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ENTROPY OF SPEED AND ACCURACY TRADE-OFF IN SPACE-TIME

A Dissertation in
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by
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ABSTRACT

The relation between movement speed and accuracy is one of the most robust phenomena in human movement performance. The essence of the speed-accuracy relation is that with an increase in movement speed there is concomitant decrease in movement accuracy. Different descriptions and explanations of the speed-accuracy relation have been proposed for characterizing time matching and time minimization movement tasks. Nevertheless, these accounts have emphasized the spatial dimension of the phenomenon providing a limited assessment given that human movement takes place in both space and time. It follows that there is a potential for both spatial error and temporal error in motor task.

Hancock and Newell (1985) proposed a space-time framework of the movement speed-accuracy relation that is based on the space-time principle that the spatial component of movement is always measured with respect to time and that the temporal component of movement is always measured with respect to space (Minkowski, 1908). However, although the spatial and temporal error was considered as complementary features, they were still considered separately, as function of other features. This dissertation investigated this problem considering the joint entropy as a way to examine the movement variability of different tasks while also integrating the space-time properties of the movement. Three experiments were conducted to investigate two hypotheses: that when space and time are integrated in one measure – the joint entropy – the phenomenon will be characterized by a U-shape function – revealing an optimal region of time/space variability; and that when space and time are integrated, different
descriptions of speed-accuracy trade-off phenomenon are not necessary given that the constraints are the same.

Experiment 1 was set-up to investigate the space-time entropy of movement outcome as a function of a range of spatial (10, 20 and 30 cm) and temporal (250 to 2,500 ms) criteria in a discrete aiming task. The joint space-time entropy was lowest when the relative contribution of spatial and temporal task criteria was comparable (i.e., mid-range of space-time constraints), and it increased with a greater trade-off between spatial or temporal task demands, revealing a U-shaped function across space-time task criteria.

In Experiment 2, two sub-experiments in an isometric single finger force task investigated the joint force-time entropy with: a) fixed time to peak force and different percentages of force level; and b) fixed percentage of force level and different times to peak force. This was done to test whether the U-shape would be generalizable across different tasks. The findings show that force error and timing error are dependent but complementary when considered in the same framework with the joint force-time entropy at a minimum in the middle parameter range of discrete impulse.

In Experiment 3, time matching and time minimization movement tasks were used to test whether these different task descriptions of speed and accuracy were due to different temporal and spatial task constraints. The results showed that the joint space-time entropy of outcome did not change across tasks and conditions – revealing a common level of space-time entropy between these two categories of aiming tasks. Overall, these results showed that the joint information entropy analysis revealed the structure of movement accuracy masked by the distributional analysis of movement data when either the spatial or temporal dimensions of movement error considered alone and
independently. The main contribution of this study was the cohesion of the methodological approach developed in terms of joint information entropy that provides an alternative perspective and a more complete account from which to describe and explain the speed and accuracy trade-off phenomenon.
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CHAPTER 1: INTRODUCTION

The speed and accuracy trade-off

The relation between movement speed and accuracy is one of the most robust phenomena in human movement. Consider the seemingly simple movement tasks of reaching out with your arm for a bottle, to finish typing a lab report before going to a class, and attempting to quickly thread a needle. Common sense teaches us that when we perform these movements faster, we make more mistakes or errors in terms of the goal we want to achieve. These situations support the adage "haste makes waste"; a long standing perspective about motor skills.

The relationship between speed and accuracy is an important topic in the field of motor control that has led to many theories and empirical findings (e.g., Fitts, 1954; Hancock & Newell, 1985; Elliott, Helsen & Chua, 2001; Plamondon & Alimi, 1997; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979; Woodworth, 1899). The essence of the speed-accuracy relation is that with an increase in movement speed there is concomitant decrease in movement spatial accuracy. In this case, we need to sacrifice movement speed for accuracy to achieve the task goal that we want to realize. That is, movement accuracy can be increased by trading movement speed.

However, human movement always take place in both space and time, therefore, there is potential for both spatial error and temporal error in motor tasks (Hancock & Newell, 1985; Newell, 1980). In movement timing tasks it has been shown that increasing movement velocity within the same movement time results in decreasing movement timing error, rather than an increasing error as in a movement spatial accuracy task (Ellis, Schmidt, & Wade, 1968; Newell, Hoshizaki, Carlton, & Halbert, 1979; Newell,
Carlton, Carlton & Halbert, 1980). Thus, a directional effect of the relation of movement speed and accuracy is a function of whether spatial or temporal accuracy is being measured, opening the idea that a paradox exists in the effect of movement speed on accuracy (Newell, 1980). For example, when a performer tries to hit a baseball he/she does not only need to know where the ball will be, but also needs to know when the ball will be in that certain location.

A fundamental question that naturally follows is - if human action is reflected in the space and time of the movement, how are we integrating the two individual dimensions of space and time into a single space-time property? In addition, how is this single space-time measurement providing empirical evidence for a new finding that could be incorporated in the relation of movement speed-accuracy trade-off?

**Focus of the Dissertation**

Hancock and Newell (1985) proposed a space-time framework of the movement speed-accuracy relation that is based on the space-time principle that the spatial component of movement is always measured with respect to time and that the temporal component of movement is always measured with respect to space (Minkowski, 1908). They outlined a space-time principle of the movement speed-accuracy relation and proposed that there are complementary spatial and temporal error functions for movement accuracy when both errors are observed in the same frame of reference. Human movement accuracy should be examined under a space-time view that requires a common frame of reference for the observation and measurement of both spatial and temporal error.
The main issue addressed in this dissertation is the development of a unified movement space-time joint entropy measure and its relation to the standard variance measures on each of the dimensions (space and time) considered separately. It is anticipated that the movement variability in terms of a probabilistic space-time approach would be different from the variability measured in the traditional single dimension distributed analysis. However, the influence of different spatial or temporal criteria is less certain, as it has been proposed that in dual space-time criterion tasks (Hancock & Newell, 1985; Newell, 1980), participants trade spatial error for timing error, rather than the traditional perspective of speed for accuracy.

In addition, according to the impulse variability theory (Carlton & Newell, 1993; Schmidt et al., 1979), the force generated by the muscular system leads to movement about a joint and the spatial and temporal variability in movement outcome could be directly inferred to variability in the force generated by the neuromuscular system. The question is whether a unified joint entropy can be generalized from space-time entropy to force-time entropy in the speed and accuracy trade-off.

Moreover, it has been proposed that there are different functions for the speed-accuracy trade-off with time matching and time minimization movement tasks. In line with the space-time perspective provided by Hancock and Newell (1985), we investigate here whether the emergent movement joint entropy of speed-accuracy properties would be the same under the same spatial and temporal constraints, independent of the classification of the task.

To address the questions outlined above, 3 experiments examined the tenets of the space-time or force-time approach to movement speed and accuracy. The experiments
will be conducted in a discrete aiming task and an isometric force production task. First, the unified spatial and temporal error measurement of the probabilistic estimates of movement outcomes is investigated by estimating the *joint* probability structure of information entropy. Second, the unified force-time variability will be investigated by estimating the *joint* probability structure of the force-time discrete impulse information entropy. Third, the movement variability is investigated under the same spatial and temporal constraints to examine the *joint* space-time entropy of movement properties independent of the task, namely, time-minimization and time-matching tasks.

Experiment 1 investigated the relation of movement space-time entropy to the traditional spatial and temporal errors under different dual space-time criteria that require moving to a fixed spatial target width in a discrete aiming task. The functions for spatial error and temporal error have been shown to be different though related when considered in the unified spatial-temporal framework (Hancock & Newell, 1985; Hsieh et al., 2013; Newell, 1980). Here we contrast the separate and unified error functions for movement temporal and spatial accuracy through the *joint* probabilistic measurement of entropy of the space-time outcome in the context of movement space-time criteria. It is hypothesized that the movement velocity in the mid-range will lead to the minimum of *joint* probabilistic measurement of entropy of the space-time outcome (Newell, 1980). A U-shaped function for entropy of the space-time outcome will be different than the traditional movement outcome measured independently in either the space or time dimension. It is also anticipated that the traditional speed-accuracy functions for spatial error and temporal error considered independently can be mapped to this space-time U-shaped function.
Experiment 2 investigated a feature of the impulse variability theory (Schmidt et al., 1979; Carlton & Newell, 1993) that assumed that a given force generated by the muscular system leads to movement about a joint(s), and that spatial and temporal variability in movement outcome could be directly inferred to variability in force generated by neuromuscular system. The question investigated here is to examine the relation between impulse properties in determining variability of response kinetics by using a unified force-time joint entropy in an isometric single finger force task. In particular, we wanted to test the degree to which the unified U-shaped space-time joint entropy in movement kinematics can be generalized to the force-time joint entropy in force production (Carlton & Newell, 1993; Hsieh et al., 2013; Hsieh et al., 2015; Kim et al., 1999; Lai et al., in press; Schmidt et al., 1979). The force-time framework of the movement speed-accuracy relation was examined in two experiments that used an isometric single finger force task with different combinations of force levels and time to peak force goals. It was hypothesized that a peak force variability function based upon a consistent time to peak force would provide a more veridical estimate of the relationship between force and force variability. Alternatively, because of the dual task criteria of peak force and time to peak force, participants will systematically deviate from the rate that is required by the task criteria when they produce force to match the task criteria. Also, we tested the hypothesis that the mid force-time range will lead to the minimum of joint probabilistic measurement of entropy of the force-time outcome. The function for entropy of the force-time outcome will be different than traditional movement outcome only in either force or time dimension.
Experiment 3 examined the space-time perspective provided by Hancock and Newell (1985), where it is expected that under the same constraints (spatial and temporal) the emergent movement properties would be the same, independent of the classification of the task. The emergent movement outcomes on both time-minimization and time-matching tasks would share the same properties of a given space-time landscape if the constraints are the same. That is, if the movement time derived from the practice in a time-minimization task is used as the goal for a time-matching task when both target width and amplitude are maintained, we expect to have the same movement properties. In addition, if the effective target width from this time-matching task is calculated and used as target-width for the time-minimization task, we would expect to have similar movement outcome properties. Thus, in the Experiment 3, we investigated whether the joint space-time entropy of movement properties under the same constraints would be the same independent of the task, namely, time-minimization and time-matching aiming tasks.

In summary, it was anticipated that an investigation of the space (force)-time perspective in the different tasks would reveal information about how the joint entropy of space (force) and time components of human control system are organized as a complementary function of speed and accuracy trade-off under dual dimensions. In addition, this study should provide further empirical evidence for a complete interpretation of this relationship examined in both spatial error (force error) and temporal dimensions of movement across the complete range of movement average velocity available to the human performer in the achievement of discrete aiming movements.
CHAPTER 2: LITERATURE REVIEW

Approaches to the "Speed-accuracy trade-off and spatial error"

The speed-accuracy trade-off has been a long standing interest in the field of motor behavior and cognitive psychology. Since the work of Fullerton and Cattell (1892), examinations and subsequent explanations of this relation have most commonly focused on movement error in a spatial dimension (e.g., Beggs & Howarth 1970; Crossman & Goodeve 1983; Fitts 1954; Woodworth 1899). Moreover, these studies have emphasized spatial error toward the maximum velocity for specific movement amplitude and target size conditions.

Woodworth (1899) is often considered the father of research to examine the speed and accuracy trade-off in movement. The main contribution of his work is that he linked a complicated relationship between movement duration, amplitude, velocity and the influence with/without vision information in the determination of movement accuracy. He proposed that the aiming movement control process is composed of an initial impulse phase and a current control phase. He also attributed the effect of movement speed on movement accuracy to the current control phase and excluded any effects on the accuracy of the initial impulse phase (Hancock & Newell, 1985; Plamondon & Alimi, 1997).

After Woodworth, many researchers pursued the description of the movement speed and accuracy relation. Fitts (1954) proposed a mathematical model that was a critical step to describing formally, the speed-accuracy phenomenon and it has become one of the most significant laws in motor control. In Fitts’ original task, a person alternately taps a handheld stylus to targets as quickly and accurately as possible over a specified duration. The movement distance (between two targets) and the target size (the width of the target)
are the independent variables, and the movement time is the dependent variable. Fitts described this relationship between movement time, amplitude and target width as $MT = a + b \log_2 (2A/W)$. In this equation, $MT$ is the time from onset to the termination of movement; $a$ and $b$ are constants; $A$ is the amplitude of the movement; and $W$ is the target width. The term of $\log_2 (2A/W)$ is called the index of difficulty.

The index of difficulty (ID) is the most important factor in Fitts’ law to determine the average movement time. According to the equation, the higher the value of ID, the longer the movement time and, therefore, the speed of movement will decrease. Thus, Fitts’ law explains the speed-accuracy trade-off by implying an inverse relationship between the movement difficulty and the movement speed. For instance, participants spend more time to achieve the task goal if amplitude becomes longer, target width becomes narrower, or both with manipulations.

Fitts explained the relation between movement speed and accuracy in relation to the limited information processing capacity of the human system (Fitts 1954). When the number of stimulus response alternatives increases, the system needs more time to process this information and resolve the uncertainty about alternatives (Fitts, 1954; Fitts & Peterson, 1964; Schmidt & Lee, 2005). Fitts’ law can be applied to a variety of contexts of movements, including everyday activities. This law also has been conducted under different situations, such as using feet, arms, hands, movements conducted underwater, and different populations (young and old) (e.g., Goggin & Meeuwsen, 1992; Langolf, Chaffin, & Foulke, 1976; Welford, 1969). In this way, Fitts’ law describes the speed-accuracy trade-off, and the performer’s capability to change their performance so that speed and accuracy are kept in an appropriate balance to achieve different task goals.
However, there are some reasons to indicate the limitations to the generality of Fitts’ law as a robust description of the movement speed-accuracy relationship. For example, as regard to its consistency, it has been found that the speed and accuracy relationship fails at very low IDs (e.g., Crossman & Goodeve, 1983; Klapp, 1975). A more important limitation of his paradigm is that movement time is a dependent variable rather than an independent variable. This factor eliminates the potential concomitant measure of temporal error into the consideration of movement accuracy (Hancock & Newell, 1985).

**Logarithmic and linear relation in speed-accuracy trade-off**

Fitts (1954) originally proposed that the movement time to complete an aiming movement to a target was a logarithmic function of the amplitude moved divided by the target width. The values of amplitude and target width were varied experimentally by changing the setup of the target board for different trial blocks and the resulting movement time was measured after subject’s completed a trial. According to the Fitts’ equation, when different combinations of amplitude and target width were used in this paradigm, the average movement time falls almost perfectly on a straight line. We should notice that the value of $\log_2 (2A/W)$ determines the time required for the aiming movements, so this index is related to the difficulty or how much information as a minimal requirement should be processed of the particular combination of amplitude and target width for the participant. In fact, the relationship is even more complicated than this, because the difficulty of the movement condition is theoretically the same for any combination of amplitude and target width that has the same ratio.

Fitts’ law implies an inverse relationship between the difficulty of a movement and the movement time with which the subject can perform. Increasing the index of
difficulty increases the movement time, in other words, the individual in some way
“trades off“ speed versus accuracy so that the rate of information processing can be held
constant. Although there has been considerable empirical support for the logarithmic
explanation suggested by Fitts (1954), the generality of this as a description of the
relation of movement speed and accuracy has been questioned.

Schmidt, Zelaznik, Hawkins, Frank, and Quinn (1979) proposed a different
relationship between speed and accuracy trade-off. From their aimed hand movement
paradigm, the relation between endpoint spatial variability (effective target width, \(W_e\))
and average movement velocity is linear. This linear speed-accuracy relation has been
replicated in a number of experiments (e.g. Wright & Meyer, 1983; Zelaznik, Mone,
McCabe, & Thaman, 1988).

According to their revised paradigm that used rapid single-aiming movements, they
instructed subjects to complete their movements not only in a particular goal movement
time specified by the experimenter rather than moving as quickly as possible, but also
aimed at a small target line that did not change in its width. In this case, both temporal
accuracy and spatial accuracy were required of subjects. An assumption of their model
of the speed-accuracy relation in aiming movements is that it did not include a feedback
corrective process. The model was based on the assumption that variability in the
muscular forces used to drive the effector toward the target increased proportionally with
the absolute force required for the movement.

There have been three hypotheses to account for the possible different relationships
for speed and accuracy. These three hypotheses have been termed as movement brevity,
feedback deprivation, and temporal precision (Wright & Meyer, 1983). First, the
movement brevity hypothesis holds that the linear speed and accuracy trade-off is observed using single aiming movement tasks where the criterion time is limited so that participants do not have time to detect (use visual feedback) an error and to issue a correction. Thus, according to this hypothesis, the linear trade-off relation occurs when there is an inability to use feedback control processes whereas the logarithmic trade-off relation occurs when it is partially governed by feedback control processes. Wright and Meyer (1983) showed that participants’ movement time when longer than the estimated 250ms minimal visual feedback processing time represented a linear relation between movement speed and accuracy.

Second, as a general version of the previous hypothesis, the feedback-deprivation hypothesis states that the linear speed-accuracy trade-off would occur when the relevant sensory feedback (visual or kinesthetic) is neither available nor used during the course of movement execution. The logarithmic trade-off would be observed if the feedback is processed to make corrections during the course of movement (Zelaznik, Shapiro, & McClosky, 1981). The third explanation is the temporal precision hypothesis. It holds that the difference between the logarithmic and the linear speed and accuracy trade-off is the intended criterion movement time. The linear relation occurs when participants’ movement time must be precisely controlled. The high temporal precision constraint causes a participant to produce a goal directed movement with a bell shape velocity profile. The single aiming paradigm design controls movement time (time-matching task) and is very different than in the Fitts’ paradigm where the movement time goal is to move as quickly as possible while maintaining a 95% successful hit rate (time-minimization task). Some evidence shows that when the temporal precision requesting the participants
to achieve a particular movement time goal is low, it diminishes the linear relation of movement speed and accuracy (Wright & Meyer, 1983; Meyer, Smith, Kornblum, Abrams, & Wright, 1990; Zelaznik et al., 1988).

Although evidence supports that there are two functions for the speed and accuracy trade-off, it is the case that the previous studies that showed the different speed-accuracy functions emerged due to the presence of different task constraints. Indeed, Carlton (1994) investigated the relationship between time-minimization and time-matching tasks. When both functions were compared, his results suggested that performing aimed movements under time minimization constraints resulted in shorter movement time, but similar movement variability compared with time-matching task (e.g., Zelanik et al., 1988). However, we would argue that not all spatial and/or temporal constraints were controlled inviting their interpretation to be a reflection of different phenomena.

**Speed-accuracy trade-off and temporal error**

Most research of the speed-accuracy trade-off relation is involved in the context of movement spatial error that is determined relative to a particular movement amplitude and target size regardless of the logarithmic trade-off relation or linear trade-off relation. In fact, the view is widespread that when we increase movement speed there is a decrease in movement spatial accuracy. However, humans generate movement in both space and time that gives the potential of a movement temporal accuracy to be measured in certain tasks.

Movement speed timing accuracy tasks have been used to examine different theoretical principles in the field of motor learning. Timing error is the difference between the criterion movement time and the observed movement time that the
participant produces for a given trial. In movement timing tasks, a consistent finding is
that temporal error decreases when average movement velocity increases (Ellis, Schmidt,
& Wade, 1968; Kim, Carlton, Liu & Newell, 1999; Newell, Hoshizaki, Carlton, &
Halbert, 1979; Newell, Carlton, Carlton, & Halbert, 1980). As a consequence, the
increment of movement velocity within the criterion movement time leads to a decrement
of movement temporal error rather than increment of error as established for movement
spatial error. Thus, there is a paradox in the effect of movement speed on movement
accuracy when both spatial and temporal dimensions are measured (Newell, 1980).

Previous literature has used movement timing tasks but had the limitation that the
independent variables, such as manipulations of duration and velocity, have been
confounded (e.g., Ellis et al., 1968). In this case, movement time has rarely been
manipulated to average movement velocity independently. Due to this, it is commonly
the condition that short movement time has high velocity and long movement time has
low velocity. The findings from early studies suggested that manipulated movement time
and velocity systematically showed that timing accuracy was affected by movement
velocity but little was made of time effect of these manipulations.

Newell et al. (1979) examined systematically movement speed timing accuracy
function over range of movement velocity from 4 to 150 cm/s. Their results suggested
that mean absolute error movement time percentage produced little or no difference
between different times at the same average velocity. In addition, timing accuracy
decreased with longer movement times and slower average velocities. The velocities
effect was independent of movement time, and suggested that average velocity is a main
factor to determine the temporal accuracy in discrete timing movements.
In line with the movement timing accuracy provided by Newell et al. (1979), the relative movement temporal error function represents to be isomorphic with the spatial error function. There are studies available to support the general timing function has an equal importance as the general spatial function (Hsieh et al., 2013; Lai et al., in press; Newell, Carlton, & Carlton, 1982). Thus, in order to completely understand and interpret the function of speed and accuracy trade-off, we should consider examining the complementarity of temporal and spatial measures in the speed-accuracy trade-off phenomenon either as a function of the other or integrated in a single measure.

**Speed-accuracy trade-off and space-time perspective**

The research on human movement accuracy has focused on the source of movement error and the explanation of the speed and accuracy relation. It is significant that human movement always takes place in both space and time dimensions. Nevertheless, most previous research of the movement speed and accuracy relation considers movement error on only one of these dimensions. Movement cannot be constrained only by spatial or temporal criteria, as movement tasks always have potential spatial and temporal boundaries. Accordingly, the influence of movement speed on movement spatial and temporal accuracy has provided a challenge as to how a task should be performed optimally in terms of the relation between spatial and temporal error when both space and time are task criteria (Hancock & Newell, 1985; Newell, 1980; Newell, Carlton, Kim, & Chung, 1993).

Hancock and Newell (1985) proposed a comprehensive space-time description of the movement speed-accuracy relation. Theoretically, human movement takes place in both dimensions and it is based on the space-time principle that the spatial component of
movement is always measured with respect to time and that the temporal component of movement is always measured with respect to space (Minkowski, 1908). When movement tasks determine that the temporal and spatial errors are in the same plane of motion, the error distributions in both dimensions affected by movement speed are complementarity. We have to carefully consider how the contributions of space and time to movement errors are calculated when invoking the space-time principle in the human behavior domain. This space-time account of movement accuracy is most strongly relevant in tasks where both spatial and temporal dimensions of movement are task criteria (e.g., Hsieh et al., 2013; Kim et al., 1999; Newell, 1980; Zelaznik et al., 1988).

In Woodworth’s (1899) dissertation, he independently manipulated 4 movement amplitudes and 10 movement durations in a line drawing task and examined over 125,000 trials to provide a comprehensive description of voluntary movement accuracy. His observations interpreted how the interaction between amplitude, movement duration and movement velocity determines the movement error. Hancock and Newell (1985) reviewed his results, re-interpreted this intricate relationship and proposed a hypothetical set of relations between movement duration, amplitude and standard error across the full range of the movement velocity continuum. Specifically, they stated that the ratio variance by the criteria (distance for spatial variance and time for temporal variance) – the standard unit error (SUE) – would show the same “curve-shape” when plotted as a function of average velocity. That is, this hypothesis states an isomorphism between time and space in terms of variance.

Hsieh, Liu, Mayer-Kress and Newell (2013) introduced a space-time weighting analysis function that affords the potential to assign the relative contribution of spatial
and temporal factors to the determination of performance outcome. This weighting strategy provides a different way to systematically change the spatial and temporal constraints in the study of the speed and accuracy relation. In other words, it confines the performance outcome to particular regions of the full range of the movement velocity continuum. Their results showed that an integrated single space-time performance score revealed a new U shaped relation of speed and accuracy from the concept of space-time (Hancock & Newell, 1985; Hsieh et al., 2013; Newell, 1980).

The integrated space-time performance score showed a U-shaped function for the relation of the movement speed and accuracy trade-off. The performance score and the within-subject variability were lowest when the contribution of space and time to the movement error had equal weighting. When the weighting of space and time became asymmetric with either spatial or temporal emphasis, the integrated performance score and its variability increased. Hence, the U-shaped speed and accuracy trade-off function is not only different from the traditional point of view for spatial and temporal error, but also can be mapped to them when considered independently (Hsieh et al., 2013; Liu, Hsieh, & Newell, 2013). Nevertheless, it can be argued that the feedback manipulation may have driven the behavior to show the observed shape. Thus, their findings inspired us to find a better candidate to unify the space-time property of the movement speed and accuracy relation (Lai, Hsieh & Newell, in press).

**Speed-accuracy trade-off and impulse variability**

In order to understand the factors that influence movement accuracy in terms of the space-time principle, we should characterize performance outcome by using kinematic and kinetic data that can lead to a description in different measurement categories.
Schmidt et al. (1979) proposed the impulse variability theory based on the motor programming view of control that the velocity of movement is the product of muscular torques acting on joints that move the limbs through space. They proposed that force generated by muscular contraction results in movement about a joint, and that kinematic variability in movement outcome could be directly related to the kinetic variability in force dynamics.

Two principles are critical. The first is that the variability in the duration of muscular contraction is proportional to the movement duration. Second, that the variability in peak force produced decreased about 65% of the maximum peak force achievable for subjects (Carlton & Newell, 1993; Schmidt & Lee, 2005; Sherwood & Schmidt 1980). Any movement errors in the movement trajectory of the limb are a result of variability in the initial muscular impulse utilized to generate the action (Carlton & Newell, 1993; Newell, Carlton, & Hancock, 1984; Urbin, Stodden, Fischman, & Weimar, 2011; Schmidt et al., 1979). To better understand a fundamental phenomenon of impulse variability, we need to know which factors are composed of the impulse.

What is impulse? Impulse is the integral area of a force and time curve. The area under the force-time curve can be implied as the aggregate of accelerative forces that act in the desired direction of the movement (Schmidt et al., 1979). The impulse duration is the duration between the initiation of force produced and the time of force termination. Based on the Newton's second law (Force = Mass x Acceleration = Mass ∆ Velocity/∆ Time), we can understand that the velocity of limb at the end of its acceleration is proportional to the impulse for acceleration when mass of the limb is constant. Hence,
any variables that influence the magnitude of force, or the time over which it acts would also influence the velocity of the movement at the end of acceleration.

Schmidt et al. (1979) suggested that variability in the impulse predicts movement spatial error and is proportional to impulse. In contrast, temporal error has been proposed to be uninfluenced by an increase in the movement amplitude and average velocity within a given movement time. However, this result was not generalized by the subsequent research and the proportional relation might appear only through using a limited range of the potential conditions for a given motor system (Carlton & Newell, 1993; Newell et al., 1979; Newell et al., 1980; Newell et al., 1993). Another fundamental problem is that their experimental design did not control the time over which participants were allowed to achieve their maximum force (Sherwood & Schmidt, 1980; Schmidt & Sherwood, 1982; Schmidt et al., 1979).

Carlton and Newell (1985) argued that the variability of movement error considered in the space dimension or time dimension separately has been shown to be a consequence of the rate of force production and the temporal properties of the impulse. For instance, holding the time to peak force constant while increasing peak force results in a co-variation in the initial rate of force production in the impulse or vice versa. This relation leads to a consideration of the relative contributions of force and time to movement error when applying the space-time viewpoint in human movement. In addition, their results suggested that even though the time to peak force was fixed in the experiment (200ms), the timing error still deviated from the rate required implicitly by task demands when the percentage of force increased.
A major contribution of the Schmidt et al. (1979) impulse variability theory for movement speed and accuracy trade-off is that it linked the kinematic data to the observed kinetic data. Nevertheless, the comprehensive relation between variability of impulse and the concomitant movement error has not been fully established. The reason is that there has not been a direct measure of impulse on the movement speed-accuracy relation in the experimental work in part because several features of the impulse tend to co-vary when the size and the shape of the initial force-time curve also change (Carlton & Newell, 1993; Newell & Carlton, 1985, 1988). Previous studies have provided considerable evidence of the non-proportionality between peak force and variability of peak force (Carlton & Newell, 1985; Newell & Carlton, 1985; Kim et al., 1999; Newell et al., 1982; Sherwood & Schmidt, 1980; Schmidt & Sherwood, 1982).

After Schmidt et al. (1979), Carlton and Newell (1993) also proposed that a complete description of variability at the kinetic level and kinematic level can be linked and used to interpret variability in movement space-time tasks (Kim et al., 1999). We clearly know that variability in the motor system can be examined at several levels. Although most of the research about movement variability is focused on the movement outcome or movement pattern, this can be related to variations in force production that are influenced by factors, such as the state of the muscle at the time activated, excitability of motor neurons, and motor signals from higher nervous centers.

We are still not clear what the primary source of motor variability is, even though we know that regulation of force is a critical function in the human muscular system (e.g., Evarts, 1968). Based on the concept that the velocity of movement through the space is an emergent property of muscular force acting on the joint and the duration of this force
acted. It has been suggested that the force-time framework for movement accuracy, a measure of movement error in both force and time dimensions, is required to analyze impulse variability rather than the independent measures of the force or time dimensions (Carlton & Newell, 1993; Hancock & Newell, 1985; Newell, 1980; Newell et al., 1984).

**Information Entropy**

The space-time principle led to the proposition that a measure of movement error in the dimensions of space and time to the analysis of movement variability is required rather than only a measure of spatial error or temporal error alone (Hancock & Newell, 1985; Newell, 1980). The construct of information entropy is a candidate to unify the space-time property of the movement speed and accuracy relation (Lai, Hsieh & Newell, in press). Originally, the concept of entropy referred to the amount of energy that was inaccessible to the work in thermodynamics (Williams, 1997). Shannon (1948) applied this concept of information entropy to the development of information theory of communication and inferred that high information entropy is parallel to high levels of uncertainty in a system.

The concept of information entropy has been used in numerous disciplines of the physical and life sciences (Cover & Thomas, 1991). Although information entropy is a straightforward and useful index that corresponds to the content of information in models of human information processing, its use as a reflection of variability in the motor system has not been applied broadly in the motor control domain (Lai, Mayer-Kress, Sosnoff, & Newell, 2005; Lai, Mayer-Kress, & Newell, 2006; Lai et al., in press).

Probabilities are the foundation of information entropy. In order to analyze discrete aiming movements by using information entropy, we need to know the probabilities of
the data distribution at each time point in the movement outcomes or movement trajectories. Most previous research has relied on normal distributional indexes of variability to describe the relationship between the functions for movement speed and accuracy, such as variable error (VE) and coefficient of variation (CV). The measure of standard deviation is based on the properties of a data distribution, and in a normal distribution it has a particular role in capturing the dispersion of deviations from the mean. The coefficient of variation provides a relative measure of variability to the spatial-temporal properties of the movement; it is also useful for comparing variability over different task conditions.

However, there are several assumptions to the use of standard deviation. A significant problem is that the assumption of a normal distribution should hold for the data to be analyzed, however, this is not always the case in most conditions of the speed-accuracy relation (Hancock & Newell, 1985; Newell, Carlton & Hancock, 1984). There are deviations from the properties of a normal distribution over the potential spatial and temporal constraints and systematic changes in skewness and kurtosis as a function of the spatial-temporal criteria of the movement task (Kim et al., 1999; Lai et al., 2006; Newell, Carlton, & Kim, 1994; Newell, Carlton, Kim, & Chung, 1993). The deviation from a normal distribution is significant because it has been found that changes in the higher order third and fourth moments that determine skewness and kurtosis can change the estimate of the standard deviation even when the dispersion of the data in the distribution remains unchanged (Hancock & Newell, 1985; Newell, Carlton & Hancock, 1984).

Lai et al. (2005) examined the variability of movement outcome in relation to the direct measure of the probabilities of the performance outcomes. Using the measures of
information entropy, their results showed that the distribution under different spatial-temporal constraints at points of the movement trajectories and performance outcomes were not normally distributed. Their study provided evidence that the entropy estimates of performance outcomes as a function of movement speed and accuracy conditions show a different property than that produced by traditional distributional estimates.

The method to obtain the probabilities in the experimental data is to utilize properties of the actual frequency distribution. The entropy estimated from actual frequency distribution of data ($H_p$) can be calculated with the following equation:

$$H_p = \sum P_i \log_2 \left( \frac{1}{P_i} \right)$$  

Where $P_i$ is defined in the frequency distribution, indicating the frequencies of data points in each bin $i$. It has been shown that probabilities through information entropy measures provide a dimensionless alternative measure of the variability of movement outcome (Lai et al., 2005; Lai et al., 2006). One advantage of information entropy measures is that they focus on probabilities of the movement outcome irrespective of the measured movement dimensions.

Lai, Hsieh and Newell (in press) investigated a unified spatial and temporal error measurement of the probabilistic estimates of movement outcomes that can be implemented even though the units for assessing movement error in spatial and temporal error are different. The measure is unified space-time entropy because it considers the joint probability structure of spatial and temporal movement error (Lai et al., in press; Williams, 1997; Scott, 1992). They examined the integrated spatial and temporal error variables of discrete aiming movements into a single joint probabilistic space-time measure to understand how different task goals, movement strategies, and movement
space-time constraints influenced this integrated information entropy in contrast to their established effect on the traditional distributional single dimension analysis.

Their findings show that the integrated measure of spatial and temporal entropy score systematically changed under the task goal and instructional bias conditions, in a way that was not found in the consideration of the spatial and temporal error on an independent basis, but is consistent with considering the combined effects of the spatial or temporal error measured separately. The spatial, temporal and both instructional movement strategy emphasis in different groups led to the same level of space-time unified entropy when individual dimension (space or time) variability showed contrasting effects of movement velocity.

In summary, most of the studies to date have shown that the phenomena in speed-accuracy trade-off are focused on either the space or time dimension considered individually. In the space-time principle, when the human system produces movements, there is the potential relation between the spatial and temporal errors because each error is measured with respect to the other (Hancock & Newell, 1985; Newell, 1980). In fact, in interceptive movements such as baseball batting, it seems counter-intuitive to say the batter missed the ball by 5ms but only missed by 1mm so he does not need to worry about missing the ball. Since probability is the basis of information entropy (Shannon & Weaver, 1949; Scott, 1992; Williams, 1997), the theoretical focus of the dissertation is on the development of an integrated joint probability measure of the spatial and temporal error of performance outcome. The primary focus of the dissertation is to contrast and systematically examine the unified movement joint entropy to the traditional spatial/force
and temporal errors observed in the context of different space-time movement or force-time production criteria.
CHAPTER 3: ENTROPY OF SPACE-TIME OUTCOME IN A MOVEMENT SPEED-ACCURACY TASK

Abstract

The experiment reported was set-up to investigate the space-time entropy of movement outcome as a function of a range of spatial (10, 20 and 30 cm) and temporal (250 to 2,500 ms) criteria in a discrete aiming task. The variability and information entropy of the movement spatial and temporal errors considered separately increased and decreased on the respective dimension as a function of an increment of movement velocity. However, the joint space-time entropy was lowest when the relative contribution of spatial and temporal task criteria was comparable (i.e., mid-range of space-time constraints), and it increased with a greater trade-off between spatial or temporal task demands, revealing a U-shaped function across space-time task criteria. The traditional speed-accuracy functions of spatial error and temporal error considered independently mapped to this joint space-time U-shaped entropy function. The trade-off in movement tasks with joint space-time criteria is between spatial error and timing error, rather than movement speed and accuracy.
Introduction

The relation between movement speed and accuracy is one of the most robust phenomena in human movement. The essence of the speed-accuracy relation is that with an increase in movement speed there is concomitant decrease in movement accuracy. The outcome is that we often need to sacrifice movement speed for spatial accuracy to achieve the task goal that we want to realize. Expressed another way, movement accuracy can be increased by trading (reducing) movement speed – hence, the phenomenon of the movement speed-accuracy trade-off. The speed-accuracy relation has been an important topic in the field of motor control that has led to many theoretical accounts and empirical findings (Crossman & Goodeve, 1983; Elliott et al., 2010; Fitts, 1954; Hancock & Newell, 1985; Meyer, Smith, Kornblum, Abrams, & Wright, 1990; Plamondon & Alimi, 1997; Schmidt, Zelaznik, Hawkins, Frank & Quinn, 1979; Woodworth, 1899).

Human movement takes place in both space and time. It follows, therefore, that there is potential for both spatial error and temporal error in motor tasks (Brouwer, Smeets, & Brenner, 2005; Hancock & Newell, 1985; Newell, 1980; Schmidt et al., 1979). Indeed, in movement timing tasks, where the goal is to move through an amplitude in a given time there can be a measurement of timing error. In these conditions it has been shown that increasing movement velocity within the same movement time results in a decrease of movement timing error, rather than an increase in error as in a movement spatial accuracy task (Ellis, Schmidt & Wade, 1968; Kim, Carlton, Liu, & Newell, 1999; Newell, Hoshizaki, Carlton & Halbert, 1979; Newell, Carlton, Carlton & Halbert, 1980). Thus, the directional effect of the relation of movement speed and accuracy is a function
of whether spatial or temporal accuracy is being measured opening the idea that a paradox exists in the effect of movement speed on accuracy (Newell, 1980).

Consider when a performer tries to hit a baseball he/she does not only need to know where the ball will be, but also when the ball will be in a certain location. Nevertheless, most previous research of the movement speed and accuracy relation accounts for movement error on only one of these dimensions. Even in closed skills, movement is not constrained only by spatial criteria as movement tasks always have spatial and temporal boundaries. It is the case that extant accounts of the speed-accuracy relation have focused largely on movement spatial error (Fitts, 1954; Woodworth, 1899) and where temporal error has also been measured, it has been considered independently of spatial error (Schmidt et al., 1979). Accordingly, the influence of movement speed on movement spatial and temporal accuracy has provided a challenge as to how a task should be performed in terms of the relation between spatial and temporal error when both space and time are task criteria (Hancock & Newell, 1985; Newell, 1980; Newell, Carlton, Kim & Chung, 1993; Newell, Carlton & Kim, 1994).

Hancock and Newell (1985) proposed a space-time framework of the movement speed-accuracy relation that is based on the space-time principle that the spatial component of movement is always measured with respect to time and that the temporal component of movement is always measured with respect to space (Minkowski, 1908). When movement tasks determine that the temporal and spatial errors are in the same plane of motion, the error distributions in both dimensions affected by movement speed are consonant. This reflects the need to carefully consider how the contributions of space and time to movement errors are calculated when invoking the space-time principle in the
human behavior domain. The space-time account of movement accuracy is most strongly relevant in tasks where both spatial and temporal dimensions of movement are task criteria (e.g., Hsieh, Liu, Meyer-Kress & Newell, 2013; Kim et al., 1999; Newell, 1980; Zelaznik, McCabe, Mone & Thaman, 1988). The variability of movement error considered in the separate dimensions of space and time has been shown to map to the rate of force production and the temporal properties of the impulse (Carlton & Newell, 1993; Kim et al., 1999; Schmidt et al., 1979).

A fundamental question that naturally follows is - if human action is reflected in the space and time of the movement, how does the participant integrate the individual dimensions of space and time into a single space-time property? In addition, how would a single space-time measurement relate to existing accounts of movement speed-accuracy trade-off? The space-time principle led to the proposition that a measure of movement error in the joint dimensions of space and time to the analysis of movement variability is required rather than only a measure of spatial error or temporal error alone (Hancock & Newell, 1985; Newell, 1980).

Hsieh et al. (2013) created a performance score as feedback that was an integrated and weighted product of spatial and temporal movement criteria. This measurement approach led to the finding of a new U-shaped function for movement speed and accuracy in contrast to the traditional accounts of the effect of movement speed on either spatial error or temporal error. Nevertheless, this approach used different weighting adding either on spatial or temporal components to see how performance influenced under different combination of space and time condition. It can be argued that the feedback manipulation may have driven the behavior to show the observed shape. Here
we investigated the construct of information entropy as another candidate approach to
unifying the space-time property of the movement speed and accuracy relation (Lai,
Hsieh & Newell, in press). Shannon (1948) applied the concept of information entropy to
the development of an information theory of communication and inferred that high
information entropy reflects high levels of uncertainty in a system. The use of entropy as
a reflection of variability in the motor system has, however, not been applied broadly in
the motor control domain (Cover & Thomas, 1991; Lai, Mayer-Kress, Sosnoff & Newell,

To obtain an estimate of entropy in the experimental data requires one to utilize the
probabilities of the actual frequency distribution. The entropy estimated from the actual
frequency distribution of data ($H_p$) can be calculated with the following equation:

$$H_p = \sum P_i \log_2 (1/P_i)$$  

(1)

where $P_i$ is defined in the frequency distribution of the dependent variable of interest,
indicating the frequency of data points in each bin $i$. It has been shown that probabilities
through information entropy provide a dimensionless alternative measure of the
used the information entropy approach in their study to avoid the mentioned problems.
Nevertheless, spatial entropy and temporal entropy still examined independently.

Lai et al. (in press) investigated a unified spatial and temporal error measurement of
the probabilistic estimates of movement outcomes that can be implemented even though
the units for assessing movement error in spatial and temporal error are different. The
measure is an unified space-time entropy because it considers the joint probability
structure of spatial and temporal movement error (Lai et al., in press; Williams, 1997;
Scott, 1992). It is anticipated that the movement variability in terms of a probabilistic space-time approach would be different from the variability measured in the traditional single dimension distribution analysis of spatial or temporal error.

The purpose of their first experiment in Lai et al. (in press) was to investigate how the joint entropy changed under different task goals (point-aiming and target aiming) and different strategies (space-emphasis, time-emphasis, or both). In the Experiment 2, they used the data from Hsieh et al. (2013) to see if using a different approach to integrate space and time would show the same U-shape. Nevertheless, it is possible that data was already biased by the weighting feedback (as discussed previously).

Danion, Bongers, and Bootsma (2014) manipulated movement time during reciprocal aiming tasks to examine how spatial variability and temporal variability vary with movement time. Their findings showed a strong negative correlation between spatial and temporal variability across different movement durations (both SD and CV). Although they characterized a trade-off between space and time dimension, the influence of different spatial or temporal criteria is still uncertain. Newell and colleagues (Hancock & Newell, 1985; Newell, 1980) has been proposed that in dual space-time criterion tasks, participants trade spatial error for timing error, rather than the traditional perspective of speed for accuracy.

In the experiment reported here we investigate the joint entropy of spatial and temporal error in a discrete aiming task over a range of amplitude and time criteria. Given the preliminary studies reported above it is hypothesized that movement velocity in the mid parameter range of spatial-temporal constraints will lead to the minimum of a joint entropy of the space-time outcome (Hsieh et al., 2013; Lai et al., in press; Newell,
1980). The finding of a U-shaped function for entropy of the space-time outcome would contrast with the speed-accuracy functions of the movement outcome measured independently in either the space or time dimension (Crossman & Goodeve, 1983; Fitts, 1954; Plamondon & Alimi, 1997; Schmidt et al., 1979; Woodworth, 1899). It is anticipated, however, that the traditional speed-accuracy functions for spatial error and temporal error considered independently can be mapped to this space-time U-shaped function.

**Methods**

**Participants**

Twelve self-reported right-handed healthy young adults (6 males and 6 females) who volunteered for the experiment. The mean age of the participants was 28.17 (range ± 3.58) years. Participants provided informed consent and the experimental procedures were approved for compliance through the policies of the Institutional Review Board of Penn State University.

**Apparatus**

A Wacom Cintiq 21UX digital tablet (Model DTZ-2100D, 561 x 421 x 61.3mm with an active surface area of 432mm x 324mm) was connected to a PC computer (the pixel range was set at 800 x 600) and used for data collection (see Figure 3.1). A handheld, cordless stylus (Model ZP-501E) with a weight of 18g was used with the digital graphic tablet. A custom computer discrete aiming program was used to present different movement time goals and different amplitudes in space-time and calculate the spatial error and temporal error for the participants immediately after each discrete aiming trial. The distance moved by the cursor represented on the screen to the actual
distance moved by the stylus on the digital board was 1:1. The sample frequency of the kinematics from the customized program was 130Hz.

![Figure 3.1. A schematic of the speed-accuracy aiming task with the target width (1mm) and movement amplitude (20cm).](image)

**Experimental design**

There were 3 different movement amplitudes (10, 20, and 30cm) between the two targets (2mm in diameter for the start point and 1mm in diameter for the target point). The criterion movement times ranged from 10cm (fast: 250ms, fast-middle: 300ms, middle: 550ms, middle accurate: 1000ms, and accurate: 1300ms), 20cm (fast: 300ms, fast-middle: 450ms, middle: 650ms, middle accurate: 1500ms, and accurate: 2000ms), and 30cm (fast: 350ms, fast-middle: 550ms, middle: 750ms, middle accurate: 1800ms, and accurate: 2500ms) for each condition. The task was to move a stylus from left (home position) to stop on the right (target position) in the criterion time. The digital tablet was positioned at the middle in front of the participant’s body.
Each participant completed 3 distance x 5 time conditions each with 100 trials of a discrete aiming task. These 15 conditions were labeled as fast, fast-middle, middle, middle-accurate, and accurate conditions over the 3 different amplitudes. It took approximately 1 hour to complete the 5 space-time conditions on each day. Each subject attended the laboratory for 3 days to complete the 15 testing conditions. The order of the 5 conditions within day and the order of amplitudes over days were randomly determined for each participant.

**Procedures**

The participant sat on a chair of standard height for working at a desk. Before the start of a trial, the home position (2mm in diameter) and target positions (1mm in diameter) were shown on the digital tablet. The participants were instructed to match the designed criterion time as accurate as possible and also be as accurate as possible to hit a center of target. The trajectory of the stylus was not shown on the board when performing the task except the cursor that is always visible during the whole trial. The algebraic temporal and algebraic spatial errors from the respective task criterion were each presented numerically on the computer screen immediately (< 2s) as information feedback after the completion of each trial. Participants were instructed that their performance (movement time and spatial error) should each be as close as possible to the space-time task criteria (temporal error and spatial error should be zero).

The participant picked up the stylus to touch the digital tablet. A cursor (1mm in diameter) showed up on the screen. Then, the participant moved the cursor onto the home position on the screen. A beep sound was given once the participant had the stylus to touch the tablet on the home position for around 1s. This was not a reaction time
experiment and the participant was instructed not to respond to the beep sound as fast as possible. He/she was to begin each trial when they were comfortable and ready after beep sound. The initiation of movement was defined by the stylus crossing the low velocity threshold of 3mm/s and stayed above that threshold for 30ms (6 frames). The stylus was to remain in contact with the tablet during the movement until the trial was completed. The trial was finished when the stylus touched the target position or the stylus came to a stop. The movement stop was defined by the stylus leaving the tablet surface or the velocity of the stylus being below 3mm/s for greater than 40 ms (8 frames).

The next trial started as soon as the participant was returned the stylus back to the home position to start the next trial. A 3-5 min break was provided after each 100 trials.

**Derivation of the space-time task criteria**

Prior to the experiment proper a pilot study of the discrete aiming task with a variety of task emphases on speed and accuracy was conducted to determine the space-time conditions of the actual experiment (see also Hsieh et al., 2013). In the pilot procedure, 3 additional participants acted as pilot participants to establish the range of space-time conditions for the experiment. These pilot participants performed 350 trials of the discrete aiming task with different emphases on spatial accuracy and movement time on each trial. For example, to move as fast as possible while aiming for the target, or to move a little slower while aiming for the target, and to move to the target as accurate as possible. All pilot procedures were the same for the 3 different amplitudes (10cm, 20cm, and 30cm).

After 350 pilot testing trials, we were able to observe the standard function of space-time performance under different speed-accuracy conditions. The data distribution of
movement time and spatial error revealed that when a longer movement time was emphasized the magnitude of spatial error was decreased, and vice versa.

We normalized the raw pilot data by using the longest movement time and largest spatial error (10cm, MT:2624ms, and SE: 10.97mm; 20cm, MT:3880ms, and SE: 20.98mm; 30cm, MT:4785ms, SE: 20.15mm). To determine the conditions of the experiment proper we used 5 slopes of the tangent lines that were spread over the speed-accuracy parameter range and close to the fast (10:1), fast middle (5:1), middle (1:1), middle accurate (1:5) and accurate (1:10). At the middle condition, the 1:1 coefficient was derived from the tangent of 45 degrees to the speed-accuracy fitting curve (pilot data) that provided equal emphasis to the contribution of movement time and spatial error. After we found the positions where the slopes were close to the determined goal values we used the corresponding movement times as the bases for the criterion of different space-time conditions that covered the range of parameter conditions.

**Data Analysis**

For each trial the algebraic error for time and space on each dimension considered independently was recorded. The spatial, temporal and joint space-time entropy was calculated in terms of probability based on an analysis of the frequency distribution of the respective movement outcome. The number of bins for the entropy analysis, based on previous analyses, was set at 20 for both the spatial and temporal data, with the last 90 trials analyzed in each of the respective dimensions. The performance outcome entropy was the summation of the spatial and temporal measures in terms of the joint probability (Williams, 1997).
To examine the distribution of the different space-time conditions, we analyzed the distribution of temporal and spatial errors of each participant by using the maximum values to normalize the temporal and spatial errors. Using normalized data, we fitted an ellipse to the distribution (85% of the raw data lie inside of the ellipse), and calculated the main angle of the ellipse to observe how it changed across different space-time conditions. All angles above 90 degrees were mirrored to situate them from 0 degrees to 90 degrees (by subtracted from 180 degrees) to make it easier for the interpretation of how the main angle of the ellipse changed.

Repeated measures ANOVAs were used to examine the effect of the space-time task movement conditions on each dependent variable. The Greenhouse-Geisser method was used to correct for violations of sphericity and the Bonferroni correction was applied for the post hoc comparisons, with eta square ($\eta^2$) (Green & Salkind, 2003) revealing the effect size. In those cases where the normal distribution assumption was not fulfilled in the data we used Friedman’s ANOVAs to analyze the effects. Repeated paired Wilcoxon tests corrected by the Holm's Sequential Bonferroni Procedure (Abdi, 2010) were used as non-parametric post-hoc tests. The effect sizes were Kendal’s W for Friedman’s ANOVA.

**Results**

**Distributional analyses**

**Spatial error**

*Spatial Constant Error.* Figure 3.2 (left column) shows spatial constant error (CE), variable error (VE), and information entropy of the discrete aiming task as a function of
the different space-time conditions. The black and grey bars indicate the different amplitudes (10, 20, and 30cm), respectively.

Figure 3.2. Left column. Spatial constant error (CE), variable error (VE), and entropy of the line drawing task as a function of the different space-time conditions. The black and gray bars indicate the different amplitudes (10, 20, and 30cm), respectively. Right column. Temporal constant error (CE), variable error (VE), and entropy of the discrete aiming task as a function of the different space-time conditions. The black and grey bars indicate the different amplitudes (10, 20, and 30cm), respectively. The error bars denote the 95% confidence interval.
The 2 way (3 amplitude x 5 space-time conditions) repeated measures ANOVA on spatial constant error showed a significant effect for space-time conditions, $F(1.35, 17.27) = 17.08, p < 0.01, \eta^2 = 0.61$. The post-hoc paired comparisons on spatial constant error showed that all of the conditions were significantly different from each other, except the fast-middle from middle, middle-accurate and accurate conditions, and middle-accurate from the accurate conditions. However, there was no amplitude effect, $F(2, 22) = 2.27, p > 0.05$, and the interaction between space-time condition and amplitude, $F(2.01, 22.15) = 0.38, p > 0.05$ for spatial CE was not significant.

**Spatial Variable Error.** The space-time condition effect was significant, $F(1.17, 12.83) = 70.43, p < 0.01, \eta^2 = 0.87$ for spatial VE. The post-hoc paired comparisons of the spatial variable error showed that all of the space-time conditions were significantly different from each other. However, there was no amplitude effect, $F(2, 22) = 1.325, p > 0.05$, and the interaction between the space-time condition and amplitude, $F(1.99, 21.86) = 1.54, p > 0.05$ for spatial VE was not significant.

**Information Entropy of Spatial Error.** The information entropy of spatial error showed large deviations from a normal distribution and thus we used non-parametric tests (see methods section). For all amplitudes, the condition effect was significant, 10 cm: $\chi^2(4) = 43.364, p < 0.001, W = 0.90$; 20 cm: $\chi^2(4) = 45.600, p < 0.001, W = 0.95$; 30 cm: $\chi^2(4) = 45.540, p < 0.001, W = 0.94$. The post-hoc analyses showed that all space-time conditions were significantly different from each other for both 10 and 20cm, except middle-accurate and accurate conditions. For 30 cm, all space-time conditions were significantly different from each other. Considering each condition, the amplitude effect was significant for middle-accurate, $\chi^2(2) = 8.000, p = .018, W = 0.33$ and accurate
conditions: $\chi^2(2) = 7.787, p = .020, W = 0.32$. The post-hoc analyses showed that the 10cm was significantly different from the 20 and 30cm.

**Temporal error**

*Temporal Constant Error.* Figure 3.2 (right column) depicts the temporal constant error (CE), variable error (VE), and entropy of the discrete aiming task as a function of the different space-time conditions. The 2 way (3 amplitude x 5 space-time conditions) repeated measures ANOVA on temporal constant error showed a significant effect for space-time conditions, $F(1.49, 16.35) = 26.7, p < 0.01, \eta^2 = 0.71$. The interaction between movement amplitude and space-time condition showed a significant difference for temporal CE, $F(8, 88) = 3.66, p < 0.001, \eta^2 = 0.25$. The post-hoc simple main effect analysis showed that temporal constant error of 10 cm was significantly different than the 30cm but only for the fast condition. Moreover, there were significant differences between all the conditions at all the amplitudes, except the middle from both middle-accurate and accurate conditions in the 10cm, fast-middle from both middle, and accurate conditions, middle from both middle-accurate and accurate conditions in the 20cm, and fast-middle from middle-accurate, middle from both middle-accurate and accurate in the 30cm.

*Temporal Variable Error.* For temporal VE, we found significant effects for space-time conditions, $F(4, 44) = 149.44, p < .001, \eta^2 = 0.93$; amplitude, $F(2, 22) = 35.65, p < 0.00, \eta^2 = 0.76$. In addition, the condition by amplitude interaction was also significant, $F(3.42, 37.71) = 7.19, p < .001, \eta^2 = 0.40$. The post-hoc simple main effect analysis showed that temporal variable error of 20 cm was significantly different than the 30cm for the fast condition, 10cm was significantly different than the 30cm for the fast-middle
condition, and all amplitudes were significantly different from each other for the middle-accurate and accurate conditions. Moreover, there were significant differences between all the conditions in all of the amplitudes, except the fast from both fast-middle and middle conditions, fast-middle from middle conditions, and middle-accurate from accurate conditions in the 10cm, the fast from both fast-middle and middle conditions, fast-middle from middle conditions in the 20cm, and also the fast from both fast-middle and middle conditions, fast-middle from middle conditions in the 30cm.

Information Entropy of Temporal Error. The 2 way (3 amplitude x 5 space-time conditions) repeated measures ANOVA on the information entropy of temporal error showed a significant effect for space-time conditions, $F(4, 44) = 162.81, p < 0.00, \eta^2 = 0.94$; and for amplitude, $F(2, 22) = 26.99, p < 0.00, \eta^2 = 0.71$. In addition, the condition by amplitude interaction was significant, $F(8, 88) = 2.67, p < 0.01, \eta^2 = 0.2$. The post-hoc simple main effect analysis showed that temporal entropy of 10 cm was significantly different from the 30cm for the fast condition, 10cm was significantly different from the 30cm for the fast-middle condition, 10cm was significantly different from both 20cm and 30cm for the middle accurate conditions, and all amplitudes were significant different from each other for the accurate conditions. Moreover, there were significant differences between all the conditions at all amplitudes, except the fast from both fast-middle and middle conditions, fast-middle from middle conditions, and middle-accurate from accurate conditions in the 10cm, the fast from both fast-middle and middle conditions, fast-middle from middle conditions, and middle-accurate from accurate in the 20cm, and also the fast from both fast-middle and middle conditions, fast-middle from middle conditions in the 30cm.
**Ellipse distribution of space-time analysis**

We used the non-parametric tests (see methods section) on the ellipse distribution (see figure 3.3) of space-time conditions at different amplitudes. For all amplitudes, the condition effect was significant, 10 cm: \( \chi^2(4) = 41.733, p < 0.001, W = 0.87 \); 20 cm: \( \chi^2(4) = 34.27, p < 0.001, W = 0.71 \); 30 cm: \( \chi^2(4) = 41.40, p < 0.001, W = 0.86 \). The post-hoc analyses showed that all space-time conditions were significantly different from each other at 10cm, except the fast from fast-middle conditions. For 20 cm, all space-time conditions were significantly different from each other, except the fast from fast-middle conditions, the fast-middle from middle conditions, and the middle-accurate from accurate conditions. For 30cm, all space-time conditions were significantly different from each other, except the fast from fast-middle conditions, and the middle-accurate from accurate conditions. However, the amplitude effect was not significant for space-time conditions, \( p > .05 \).
Figure 3.3. Examples of data distribution for different space-time conditions from three participants, respectively. The different rows indicate different amplitudes (10cm, 20cm, and 30cm).

**Joint entropy space-time analysis**

Figure 3.4 shows the joint space-time entropy of the discrete aiming tasks as a function of the movement amplitudes. The 2 way (3 amplitude x 5 space-time conditions) repeated measure ANOVA on the joint space-time entropy showed a significant effect for space-time conditions, $F(4, 44) = 21.72, p < 0.00, \eta^2 = 0.66$, and for amplitude, $F(2, 22) = 4.34, p < .020, \eta^2 = 0.28$. Additionally, the interaction between condition and amplitude was significant, $F(8, 88) = 47.6, p < .001, \eta^2 = 0.81$. The post-hoc simple main effect analysis showed that joint entropy of 10 cm and 20cm were both significantly different from the 30cm for the fast condition, 10cm and 20cm were both significantly different from the 30cm for the fast-middle condition, 20cm was significantly different
from both 10cm and 30cm for the middle conditions, and 10cm and 20cm were both significantly different from 30cm for the middle-accurate conditions. All amplitudes were significantly different from each other for the accurate conditions. In addition, there were significant differences between all the conditions at all amplitudes, except the fast from both middle and middle-accurate conditions, and fast-middle from middle and accurate conditions in the 10cm, the fast from both fast-middle, middle and accurate conditions, and middle-accurate from accurate in the 20cm, and also the fast-middle from both middle and accurate conditions, middle from middle-accurate conditions in the 30cm.

To test the U-shaped function of the joint space-time entropy for the 5 different space-time conditions at 3 different amplitudes (10, 20 and 30cm), we fitted the joint entropy from the last 90 trials of each individual participant with a quadratic function: \( y = a + bx + cx^2 \) where \( y \) is the joint entropy and \( x \) is the logarithm of the weighting ratio between movement time and spatial accuracy. The \( R^2 \) values of the group mean fit results ranged from .94-.98 for the joint space-time entropy (see figure 3.4). Table 3.1 shows the \( R^2 \) values of the individual fit results for the joint space-time entropy.
Figure 3.4. The joint space-time entropy of the discrete aiming task at different space-time conditions. The quadratic function fit only for the joint space-time entropy from group mean. The logarithm of the weighting ratio was used as the independent variable, therefore $\log_{10}(1/10) = -1$, $\log_{10}(1/5) = -0.7$, $\log_{10}(1/1) = 0$, $\log_{10}(5/1) = 0.7$, $\log_{10}(10/1) = 1$ were used as the 5 values for the fitting variable. The error bars denote the 95% confidence interval.

<table>
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<th>Participants</th>
<th>10cm</th>
<th>20cm</th>
<th>30cm</th>
</tr>
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<td>0.76</td>
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</tr>
<tr>
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<td>0.70</td>
<td>0.95</td>
</tr>
<tr>
<td>P3</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
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</table>
Discussion

The experiment reported was set-up to investigate the space-time entropy of movement outcome as a function of a range of spatial (10, 20 and 30 cm) and temporal (250 to 2,500 ms) criteria in a movement speed-accuracy discrete aiming task. The goal-directed discrete aiming task has been central to experiment and theory of the movement speed and accuracy trade-off (Elliott et al., 2010; Fitts, 1954; Hancock & Newell, 1985; Plamondon & Alimi, 1997; Schmidt et al., 1979; Woodworth, 1899). However, typically in movement speed-accuracy trade-off experiments the task criteria have focused on either the spatial or temporal dimensions of movement error considered alone and/or independently.

In the current study, we integrated the individual spatial and temporal error data into a joint space-time entropy to investigate both dimensions in the same frame of reference. The joint space-time entropy revealed a U-shaped function for the movement speed and accuracy relation at all of the 3 movement amplitudes. The lowest entropy was located at the region where the relative contributions of space and time to movement outcome were comparable (see also Hsieh et al., 2013 with a different approach to space-time integration). When the spatial and temporal constraints increased the asymmetry in the influence of either the space or time dimensions the joint entropy increased. This was particularly apparent at the short movement time and long movement time conditions (see also Lai et al., 2006). This U-shaped space-time speed-accuracy relation is different from the traditional functions for both spatial (Fitts, 1954; Fitts & Peterson, 1964; Schmidt et al., 1979) and temporal (Newell et al., 1979) error considered independently.
Fitts (1954) used information theory to describe and explain how the movement time as a dependent variable changed according to different combinations of amplitude and target width (index of difficulty, ID). This approach assumes a fixed information transmission capacity of the motor system so that when ID increases the perceptual-motor system requires more time to process information to preserve accuracy leading to an increase in movement time. However, in the classic Fitts (1954) approach only spatial dimension of movement accuracy is considered, and even in this protocol the spatial error has often not been reported. Fitts (1954) did not discuss how the motor system processes information in the temporal dimension and thus does not explain changes in temporal error given different task constraints. Indeed, most of previous accounts of the movement speed and accuracy relation have ignored the influence of the time criteria on the resultant space-time movement error (Crossman & Goodeve, 1983; Fitts & Peterson, 1964; Wright & Meyer, 1983; Woodworth, 1899).

Our findings show that the space-time U-shaped function can be mapped to each of the respective errors on the individual dimensions of space or time. That is, the standard speed-accuracy relations are shown when they are considered independently in that increased velocity led to greater spatial movement error (Schmidt et al., 1979; Woodworth, 1899), but decreased movement error in timing (Ellis et al., 1968; Newell et al., 1979). With respect to the traditional distributional analysis, the findings showed that the space-time conditions influenced constant, variable and information entropy of both spatial and temporal errors. However, the temporal variable error and information entropy of temporal error were also influenced by movement amplitude. It seems that the movement amplitude contributed more influence to movement outcome in the temporal
dimension than to movement outcome in the spatial dimension. This finding suggests that movement temporal error is more easily influenced compared to the movement spatial error by the different space-time constraints and movement amplitudes (Hsieh et al., 2013; Lai et al., in press).

The U-shaped speed and accuracy function of space-time joint entropy shows that different spatial/temporal constraints are related to the trade-off in movement accuracy when the space and time dimensions of performance are integrated together. The error distributions were influenced qualitatively by the parameter range of the different space-time constraints on movement outcome. The movement outcome data cloud was elongated on the spatial dimension when the task criteria required participants to move fast (see figure 3.3). In this case, participants maintained temporal accuracy with high movement velocity in the temporal dimension but this resulted in decreased spatial accuracy, and vice versa when participants performed in the accurate condition. From a dynamic point of view, constraints would change the stability landscape channeling the intrinsic dynamics that has not been well established for discrete movement tasks. For instance, under dual (space and time) criteria there is a parameter combination where the joint entropy of movement outcome can be minimized and this emerges when the relative contribution of space and time constraints to movement outcome is comparable. Moving faster or slower from this critical relation increased the joint entropy on the space-time dimension (Hancock & Newell, 1985).

In this space-time perspective, the U-shaped function could be influenced by the cost and pay-offs associated with errors of space and time. For example, when we sacrifice speed to increase spatial accuracy we lose timing accuracy to jointly influence the
movement performance (Newell, 1980). In the case of considering both space and time dimensions together, the most efficient way to minimize movement error is to operate in the region where the intersection between the spatial and temporal entropy curves occur. To understand the general problem of speed-accuracy trade-off requires a combining of spatial and temporal measures into a space-time explanation of the movement relation between the speed and accuracy trade-off. The cohesion of the methodological approach developed and investigated in this study provides an alternative perspective and a more complete account from which to describe and explain the speed and accuracy trade-off phenomenon.

In summary, our approach to the joint entropy of space and time was based on the probability distribution between spatial and temporal errors under different space-time criteria. This allows the examination of a space-time approach to understanding the movement speed and accuracy trade-off relation. The theoretical approach developed and investigated in the current study emphasizes the movement speed and accuracy trade-off relation in movement aiming tasks that have explicit dual space and time criteria, but it holds general principles for the range of aiming tasks. The independent speed-accuracy functions for spatial and temporal error map to this U-shaped function of the entropy of movement space-time.
CHAPTER 4: FORCE-TIME ENTROPY OF ISOMETRIC IMPULSE

Abstract

The relation between force and temporal variability in discrete impulse production has been viewed as independent (Schmidt, Zelaznik, Hawkins, Frank & Quinn, 1979) or dependent on the rate of force (Carlton & Newell, 1993). Two experiments in an isometric single finger force task investigated the joint force-time entropy with: a) fixed time to peak force and different percentages of force level; and b) fixed percentage of force level and different times to peak force. The results showed that the peak force variability increased either with the increment of force level or through a shorter time to peak force that also reduced timing error variability. The peak force entropy and entropy of time to peak force increased on the respective dimension as the parameter conditions approached either maximum force or a minimum rate of force production. These findings show that force error and timing error are dependent and complementary when considered in the same framework with the joint force-time entropy at a minimum in the middle parameter range of discrete impulse.
Introduction

There have been many descriptions and explanations of the speed-accuracy trade-off through kinematic examinations of movement and its outcome (e.g., Fitts, 1954; Meyer, Smith, Kornblum, Abram, & Wright, 1988; Schmidt et al., 1979; Woodworth, 1899). However, the spatial and temporal variability in movement kinematics can also be understood in terms of the variability in force production of the neuromuscular system (Carlton & Newell, 1993; Fullerton & Cattell, 1892; Schmidt et al., 1979). Indeed, there have been a variety of theoretical perspectives developed to account for the relation between the magnitude of force generated by the muscular system and force variability (e.g., Carlton & Newell, 1988; Newell, Carlton, & Carlton, 1982; Urbin, Stodden, Fischman, & Weimar, 2011; Urbin, Stodden, Boros, & Shannon, 2012; Sherwood & Schmidt, 1980; Schmidt et al., 1979).

Schmidt et al. (1979) outlined the impulse variability theory based on the motor programming view of control in which the velocity of movement is the product of muscular torques acting on joints that move the limbs through space. They proposed that force generated by the muscular contraction results in movement about a joint, and that kinematic variability in movement outcome was directly related to the kinetic variability in the force dynamics. Error in the movement trajectory of a limb was interpreted as a result of variability in the initial muscular impulse utilized to generate the action (Carlton & Newell, 1993; Newell, Carlton, & Hancock, 1984; Urbin et al., 2011; Schmidt et al., 1979).

Schmidt et al. (1979) also proposed that variability in impulse is proportional to impulse and predicts spatial error in a movement-aiming task. In contrast, temporal error
was hypothesized to be uninfluenced by an increase in the movement amplitude or average velocity within a given movement time. However, this prediction has not been supported by subsequent research and the proportional relation might appear appropriate only through using a limited range of the potential space-time conditions for a given effector system (Carlton & Newell, 1985; Hancock & Newell, 1985; Kim, Carlton, Liu, & Newell, 1999). Another limitation was the failure to control the time over which participants were allowed to achieve their maximum force (Sherwood & Schmidt, 1980; Schmidt & Sherwood, 1982; Schmidt et al., 1979). The results of subsequent studies of force variability have shown that the rate of force generated plays an important role in the variability of impulse properties (Newell & Carlton, 1985; Newell et al., 1984).

The variability of movement error considered in the space dimension or time dimension separately has been shown to be a consequence of the rate of force production and the temporal properties of the impulse (Carlton & Newell, 1993; Kim et al., 1999). For instance, holding the time to peak force constant while increasing peak force results in a co-variation in the initial rate of force production in the impulse or vice versa. This relation leads to a consideration of the relative contributions of force and time to movement error when applying the space-time viewpoint in human movement.

Following the force (space)-time framework for movement accuracy, a measure of movement error in both force and time dimensions is required to analyze impulse variability rather than the independent measures of the force or time dimensions (Hancock & Newell, 1985; Newell, 1980; Newell et al., 1984). Information entropy is a candidate measure within the concept of information theory models of human information processing (Williams, 1997; Shannon & Weaver, 1949) and it is based on
probabilities as a foundation to unify both force and time error of the movement speed and accuracy relation (Lai, Hsieh & Newell, in press; Hsieh, Pacheco & Newell, 2015). An advantage of an information entropy measure is that it is based upon the probability of the actual frequency distribution irrespective of the measured movement/force error dimensions and assumptions about a normal distribution. The entropy estimated from the actual frequency distribution of data (Hp) can be calculated, and expressed by the following equation:

$$Hp = \sum Pi \log_2 (1/Pi)$$  \hspace{1cm} (1)

Where $Pi$ is defined in the frequency distribution, indicating the frequencies of data points in each bin $i$.

Hsieh et al. (2015) investigated a unified joint probabilistic entropy estimate of spatial and temporal movement error (Lai et al., in press; Scott, 1992; Williams, 1997). The results showed a U-shaped joint entropy function under the movement space-time conditions (different spatial and temporal criteria) for all of the 3 movement amplitudes. The lowest entropy was located at the condition where the relative contributions of space and time to movement error were comparable (Hsieh et al., 2013; Lai et al., in press). The joint entropy increased when the relative contribution of space and time deviated in the influence from that comparable region in either the space or time dimensions. This U-shaped spatial and temporal error trade-off is different from the traditional functions for both spatial (Fitts, 1954; Fitts & Peterson, 1964; Schmidt et al., 1979) and temporal (Newell et al., 1979) error when considered individually.

In the current study, our central focus was to examine the relation between force and time properties in determining variability of response kinetics by using a unified
force-time joint entropy in an isometric discrete force-time task. In particular, we wanted to test the degree to which the unified U-shaped space-time joint entropy in movement kinematics can be generalized to the force-time joint entropy in force production (Carlton & Newell, 1993; Hsieh et al., 2015; Kim et al., 1999). The force-time framework of the movement speed-accuracy relation was examined in two experiments that used an isometric single finger force task with different combinations of force level and time to peak force goals. Participants took part in both experiments that were counterbalanced in order of presentation over 2 days.

Experiment 1 had a fixed time to peak force (400ms) over different percentages of the level force (10%-70% MVC). Experiment 2 had a fixed percentage of level force (25% MVC) across a range of times to peak force (35ms-580ms). The unified force-time variability was investigated by estimating the joint probability structure of force-time information entropy (Scott, 1992; Williams, 1997).

**Experiment 1**

Our primary goal in this experiment was to investigate the entropy of force-time through an isometric single finger force task that required the production of a particular force-time impulse relation. We manipulated a range of peak force conditions but in addition analyzed the effect of time to peak force on peak force variability. We also investigated whether a single joint probabilistic force-time entropy analysis provides a different function for the relation between force error and temporal error than the functions elaborated from the traditional movement outcome measured and analyzed independently in force/amplitude or time dimensions (Crossman & Goodeve, 1983; Fitts, 1954; Plamondon & Alimi, 1997; Schmidt et al., 1979; Woodworth, 1899). Thus, we
investigate whether there is a force-time parameter range that will lead to the minimum of joint entropy of the force-time outcome (Hsieh et al., 2013, Lai et al., in press; Newell, 1980).

Methods

Participants

Twelve self-reported right-handed healthy participants (10 males and 2 females) ranging in age from 28 to 31 years old volunteered from The Pennsylvania State University for the experiment. The experimental procedures were approved by the Penn State University IRB committee, and each participant read and signed a consent form prior to the experimental tests.

Apparatus

The participant sat on a chair in front of the monitor. An Eltran ELFS-B3 load cell (1.27cm in diameter) that was vertically fixed to a wooden block recorded the force data. Analog output from load cell was amplified through a Coulbourn (V72-25) resistive bridge strain amplifier with an excitation voltage of 10 V and an amplifier gain of 100. A 16 bit A/D converter was used to sample the force output at 1000 Hz. A laptop computer that was placed on an adjacent table collected the data. The target line was presented as a red line that was centered on the screen of the width of a 43.18cm LCD monitor with a resolution of 1080 vertical pixels and 1920 horizontal pixels. The ratio of pixel to N was set at 33.33p/N, that is every N of force that the participant applied to the load cell represented 33.33 pixels of change, providing force feedback to the participant. The customized isometric force collection program was coded by visual basic software (see figure 4.1).
Experimental design

There were 7 different percentages of force level (10, 20, 30, 40, 50, 60 and 70% maximum voluntary contraction (MVC)). The criterion of movement time to peak force was 400ms for each condition. Each participant completed 7 (force levels) at the same time to peak force condition, and each condition had 80 trials of the isometric finger force task. It took approximately 2 hours to complete the 7 force-time conditions on a day. The order of the 7 conditions was randomly determined for each participant.

Participants had the maximal peak force obtained at the 400ms time to peak force set as 100% performance and values that represented 10, 20, 30, 40, 50, 60 and 70% of peak force were calculated. These % MVC values were chosen to cover much of the full range of the force-force variability function.
Procedure and Task

Participants sat comfortably on a chair approximately 60cm in front of a monitor with their right hand placed in a pronated position on the table with no external constraints during testing procedures. He/she put the lateral aspect of the distal phalange joint of the right index finger against a vertical oriented load cell with all fingers comfortably extended while keeping their palm and wrist flat against the table, and was instructed not to move the forearm position throughout the experiment.

Before the data collection, participants were given 20 trials for familiarization to learn the appropriate time to peak force and different levels of peak force. Then, they were instructed to produce maximal isometric abduction force with the distal phalange of the index finger in contact with the load cell with time to peak force of 400ms. Visual feedback of the force trajectory was displayed on the monitor. Three maximal contraction trials were recorded with 2s duration and 5s rest between each trial. The highest force level achieved over all trials defined the participant's MVC.

The experimental protocol was identical regardless of the experimental condition. Before a trial started, there were two different lines presented on the screen. The yellow line was represented as a base line (0.1N) and the red line was represented as a target line (different levels of force). The shift of the white bar (going up and down) provided the information to exert external force against the load cell once the white bar above the yellow line, the program started to record data. The task goal was to try and match a criterion peak force (different percentages of force level) and time to peak force (400ms). Participants received the feedback (peak force error and temporal error) on the screen within 1.5s of finishing the trial and a few seconds later the two lines (base line and target
line) were presented to start a new trial. Each participant attended the laboratory for 1 day to complete the 7 testing conditions, these conditions varying in the percentage of peak force at a fixed time to peak force. A 1 min break was provided after each block of 20 trials.

**Data analysis**

For each trial the algebraic error for time and force on each dimension considered independently was recorded. The force, temporal and joint force-time entropy was calculated in terms of probability based on an analysis of the frequency distribution of the respective movement outcome. The number of bins for the entropy analysis, following pilot analysis, was set at 20 for both the force and temporal data, with the last 75 trials analyzed in each of the respective dimensions. The performance entropy was the summation of the force and temporal measures in terms of the joint probability. A detailed explanation of the determination of the joint force-time entropy measure is provided in Appendix A.

Repeated measures ANOVAs were used to examine the effect of the force-time task movement conditions on each dependent variable. The Greenhouse-Geisser method was used to correct for violations of sphericity and the Bonferoni correction was applied for the post hoc comparisons, with eta square (\(\eta^2\)) (Green & Salkind, 2003) revealing the effect size. In those cases where the normal distribution assumption was not fulfilled in the data we used Friedman’s ANOVAs to analyze the effects. Repeated paired Wilcoxon tests corrected by the Holm's Sequential Bonferroni Procedure (Abdi, 2010) were used as non-parametric post-hoc tests. The effect sizes were Kendal’s W for Friedman’s ANOVA. We used Matlab Version 8.0 (Mathworks, 2012) for data preparation and
analysis, SPSS for Window Version 13.0 for statistical analysis and SigmaPlot Version 10.0 for the examination of function fitting of force and temporal errors, and joint force-time entropy.

Results

**Distributional analyses**

**Force error**

Figure 4.2 (right column) shows force constant error (CE), variable error (VE), and information entropy of the isometric single finger force task as a function of the different force-time conditions. The one way repeated measure ANOVA revealed a significant effect for condition, $F(2.19, 24.1)=34.537, p<.05, \eta_p^2 =0.76$ in force variable error, but not in force constant error, $F(1.31, 14.38)=0.729, p>.05$. The post-hoc paired comparisons in force variable error showed that all of the conditions were significantly different from each other, except the 10% of force level from 20%, 30%, and 40% conditions, 30% of force level from 40% condition, 40% of force level from 50% condition, 50% of force level from 60% condition and 60% of force level from 70% condition.
Figure 4.2. Right column. Force constant error (CE), variable error (VE), and entropy of the isometric single finger force task as a function of the different force-time conditions. Left column. Temporal constant error (CE), variable error (VE), and entropy of the isometric single finger force task as a function of the different force-time conditions. The error bars stand for the 95% confidence interval.

The information entropy of force error data showed deviations from normality and thus we used the non-parametric tests (see methods section). The condition effect was significant, $\chi^2(6) = 64.071$, $p < 0.001$, $W = 0.89$. The post-hoc analyses showed that all
force-time conditions were significantly different from each other, except the 10% force level from 20%, 30%, 40%, 50% conditions, and the 30% force level from the 40% condition.

**Temporal error**

Figure 4.2 (left column) depicts the temporal constant error (CE), variable error (VE), and entropy of the isometric single finger force tasks as a function of different force-accuracy constraints. The one way repeated measure ANOVA revealed a significant effect for force level, $F(6, 66)=2.569, p<.05, \eta^2_p=0.19$ in the temporal constant error, and also $F(6, 66)=11.005, p<.05, \eta^2_p=0.5$ in the temporal variable error. The post-hoc paired comparisons in the temporal variable error showed that all of the conditions were significantly different from each other, except the 20% of force level from 30%, 40%, 50%, 60%, 70 conditions, the 30% force level from 40%, 50%, 60%, 70% conditions, the 40% force level from 50%, 60%, 70% conditions, the 50% force level from 60%, 70% conditions, and the 60% force level from the 70% condition.

The one way repeated measure ANOVA on temporal entropy revealed a significant effect for force level, $F(6, 66)=9.811, p<.05, \eta^2_p=.471$. The post-hoc paired comparisons on temporal entropy showed only that the 10% force level was significantly different from other force levels.
Figure 4.3. The joint force-time entropy of the isometric single finger force task at different space-time conditions. The error bars stand for the 95% confidence interval.

**Joint entropy force-time**

Figure 4.3 shows the joint force-time entropy of the isometric single finger force tasks as a function of the different force and temporal constraints. The information entropy of force-time error data showed deviations from normality and thus we used the non-parametric tests. The condition force level effect was significant, $\chi^2(6) = 48.25, p < 0.001, W = 0.67$. The post-hoc analyses showed that only the 20% force level was significantly different from the 50%, 60%, and 70% conditions, the 30% force level was significantly different from the 60% and 70% conditions, and the 40% force level was significantly different from the 60% and 70% conditions.

**Experiment 2**

In Experiment 2, we tested the force-time principle in a task that had a fixed percentage of level force (amplitude) over a range of times to peak force (MT criteria:
It was hypothesized that there will be a minimum of joint entropy of the force-time outcome in the mid-range of force-time constraints (Hsieh et al., 2015, Lai et al., in press; Newell, 1980). This experiment provides a test of the generalization between kinematic to kinetic approaches to the relation of the speed and accuracy trade-off (Carlton & Newell, 1993; Schmidt et al., 1979).

Methods

Participants

The participants were 12 right-handed volunteers from the Pennsylvania State University. Each of the participants had also participated in Experiment 1.

Apparatus

The apparatus was the same as that used in Experiment 1.

Experimental design

There were 5 different time to peak force (35ms, 55ms, 140ms, 375ms, and 580ms) conditions and the criterion percentage of peak force was 25% of MVC for each condition. Each participant completed 5 (time to peak force) x 1 (peak force) conditions that were characterized by the force-time requirements, and each condition with 100 trials of a isometric finger force task. It took approximately 1 hour to complete the 5 force-time conditions on a day. The order of the 5 conditions was randomly determined for each participant.

Derivation of the temporal task criteria

Prior to the experiment proper a pilot study of the isometric finger force with a variety of task emphases on speed and accuracy was conducted to determine the force-
time conditions of the actual experiment. In the pilot procedure, 2 additional participants acted as pilot participants to establish the range of force-time conditions for the experiment. These pilot participants performed 350 trials of the isometric finger force task with different emphases on force accuracy and movement time on each trial in a fixed peak force target (25% of MVC). For example, to reach the target line as fast as possible while aiming for the target, or to reach the line a little slower while aiming for the target, and to reach the line to the target as accurate as possible by pressing the load cell.

After the 350 pilot testing trials, we were able to observe the standard function of force-time performance under different speed-accuracy conditions. The data distribution of movement time and force error revealed that when a longer impulse duration was emphasized the magnitude of force error was decreased, and vice versa. We normalized the raw pilot data by using the longest impulse duration and largest force error (MT: 2534 ms, and FE: 3.145 N). The hyperbolic function of movement time and force error formed the basis for determining the conditions of this experiment in which we used 5 slopes of the tangent lines that were close to the fast (10:1), fast middle (5:1), middle (1:1), middle accurate (1:5) and accurate (1:10). We fitted the normalized data with a power law function (2 parameter), \( y = ax^b \); \( a = 0.012 \), and \( b = -0.675 \). At the middle condition, the 1:1 coefficient was derived from the tangent of 45 degrees to the speed-accuracy fitting curve (pilot data) that provided equal emphasis to the contribution of movement time and force error. After we found the positions where the slopes were close to the determined goal values we used the corresponding peak force times as the bases for the criterion of different force-time conditions that were fast:35ms, middle-fast:55ms, middle: 140ms,
middle-accurate: 375ms, and accurate: 580ms. The principles for the derivation of the temporal task criteria were similar to those in Hsieh et al. (2013) with a discrete movement aiming task.

**Procedures and task**

The general experimental protocol was identical to Experiment 1. Before a trial started, there were two different lines presented on the screen. The yellow line was represented as a base line (0.01N) and the red line was represented as a target line (level of force, 25%). The shift of the white bar (go up and down) was the information to exert an external force against the load cell once the white bar above the yellow line, the program started to record data. The task goal was that participants attempted to match a criterion peak force and different time to peak force (35ms, 55ms, 140ms, 375ms, and 580ms). Participants received the feedback (algebraic peak force error and temporal error) on the screen within 1.5s of finishing the trial and a few seconds later the two lines (base line and target line) were presented to start a new trial. Five conditions varied in the time to peak force at a fixed 25% of peak force. A 1 min break was provided after each 25 trials.

**Data analysis**

The data analyses used were identical to those conducted in Experiment 1.

**Results**

**Distributional analyses**

**Force error**

Figure 4.4 (left column) shows force constant error (CE), variable error (VE), and information entropy of the isometric single finger force task as a function of the different
force-time conditions. The force variable error data showed deviations from normality and thus we used the non-parametric tests. The condition effect was significant, \( \chi^2(4) = 44.467, p < 0.001, W = 0.93 \). The post-hoc analyses showed that all force-time conditions were significantly different from each other.

Figure 4.4. Left column. Force constant error (CE), variable error (VE), and entropy of the isometric single finger force task as a function of the different force-time conditions. Right column. Temporal constant error (CE), variable error (VE), and entropy of the isometric single finger force task as a function of the different force-time conditions. The error bars stand for the 95% confidence interval.
The condition effect of force level on information entropy of force error data was significant, \( \chi^2(4) = 45.133, p < 0.001, W = 0.94 \). The post-hoc analyses showed that all force-time conditions were significantly different from each other, except the 35ms of time to peak force from the 55ms condition.

**Temporal error**

Figure 4.4 (right column) depicts the temporal constant error (CE), variable error (VE), and entropy of the isometric single finger force tasks as a function of different force-accuracy constraints. The temporal constant error data showed large deviations from normality and thus we used the non-parametric tests. The condition effect was significant, \( \chi^2(4) = 28.933, p < 0.001, W = 0.60 \). The post-hoc analyses showed that all force-time conditions were significantly different from each other, except 35ms of time to peak force from 55ms condition, 55ms of time to peak force from 140ms and 375 conditions, 140ms of time to peak force from 375ms condition, and 375 of time to peak force from the 580ms condition.

The one way repeated measure ANOVA revealed a significant effect for condition, \( F(1.93, 21.18) = 211.367, p < .05, \eta^2_p = 0.95 \) in temporal variable error. The post-hoc paired comparisons showed that all of the conditions were significantly different from each other, except the 35ms of time to peak force from 55ms and 140ms conditions, and 55ms of time to peak force from the 140ms condition.

The one way repeated measure ANOVA on the temporal entropy revealed a significant effect for condition, \( F(4, 44) = 175.598, p < .05, \eta^2_p = 0.941 \). The post-hoc paired comparisons in the temporal entropy showed all of the conditions were significantly
different from each other, except the 35ms of time to peak force from 55ms and 140ms conditions, and 55ms of time to peak force from 140ms condition.

Figure 4.5. The joint force-time entropy of the isometric single finger force at different force-time conditions. The quadratic function fit only for the joint force-time entropy from group mean. The logarithm of the weighting ratio was used as the independent variable, therefore $\log_{10}(1/10) = -1$, $\log_{10}(1/5) = -0.7$, $\log_{10}(1/1) = 0$, $\log_{10}(5/1) = 0.7$, $\log_{10}(10/1) = 1$ were used as the 5 values for the fitting variable. The error bars stand for the 95% confidence interval.

**Joint entropy force-time analysis**

Figure 4.5 shows the joint force-time entropy of the isometric single finger force force tasks as a function of the different force and temporal constraints. The one way repeated measure ANOVA on the temporal entropy revealed a significant effect for condition, $F(4, 44)=22.609, \ p<.05, \ \eta^2_p = .673$. The post-hoc paired comparisons in the temporal entropy showed all of the conditions were significantly different from each other, except 35ms of time to peak force from 55ms, 140ms, and 375ms conditions, and 55ms of time to peak force from 140ms and 375ms conditions.
To test the U-shaped function of the joint force-time entropy for the 5 different force-time conditions, we fitted the joint entropy from the last 90 trials of each individual participant with a quadratic function: $y = a + bx + cx^2$ where $y$ is the joint entropy and $x$ is the logarithm of the weighting ratio between movement time and force accuracy. The $R^2$ values of the group mean fit results was .99 for the joint force-time entropy (see figure 5) and the effect was significant, $F (1, 11) = 26.6, p < .0001, \eta^2_p = .71$. Table 4.1 shows the $R^2$ values of the individual fit results for the joint force-time entropy.

<table>
<thead>
<tr>
<th>Participants</th>
<th>25% of Force level</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.98</td>
</tr>
<tr>
<td>P2</td>
<td>0.97</td>
</tr>
<tr>
<td>P3</td>
<td>0.94</td>
</tr>
<tr>
<td>P4</td>
<td>0.87</td>
</tr>
<tr>
<td>P5</td>
<td>0.83</td>
</tr>
<tr>
<td>P6</td>
<td>0.88</td>
</tr>
<tr>
<td>P7</td>
<td>0.83</td>
</tr>
<tr>
<td>P8</td>
<td>0.96</td>
</tr>
<tr>
<td>P9</td>
<td>0.83</td>
</tr>
<tr>
<td>P10</td>
<td>0.93</td>
</tr>
<tr>
<td>P11</td>
<td>0.89</td>
</tr>
<tr>
<td>P12</td>
<td>0.91</td>
</tr>
</tbody>
</table>
Discussion

In the present study, we investigated in two experiments the joint force-time entropy as a function of a range of force-time discrete impulse conditions in a single finger isometric force task. The discrete isometric force protocol has been used to investigate the contribution of force characteristics (e.g., peak force, time to peak force, and impulse) and their interactions to impulse variability and also to provide a potential link between kinematic and kinetic accounts of the speed and accuracy trade-off phenomenon (Carlton & Newell, 1993; Newell et al., 1984; Schmidt et al., 1979). Furthermore, we used the concept of movement error in space-time framework to measure and analyze force and temporal error in the discrete isometric task (Carlton & Newell, 1993; Hancock & Newell, 1985).

In Experiments 1 and 2, the mean temporal constant error, tended to undershoot or overshoot across different percentages of force level and different force-time conditions, respectively. However, the mean force constant error always overshot the target whether the force-time conditions were manipulated by the fixed time to peak force or force level. These findings on constant error properties of impulse extrapolated to the movement domain did not follow the prediction of Hancock and Newell’s model (1985) that as movement velocity increased within a given movement amplitude, the constant error would shift from a positive to a negative value in the spatial dimension and vice versa in the temporal dimension.

Some previous studies of aiming movements have shown that primary movement endpoints always undershoot the target for two reasons. First, is that a longer movement time is needed if the primary movement is an overshoot. Second, that target overshoot is
more costly in terms of both energy and attention than a target undershoot (Carlton, 1979; Elliott & Khan, 2010; Elliott, Hansen, Mendoza, & Tremblay, 2004). However, there are also experimental outcomes showing different results of constant error than the prediction that overshooting the target is a general phenomenon when participants aim at a small target (Carlton, 1994; Hsieh et al., 2015; Lai et al., 2006). These discrepancies in the pattern of constant error require a further study over a broad range of conditions including the systematic change in the target size to better understand the characteristics of constant error in the speed-accuracy trade-off (which is often not reported).

The results from both experiments showed that the peak force variability increased either with the increment of force level or through a shorter time to peak force that also reduces timing error variability (Carlton & Newell, 1993; Kim et al., 1999; Schmidt et al., 1979). However, the change in timing error variability did not occur proportionally to force level or time duration (e.g., Ghez & Vicario, 1978; Schmidt et al., 1979). Rather, even though the time to peak force was fixed in Experiment 1 (400ms), the timing error still deviated from the rate required implicitly by task demands when the percentage of force increased (Carlton & Newell, 1985; Newell & Carlton, 1985). The findings showed that participants produce higher percentages of force level with a deviation in time to peak force (Carlton & Newell, 1993; Newell & Carlton, 1985; Newell et al., 1984; Kim et al., 1999) and confirm that time to peak force is a critical variable that influences the peak force variability function. The present findings also provided additional evidence that when the measures of force error and timing error were considered in the same framework, the force and time errors are complementary (Carlton, Kim, Liu & Newell, 1993; Kim et al., 1999; Newell et al., 1982).
Information entropy is based on the concept of probability (Williams, 1997). It provides an index of motor output uncertainty to examine movement variability other than the dependent variables that are derived from the distributional account of movement variability. The findings from both experiments showed that force entropy increased and temporal entropy decreased as the percentage of force increased or the rate of force change increased when considering force and time dimension separately (Lai et al., 2006; Kim et al., 1999). These results imply that when the percentage of force level increased the uncertainty of the motor output is increased. The difference between standard deviation and entropy analysis is that entropy is based on the actual probabilities of the data distribution rather than the implicit assumptions about the characteristics of the data distribution should be normal, such as standard deviation (Hancock & Newell 1985).

In the present study, a novel contribution was that the joint force-time entropy was defined from the individual force and temporal error data that participants performed across different parameter conditions of the force-time dimensions. In Experiment 1, the joint entropy at the 20% force level was lowest. As the percentage of force level deviated from 20%, the joint force-time entropy increased, especially over the 30%-70% range, and higher joint entropy was contributed by the variability of force dimension. In Experiment 2, we sought to provide a more direct test than Experiment 1 to find a potential linkage between the kinematic and kinetic approach to the relation between speed and accuracy by manipulating a fixed percentage of one force level with a range of time to peak force conditions (Carlton & Newell, 1993; Schmidt et al., 1979). The findings revealed a U-shaped function and the lowest joint entropy was situated at the
parameter range where the weighting of force and time was comparable (Hsieh et al., 2013; Hsieh et al., 2015; Lai et al., 2015). In addition, the joint entropy increased as the contributions of force-time deviated from that region.

The U-shaped joint entropy function leads to the inference that there is a trade-off between force error and temporal error when the force and time dimensions of movement outcome errors are considered together. Based on the concept that the velocity of movement through the space is an emerged property of muscular force acting on the joint and the duration of this force acted. Producing a long movement time/lower force level or short movement time/ higher force level changes the contribution of either force or time from this middle parameter range and leads to an increase of the joint entropy on the force-time dimension (Carlton & Newell, 1993; Hancock & Newell, 1985).
CHAPTER 5: MATCHING AND MINIMIZING MOVEMENT TIME IN SPEED-ACCURACY TASKS

Abstract

It has been proposed that there are different functions for the speed-accuracy trade-off with time matching and time minimization movement tasks. The goal of present experiment was to test whether these different task descriptions of speed and accuracy were due to different temporal and spatial task constraints. Fifteen participants twice performed 100 trials of time minimization and time matching tasks with the yoked temporal and spatial requirements (criterion time and target width). The results showed that performing an aiming movement under the same spatial and temporal constraints resulted in similar outcomes with distributional properties (skewness and kurtosis) being slightly affected by practice effects. There was a trade-off in the information entropy for space and time (temporal information entropy decreased as spatial information entropy increased) with practice. Nevertheless, the joint space-time entropy of outcome did not change across tasks and conditions – revealing a common level of space-time entropy between these two categories of aiming tasks. These findings support the hypothesis that under the same spatial and temporal constraints the movement speed-accuracy function shares the same properties independent of task category.
Introduction

Haste makes waste is an old adage handed down from generation to generation. It suggests that when we try to complete a movement task quickly, we make mistakes. This assumption sits behind the concept of the speed-accuracy trade-off in motor behavior. Increasing the movement speed when performing a task leads to poorer accuracy. Since the work of Woodworth (1899), there have been numerous attempts to examine the relation between distance, time and movement error (spatial or temporal error) (Crossman & Goodeve, 1983; Elliott et al., 2010; Hancock & Newell, 1985; Keele, 1981; Meyer, Smith, Kornblum, Abrams, & Wright, 1990; Plamondon & Alimi, 1997; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979).

Fitts (1954) provided the first formal description of the relation between movement speed and accuracy. He asked individuals to perform reciprocal movements between two targets as fast and accurate as possible, in conditions with different movement amplitude and target size (i.e., time-minimization task). Fitts observed that the time to complete the movement was described by a logarithmic function of the movement amplitude (A) divided by the width of the target (W),

\[ MT = a + b \log_2 \left( \frac{2A}{W} \right) \]  

with a and b being empirically derived constants (Fitts, 1954; Fitts & Peterson, 1964). Given the high reliability and generality of this equation in time-minimization tasks (Crossman & Goodeve, 1983; Keele, 1968; Langolf, Chaffin, & Foulke, 1976; Wallace & Newell, 1983), this relation has ultimately become known as Fitts’ law. Although a considerable amount of experimental data has provided support for Fitts’ law, the
sufficiency and generality of the proposition has still been questioned (Hancock & Newell, 1985; Hoffman, 2013; Meyer et al., 1990; Sheridan, 1979).

The logarithmic account of movement speed and accuracy trade-off is not the only function to describe this phenomenon. Schmidt et al. (1979) used a different speed-accuracy paradigm by asking participants to perform rapid discrete aiming movements to a target line while matching a criterion time (i.e., time-matching task). They found a linear relation between spatial variability and the generated impulse, and proposed the impulse variability model for rapid movements interpreting the speed-accuracy trade-off as a result of “noise” in the muscular system. This linear relation of movement speed and accuracy has been replicated in a number of different experiments (Wright & Meyer, 1983; Zelaznik, Mone, McCabe, & Thaman, 1988; Zelaznik, Shapiro, & McClosky, 1981).

Three main hypotheses have been considered to account for these different functions (linear and logarithmic) of movement speed and accuracy (Carlton, 1994; Wright & Meyer, 1983). First, the movement-brevity hypothesis states that the linear speed-accuracy trade-off would be observed when a person tries to match very short criterion movement times (under 200ms) without any corrective sub-movements (Schmidt et al., 1979). The logarithmic trade-off would hold in the case where more time is available since this person could use visual feedback to correct the movement error (Crossman & Goodeve, 1983; Keele, 1968). Second, as a general version of the previous hypothesis, the feedback-deprivation hypothesis states that the linear speed-accuracy trade-off would occur when the relevant sensory feedback (visual or proprioceptive) is neither available nor used during the course of movement execution. The logarithmic trade-off would be
observed if the feedback is processed to make corrections during the course of movement (Zelaznik et al., 1981). Third, the temporal precision hypothesis states that a linear trade-off occurs when precise movement duration is required (e.g., in time-matching tasks). In this situation, a person would produce an aimed movement with minimum temporal variability by a single pair of opposing impulses. In contrast, the logarithmic trade-off would appear when the emphasis is on precise spatial movements, which would be characterized by overlapping impulses (Meyer, Abrams, Kornblum, Wright & Smith, 1988; Zelanik et al., 1988).

Hancock and Newell (1985) proposed that the space-time function is an emergent property from the constraints of the individual, environment, and task within a space-time frame of reference. This space-time perspective provides a comprehensive interpretation of spatial and temporal movement error across a broad spectrum of the kinematic continuum (with movement times ranging from as short as 140ms to as long as 3000ms). Using the standard unit error (i.e., the variable error of space or time, divided by the respective spatial or temporal movement criterion), they provide three predictions in the relation of speed-accuracy for movement variability. First, the standard unit error decreases nonlinearly with constant increments in average movement velocity within a given criterion time. Second, the standard unit error increases nonlinearly with increments in average movement velocity within a given amplitude. And third, the increase in the standard unit error is not proportional to in either the respective amplitude or criterion time. These predictions are consistent with the reinterpretation by Hancock and Newell (1985) of the data of Schmidt et al. (1979) and Woodworth (1899).
According to the space-time perspective, the different speed-accuracy functions in tasks emerged due to the presence of different task constraints arising from the manipulations. Although both logarithmic and linear descriptions considered the upper end of the velocity continuum, one observed the emergent movement time when specific spatial constraints were imposed (i.e., spatial variability – target width, and movement amplitude) (e.g., Fitts, 1954) while the other characterized the movement variability when emphasis is given to matching both amplitude and time (e.g., Schmidt et al., 1979). Additionally, when the experimental conditions are considered under which both functions were compared, not all spatial and/or temporal constraints were controlled inviting their interpretation to be a reflection of different phenomena (e.g., Carlton, 1994; Zelanik et al., 1988).

Carlton (1994) was the first to directly examine the difference in movement speed and accuracy between matching and minimizing tasks. Participants produced aimed hand movements to hit a target (crosshairs) in a time-matching task (goal MT = 400ms). Then the dispersion of 95% of the aimed movement outcome of each individual was used as a target size for the time-minimization task. However, participants actually produced their movements under different spatial or temporal constraints (i.e., crosshairs versus circular target or 400ms versus as fast as possible).

In line with the space-time perspective provided by Hancock and Newell (1985), we investigate here whether the emergent movement speed-accuracy properties would be the same under the same spatial and temporal constraints, independent of the classification of the task. In this view, both time-minimization and time-matching tasks would share the same region in a given speed-accuracy description if the movement spatial-temporal
constraints are the same. That is, if the movement time derived from the practice in a time-minimization task is used as the goal for a time-matching task when both target width and amplitude are maintained, then the conditions would have the same movement properties. In addition, if the effective target width from the time-matching task is calculated and used as the target-width for the time-minimization task, we would also expect similar movement outcome properties.

Following the space-time perspective, Lai, Hsieh and Newell (2015) used joint information entropy to unify the probability estimates of space-time movement outcomes in the movement speed and accuracy relation. Information entropy is one candidate approach to reflect both the space and time movement variability in motor output (Lai, Mayer-Kress, Sosnoff & Newell, 2005; Lai, Mayer-Kress & Newell, 2006). It is anticipated that the movement variability in terms of a single dimension (space or time) would be different from the movement variability measured in the joint probabilistic space-time entropy under the dual dimension (both space and time). Thus, in the present study, we investigated whether the joint space-time entropy of movement properties under the same constraints would be the same independent of the task, namely, time-minimization and time-matching aiming tasks.

Methods

Participants

Fifteen self-reported right-handed healthy students (8 males and 7 females) of the University of Georgia whose mean age was 27.35 (range ± 7.09) years old volunteered for the experiment. Informed consent was obtained prior to the experiment and the
Institutional Review Board at the University of Georgia approved all the experimental procedures.

**Apparatus**

A line drawing system that included a Wacom Cintiq 21UX digital tablet (Model DTZ-2100D, 561 x 421 x 61.3mm with an active surface area of 432mm x 324mm), a PC computer (the pixel range was set at 800 x 600), a handheld cordless stylus (Model ZP-501E) with a weight of 18g was used. A custom computer aiming task program was used to preset different movement time goals and different amplitudes in space-time and calculate the spatial error and temporal error for the participant immediately after each discrete aiming trial (see figure 5.1). The sample frequency of the kinematics from the customized program was 130Hz and the ratio of the distance moved by the cursor representing on the screen and the actual distance moved by the stylus tip on the tablet was 1:1.

![Figure 5.1. A schematic of the discrete line drawing experimental setup.](image)

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**Experimental design and task**

Time minimization and time matching procedures were used in the present experiment that ran over 4 days in two segments (first segment: Day 1 and 2; second segment: Day 3 and 4). On Day 1 of the first segment, participants performed the time minimization task in a discrete aiming movement of 20cm amplitude from left to right as fast and accurate as possible to a target (1cm in diameter for the target point with an index of difficulty of 4.32). The middle of movement amplitude was approximately on the midline of participants’ body. On Day 2, the criterion time of a time matching task was defined as the movement time that the participant performed most frequently (mode) from the time minimization task (movement time data showed non-normal distribution). Participants were instructed to match this designated movement time with the same amplitude and target width in a time minimization task.

On Day 3 of the second segment, based on the spatial error produced on the time matching task that has been named an effective target width ($W_e$), it was defined as the mean spatial error in the horizontal direction (main direction of movement, x axis) plus two standard deviations that encompass 95% of the movement outcomes, if these performances represented a normal distribution (Carlton, 1994; Lai et al., in press). According to this strategy, the individual’s $W_e$ was used to construct a new target width in the time minimization task. On Day 4, the criterion time of time matching task was defined as the movement time that participant performed most frequently (mode) that resulted from the time minimization task (movement time data showed non-normal distribution) as on Day 2. Overall, each participant completed 4 tasks over 2 days (1. time-minimization task, 2. time-matching task, 3. time-minimization task, and 4. time-
matching task) each with 130 trials of aiming movement. It took approximately 20 min
to complete a task on each day.

Procedures

A discrete aiming movement was made with the dominant hand (right hand) in a left
to right direction. The home point (2-mm circle, left) and target point (1-cm circle, right)
were shown on the digital tablet before the start of a trial in the time minimization task on
Day 1 (first session). The participants were instructed to look at the tablet when they
used the stylus to move from the left home point to the right target point and to be as fast
and accurate as possible in hitting the target. There was a black, small cursor (1-mm
circle) representing to a stylus tip that was displayed on the tablet screen; although, no
trajectory of the stylus shown on the tablet when performing the task.

An auditory tone was given once the participant held the stylus to touch the tablet
within the home point around 600ms. The participant was instructed not to respond to
the auditory tone as fast as possible because this was not a reaction time experiment. The
participant was to begin a trial when he/she was ready and comfortable after hearing the
tone. After the trial start, the stylus was to remain in contact with the tablet during the
whole movement until the trial was completed. The trial was finished when the black
cursor touched the target point or the stylus came to stop after passing the half way of the
amplitude.

The initiation of movement was defined as the point at which the stylus crossed the
low velocity threshold of 3mm/s for more than 30ms and the end of movement was the
initial point when stylus left the tablet and for the stylus’s velocity was less than 3mm/s
for more than 30ms. The movement time/temporal error (time in ms between the start
and end of the movement for the time minimization or the difference between movement
time and time criteria in the time matching task) and spatial score (the Euclidean distance
between the center of target and the end point of each trial) were displayed on the tablet
screen immediately to provide information feedback after finishing each trial. The
participant moved the stylus back to the home point to start the next trial as soon as the
participant was ready. A 3-5 min break was provided after every 50 trials. Trials on the
time-matching and time-minimization (new target size and new criterion time) tasks were
performed on the 2, 3, and 4 days following testing on the first time-minimization task.
All the procedures were identical through the two segments over four days.

Data analysis

For each trial the movement time and spatial movement outcome were recorded and
analyzed. For spatial movement, we considered the Euclidean distance between the
center of target and the movement endpoint. The deviation from the center of target
(spatial movement outcome), standard deviation of movement time and spatial movement
outcome, skewness and kurtosis of movement time and spatial movement outcome were
calculated. The spatial entropy, temporal entropy and joint space-time entropy were
calculated in terms of probability based on an analysis of the frequency distribution of the
respective movement outcome. The entropy estimated from the actual frequency
distribution of data ($H_p$) was calculated with the following equation:

$$H_p = \sum P_i \log_2 (1/P_i)$$  \hspace{1cm} (1)

where $P_i$ is defined in the frequency distribution, indicating the frequency of data points
in each bin $i$. The number of bins for the entropy analysis was set at 20 for both the
spatial and temporal data, with the last 100 trials analyzed in each of the respective
dimensions. The performance outcome was the summation of the spatial and temporal entropy in terms of the joint probability. A detailed explanation of the determination of the joint space-time entropy measure is provided in Lai et al., (in press) and Williams (1997).

Repeated measures ANOVAs were used to examine the effect of the time-minimization and time-matching tasks on each dependent variable. The Greenhouse-Geisser method was used to correct for violations of sphericity and the Bonferroni correction was applied for the post hoc comparisons, with eta square (\(\eta^2\)) (Green & Salkind, 2003) revealing the effect size. In those cases where the normal distribution assumption was not fulfilled in the data we used Friedman’s ANOVAs to analyze the effects. Repeated paired Wilcoxon tests corrected by the Holm's Sequential Bonferroni Procedure (Abdi, 2010) were used as non-parametric post-hoc tests. The effect sizes were Kendal’s W for Friedman’s ANOVA. We used Matlab Version 8.2 (Mathworks, R2013b) for data preparation and analysis, SPSS Window Version 19.0 for statistical analysis and SigmaPlot Version 10.0 for plotting figures of dependent variables.

In the present study, we tested the null hypothesis that under the same spatial and temporal constraints the participants’ movement aiming behavior would show similar properties. This required that the statistical power is high enough to adequately test the null hypothesis (Field, 2009). We chose a sample size that would provide at least 90% power (specifically, 94%) to show significant results in the case of a medium effect size (\(\eta^2= 0.13\)).
Results

Distributional Analyses

Movement Time

Mode of movement time. Figure 5.2 (top left) shows the mode of movement time of the aiming task as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on the mode of movement time showed a significant effect for segments, $F(1, 14) = 12.30, p < .003, \eta^2 = .46$. The post-hoc comparison on mode of movement time showed that the first segment was significantly longer than the second segment. However, there was no task effect, $F(1, 14) = .45, p > .05$, and the interaction between segment and task, $F(1, 14) = .05, p > .05$ was not significant.

Average movement time. Figure 5.2 (top right) shows the average movement time of the aiming task as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on average movement time showed a significant effect for segments, $F(1, 14) = 30.68, p < .0001, \eta^2 = .68$. The post-hoc comparison on average movement time showed that the first segment was significantly longer than the second segment. However, there was no task effect, $F(1, 14) = 2.15, p > .05$, and the interaction between segment and task, $F(1, 14) = 2.21, p > .05$ was not significant.
Figure 5.2. Left Column. Mode of movement time, SD of movement time, and skewness of movement time in the time-minimization and time-matching tasks as a function of the different segments. Right Column. Average movement time, entropy of movement time, and kurtosis of movement time in the time-minimization and time-matching tasks as a function of the different segments. The error bars stand for the 95% confidence interval.
**Standard deviation of movement time.** Figure 5.2 (middle left) shows the standard deviation of movement time of the aiming task as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on variable movement time was not significant for segment, $F(1, 14) = 1.51, p > .05$, task, $F(1, 14) = .64, p > .05$, and their interaction, $F(1, 14) = 1.30, p > .05$.

**Information entropy of movement time.** Figure 5.2 (middle right) shows the entropy of movement time of the aiming task as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on information entropy of movement time was significant for segment, $F(1, 14) = 16.70, p < .001, \eta^2 = .54$. The post-hoc paired comparisons of information entropy of movement time showed that the first segment was significantly larger than that of the second segment. An interaction between segment and task was significant, $F(1, 14) = 9.47, p < .008, \eta^2 = .40$. The post-hoc simple main effect analysis showed that information entropy of movement time in the time-minimization task was significantly longer than time-matching but only in the first segment. Moreover, there was a significant difference between the first segment and second segment only in time-minimization task. However, there was no task effect, $F(1, 14) = 1.92, p > .05$.

**Skewness and kurtosis of movement time.** Figure 5.2 (bottom left and right) shows the skewness and kurtosis of movement time of the aiming task as a function of the different segments and tasks. The skewness of movement time and kurtosis of movement time data showed large deviations from a normal distribution and thus we used non-parametric tests (see methods section).
For skewness of movement time, the segment effects were significant for the time-minimization task; $Z = -3.06$, $p < .002$, $r = .79$, and for the time-matching task; $Z = -2.27$, $p < .02$, $r = .58$. The task effect reached a significant level in the first segment; $Z = -2.15$, $p < .03$, $r = .55$, but not for second segment; $Z = -.45$, $p > .05$.

For kurtosis of movement time, the segment effects were significant for the time-minimization task: $Z = -3.06$, $p < .002$, $r = .79$, and for the time-matching task; $Z = -2.55$, $p < .01$, $r = .65$. However, the task effect did not reach the significance level, in the first segment: $Z = -1.59$, $p > .05$; or in the second segment: $Z = -.68$, $p > .05$.

**Spatial dispersion**

*Spatial distance from the target center.* Figure 5.3 (top left) shows the spatial distance from the target center of the aiming task as a function of the different segments and tasks. The spatial distance from the target center showed large deviations from a normal distribution and thus we used non-parametric tests (see methods section). For spatial distance from the target center, the segment effects were significant for the time-minimization task; $Z = -2.66$, $p < .008$, $r = .68$, and for the time-matching task, $Z = -3.23$, $p < .01$, $r = .83$. The task effect did not reach a significant level in either the first segment; $Z = -.05$, $p > .05$, or the second segment; $Z = -.11$, $p > .05$.

*Standard deviation of spatial distance from the target center.* Figure 5.3 (top right) shows the standard deviation of spatial distance from the target center of the aiming task as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on SD of spatial distance from the target center showed a significant effect for segments, $F(1, 14) = 22.71$, $p < .001$, $\eta^2 = .61$. The post-hoc comparison on the standard deviation of spatial distance from the target center showed
that the first segment was significantly smaller than that of the second segment. However, there was no task effect, $F(1, 14) = .25, p > .05$, and the interaction between segment and task, $F(1, 14) = .10, p > .05$ was not significant.

Figure 5.3. Left Column. Spatial distance from the target center, entropy of spatial distance from the target center, and kurtosis of spatial distance from the target center of the aiming task as a function of the different segments and tasks. Right Column. SD of spatial distance from the target center, and skewness of spatial distance from the target center. The error bars denote the 95% confidence interval.

Information entropy of spatial distance from the target center. Figure 5.3 (middle left) shows the entropy of spatial distance from the target center of the aiming task as a
function of the different segments and tasks. The 2 way (2 sessions x 2 tasks) repeated measures ANOVA on information entropy of spatial distance from the target center was significant for segment effect, $F(1, 14) = 31.64, p < .001, \eta^2 = .69$. The post-hoc paired comparisons of information entropy of spatial distance from the target center showed that the first segment was significantly lower than the second segment. There was no task effect, $F(1, 14) = .08, p > .05$ and also an interaction between segment and task was not significant, $F(1, 14) = .001, p > .05$.

**Skewness of spatial distance from the target center.** Figure 5.3 (middle right) shows the skewness of spatial distance from the target center as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on skewness of spatial distance from the target center was not significant for segment, $F(1, 14) = .38, p > .05$, task, $F(1, 14) = .12, p > .05$, and their interaction $F(1, 14) = 1.98, p > .05$.

**Kurtosis of spatial distance from the target center.** Figure 5.3 (bottom left) shows the kurtosis of spatial distance from the target center of the aiming task as a function of the different segments and tasks. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on kurtosis of spatial distance from the target center was not significant for segment, $F(1, 14) = .19, p > .05$, task, $F(1, 14) = .55, p > .05$, and the interaction $F(1, 14) = 3.51, p > .05$.

**Joint entropy space-time analysis**

The figure 5.4 showed the joint space-time entropy of the outcome of aiming as a function of task and at different testing segments. The 2 way (2 segments x 2 tasks) repeated measures ANOVA on joint space-time entropy was not significant for segment,
\( F(1, 14) = .12, p > .05, \) task, \( F(1, 14) = .06, p > .05, \) and their interaction \( F(1, 14) = 3.76, p > .05. \)

Figure 5.4. The joint space-time entropy of the aiming task in the time-minimization and time-matching tasks as a function of the different testing segments. The error bars denote the 95% confidence interval.

Discussion

We investigated the relation between movement speed and accuracy under the same spatial and temporal constraints in time-minimization and time-matching aiming tasks. The findings showed that performing an aiming movement under the same spatial and temporal constraints (criterion time and target width) resulted in similar accuracy and variability of movement time under both time-minimization and time-matching constraints that is most strongly revealed in the joint entropy of space and time. This contrasts with the original observation of Carlton (1994) that the task categories of
matching and minimizing movement time lead to different speed-accuracy outcomes in aiming tasks.

A limitation of the Carlton (1994) study is that it controlled the equality of the spatial constraints by using a dispersion of 95% of aimed movement to be a target width between time-matching and time-minimization task, but ignored the temporal constraints that were not equally controlled for two tasks (e.g., 400ms for time-matching task; move as fast as possible for time-minimization). This could be a reason that participants’ movement time in time-matching task was different than in the time-minimization task. In addition, the findings of spatial dispersion of the time-minimization task only showed the percentage of hitting a target and there might be a trade-off between movement time and movement endpoint within an individual, as is shown here.

Under the time-minimization instruction, participants performed with a slightly lower mode of movement time and average movement time than under the time-matching condition. The reduced movement time in the time-minimization task might be related to the participants’ ability to approach more closely to their maximum velocity, while participants do not push themselves to the maximum velocity limit in the time-matching task, as long as they can match the time goal. It has been proposed that this strategy leads to a slower movement time when participants try to hit the target accurately in time-matching (Carlton, 1994). In addition, participants performed on average shorter movement times in the last two days that might be due to a familiarity with the experimental situation or practice effect.

The analysis of the skewness and kurtosis of movement time revealed that the movement outcome distribution shifted toward high positive skewness and
leptokurtic at high average velocity. In other words, participants tried to approach their limit to minimize the movement time when moving as fast as possible or matching the criterion time followed the task instructions. This result supports the prediction of Hancock and Newell (1985) that these derivatives of the 3rd and 4th moments are systematically influenced by the task constraints. Our comprehensive analysis of the movement outcome distribution provides a more complete interpretation of the relation between speed and accuracy that can also reveal the interaction between these dependent variables rather than only considering constant error or variable error alone. The complete movement speed-accuracy description needs to consider how the four moments of the distributions vary under the velocity continuum conditions.

The spatial bias gradually closed to the center of target through different days of practice while maintaining a short movement time, although this effect did not reach a significant level. This is consistent with the general finding that participants often undershoot at high movement velocity conditions (Elliott et al., 2004; Hancock & Newell, 1985). Additionally, there is a trade-off within a shorter movement time and larger movement variability in terms of information entropy in two tasks through different days.

The information entropy measure of the variability of movement outcome is based on the actual frequency distribution despite the movement dimension or shape of the distribution (William, 1997). Traditionally, the explanation of movement variability implicitly assumes a normal distribution of movement outcome (e.g., Welford, 1968; Schmidt et al., 1979). Skewness and kurtosis may bias the estimate of standard deviation of movement outcome particularly if the participant’s performance is at or toward the extreme conditions (e.g., high movement velocity). As described above, the information
entropy of movement time decreased in the last two days and led to an increase in information entropy of spatial bias. This result reveals that movement outcome is being distributed more over the center of the target when the participant moves faster. The joint information entropy of movement variability between space and time essentially represents a trade-off rather than that shown previously in formulation of the movement speed-accuracy function, trading speed for accuracy (e.g., Elliott et al., 2010; Fitts, 1954; Meyer et al., 1990; Plamondon & Alimi, 1997; Schmidt et al., 1979).

The joint entropy represents an alternative view of the trade-off between movement time and movement spatial outcome variability (Lai et al., in press). It is possible that information entropy of movement time decreased in the last two days due to a practice effect or it might be that participants just moved faster because the information entropy of spatial bias increases at the same time as well. However, the joint entropy did not change significantly through the four days of practice over the different tasks thus confirming the null effect of task category. The similar joint entropy is the resultant outcome between the variability of movement time and spatial bias complementing each other, that is, accommodating a trade-off. In this case, as information entropy of movement time is reduced the information entropy of spatial bias is increased, thus compensating each other leading to a similar joint space-time entropy. This is consistent with the Hancock and Newell (1985) prediction that under the same spatial (amplitude, target width) and temporal (criterion time) constraints, the participants’ movement outcome properties would be similar.

A postulation from our findings is that the joint entropy is a measure that is able to capture the invariant of the interaction between constraints and individual. Invariants are
usually described in terms of informational or topological properties of the system under scrutiny (e.g., Kugler, Kelso, & Turvey, 1980). Nevertheless, as an emergent property that captures the distributional features of the system encompassing both time and space, it is a strong candidate to be a measure to describe the potential of an individual when temporal and spatial demands are imposed. Indeed, as we observed in the present study, there was a trade-off between spatial and temporal movement properties when constraints are maintained but the joint entropy was unaltered. This is evidence that the distributional properties of the system under a given set of constraints are invariant reflecting an encompassing dynamical landscape that characterizes the task-individual interaction (Hsieh et al., 2015).

In summary, this study provided a new approach to examining the relation between movement speed and accuracy that provided an alternative analysis (joint information entropy) to interpret movement variability under the same spatial and temporal constraints but in tasks that are different (time-minimization vs. time-matching). The findings showed that under the same spatial and temporal constraints the emergent movement outcomes share the same properties, independent of tasks. Moreover, the unified joint entropy provides an alternative description of the speed and accuracy trade-off as a function of movement time and movement amplitude.
CHAPTER 6: GENERAL DISCUSSION

The focus of this dissertation was to examine the variability of discrete aiming movements and isometric force production through the probabilistic approach of joint information entropy (Williams, 1997). There were two important issues that were explicitly addressed. The first issue of this dissertation focused on the application of the foundation of space-time principle that spatial and temporal error are considered as complementary features (Hancock & Newell, 1985) through the methodology of information entropy - by using the concept of joint probability to unify the spatial and temporal movement variables into a single parameter to investigate both dimensions in the same frame of reference. This approach follows from the assumption that time and spatial properties of error should not be considered isolated. In this sense, information entropy is the strongest candidate to represent this function and is based on the probability approach that is irrespective of the measured movement dimensions, unit, and the assumption of a normal distribution (e.g., standard deviation).

The second issue of the thesis focused on the different functions of the speed-accuracy trade-off (logarithmic function and linear function) and the proposition that they were emergent properties under same constraints (spatial and temporal constraints) in different tasks (time-minimization and time-matching). In consonance with our previous experiments, the constraints would define the spatial-temporal relation in a given task. In this section, we expanded our concern to test the idea of different functions that would follow from the constraints-based proposition of our previous experiments.
Movement variability as a white Gaussian noise

Fitts (1954) used information theory to describe and explain how the movement time as a dependent variable changed accordingly to different combinations of amplitude and target width (index of difficulty, ID). This approach assumes a fixed information transmission capacity of the motor system. It implies that when ID increases, the perceptual-motor system requires more time to transmit a signal to preserve movement accuracy. From his equation of an index difficulty, he assumed that participants’ movement outcome could be characterized as white Gaussian noise. This noise is analogous to be a target width (W). The information capacity brought by Fitts is resultant from his view of the human motor system in respect to Shannon’s information theory.

Schmidt’s impulse variability theory (Schmidt et al., 1979) described the movement variability by measuring within-subject standard deviation (effective target width, \( W_e \)) of the movement outcome. Effective target width was defined as the dispersion or inconsistency of performance when aiming to a target. Following this concept, the impulse variability theory predicted that the standard deviation of movement endpoint is a linear function of movement velocity. The main hypothesis was that when the performer produces larger impulses (either produced large force or shorten the movement duration in fixed impulse size), there is a concomitant increase in movement outcome variability resulting from enhanced noise within the muscular system. In sum, both approaches deduced that movement variability was influenced by white Gaussian noise within the human motor system. We also interpret that inherent motor noise is one of the factors that cause movement variability. However, the assumption of a normal distribution of movement outcome is unwarranted (Lai et al., 2005; Newell et al., 1993).
Hancock and Newell (1985) suggested that the distribution of movement outcome would be affected by average velocity. Moreover, only reporting constant error (deviation relative to target) and standard deviation (inconsistent) is not enough to characterize the movement distribution. They pointed out that the performance distribution shifts from high leptokurtic and positive skewness at low velocity through a normal distribution that approximated 50% average velocity to high platykurtic and negative skewness. Adding the analysis of skewness and kurtosis leads to a comprehensive description of the constant error and variable error in the relation of speed and accuracy.

The findings of Experiment 3 reflected this concern in that the movement outcome was not always normally distributed. Participants gradually decreased their movement time (average movement velocity increased) to hit a center of target from the 1st day to the 2nd day. In consonance, the value of skewness and kurtosis of movement time and spatial distance from the target center also increased across different days. Thus, the distribution of movement outcome deviated from normality, supporting the proposition of Hancock and Newell (1985). They pointed out that the distribution of movement outcome within a given amplitude will change over the range of average movement velocity. In this case, the previous theoretical assumption that movement variability could be represented as a white Gaussian noise should be reconsidered.

The use of information entropy focused on the actual frequency distribution of data. By using this method, we avoid the problem of the spatial or temporal constraints channeling the distribution of movement outcome and deviating its distribution from
normality. This would bias the estimation of the movement variability if the standard deviation (or its transformations: CV, SUE, etc.) measure was used.

**The complementarity of spatial and temporal variability**

The ability to control our performance in space and time is essential in daily life. Most of the attention in the speed-accuracy literature has focused on the relation between movement duration and spatial variability. Researchers in general have ignored the importance of temporal variability influencing the movement accuracy. In the classic Fitts (1954) approach, only the spatial dimension of movement accuracy is considered, and even in this protocol the spatial error has often not been reported (i.e., percentage of missing the target). Fitts (1954) did not discuss how the motor system processes information in the temporal dimension.

Similarly, impulse variability brought the concept of force and time relation to explain the movement variability (Meyer et al., 1988; Schmidt et al., 1979). They suggested that the predicted spatial error is proportional to the variability in impulse. In contrast, temporal error was hypothesized to be uninfluenced by an increase in the movement amplitude or average velocity within a given movement time. Overall, the previous literature emphasized the influence of movement variability on spatial dimension. When considered, the temporal dimension has generally been studied independently, using a variety of experimental paradigms and motor tasks.

It is noted that peak force is a result of the rate of force production and the amount of time that rate is produced (Carlton & Newell 1985). This relation leads to a consideration of the relative contributions of force and time to movement error when applying the space-time viewpoint in human movement. The results in Experiment 2
showed that even though the time to peak force was fixed in one of the experiments (400ms), the timing error still deviated from the rate required implicitly by task demands when the percentage of force increased (Carlton & Newell, 1985; Newell & Carlton, 1985). The findings confirm that time to peak force is a critical variable that influences the peak force variability function.

Moreover, the results of Experiment 1, 2 and 3 reveal the trading of spatial/force error with temporal error leads to a consideration of the relative contributions of two dimensions (space-time or force-time) to movement accuracy when applying the space-time principle to human movement. This observation indicates that the space and time dimensions were complementary rather than supplementary. In other words, these two dimensions are dependent of each other, rather than independent.

**The unified space-time movement variable**

It is intuitive that human movement takes place in both space and time. Specifically, if we think about when a performer tries to hit a baseball or to catch a Frisbee, he/she does not only need to know where the ball will be, but also when the ball will be in a certain location. After Hancock and Newell (1985) proposed that spatial and temporal error should be considered as complementary features, there there have been studies that followed this concept to interpret the relationship of movement accuracy between space and time dimensions (e.g., Danion et al., 2014; Lai et al., 2006). However, temporal and spatial error were still separated as two different dimensions.

In the present study, we integrated the individual spatial and temporal error data into a joint space-time entropy to investigate both dimensions in the same frame of reference. Our Experiments 1 and 2 revealed a U-shaped function for the movement speed and
accuracy relation. This U-shaped function shows that different spatial/temporal constraints influenced the trade-off in movement accuracy when the space and time dimensions of performance are integrated with each other. In this case, participants maintained temporal accuracy with high movement velocity in the temporal dimension but resulted in decreased spatial accuracy, and vice versa when participants performed in the accurate condition.

In addition, the lowest entropy was located at the region where the relative contributions of space and time constraints to movement outcome were comparable (Hsieh et al., 2013; Lai et al., in press). This region corresponds to the idea of optimal condition where both spatial and temporal entropy is minimal. Moving faster or slower from this critical relation increased the joint entropy on the space-time dimension (Hancock & Newell, 1985). This result follows from the Newell (1980) proposition that there is an appropriate point on the velocity continuum at which to operate. This would vary accordingly to the task and the cost and pay-offs associated with errors of space and time.

To understand the general phenomenon of speed-accuracy trade-off, it is required that there is a combination of spatial and temporal measures into a space-time description. The cohesion of the methodological approach in this study provides an alternative perspective and a more complete account from which to describe and explain the relation between movement speed and accuracy.

**Movement outcome as an emergent property**

Human movement has been proposed as being the emergent product of organism, task and environment constraints interaction (Newell, 1986). To date, there is still a
debate on the relation between speed and accuracy trade-off as a linear or logarithmic function. The argument is that the different functions arise from different experimental paradigms (timing-minimization or time-matching). Fitts (1954) found that the logarithmic relation exists between the average movement time and the task difficulty when participants were instructed to move as fast as possible while maintaining a given percentage of success (in his case, 95%). Schmidt et al. (1979) discovered the linear relationship between average movement velocity and effective target width when participants were instructed to match the specified movement time.

Nevertheless, Hancock and Newell (1985) suggested that the general speed and movement accuracy function would be a result of the interaction between movement duration, amplitude and movement accuracy. We argue that this general description of accuracy as a function of movement duration and movement amplitude has not been found due to researchers employing different constraints in the task. For instance, researchers preferred either to employ target widths and maximal velocity (e.g. Fitts, 1954) or limited range of velocity with the actual movement error been recorded within a given movement amplitude (e.g. Schmidt et al. 1979).

The findings in Experiment 3 showed that discrete aiming movements under the same spatial and temporal constraints (criterion time and target width) resulted in similar movement properties in both time-minimization and time-matching tasks. This is shown most clearly when the movement outcome is described by the joint entropy. These results support the idea of constraints as the main factors to influence the movement outcome, independent of the task.
In this experiment, the assigned criterion time in time-matching task pushed the participants to achieve their maximum movement velocity. This can be observed from the analysis of the movement distribution that shifted toward high positive skewness and kurtosis at high average velocity and also led to an increase in information entropy of spatial bias. This contrasts with the original observation of Carlton (1994) that the task categories of matching and minimizing movement time lead to different speed-accuracy outcomes in aiming tasks. The limitation of Carlton’s study was that it controlled the equality of the spatial constraints by using a dispersion of 95% of aimed movement to be the target width between time-matching and time-minimization task, but it ignored the temporal constraints. That is, the time criterion was not controlled for those tasks (i.e., 400ms for time-matching task; move as fast as possible for time-minimization). In addition, he only provided the percentage of success rate without examining the actual distribution of movement endpoint limiting the possibility of a trade-off between time and space. In fact, our third experiment showed that there was a trade-off between spatial and temporal movement properties when constraints are maintained but the joint entropy was unaltered.

A conclusion from our findings is that joint entropy of spatial and temporal errors is an appropriate variable that can capture the main properties of movement outcome. The joint entropy would characterize the outcome from the task-individual interaction.

**Limitations and future directions**

The findings of this dissertation provided the examination of a space-time approach to understanding the movement speed and accuracy trade-off relation. To understand the general problem of speed-accuracy trade-off requires a combining of spatial and temporal
measures into a space-time explanation of the movement relation between the speed and accuracy trade-off. However, there are limitations and suggestions should be mentioned in this last section for future studies.

First, in order to calculate the information entropy, larger sample sizes are encouraged. This sample size has an effect on the distributions of movement outcome and it has a significant role on the number of bins used in the data analysis. There is no specific number of bins or bin size that is supposed to fit for a certain sample size (William, 1997; Scott, 1992). The appropriate way for a bin number or bin size is to consider varying the number of bins and also cover the whole range of performances across different individuals.

Second, only a limited range of movement speed-accuracy conditions was examined in this work. In order to understand the complete description of space-time error function, one should examine not only a broader range of the continuum of movement velocity but also include the time-minimization and time matching tasks in different distances to actually confirm the U-shaped and common function of speed-accuracy trade-off. In addition, only performance outcome reported in this study, examination of the submovements through movement trajectory across changes in different conditions and tasks might lead to the discovery of the mechanism that are directly related to the general space-time error function (Carlton, 1994; Wright & Meyer, 1983). In these two cases, systematically examining a broad range of conditions within two different tasks can lead to fully understanding the phenomenon and control strategy under these two functions.
REFERENCES


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FOOTNOTES

1. The analysis of movement variability here only focused on the Euclidean distance from the target though the findings show the same effects for the primary movement direction (x-axis).
APPENDIX A: Calculation of space-time entropy from the joint force and temporal error probabilities

In this Appendix A, we provide an example of the calculation of the joint probability of force and temporal error for the isometric single finger force task of a single participant in the fast condition in Experiment 2 (25% of force level). The upper left panel and upper right panel of Figure A1 show the distribution of temporal error and force error individual trial data of the 90 trials used in the analysis of this condition, respectively. The lower left and right panel shows the 2 dimensions and 3 dimensions of force-temporal error distribution.

Scott’s (1992) rule holds that the number of bins should be at least $a \times s \times n^{-1/3}$, where $a = 3.49$, $s$ is an estimate of the standard deviation and $n$ is number of trials. As Scott noted many authors suggested that for real data sets histograms based on 5-20 bins usually suffice. With the 90 trials in the individual force and temporal dimensions 20 bins were used to cover the range of the force and temporal error data.

To calculate the performance entropy of isometric single finger force task, one needs to know the probabilities of the data distribution of the movement outcome. In the method for obtaining the probabilities in the experimental data, we used properties of the actual frequency distribution and calculated the entropy ($H_p$) obtained with the following equation:

$$H_p = \sum P_i \log_2 (1/P_i)$$  \hspace{1cm} (A1)

where $P_i$ is defined in the frequency distribution, indicating the relative frequencies of data points in the $i$th bin (Shannon & Weaver, 1949; Williams, 1997). Here we used the concept of joint probability to calculate the joint probability of force and temporal error.
for the isometric single finger force task and to observe how different force-time constraints influence the values of information entropy. \( P(x, y) \) is the joint probability that can be obtained from the following equation (Williams, 1997, Eq. 27.12, p.413).

\[
P(x, y) = P(x)P(y|x) \quad \text{(A2)}
\]

and therefore,

\[
H_p = \sum P(x_i, y_i) \log_2 \left( \frac{1}{P(x_i, y_i)} \right) \quad \text{(A3)}
\]

Figure A1. The force (upper right panel) and temporal (upper left panel) distributions of the individual trial error data of the 90 trials of a single participant in the fast condition in Experiment 2 (25% of force level). The distribution of the unified force-temporal errors is organized for 2 dimensions (lower left panel) and 3 dimensions (lower right panel with 20 x 20 bins).
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