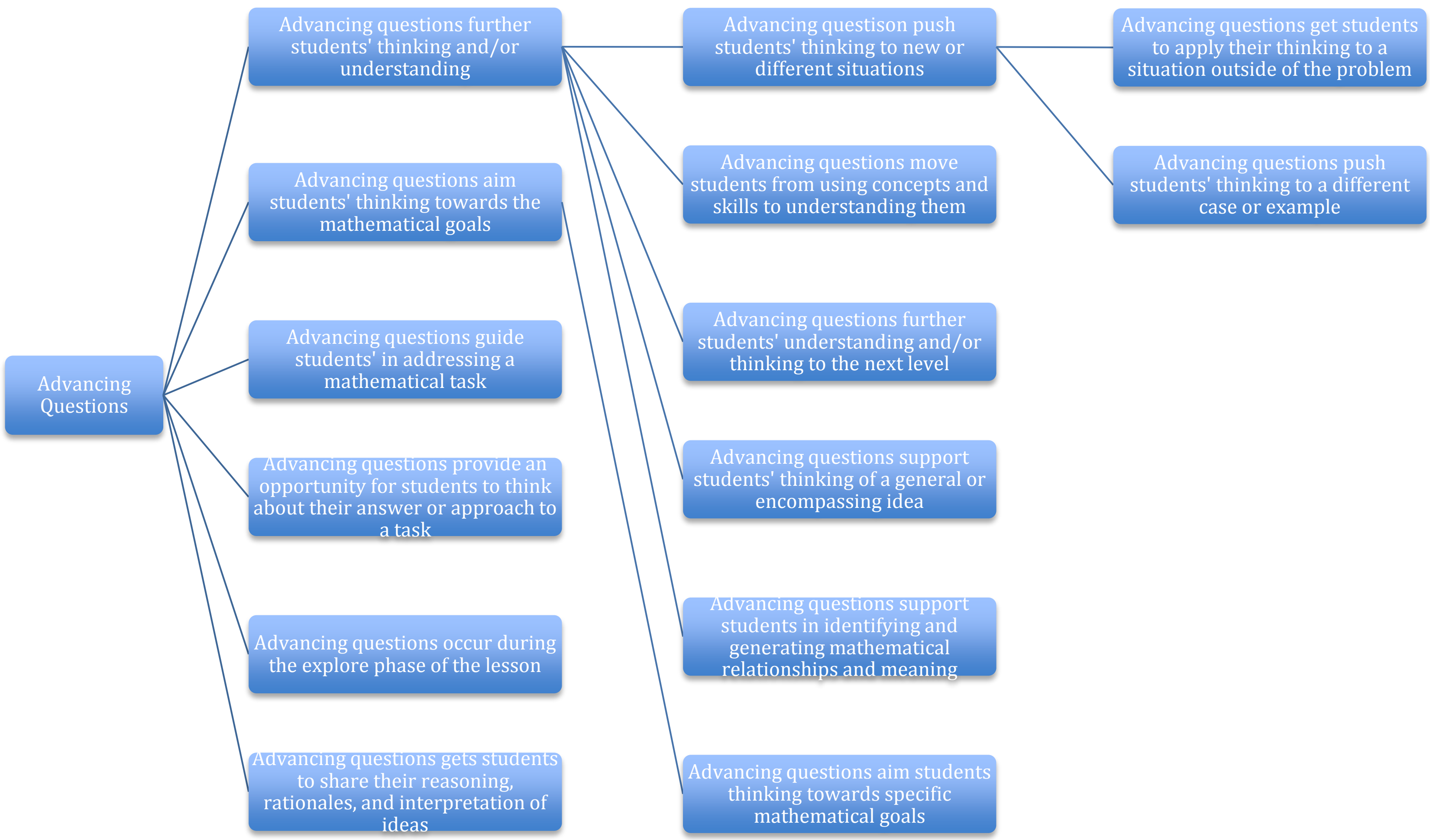


Figure 4-2: Steve’s conception of advancing questions

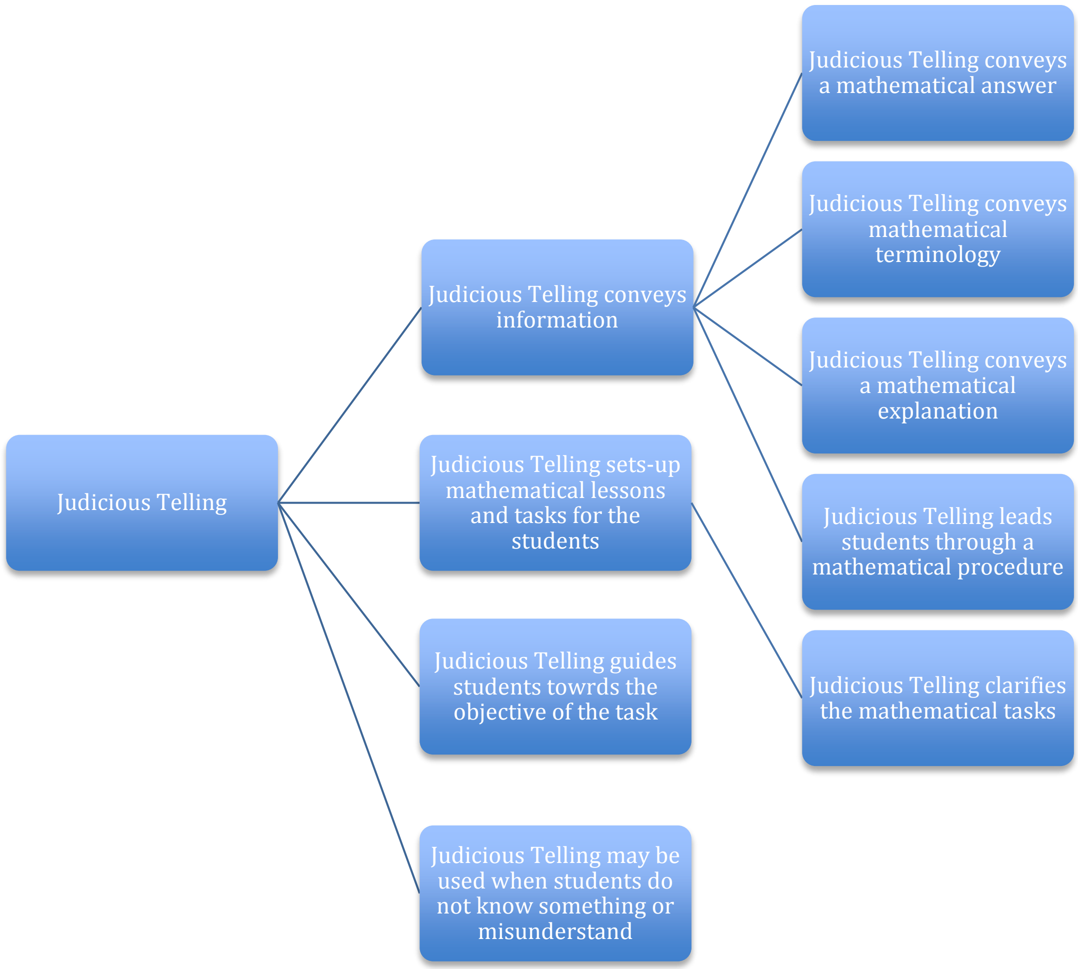


Figure 4-3: Steve's conception of judicious telling

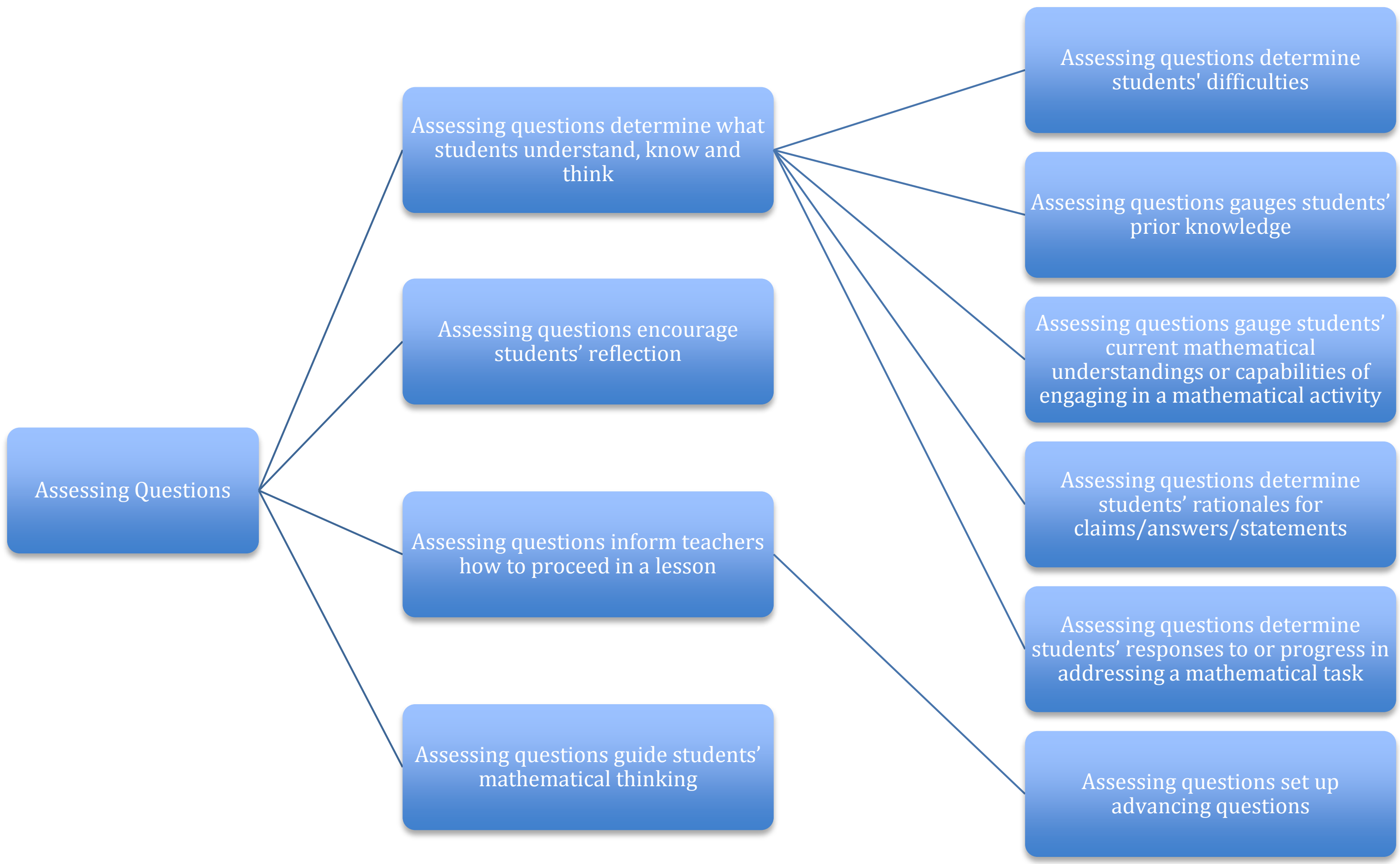


Figure 4-4: Leslie's conception of assessing questions

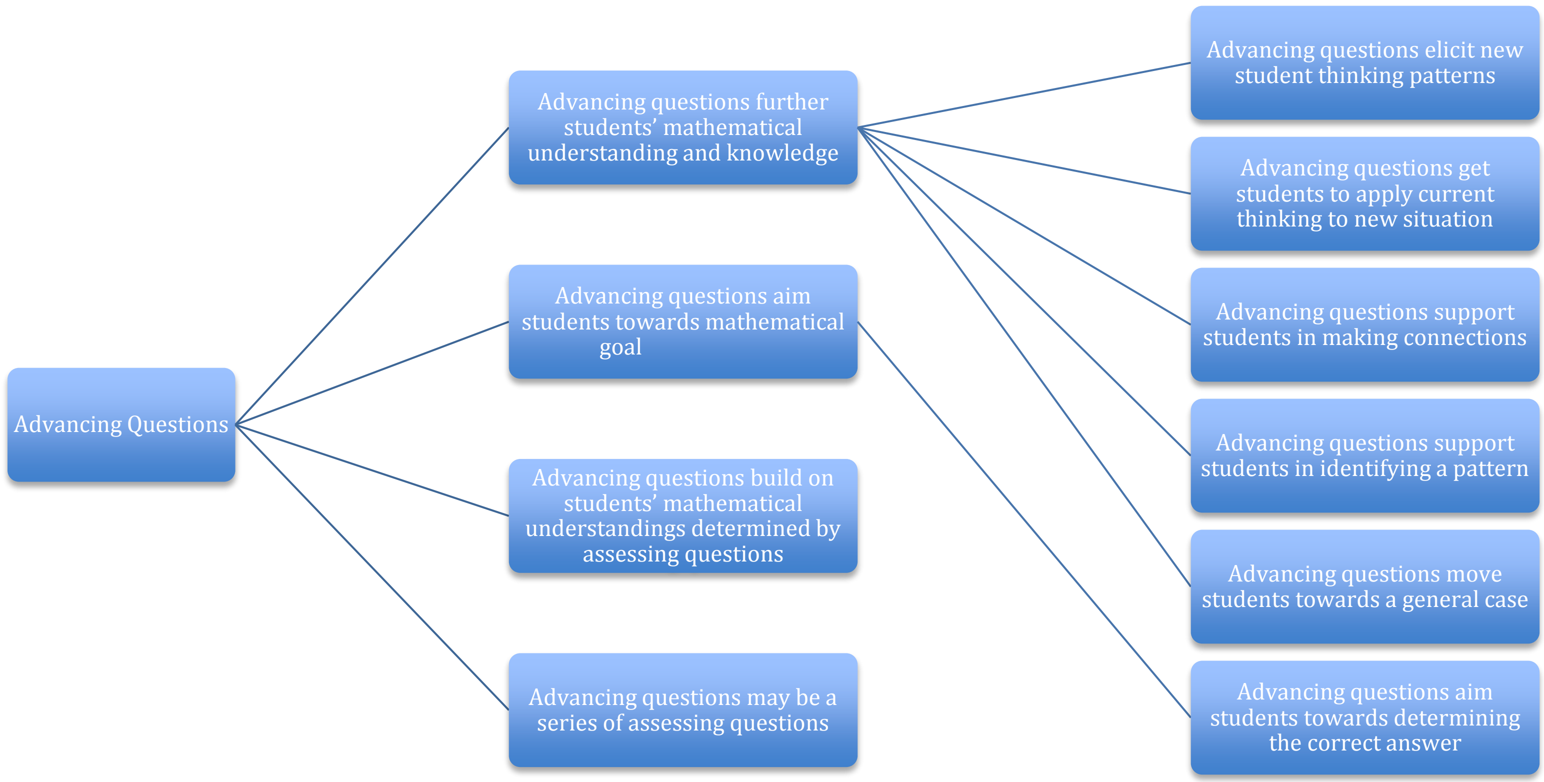


Figure 4-5: Leslie's conception of advancing questions

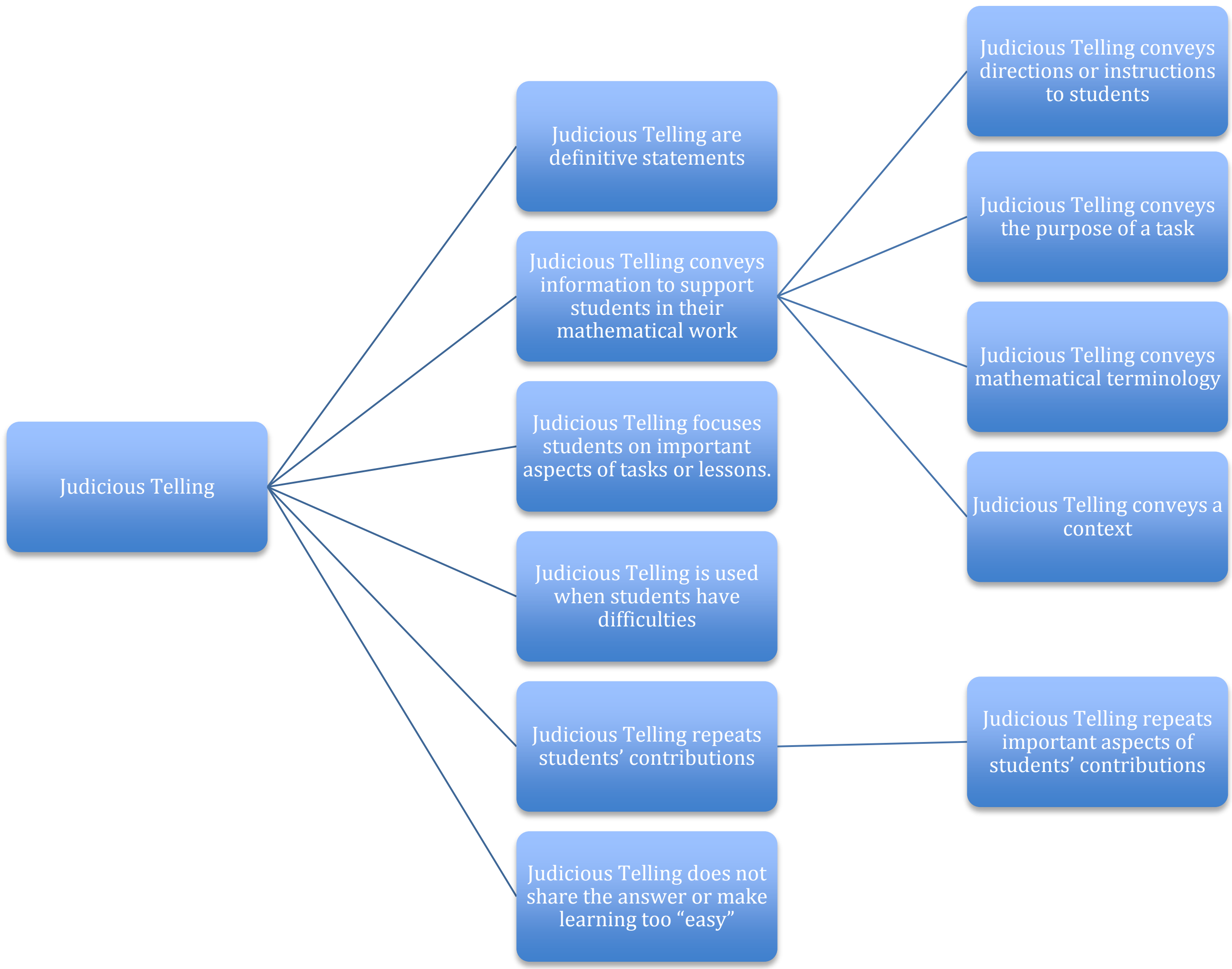


Figure 4-6: Leslie's conception of judicious telling

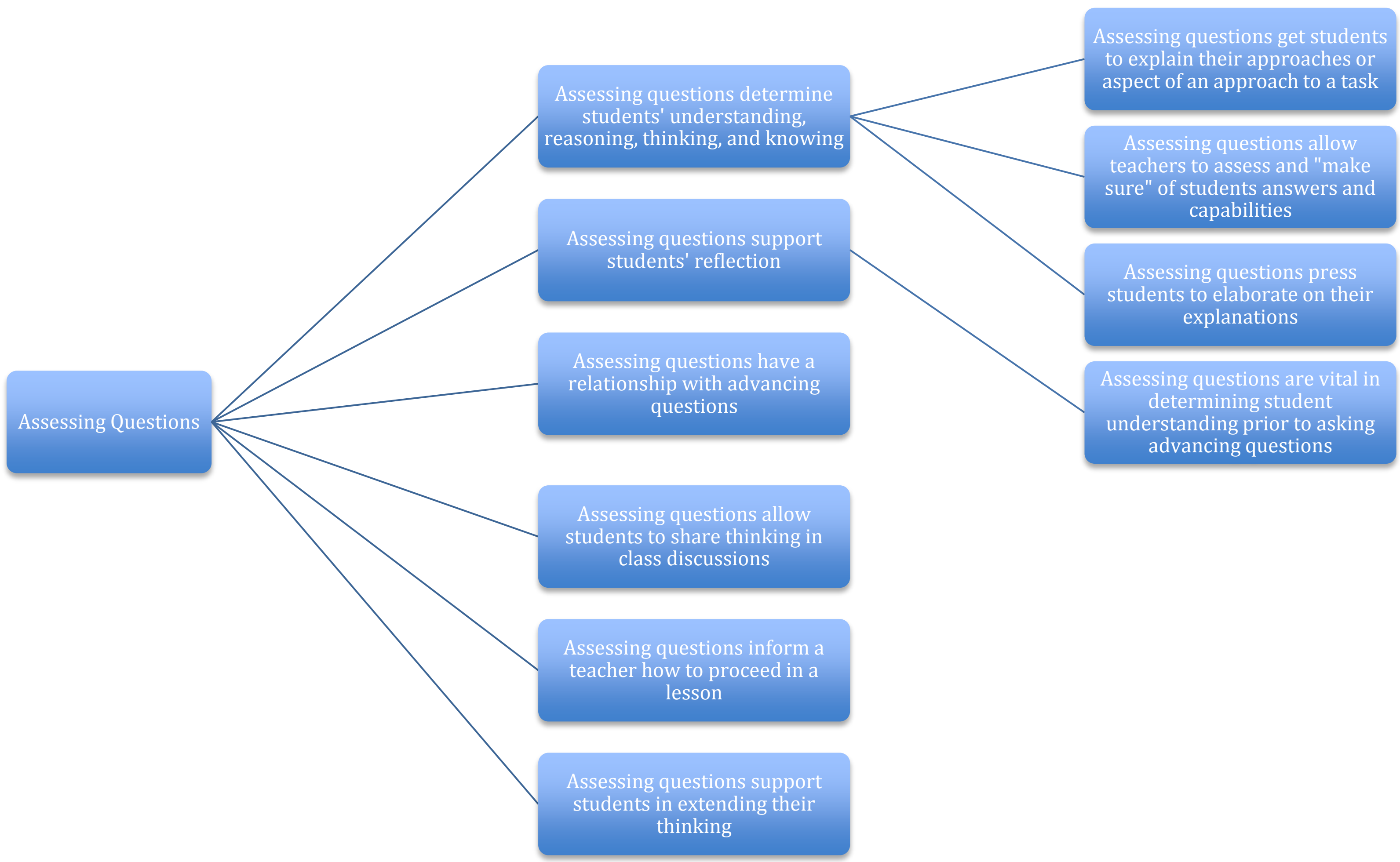


Figure 4-7: Gretchen's conception of assessing questions

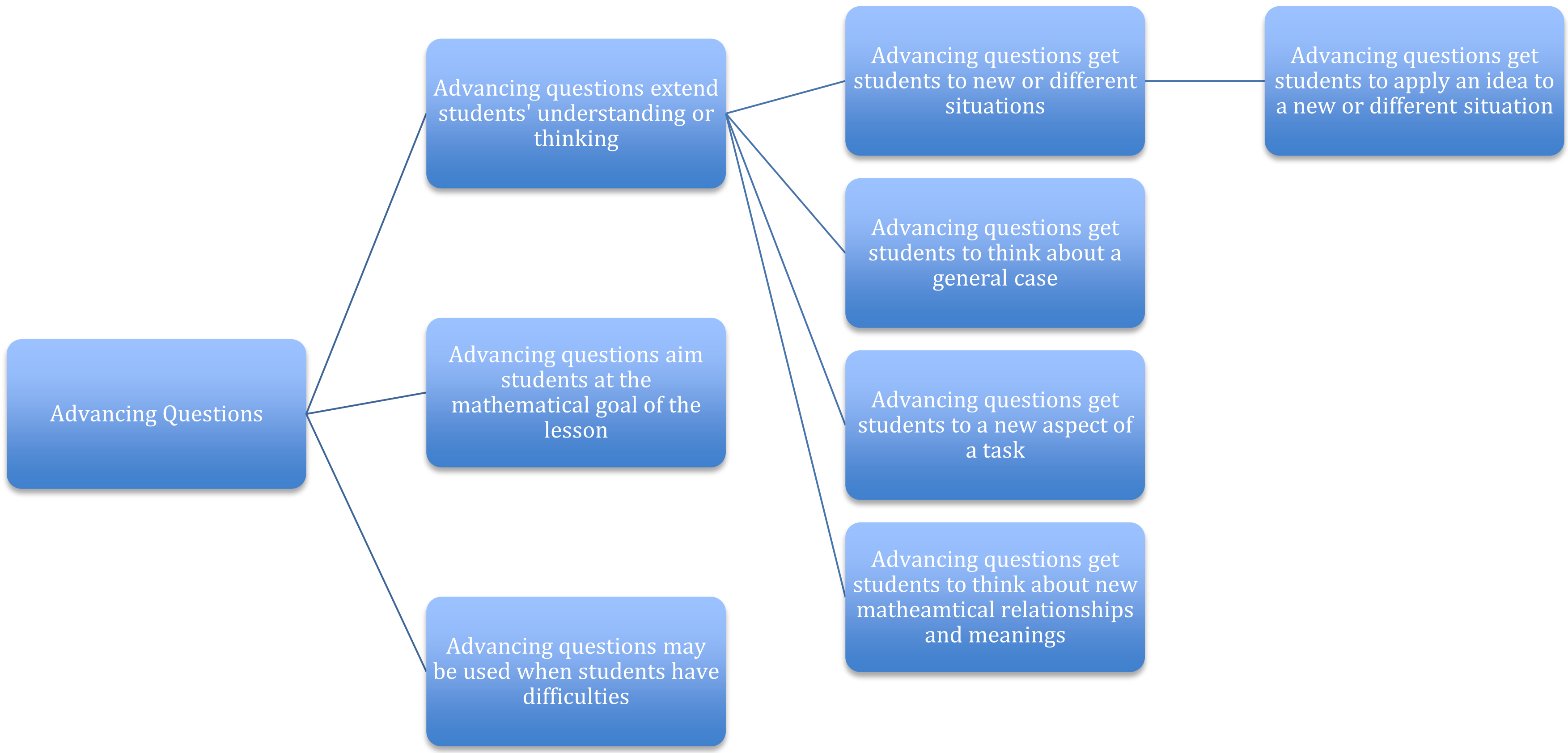


Figure 4-8: Gretchen's conception of advancing questions

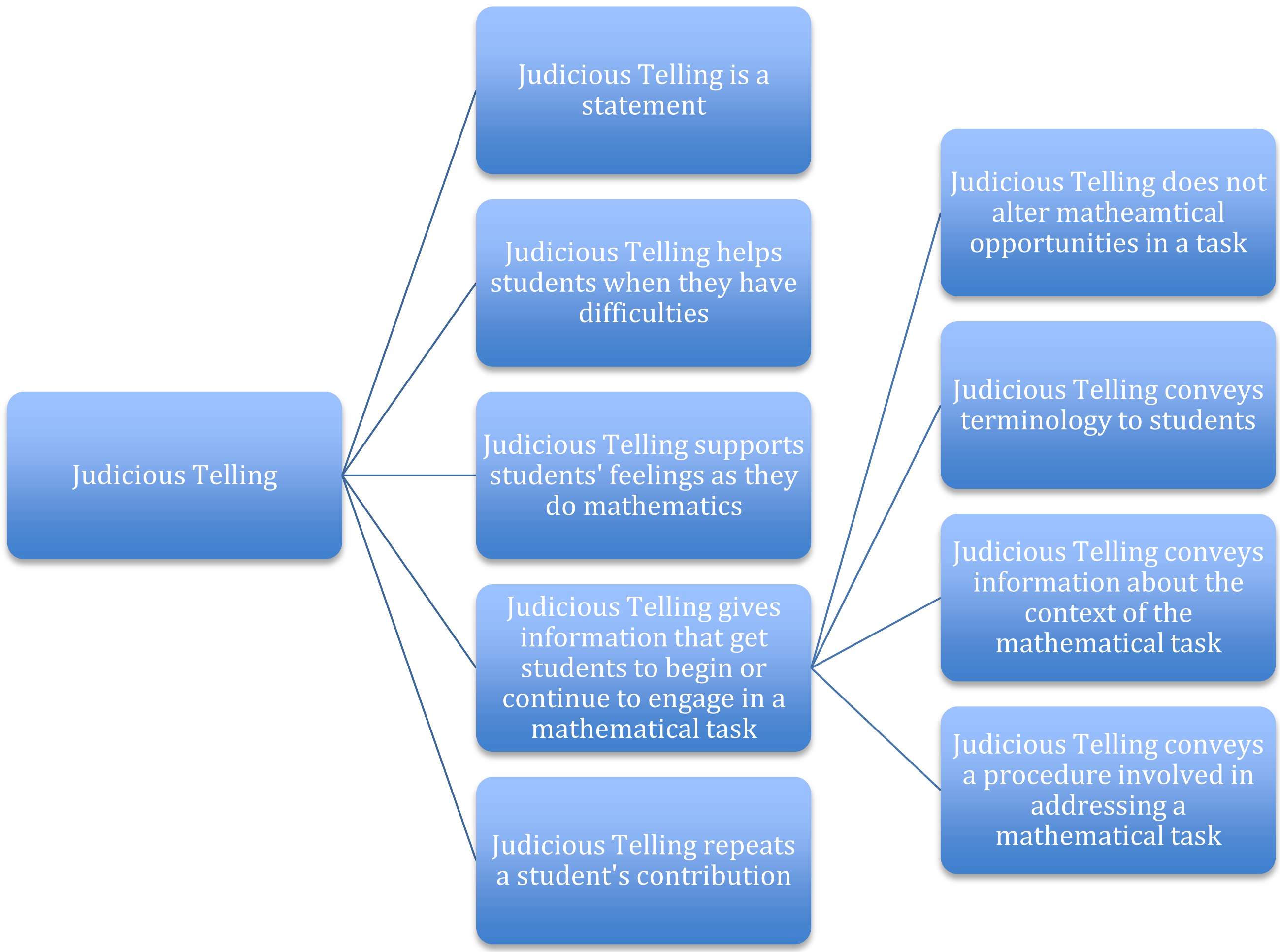


Figure 4-9: Gretchen's conception of judicious telling

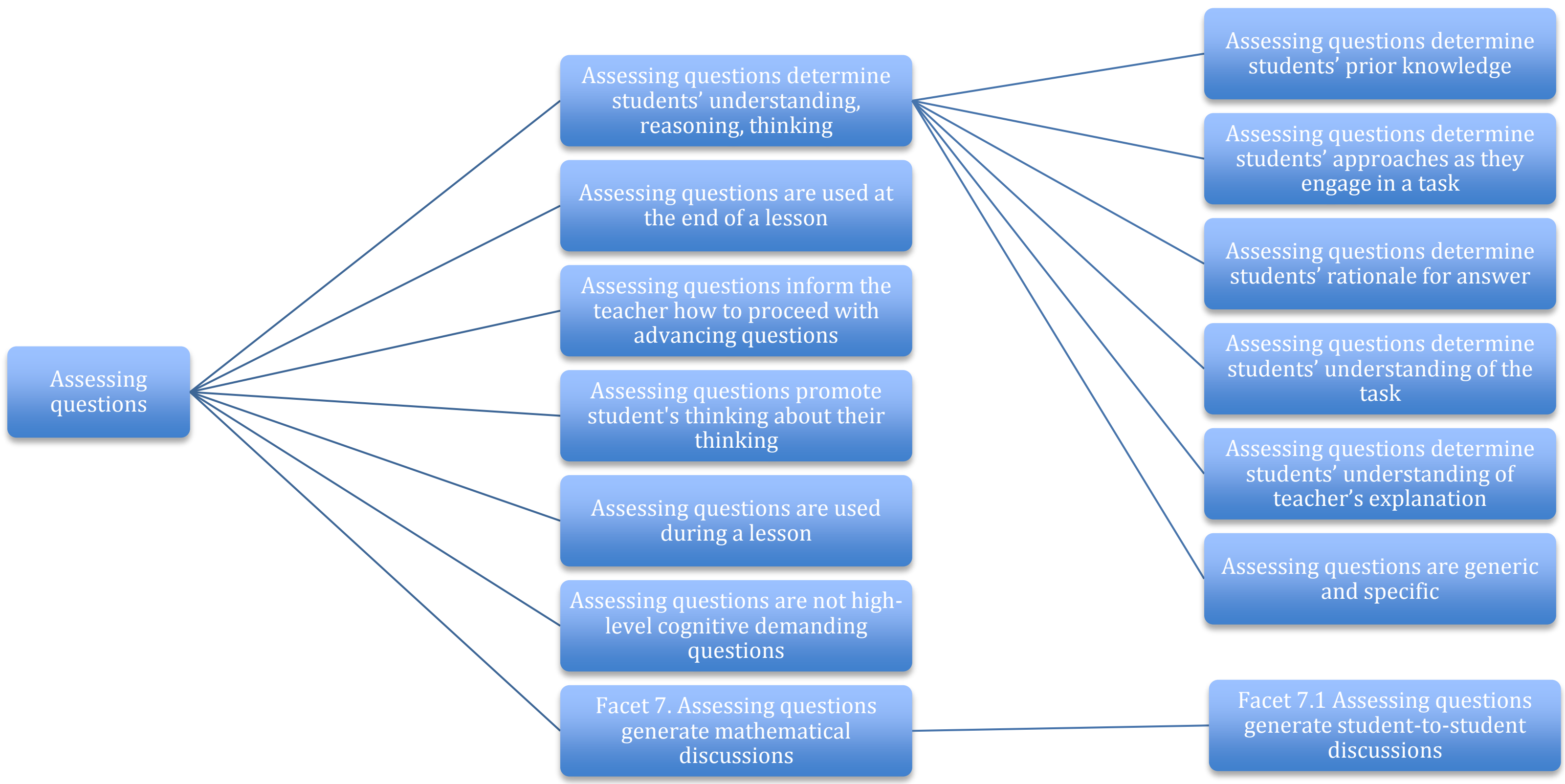


Figure 4-10: Nick's conception of assessing questions

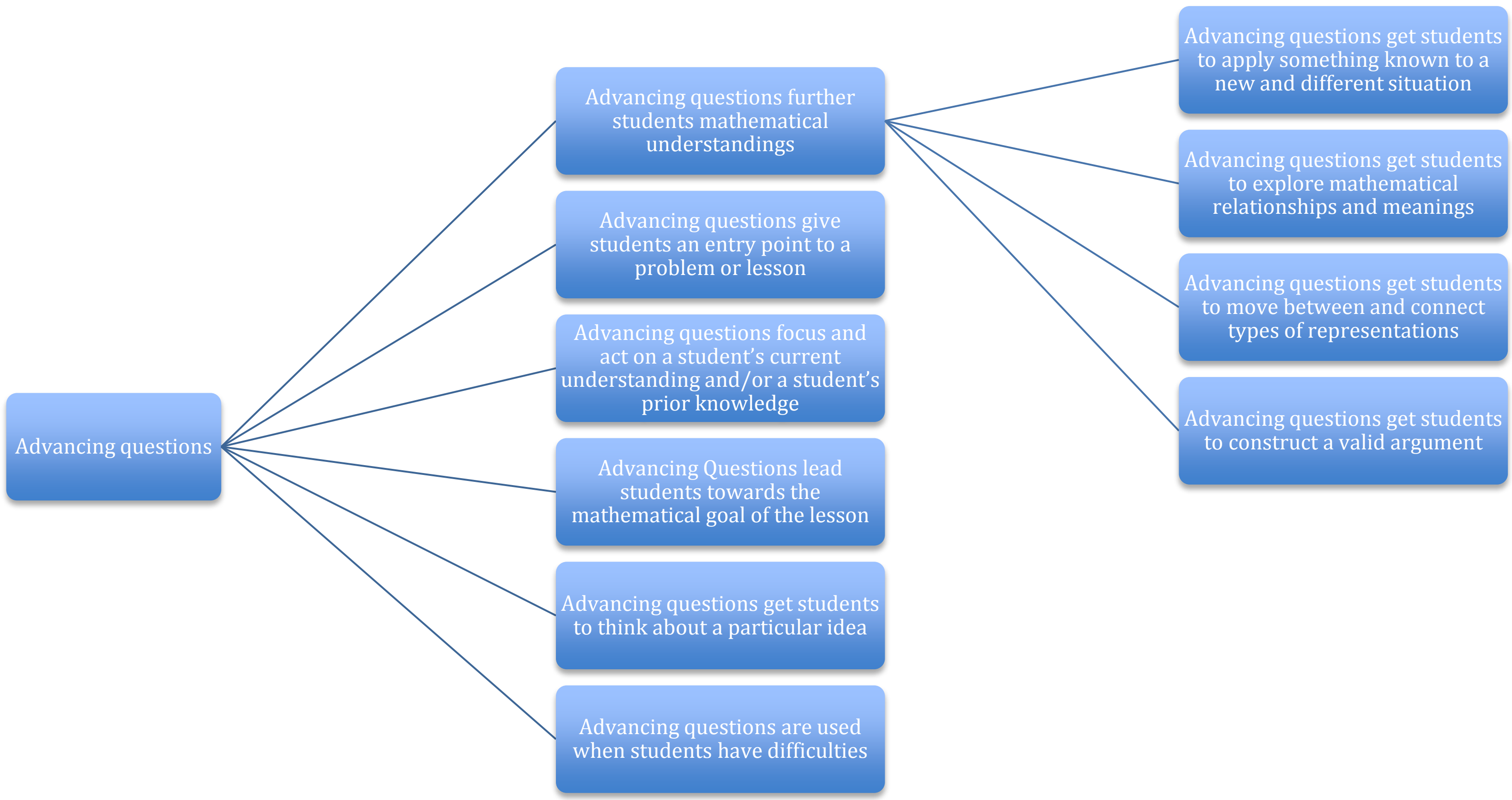


Figure 4-11: Nick's conception of advancing questions

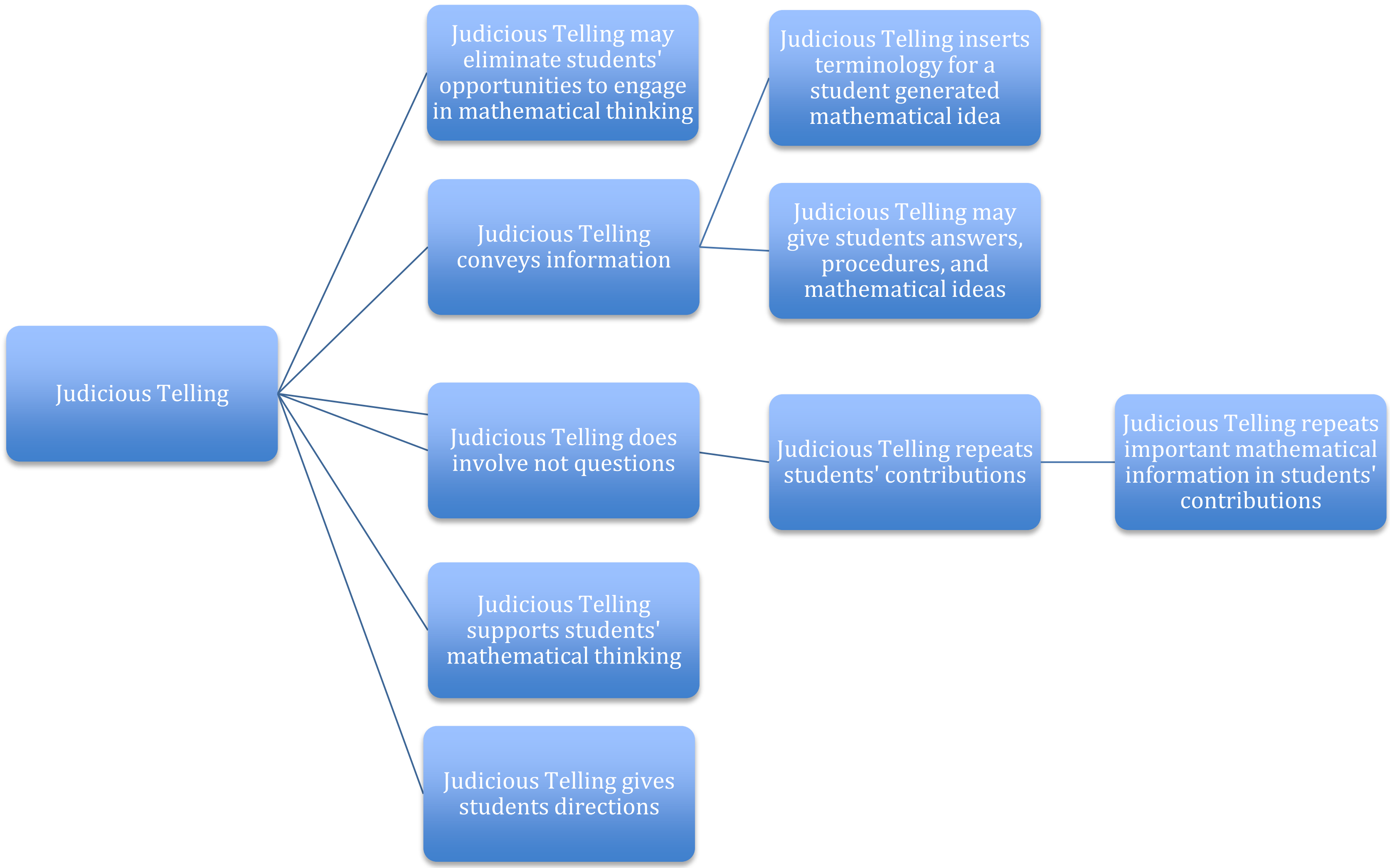


Figure 4-12: Nick's conception of judicious telling

In summary, the four PSMTs constructed unique conceptions of assessing questions, advancing questions, and judicious telling that included facets that addressed both qualities of TTT and the function of TTT. While unique, the PSMTs' conceptions included similar, but nuanced facets. In order to address the similar facets and nuances among the PSMTs' conceptions of TTT, I first need to explain the construct *features of TTT* as a way to communicate my findings from the cross-case analysis.

Features of TTT

I consider *features of TTT* to be commonalities among the PSMTs' conceptions of TTT. These commonalities are similar facets of the PSMTs' conceptions of types of teacher. While similarities exist among the PSMTs' facets, I consider the *feature* to be a broader category and not a way of replacing the way a facet characterizes a specific aspect of the PSMTs' conception of types of teacher. This is because the PSMTs' conceptions are unique to each PSMT and nuances exist among the PSMTs' individual conceptions. Hence, when I talk about *features* in this chapter, I talk about facets associated with a feature. The reason for this phrase is to emphasize that the term feature is not a one-size-fits all objective/normative piece of knowledge but an organizational tool for collectively presenting the unique PSMTs' conceptions.

In the following three sections, I report findings from the cross-case analysis addressing similarities and differences among the PSMTs' conceptions of assessing questions, advancing questions, and judicious telling.

Assessing Questions

The four PSMTs constructed conceptions of assessing questions that included facets associated with ten different features, four of which are more particular features of the first feature:

- *Assessing questions determine students' mathematical thinking, understanding, and/or knowledge*
    - *Assessing questions determine students' approaches or rationales for work as they engage in a task*
    - *Assessing questions determine students' rationales for answers to a mathematical task*
    - *Assessing questions gauge students' mathematical understanding and/or capabilities*
    - *Other facets associated with the first feature*
- *Assessing questions inform teachers' subsequent instruction*
- *Assessing questions support students' reflection*
- *Role of assessing questions in whole class discussions*
- *Assessing questions guide students' mathematical thinking*
- *Assessing questions relation to advancing questions*
- *Other facets associated with assessing questions*

The more particular features of the first feature address *what* about the students' mathematical thinking, understanding, and/knowledge that the assessing questions support teachers in determining.

In the following subsections addressing each of the features of the assessing questions, I illustrate the similarities and nuances among the PSMTs' facets associated with the feature using data that are indicators of their facets.

Assessing Questions Determine Students' Thinking, Understanding, and/or Knowledge

All four PSMTs' conceptions of assessing questions included facets associated with the feature *assessing questions determines students' mathematical thinking, understanding, and/or knowledge*. For example, during Leslie's second interview, she was asked to describe her current thinking about assessing questions, to which she responded,

> The purpose of [asking] assessing [questions] would be to figure out where the students are in their thinking … So, I think the main purpose of a teacher asking them that, especially in mathematics, is just to figure out where they are and get the general knowledge of where they are so that you can move them along. (Second Interview, audio-recording)

In addition, all four PSMTs constructed facets associated with more particular features of the first feature. The next four subsections report the PSMTs' facets associated with three more particular features – *assessing questions determine students' approaches or rationales as they engage in a task, assessing questions determine students' rationales for answers to tasks, assessing questions gauge students' mathematical understandings and/or capabilities* – and a set of facets in the *other* category.

Assessing Questions determine students' approaches or rationales as they work. All four PSMTs' conceptions of assessing questions included facets that were associated with the feature assessing questions *determine students' approaches or rationales as they engage,* which involved asking assessing questions to students as they engage in a mathematical task in order to determine: (a) students' progress in addressing a task; (b) the students' approaches to a task; and (c) the students' reasoning for their approaches to the task. For example, after the first rehearsal, Leslie wrote, "The instructional purpose of assessing questions for a teacher who allows mathematics to be problematic is to see where the students are in their learning, whether it is their previous knowledge or their progression in the given activity" (Leslie's Homework Assignment after Class 10). Another example, at the end of the course, Nick identified the following set of questions from his Class 25 Approximation of Practice as assessing questions, "So explain to me what you are doing or what you are thinking right now.; So why are you dividing fifteen by four?; And why would?; What do you mean by a length of one?" (Class 25 Approximation of Practice, audio-recording). In his StudioCode Timeline memo associated with this instance, Nick wrote,

> All of these questions gathered information from the student about what he was thinking or doing or why he was doing it. Assess what the student was doing and why he was doing it … The student was describing an erroneous method he was attempting. Instead of trying to correct him right away, I decided to gather information about what he was doing and why he was doing it so I could have an opportunity to teach when he realized the method was faulty. The evidence I have of this teacher talk working is that the student described his thought process to me - including what he was doing and why he was doing it. (Nick's Homework Assignment after Class 25, StudioCode Timeline)

Assessing Questions determine students' rationales for answers. All four PSMTs' conceptions of assessing questions included facets associated with the feature *assessing questions determine students' rationales for answers to a mathematical task,* which involved determining students' mathematical justifications, reasoning, and/or approaches for their answers to tasks. For example, after the Class 25 approximation of practice, Steve categorized his question, "What did you have to do to get from here to here?" (Class 25 Approximation of Practice, audio-recording) as an Assessing Question. Steve wrote in the corresponding StudioCode Timeline memo,

> This is an Assessing Question because I have the student explain to me the whole process as to how he got the answer as a whole, complete statement instead of all of his steps mixed together. I wanted the student to be able to explain his work in order to solidify his understanding of the solution he came up with. (Steve's Homework Assignment after Class 25, StudioCode Timeline)

Another example, after the Class 14 Second Rehearsal, Gretchen and her peers coded her Class 25 Approximation of Practice question, "Okay, Student 3. So can you explain to me your thinking here with these numbers at the top?" (Class 14 Second Rehearsal, audio-recording) as an assessing question. Gretchen wrote in the StudioCode Timeline Memo associated with this question, "I wanted Student 3 to explain what his strategy was for solving the problem at hand" (Class 15 StudioCode Analysis of Second Rehearsal, StudioCocde Timeline).

Assessing questions gauge students' understanding and/or capabilities. Three of the four PSMTs' (Leslie, Steve, and Gretchen) conceptions of assessing questions included facets associated with the feature *assessing questions gauge students'* *mathematical understanding and/or capabilities*, which involved teachers using assessing questions to gauge or verify students' understanding of a mathematical idea or their capabilities in doing mathematics. The facets of Leslie's and Gretchen's conceptions of assessing questions associated with this feature were subtly different from Steve's facet. Steve's facet focused on gauging something the student had already done in a lesson or some mathematical idea that the student had learned previously. For example, during the second interview, Steve was asked to review his Second Rehearsal assessing question, "How'd you get expression 4k + 4?" (Class 14 Second Rehearsal, audio-recording) and explain his thinking for asking the question. Steve stated:

> That's assessing as well. I know what she did. She added the two numbers together, but can she explain to me what she did? Or will she say, "I think it's this." And that was the only reason why I was asking this question - does she know what she did here to get this expression. (Second Interview, audio-recording)

In addition to focusing on something the student had already done or knew, Gretchen and Leslie also focused on gauging the students' capabilities to engage in some mathematical process, procedure, and/or operation (e.g., calculating the difference between numbers, interpreting a diagram, constructing an explanation) related to a current task. For example, during the second interview, Leslie is asked to reflect on a series of questions she asked during the first rehearsal in which the doctoral students were asked to calculate the differences in distances corresponding to consecutive hours,

> We want to go through it together. At one hour, [the difference is] five miles. How many at two hours? Ten miles. Now what's the difference between that? … I

> don't think it's *[referent is the series of questions]* necessarily separately all advancing, because you're just seeing what they know. Seeing if you [Student 3] can figure out that the difference between ten and five is five. (Second Interview, audio-recording).

The nuance between Steve's facet and the facets of Leslie and Gretchen was that Leslie and Gretchen addressed an additional aspect of *what* teacher's gauge about students' mathematical thinking and understanding with assessing questions – students' capabilities to engage in a mathematical process or procedure.

Other facets associated with the first feature. Two of the four PSMTs' (Leslie and Nick) conceptions of assessing questions included additional more particular facets associated with the feature *assessing questions determine students' mathematical thinking and understanding*, but the facets were not similar. Leslie's conception included the more particular facet *assessing questions determine student difficulties* that involved assessing questions determining what students do not understand or eliciting students' thinking when the teacher perceives that the student may be "stuck." For example, during Leslie's third interview, she was asked how her understanding of TTT influenced her interaction with the high school student in the Class 25 Approximation of Practice, to which she stated, "I began with assessing questions to see if he knew what he was doing. Any time that he got stuck or he was quiet for a while and really thinking and a decent amount of time passed" (Third Interview, audio-recording).

Nick's conception included three more particular facets. First, the facet *assessing questions determine students' understanding of the task* involved assessing questions determining students' understanding of the directions and components of the written mathematical task or the teacher's set-up of the mathematical task. For example, during the third interview, Nick was asked to explain how his understandings of the TTT influenced his instruction of the high school student, to which he responded,

> I wanted to do a lot of assessing right off the bat. Like, does he understand the problem? What is his interpretation of the problem? Make sure he understands what a beam is in the problem, things like that. So, I remember I did do a lot of assessing right off the bat. (Third Interview, audio-recording)

Second, the facet *assessing questions determine students' understanding of an explanation* involved assessing questions determining what the students understand mathematically after a teacher shares a mathematical explanation. For example, early in the semester, Nick wrote an instructional purpose for assessing questions, "Another instructional goal of an assessing question is to see what students understand after an explanation has been given" (Nick's Homework Assignment after Class 6). Last, the facet *assessing questions may be generic or specific* involved assessing questions determining students' general progress in a task or the students' thinking about a specific mathematical idea in a task. For example, during the second interview, Nick was asked about his group's crafting of assessing questions for the first rehearsal. The following is an excerpt from the conversation in which the interviewer asked Nick to talk more about his response,

> Nick: And that's how we got the assessing questions. Some of them were generic, like, "What is happening here?" and "Can you tell me where this came from?" We thought those were generic assessing questions that we may or may not have to ask.

> Interviewer: Those weren't necessarily - so when you say generic, were they necessarily for a specific question?
>
> Nick: No, we thought they could be thrown if needed. They could be thrown in somewhere. Those ones weren't specific, but, like, "How did you find the rate?" was specific … As we worked through some of them *[referent is the CMP problem set]*, we came up with specific [questions]. (Second Interview, audio-recording)

In summary, Leslie's conception of assessing questions included one additional more particular facet associated with the first feature and Nick's conception of assessing questions included three additional more particular facets associated with the first feature. In comparison to Steve and Gretchen, Nick's and Leslie's conceptions of assessing questions, especially Nick's conception, included a greater variety of the different aspects of students' mathematical thinking that assessing questions afford a teacher to determine.

Assessing Questions Inform Teachers' Subsequent Instruction

All four PSMTs' conceptions of assessing questions included facets associated with the feature *assessing questions inform teachers' subsequent instruction,* which involved eliciting students' mathematical thinking, mathematical understanding, prior mathematical knowledge, and/or progress in a mathematical task so that teachers may make instructional decisions that are based on their interpretations of students' mathematical thinking. For example, during the third interview, Nick was asked to explain his understanding of assessing questions, to which he responded,

> Assessing is gathering information from the student so that they *[referent is teachers]* can ask a good advancing question … For the most part, the teacher

> gathering information - getting a plan about where to go next in the lesson. (Third Interview, audio-recording)

Another example, during the third interview, Gretchen was asked to categorize the gathering information question type as a type of teacher talk. Gretchen categorized the question type as an assessing question and stated,

> Okay, gathering information and leading students through a procedure. That’s also assessing because you have to get information to know where a student is at and what they know and whether they're ready to be advanced or they're ready to move on. Or they don't understand and they need more help. (Third Interview, audio-recording)

These two examples illustrate another aspect of these facets of the PSMTs’ conceptions – the PSMTs acknowledged that the subsequent instruction might include advancing questions.

Gretchen was subtly different than the other three PSMTs in that Gretchen’s conception of assessing questions included a facet that emphasized the importance of not assuming a student’s mathematical thinking prior to subsequent instruction. The following instance, in which Gretchen wrote about her experience in the Second Rehearsal, illustrates the importance for her in first determining student’s mathematical thinking,

> I assumed that Student 3 was looking at the different cases of numbers in the ones place for three consecutive odd numbers, but I didn’t explicitly ask him about that. I was so focused on where *I* wanted him to get that I didn’t take enough time to listen to him explain his thinking and reasoning about where he already was …

> Dr. A coached me to have Student 3 explain more of the work he had already done before I moved on to advancing him towards the mathematical goals. I thought I had understood what Student 3 was trying to show with his work so then I didn't question him about it, but I needed to hear him explain it to know what he was thinking and why he made the choices that he did. I learned that I needed to assess students more even if I have an idea of what they might be getting at. I only had Student 3 explain on of his examples when I should have had him explain all of his examples. By Dr. A stepping in [with an assessing question] Student 3 was also able to understand that just writing out the number of threes in each sum wasn't enough and that he needed to explain why the sum of those threes was, in fact, divisible by 3. (Gretchen's Homework assignment after Class 15)

This instance is an indicator of two things. First, Gretchen's reflection indicates that, for Gretchen, teachers need to carefully listen to their students' responses to assessing questions so that the teachers can truly understand the students' mathematical thinking. Second, Gretchen's reflection indicates that, for Gretchen, it is vital for teachers' to truly understand the students' mathematical thinking so that they may *advance* students' mathematical thinking based on the students' current mathematical thinking and understanding.

The data clearly indicates that all four PSMTs' conceptions of assessing questions addressed the importance of focusing on students' mathematical thinking to inform subsequent instruction. However, Gretchen's more particular facet associated with this feature suggests, in comparison her peers, she paid greater attention to the role that assessing questions have in determining students' mathematical thinking that informs subsequent instruction.

Assessing Questions Support Students' Reflection

All four PSMTs' conceptions of assessing questions included facets associated with the feature *assessing questions support students' reflection*, which involved students' thinking about their own thinking or about their approaches to mathematical tasks. For example, early in the semester, Leslie wrote the following as a defining feature of assessing questions, "encourages 'self-thinking' or metacognition" (Leslie's Homework Assignment after Class 5). Another example, Steve wrote in his instructional purpose for assessing questions, "The assessing questions that are often asked in this context also provide the student a means for self-reflection" (Steve's Homework Assignment after Class 6).

While the four PSMTs' facets all addressed self-reflection as students' thinking about thinking, there was a difference among the facets. The facets of Leslie's, Gretchen's, and Nick's conceptions of assessing questions were more elaborate than Steve's in that the three PSMTs addressed assessing questions as a way to get students to review and think about the approaches they used to address a task. For example, Leslie wrote the following in her analysis of the Case of Edith Hart, "In line 10, she asks Heather, "What do you mean by same cost?" in order to encourage metacognition and make sure that Heather knows what she is saying and why she is saying it" (Leslie's Homework Assignment after Class 6). Similarly, during the second interview, Gretchen was asked how "context" may influence whether a teacher's question or statement is a certain type of teacher talk, to which she responded, "They're assessing still, because they know that the student knows more about what they did and they're just getting them to verbalize it and think about what they did and everything" (Second Interview, audio-recording).

Role of Assessing Questions in Whole Class Discussions

Three of the four PSMTs' (Steve, Gretchen, and Nick) conceptions of assessing questions included facets associated with the feature *role of assessing questions in whole class discussions,* which involved assessing questions as a way for the teacher to promote classroom mathematics discourse and student-to-student interaction. For example, Steve wrote in his second instructional purpose for assessing questions,

> Edith Hart does this well on many occasions; for example in paragraph 12 on page 55 Danielle said, "I noticed they all have different slants." Edith responded by asking Danielle "to say more on that." This prompt allows Danielle to explain what she is talking about and it provides a basis for a discussion further on in the lesson. (Steve's Homework Assignment after Class 10)

Gretchen's and Nick's facets differed from Steve in that they included that the student-to-student interaction in the classroom discourse may involve a debate or critique of students' answers. For example, during the third interview, Gretchen was asked to categorize the generating discussion question type, to which Gretchen stated,

> Generating discussions. I think a lot of times when you're generating discussion you might already know what the student has written for you know certain students have the right or wrong answer and you need them to share those answers with the class or with their group for the benefit of others. So, you're

> kind of assessing their ability to share their work and for others to kind of critique them. (Third Interview, audio-recording)

Hence, Gretchen's and Nick's facets associated with this feature more specifically addressed the role of assessing questions in promoting a whole class discussion. That is, they addressed assessing questions as a way for a teacher to facilitate mathematical communication among the students.

Assessing Questions Relation to Advancing Questions

Three of the four PSMTs' conceptions of assessing questions included facets associated with the feature *assessing questions relation to advancing questions*, which involved assessing questions serving the function of determining students' mathematical understanding in order for the teacher to ask advancing questions based on the students' mathematical understandings. For example, during the third interview, when pressed to describe the purposes for assessing questions, Nick stated, "Assessing is gathering information from the student so that they [the teachers] can ask a good advancing question" (Third Interview, audio-recording). Another example, during Leslie's second interview, she was asked to explain how she envisioned the role of TTT in her future classroom, to which she responded, "So, I think these *[referent is TTT]* are definitely ways to gain the students' full understanding and have them work through the problems and me just trying to use assessing questions to figure out where they are. Then, based on that, use the advancing questions" (Second Interview, audio-recording).

Assessing Questions Guide Students' Mathematical Thinking

Prior to reporting the facets associated with this feature, it is necessary to address an observation about these facets of the PSMTs' conceptions of assessing questions.

Data analysis indicates that the PSMTs constructed facets associated with the feature *assessing questions guide students' mathematical thinking* as viable facets for their conceptions of assessing questions. However, the instructors of the course as well as researchers and practitioners in mathematics education would likely consider this feature to be a feature of advancing questions. Hence, the PSMTs seem to have constructed a feature that is considered by the mathematics teacher education community as a misconception of assessing questions.

Three of the four PSMTs' (Steve, Leslie, and Gretchen) conceptions of assessing questions included facets associated with the feature *assessing questions guide students' mathematical thinking,* which involved questions that orient students towards mathematical ideas, representations, or procedures as they engage in a task. However, there was a difference among the way in which the PSMTs addressed teacher's guiding. Gretchen addressed a teacher's guiding as orienting or focusing students on a mathematical idea (e.g., an situation other than the one with which they were currently working). For example, Gretchen and her peers coded her Second Rehearsal question "Okay, so let's take a step back for a second. Can you write a couple other examples of consecutive odd integers?" (Class 14 Second Rehearsal, audio-recording) as an assessing question. Gretchen wrote in her StudioCode Timeline memo associated with the instance, "I wanted Student 3 to consider another example of three consecutive odds so that he could possibly attempt to see the relationship between the ones places" (Class 15 StudioCode Analysis of the Second Rehearsal, StudioCode Timeline). Leslie's and Steve's facets differed from Gretchen's facet. Leslie's and Steve's facets addressed assessing questions as directing/prompting the students through steps of the mathematical tasks. For example, during the third interview, Steve was asked to explain his understanding of assessing questions, to which he stated, "I'd like to go with the gathering the information for the student. I like asking questions to get a student through a problem. 'Okay so what's your next step? And then what?'" (Third Interview, audio-recording). One could argue that guiding students' thinking using a series of questions that direct students' through steps for carrying out a task limits the amount of thinking on the part of the students. Hence, Leslie's and Steve's facets addressed guiding as a teacher's use of questioning that may constrain students' mathematical thinking, where as Gretchen's facet addressed guiding as questioning that may promote students' mathematical thinking.

Other Facets of Assessing Questions

Two of the four PSMTs' (Steve and Nick) conceptions of assessing questions included additional facets not associated with any of the features. Steve's additional facet – *assessing questions are open-ended questions* – addressed the form of an assessing question. For example, during the third interview, Steve was asked to explain his understanding of assessing questions, to which he responded, "When we were writing a definition for assessing in class, one of the points that we put in that category was that this question is generally open-ended" (Third Interview, audio-recording). Nick's additional facet – *assessing questions are not high-level cognitive demanding questions* – positioned assessing questions as relatively simple questions that often involve asking students questions about something they already know and do not push students' mathematical thinking. For example, early in the course, Nick wrote, "Students don't have to struggle to get answer" (Nick's Notebook) and "more simple questions ('what')" (Nick's Notebook) on his list of defining features for assessing questions.

Advancing Questions

The four PSMTs constructed conceptions of advancing questions that included facets associated with ten different features of advancing questions, five of which are more particular features of the first feature:

- *Advancing questions extend students' understanding and/or thinking*
    - *Advancing questions support students' thinking about new or different situations*
    - *Advancing questions support students' thinking about a general case/idea*
    - *Advancing questions support students thinking about new mathematical relationships and/or meanings*
    - *Advancing questions support students in engaging in mathematical processes*
    - *Other facets associated with the first feature*
- *Advancing questions aim students' towards the mathematical goal of a lesson*
- *When teachers may ask advancing questions*
- *Advancing questions build on students' mathematical understandings*
- *Advancing questions guide students' mathematical thinking*

The five more particular features of the first feature address the ways in which students' thinking is extended towards the mathematical goal of the lesson by advancing questions.

In the following subsections addressing each of the features of the advancing questions, I illustrate the similarities and differences among the PSMTs' facets associated with the feature using data that are indicators of their facets.

Advancing Questions Extend Students' Understanding and/or Thinking

All four PSMTs' conceptions of advancing questions included facets associated with the feature *advancing questions extend students' mathematical understanding and/or thinking*, which involved advancing questions furthering students' mathematical knowledge, understanding, and/or thinking beyond the current state of the students' knowledge, understanding, and/or thinking. For example, after the first rehearsal, Gretchen wrote a second instructional purpose for advancing questions, "To extend or deepen student thinking and understanding towards the mathematical goals of the lesson" (Gretchen's Homework Assignment after Class 10). In addition, all four PSMTs constructed more particular facets associated with the first feature in that the PSMTs focus on the ways in which the students' thinking may be extended by advancing questions. I report these features – *advancing questions support students' thinking about new or different situations*, *advancing questions support students' thinking about a general case/idea*, *advancing questions support students thinking about new mathematical relationships and/or meanings*, *advancing questions support students in engaging in mathematical processes*, *and other facets associated with the first feature* – in the following five subsections.

Advancing questions support students' thinking about new or different situations. All four PSMTs constructed conceptions of advancing questions that included facets associated with the feature *advancing questions support students'* *thinking about new or different situations,* which involved advancing questions extending students' thinking by moving students beyond the current example(s), case(s), and/or situation(s) with which they are currently working and think about a different or new example(s), case(s), or situation(s). For example, after the first rehearsal, Nick wrote a second instructional purpose for advancing questions, "an advancing question can move a student who does have a mathematical understanding of a formula he/she produced to a new level of thinking, such as if this formula would work in a different situation or context" (Nick's Homework assignment after Class 10). A facet of Steve's conception of advancing questions was subtly different than those of his peers in that his facet was more particular about what constituted the new situation. Steve's facet addressed advancing questions furthering students' thinking by getting the students to think about or apply their thinking about a mathematical idea to the real world. For example, during the second interview, Steve was asked to explain his understanding of advancing questions, to which he responded,

> Overall, or even just thinking outside the box in how this one topic can apply to several different things here or there. And have the students come to the understanding, not just me telling them, "Oh, well you know you can apply this to bug reproduction or whatever you are working on." (Second Interview, audio-recording)

Advancing questions support students' thinking about the general case/idea. Three of the four PSMTs (Leslie, Steve, and Gretchen) constructed conceptions of advancing questions that included facets associated with the feature *advancing questions support students' thinking about the general case/idea,* which involved advancing questions supporting students to think about an idea that is more broad or general that encompasses the set of mathematical ideas with which they are currently working. For example, after the Class 25 Approximation of Practice, Leslie coded the question, “And we want to find a way to be able to express that in variables, not just numbers. So, is there a way that you can ... use what you wrote up here to, uh, make it in a form with variables? A more general formula?” (Class 25 Approximation of Practice, audio-recording) as an advancing question. Leslie wrote in her analysis of the questions,

> This is an advancing question because I asked him to take what he was doing on the first part of the problem, where he was strictly working with numbers, and express it in a way using variables. I was asking him to represent the material using variables.
>
> …
>
> I wanted to move him towards a more general formula that can be applied to multiple situations in which one could change the length to anything in order to find the number of rods used or vice versa. The more general equation can be used for any values of N instead of working with a formula based off of a picture that is given to you or simply adding on rods and then going through and counting. (Steve’s Homework Assignment after Clas 25, StudioCode Timeline)

Another example, Steve coded his Class 25 Approximation of Practice question, “Do you think you could generalize a rule for that?" (Class 25 Approximation of Practice, audio-recording) as an advancing question. Steve wrote in the corresponding StudioCode Timeline memo, “I am advancing the student's thinking towards the next step of the problem … [The instructional purpose is] take his current thinking and channel that into the broader thinking of creating a general rule that would work for all numbers" (Homework Assignment after Class 25, StudioCode Timeline).

Advancing questions support students' thinking about new relationships. All four PSMTs constructed conceptions of advancing questions that included facets associated with the feature *advancing questions support students' thinking about new mathematical relationships and/or meanings*. However, the nature of the relationships differed among the PSMTs' facets. Nick's, Gretchen's, and Steve's facets associated with this feature addressed relationships among mathematical ideas and relationships among different mathematical representations. For example, during the Class 10 StudioCode Analysis, Steve and his group coded PT13's question, "Would that be difficult for someone to see if they didn't have that table. What that line would be?" (Class 9 First Rehearsal, audio-recording) as an advancing question. The following is an excerpt from their conversation:

> PT13: So it's similar, but different. Because now I'm kind of having him think about -
>
> Steve: Generate a relationship between the table and the graph.
>
> PT14: She's asking him to think about -
>
> Steve: It's advancing because we want him to develop good relationships. And the purpose is to see if he understands the relationships. (Class 10 StudioCode Analysis of the First Rehearsal, audio-recording)

Leslie's facet of advancing questions differed, because her facet did not address relationships among representations and was limited to relationships among mathematical ideas and other contexts. For example, after the first rehearsal, Leslie and her group coded PT5's question, "What would that be?" (Class 9 First Rehearsal, audio-recording) as an advancing question. The following is an excerpt from their conversation,

> PT2: So, what are we going to say this one is going to be?
>
> PT4: It's trying to connect this to something like real world.
>
> PT2: We'll give this a five.
>
> Leslie: Something they know already.
>
> PT16 and PT4 agree.
>
> PT4: Like how they connect.
>
> Leslie: Goes from a graph to a real life application. (Class 10 StudioCode Analysis of the First Rehearsal, audio-recording)

Nick's facet of advancing questions associated with this feature was subtly different from Steve's and Gretchen's facets in that he was more particular about the nature of the relationship among the representations. For example, Nick wrote a second instructional purpose for advancing questions,

> The instructional purpose of an advancing question for a teacher who wants to make mathematics problematic is to push students to the next step, whatever that next step may be. For instance, if a student has produced a correct formula but does not really know the mathematical relationships behind it, an advancing question can be used to allow the student to explore the relationships and reach that next level of understanding on their own. (Nick's Homework assignment after Class 10)

Advancing questions support students in engaging in mathematical processes. Two of the four PSMTs (Leslie and Nick) constructed conceptions of advancing questions that included facets associated with the feature *advancing questions support students in engaging in mathematical processes,* which involved advancing questions supporting students engaging in mathematical processes such as identifying patterns or constructing arguments. For example, during the second interview, Leslie is asked to reflect on a set of questions she asked in the first rehearsal, to which she stated,

> And putting those assessing questions together is considered advancing, because I'm getting them to the next box through a series of assessing questions so … I think my main goal was to find the pattern and actually finding a pattern is more advancing than asking them what they see at this moment. (Second Interview, audio-recording)

Another example, Nick wrote about the Second Rehearsal in his reflection assignment,

> I would say my advancing questions served their purpose in the sense that they got Student 2 to logically move through and explain steps of his proof. For example Student 2 did show me how a^2 * b^2 expanded to aabb and, using the commutative property, could be reordered as abab. (Nick's Homework assignment after Class 15)

Other facets associated with the first feature. Two of the PSMTs' (Steve and Leslie) conceptions of assessing questions included additional more particular facets associated with the feature *advancing questions extend students' mathematical thinking and understanding.* Leslie's facet *advancing questions elicit new thinking patterns* addressed advancing questions prompting students to think in new or different ways. For example, during the second interview near the middle of the semester, Leslie was asked to explain her understanding of defining features of advancing questions, to which she stated, “To grow, to find different thinking patterns, different – to get them to get out of their previous pattern of thought and move on to something new or different” (Second Interview audio-recording).

Steve’s conception included two additional more particular facets. First, Steve’s facet *advancing questions move students from using mathematical ideas to understanding the ideas* addressed advancing questions supporting students in developing a conceptual understanding of mathematical ideas, such as concepts and skills rather than using mathematical ideas without knowing conceptual underpinnings. For example, early in the course, Steve wrote, “getting students from using rules to understanding concepts” (Steve’s Homework Assignment after Class 5) as a defining feature for assessing questions. Second, Steve’s facet *advancing questions further students’ understanding and/or thinking to the next level* addressed advancing questions furthering a students’ thinking and/or thinking to a level of thinking or understanding beyond the level at which the student is currently. For example, early in the semester, Steve wrote, “Sometimes more difficult questions that require the student to go to the next level of thinking” (Steve’s Homework assignment after Class 5) as a defining feature for advancing questions.

In summary, Leslie’s conception of advancing questions included one additional more particular facet associated with the first feature and Steve’s conception of advancing questions included two additional more particular facets associated with the first feature. In comparison to Nick and Gretchen, Steve’s and Leslie’s conceptions of advancing questions, especially Steve’s conception, included a greater variety of the ways in which students’ mathematical thinking may be promoted by advancing questions.

Advancing Questions Aim Students Towards the Mathematical Goal

All four PSMTs constructed conceptions of advancing questions that included facets associated with the feature *advancing questions aim students towards the mathematical goal of the lesson.* For example, after the first rehearsal, Gretchen wrote a second instructional purpose for advancing questions, “The instructional purpose of advancing questions is to press a student toward the mathematical goals of the lesson” (Gretchen’s Homework Assignment after Class 10). Steve’s and Leslie’s facets of advancing questions associated with this feature differed from Nick’s and Gretchen’s facets in that Steve and Leslie specifically identified mathematical goals in their facets. For example, during the second interview, Steve was asked to explain his understanding of advancing questions, to which he responded, “Advancing questions are going to be the way for the teacher to get the students to the mathematical goal of the lesson. Whether it’s being able to provide a valid or correct answer to the problem posed or just getting that correct answer depending on what you’re working with” (Second Interview, audio-recording). Steve’s description positions the correct answer as the goal of the mathematics lesson, which was common among many of his descriptions towards the end of the semester and common to Leslie’s descriptions.

One may argue the mathematical goal that Steve and Leslie included in their descriptions may be a limiting way for a teacher to think about the target of mathematics instruction if the teacher aspires to extend students’ thinking in the more particular ways characterized by the PSMTs in the previous subsections.

When Teachers May Ask Advancing Questions

Three of the four PSMTs (Gretchen, Steve, and Nick) constructed conceptions of advancing questions that included facets associated with the feature *when teachers may ask advancing questions,* which involved the different points of time during lesson when the teacher uses an advancing question. Gretchen's and Nick's facets associated with this feature focused on advancing questions as a question teachers may ask to support students who are engaged in unproductive struggle (e.g. uncertainty as to how to begin a task, non-efficient approach to a task) as they engage in a mathematical task and/or support a student in addressing an incorrect aspect of their approach or answer to a task. For example, during the second interview, Gretchen was asked to explain how she and her group wrote the TTT for the first rehearsal, to which she responded,

> I'm trying to think how we did them. I think we thought about them as most likely questions to cause the students trouble… The ones that would have caused us the most trouble. One thing that we focused on is that the graphs would likely not reflect the answers that they needed. They wouldn't be big enough when they initially graphed them…That seventy miles on our graphs wasn't reflected. It didn't go up high enough, because we only did it until like um seven or something, seven hours and it required more hours. So, that was one of our big advancing things. It was helping the students to think of a way to remedy that situation if they get stuck on that - is there a way to extend the graph without just saying take another piece of paper out or redo it completely, because then that would be telling. (Second Interview, audio-recording)

Nick's and Steve's facets associated with this feature involved advancing questions being used at a temporal point in the lesson, such as during the explore phase of the lesson or the beginning of a lesson. For example, during Class 5, Nick shared with his peers that advancing questions, "can give them an entry point to a problem or lesson. At the beginning or during a lesson" (Class 5 Working Definition Discussion, audio-recording).

Advancing Questions Build on Students' Understandings

All four PSMTs constructed conceptions of advancing questions that included facets associated with the feature *advancing questions build on students' mathematical understandings*, which involved teachers creating advancing questions based on the teacher's inference of the students' current mathematical thinking and/or understanding. For example, during the third interview, Nick was asked to explain how his understanding of TTT influenced his planning for the Class 25 Approximation of Practice, to which he responded,

> I guess I tried to prep more advancing questions than anything in the third rehearsal *[referent is the Class 25 Approximation of Practice]*… I still feel like that's the hardest type of question to ask, because it's so dependent on what you are doing in the class – depending on what your students are doing and where they're at. (Third Interview, audio-recording)

Leslie's and Gretchen's facets associated with this feature differed from Nick's and Steve's facets in that their facets addressed the role of assessing questions in determining students' understanding so that the advancing question may be based on the student's understanding. For example, during the second interview, Leslie is asked to explain her understanding of advancing questions, to which she stated, "Based on my own understanding they [advancing questions] work off of assessing questions. So you would have to figure out based on students' answers" (Second Interview, audio-recording).

It is important to note that it would be unfair and untrue to claim that Nick and Steve overlooked the relation of assessing questions to advancing questions and the way in which this relation supports PSMTs' instruction (see previous section addressing the PSMTs' conceptions of assessing questions, specifically assessing questions inform teachers' subsequent instruction).

Advancing Questions Guide Students' Mathematical Thinking

All four PSMTs constructed conceptions of advancing questions that included facets associated with the feature *advancing questions guide students' mathematical thinking*, but the nature of the facets differed among the PSMTs. Gretchen's and Nick's facets characterized guiding as a less constraining teacher action than Steve's and Leslie's facets. Gretchen's facet addressed advancing questions as a way to get students to move beyond an aspect of a mathematical task with which they are currently working and to think about an aspect of the task that they have not yet reached. For example, during the second interview, Gretchen was asked to explain her understanding of advancing questions, to which she responded, "They challenge the student to see something about the problem that they haven't yet seen" (Second Interview, audio-recording). Nick's facet addressed advancing questions as a way to direct, orient, and/or focus students to particular mathematical ideas and elements of a task that supports the students in their mathematical thinking. For example, during the third interview, Nick was asked to code the orienting and focusing question type, to which he responded, "Orienting and focusing, probably I would put it in advancing. Because it's kind of getting the students to think about key elements, but you're doing that in order for students to solve the problem getting started" (Third Interview, audio-recording).

Both Leslie's and Steve's facets characterized the action of guiding as a series of questions that funneled students through a task. For example, during the second interview, Leslie was asked to talk about a series of questions in which she asked the student to determine the difference in the distance traveled between hours 0 and 1, 1 and 2, 2 and 3. Leslie stated,

> But during this part I think I was just advancing, essentially getting them to think outside their current thinking pattern … I think I almost use a series of assessing questions … And putting those assessing questions together is considered advancing because I'm getting them to the next box through a series of assessing questions … I definitely think my main goal was to find the pattern. Actually going and finding a pattern is more advancing than asking them maybe what they see at this moment. (Second Interview, audio-recording)

Another example, during the third interview, Steve was asked to explain his definition for advancing questions, to which he responded,

> If I want them to complete the problem and understand why the problem has this specific answer, I would ask them questions throughout the problem. "So what's the first thing you're going to do? Okay, what's the next one? Next? All right." Then, eventually they just talk through it after a while. Talk through the problem. (Third Interview, audio-recording)

One could argue that using a series of questions that direct students' through steps for carrying out a task limits the amount of thinking on the part of the students. Hence, Leslie's and Steve's facets addressed guiding as a use of questioning that may constrain students' mathematical thinking. Where as Gretchen's and Nick's facets addressed guiding as questioning that may promote students' mathematical thinking.

Judicious Telling

The four PSMTs constructed conceptions of judicious telling that included facets associated with twelve different features of judicious telling, five of which are more particular features of the first feature:

- *Judicious telling conveys information*
    - *Judicious telling conveys terminology*
    - *Judicious telling conveys directions*
    - *Judicious telling conveys contexts*
    - *Judicious telling conveys answers*
    - *Judicious telling conveys procedure*
- *Judicious telling is a statement and not a question*
- *Judicious telling repeats students' contributions*
- *Judicious telling shouldn't take away students' opportunities to do mathematics*
- *Judicious telling is used when students have difficulties*
- *Judicious telling sets up mathematical tasks*
- *Judicious telling supports students when they engage in mathematical tasks*

The five more particular features of the first feature address what information is conveyed by judicious telling. In the following subsections addressing each of the features of judicious telling, I illustrate the similarities and differences among the PSMTs' facets associated with the feature using data that are indicators of their facets.

Judicious Telling Conveys Information

All four PSMTs' conceptions of judicious telling that included facets associated with the feature *judicious telling conveys information to the student.* For example, Leslie wrote early in the course, "They [judicious telling] may be the teacher 'telling' the group information in order for them to be able to complete an assignment" (Leslie's Homework Assignment after Class 5) as a defining feature for judicious telling. Further, all four PSMTs constructed conceptions of judicious telling that included facets that specifically addressed the type of information conveyed through judicious telling. In the following five subsections I report these features - *judicious telling conveys terminology*, *judicious telling conveys directions*, *judicious telling conveys contexts, judicious telling conveys answers*, and *judicious telling conveys procedure*.

Judicious telling conveys terminology. All four PSMTs' conceptions of judicious telling that included facets associated with the feature *judicious telling conveys mathematical terminology,* but there were two differences among the PSMTs' facets. First, Leslie's, Gretchen's, and Nick's facets addressed conveying mathematical terminology that corresponded to student generated mathematical ideas. For example, during her third interview, Leslie stated during the card sort activity, "Inserting terminology. Most of the time I'd say that's telling. …So they get the idea and are getting to the idea by themselves, but when you are inserting terminology, you're just giving them a name for the face" (Third Interview, audio-recording). Steve's facet addressed conveying terminology and definitions as something given to students when they are talking about an idea but do not know the word associated with the idea. For example, during the second interview, Steve was asked to explain his understanding of judicious telling, “So, telling can be the teacher telling the student the word they are looking for. They know what they are talking about, but they don’t know, maybe, the certain vocabulary for the lesson” (Second Interview, audio-recording). A second difference was Nick’s facet addressed terminology enabling communication of mathematical ideas among the students in the classroom. During the third interview, Nick categorized inserting terminology as telling and stated,

> Students have a good idea of what's going on and your just giving them mathematical language, which would make conveying mathematical ideas possible. [inaudible] universal thing or whatever environment your in, that’s a way for people to communicate and you can't expect [the students] just to know that right off the bat. (Third Interview, audio-recording)

The key difference among the PSMTs’ facets, was that Nick addressed the role conveying terminology has in promoting student-to-student mathematics discourse.

Judicious telling conveys directions. Three of the four PSMTs’ (Leslie, Steve, and Nick) conceptions of judicious telling included facets associated with the feature *judicious telling conveys directions*, but there was a difference among the PSMTs’ facets. Both Leslie’s and Steve’s facets addressed the way in which teachers may use directions or instructions to clarify aspects of the tasks during either a teacher’s set-up of the tasks or while students work on the task. For example, after the Class 25 Approximation of Practice, Steve coded the following statement from Class 25 as telling, “He's putting four tacks here on the end - on each of the ends. So if he has a long stream of posters he puts four tacks on either end of that" (Class 25 Approximation of Practice, audio-recording). Steve wrote in the corresponding StudioCode Timeline,

> During the lesson the student read the introduction and the problem, but was confused by what it meant. I explained to the student that in the poster problem there will be four tacks on the end of a long string of posters … The purpose of this telling statement was to clarify the instructions for the student in order for him to successfully work on the rest of the problem. (Steve's Homework Assignment after Class 25, audio-recording)

Nick's facet was limited to conveying directions or instructions to a student, and did not include the purpose for conveying directions or instructions (e.g., setting up a task, clarifying a task). For example, early in the course, Nick wrote, "directions/giving the problem" (Nick's Homework assignment after Class 5) as a defining feature for judicious telling.

The difference between Nick's facet and the facets of Leslie and Steve is important to recognize. In comparison to Nick's facet, Leslie's and Steve's facets more thoroughly addressed the role judicious telling has for teachers in setting up mathematical tasks (e.g., judicious telling clarifies aspects of a written task that allows students to engage in a task).

Judicious telling conveys a context. Two of the four PSMTs' (Gretchen and Leslie) conceptions of judicious telling included facets associated with the feature *judicious telling conveys or establishes a context,* but there was difference in the way Leslie and Gretchen described the teacher's rationale for conveying a context. Leslie's facet associated with this feature involved sharing a context to illustrate the importance of a mathematical idea or a reason why the students are learning a mathematical idea. For example, during the third interview, Leslie is asked to talk more about the establishing context question type, to which she responded, “If your purpose is just to provide them with context – “This is why it's important.”, “Where you’ll see it.”, and “How it came to be.” – making links that way is telling” (Third Interview, audio-recording). Gretchen’s facet associated with this feature addressed conveying information about the context in which the mathematical task is situated so that the student may be able to engage in the task. For example, during the third interview, Gretchen categorized the establishing context question type as a telling statement and stated,

> If you're doing the question about the lottery, and you [the student] don't know about the lottery, I'm going to tell you what the lottery is … And how you win and what's going on for that mathematics to make sense to you. Because if you don't know about the lottery, then you can't do a math problem about it and you’re not going to be able to discover that through a question. (Third Interview, audio-recording)

Hence, the difference between these two PSMTs’ facets concerned the way in which Leslie and Gretchen interpreted *context*.

Judicious telling conveys answers. Two of the four PSMTs’ (Nick and Steve) conceptions of judicious telling included facets associated with the feature *judicious telling conveys answers to mathematical tasks.* Both Nick and Steve understood that this function of judicious telling was not necessarily an aspect of instruction that promoted students’ mathematical thinking. For example, after the Class 9 First Rehearsal, during Class 10, Nick and his group coded one of his explanations in the Class 9 First Rehearsal, “So, Student 3, if you look at your table. You used the equation, times five is equal to distance. So, one times five is equal to five, two times five is equal to ten, and so on down the table" (Class 9 First Rehearsal, audio-recording), as judicious telling. Nick explained to his group, "I was basically showing him how the symbolic representation related to the table. But I should have asked about it" (Second Interview, audio-recording).

Judicious telling conveys mathematical procedures. Three of the four PSMTs' (Nick, Steve, and Gretchen) conceptions of judicious telling included facets associated with the feature *judicious telling conveys mathematical procedures*, which involved conveying mathematical procedures and leading students through a mathematical procedure. For example, Gretchen and her peers categorized her Second Rehearsal statement, "Well let's look at this. So if we have the one, three, and five, we can think like this … But then, for each of these numbers" (Class 14 Second Rehearsal, audio-recording) as a telling. Gretchen wrote in the corresponding StudioCode Timeline memo,

> I wanted to tell Student 3 how to look at the ones place and relate it to the sums he had already figured out. This worked because Student 3 was able to realize the ones place sums would be the same and then he just had the tens to deal with. Leading students through a method. (Class 15 Second Rehearsal Analysis, StudioCode Timeline)

Another example, during the Class 26 small group analysis, Nick and his group analyzed PT2's statement during that Class 25 Approximation of Practice in which PT2 instructs his student to draw beams of length one and two. The following is an excerpt from the conversation,

> Nick: It sounds like your purpose was more you wanted her to see that she was wrong. You wanted her to get the right answer to see that her answer was wrong.

> PT2: My purpose was for them to make a larger representation. The student was staring – she was just kind of staring at it and not making any progress so I gave an alternative way to solve the problem.
>
> Nick: Yeah, okay. Then that is more leading through a method … If that's your purpose. You're beginning the method for her … You're giving her an entry point. (Class 26 StudioCode Analysis of Class 25 Approximation of Practice, audio-recording)

Judicious Telling is a Statement and Not a Question

Three of the four PSMTs' (Leslie, Gretchen, and Nick) conceptions included facets associated with the feature *judicious telling is a statement and not a question.* For example, as Nick and his peers analyzed a narrative case, he stated, "I think that's how I label things as telling. If they are not questions, usually" (Class 13 small group analysis). Another example, Gretchen told her peers during a small group discussion that "Telling has to be a statement and a question is assessing" (Class 11 Categorization of 9QTs, audio-recording).

Judicious Telling Repeats Students' Contributions

Three of four PSMTs' (Leslie, Gretchen, and Nick) conceptions of judicious telling included facets associated with the feature *judicious telling repeats students' contributions*, but there is a difference between Gretchen's purpose for repeating and the other two PTs' purposes for repeating. Both Leslie's and Nick's facets included the teacher's purpose for judicious telling, which was to highlight for the students key mathematical elements pertaining to the task or lesson goal. For example, during the third interview, Nick stated, "I think the one that sticks out to me was repeating what a student said. That could be when they said something important and you just want to highlight that" (Third Interview, audio-recording). Gretchen's purpose was to give students an opportunity to hear or confirm a student's contribution. For example, after the Second Rehearsal, Gretchen wrote in her StudioCode Timeline memo corresponding to a telling statement, "Just repeating what Student 3 told me to confirm this understanding and my own of this work" (Class 15 StudioCode Analysis of the Second Rehearsal, StudioCode Timeline).

The difference between Gretchen's facet and the facets of Nick and Leslie speaks to the multiple functions that this feature of judicious telling may serve in the mathematics instruction. I argue that Nick's and Leslie's purpose for repeating student contributions is oriented towards promoting students' mathematical thinking by highlighting aspects of their current thinking. While Gretchen's purpose for repeating student contributions is oriented towards recognizing and confirming students' mathematical thinking.

Judicious Telling Shouldn't Remove Students' Opportunities to Do Mathematics

Three of the four PSMTs (Leslie, Gretchen, and Nick) constructed conceptions of judicious telling that included facets associated with the feature *judicious telling shouldn't take away students' opportunities to do mathematics,* which involved judicious telling not making tasks too easy or eliminating students' opportunity to engage in productive struggle in mathematics (productive struggle described by NCTM, 2014). For example, at the end of the semester, Leslie wrote,

> Judicious telling are also important but should be used wisely. You, as the teacher, do not want to give away too much information that you hinder the

> development of the student. A telling statement, in which you flat out tell them the equation to use and expect them to memorize it, would be an ineffective and the exact opposite of what we have learned all semester. (Final Project Paper)

Judicious Telling is Used When Students Have Difficulties

Three of the four PSMTs' (Leslie, Steve, and Gretchen) conceptions of judicious telling included facets associated with the feature *judicious telling is used when students have difficulties.* For example, during the second interview, Steve was asked to explain his understanding of judicious telling, to which he responded,

> So, telling can be the teacher telling the student the word they are looking for. They know what they are talking about they don't know, maybe, the certain vocabulary for the lesson … And then telling is when the students because they don't really don't know. (Second Interview, audio-recording)

Another example, Leslie wrote in a StudioCode timeline memo associated with a telling statement in her Class 25 Approximation of Practice, "I wanted him to start thinking about his formula - which he was getting stuck on - in terms of multiplying something by 4" (Leslie's Homework Assignment after Class 25, StudioCode Timeline).

Judicious Telling Sets Up Mathematical Tasks

All four PSMTs' conceptions of judicious telling included facets associated with the feature *judicious telling sets up mathematical tasks,* which involved the purpose for teachers conveying a task, directions, or other information to students at the beginning of a lesson. Nick's facet differed from the other three PSMTs because his facets focused on sharing a mathematical task. For example, early in the course, Nick wrote, "directions/giving the problem (Are they Judicious telling?)" (Nick's Homework assignment after Class 5) as a defining feature for judicious telling. This is in contrast to the other three PSMTs, whose facets addressed the ways in which the directions set up a task. For example, after the Class 9 First Rehearsal, Gretchen wrote, “The instructional purpose of Judicious telling is to provide students with enough information, support, or terminology to begin a problem and understand which direction they are supposed to take it in” (Gretchen’s Homework Assignment after Class 6).

Judicious Telling Supports Students as they Engage in a Task

All four PSMTs’ conceptions of judicious telling included facets associated with the feature *judicious telling supports students as they engage in a task.* However, there were differences among the facets in the way the judicious telling supported the students. Steve’s facets focused on using directions or recommendations as ways to guide students through steps of a mathematical procedure and to the objective of a task. For example, during the second interview, Steve was asked to talk about his understanding of judicious telling, to which he stated,

> It could be … the teacher is teaching a procedure of how to distribute a number. So, “The first thing you have to do is multiply this number by the first number in a binomial, trinomial, or, whatever. The second thing you are going to do is multiply by the second number in the trinomial, or whatever.” (Second Interview, audio-recording)

This use of judicious telling in mathematics teaching aligns more with direct instruction in which a teacher is transmitting a particular approach to a mathematical task or leading a student through the execution of a mathematical procedure.

Gretchen's and Leslie's facets addressed judicious telling as conveying necessary information to students who are unable to proceed in a task. For example, early in the course Leslie wrote,

> The instructional purpose of telling is to provide direction or instruction so the students can do their work in the most effective way possible. They are also used as follow-ups to assessing questions in which the students may not know the answer and the teacher says it in order to waste as little time as possible. They are used in order to provide the students with information that they may not know otherwise that is useful to the current activity. (Second Interview, audio-recording)

Nick's facet was similar in that the teachers were conveying information, but Nick's facet did not address students' difficulties. Rather, his facet focused on judicious telling supporting students in exploring and struggling through mathematics. For example, during the third interview, Nick stated,

> I know that was a good thing for the telling statement, because when we first learned about Judicious telling my idea of them was giving students information and I wanted to avoid them completely. By the end of the class, I found like ten different ways how I could effectively use telling in the classroom and not be directly giving my students information – they could still be struggling through the mathematics. (Third Interview, audio-recording)

The difference among the PSMTs' facets is important to recognize. Steve's facet of judicious telling seems to align with direct instruction and may constrain students' opportunities to engage in productive struggle (productive struggle as described by

NCTM, 2014). The other three PSMTs' facets, particularly Nick's facet, seem to convey judicious telling as a way to support students' opportunities to engage in productive struggle and/or a way to support their work on a task.

Chapter Conclusion

In summary, the cross-case analysis of the four PSMTs' conceptions of assessing questions, advancing questions, and judicious telling determined that the PSMTs constructed sets of similar facets for each TTT. These findings are the basis for two assertions.

First, the PSMTs' conceptions of assessing questions, advancing questions, and judicious telling included facets that were oriented towards focusing on and promoting students' mathematical thinking. For example, the more particular facets of the PSMTs' conceptions of assessing questions associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge* were oriented towards teachers focusing on students' mathematical thinking. That is, the more particular facets associated with the more particular features – *assessing questions determine students' approaches or rationales for work as they engage in a task, assessing questions determine students' rationales for answers to a mathematical task,* and *assessing questions gauge students' mathematical understanding and/or capabilities* – addressed *what* about students' mathematical thinking, understanding, and/or knowledge that assessing questions support teachers in determining.

Another example, the more particular facets of the PSMTs' conceptions of advancing questions associated with the feature *advancing questions extend students' understanding and/or thinking* were oriented towards promoting students' mathematical thinking. That is the, the more particular facets associated with the more particular features – *advancing questions support students' thinking about new or different situations, advancing questions support students' thinking about a general case/idea, advancing questions support students' thinking about new mathematical relationships and/or meanings,* and *advancing questions support students in engaging in mathematical processes* – addressed the ways in which students' thinking is extended towards the mathematical goal of the lesson by advancing questions.

A final example, some of the facets of the PSMTs' conceptions of judicious telling focused on promoting students' mathematical thinking by either conveying information – *judicious telling conveys directions* and *judicious telling conveys terminology* – or by focusing on an aspect of students' contributions – *judicious telling repeats students' contributions*. Also, some of the facets of the PSMTs' conceptions of judicious telling included facets associated with the features *judicious telling shouldn't take away students' opportunities to do mathematics, judicious telling sets up mathematical tasks,* and *judicious telling supports students when they engage in mathematical tasks* were oriented towards ways teachers may use judicious telling to promote students' engagement in productive struggle.

A second assertion, while PSMTs constructed similar facets as a group, they individually constructed nuanced facets that were different from each other. One common difference was the way in which PSMTs conceived of or interpreted particular constructs in the facets of their conceptions of TTT. For example, both Gretchen's and Leslie's conceptions of judicious telling included facets associated with the feature *judicious telling conveys a mathematical context,* but the way in which the PSMTs' interpreted context differed.

A more important difference was in the degree to which the PSMTs' conceptions of the TTT addressed the way the TTT supported the students' mathematical thinking and learning. For example, Leslie's and Steve's conceptions of advancing questions and assessing questions included facets – *assessing questions guide students' mathematical thinking* and *advancing questions guide students' mathematical thinking* – that characterize guiding as a teacher action that could constrain the amount of mathematical thinking on the part of students. Another example, all the PSMTs' conceptions of assessing questions and advancing questions included facets that addressed the relation between the two TTT and conceptions of assessing questions that included the facet *assessing questions inform teacher's subsequent instruction*. However, in comparison to the other three PSMTs' conceptions, Gretchen's conceptions of both of these TTT more intricately addressed the importance of assessing questions in teacher's instruction and the relation between assessing questions and advancing questions.

In the next chapter, I address the PSMTs' pathways for constructing their conceptions of TTT that were presented in this chapter.

Chapter 5

PSMTs' Pathways for Constructing Conceptions

In this chapter, I report my findings in response to the second research question concerning the pathways in which the PSMTs constructed their conceptions of assessing questions, advancing questions, and judicious telling. I begin with an introduction that briefly reminds the reader of the constructivist learning perspective and explains terminology (e.g., pathway for constructing) as well as the diagrams used to model the PSMTs' pathways for constructing conceptions of TTT. Then, I share my findings concerning the PSMTs' pathways for constructing conceptions of the TTT. These findings are organized into three sections around assessing questions, advancing questions, and judicious telling.

Introduction

As I explained at the end of Chapter 1, from a constructivist learning perspective, an accommodation to one's conception of an entity is an act of learning. More specifically, an accommodation involves adding conditions to or changing conditions of one's recognition pattern, which I interpret as representative of changes to one's conception of the entity. Hence, the PSMTs' pathways for constructing conceptions of TTT are sequences of accommodations to the PSMTs' conceptions of TTT.

I use the phrase *pathway for constructing* because, as the researcher, I retrospectively analyzed the way in which the PSMTs constructed their conceptions of TTT. That is, I examined the sequence in which the PSMTs added facets to their conceptions of TTT and I proposed a model to characterized the totality of the sequence – a *pathway*. The pathway is the sequential order in which a PSMT added facets to their conception of a TTT. An important note for the reader, the more particular facets of assessing questions, advancing questions, and judicious telling identified in Chapter 4 are not included in these pathways. I made this decision because I wanted to focus the analysis of the pathways on the key features of the TTT, as identified by the PSMTs, in order to make similarities and differences among the pathways more salient for the reader. Hence, I will be focusing on the addition of the facets associated with features and not address the changes to facets associated with the features (see Stage 4 of the Analysis in Chapter 3 for an explanation of addition of a facet and changes to a facet).

The four PSMTs' pathways for constructing each TTT are represented in diagrams organized in a sequential group of rectangular cells. Each of the cells in the diagram contains one or more symbols that represent features of a TTT, which I refer to as a *set*. The symbols are indicators of the features of the TTT and the key for the symbols can be found at the beginning of each section in Table 5-1, Table 5-2, and Table 5-3. Each set represents an accommodation and the addition of new symbol(s) to the set represents the addition of facet(s) to the PSMTs' conceptions. The arrows between the sets are meant to help direct the reader through the sequence of accommodations – the PSMTs' pathway for constructing conceptions of a TTT. The sets in the pathway do not necessarily align temporally. For example, consider Steve's and Nick's pathways for constructing assessing questions (see Figure 5-1 and Figure 5-4, respectively). Steve's final set and Nick's final set both are the fourth set in the sequence of accommodations, but the accommodations occurred at different times in the semester for Steve and Nick (I talk more thoroughly about this in Chapter 6). A note of clarification, since the PSMTs constructed facet(s) associated with feature, I will refer to *sets of facets* or *sets of facets associated with features* rather than *sets of features*.

PSMTs' Pathways for Constructing Conceptions of TTT

This section is organized into three subsections around assessing questions, advancing questions, and judicious telling. I begin each subsection with a table that both reminds the reader of the features of each of TTT and provides a key for interpreting the diagrams of the models of the PSMTs' pathways for constructing conceptions of TTT. Then, I present the models' of the PSMTs' pathways and conclude each section with a comparison of the four PSMTs' pathways.

PSMTs' Pathways for Constructing Assessing Questions

The PSMTs constructed conceptions of assessing questions that included facets associated with six features and the other category (see Table 5-1).

Table 5-1: Assessing question features and associated symbol for diagrams.

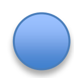
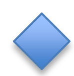
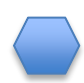
Assessing questions determine students' mathematical thinking, understanding, and/or knowing	
Assessing questions inform teachers' subsequent instruction	
Assessing questions support students' reflection	
Assessing questions role in whole class discussions	
Assessing questions guide students' mathematical thinking	
Assessing questions relation to advancing questions	
Other	

Figure 5-1 to Figure 5-4 represent the four PSMTs' pathways for constructing conceptions of assessing questions. While each of the PSMTs' pathways is unique, there are some similarities in the way in which the PSMTs' constructed their sets of facets. I will discuss these similarities in the subsection following the diagrams.

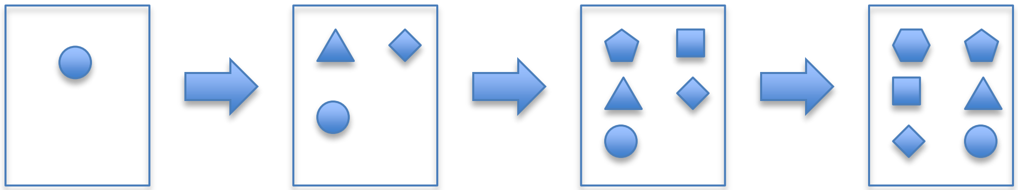


Figure 5-1: Steve's pathway for constructing assessing questions

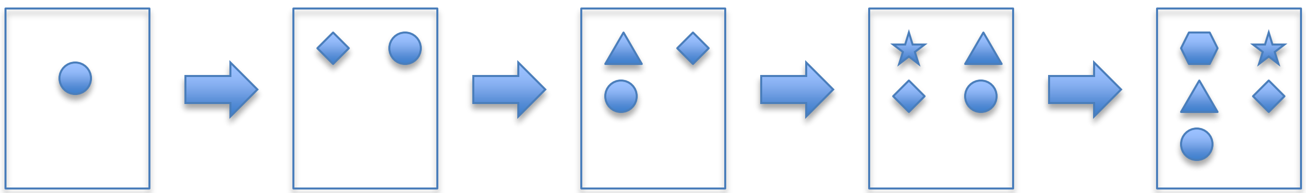


Figure 5-2: Leslie's pathway for constructing assessing questions

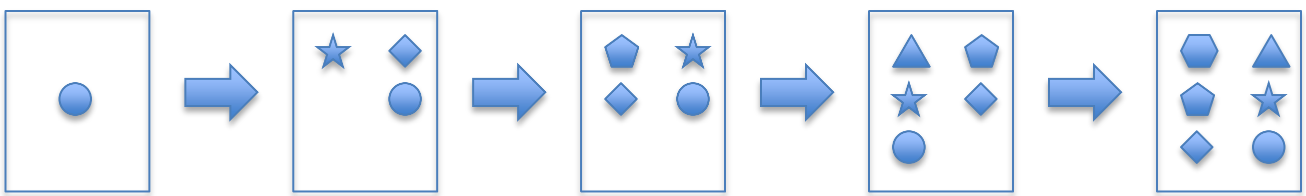


Figure 5-3: Gretchen's pathway for constructing assessing questions

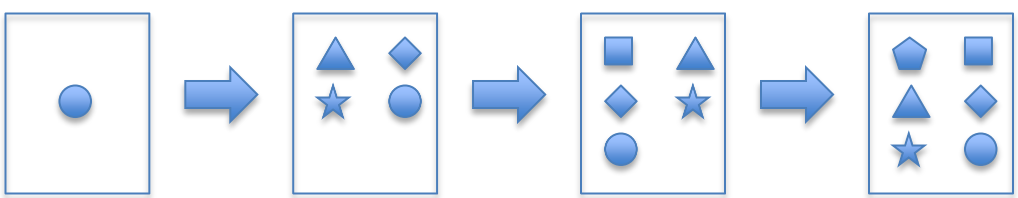


Figure 5-4: Nick's pathway for constructing assessing questions

Comparison of PSMTs' Pathways for Conceptions of Assessing Questions

The similar symbols among the PSMTs' pathways in Figures 5-1 to 5-4 indicate several similarities among the PSMTs' accommodations. The first set of facets in the PSMTs' pathways indicates a similarity among the four PSMTs' initial conceptions – all four PSMTs' included facets associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge.* This similarity is not surprising, as indicators of the facets for all four PSMTs came from their descriptions of assessing questions in the Class 4 Homework Assignment. The assignment introduced assessing questions through a course reading (Smith et al., 2008) and the PSMTs' facets associated with the first feature were nearly identical to Smith et al.'s description of assessing questions (I more thoroughly address this key course activity and evidence of impact in Chapter 6).

The second set of facets in the PSMTs' pathways indicates the following similarities among the PSMTs' accommodations: (a) all four PSMTs added facets associated with the feature *assessing questions support students' reflection*; (b) Nick and Steve both added facets associated with the feature *assessing questions inform teachers' subsequent instruction*; and (c) Nick and Gretchen added facets associated with the feature *assessing questions relation to advancing questions.* As reported more thoroughly in Chapter 6, the PSMTs added the facets associated with the features in the second set during or after the Class 5 Defining Features Discussions. In the Class 5 small and whole group discussions, the members of the class introduced several new qualities and functions of assessing questions.

The third set of facets in the PSMTs' pathways indicates one similarity among the PSMTs' accommodations – two of the PSMTs (Nick and Steve) added facets associated with the other category. That is, Steve and Nick each added a facet to their conceptions of assessing questions that were not associated with any of the features in Table 5-1 (the specific facets are addressed in Chapter 4).

The final set of facets in the PSMTs' pathways indicates one similarity among the PSMTs' accommodations. Gretchen, Leslie, and Steve all added facets associated with the feature *assessing questions guide students' mathematical thinking.* Interestingly, while these final accommodations occurred at different times during the semester for these three PSMTs (see Chapter 6 for the key course activities connected to the PSMTs accommodations), the PSMTs' accommodations occurred during or were preceded by a rehearsal.

The diagrams of the PSMTs' pathways for constructing conceptions of assessing questions also allows one to compare the complexity[8] of the PSMTs' final conceptions of assessing questions after a sequence of accommodations to their initial conceptions. For example, consider the case of Steve. Steve's initial conception of assessing questions included a single facet associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowing*. After the final accommodation in his pathway, Steve's conception of assessing questions included facets associated with five key features of assessing questions – *assessing questions inform teachers' subsequent instruction, assessing questions support students' reflection, assessing questions role in whole class discussions,* and *assessing questions guide*

[8] I consider *complexity* to refer to the multi-faceted nature of a PSMT's conception AND the number of facets orienting towards teachers focusing on and promoting students' mathematical thinking.

students' mathematical thinking – orienting towards focusing on and promoting students' mathematical thinking (I more thoroughly address the way in which these features are oriented towards focusing on and promoting students' mathematical thinking in Chapter 4). Further, the pathway indicates that Steve constructed this conception through a sequence of three accommodations following his initial conception (represented by the four sets of symbols). This sequence indicates that Steve constructed his conception at different points in time during the semester, preceded by different key course activities (the key course activities and their connection to the PSMTs' accommodations are addressed in Chapter 6).

PSMTs' Pathways for Constructing Advancing Questions

The PSMTs constructed conceptions of advancing questions that included similar facets associated with five features of advancing questions (see Table 5-2).

Table 5-2: Advancing Question features and associated symbol in diagrams

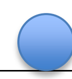
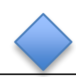
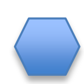
Advancing questions extend students' mathematical thinking and/or understanding	
Advancing questions aim students' towards the mathematical goal of a lesson	
When teachers may ask advancing questions	
Advancing questions build on students' mathematical understandings	
Advancing questions guide students' mathematical thinking	

Figure 5-5 to Figure 5-8 represent the four PSMTs' pathways for constructing conceptions of advancing questions. While each of the PSMTs' pathways are unique, there are some similarities in the way in which the PSMTs' constructed their sets of facets. I will discuss these similarities in the subsection following the diagrams.

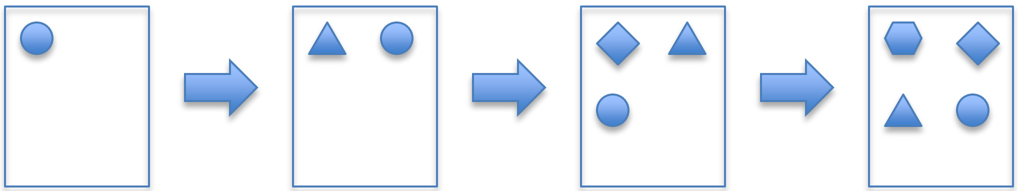


Figure 5-5: Steve's pathway for constructing advancing questions

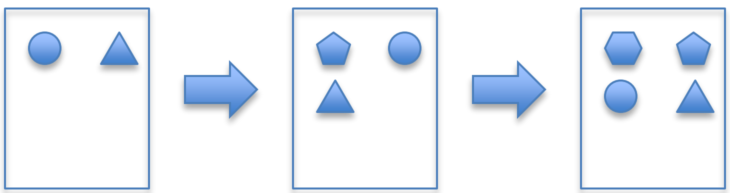


Figure 5-6: Leslie’s pathway for constructing advancing questions

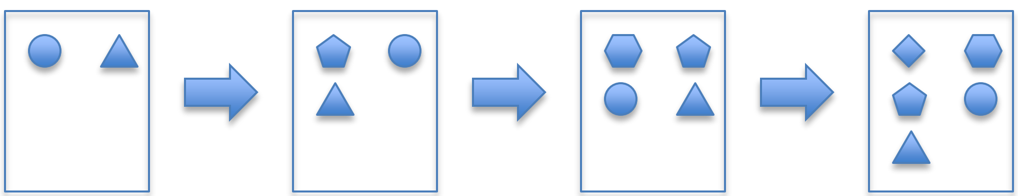


Figure 5-7: Gretchen’s pathway for constructing advancing questions

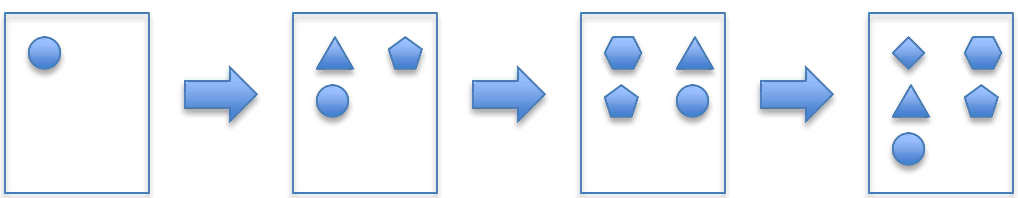


Figure 5-8: Nick's pathway for constructing advancing questions

Comparison of PSMTs' Pathways for Conceptions of Advancing Questions

The similar symbols among the PSMTs' pathways in Figures 5-5 to 5-8 indicate several similarities among the PSMTs' accommodations. The first set of facets in the PSMTs' pathways indicates a similarity among the four PSMTs' initial conceptions – all four PSMTs' included facets associated with the feature *advancing questions extend students' mathematical thinking and/or understanding.* Also, Gretchen and Leslie initially constructed facets associated with the feature *advancing questions aim students at the mathematical goal of the lesson*. These similarities are not surprising, as indicators of the facets for all four PSMTs came from their descriptions of assessing questionings in either the Class 4 Homework Assignment (Gretchen, Nick, and Steve) or immediately after the assignment in the Class 5 Working Definition Discussion (Leslie). The assignment introduced advancing questions through a course reading (Smith et al., 2008) and the PSMTs' facets associated with both of these features were nearly identical to Smith et al.'s description of advancing questions (I more thoroughly address this key course activity and evidence of impact in Chapter 6).

The second set of facets in the PSMTs' pathways indicates two similarities among the PSMTs' accommodations. First, three of the four PSMTs (Leslie, Gretchen, and Nick) added facets associated with the feature *advancing questions build on students' mathematical understanding and/or thinking.* Second, both Steve and Nick added facets associated with the feature *advancing questions aim students at the mathematical goal of the lesson*. As reported more thoroughly in Chapter 6, Gretchen, Nick, and Steve added the facets associated with these features during or after the Class 5 Defining Features Discussions. In the Class 5 small and whole group discussions, the members of the class introduced new qualities of advancing questions that were similar to the PSMTs' descriptions that were indicators of the facets associated with the feature *advancing questions build on students' mathematical understanding and/or thinking*. Interestingly, even though Leslie was present in Class 5, she did not add a facet associated with this feature of advancing questions until a later key course activity (see Chapter 6).

The third set of facets in the PSMTs' pathways indicates one similarity among the PSMTs' accommodations – three of the PSMTs (Leslie, Gretchen, and Nick) added facets associated with the feature *advancing questions guide students' mathematical thinking*. Interestingly, this third accommodation occurred at a different time for Leslie than it did for Nick and Gretchen. For Nick and Gretchen, the accommodation occurred as they engaged in the Class 5 and Class 6 StudioCode Analysis. For Leslie, the accommodation occurred later during the Class 9 First Rehearsal (these key course activities and their connections to PSMTs' accommodation are addressed in Chapter 6).

The fourth set of facets in the PSMTs' pathways indicates one similarity among the PSMTs' accommodations – two PSMTs (Nick and Gretchen) added facets associated with the feature *when teachers may ask advancing questions.* This similarity is not surprising, as indicators of the facets for both Nick and Gretchen came from data around a conversation the two PSMTs had with Dr. A in Class 8 (Class 8 audio-recordings, Second Interview audio-recordings). While Nick and Gretchen collectively planned for the First Rehearsal, they sought Dr. A's advice as to whether judicious telling would be an appropriate way to address an aspect of the task they considered to be problematic for the students. Dr. A role-played a scenario with Gretchen and Nick that ultimately resulted in the PSMTs' describing the way in which advancing questions could be used to support students' mathematical thinking if the students' encountered a difficulty.

Similar to the pathways for advancing questions, the diagrams of the PSMTs' pathways for constructing conceptions of assessing questions allows one to compare the complexity of the PSMTs' final conceptions of advancing questions with their initial conceptions of advancing questions. For example, consider the case of Nick. Nick's initial conception of advancing questions included a single facet associated with the feature *advancing questions extend students' mathematical thinking and/or understanding*. After the final accommodation in his pathway, Nick's conception of advancing questions included facets associated with four additional key features of assessing questions – *advancing questions aim students' towards the mathematical goal of a lesson, when teachers may ask advancing questions, advancing questions build on students' mathematical understandings,* and *advancing questions guide students' mathematical thinking.* Further, the pathway indicates that Nick constructed this conception through a sequence of three accommodations following his initial conception (represented by the four sets of symbols). This sequence indicates that Nick constructed his conception at different points in time during the semester, preceded by different key course activities (the key course activities and their connection to the PSMTs' accommodations are addressed in Chapter 6).

PSMTs' Pathways for Constructing Conceptions of Judicious Telling

The PSMTs constructed conceptions of judicious telling that included similar facets associated with seven features (see Table 5-3).

Table 5-3: Judicious telling features and associated symbol used in diagrams

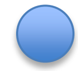
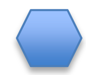
Judicious telling conveys information	
Judicious telling are statements and not questions	
Judicious telling repeats students' contributions	
Judicious telling shouldn't take away students' opportunities to do mathematics	
Judicious telling is used when students have difficulties	
Judicious telling sets up mathematical tasks	
Judicious telling supports students when they engage in mathematical tasks	

Figure 5-9 to Figure 5-12 represent the four PSMTs' pathways for constructing conceptions of judicious telling. While each of the PSMTs' pathways is unique, there are some similarities in the way in which the PSMTs' constructed their sets of facets. I will discuss these similarities in subsection following the diagrams.

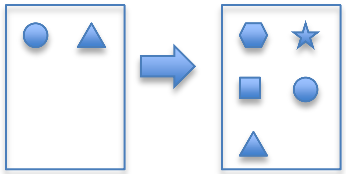


Figure 5-9: Steve's pathway for constructing judicious telling

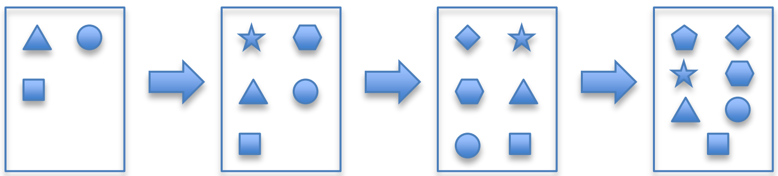


Figure 5-10: Leslie's pathway for constructing judicious telling

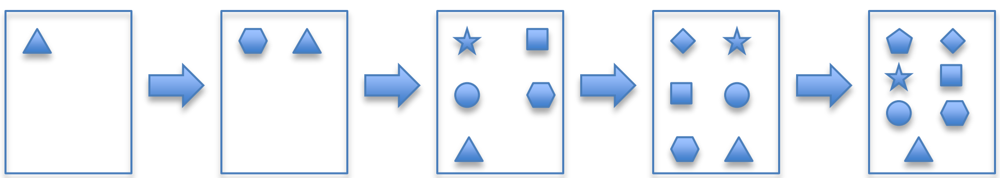


Figure 5-11: Gretchen's pathway for constructing judicious telling

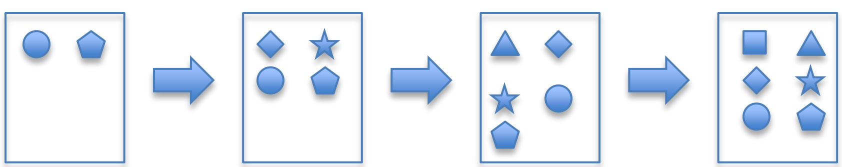


Figure 5-12: Nick's pathway for constructing judicious telling

Comparison of PSMTs' Pathways for Conceptions of Judicious Telling

Unlike the PSMTs' pathways for assessing questions and advancing questions, there were few similarities in the sequential order in which the PSMTs constructed facets of their conceptions of judicious telling. The first set of facets in the PSMTs' pathways indicates a similarity among Steve's, Leslie's, and Nick's initial conceptions – the three PSMTs' included facets associated with the feature *judicious telling conveys information.* Also, Gretchen's and Leslie's initial conceptions included facets associated with the feature *judicious telling is a statement and not a question*. The indicators of the facets associated with the two features came from the PSMTs' descriptions of judicious telling during or immediately after the Class 5 Working Definition Discussion. During the whole group discussion, Dr. A introduced the construct of judicious telling and the PSMTs' descriptions closely align with Dr. A's description of judicious telling (more thoroughly addressed in Chapter 6).

The second set of facets in the PSMTs' pathways indicates two similarities among the PSMTs' accommodations: three PSMTs (Steve, Leslie, and Gretchen) added facets associated with the feature *judicious telling is used when students have difficulties*; and three PSMTs (Steve, Leslie, and Nick) added facets associated with the feature *judicious telling sets up mathematical tasks.* As reported more thoroughly in Chapter 6, the PSMTs added the facets associated with the features in the second set during the Class 5 and Class 6 StudioCode Analysis. Interestingly, during this key course activity, none of the four PSMTs' were members of the same small group and the small groups were separated from each other.

Similar to the pathways for assessing questions and advancing questions, the diagrams of the PSMTs' pathways for constructing conceptions of judicious telling also allows one to compare the complexity of the PSMTs' final conceptions of judicious telling with their initial conceptions. For example, consider the case of Gretchen. Gretchen's initial conception of judicious telling included a single facet associated with the feature *judicious telling conveys information*. After the final accommodation in her pathway, Gretchen's conception of judicious telling included facets associated with six additional key features of judicious telling – *judicious telling are statements and not questions, judicious telling repeats students' contributions, judicious telling shouldn't take away students' opportunities to do mathematics, judicious telling is used when students have difficulties, judicious telling sets up mathematical tasks, and judicious telling supports students when they engage in mathematical tasks.* Further, the pathway indicates that Gretchen constructed this conception through a sequence of four accommodations following her initial conception (represented by the five sets of symbols). This sequence indicates that Gretchen constructed her conception at different points in time in the semester, preceded by different key course activities (the key course activities and their connection to the PSMTs' accommodations are addressed in Chapter 6).

Interestingly, Steve's pathway, unlike the other PSMTs' pathways, included only two sets. This pathway indicates there was only one accommodation after the initial conception in which Steve added facets. However, Steve's second set of facets does include facets associated with three additional features, which indicates a difference in complexity between his initial conception and his conception of judicious telling after the accommodation.

Chapter Conclusion

The findings in this chapter are the basis for two assertions. First, as one may expect, the PSMTs' conceptions of assessing questions, advancing questions, and judicious telling became more complex after a sequence of accommodations. Looking across the diagrams of PSMTs' pathways for each TTT, one can recognize an increase in the number of facets in each set, which indicates an increase in the complexity of the PSMTs' conceptions after each accommodation.

Second, although the PSMTs constructed conceptions of assessing questions, advancing questions, and judicious telling that included similar facets (as reported in Chapter 4), each of the PSMTs' pathways for constructing the TTT differed. One difference was in the number of sets of facets across the PSMTs' pathways (e.g., Gretchen's and Leslie's pathways for constructing assessing questions had four sets, where as Nick's and Steve's pathways had five sets). The difference in the number of sets among the PSMTs' pathways indicates that the PSMTs differed in the number of accommodations to their conceptions of TTT during the semester. Second, while similar facets were added during accommodations in the same ordinal position of the PSMTs' pathways, the accommodations occurred at different times during the semester (more thoroughly addressed in Chapter 6).

Chapter 6

Course Activities that Promoted PSMTs' Conceptions of TTT

In this chapter, I report my findings in response to my third research question concerning which course activities were key course activities. In Chapter 3, I described key course activities as course activities that were *connected*[9] to an addition of a facet or change in a facet (accommodation) of the PSMTs' conceptions of TTT. Hence, key course activities may have been perturbing events for the PSMTs' conceptions of TTT.

Chapter 6 is organized around the key course activities. For each of the key course activities, I first report the accommodations to the PSMTs' conceptions of TTT – additions of and changes to facets – that were connected to the key course activity. Then, I describe the nature of the key course activity. Last, I use findings from one of the four case study analyses to provide evidence that the course activity was a key course activity and may been a perturbing event for the PSMTs' conceptions of TTT.

An important note, key course activities, like all course activities (see Appendix A for a complete list of course activities), were designed around decompositions of, representations of, and approximations of practice (see Chapter 1 for description of these pedagogies of practice). Further, the instructors organized the course activities into three modified CEIs (see Appendix A for a thorough description of this design). In each description of the nature of the key course activities, I will explain the way in which the course activity was framed by pedagogies of practice.

[9] When I use *connected* in this section, I mean that the course activity either included or immediately preceded data indicating an accommodation to the PSMTs' conceptions.

Course Reading Assignment after Class 4

The Course Reading Assignment after Class 4 was the first key course activity. The Course Reading Assignment was connected to all four PSMTs' initial conceptions of assessing questions and three of the four PSMTs' (Steve, Gretchen, and Nick) initial conceptions of advancing questions (see Table 6-1).

Table 6-1: The features connected to Course Reading after Class 4

	Steve	Gretchen	Leslie	Nick
Assessing Questions	Assessing questions determine students' mathematical thinking, understanding, and/or knowledge	Assessing questions determine students' mathematical thinking, understanding, and/or knowledge	Assessing questions determine students' mathematical thinking, understanding, and/or knowledge	Assessing questions determine students' mathematical thinking, understanding, and/or knowledge
Advancing Questions	Advancing questions extend students' understanding and/or thinking	Advancing questions extend students' understanding and/or thinking	None	Advancing questions extend students' understanding and/or thinking
	Advancing questions support students' thinking about new or different situations	Advancing questions aim students' towards the mathematical goal of a lesson		
		Advancing questions support students' thinking about new or different situations		
Judicious Telling	None	None	None	None

Construction of PSMTs' Conceptions of Assessing Questions

The Course Reading Assignment was a key course activity connected to Steve's, Gretchen's, Leslie's, and Nick's initial conceptions of assessing questions that included facets associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge.*

Construction of PSMTs' Conceptions of Advancing Questions

Course Reading Assignment was a key course activity connected to three of the four PSMTs' (Steve, Gretchen, and Nick) initial conceptions of advancing questions that included facets associated with one or more features. All three of the PSMTs constructed initial conceptions that included facets associated with *advancing questions extend students' mathematical understanding and/or thinking.* In addition, both Gretchen and Steve constructed initial conceptions that included facets associated with the feature *advancing questions support students' thinking about a new or different situation*, which is a more particular feature of the first feature *advancing questions extend students' mathematical understanding and/or thinking.* That is, supporting students' thinking about a new or different situation is a way in which the advancing questions might extend students' mathematical understanding and/or thinking. Also, Gretchen constructed an initial conception that included a facet associated with the feature *advancing questions aim students' towards the mathematical goal of the lesson.*

Description of Course Reading Assignment after Class 4

The Course Reading Assignment after Class 4 was the course activity that first introduced components of the TTT framework, which served as the main decomposition of practice throughout the semester. In subsequent course activities, the PSMTs further defined the components of the TTT framework. In addition, they used this decomposition of practice as a framework to focus their analyses of representations of practice and their instruction during approximations of practice.

The Course Reading Assignment involved reading the article "Thinking Through the Lesson: Successfully Implementing High-Level Tasks" (Smith et al., 2008) and a brief writing assignment. The article introduced a protocol for crafting mathematics lessons grounded on students' thinking and introduced two of the three TTT – assessing questions and advancing questions. Smith et al. (2008) described assessing questions as, "questions that can *assess* what students understand about the problem (e.g., clarify what the student has done and what the student understands)" (p. 136) and described advancing questions as,

> questions that help students *advance* toward the mathematical goals of the less. Teachers can extend students beyond their current thinking by pressing them to extend what they know to a new situation or think about something they are not currently thinking about. (p. 136)

In addition to reading the article, the PSMTs were instructed to write descriptions for assessing questions and advancing questions in their notebooks.

Evidence Course Reading Connected to PSMTs' Accommodations

I will use the case of Gretchen to illustrate the way in which the Course Reading Assignment was a key course activity. This key course activity was connected to Gretchen constructing her initial conception of assessing questions that included a facet associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge.*

Prior to reading Smith et al. (2008), two pieces of evidence indicated that Gretchen was aware of teacher questioning in mathematics instruction, but not aware of functions or qualities of assessing questions and advancing questions. First, in Class 4, Gretchen agreed with PT12's statement, "the teacher knowing what questions to ask … ask them a question that's going to make them about how to get to the answer" (Class 4 small group discussion). Second, Gretchen stated in the second interview that she and her peers "didn't know anything about the teacher talk" (Second Interview) while they worked on the Staircase Problem in the first three classes of the semester.

In addition to reading Smith et al. (2008) after Class 4, the PSMTs wrote initial descriptions for assessing and advancing questions. Gretchen's initial description of assessing questions in her notebook after Class 4, "Questions that clarify what the student has done and what the student understands (so that teacher can assess what the student understands)" (Notebook after Class 4), is the first time she mentioned assessing questions in the course and is almost a direct restatement of the passage in Smith et al. (2008). The similarity and proximity of Gretchen's initial description and the description in Smith et al. (2008) indicate that the course reading was a key course activity for Gretchen. This key course activity was connected to Gretchen constructing of her initial conception of assessing questions that included a facet associated with *assessing questions determine students' mathematical thinking, understanding, and/or knowledge.*

Class 5 Working Definition Discussion

The Class 5 Working Definition Discussion was the second perturbing key course activity connected to: (a) all four PSMTs' accommodating their conceptions of assessing questions; (b) three of the four PSMTs' accommodating their conceptions of advancing questions; (c) Leslie's initial conception of advancing questions; (d) and all four PSMTs' initial conceptions of judicious telling (see Table 6-2).

Table 6-2: The features connected to 5 Working Definition Discussion

	Steve	Gretchen	Leslie	Nick
Assessing Questions	Assessing questions inform teachers' subsequent instruction	Assessing questions support students' reflection	Assessing questions support Students' reflection	Assessing questions inform teachers' subsequent instruction
	Assessing questions support students' reflection	Assessing questions relation to advancing questions	Other	Assessing questions support students' reflection
	Assessing questions determine students' rationales for answers to a mathematical task			Assessing questions relation to advancing questions
Advancing Questions	Advancing questions aim students' towards the mathematical goal of a lesson	Advancing questions build on students' mathematical understandings	Advancing questions extend students' understanding and/or thinking	Advancing questions build on students' mathematical understandings
				Advancing questions aim students' towards the mathematical goal of a lesson
	Other	Other	Advancing questions aim students' towards the mathematical goal of a lesson	Support students to think about new or different situations
Judicious Telling	Judicious telling conveys information	Judicious telling is a statement and not a question	Judicious telling is a statement and not a question	Judicious telling conveys information

	Judicious telling conveys answers		Judicious telling conveys information	Judicious telling shouldn’t take away students’ opportunities to do mathematics
			Judicious telling supports students when they engage in mathematical tasks	Judicious telling conveys answers
				Judicious telling conveys procedure

Accommodations to PSMTs’ Conceptions of Assessing Questions

The Working Definition Discussion was a key course activity connected to all four of the PSMTs accommodating their conceptions of assessing questions to include facets associated with multiple features. First, all four PSMTs added facets associated with the feature *assessing questions support students’ reflection*. Second, both Steve and Nick added facets associated with the feature *assessing questions inform teachers’ subsequent instruction.* Third, both Gretchen and Nick added facets associated with the feature *assessing questions relation to advancing questions*. Fourth, Steve changed a facet associated with the feature *assessing questions determine students’ mathematical understanding, knowing, and/or thinking* in that he differentiated the facet into a more particular facet that addressed what teachers aim to determine with assessing questions. The more particular facet was associated with the more particular feature *assessing questions determine students’ rationale for answers.* Last, Leslie added a facet associated with the *other* category.

Accommodations to PSMTs’ Conceptions of Advancing Questions

The Working Definition Discussion was a key course activity connected to Steve, Gretchen, and Nick accommodating their conceptions of advancing questions to include facets associated with multiple features and Leslie's initial conception of advancing questions that included facets associated with multiple features. First, Steve, Nick, and Leslie added facets associated with the feature *advancing questions aim students towards the mathematical goal of the lesson.* Second, both Nick and Gretchen added facets associated with the feature *advancing questions build on students' mathematical understandings.* Third, Nick, Steve, and Gretchen changed facets associated with the feature *advancing questions extend students' mathematical understanding and/or thinking* in that they differentiated the facets into more particular facets that addressed the way in which students' thinking is extended through advancing questions. For Nick, the more particular facets were associated with the more particular feature *advancing questions support students' thinking about new and/or different situations*. For Steve and Gretchen, the more particular facets were associated with the *other* category.

Construction of PSMTs' Conceptions of Judicious Telling

The Working Definition Discussion was a key course activity connected to all four PSMTs' initial conceptions of judicious telling that included facets associated with multiple features. First, Steve's, Leslie's, and Nick's conceptions included facets associated with the feature *judicious telling conveys information*. In addition, Nick's conception included a more particular facet associated with this feature in that he addressed the information conveyed by judicious telling. The more particular facet was associated with the more particular feature *judicious telling conveys answers* and the more particular feature *judicious telling conveys procedures.* Third, both Gretchen's and Leslie's conceptions included facets associated with the feature *judicious telling is a statement and not a question.* Fourth, Nick's conception included a facet associated with the feature *judicious telling does not take away students' opportunities to do mathematics.* Fifth, Leslie's conception included a feature *judicious telling supports students as they engage in a mathematical task.*

Description of the Class 5 Working Definition Discussions

The Class 5 Working Definition Discussions was the first course activity in which the instructors and PSMTs, collectively, furthered defined the constructs in the decomposition of practice – the TTT framework. The course activity provided an opportunity for the course instructor to introduce a third TTT (judicious telling) and for the members of the classroom to discuss qualities of each TTT.

In Class 5, the PSMTs were organized into four small groups of four to five students and the students shared their initial descriptions for assessing and advancing questions. As the groups discussed commonalities and differences among their descriptions, Dr. A monitored the conversations and instructed the groups to write their description of assessing questions and advancing questions on the whiteboard (see Figure 6-1). The activity culminated in a whole class discussion in which Dr. A pressed the groups to explain their meaning of the quality/characteristic that they wrote on the whiteboards. At the end of the conversation, Dr. A introduced the third type of teacher talk – judicious telling – that completed the decomposition of practice (e.g., the TTT framework). Dr. A explained that the PSMTs would use the TTT to analyze her utterances in an audio-recording from the first class session (a representation of practice) in which she taught the PSMTs in a lesson centered on the Staircase Problem (see Appendix E).

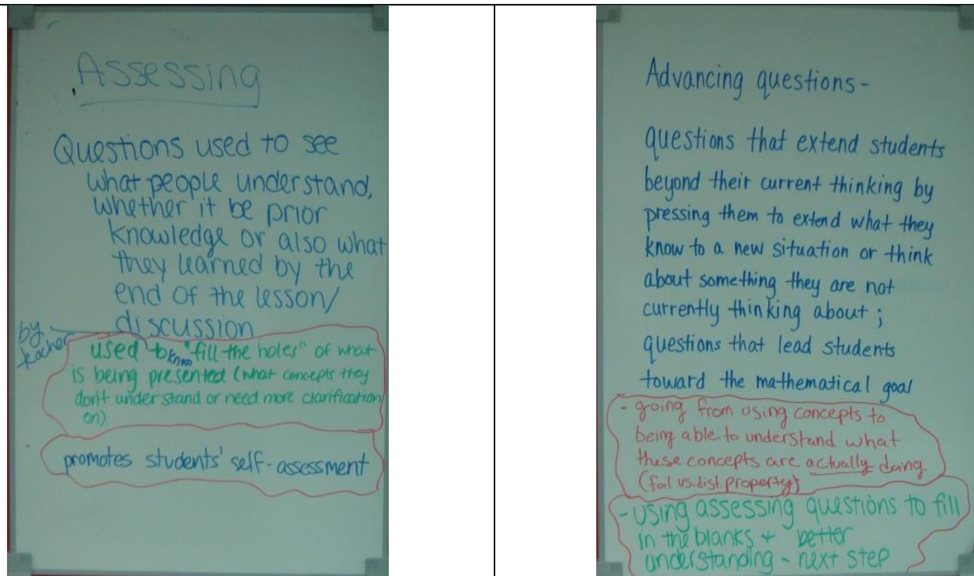


Figure 6-1: Photos of the white boards from Class 5

Evidence Class 5 Discussions Connected to PSMTs' Accommodations

I will use the case of Steve to illustrate the way in which the Class 5 Working Definition Discussion was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Steve adding a facet associated with the feature *assessing questions support students' reflection*.

During the Class 5 Working Definition Discussions, Dr. A and Gretchen introduced a new quality of assessing questions – assessing questions promote students' metacognition – to the whole class discussion,

> Dr. A: And to promote students' self assessment *[Dr. A is pointing to the whiteboard]*, talk about that a little bit.
>
> Gretchen: Yeah, we kind of talked about how the teacher asks an assessing question as the student answers and thinks about their answer, because also helps to understand potential gaps they need to acknowledge and potentially fill with the rest of the lesson. *[PTs writing at tables.]*

…

Dr. A: Yeah, it's thinking about your own thinking. So one of your outcomes of assessing questions is encouraging students to think about their own thinking. Not only do you get to know what they are thinking, but they actively have to think about their own thinking. So it encourages metacognition. (Class 5 Whole Class Working Definition Discussion, audio-recording and video-recording)

Dr. A’s statement at the end – “thinking about your own thinking … So it encourages metacognition” – indicates the way in which she interpreted Gretchen’s description – “promotes self-assessment.” Dr. A’s description and Gretchen’s description are similar to a defining feature Steve wrote after Class 5, **“**Helps student self-assessment … metacognition – think about own thinking” (Steve’s Homework assignment after Class 5). This defining feature is an indicator of Steve’s facet associated with the feature *assessing questions support students’ reflection*. In addition, while the conversation took place, Steve was observed in the video writing at his desk (Class 5 Working Definition Discussions, video-recording). The similarity between Steve’s defining feature and the statements during class in conjunction with the proximity of descriptions and Steve writing while the conversation took place, indicates that the Class 5 Whole Class Working Definition discussion was a key course activity for Steve. The key course activity was connected to Steve accommodating his conception of assessing questions to include a facet associated with the feature *assessing questions support students’ reflection.*

Class 5 and Class 6 StudioCode Analysis

The Class 5 and Class 6 StudioCode Analysis was a key course activity that was connected to all four PSMTs accommodating their conceptions of assessing questions, advancing questions, and judicious telling (see Table 6-3).

Table 6-3: The features connected to Class 5 and 6 StudioCode Analysis

	Steve	Gretchen	Leslie	Nick
Assessing Questions	Role of Assessing Questions in whole class discussions	Assessing Questions determine students' approaches or rationales for work as they engage in a task	Assessing Questions inform teachers' subsequent instruction	Assessing Questions determine students' approaches or rationales for work as they engage in a task
	Other facets	Assessing Questions determine students' rationales for answers to a mathematical task	Assessing Questions gauge students' mathematical understanding and/or capabilities	Assessing Questions determine students' rationales for answers to a mathematical task
		Assessing Questions gauge students' mathematical understanding and/or capabilities		
Advancing Questions	Advancing questions guide students' mathematical thinking	Advancing questions guide students' mathematical thinking	Advancing questions support students' thinking about new or different situations	Advancing questions guide students' mathematical thinking
		Other facets		
Judicious Telling	Judicious telling is used when students have difficulties	Judicious telling is used when students have difficulties	Judicious telling is used when students have difficulties	Judicious telling repeats students' contributions

	Judicious telling sets up mathematical tasks		Judicious telling sets up mathematical tasks	Judicious telling sets up mathematical tasks
	Judicious telling supports students when they engage in mathematical tasks		Judicious telling conveys directions	Judicious telling conveys directions
	Judicious telling conveys directions			

Accommodations to PSMTs' Conceptions of Assessing Questions

The Class 5 and 6 StudioCode Analysis was connected to all four PSMTs accommodating their conceptions of assessing questions to include facets associated with multiple features. First, Gretchen, Leslie, and Nick changed their facets associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge* in that they differentiated the facets into more particular facets that addressed what teachers aim to determine with assessing questions. For both Gretchen and Nick, the more particular facets were associated with the more particular features *assessing questions determine students' approaches or rationales for their work as they engage in a task* and *assessing questions determine students' rationales for answers to a task.* Also, for both Leslie and Gretchen, the more particular facets were associated with the more particular feature *assessing questions gauge students' mathematical understanding and/or capabilities.* Second, Leslie added a facet associated with the feature *assessing questions inform teachers' subsequent instruction.* Last, Steve added facets associated with the *other category* and with the feature *role of assessing questions in whole class discussions.*

Accommodations to PSMTs' Conceptions of Advancing Questions

The Class 5 and 6 StudioCode Analysis was connected to all four PSMTs accommodating their conceptions of assessing questions to include facets associated additional features of advancing questions. First, Steve, Gretchen, and Nick all added facets associated with the feature *advancing questions guide students' thinking.* In addition, Leslie and Steve changed their facets associated with the feature *advancing questions extend students' mathematical thinking and/or understanding* in that they differentiated the facets into more particular facets that addressed the way in which students' thinking was extended by advancing questions. For both Leslie and Steve, the more particular facets were associated with the more particular feature *advancing questions support students to think about new or different situations*, and, for Steve, the more particular facets were also associated with the other category.

Accommodations to PSMTs' Conceptions of Judicious Telling

The Class 5 and 6 StudioCode analysis was connected to all four PSMTs accommodating their conceptions of judicious telling to include facets associated with at least one additional feature of judicious telling. First, Steve, Leslie, and Nick all added facets associated with the feature *judicious telling is used to set-up mathematical tasks*. Second, the three PSMTs changed facets associated with the feature *judicious telling conveys information to students* in that they further differentiated the facets into more particular facets that addressed what information was conveyed. For all three PSMTs, the more particular facets were associated with the more particular feature *judicious telling conveys directions.* Third, Steve, Gretchen, and Leslie all added facets associated with the feature *judicious telling is used when students have difficulties*. Last, Steve added a facet associated with the feature *judicious telling supports students when they work on tasks* and Nick added a facet associated with the feature *judicious telling repeats students' contributions*.

Description of the Class 5 and 6 StudioCode Analysis

The Class 5 and 6 StudioCode Analysis was the students' first experience analyzing a representation of practice – an audio-recording of Dr. A teaching the Staircase Problem. The PSMTs were organized in small groups, four to five PTs, and collectively analyzed the representation of practice using the decomposition of practice (TTT framework), which the members of the group began to collectively define earlier in Class 5 (see description of Class 5 Working Definition Discussions).

The analysis activity began with a brief explanation of directions on how to use StudioCode as well as instructions for the activity. Then, in their small groups, the PSMTs used StudioCode to analyze an eight minute audio-recording of Dr. A interacting with the small groups of PSMTs as they engaged in the Staircase Problem. The PSMTs' coded instances of Dr. A's utterances in the audio-recording were represented in the StudioCode Timeline (see bottom of the screen shot in Figure 6-2). For each coded instance in the StudioCoce Timeline, the PSMTs were instructed to write memos (see upper right of screenshot in Figure 6-2) that included: (a) a brief rationale as to why the group classified the statement/question as Telling, Assessing, Advancing, or Other; and (b) what your group thinks was Dr. A's purpose in asking the question or saying a statement.

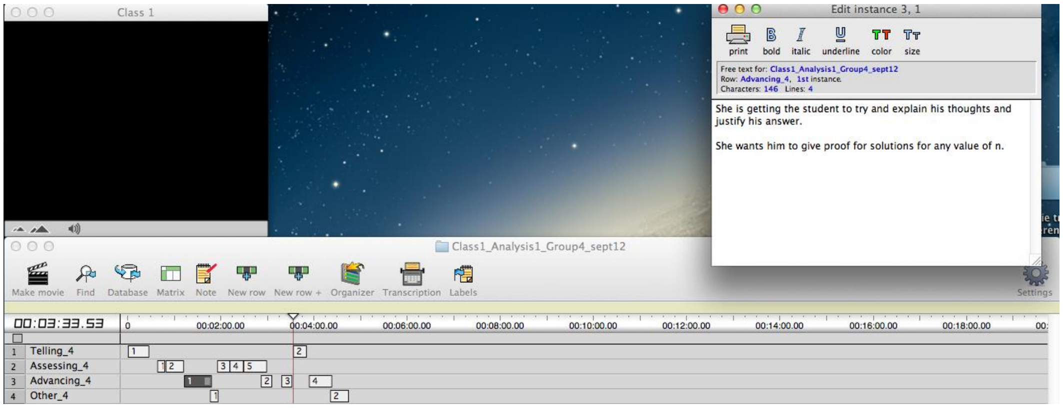


Figure 6-2: Example of StudioCode analysis window from Class 5 and Class 6

Evidence StudioCode Analysis Connected to PSMTs' Accommodations

I will use the case of Nick to illustrate the way in which the Class 5 and 6 StudioCode Analysis was connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Nick adding a facet associated with the feature *advancing questions guide students' mathematical thinking.*

During the analysis in Class 5, Nick was grouped with three other PSMTs – PT1, PT4, and PT5. PT 4 and PT5 coded Dr. A's questions "And what kind of thinking is that called? What kind of thinking is that called?" (Dr. A and the Staircase Problem, audio-recording) as advancing questions. Later, the group returned to the question in order to write the memo. The following is an excerpt from the conversation,

Nick: So.

PT1: It gets you to think about your own thinking. So metacognition.

[Pause while Nick types.]

Nick: Purpose for asking this. I think she was trying to ask this to relate it. This group was talking about their previous class about how they already had seen this example.

PT1: Yeah.

Nick: I think she was trying to get them to think about -

PT4: Tie the two together.

Nick: Yeah, because they were talking about combinatorics and they definitely would have talked about that. (Class 5 StudioCode Analysis, audio-recording)

Nick's statement "I think she was trying to get them to think about" introduced a quality of advancing questions that Nick had not included in his prior descriptions – advancing questions get students to think about a particular mathematical idea. Nick's peers' identification of Dr. A's question as an advancing question in Class 5 afforded the group an opportunity to further analyze her two questions. The subsequent analysis in which Nick's group discussed the purpose of the advancing question resulted in Nick introducing the new quality of advancing questions. Hence, the small group analysis of Dr. A and the Staircase Problem was a key course activity for Nick. This key course activity was connected to Nick accommodating his conception of advancing questions to include a facet associated with the feature *advancing questions guide students' mathematical thinking*

Class 6 Defining Features Discussions

The Class 6 Defining Features Discussions was a key course activity that was connected to: (a) Nick and Steve accommodating their conceptions of assessing questions; (b) Steve accommodating his conceptions of advancing questions; and (c) three PSMTs (Steve, Gretchen, and Nick) accommodating their conceptions of judicious telling (see Table 6-4).

Table 6-4: The features connected to Class 6 Defining Features Discussions

	Steve	Gretchen	Leslie	Nick
Assessing Questions	Assessing Questions gauge	None	None	Other facets

	students' mathematical understanding and/or capabilities			
Advancing Questions	Advancing questions build on students' mathematical understandings	None	None	None
Judicious Telling	Judicious telling conveys terminology	Judicious telling conveys information	None	Judicious telling is a statement and not a question
		Judicious telling sets up mathematical tasks		Judicious telling conveys terminology
		Judicious telling supports students when they engage in mathematical tasks		

Accommodation to PSMTs' Conceptions of Assessing Questions

The Defining Features Discussion was connected to both Nick and Steve changing facets associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge* in that they differentiated the facets into more particular facets that addressed what teachers aim to determine with assessing questions. For Nick, the more particular facets were associated with the other category and, for Steve, one of the more particular facets was associated with the more particular feature *assessing questions gauge students' mathematical understandings and/or capabilities*.

Accommodation to PSMTs' Conceptions of Advancing Questions

The Defining Features Discussion was connected to only one of the PSMTs, Steve, accommodating his conception of advancing questions. Steve added a facet associated with the feature *advancing questions build on students' mathematical thinking*.

Accommodations to PSMTs' Conceptions of Judicious Telling

The Defining Features Discussion was connected to three of the four PSMTs, Steve, Gretchen, and Nick, accommodating their conception of judicious telling. First, Steve and Nick changed their facets associated with the feature *judicious telling conveys information to students* in that they differentiated the facets into more particular facets that addressed what information was conveyed. For both Steve and Nick, the more particular facets were associated with the more particular feature *judicious telling conveys terminology*. In addition, Nick added a facet associated with the feature *judicious telling is a statement and not a question*. Last, Gretchen added facets associated with three features – *judicious telling conveys information, judicious telling sets up mathematical tasks,* and *judicious telling supports students when they work on tasks.*

Description of the Class 6 Defining Features Discussion

The Class 6 Defining Features Discussion was a second opportunity for the PSMTs to collectively further define the components of the TTT framework (decomposition of practice) – assessing questions, advancing questions, and judicious telling. This course activity built on two prior course activities. First, the Defining Features Discussion built on the PSMTs' defining features for the TTT that they wrote in the homework assignment after Class 5. Second, the Defining Features Discussion was conducted after the Class 5 and 6 StudioCode Analysis so that the members of the classroom could use their analyses of the representation of practice to support their assertions for defining features of the TTT framework.

Class 6 began with the PSMTs, in small groups of four to five, sharing and comparing their defining features from the homework assignment after Class 6. While the PSMTs engaged in these small group discussions, the instructors compiled all the groups' StudioCode Timelines into a single Timeline. The instructors intended to use the Timeline as a way to initiate a whole class discussion of the coded instances in order to generate a list of defining features for assessing questions, advancing questions, and judicious telling.

The whole class discussion began with the whole group discussing three of instances of Dr. A's utterances that the groups all coded the same way. After a recording was played for the class, Dr. A asked the students for their coding rationales and what this particular instance meant for the defining features of assessing questions, advancing questions, and judicious telling. Throughout the discussion, Dr. A recorded the defining features in a publicly displayed Word document. After discussing utterances that the group all agreed were advancing, assessing, and judicious telling, the discussion focused on several instances that the groups did not code the same. During this portion of the discussion, the PSMTs' shared their rationales for coding an utterance and Dr. A encouraged the PSMTs to respond to their peers' rationales, questions, and comments. In addition, throughout the conversation Dr. A emphasized that the PSMTs use evidence from the recording and course readings to support their arguments for coding an utterance as a particular TTT. The discussion concluded with several of the PSMTs sharing their observations about the structure/pattern of Dr. A's TTT in the audio-recording.

Evidence Class 6 Discussions Connected to PSMTs' Accommodations

I will use the case of Nick to illustrate the way in which the Class 6 Defining Features Discussions was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Nick changing a facet associated with the *judicious telling conveys information* in that he differentiated the facet into a more particular facet associated with the more particular feature *judicious telling conveys mathematical terminology.*

During the whole class discussion about defining features, Dr. A asked for the small groups' rationales for coding, "That's recursive thinking, right?" (Dr. A and the Staircase Problem, audio-recording) as judicious telling. The following is an excerpt from the discussion in which Dr. A and PT2 introduce a new quality of judicious telling – conveys definitions and terminology,

> Dr. A: Yeah, a mathematical definition. Right? *[PTs are writing at their tables.]* So, telling may have a feature of conveying a mathematical definition or putting a mathematical word, or vocabulary, to an idea.
>
> PT2: I would just like to say that you may also do that without giving a definition. Because when you said recursive.
>
> Dr. A: Mmhmm.
>
> PT2: If you listen almost the whole table said, "Oh well, that's this, this, this, and this."
>
> *[Nick and his peers are writing at table.]*

Dr. A: So is that more putting mathematical vocabulary to an idea?

PT2: Yeah.

Dr. A: Sometimes it might give definition and sometimes it just might say, "Oh, here's what you are talking about mathematically. Here's what this is called in mathematics." So let's put that on a different bullet. (Class 6 Defining Features Discussions, audio-recording and video-recording)

Dr. A and PT2 both assert that judicious telling may introduce a mathematical term to an idea generated by the students. As, Dr. A and PT2 are talking to each other, Nick wrote at his table, presumably, the following addition to his list of defining features, “Convey mathematical definition/putting mathematical vocab to an idea” (Nick’s notebook). The similarity between Nick’s feature and the quality of judicious telling introduced by members of the classroom in conjunction with Nick writing while the members of the classroom spoke, indicates that the Defining Features Discussion was a key course activity for Nick. This key course activity was connected to Nick accommodating his conception of judicious telling to include a more particular facet associated with the more particular feature *judicious telling conveys mathematical terminology*.

Case of Edith Hart Analysis

The Case of Edith Hart (Smith, Silver, & Stein, 2005) Analysis was a key course activity connected to all four PSMTs accommodating their conceptions of the TTT. The perturbing course activity was connected to: (a) Nick accommodating his conceptions of all three TTT; (b) Gretchen accommodating her conceptions of assessing questions and judicious telling; (c) Steve accommodating his conceptions of assessing questions and advancing questions; and (d) Leslie accommodating her conception of judicious telling (see Table 6-5).

Table 6-5: The features connected to Edith Hart Analysis

<table>
<tr><th></th><th>Steve</th><th>Gretchen</th><th>Leslie</th><th>Nick</th></tr>
<tr><td>Assessing Questions</td><td>Assessing questions determine students’ approaches or rationales for work as they engage in a task</td><td>Role of assessing questions in whole class discussions</td><td>None</td><td>Role of assessing questions in whole class discussions</td></tr>
<tr><td rowspan="4">Advancing Questions</td><td>When teachers may ask advancing questions</td><td rowspan="4">None</td><td rowspan="4">None</td><td rowspan="4">Advancing questions support students thinking about new mathematical relationships and/or meanings</td></tr>
<tr><td>Advancing questions support students’ thinking about a general case/idea</td></tr>
<tr><td>Advancing questions support students thinking about new mathematical relationships and/or meanings</td></tr>
<tr><td>Advancing questions support students’ thinking about new or different situations</td></tr>
<tr><td rowspan="2">Judicious Telling</td><td rowspan="2">None</td><td rowspan="2">Judicious telling repeats students’ contributions</td><td rowspan="2">Judicious telling repeats students’ contributions</td><td>Judicious telling repeats students’ contributions</td></tr>
<tr><td>Judicious telling supports students when they engage in mathematical tasks</td></tr>
</table>

Accommodations to PSMTs' Conceptions of Assessing Questions

The Edith Hart Analysis was connected to three of the four PSMTs (Steve, Gretchen, and Nick) accommodating their conceptions of assessing questions. First, both Gretchen and Nick added facets associated with the feature *role of assessing questions in whole class discussions*. Second, Steve changed his facet associated with the feature *assessing questions determine students' mathematical thinking, understanding, and/or knowledge* in that Steve differentiated the facet into a more particular facet that addressed what teachers aim to determine with assessing questions. The more particular facet was associated with the more particular feature *assessing questions determine students' approaches or rationales for their work as they engage in a task.*

Accommodations to PSMTs' Conceptions of Advancing Questions

The Edith Hart Analysis was connected to two of the four PSMTs (Steve and Nick) accommodating their conceptions of advancing questions. First, Steve added a facet associated with the feature *when teachers may ask advancing questions*. Second, Steve and Nick changed their facets associated with the features *advancing questions extend students' mathematical thinking and/or understanding* in that they differentiated the facets into more particular facets about the ways students' thinking may be extended by advancing questions. For both Nick and Steve, the more particular facets were associated with the more particular feature *advancing questions support students in thinking about new mathematical relationships and/or meanings* and, for Steve, the more particular facets were also associated with the more particular feature *advancing questions support students in thinking of a general case/idea* and the more particular feature *advancing questions support students' thinking about new or different situations.*

Accommodations to PSMTs' Conceptions of Judicious Telling

The Edith Hart Analysis was connected to three of the four PSMTs (Gretchen, Leslie, and Nick) accommodating their conceptions of judicious telling. All three of these PSMTs added facets associated with the feature *judicious telling repeats students' contributions.* Also, Nick added a facet associated with the feature *judicious telling supports students as they work on a task.*

Description of the Edith Hart Analysis

The Edith Hart Analysis was the PSMTs' second opportunity to analyze a representation of practice using the TTT framework (decomposition of practice) that the members of the classroom had collectively and individually defined in course activities following Class 4. The Case of Edith Hart (Smith et al., 2005) was a representation of practice that both modeled for the PSMTs how a teacher may enact TTT and engaged the PSMTs in the analysis of the ways in which the TTT may support students' thinking and engagement in mathematics.

The Edith Hart Analysis began in the homework assignment after Class 6. The individual analysis assignment involved the PSMTs reading the Case of Edith Hart (Smith et al., 2005), a narrative case of secondary mathematics teaching, and responding to the following prompts,

> (a) "code" for *Telling Statements, Assessing Questions*, and *Advancing Questions*; (b) What were Edith Hart's mathematical goals for her students?; (c) What did her students appear to learn during this class period? Provide specific evidence from the case (line numbers) to support your contentions about student learning; and (d) What did Edith Hart do (specifically) to support her students' learning?

Provide specific evidence from the case (line numbers) to support your contentions about her teaching practices. (Homework assignment after Class 6)

During most of Class 7, the PSMTs, in small groups of three to four PTs, shared and discussed their responses to the prompts in the homework assignment. During the small group discussions, Dr. A and the two teaching assistants monitored the PSMTs' conversations, pressed the PSMTs for evidence to support their assertions, and pressed PSMTs for ways in which their observations of Edith Hart's teaching connected with other readings.

Evidence Edith Hart Analysis Connected to PSMTs' Accommodations

I use the case of Gretchen to illustrate the way in which the Edith Hart Analysis was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Gretchen adding a facet associated with the feature *judicious telling repeats a students' contribution* to her conception of judicious telling.

During the individual analysis, Gretchen highlighted the following portion of her copy of the Edith Hart narrative case as a telling statement, "I repeated exactly what Patrick said so students could write it down" (Smith et al., 2005) (Gretchen's Notebook). During the Class 7 small group Edith Hart analysis, Gretchen was grouped with Nick and PT12. As the group discussed the Case of Edith Hart, Gretchen questioned whether repeating a student contribution was a quality of judicious telling,

Gretchen: I also didn't know when she repeats what the student said to reassure them that that's correct or make sure that everyone heard it. Is that telling?

[Nick and PT12 agree.]

Gretchen: It really wasn't her telling, but she's like reassuring a student.

PT12: Yeah. Reinforcing what she is saying.

Gretchen: Yeah, telling the rest of the class.

Nick: It's kind of highlighting important parts. (Class 7 Edith Hart Analysis, audio-recording)

Gretchen's question, about whether Edith Hart repeating a student's contribution was judicious telling, indicates that Gretchen was uncertain whether repeating was a quality of judicious telling. However, her peers' subsequent statements supporting this quality indicate that Gretchen and her group accepted repeating as a quality of judicious telling. The combination of the Class 7 discussion, Gretchen's highlighted statements in the narrative case of Edith Hart (Gretchen's notebook), and Gretchen's remarks that the Edith Hart Analysis was impactful on her learning of TTT (Second Interview, audio-recording) indicates that the Edith Hart Analysis was a key course activity for Gretchen. This key course activity was connected to Gretchen accommodating her conception of judicious telling to include a facet associated with the feature *judicious telling repeats students' contributions*.

Class 8 Planning Session for the First Rehearsal

The Class 8 Planning Sessions for the First Rehearsal was a key course activity that was connected to: three of the PSMTs (Gretchen, Leslie, and Nick) accommodating their conceptions of at least two TTT (see Table 6-6).

Table 6-6: The features connected to the Class 8 Planning Session

	Steve	Gretchen	Leslie	Nick
Assessing Questions	None	Assessing questions inform teachers'	Assessing questions inform teachers' subsequent instruction	Other facets

		subsequent instruction	Assessing questions relation to advancing questions	
			Assessing questions determine students' approaches or rationales for work as they engage in a task	
Advancing Questions	None	When teachers may ask advancing questions	Advancing questions build on students' mathematical understandings	When teachers may ask advancing questions
			Advancing questions support students thinking about new mathematical relationships and/or meanings	Advancing questions support students thinking about new mathematical relationships and/or meanings
			Advancing questions support students in engaging in mathematical processes	
Judicious Telling	None	Judicious telling shouldn't take away students' opportunities to do mathematics	None	None

Accommodations to PSMTs' Conceptions of Assessing Questions

The Class 8 Planning Session was connected to three of the four PSMTs (Gretchen, Leslie, and Nick) accommodating their conceptions of assessing questions. First, both Gretchen and Leslie added facets associated with the feature *assessing questions inform teachers' subsequent instruction.* Second, Leslie added a facet associated with the feature *assessing questions relation with advancing questions*. Last, both Leslie and Nick changed their facets associated the feature *assessing questions* *determine students' mathematical thinking, understanding, and/or knowledge* in that they differentiated the facets into more particular facets that addressed what teachers aim to determine with assessing questions. For Leslie, the more particular facet was associated with the more particular feature *assessing questions determine students' approaches or rationales for their work as they engage in a task* and, for Nick, the more particular facet was associated with the other category.

Accommodations to PSMTs' Conceptions of Advancing Questions

The Class 8 Planning Session was connected to three of the four PSMTs (Gretchen, Leslie, and Nick) accommodating their conceptions of advancing questions. First, both Gretchen and Nick added facets associated with the feature *when teachers may ask advancing questions*. Second, Leslie added a facet associated with the feature *advancing questions build on students' mathematical understandings*. Last, both Leslie and Nick changed their facets associated with the feature *advancing questions extend students' mathematical thinking and/or understanding* in that they differentiated the facets into more particular facets that addressed the ways students' mathematical thinking may be furthered by advancing questions. For both Leslie and Nick, the more particular facets were associated with the more particular feature *advancing questions support students to think about new mathematical relationships and/or meanings*. In addition, for Leslie, one of the more particular facets was associated with the more particular feature *advancing questions support students in engaging in mathematical processes*.

Accommodations to PSMTs' Conceptions of Judicious Telling

The Class 8 Planning Session was connected to only one PSMT (Gretchen) accommodating her conception of judicious telling. Gretchen added a facet associated with the feature *judicious telling shouldn't take away students' opportunities to do mathematics.*

Description of Class 8 Planning Sessions for First Rehearsal

The Class 8 Planning Session for the First Rehearsal was the PSMTs first opportunity to engage in an approximation of practice. This course activity was connected with several other course activities. First, the TTT framework (decomposition of practice) developed in prior course activities was used to focus the PSMTs' planning of a mathematics lesson on a particular aspect of instruction. Second, the analyses of the representation of practices (Case of Edith Hart, Dr. A and the StairCase problem) provided a set of examples of TTT from which the PSMTs used in planning their lessons. Last, the PSMTs used the lesson plans from Class 8 in their enactment of the lessons in the Class 9 First Rehearsal (approximation of practice).

At the start of the planning session, Dr. A briefly explained to the PSMTs the structure and organization of the First Rehearsal. She explained that two of the PSMTs in each of the small groups of four to five PSMTs would teach three doctoral students (Student 1, Student 2, and Student 3) for a 5-6 minute segment of a lesson involving a set of problems from *Connected Mathematics Project* (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002) (see Appendix C). She emphasized that the even though the First Rehearsal would last 5-6 minutes for each pair of PSMTs, the small groups needed to plan ways in which they would enact the entire problem set. Next, she explained that the planning session would focus on facilitating classroom mathematics discourse using the TTT framework (decomposition of practice) – asking good assessing questions and advancing questions as well as judicious telling. Dr. A's purpose for this type of planning was to focus PSMTs on how the PSMTs can prepare to interact with a small group of students by focusing on the students' mathematical thinking and understanding. Last, Dr. A introduced the modified "Thinking Through the Lesson Protocol" (see Appendix D) as a way of structuring the PSMTs planning for a mathematical lesson in which the PSMTs would be expected to enact instruction framed by the TTT framework (decomposition of practice). The protocol involved: (a) establishing the learning goal for the lesson; (b) identifying the CCSS-M content and practice standards involved in the task; anticipating student approaches, errors, and misconceptions; and (c) TTT they may use to support students progress towards the learning goal.

Evidence the Planning Session Connected to PSMTs' Accommodations

I use the case of Leslie to illustrate the way in which the Class 8 Planning Session was a key course activity that was connected to PSMTs accommodating their conceptions of TTT. This key course activity was connected to Leslie changing her facet associated with the feature *advancing questions extend students' mathematical thinking and understanding* in that she differentiated the facet into two more particular facets associated with two more particular features of advancing questions – *advancing questions support students in engaging in a mathematical process* and *advancing questions support students' thinking about new mathematical relationships and/or meanings.*

At the beginning of Class 8 Planning Session, Leslie and her small group (PT2 and PT5) discussed the mathematical processes and concepts involved in the problem set. Prior to Class 8, Leslie's conception of advancing questions included facets associated with the feature *advancing move students towards the mathematical goal.* This facet of her conception seemed to focus Leslie on not only the mathematics involved in the task, but also the ways in which advancing question may engage the students in the mathematics.

Leslie and her peers spent a significant amount of time during the Class 8 Planning Session talking about the mathematical opportunities in the CMP problem set and referred back to these opportunities as they planned their assessing questions and advancing questions. The following is an excerpt from a conversation, in which PT2 introduced two mathematical opportunities in the CMP Problem Set,

> PT2: A pattern, they are looking for the pattern first.
>
> Leslie: Okay, so ... [Leslie is typing.]
>
> PT5: To find patterns [inaudible.]
>
> PT2: Right.
>
> Leslie: Yeah.
>
> ...
>
> Leslie: Should we also include being able to read a graph and a table? Do we have something like that already?
>
> PT2: Relate the graph to the table.
>
> Leslie: Okay. Being able to plot a table of points on a graph. *[Leslie is typing]*.
>
> (Class 8 Planning Session for First Rehearsal, audio-recording)

PT2 introduced into the small group conversation the idea that the problem set involved students "looking for the pattern first", to which Leslie and PT5 agreed. In the second portion of the conversation, Leslie agreed with PT2 that one of the mathematical processes involved in the set of problems was being able to "relate the graph to the table." Both of PT2's statements share similarities with Leslie's entries in the modified TTLP - Leslie included "be able to recognize patterns in tables" (Homework assignment after Class 8) and "be able to plot points on a graph" (Homework assignment after Class 8) in the modified TTLP as "mathematical content and processes" that the students will learn from the set of tasks. Additionally, PT2's statements are similar to aspects of Leslie's descriptions of advancing questions that emerge after the Class 9 First Rehearsal (Class 10 StudioCode Analysis, audio-recording).

The proximity and similarity between the qualities of advancing questions PT2 introduced in the Class 8 conversation and both Leslie's entries in the TTLP and the emergence of the qualities in Leslie's descriptions of advancing questions after the Class 9 First Rehearsal, indicates that the Class 8 Planning Session was a key course activity for Leslie. This key course activity was connected to Leslie accommodating her conception of advancing question to include two more particular facets that addressed two new ways advancing questions further students' mathematical understanding – *advancing questions support students in engaging in a mathematical process* and *advancing questions support students' thinking about new mathematical relationships and/or meanings*.

Class 9 First Rehearsal

The Class 9 First Rehearsal was a key course activity that was connected to one of the PSMTs (Leslie) accommodating her conceptions of assessing questions and advancing questions (see Table 6-7).

Table 6-7: The features connected to the Class 9 First Rehearsal

	Steve	Gretchen	Leslie	Nick
Assessing Questions	None	None	Assessing questions guide	None

			students' mathematical thinking	
Advancing Questions	None	None	Assessing questions guide students' mathematical thinking	None
Judicious Telling	None	None	None	None

Accommodations to PSMTs' Conceptions of Assessing Questions

The Class 9 First Rehearsal was connected to Leslie adding a facet associated with the feature *assessing questions guide students' mathematical thinking.*

Accommodations to PSMTs' Conceptions of Advancing Questions

The Class 9 First Rehearsal was connected to Leslie to adding a facet associated with the feature *advancing questions guide students' mathematical thinking.*

Description of Class 9 First Rehearsal

The Class 9 First Rehearsal was the second opportunity that the PSMTs had to engage in an approximation of practice, and the first approximation of practice that involved teaching mathematics in a setting of reduced complexity. The TTT framework (decomposition of practice) focused the PSMTs on enacting a specific aspect of instruction in the approximation of practice. The lesson planning in the prior class – Class 8 (approximation of practice) – was intended to provide the PSMTs an opportunity to anticipate students' approaches to the mathematical tasks in the Class 9 First Rehearsal. More importantly, Class 8 engaged the PSMTs' in planning the assessing questions, advancing questions, and judicious telling in response to the students' approaches in the First Rehearsal.

The First Rehearsal involved four pairs of PSMTs instructing (teaching pairs) the three doctoral students while Dr. A coached the PSMT teaching pair and the remaining PSMTs observed the lesson. The doctoral students were situated at a table in the center of a fishbowl arrangement of the classroom (see Figure 6-3) surrounded by the PSMTs. The teaching pair of PSMTs and Dr. A worked with the doctoral students in the center of the fishbowl arrangement.

Figure 6-3: Photo of a teaching pair instructing the three doctoral students

The Frist Rehearsal was originally planned to provide each of the four PSMTs 6 minutes to instruct the three doctoral students on a portion of the CMP problem set (see Table 6-8 for details on length of time and CMP problems). An alarm was used to indicate the end of the rehearsal for the teaching pair, at which time another teaching pair of PSMTs took over teaching the lesson. Since the doctoral students focused mainly on the first problem in the problem set during the rehearsal segments for the first two teaching pairs, Dr. A made an adjustment to the remaining rehearsal segments. Dr. A instructed the doctoral students to move on to different tasks in the problem set and extended the rehearsal time for the third and fourth teaching pairs. Dr. A instructed the third teaching pair and the fourth teaching pair to give the students a few minutes at the beginning of their segments to work on the tasks so that the teaching pair may be able to observe what the doctoral students wrote for the second problem and third problem, respectively.

Table 6-8: First Rehearsal Schedule

Teaching Pair	**Time**	**Mathematical Task**
PT5, Leslie	6 ½ minutes	Problem 1
PT8, PT9	6 ½ minutes	Problem 1
PT1, PT13	10 minutes	Problem 2
Gretchen, Nick	11 ½ minutes	Problem 3

Evidence First Rehearsal Connected to PSMTs' Accommodations

The First Rehearsal was a key course activity for only one of the PSMTs – Leslie. I focus on Leslie's conception of advancing question to illustrate the way in which the Class 9 First Rehearsal was connected to Leslie accommodating her conceptions of TTT. This key course activity was connected to Leslie adding a facet associated with the feature *advancing questions guide students' mathematical thinking.*

During the second interview, Leslie was asked to reflect on a series of questions that she and her group coded in the Class 10 StudioCode Analysis of the First Rehearsal as advancing questions. Leslie began her reflection explaining why she asked the series of questions,

> Leslie: Looking at the table as a whole and that's what I said, let's start over. Let's look at the whole table. Then, I realized that that's not going to work. You got to break it down to one person, find one pattern per person. Or, find how to find the pattern for the first one and then it will be easier to find the pattern for the other two. (Second Interview, audio-recording)

Leslie's statements "Let's start over" and "Then I realized that that's not going work" indicate that during the rehearsal she adjusted her instruction given her interpretation of what the student was saying and doing. Her statement "You got to break it down to one person find one pattern per person" indicates what she intended her student to focus on. In the following excerpt Leslie, is asked to talk more about her use of the series of questions and what she meant by the word, pattern,

> For example, focusing on Jose *[person in CMP problem set]*… You go from zero to five. The next time you go from five to ten. Well, what's the difference between five and ten. Five. What's the difference between zero and five? Five. Then, go on to the next one, what's the difference between fifteen [inaudible]. So what's it going up by each time? … The pattern, it's increasing by five every time so. Eventually, breaking it down to how fast they are going per hour. (Second Interview, audio-recording)

Leslie's response indicates that she intended for the student to focus on the pattern of differences in the distance traveled that corresponds with consecutive hours. As Leslie continued to reflect on the series of assessing questions, she addressed the questions themselves and the interaction with Student 3,

> During this part, I think it is just advancing. It is essentially getting them to think outside their current thinking pattern. And I used a series [of questions]. This is where I think I almost use a series of assessing questions. And putting those assessing questions together is considered advancing because I'm getting them to the next box through a series of assessing questions. But I definitely think my main goal was to find the pattern and actually finding a pattern is more advancing

then asking them maybe what they see at this moment. (Second Interview, audio-recording)

In this excerpt Leslie repeatedly described the series of assessing questions as advancing questions. Her statement “my main goal was to find the pattern and actually going and finding a pattern is more advancing” is an indicator that her purpose for asking the series of assessing questions was to extend the students’ mathematical thinking.

Leslie’s reflection indicates she purposefully adjusted her instruction so that she asked a series of questions that required students to calculate values in order to “break down” a pattern for the students. Leslie’s overarching purpose for this decision was to support students in identifying the pattern among the difference. Hence, Leslie’s reflection indicates that the Class 9 First Rehearsal was a key course activity. This key course activity was connected to Leslie accommodating her conception of advancing questions to include a facet associated with the feature *advancing questions guide students’ mathematical thinking.*

Class 10 StudioCode Analysis of the First Rehearsal

The Class 10 StudioCode Analysis of the First Rehearsal was a key course activity that was connected to Leslie accommodating her conception of judicious telling. Also, the key course activity was connected to Steve and Nick accommodating their conceptions of assessing questions (Table 6-9).

Table 6-9: The features connected to Class10 StudioCode Analysis

	Steve	Gretchen	Leslie	Nick
Assessing Questions	Assessing questions guide students’ mathematical thinking	None	None	Role of assessing questions in whole class discussions

Advancing Questions	None	None	None	None
Judicious Telling	None	None	Judicious telling shouldn't take away students' opportunities to do mathematics	None
			Judicious telling sets up mathematical tasks	

Accommodations to PSMTs' Conceptions of Assessing Questions

The Class 10 StudioCode Analysis was connected to both Steve and Nick accommodating their conceptions of assessing questions. Steve added a facet associated with the feature *assessing questions guide students' mathematical thinking* and Nick added a facet associated with the feature *role of assessing questions in whole class discussions.*

Accommodations to PSMTs' Conceptions of Judicious Telling

The Class 10 StudioCode Analysis was connected to Leslie adding facets associated with two features – *judicious telling shouldn't take away students opportunities to do mathematics* and *judicious telling sets up mathematical tasks.*

Description of Class 10 StudioCode Analysis of the First Rehearsal

The Class 10 StudioCode Analysis of the First Rehearsal was the PSMTs' third opportunity to analyze representations of practice, an audio-recording of First Rehearsal and the collection of the students' work. This course activity built on several of the previous course activities. First, the audio-recordings of the PSMTs' instruction in the Class 9 (approximation of practice) were the representations of practice that the PSMTs' analyzed in Class 10. Second, the PSMTs' again used the TTT framework (decomposition of practice), developed in prior classes, as a lens for analyzing the PSMTs' instruction. Since the PSMTs were analyzing instruction that they themselves enacted, they had an opportunity to share their intended purposes for the TTT and discuss with their peers the outcomes of the TTT on the doctoral students' mathematical work.

The Class 10 StudioCode Analysis involved the PSMTs analyzing the teaching pair of PSMTs enactment of their First Rehearsal using the TTT framework (decomposition of practice). The PSMTs were organized in small groups of four to five PSMTs. The group analyzed the audio-recordings using StudioCode and an analysis protocol for writing Timeline Memos that involved the following prompts,

- Identify interactions between the teacher and the student(s) during the segment.
- Discuss with the group if the interaction was assessing, advancing, telling, or other.
- Code the interaction so that it includes the teacher's utterance and the student's response.
- Write a memo that includes the rationale for coding the instance, the teacher's purpose, the quality of each statement on a scale of 1-5, and, if needed, a modification of the utterance. (Class 10 Analysis Assignment)

The small groups were able to work separately in different rooms. During the analysis, the PSMTs frequently agreed on how to code a teacher's utterance. However, the PSMTs spent a majority of the analysis section negotiating the rationale, purpose, and quality of the coded instance. The negotiations often involved PSMTs sharing their understanding of the purposes of TTT, use of evidence from the recordings, and their perspectives on the interaction as either the teacher or the observer.

Evidence Class 10 Analysis Connected to PSMTs' Accommodations

I use the case of Steve to illustrate the way in which the Class 10 Analysis was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Steve adding a facet associated with the feature *assessing questions guide students' mathematical thinking.*

In Class 10, Steve was grouped with PT1 and PT13, which were one of the four teaching pairs during the Class 9 First Rehearsal. During the Class 10 StudioCode Analysis, Steve observed that PT13's statement, "Show me how some of these points correspond with your graph" (Class 9 First Rehearsal, audio-recording) was "a good question" (Class 10 StudioCode Analysis of First Rehearsal, audio-recording). He agreed that the question was an assessing question, because Student 2 was asked to show his work for a problem he had completed (Class 10 StudioCode Analysis of First Rehearsal). As the group began to discuss the purpose, Steve asked for the audio-recording of PT13's question to be played again. After the recording is played, Steve stated, "Oh, you are asking him to connect the table to the graph. The points are the points on the table and you're asking him to explain how they correspond to the graph. Does that make sense? Is that what we think?" (Class 10 Small Group Analysis of First Rehearsal, audio-recording). Steve's description was first time he included this function of assessing questions in his descriptions of assessing questions and is the initial indicator of his facet associated with the feature *assessing questions guide students' mathematical thinking.* Steve's description was a result of him interpreting PT13's purpose for asking a question that group identified as an assessing question. Hence, the Class 10 StudiocCode Analysis was a key course activity for Steve. This key course activity was connected to Steve accommodating his conception of assessing questions to include a facet associated with the feature *assessing questions guide students' mathematical thinking.*

Categorization of the 9QTs

The Categorization of the 9QTs (Boaler & Brodie, 2004) was a key course activity that was connected to Steve, Gretchen, and Leslie accommodating their conceptions of judicious telling. In addition, the key course activity was connected to Gretchen and Steve accommodating their conceptions of advancing questions (see Table 6-10).

Table 6-10: The features connected to the Categorization of the 9 QTs

	Steve	Gretchen	Leslie	Nick
Assessing Questions	None	None	None	None
Advancing Questions	Advancing questions support students' thinking about new or different situations	Advancing questions support students thinking about new mathematical relationships and/or meanings	None	None
Judicious Telling	Judicious telling conveys procedure	Judicious telling conveys terminology	Judicious telling conveys terminology	None
		Judicious telling conveys contexts	Judicious telling conveys contexts	

Accommodations to PSMTs' Conceptions of Advancing Questions

The Categorization of the 9QTs was connected to two of the PSMTs (Steve and Gretchen) accommodating their conception of advancing questions. Both Steve and Gretchen changed facets associated with the feature *advancing questions extend students' mathematical thinking and understanding* in that they differentiated the facets into more particular facets that addressed the way in which students' mathematical thinking was extended by advancing questions. For Steve, the more particular facet was associated with the more particular feature *advancing questions support students' thinking about new or different situations* and, for Gretchen, the more particular facet was associated with the more particular feature *advancing questions support students' thinking about new mathematical relationships and/or meanings*.

Accommodations to PSMTs' Conceptions of Judicious Telling

The Categorization of the 9QTs was connected to three of the four PSMTs (Steve, Gretchen, and Leslie) accommodating their conceptions of judicious telling. First, the three PSMTs changed facets associated with the feature *judicious telling conveys information* in that they differentiated the facets into more particulars facets that addressed what information was conveyed. For both Gretchen and Leslie, the more particular facets were associated with two more particular features – *judicious telling conveys mathematical terminology* and *judicious telling conveys contexts* – and, for Steve, the more particular facet was associated with the more particular feature *judicious telling conveys a mathematical procedure.*

Description of Categorization of the 9QTs

The 9QTs was a second decomposition of practice and was a way for the PSMTs to collectively and individually further define assessing questions, advancing questions, and judicious telling. The 9QTs were introduced in two course readings (Boaler & Humphreys, 2005; Smith & Stein, 2011) that were a part of the homework assignment after Class 10. In addition to the readings, the homework assignment involved: (a) categorizing the 9QTs as assessing questions, advancing questions, and/or judicious telling; (b) coding the case of Edith Hart (Smith et al., 2005) using the 9QT framework; and (c) writing a second set of instructional purposes for the TTT (Homework assignment after Class 10).

The categorization activity continued in Class 11 as the PSMTs, organized in small groups of three to four PSMTs, discussed the following: (a) their categorization of the 9QTs as assessing, advancing, and telling; and (b) their individual analysis of the Case of Edith Hart using the 9QTs. The discussions involved the PSMTs using the sample questions associated with the 9QTs, the descriptions of the 9QTs, and the PSMTs' definitions for TTT as rationales for categorizing the 9QTs as TTT. Overall, the PSMTs in the small groups did not immediately or ultimately agree how to categorize all the 9QTs. Nearly all the PSMTs observed that the sample questions included in the articles were not helpful and they preferred to use the descriptions (see Figure 3-1). In addition, the collective analysis of Edith Hart in the small groups promoted some of the groups to further discuss qualities of the 9QTs and the TTT. The class concluded with a brief whole class discussion in which the PSMTs shared with Dr. A that the descriptions of the 9QTs in the course readings (Boaler & Humpreys, 2005; Smith & Stein, 2011) significantly impacted how they categorized the 9QTs as TTT.

A note that is important for the next subsection. Dr. A instructed the PSMTs during the small group discussions to make additions to their notebook categorizations of the 9QTs using a pen with a different colored ink than the one with which they wrote their original categorizations of the 9QTs. Her purpose was for the PSMTs to be able to confront any conflicting ideas around the 9QTs and TTT.

Evidence Categorization of 9QTs Connected to PSMTs' Accommodations

I use the case of Leslie to illustrate the way in which the Categorization of the 9QTs was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Leslie changing her facet associated with the feature *judicious telling conveys information* in that Leslie differentiated the facet into two, more particular facets associated with the features *judicious telling conveys terminology* and *judicious telling conveys contexts.*

In Class 11, Leslie was grouped with PT17, PT8, and PT9. During the small group discussion in Class 11, Leslie's peers categorized the inserting terminology question types and establishing context questions types as judicious telling. The following excerpt is from the conversation in which the group first discussed inserting terminology,

PT17: Inserting terminology.

Leslie: I didn't know what that one was.

PT9: I have assessing or telling depending on -

PT17: I have telling.

PT9: It depends on the way you do it.

PT8: I had advancing.

Leslie: I think it'd be more telling. (Class 11 small group discussion)

Leslie's response to PT17, "I didn't know what that one was", indicates she had not categorized the question type. Leslie's response, "I think it'd be more telling", is an indicator that she agreed with PT17 that inserting terminology was judicious telling. The following excerpt is from later in the conversation as the group categorized establishing context,

PT17: Establishing context. I think that's telling and assessing.

PT9: That's exactly what I had.

PT8: I had assessing and advancing.

PT17: Boo!

Leslie: I don't have anything.

PT8: But I can see why it's telling.

Leslie: I put I'm not sure.

PT9: If you are establishing context, there has to be some telling there.

PT8: Yeah.

PT9: Because you have to tell them what the context is.

PT8: Yeah.

Leslie: All right, all right. I feel you. (Class 11 small group discussion)

Leslie's statement, "I don't have anything", is an indicator that Leslie had not yet categorized the establishing context question type. Leslie's response to PT9's explanation for categorizing the question type as telling is an indicator that Leslie agreed with PT9 that establishing context might be judicious telling.

In addition, prior to Class 11, Leslie wrote entries in her notebook – using blue ink – for the 9QTs that included descriptions of the question types that were identical to the descriptions of the questions types in Smith and Stein (2011). Some of the question types did not have categorizations in blue ink, including inserting terminology and establishing context. Instead, both inserting terminology and establishing context, had “telling” written next them in black ink (Leslie’s Notebook). This indicates that Leslie categorized these question types during or following Class 11 using a different pen with a different color of ink.

Leslie not writing categorizations for the inserting terminology and establishing context question types in her notebook prior to Class 11 in conjunction with her stating “I don’t know” for both question types during the small group Class 11 conservation, indicate that Leslie did not know how to categorize these question types prior to Class 11. Leslie agreeing with her peers that both the question types inserting terminology and establishing context are judicious telling in conjunction with her categorizing these questions types as telling in her notebook during or after Class 11, indicates that the Categorization of the 9QTs was a key course activity for Leslie. This key course activity was connected to Leslie accommodating her conception of judicious telling to include two, more particular facets associated with more particular features *judicious telling conveys terminology* and *judicious telling conveys contexts.*

The Class 14 Second Rehearsal

The Class 14 Second Rehearsal was a key course activity that was connected to Steve, Gretchen, and Nick accommodating at least one of their conceptions of the TTT. The key course activity was connected to: (a) Gretchen and Steve accommodating their conceptions of assessing questions and advancing questions; (b) Gretchen accommodating her conception of judicious telling; and (c) Nick accommodating his conception of advancing questions (see Table 6-11).

Table 6-11: The features connected to the Class 14 Second Rehearsal

	Steve	Gretchen	Leslie	Nick
Assessing Questions	Assessing questions guide	Assessing questions inform	None	None

	students' mathematical thinking	teachers' subsequent instruction		
		Assessing questions relation to advancing questions		
Advancing Questions	Other	Advancing questions build on students' mathematical understandings	None	Advancing questions support students in engaging in mathematical processes
Judicious Telling	None	Judicious telling conveys procedure	None	None

Accommodations to PSMTs' Conceptions of Assessing Questions

The Second Rehearsal was connected to two of the four PSMTs (Steve and Gretchen) accommodating their conceptions of assessing questions. First, Steve added a facet associated with the feature *assessing questions guide students' mathematical thinking*. Second, Gretchen added facets associated with the features *assessing questions inform teachers' subsequent instruction* and *assessing questions relationship with advancing questions*.

Accommodations to PSMTs' Conceptions of Advancing Questions

The Second Rehearsal was connected to three of the four PSMTs (Steve, Gretchen, and Nick) accommodating their conceptions of advancing questions. First, Gretchen added a facet associated with the feature *advancing question build on students' mathematical thinking*. Second, Steve and Nick changed facets associated the feature *advancing questions extend students' mathematical thinking and understanding* in that they differentiated the facets into more particular facets addressing the way in which students' mathematical thinking was extended by advancing questions. For Nick, the more particular facet was associated with the more particular feature *advancing questions support students engaging in mathematical processes* and, for Steve, the more particular facet was associated with the other category.

Accommodations to PSMTs' Conceptions of Judicious Telling

The Second Rehearsal was connected to Gretchen accommodating her conception of judicious telling. Gretchen changed her facet associated with the feature *judicious telling conveys information* in that she differentiated the facet into a more particular facet associated with the more particular feature *judicious telling conveys a procedure to a mathematical task.*

Description of the Class 14 Second Rehearsal

The Second Rehearsal was the PSMTs' fourth opportunity to engage in an approximation of practice, which was the second approximation of practice in which the PSMTs were able to teach mathematics in a setting of reduced complexity. Similar to the First Rehearsal in Class 9, the second rehearsal built on a number of earlier course activities. First, the TTT framework (decomposition of practice) focused the PSMTs on enacting a specific aspect of instruction in the approximation of practice. Second, the lesson planning in the prior class – Class 13 (approximation of practice) – provided the PSMTs an opportunity to anticipate students' approaches to the mathematical tasks and plan TTT that they would use in response to these approaches during the Second Rehearsal. Last, the Second Rehearsal gave the PSMTs' an opportunity to enact the TTT modeled in the representations of practice (e.g., Case of Nancy Edwards, Case of Edith Hart).

At the beginning of Class 14, Dr. A emphasized to the PSMTs that the goal of their instruction in the Second Rehearsal was to use assessing questions, advancing questions, and judicious telling to support a doctoral student in making progress on his or her argument for one of the six tasks in Figure 6-4. Also, Dr. A emphasized at the start of the class session that it was not necessary or realistic for the PSMT to expect to support the students in completing the argument in the short amount of time allotted for the rehearsal.

Write a convincing argument for the following:

1. The product of any two perfect squares is a perfect square.
2. The sum of any two positive consecutive odd numbers is divisible by 4.
3. The product of two positive even numbers is even.
4. The sum of any three positive consecutive odd numbers is divisible by 3.
5. The product of any three positive consecutive numbers is always a multiple of 6.
6. For every counting number N (1, 2, 3, ….), N^2 + N is always even.

Figure 6-4: List of Second Rehearsal mathematical tasks

Each of the PSMTs had 3 minutes to individually work with one of the doctoral students about their argument (see Table 6-12 for Second Rehearsal schedule). Also, Dr. A emphasized at the start of the class session that it was not necessary or realistic for the PSMT to expect to support the students in completing the argument in three minutes. The PSMTs not teaching were organized in a fishbowl arrangement that allowed them to observe and take notes on the teaching episode. The student's argument was projected on the white board, which allowed the PSMT and the doctoral student to augment and alter his/her argument (see Figure 6-5). Dr. A, as in rehearsal one, was on the perimeter of the interaction. She entered the rehearsal and coached the PSMTs if the opportunity presented itself, such as the PSMTs not using precise mathematical language, the PSMTs requesting assistance, or if the PSMTs spent a significant amount of time explaining and talking at the student.

Table 6-12: Second Rehearsal schedule

	Student 1	Student 2	Student 3
Problem 1	PT 3	PT 14	PT 12
Problem 2	PT 15	PT 16	PT 6
Problem 3	PT 4	PT 2	PT 17
Problem 4	PT 10	PT 1	PT 13
Problem 5	PT 5	Unidentified PT	PT 11
Problem 6	PT9	PT 7	PT 8

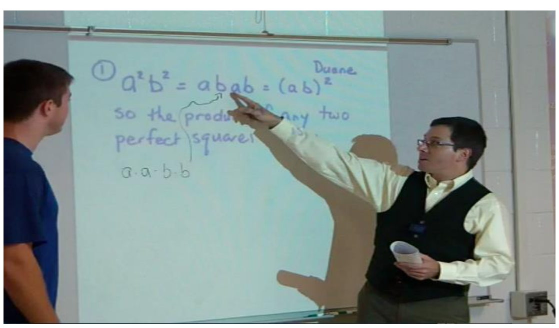


Figure 6-5: PSMT interacting with Student 2 during the Second Rehearsal

After the rehearsals, Dr. A shared with the group that many of their interactions with the students did not focus on supporting the students in making progress with their existing argument. Rather, the PSMTs directed students in crafting an algebraic argument. She assured students that their teaching was a good representation of beginning teachers' discourse with students. Dr. A encouraged the students to think about ways that they may improve their teaching rather than what they did not do perfectly in their teaching during the Second Rehearsal. This included ways they can use TTT to determine what students' are mathematically thinking about an argument and how they as teachers may continue to balance supporting the student in their argument and supporting the student in constructing a valid argument.

Evidence Second Rehearsal Connected to PSMTs' Accommodations

I use the case of Gretchen to illustrate the way in which the Class 14 Second Rehearsal was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Gretchen changing her facet associated with the feature *judicious telling conveys information* in that she differentiated the facet into a more particular facet associated with the more particular feature *judicious telling conveys a procedure for a task.*

Prior to the Class 14 Second Rehearsal, Gretchen's conception of judicious telling included facets associated with more particular features of *judicious telling conveys information.* However, none of her descriptions, which these facets were inferred, included conveying: methods, procedures, or ways of thinking for addressing mathematical tasks. This changed after the Class 14 Second Rehearsal. During the Class 15 StudioCode Analysis of the Second Rehearsal, Gretchen admitted that she really wanted Student 3 to construct an argument during the Second Rehearsal in a particular way. The following is an excerpt from the conversation:

> Gretchen: Yeah, now that I look back on mine I probably should have asked him way more questions about why he only did three examples instead of assuming he was thinking of those as all of the cases.

> *PT13 and PT1 agree.*
>
> Gretchen: Because I was like just thinking, "Oh, he wrote the ones place."
>
> PT1: You just really wanted to do that problem.
>
> Gretchen: I know! It was what I was getting at. (Class 15 StudioCode Analysis of the Second Rehearsal, audio-recording)

Gretchen's initial statement and second statement indicate that she was assuming a particular way the student was thinking. Gretchen eagerly agreed with PT1 that she "really wanted to do that problem", which indicates that Gretchen wanted to pursue a particular argument. In addition, during the third interview, Gretchen stated,

> I think in that [second] rehearsal I had like read the student work and gotten in my head how I thought they were thinking about it. And so when I jumped in I was like already to go from where they were. (Third Interview, audio-recording)

Gretchen's statement "gotten in my head how I thought they were thinking about it" indicates that Gretchen had inferred the students' argument as one similar to her own. Also, Gretchen's statement "I was like already to go from where they were" indicates that Gretchen wanted to support the students in constructing the particular argument. Hence, it seems reasonable that one way Gretchen would support her student in the Second Rehearsal task is to provide him with a method he might use to construct the argument. Gretchen's remarks in Class 15 combined with her remarks during the third interview, indicates that the Class 14 Second Rehearsal was a key course activity for Gretchen. This key course activity was connected to Gretchen accommodating her conception of judicious telling to include a more particular facet associated with the more particular feature *judicious telling conveys a procedure to a mathematical task.*

Class 25 Approximation of Practice

The Class 25 Approximation of Practice was a key course activity that was connected to both Leslie and Steve accommodating their conceptions of judicious telling. In addition, the key course activity was connected to Leslie accommodating her conception of advancing questions (see Table 6-13).

Table 6-13: The features connected to the Class 25 Approximation of Practice

	Steve	Gretchen	Leslie	Nick
Assessing Questions	None	None	None	None
Advancing Questions	None	None	Advancing questions support students' thinking about a general case/idea	
Judicious Telling	Judicious telling supports students when they engage in mathematical tasks	None	Judicious telling repeats students' contributions	None

Accommodations to PSMTs' Conceptions of Advancing Questions

The Class 25 Approximation of Practice was connected to Leslie accommodating her conception of advancing questions. Leslie changed her facet associated with the feature *advancing questions extend students' mathematical thinking and understanding* in that she differentiated the facet into a more particular facet associated with the more particular feature *advancing questions support students' thinking of a general case/idea.*

Accommodations to PSMTs' Conceptions of Judicious Telling

The Class 25 Approximation of Practice was connected to Steve and Leslie accommodating their conceptions of judicious telling. First, Steve added a facet associated with the feature *judicious telling supports students as they engage in a mathematical task*. Second, Leslie added a facet associated with the feature *judicious telling repeats students' contributions.*

Description of the Class 25 Approximation of Practice

The Class 25 Approximation of Practice was the sixth approximation of practice, but the first approximation of practice that provided the PSMTs an opportunity to work with actual high school students in a setting of reduced complexity. This approximation of practice was not considered a rehearsal, because the approximation of practice did not involve a course instructor "coaching" the PSMTs in their teaching. Similar to the First Rehearsal and Second Rehearsals, the Class 25 Approximation of Practice built on a number of earlier course activities (see Description of the Class 9 First Rehearsal and Description Class 14 Second Rehearsal).

The Class 25 Approximation of Practice took place at Field High School (a pseudonym). The PSMTs taught one to two grade 9-12 students in a thirty-minute mathematics lesson. The PSMTs implemented their lesson plans for three problems that involved students generalizing a rule for an "nth" case given a set of figures - The Beam Task, The Poster Task, and The Staircase Problem (see Figure E-1 to Figure E-3 in Appendix E). Many of the PSMTs were able to work with their student on both the Beam Task and the Poster task, but were not able to work with students on the Staircase Problem. The PSMTs audio-recorded their rehearsals and, with permission from the student, retained their students' work on the three mathematical tasks for subsequent analysis.

Evidence Class 25 Connected to PSMTs' Accommodations

I use the case of Leslie to illustrate the way in which the Class 25 Approximation of Practice was a key course activity connected to the PSMTs accommodating their conceptions of TTT. This key course activity was connected to Leslie changing her facet associated with the feature *advancing questions extend students' mathematical thinking and understanding* in that Leslie differentiated the facet into a more particular facet associated with the more particular feature *advancing questions support students' thinking about a general case/idea.*

Prior to planning for the Class 25 Approximation of Practice, Leslie had two opportunities during the semester that provided opportunities to plan and enact sets of tasks that involved generalizing and/or a general case. On neither occasion did Leslie write in her lesson plan documents, discuss in the planning sessions, or write in her analysis of the rehearsals about a general case or general equation. During the Class 24 Planning Session for the Class 25 Approximation of Practice, Leslie was silent nearly the entire time (Class 24 Planning Session for the Class 25 Approximation of Practice, audio-recording). However, Leslie wrote the following in response to the Class 24 modified TTLP's prompt about what CCSSM mathematical practices in which the students may engage:

> The entire task at hand is a matter of making sense of problems and being able to work to solve them. They all provide a problem in which one needs to take the given information and come up with a general formula to solve that problem more easily as well as problems involving large numbers. (Leslie's Homework Assignment after Class 24).

Leslie's statement "take the given information and come up with a general formula" indicates that she identified generating a general formula as a mathematical goal of the lesson. Leslie's use of "general formula" is similar to her use of "general formula" in her description of an advancing question after the Class 25 Approximation of practice,

> I wanted to move him towards a more general formula that can be applied to multiple situations in which one could change the length to anything in order to find the number of rods used or vice versa. The more general equation can be used for any values of n instead of working with a formula based off of a picture that is given to you or simply adding on rods and then going through and counting. (Homework Assignment after Class 25, StudioCode Timeline)

In my case analysis of Leslie, this description is the initial indicator of the facet associated with the feature *advancing questions support students' thinking about a general case/idea.* The similarity and proximity of this data indicates that the Class 24 Planning Session in conjunction with the Class 25 Approximation of Practice provided Leslie an opportunity to recognize another way in which students' mathematical thinking maybe advanced – generating a general case. Hence, the Class 24 Planning Session and the Class 25 Approximation of Practice were key course activities for Leslie. This key course activity was connected to Leslie accommodating her conception of advancing questions to include a more particular facet associated with the more particular feature *advancing questions support students' thinking about a general case/idea.*

Chapter Conclusion

In summary, I reported my findings around the nature of the eleven key course activities in the methods course, the accommodations to the PSMTs' conceptions of TTT connected to key course activities, and evidence from the case analyses illustrating the connection of the key course activities to the PSMTs' accommodations. Based on these findings, I make two assertions.

First, the findings in which I described the nature of the key course activities indicate the key course activities designed around the decompositions of, representations of, and approximations of practice coordinated and complemented each other across the semester. For example, the early course decomposition of practice, TTT framework, served as a structure that focused the PSMTs' analyses and enactment of mathematics instruction on a specific set of component core practices of facilitating classroom mathematics discourse – assessing questions, advancing questions. More specifically, the early course activities (prior to the first rehearsal) in which the PSMTs moved between analyzing representations of practice and collectively discussing qualities and functions of the TTT, repeatedly focused the PSMTs on further defining the TTT and the ways in which TTT were related to students' mathematical thinking and learning. Another example, the PSMTs engaging in approximations of practice (e.g., First and Second Rehearsals) were designed in proximity to the course activities involving the analyses of representations of practice so that the PSMTs' had an opportunity to enact the TTT modeled in their analyses of representations of practice (e.g., Case of Nancy Edwards, Case of Edith Hart). A final example, the analyses of representations of the PSMTs' approximations of practice (e.g., Class 10 StudioCode Analysis) engaged the PSMTs in opportunities to interrelate the conceptual aspects of the TTT framework (e.g., relation of the TTT to students' mathematical thinking and learning, principles undergirding mathematics instruction) with the practical aspects of enacting assessing questions, advancing questions, and judicious telling (e.g., phrasing of questions, using wait time, actually speaking with a person about his/her work).

Second, while the PSMTs' conceptions included similar facets, different key course activities were connected to their accommodations. For example, consider the three facets of the PSMTs' conceptions of TTT in Table 6-14. Each row in the table presents the different key course activity connected to the PSMTs' facets associated with the features in the far left column. For example, row one indicates that three different key course activities connected to the PSMTs' facets associated with the feature *assessing questions determine students' approaches or rationales as they work on a mathematical task.*

Table 6-14. Key course activities connected to PSMTs' conceptions of TTT

	Steve	Leslie	Gretchen	Nick
Assessing questions determine students' approaches or rationales for their work as they engage in a task	Case of Edith Hart Analysis	Class 8 Planning Session	Class 5 & Class 6 Studio Code Analysis	Class 5 & Class 6 Studio Code Analysis
Advancing questions support students' thinking about new mathematics relationships and meanings	Case of Edith Hart Analysis	Categorization of 9 QTs	Class 8 Planning Session	Case of Edith Hart Analysis, Class 8 Planning Session
Judicious telling conveys mathematical terminology	Categorization of 9 QTs	Class 6 Defining Features Discussion	Categorization of 9 QTs	Class 6 Defining Features Discussion

It is important to note, while there were key course activities throughout the semester, most of the PSMTs' accommodations were connected to key course activities through the Categorization of the 9QTs, which occurred mid-semester (see Appendix A for full listing of course activities and when they occurred during the semester). I will more thoroughly address this observation in the later portion of the discussion in Chapter 7.

Chapter 7

Conclusions, Implications, & Directions for Future Research

In Chapter 1 of this dissertation, I argued that this study was needed in order for the mathematics teacher education community to better understand *what* PSMTs enrolled in mathematics methods courses learn about component core practices of facilitating classroom mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling) and *how* teacher education course activities designed around decompositions of practice, representations of practice, and approximations of practices connected to the PSMTs' learning. In Chapters 4 – 6, my findings from my cross-case analysis address both what the PSMTs learned about the component core practices and how teacher education course activities supported the PSMTs' learning of the component core practices.

In this chapter, I report the ways in which my findings contribute to the literature body that established my rationale for the dissertation study in Chapter 1 and the literature body reviewed in Chapter 2. I organize this chapter around the six assertions I made at the end of Chapters 4-6. For each assertion, I begin with a brief discussion of the assertion and its relation to the findings. Then, I explain the way in which findings associated with the assertion contribute to the existing research on PSMTs' learning of mathematics pedagogy and the importance of these findings for the field of mathematics education. I conclude this chapter with the implications of this study and directions for future research.

Assertion 1: Conceptions of TTT Oriented on Students' Mathematical thinking

The PSMTs in this study constructed multi-faceted conceptions of assessing questions, advancing questions, and judicious telling that were oriented towards focusing on and promoting students' mathematical thinking. In order to remind the reader of the PSMTs' facets associated with features of TTT, Tables 7-1 to 7-3 present the features of the TTT in the right column and the initial description of the TTT as it appeared in course materials in the left column.

Table 7-1: Initial description and features of the conceptions of assessing questions

Assessing questions:
• "Can assess what students understand about the problem (e.g., clarify what the student has done and what the student understands)" (Smith et al., 2008, p. 136)
	Assessing questions:
	• Determine students' mathematical thinking understanding, and/or knowledge
	  ○ Determine students' approaches or rationales for work as they engage in a task
	  ○ Determine students' rationales for answers to a mathematical task
	  ○ Gauge students' mathematical understanding and/or capabilities
	  ○ Other facets associated with the first feature
	• Inform teachers' subsequent instruction
	• Support students' reflection
	• Role in whole class discussions
	• Guide students' mathematical thinking
	• Relation to advancing questions
	• Other facets

Table 7-2: Initial description and features of the conceptions of advancing questions

Advancing questions:
• "Help students advance towards the mathematical goals of the lesson. Teachers can extend student beyond their current thinking by pressing them to extend what they
	Advancing questions:
	• Extend students' understanding and/or thinking
	  ○ Support students' thinking about new or different situations
	  ○ Support students' thinking about a general case/idea
	  ○ Support students' thinking about new mathematical relationships and/or meanings
	  ○ Support students in engaging in

know to a new situation or think about something they are not currently thinking about" (Smith et al., 2008, p. 136)	mathematical processes • Aim students' towards the mathematical goal of a lesson • When teachers may ask these questions • Build on students' mathematical understandings • Guide students' mathematical thinking • Other facets

Table 7-3: Initial description and features of the conceptions

Judicious telling:
• "Telling statements are more declarative in nature, they are not questions. They are just statements. It is a conveyance of information" (Dr. A, Class 5, audio-recording)
	Judicious telling:
	• Conveys information
	  ○ Conveys terminology
	  ○ Conveys directions
	  ○ Conveys contexts
	  ○ Conveys answers
	  ○ Conveys procedures
	• Is a statement and not a question
	• Repeats students' contributions
	• Shouldn't take away students' opportunities to do mathematics
	• Is used when students have difficulties

I found three things to be interesting around the findings associated with this assertion. First, it is interesting that the PSMTs' conceptions of both assessing questions and advancing questions included more particular facets of the key functions for the questions than were described in the course reading Smith et al. (2008) (determining students' mathematical thinking and extending students' thinking towards the mathematical goal of the lesson, respectively). For example, the PSMTs constructed facets of assessing questions that addressed at least four different ways the questions determined students' mathematical thinking and understanding – *assessing questions determine students' approaches or rationales for their work as they engage in a task, assessing questions determine students' rationales for answers, assessing questions gauge students' mathematical understandings and/or capabilities, and determining* *students' difficulties.* Also, the PSMTs constructed facets of advancing questions that addressed at least five different ways in which students' mathematical thinking may be extended by the questions – *advancing questions support students' thinking about new or different situations, advancing questions support students' thinking about a general case/idea, advancing questions support students' thinking about new mathematical relationships and/or meanings, and advancing questions support student in engaging in mathematical processes*.

Second, it is interesting that the PSMTs' conceptions of judicious telling included facets that were oriented on promoting students' mathematical thinking, because the instructor originally explained judicious telling as conveying information and a number of the PSMTs initially thought judicious telling involved explaining an answer or a procedure to a task. However, throughout the semester, the PSMTs constructed conceptions that included ways in which judicious telling promoted students' mathematical thinking as students engaged in a mathematical task (e.g., repeating students' contributions to highlight key mathematical information, inserting terminology for a student generated idea to enable communication of mathematical ideas in the classroom).

Last, it was interesting that the PSMTs constructed conceptions of assessing questions and advancing questions that included facets that addressed relationships between the two types of questions. For example, the PSMTs' conceptions of the questions included facets that addressed assessing questions enabling teachers to determine students' mathematical thinking so that they could extend students' mathematical thinking with advancing questions.

My findings associated with the first assertion complements research examining PSMTs' learning to analyze mathematics teaching (Stockero, 2008; Van Es & Sherin, 2002) and research examining changes in PSMTs' mathematical knowledge for teaching (Jenkins, 2010; Stump, 2001) in that the PSMTs in this study learned instructional strategies that supported students' mathematical thinking. In addition, my findings extend these two bodies of literature in that the psychological perspective enabled me to examine and report the PSMTs' nuanced conceptions instructional strategies (e.g., assessing questions, advancing questions, judicious telling) that promote students' mathematical thinking. This is important for the field because it provides mathematics teacher educators with an understanding of the detailed and different ways PSMTs understand the relationship between mathematics instruction and students' mathematical thinking.

Assertion 2: Differences among the Facets Associated with Similar Features

While the PSMTs' conceptions included facets associated with similar features of TTT as a group, they individually constructed nuanced facets that were different from each other. These differences among the PSMTs' similar facets are characteristics of the PSMTs' viable, unique conceptions of assessing questions, advancing questions, and judicious telling. The most important of these differences was the way the PSMTs' facets of the TTT addressed the relation of the TTT in supporting students' mathematical thinking and learning.

As reported in the cross-case analysis of the PSMTs' conceptions in Chapter 4, Nick's and Gretchen's facets of TTT consistently addressed ways in which the TTT supported students' opportunities to engage in productive struggle in learning mathematics (productive struggle in learning mathematics as described in NCTM, 2014). Where as Steve's and Leslie's conceptions of TTT included multiple facets (e.g., *assessing questions guide students' mathematical thinking, judicious telling supports students engaging in a mathematical task*) that addressed the relation between TTT to students' work mathematical thinking and learning as one that may constrain students' opportunities to engage in productive struggle in learning mathematics. This difference is not meant to position Nick's and Gretchen's conceptions of the TTT as superior to Steve's and Leslie's conceptions of TTT. In fact, Steve's and Leslie's conception of TTT included some facets that did address ways TTT promoted students' mathematical thinking that would support students to engage in productive struggle in mathematics (e.g., more particular facets associated with both *assessing questions determine students' mathematical thinking, understanding, and/or knowledge* and *advancing questions extend students' mathematical thinking and/or understanding*). Instead, this difference indicates that PSMTs' conceptions of TTT may include facets that both do and not *yet* address the way in which TTT support students in productive struggle in learning mathematics.

The findings associated with the second assertion support and extend the work of researchers examining PSMTs' learning to facilitate classroom mathematics discourse. My findings complement Boerst et al. (2011), Rahal and Melvin (1998), and Boerst et al. (2011) in that the PSMTs learned different ways questioning may support classroom mathematics discourse. More specifically, my findings support Boerst et al.'s (2011) and Tyminski et al.'s (2014) findings that PSMTs enrolled in methods courses learn different types of questions for facilitating classroom mathematics discourse. My findings build on those of Boerst et al. (2011) and Tyminski et al. (2014) in two ways. First, I examined

PSMTs' learning rather than PEMTs' learning. This is important for the field because it affords secondary mathematics teacher educators an understanding of what PTs in their methods courses might be learning about practices that constitute facilitating classroom mathematics discourse. Second, whereas Tyminski et al. found "many prospective elementary teachers were successful in writing clarifying and leading questions, there was a comparative dearth of questions providing students opportunity to make mathematical connections" (p.483), the PSMTs in this study constructed multi-faceted conceptions of advancing questions that provide students opportunities to make mathematical connections. This is important for the field because it provides mathematics teacher educators with an initial understanding of what mathematics PTs may learn about questions that extend students' mathematical thinking and understanding.

Assertion 3: Defining Features of TTT

The PSMTs' similar facets contribute defining features for component core practices of facilitating classroom mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling). The PSMTs in this study, although novices, are members of the secondary mathematics teaching community. I argue that the similar facets these PSMTs generated provide the community with more technical definitions of assessing questions, advancing questions, and judicious telling that may be used in secondary mathematics teaching (see Table 7-4).

Table 7-4: Definitions of TTT

Assessing questions enable teachers to elicit students' mathematical thinking and understanding during a lesson, which serves several functions for the teacher, such as: (a) determine students' mathematical thinking, understanding; (b) determine students' rationales for answers and approaches to mathematical tasks as well as students' progress on mathematical tasks; (b) gauge students' understandings of mathematical ideas and capabilities to engage in mathematical processes; and (c) enable teachers to purposefully position students in classroom mathematical discussions by supporting students' entry into

the discussion and supporting student-to-student interaction focused on students' contributions. In addition, the understanding of students' thinking, understanding, progress on tasks, and capabilities informs teachers' subsequent instruction. Besides serving a function for teachers, assessing questions serve a function for students – assessing questions support students' reflecting on their approaches to mathematical tasks and reflecting on their thinking.
Advancing questions are questions that build on students' current mathematical thinking and/or understanding and aim students' towards the mathematical goal of a lesson by extending students' thinking in the following ways: (a) supporting students' thinking about new or different situations; (b) supporting students in engaging in mathematical processes (e.g., representing, generalizing, justifying); and (c) support students thinking about new mathematical relationships and/or meanings. In addition, advancing questions may be used at various times during a lesson such as when students' have difficulties and to support students as they engage in mathematical tasks. Last, advancing questions orient and focus students on key mathematical ideas that aim students towards the mathematical goals of the lesson.
Judicious telling sets up mathematical tasks and supports students as they engage in a mathematical task by conveying information that does not take away students' opportunities to do mathematics, which includes: (a) terminology that will enable mathematical communication; (b) clarification of directions or contexts in a mathematical task; and (c) procedures that students would not have been able to "discover" on their own. In addition, repeating key mathematical ideas in students' contributions is way judicious telling can be used to position students as an authority of the mathematics in the classroom.

The findings associated with the third assertion support and extend the secondary mathematics education literature focusing on facilitating classroom mathematics discourse. The PSMTs' defining features of the assessing questions and advancing questions complement Smith et al.'s (2008) brief descriptions of these two types of questions. That is, assessing questions determine students' mathematical thinking and advancing questions extend students' mathematical thinking towards the mathematical goal of a lesson. In addition, the PSMTs' defining features of judicious telling, generated by the PSMTs in this study, complements the work of Lobato et al. (2005) in that "there are multiple sophisticated teacher actions beneath the umbrella term 'telling'" (p.132) and judicious telling supports students construction of mathematical understandings. The PSMTs' constructed and nuanced understandings of assessing questions, advancing questions, and judicious telling enhance the definitions of Smith et al. (2008) and Lobato et al. (2005) in that the defining features provide the field of secondary mathematics teaching with a more robust way to conceive of purposeful questioning and judicious telling involved in facilitating classroom mathematics discourse, thus making an important contribution to the literature. Also, as advocated by reformers in teacher education (e.g., Grossman & McDonald, 2008), these enhanced definitions of this particular decomposition of practice enables the field to better communicate about, as well as design research on, specific components of teaching.

Assertion 4: Conceptions of TTT Changed in Complexity

Throughout the semester, the PSMTs' conceptions of assessing questions, advancing questions, and judicious telling changed in complexity[10]. In order to summarize the change in complexity of the PSMTs' conceptions, Tables 7-5 to 7-7 present the PSMTs' initial conceptions of TTT and their final conceptions of TTT. The *X* symbol in each cell indicates the presence of the facet in the PSMTs' initial conceptions and the final conceptions.

Table 7-5. PSMTs' initial and final conceptions of assessing questions

	Initial				Final			
Assessing questions:	Steve	Leslie	Gretchen	Nick	Steve	Leslie	Gretchen	Nick
Determine students' mathematical thinking, understanding, and/or knowing	X	X	X	X	X	X	X	X
Inform teachers' subsequent instruction					X	X	X	X

[10] *Complexity* refers to the multi-faceted nature of a PSMTs' conception AND the number of facets oriented towards focusing on and promoting students' mathematical thinking (see Chapter 5).

Support students' reflection					X	X	X	X
Role in whole class discussions					X		X	X
Guide students' mathematical thinking					X	X	X	
Relation to advancing questions						X	X	X
Other					X			X

Table 7-6. PSMTs' initial and final conceptions of advancing questions

	Initial				Final			
Advancing questions:	Steve	Leslie	Gretchen	Nick	Steve	Leslie	Gretchen	Nick
Extend students' mathematical thinking and/or understanding	X	X	X	X	X	X	X	X
Aim students' towards the mathematical goal of the lesson		X	X		X	X	X	X
When teachers may ask these questions					X		X	X
Build on students' mathematical understandings						X	X	X
Guide students' mathematical thinking					X	X	X	X

Table 7-7. PSMTs' initial and final conceptions of judicious telling

	Initial				Final			
Judicious telling:	Steve	Leslie	Gretchen	Nick	Steve	Leslie	Gretchen	Nick
Conveys information	X	X		X	X	X	X	X
Are statements and not	X	X	X		X	X	X	X

questions								
Repeats students' contributions						X	X	X
Shouldn't take away students' opportunities to do mathematics				X		X	X	X
Used when students have difficulties					X	X	X	
Set-up mathematical tasks					X	X	X	X
Supports students when they engage in mathematical tasks		X			X	X	X	X

Although it is important to recognize the change in complexity of the PSMTs' conceptions, one may expect the PSMTs' conceptions of TTT to change in complexity over the course of a semester-long methods course. The more important finding from Chapter 5 was that the changes in complexity of the PSMTs' conceptions of TTT (with the exception of Steve's conception of judicious telling) occurred over a sequence of three to five accommodations (see the diagrams of the PSMTs' pathways in Figures 5-1 to 5-12). Further, while each of the PSMTs' sequence of accommodations was unique, there were similarities among the sequential order in which the PSMTs added facets. Interestingly, the four PSMTs' pathways for constructing conceptions of the two types of questions and all but one of the PSMTs' pathways for constructing conceptions of judicious telling began with facets associated with the same feature.

While there were similarities among the sequential order in which the PSMTs added facets, the PSMTs differed temporally as to when they added the facets. For example, the four PSMTs' initial conceptions of advancing questions included facets associated with *advancing questions extend students' mathematical thinking*. These facets occur first in the sequence of accommodations for all four PSMTs' conceptions of advancing questions. However, for Leslie, the description that was the initial indicator of this facet came from the Class 5 Working Definition Discussion, where as the other three PSMTs' initial indicators of the facet came from the Course Reading Assignment after Class 4. Hence, while all four PSMTs' sequence of accommodations began with facets associated with the same feature, the PSMTs' differed temporally as to when they constructed the facets.

The set of findings associated with this fourth assertion support and extend the work of researchers in three areas of research on mathematics PTs: PSMTs analysis of teaching (e.g., Stockero, 2008); PSMTs' mathematical knowledge for teaching (e.g., Stump, 2001); and mathematics PTs' learning of core practices (e.g., Norton & Kastberg, 2012) – specifically facilitating classroom mathematics discourse (e.g., Rahal & Melvin, 1998). My findings complement these studies in that I report the nature of the PSMTs' learning (e.g., changes in the PSMTs' conceptions) about particular aspects of mathematics pedagogy (e.g., instruction that promotes students' mathematical thinking), while enrolled in methods courses. My findings around the PSMTs' pathways of construction extend these studies, because the pathways provide a more detailed, precise reporting of the mathematics PTs' learning of particular aspects of mathematics pedagogy (e.g., assessing questions, advancing questions, and judicious telling) throughout a semester in a mathematics methods course, indicating that learning of similar content occurs at different times for different mathematics PTs. This is important for the field because it cautions mathematics teacher educators that while mathematics PTs may ultimately construct similar conceptions of component core practices, the pathways of their learning might differ.

Assertion 5: Key Course Activities Coordinated and Complimented Each Other

The findings presented in Chapter 6 that addressed the nature of the key course activities indicate that the key course activities, which were designed around pedagogies of practice (decompositions of, representations of, and approximations of practice), coordinated and complemented each other across the semester. To remind the reader, key course activities were course activities (e.g., homework assignments, small group discussions, in-class analysis of teaching) that included or immediately preceded data indicating an addition of a facet or change in a facet of the PSMTs' conceptions (accommodation) of assessing questions, advancing questions, and judicious telling. The design of this study does not allow me to definitively assert that the coordination of the key course activities caused the course activities to be perturbing events for the PSMTs or that the coordination of the key course activities was the sole reason for the PSMTs' learning opportunities. However, the findings in Chapter 6 do indicate *which* key course activities were connected to the PSMTs' accommodations and the *way* in which the key course activities coordinated with each other during the semester.

In order to more specifically address the coordination of the key course activities, consider the diagram in Figure 7-1. The loops in the diagram represent the four different collections of course activities – Introduction to TTT, modified CEI 1, modified CEI 2, and modified CEI 3 – across the semester and the arrows in the diagram represent the coordination of the collection of course activities. I first address which key course activities, identified in Chapter 6, were included in each collection of course activities. Then, I give an overview of the ways the course activities designed around pedagogies of practice, which include key course activities, coordinate with each other across the semester.

The Introduction to TTT collection of course activities included the following key course activities: Course Reading Assignment after Class 4, Class 5 Working Definition Discussion, Class 5 and Class 6 StudioCode Analysis, and Class 6 Defining Features discussion. The modified CEI 1 collection of activities included the following key course activities: Case of Edith Hart Analysis, Class 8 Planning Session for the First Rehearsal, Class 9 First Rehearsal, Class 10 StudioCode Analysis. The modified CEI 2 included two key course activities – Categorization of the 9QTs and Class 14 Second Rehearsal – and the modified CEI 3 included a single key course activity – Class 25 Approximation of Practice.

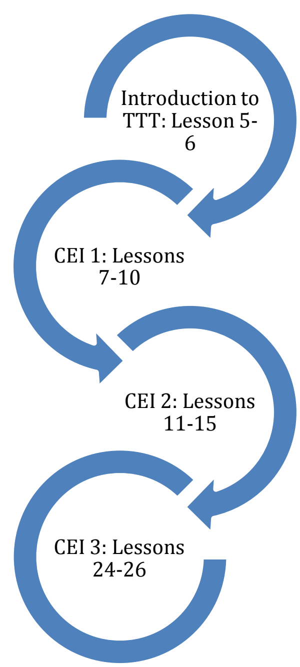


Figure 7-1: The coordination of four phases of course activities that focused on TTT

The key course activities in the Introduction to TTT first introduced the PSMTs to the decomposition of practice (TTT framework) that was used to analyze mathematics instruction in representations of practice and enact mathematics instruction in approximations of practice during the course activities in modified CEIs 1 – 3. The course activities, including key course activities, during each CEI built on the PSMTs' experiences with the decomposition of practice (TTT framework) across the collections of course activities. That is, the PSMTs continued to define the components of the decomposition of practice (assessing questions, advancing questions, and judicious telling), used the TTT framework to analyze representations of practice and engage in approximations of practice that increased in complexity and degree of authenticity.

The set of findings associated with the fifth assertion supports the work of researchers examining what mathematics PTs learn about facilitating classroom mathematics discourse (Ghousseini, 2008; Ghousseini & Herbst, 2014; Tyminski et al., 2014) and research examining mathematics PTs' learning core practices (e.g., Norton and Kastberg, 2012) in that the teacher education course activities, designed around approximations of practice, were connected to the mathematics PTs' learning of a core practice(s). More specifically, my findings support Ghousseini's (2008), Ghousseini's and Herbst's (2014), and Tyminski et al.'s (2014) findings about the ways in which different pedagogies of practice, centered on practices that constitute facilitating classroom mathematics discourse, coordinated with each other in methods courses.

My findings extend research examining mathematics PTs' learning core practices (e.g., Crespo, 2003; Crespo & Sinclair, 2008), which has focused mainly on changes in PEMTs' capacities to enact core practices, in that I provide accounts of the ways in which the approximations of practice (e.g., planning for rehearsals, rehearsals) were connected to the PSMTs learning of the component core practices (e.g., assessing questions, advancing questions, and judicious telling). In addition, this study extends research that has examined ways in which pedagogies of practice promote mathematics PTs learning to facilitate classroom mathematics discourse (e.g., Ghousseini & Herbst, 2014; Tyminski et al., 2014) in that I examined the ways in which multiple iterations of decompositions of, representations of, and approximations of practice coordinated with each other over the course of a semester, as compared to a specific intervention (Tyminski et al., 2014) or two class sessions (Ghousseini & Herbst, 2014). This is important to the field of mathematics teacher education because it illuminates for mathematics teacher educators different teacher education activities designed using pedagogies of practice and the ways in which the teacher education course activities coordinate and complement each other over an entire semester.

Assertion 6: Similar Facets Connected to Different Key Course Activities

While the PSMTs in this study constructed conceptions of TTT that included facets associated with similar features (see Chapter 4), the findings in Chapter 6 indicate that different key course activities at different times in the semester were connected to the addition of or changes in the facets (accommodation) associated with the similar features. For example, all four PSMTs constructed facets associated with the feature *assessing questions inform teachers' subsequent instruction*. The Class 5 Working Definition Discussion was the key course activity connected to both Steve and Nick adding facets associated with this feature, but different key course activities were connected to Leslie and Gretchen adding and changing facets associated with this feature. That is, the Class

5 and 6 StudioCode Analysis was the key course activity connected to Leslie adding a facet, the Class 8 Planning Session for the First Rehearsal was connected to Leslie changing the facet and Gretchen adding a facet, and the Class 14 Second Rehearsal was connected to Gretchen changing the facet. Hence, four key course activities across the semester were connected to the PSMTs' accommodating their facets associated with the same feature of assessing questions.

Another important finding related to this assertion, was that nine of the eleven key course activities occurred in the first half of the semester and only two key course activities occurred in the second half the semester (Class 14 Second Rehearsal and the Class 25 Approximation of Practice). One possible explanation for this difference in the number of key course activities early and later in the semester is saturation. That is, the early course activities provided the PSMTs' opportunities to learn all that there was to learn about the TTT and the course activities later in the semester became redundant. A second explanation for this difference is that course instructor, Dr. A, differed in the degree to which she monitored the PSMTs' understanding of the TTT during portions of the semester and lost an opportunity later in the semester to push the PSMTs' learning of the TTT. I argue in favor of the second explanation. There was some evidence in the interviews that the PSMTs found the continual focus on TTT in the course activities redundant. However, the differences among the facets PSMTs' conceptions of TTT associated with similar features (see Chapter 4) suggests that all the PSMTs had room for growth in their understandings of TTT. Specifically, in the way TTT may be used to promote students' engagement in productive struggle in learning mathematics.

The set of findings associated with the sixth assertion complements findings from the literature body examining mathematics PTs' learning of mathematics pedagogy (e.g., Stump, 2001; van Es & Sherin, 2002; Norton & Kastberg, 2012), specifically what mathematics PTs learn about component core practices of facilitating classroom mathematics discourse (e.g., Ghousseini & Herbst, 2014; Tyminski et al., 2014). Similar to these researchers, I Identified the teacher education course activities that were connected to the PSMTs' learning of component core practices and the nature of the teacher education course activities that supported the PSMTs' learning. My findings extend this literature body because the psychological perspective enabled me to examine and report some of the ways in which different teacher education activities were connected to the PSMTs accommodating their conceptions of component core practices of facilitating classroom mathematics discourse. In addition, the psychological perspective enabled me to report the differences in what the PSMTs learned about the component core practices as a result of the teacher education activities. This is important for the field of mathematics teacher education because it affords insight as to how teacher education activities, designed around pedagogies of practice, are connected to the mathematics PTs' learning of mathematics pedagogy. In addition, it cautions mathematics teacher educators that teacher education course activities may impact the PSMTs' learning of mathematics pedagogy in different ways or may not impact the PSMTs' learning at all.

Implications and Directions for Future Research

The findings of this research study have several implications for the practice of teacher education. First, the findings strongly suggest that if we want PTs to learn core practices with meaning, then they need to engage in multiple and extended experiences with each core practice. As we know from the professional development literature, "one-shot" learning experiences have little impact on inservice teachers' knowledge and practice. This study strongly suggests that the same holds true for the preservice teacher education. Throughout their professional coursework, PTs need multiple experiences in which there is sustained focused on core practice(s) and the way in which core practices interweave to constitute classroom instruction centered on students' thinking.

An implication that follows this first one concerns the current curriculum of teacher education. If PTs need multiple and extended opportunities in order to construct deep knowledge of core practices, then the field must work to identify a small set of core practices on which to focus preservice teacher education. One only needs to consider teacher education accreditation standards to see that the teacher education curriculum is "a mile wide and an inch deep." The current organization of most preservice teacher education programs does not allow PTs to have multiple and extended experiences with a large number of core practices. Also, while core practices are considered to be accessible constructs for novice teachers, they are nonetheless complex. Core practices are composed of more particular practices and inter-related with each other to constitute classroom instruction that centers on students' thinking. In order for professional coursework to appropriately engage PTs in multiple and extended opportunities that are needed to construct deep knowledge of the complexity of core practices, the field needs to clarify manageable sets of core practices around which the teacher education curriculum may be organized.

Therefore, an implication of this study for mathematics teacher education researchers is to develop a coordinated set of studies, with questions similar to this study but about different core practices, that enables researchers to assess the viability of methods courses focused on different core practices. A coordinated research agenda in this area would move the field forward in substantive ways by identifying what PTs learn about different core practices and the ways in which course activities supported or inhibited the PTs' learning. This research would contribute to a knowledge base that could better inform both policy and practice in teacher education.

The findings of this study also suggest a number of avenues for future research. One direction is to examine the ways in which the PSMTs assimilated their experiences in teacher education course activities designed around pedagogies of practice. This study used a constructivist learning perspective to examine the PSMTs' learning of component core practices of facilitating classroom mathematics discourse (e.g., assessing questions, advancing questions, and judicious telling). However, I focused mainly on the accommodations and the connection of course activities to the PSMTs' accommodations to their conceptions of component core practices. This addresses one of the key tenets of constructivism – accommodation – but does not give sufficient attention to another key tenet – assimilation. Hence, it would be interesting in future research to examine the ways in which PSMTs' existing conceptions contributes to their interpretation of experiences (assimilation) with core practices or component core practices as they engage in teacher education activities designed using decompositions of, representations of, and approximations of practice.

Another direction is to examine the ways in which the PSMTs' conceptions of a core practice influenced their enactment of the core practice. This study focused on the PSMTs' conceptions of the TTT, but did not include the ways in which the PSMTs enacted the TTT during the approximations of practice. It would be interesting in future research to examine the ways in which the PSMTs' conceptions of TTT influenced their planning and enactment of TTT in the approximations of practice over the course of the semester.

A third direction is to conduct longitudinal study of the PSMTs learning of a core practice and their enactment of the core practice as novice teachers in their own classroom. It would be interesting in future research to examine whether the PSMTs' conceptions of the core practice, constructed in professional coursework, were maintained during student teaching and in their classrooms as novice teachers. Perhaps more important, it would be interesting in future research to examine these participants' instruction as novice teachers to determine whether the participants' instruction incorporates the TTT and, if so, to examine the characteristics of the TTT and the way it shapes the classroom mathematics discourse.

Conclusion

I began this dissertation report with a quote that captured the current landscape of the professional education of teachers in the United States. Contemporary reformers in teacher education advocate professional coursework in which the curricula are focused on core practices and courses are designed around pedagogies of practice (e.g., Grossman et al., 2009a; McDonald et al., 2013). Both reformers and researchers emphasize the need for our field to examine not only the design of courses but the what and how PTs learn core practices as they engage in this professional coursework designed around pedagogies of practice. This study joins the work of other researchers in building the body of literature that addresses what and how PTs learn about core practices in courses designed around pedagogies of practice, particularly in the area of mathematics teacher education. In addition, growth of this body of literature helps the field of teacher education establish and explicate the meaning for common frameworks (e.g., pedagogies of practice) and language (e.g., core practice) in the field of teacher education. More importantly, the growth of this body of literature is necessary for the field of teacher education if we want to heed the caution of teacher education researchers (e.g., McDonald et al., 2013) in that professional coursework focused on core practices and designed around pedagogies of practice is not just the latest fad in teacher education, but instead is a worthwhile and effective model of teacher education.

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Appendix A

Teacher Education Activities in the Fall 2013

The Modified Cycle of Enactment and Investigation

The instructor structured the course activities using Lampert and colleagues' Cycle of Enactment and Investigation (CEI) (see Figure A-1). The modified CEI has five phases: doing a mathematical task; reading a narrative case and/or watching a video case, then collectively analyzing the teaching and learning in the case; preparing to enact the practices that constitute facilitating discussions; rehearsing/performing the practices; and collectively analyzing the teaching and learning in the rehearsal. Table A-1 includes a more thorough explanation of each phase in the modified CEI and Figure A-2 represents an example of a modified CEI.

Table A-1: The modified CEI

Phase of the Cycle	Description
Phase 1. Doing the mathematical task	PSMTs do a mathematical task, identify mathematical ideas and processes in the task, and possible learning goals for a teacher using the task.
Phase 2. Analyzing *representations of practice*	PSMTs individually and collectively *analyze a representation of practice* (e.g., narrative case) involving the mathematical task from Phase 1 using a *decomposition of practice* as a framework.
Phase 3. Engaging in *approximation of practice*: Planning	Small groups of PSMTs plan to teach a rehearsal emphasizing an aspect of teaching characterized by the *decomposition of practice* and exemplified by the *representation of practice* in Phase 2.
Phase 4. Engaging in *approximation of practice:* Teaching	PSMTs will rehearse their plans from Phase 3 with different people (e.g. instructors, peers, high school students) playing the role of students. The lead instructor providing feedback during

	rehearsal.
Phase 5. Analyzing *representations of practice*	Small groups of students collectively and individually analyze a representation of the rehearsal in Phase 4 using a *decomposition of practice*.

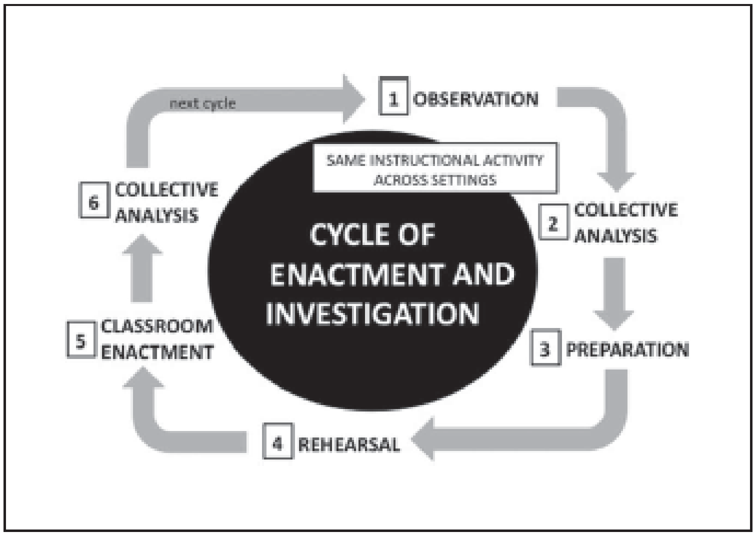


Figure 1. Cycle of enactment and investigation

Figure A-1: CEI from Lampert et al. (2013)

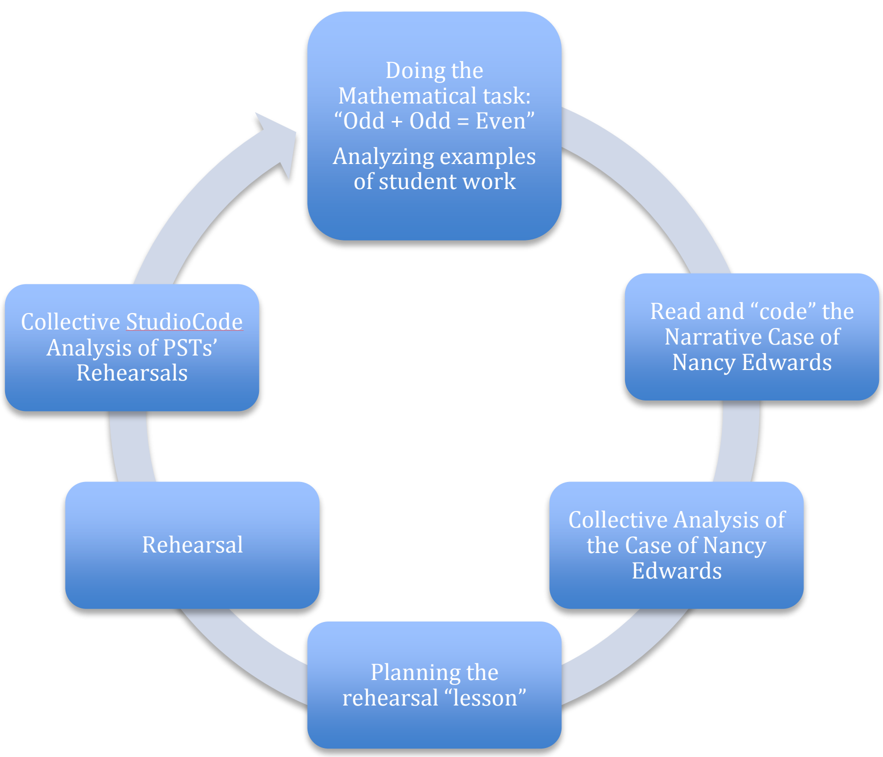


Figure A-2: The second modified CEI during Secondary Mathematics Teaching I

The course instructors agreed with the overall structure of the CEI and the implementation of multiple iterations of a CEI proposed by Lampert and colleagues, but the instructors made two modifications to the cycle. First, because Teaching Secondary Mathematics I did not have a field experience component, the instructors did not include a phase after the rehearsal in which the PTs taught a group of secondary students. Second, the instructors included one phase to the cycle – *doing mathematical activity*. The instructors modified the cycle to include *doing mathematics* because they considered it important for the PSMTs to have opportunities to think about the important mathematical ideas and mathematical processes embedded in the task used in the *representations of practice.*

In the next section, I describe the way the different iterations of the modified CEIs were coordinated with another set of course activities during the course.

Coordination of Iterations of Modified CEIs.

Figure A-3 illustrates the way in which the initial set of course activities – Introduction to TTT– was coordinated with the sets of course activities in the three iterations of the modified CEIs. The arrows in the diagram represents that each of the sets of course activities "feeds" the subsequent set. For example, the first set was the collection of course activities that introduced the PSMTs to the TTT and this framework provided the students a *decomposition of practice* that they used to analyze and enact the core practices in CEI 1 – 3. Another example, each of the PSMTs' experiences within each of the CEIs were built on in subsequent CEIs as they continued to define the TTT, use the TTT to analyze *representations of practice,* and engage in *approximations of practice* that increased in complexity and degree of authenticity.

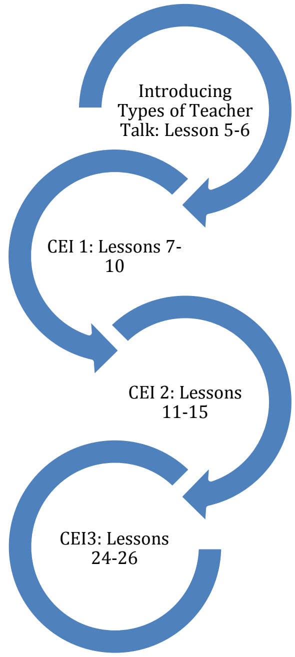


Figure A-3: The coordination of four phases of course activities that focused on TTT

Introduction of Types of Teacher Talk

Table A-2 presents the course activities that the instructor selected and sequenced to introduce the PSMTs to assessing questions, advancing questions, and judicious telling. The course activities are in class course activities and homework assignments that occur after Class 4 to the end of Class 6.

Table A-2: Course activities during the introduction to TTT

Class # and Date	**Purpose of Course Activity**	**Description of Course Activity**
Homework Assignment after Class 4 Due on Tuesday 9/10/13	Introduction of *decomposition of practice*	PTs were assigned a course reading (Smith, Bill, & Hughes, 2008) and wrote initial descriptions in their notebooks of assessing questions and advancing Questions.

Class 5: Tuesday 9/10/13	Defining Types of Teacher Talk	In small groups, PTs discussed initial descriptions of assessing questions and advancing questions. The small groups publicly displayed their discussions on whiteboards. Then, Dr. A facilitated a whole class discussion about the working definitions of the three types of teacher talk (TTT) - assessing questions, advancing questions, and judicious telling.
	Analyzing a *Representation of Practice*	In small groups, the PTs conducted a StudioCode analysis of Dr. A and the Staircase problem using the TTT framework.
Class 5 Homework Assignment: Due on Thursday 9/12/13	Defining Types of Teacher Talk	PTs wrote the defining features of the TTT that in response to the prompt – "How do you know it's telling/assessing/advancing when you hear it?" (MTHED 411 HW Assignment 9/10/13).
Class 6: Thursday 9/12/13	Analyzing a *representation of p*ractice	In small groups, the PTs continued the StudioCode analysis of Dr. A and the Staircase problem using the TTT framework.
	Individual reflection on learning	Individually, the PTs completed an assignment that in which they were asked to identify ways in which the StudioCode analysis supported their learning of TTT.
	Defining Types of Teacher Talk	In small groups, the PTs discussed defining features of the TTT. Then, Dr. A orchestrated a whole group discussion about the small group's coded segments in StudioCode from which the group constructed a list of defining features of TTT.

First Modified CEI

Table A-3 presents the course activities involved in the first modified CEI. The course activities began with the homework assignment after Class 6 and continue to the end of Class 10. In this modified CEI, the PSMTs have opportunities to further define the Types of Teacher Talk, use the TTT to analyze two different *representations of practice*, and learn to enact TTT in the two *approximations of practice* – planning for and teaching in the First Rehearsal.

Table A-3: Course activities during the first modified CEI

Class # and Date	**Purpose of Course Activity**	**Description of Course Activity**
Homework assignment after Class 6 Due on Tuesday 9/17/13	Doing mathematical task	PTs were assigned the “Cal’s Dinner Card Deal” task (Smith, Silver, & Stein, 2005).
	Analyzing representation of practice	PTs were assigned the narrative case of Edith Hart (Smith et al., 2005). Also, PTs were instructed to individually code (assessing, advancing, & telling) the narrative case of Edith Hart and answer a set of questions about: Edith’s mathematical goals, what Edith’s students learned, and how Edith’s teaching supported the learning.
	Defining Types of Teacher Talk	PTs were instructed to write instructional purposes for each TTT.
Class 7 Tuesday 9/17/13	Analyzing representation of practice and defining Types of Teacher Talk	PTs collectively discussed and analyzed Edith Hart in small groups. Then, the PTs discussed instructional purposes for TTT in small groups. Last, Dr. A orchestrated a whole class discussion about instructional purposes for TTT and analyses of Edith Hart
Homework assignment after Class 7 Due 9/19/13	Doing mathematical task	PTs were assigned to the *Connected Mathematics Project* (CMP) (CITE) problem set.

Class 7 Thursday 9/19/13	Engaging in an Approximation of Practice	Dr. A’s explained the structure of the First Rehearsal and organized groups of students for the First Rehearsal. Then, the rehearsal groups used a modified Thinking Through a Lesson Protocol (TTLP) to plan how they would interact with a group of students doing the CMP set. The modified TTLP included a section in which the PTs prepared TTT. Last, Dr. A facilitated a whole class discussion about the TTLPs and the First Rehearsal
Class 9: Tuesday 9/24/13	Engaging in an approximation of Practice	The PTs, with the support of Dr. A who is acting as a co-teacher, interacted with three course assistants (doctoral students) who are working on the CMP problem set. The rehearsal lasted 5 – 8 minutes for each rehearsal pair. The non-rehearsing PTs used an observation protocol to monitor and document the frequency and effectiveness of the rehearsing PTs’ TTT. Four groups of pairs rehearse – thirteen of the PTs did not rehearse.
Class 10 Thursday 9/26/13	Analyzing representation of practice	The small rehearsal groups collectively coded the rehearsing PTs’ teaching using TTT and StudioCode. For each coded instance, the small groups also wrote memos about the purpose, quality, and possible alternative instruction.

Second Modified CEI

Table A-4 presents the course activities in the second modified CEI. The course activities associated with the second modified CEI began after Class 10 and continued through the homework assignment after Class 15. The second modified CEI differed from the first because the course activities after Class 10 and through the end of Class 11 were used to support the PTs in furthering their understanding of Types of Teacher Talk.

Hence, the first phase of the modified CEI in which the PSMTs' "do the mathematical task" takes place in Class 12.

Table A-4: Course activities in the second modified CEI

Class # and Date	Purpose of Course Activity	Description of Course Activity
Homework assignment after Class 10 Due Tuesday 10/1/13	Defining TTT and analyzing representation of practice.	PTs were assigned course readings (Boaler & Humphreys, 2005; Smith & Stein, 2011). Categorized the nine-question typology (Boaler & Brodie, 2004), which was introduced in the readings, as one or more of the TTT, as Types of Teacher Talk. Last, analyzed Edith Hart (Smith et al., 2005) using the nine-question typology.
Class 11 10/1/13	Defining TTT and analyzing representation of practice	PTs collectively categorized nine-question typology as TTT in small groups and discussed analysis of Edith Hart. Then, Dr. A orchestrated a whole class discussion about the small groups categorization of the nine-question typology and analyses of Edith Hart.
Class 12 Thursday 10/3/13	Analyzing representation of practice	PTs, in small groups, read Student A-J Work Samples for the Odd + Odd = Even task (CORP materials) and determined whether the arguments in Student Work Samples A – J were valid arguments.
Homework assignment after Class 12 Due on Tuesday 10/8/13	Analyzing representation of practice	PTs individually coded and analyzed the Case of Nancy Edwards (CITE) using the TTT and nine-question typology.
Class 13 Tuesday 10/8/13	Defining valid arguments	Dr. A orchestrated a whole group discussion in which the group created a list of criteria for "valid arguments" using evidence from the Student Work Samples A – J.
	Analyzing representation of practice	PTs discussed their analyses of Nancy Edwards in small group and as a whole class discussion.

	Engaging in an approximation of practice	In small three-member rehearsal groups, PTs did 1 of 6 problems in order to generate at least 4 possible “student” arguments. Then, the rehearsal groups then completed the first portion of a modified TTLP (goal of problem, standards problem covers, possible student errors & misconceptions.)
Homework Assignment after Class 13 Due on Tuesday 10/10/13	Engaging in approximation of practice	PTs individually completed the second portion of the modified TTLP (included TTT prompts). In order to complete the TTLP, the PTs downloaded their respective doctoral student’s work from the course website.
Class 14 Tuesday 10/10/13	Engaging in approximation of practice	The PTs, with the support of the Dr. A who acted as a coach, helped a student (doctoral student) in “making progress” on their argument (projected on overhead so that the audience can see) for three minutes. Each of the 18 PTs have opportunities to rehearse.
Class 15 Tuesday 10/15/13	Analyzing representation of practice	Using StudioCode, the rehearsal group collectively coded each of the PTs’ rehearsals using TTT. Then, individually, each of the PTs’ wrote memos for their coded instances that included rationales for the TTT, purposes for the TTT, and categorized the TTT using the nine-question typology.
Homework Assignment after Class 15 Due Thursday 10/17/13	Individual reflection on learning	Individually, PTs reflected on what they learned in the Second Rehearsal.

Third Modified CEI

Table A-5 presents the course activities that constitute the Class 25 Approximation of Practice. Unlike the first and second modified CEI, the third CEI did not include a phase two in which the PSMTs analyzed a representation of practice prior to planning for and teaching in an approximation of practice. The third modified CEI also differed in that the approximation of practice (teaching portion) was not a rehearsal and it involved actual high school students.

Table A-5: Course activities in the third modified CEI

Class # and Date	**Pedagogy of Practice**	**Course Event**
Class 24 and Homework assignment after Class 24 Tuesday 12/03/13	Doing mathematical task	PTs, in small rehearsal groups, collectively did the three rehearsal problems - Beam, Poster, and Staircase – in order to generate multiple approaches to a solution.
	Engaging in approximation of practice	The rehearsal groups began working through a modified TTLP. PTs individually completed the modified TTLP and wrote a 1-page plan for enacting the tasks with the students that utilized a "launch, explore, summarize" teaching sequence.
Class 25 Thursday 12/05/13	Engaging in approximation of practice	The PTs individually enacted their plans with one or two high schools students around three mathematical tasks (Beam Problem, Poster Problem, & Staircase Problem).
Homework Assignment Due on Tuesday 12/10/13	Analyzing representation of practice	PTs individually used StudioCode to identify, code, and write memos for a total of five-minutes of your instruction that the PTs believed "went well". Then, PTs coded and wrote memos for three examples of each TTT (assessing, advancing, and telling).
Class 26 Tuesday 12/10/13	Analyzing representation of practice	Using StudioCode, the rehearsal group collectively coded each of the 3 PTs' rehearsals. Each member of the group selected for discussion with the group an assessing, advancing, and telling statement from those they individually already coded. The group categorized the TTT using the nine-questions typology. Then, they wrote an explanatory rationale for they you coded categorized the TTT as a

		particular question type from the nine-question typology.
Homework assignment after Class 26	Analyzing representation of practice	The PTs wrote Final Project Papers that included their analyses of the Class 25 Approximation of Practice and the characterizations of three learning experiences that support students' learning of mathematics.

Appendix B

Card Sort Activity from Interview 3

Question Type	Description
Gathering information, leading students through a procedure	Requires immediate answer. Rehearses known facts/procedures. Enables students to state facts/procedures.

Question Type	Description
Inserting Terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them.

Question Type	Description
Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations.

Question Type	Description
Probing, getting students to explain their thinking	Asks student to articulate, elaborate, or clarify ideas.

Question Type	Description
Generating discussion	Solicits contributions from other members of the class.

Question Type	Description
Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life.

Question Type	Description
Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used.

Question Type	Description
Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem solving.

Question Type	Description
Establishing context	Talks about issues outside of mathematics in order to enable links to be made with mathematics.

Appendix C

First Rehearsal CMP Problem Set from Lappan et al., (2002)

In 1–3, use the following information: José, Mario, and Melanie went on a weeklong cycling trip. The table below gives the distance each person traveled for the first 3 hours of the trip. The table shows only the time when the riders were actually biking, not when they stopped to rest, eat, and so on.

	Distance (miles)		
Cycling time (hours)	José	Mario	Melanie
0	0	0	0
1	5	7	9
2	10	14	18
3	15	21	27

1. **a.** How fast did each person travel for the first 3 hours? Explain how you got your answer.

b. Assume that each person continued at this rate. Find the distance each person traveled in 7 hours.

2. **a.** Graph the time and distance data for all three riders on the same coordinate axes.

b. Use the graphs to find the distance each person traveled in $6\frac{1}{2}$ hours.

c. Use the graphs to find the time it took each person to travel 70 miles.

d. How does the rate at which each person rides affect the graphs?

3. **a.** For each rider, write an equation that can be used to calculate the distance traveled after a given number of hours.

b. Use your equations from part a to calculate the distance each person traveled in $6\frac{1}{2}$ hours.

c. How does a person's biking rate show up in his or her equation?

Figure C-1: First rehearsal problem set, page 1, from Lappan et al., (2002)

4. Mike was on the bike trip with José, Mario, and Melanie (from questions 1–3). He made the following table of the distances he traveled during day 1 of the trip.

Time (hours)	Distance (miles)
0	0
1	6.5
2	13
3	19.5
4	26
5	32.5
6	39

a. Assume Mike continued riding at this rate for the entire bike trip. Write an equation for the distance Mike traveled after t hours.

b. Sketch a graph of the equation.

c. When you made your graph, how did you choose the range of values for the time axis? For the distance axis?

d. How can you find the distance Mike traveled in 7 hours and in $9\frac{1}{2}$ hours, using the table? The graph? The equation?

e. How can you find the number of hours it took Mike to travel 100 miles and 237 miles, using the table? The graph? The equation?

f. For parts d and e, give the advantages and disadvantages of using each form of representation—a table, a graph, and an equation—to find the answers.

g. Compare the rate at which Mike rides with the rates at which José, Mario, and Melanie ride. Who rides the fastest? How can you determine this from the tables? From the graphs? From the equations?

Figure C-2: First rehearsal problem set, page 2, from Lappan et al., 2002

Appendix D

Thinking Through a Lesson Protocol (adapted from Smith et al., 2008)

The main purpose of the Thinking Through a Lesson protocol is to prompt you in thinking deeply about a specific lesson that you will be teaching. The goal here is to move beyond the structural components associated with lesson planning (e.g., listing the materials you will need, describing the way students will be grouped, determining teacher actions during the lesson) to a deeper consideration of how you are going to advance students' mathematical understanding during the lesson. This is not to say that structural components of a lesson are not important, but rather that a focus on structural components alone is *not sufficient* to ensure that students learn mathematics.

The Mathematical Task(s)

- **What mathematical content and processes do you hope students will learn from their work on these tasks?**

- **What specific *content standards* from the Common Core State Standards (High School Functions Standard) will your students learn from their work on these tasks?**

- **What specific *mathematical practice standards* from the Common Core State Standards – Mathematics (CCSS-M) will your students have the opportunity to engage in during their work on these tasks?**

- **What are all the ways the task can be solved?**
    - **Which of these methods do you think your students will use?**
    - **What misconceptions might students have?**
    - **What errors might students make?**

Supporting Students' Exploration of the Task(s)

- **As students are working in small groups:**

- **What questions will you ask to *assess* students' understanding of key mathematical ideas, problem-solving strategies, or the representations?**
- **What questions will you ask to *advance* students' understanding of the mathematical ideas?**
- **What mathematical concepts/definitions/constructs might you need to introduce to the students by *telling*?**

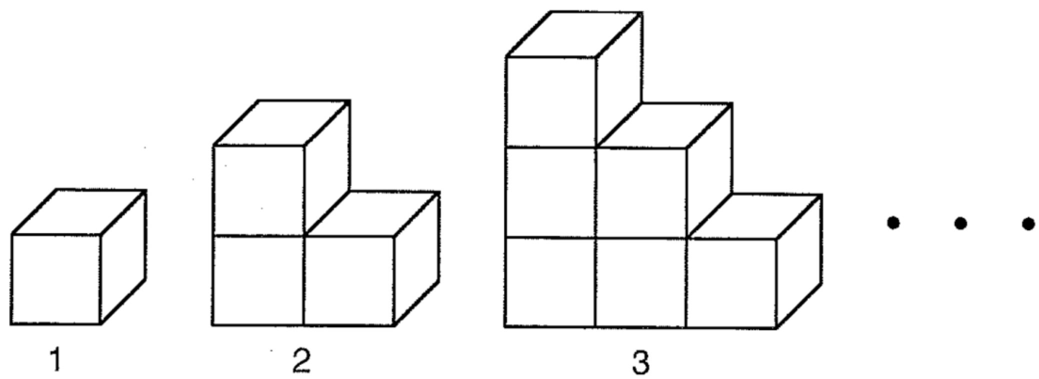
Appendix E

Class 25 Approximation of Practice Problem Set

The Staircase Problem

Adapted from Annenberg Learner's Teaching Math: A Video Library, 9-12

Look over the following sequence of figures. (The small cubes used to build each of the figures measure one inch along each edge.)



Complete the table below.

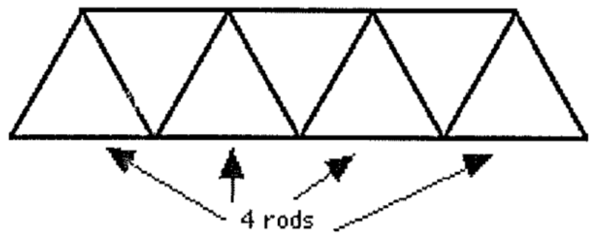
Height of Staircase	1	2	3	4	5	6	7	8	9	10
Number of Blocks										

How many blocks would you need to build a staircase that has a height of 20? 50? 100? n?

Figure E-1: Class 25 Approximation of Practice Staircase Task

Beam Design Problem

Beams are designed as a support for various bridges. The beams are constructed using rods. The length of the beam is determined by the number of rods used to construct the bottom of the beam. Below is a beam of length 4.



1. How many rods are needed to make a beam of length 5? Of length 8? Of length 10? Of length 20? Of length 34? Of length 76?

2. How many rods are needed for a beam of length 223?

3. Write a rule or a formula for how you could find the number of rods needed to make a beam of any length. Explain your rule or formula.

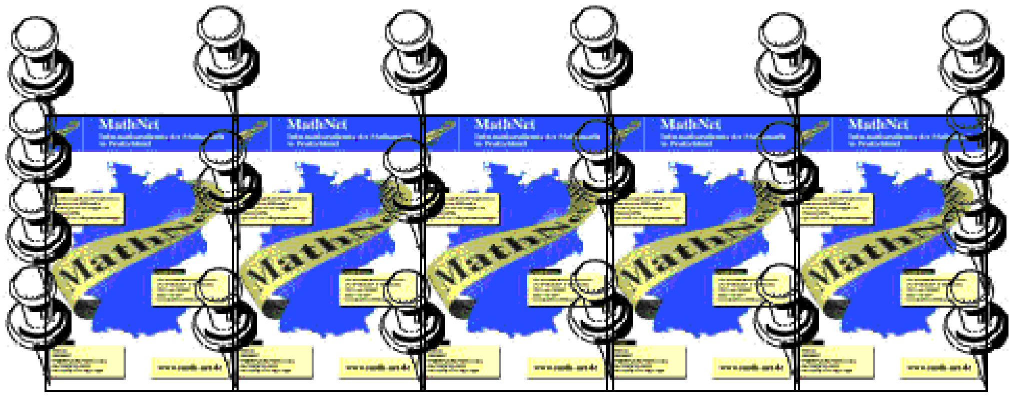
4. What would the length of a beam be that requires 691 rods?

Figure E-2: Class 25 Approximation of Practice Beam Task

Poster Problem

Developed by Dr. John K. Lannin, University of Missouri

Deck R. Ater has decided that three tacks is not enough. He decides to reinforce his posters by placing four tacks on each of the ends. He continues to place three tacks on the overlap between the posters.



1. How many tacks will Deck need to hang 3 posters? 6 posters? 10 posters? 20 posters?

2. Explain how you would determine the number of tacks for any number of posters. Write a rule for how you would find the number of tacks given the number of posters.

3. How many tacks will Deck need to hang 439 posters?

4. Suppose that Deck used 566 tacks. How many posters did he hang?

Figure E-3: Class 25 Approximation of Practice Poster Task

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Publications

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Grants, Fellowships, & Awards

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