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MEASURING TWO-STEP ALGEBRA EQUATION FLUENCY OF SECONDARY PARTICIPANTS

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Abstract

Mathematical fluency is the speed and accuracy of basic addition, subtraction, multiplication and division. Fluency in these areas leads to freeing of working memory with the eventual goal of allowing participants to attain the skills needed to complete multi-step problems, work more efficiently, exercise higher order thinking skills, and complete secondary algebra course work.

This study examines 131 high school participants with and without disabilities that are enrolled in algebra one classes and measures their fluency in solving two-step algebra equations. Additionally, the study determines if there are commonalities in demographic information and participant history of academic performance that correlate with the participant's performance on the fluency probes. The data show a connection between the PSSA scores, course grades, and performance on the fluency probes.

Implications for practice are to identify specific fluency rates for solving algebraic equations for both participants with and without disabilities, develop fluency measures targeting the solving of algebraic equations, and to recognize the relationship between fluency rates of solving algebraic equations and end of year state assessments for participants in grades 9-12.

Keywords:

algebra, fluency, two-step algebra equations, secondary participants, mathematical instruction, automaticity

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Math is important and as a result national reforms have been initiated in the form of the Common Core State Standards in Mathematics (CCSS-M) with a focus on the need for skill and knowledge in algebra, more rigorous assessments, and added emphasis for participants to complete at least one algebra course as a graduation requirement. The purpose of the CCSS-M is to use research-based instructional strategies to teach students mathematical concepts in an organized way across the school year and grades. The standards require students to solve real-world problems in order to develop the knowledge and skills they will need to be prepared for mathematics in college, career, and life.

The data supplied by The National Assessment of Educational Progress (NAEP) of public school students in the United States indicate that students in the United States are under-performing in mathematics. The NAEP mathematics scores range from zero to five hundred for grades four and eight. In 2013, the average NAEP mathematics scores for fourth grade and eighth grade students were higher than the average scores in all previous assessment years. From 1990 to 2013, the average fourth grade NAEP mathematics score increased by twenty-eight points, from two hundred-thirteen to two hundred forty-two. Also during 1990 to 2013, the average eighth grade score increased by twenty-two points, from two hundred sixty-three to two hundred eighty-five. Twelfth graders were most recently assessed in 2009; in that year, the average twelfth grade mathematics score was three points higher than in 2005, the first year that the revised assessment was administered.

The NAEP study states that in 2013, some 83% of fourth grade students performed at or above the Basic achievement level and 42% performed at or above the proficient level in mathematics. While the percentage of students at or above the basic level in 2013 was not measurably different from that in 2011 (82%), it was higher than the percentage in 1990 (50 %).

A higher percentage of fourth grade students performed at or above proficient in 2013 than in all previous assessment years. In 2013, some 74% of eighth grade students performed at or above basic and 35% performed at or above proficient in mathematics. The percentages at or above basic and at or above proficient in 2013 showed no measurable change from 2011, but they were higher than the percentages in all assessment years prior to 2011. The percentages of twelfth grade students performing at or above basic (64%) and at or above proficient (26%) in mathematics were each three percentage points higher in 2009 than in 2005. Testing accommodations (e.g., extended time, small group testing) for children with disabilities and English language learners were not permitted in 1990 and 1992 (U.S. Department of Education, National Center for Education Statistics, 2014).

The NAEP statistics clarify the need for more rigorous curriculum and interventions for students in the United States. To be specific, the language of algebra fluency is increasingly becoming an essential life skill for adults. Alan Schoenfeld describes (in Lamcampagne, Blair, and Kaput [1995] p.231) algebra as "an academic passport for passage into virtually every avenue of the job market and every street of schooling." Further, Hyman Bass (2006) specifies that algebra is viewed as a foundation for all mathematics and science. The National Council for Teachers Mathematics (NCTM) found that completion of Algebra II doubles the probability of college graduation and stresses teaching mathematics should be based on conceptual understanding with an emphasis on computational fluency, and “computational fluency should develop in tandem with understanding of the role and meaning of arithmetic operations in number systems” (NCTM, 2000, p. 32).

The Partnership for Assessment of Readiness for College and Careers (PARCC) Model Content Frameworks for Mathematics states that fluency is important in high school mathematics.   The frameworks note that fluency in algebra will lead students to the ability to observe structure and patterns in problems. PARCC further suggests that fluency in algebra can allow for progress beyond the college and career readiness threshold toward readiness for further study and careers in science, technology, engineering, and mathematics (STEM) fields (PARCC Model Content Frameworks for Mathematics, 2012). President Barack Obama stated that, "... Leadership tomorrow depends on how we educate our students today—especially in science, technology, engineering and math." Obama has articulated a clear priority for STEM education: within a decade, American students must "move from the middle to the top of the pack in science and math." (U.S Department of Education, 2015)

The President's 2015 Fiscal Budget Proposal has designated one hundred seventy million dollars in STEM subjects for teachers and students in the nation's schools. Specifically, he has called on the nation to develop, recruit, and retain one hundred thousand excellent STEM teachers over the next ten years. He also has asked colleges and universities to graduate an additional one million students with STEM majors. These improvements in STEM education will happen only if Hispanics, African-Americans, and other underrepresented groups in the STEM fields—including women, people with disabilities, and first-generation Americans—participate. (U.S. Department of Education, 2015)

The following CCSS-M standards note the importance mathematical fluency to lay the foundation for algebraic problem solving at the secondary level: Operations & Algebraic Thinking K.OA.A.5: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from; fluently add and subtract within five; Operations & Algebraic Thinking 2.0A.2:   fluently add and subtract within 20 using mental strategies; by end of grade two, know from memory all sums of two one-digit numbers; Operations & Algebraic

Thinking 3.0A.7: fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., knowing that 8x5=40, one knows 40/5=8 or properties of operations; by the end of third grade: know from memory all products of two one-digit numbers; Number & Operation Base Ten Three. NBT: use place understanding to round numbers to the nearest 10 or 100; fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction; 5.NBT.5: Perform operations with multi-digit whole numbers and with decimals to hundredths; fluently multiply multi-digit whole numbers using the standard algorithm; 6.NS.2: compute fluently with multi-digit numbers and find common factors; fluently divide multi-digit numbers using the standard algorithm; 6.NS.3: compute fluently with multi-digit numbers and find common factors and multiples; and 7.EE.4: equations use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. (CCSS-M, 2010).

Wherever the word fluently appears in a content standard, the word means quickly and accurately. It means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself. A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to participant understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all participants to meet the standards that call explicitly for fluency (PARCC, 2012, p. 9). Mathematical fluency or automaticity will free up the working memory. Grover Whitehurst, the Director of the Institute for Educational Sciences (IES), noted this research during the launch of the federal Math Summit in 2003 that cognitive psychologists have revealed that humans have fixed limits on the attention and memory that can be used to solve problems. Whitehurst believes that these limits can be circumvented by having certain components of a task become so routine and over-learned that they become automatic (Whitehurst, 2003).

The final report of the National Advisory Panel of Mathematics (NAPM) suggests that we must prepare students for algebra curriculum that will simultaneously develop conceptual understanding, computational fluency, factual knowledge and problem solving skills. However, limitations in the ability to keep many things in mind (working-memory) can hinder mathematics performance. Practice can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand. Learning is most effective when practice is combined with instruction on related concepts. Conceptual understanding promotes transfer of learning to new problems and better long-term retention (NAPM, 2008).

In order for participants to attain higher order math skills, they must be able to recall basic math facts fluently that will free up working memory for more complex math problems. A 2014 study, *Hippocampal-Neocortical Functional Reorganization* emphasizes the shift to memory-based problem solving as the assurance to children's cognitive development in arithmetic as well as other domains. The hippocampus portion of the medial temporal lobe (MTL) appears to be critical for children's learning of mathematics in ways not as evident in adults. Evidence supports as the hippocampal becomes increasingly engaged in its known role in learning and memory, which supports the theory hypothesis that hippocampal-dependent memory processes is important in the development of children's memory based problem-solving strategies. The memory increases during child development and stabilize through adolescence and adulthood. This 2014 study supports two-step equation fluency research as it reiterates the idea that freeing the working memory will lead to automaticity in basic factual recall and fluency.

Impecoven-Lind and Foegen (2010) describe the importance of teaching algebra to all students: achievement in advanced mathematics, college entrance, and economic equity in the workforce. In particular, these authors studied the five to eight percent of the school-aged population impacted by a specific learning disability in mathematics. The researchers note while these students may be performing at an average or above average intelligence level, they likely have deficits in language, memory, attention, or metacognition that impact their procurement of mathematic skills. In regard to fluency and solving two-step algebra equations, memory is relevant remember the steps of algorithms. Without automaticity in algorithms and basic skills, fluency will not be achieved. Language deficiencies impede fluency in two-step algebraic equations as it leads to impairments in recall arithmetic facts, calculations, and solving multi-step problems; all skills necessary to become fluent in two-step algebra equations.

Given importance of algebra and needs of students of students with a specific learning disability in mathematics, teachers need to have access to appropriate assessments for achievement in mathematics. According to Developing Progress Monitoring Measures for Secondary Mathematics: An Illustration in Algebra (Foegen, Olson, and Impecoven-Lind, 2008), there is a deficiency in access to assessments in algebra, geometry, and advanced algebra. This study follows the work of teachers and researchers working collaboratively to develop curriculum based measured for algebra one know as, Project AAIMS, which will be used for progress monitoring. There is a need for robust indicators that integrate a variety of concepts and skills from the curriculum. Some researchers have demonstrated that during the elementary and middle school grades fluency with the basic facts serves as a robust indicator (Foegen, 2000; Foegen & Deno, 2001,). These same authors believe that while measuring fluency is not a full representation of the total curriculum, it is a competent proxy of global outcomes as it correlates with other indicators of mathematics proficiency. Assessments grounded in evidenced based research are necessary for data driven interventions.

Using Project AAIMS progress monitoring data, Foegen completed her next body of research, *Algebra Progress Monitoring and Interventions for Students with Learning Disabilities* in 2008. Foegen collected sufficient progress monitoring measures to suggest that these tools may have sufficient technical adequacy to be used as indicators of student development in algebra. Foegen is suggesting that Project AAIMS can be used to notify students when a student is not making adequate progress in algebra and implement or change an intervention. Foegen has demonstrated each stage of development leading to data driven instruction for students with special needs in algebra. As Foegen notes in this 2008 body of research, "the research reported and reviewed her suggests that there is a growing need for mathematics assessment and intervention tools for secondary students with learning disabilities." The goal of this measuring the fluency of two-step algebra equations is further the field of research in algebra assessments, interventions, and instruction for students with and without disabilities.

Geary's research (2004) specifically draws attention to the need for the attention for more research in the area of mathematics and learning disabilities as five to eight percent of school children are identified with a mathematics learning disability (MLD). A MLD would manifest in conceptual or procedural aptitudes that define the mathematical domain which are due to underlying deficits in areas such as processing, working memory, retention, spatial relationships, or long-term memory. In his study, Geary discusses the difference between conceptual and procedural mathematics problems and that errors appropriate instruction and assessments for students can lead to a misidentification of a mathematics learning disability (MLD). Students with MLD exhibit developmental delays in basic math skills such as counting, adding, subtraction, and multiplication. As a result of these delays, multi-step word problems become laborious, time consuming, and more procedural errors occur. Students with a MLD often have comorbidity issues which effect long-term memory and mathematic procedures that require memorization can cause more student errors. Algebra fluency may be affected when the student has a MLD. Procedural errors, processing issues, struggles with memorization of basic facts, or an inability to attend to task can tax the working memory and impede algebra fluency from developing.

Geary's research draws attention for the need for more appropriate diagnostic assessments for mathematics learning disabilities, assessments for data driven instruction, and research in more complex mathematics such as algebra. Geary's findings support the need for this study's efforts to improve algebra one two-step equation fluency assessments and interventions.

Anne Foegen's study (2008) further reiterates the importance of algebra on a participant's ability to progress in their post-secondary career or education and focus' attention on progress monitoring in algebra. This study establishes a strong case for further research in instructional interventions in algebra progress monitoring for participants with specific learning disabilities. Foegen's recommendations for further research is to focus attention on accessing assessment tools, finding methods of monitoring participant progress, use interventions that are evidence-based, and make data driven decisions. Finally, Foegen supports the use of instructional methods rooted in evidence based research and data that are especially effective with participants with specific learning disabilities.

Due to the importance of mathematical fluency, change in the national standards, and lack of research in the area, this study measures the current algebra fluency secondary participants sample from a rural school in Pennsylvania and establishes the need for further research in algebra fluency. The research questions addressed: (a) what is the two-step algebra equation fluency among a sample of secondary participants? (b) are there commonalities in demographic information and participant history of academic performance that correlate with the participant's performance on the fluency probes?

Methods

Participants and Setting

The participants are one hundred thirty-one participants enrolled in algebra one, who gave parental consent and participant assent for this project. The sample represents 78% of the total number of participants currently taking algebra one. The sample population includes participants between the ages of thirteen and eighteen, participants with and without disabilities, mixed race, and mixed gender. For confidentiality purposes, the study does not include socio-economic information about the sample. However, according to the 2012 United States Census Bureau, this community has a population of 5,904 people. 83.2% of the community are high school graduates and 12.7% of the population has a bachelor's degree or higher. The median household income is $33,569 and 21.6% of families are living below the poverty level. Further, the population is inclusive of participants who performed in the full range of the Pennsylvania State Standardized Assessment (PSSA) in mathematics from below basic to advanced levels. Since the participants are from diverse grade levels, the most recent math PSSA score available was used for each participant. In the case of the eighth graders, their 2013 PSSA seventh grade scores were used for this study. For all of the other subjects, the most recent eighth grade PSSA score was recorded. If a participant did not have a score available, the participant attended a private parochial school and did not take the PSSA exams. Eighty-one participants (62%) in this sample are being exposed to the standard algebra one curriculum, thirty-one participants (24%) are eighth graders participating in the advanced curriculum, and nineteen (15%) of the ninth graders are participating in the academic curriculum. There are seventy one (54%) female participants and sixty (46%) male participants. One hundred twenty eight (98%) of this sample population is Caucasian and seven (5%) are identified with a disability.

The setting is the algebra one classes in a rural western Pennsylvania high school, which includes eighth through twelfth grades. The algebra one classes are full-year courses assessed at the end of the year by the Keystone Exam. The high school has four levels of algebra one: advanced, academic, standard, and co-taught. Eighth grade participants have the option of taking advanced algebra one based upon their performance in seventh grade pre-algebra and teacher recommendation. Academic algebra one is for ninth grade participants who demonstrate exceptional performance in eighth grade pre-algebra or need to repeat the course from eighth grade. Standard algebra one is for all remaining participants' in grades nine to twelve. According to the mathematics department head, the same objectives are followed in all of the algebra one courses, but the advanced and academic courses are "more in depth". The co-taught class is also a standard algebra one course, but combines general education and special education participants. It is co-taught by a certified secondary mathematics teacher and a certified secondary special education teacher using the one teach and one assist co-teaching model.

Materials

Three parallel form thirty question probes with two-step algebra equations using positive and negative numbers were created by the principal researcher. The development of the probes was guided by the Common Core State Standards for Mathematics (CCSS-M), specifically the Algebra standards. The specific standard targeted on the probes is CCSS-M 7.EE.4. The probes were reviewed by a faculty researcher with expertise in this area, then the principal, and high school mathematics department head also reviewed the probes to ensure that they were appropriate for the study participants. After consulting with the department head, the number (thirty) of questions was selected to be more than the students could complete in one minute to avoid the ceiling effect.

As noted by Foegen's 2007 literature review on mathematics progress monitoring, her team was unable to find any measures developed and used at the high school level. According to Foegen, most curriculum based mathematic measures and studies are at the elementary and middle grade levels. There are no published algebra one probes that measure fluency in two-step equations to model or use for this study. Foegen stresses this gap in knowledge and need for further research in measures in high school mathematics such as algebra (Foegen, Jiban, & Deno, 2007).

Independent and Dependent Variables

This study measured four independent variables. These independent variables are the grade level of the student, the Algebra One course in which the student was enrolled, performance on the eighth grade Pennsylvania State Standardized Assessment in Mathematics, and whether or not the student has a learning disability. The dependent variable is the average fluency score of secondary students' ability to solve two-step algebra equations.

Procedural Integrity

The primary researcher established procedural integrity by implementing the probes in each classroom environment and using standardized directions which she read to the students for each of the three probes. The same materials were available to students during each session. Students were assessed in the same environment with the same instructional staff for each probe. The primary researcher followed the same assessment schedule with each student group on the days students were assessed. The one minute timer on the primary researcher's cell phone was used in across all environments for each probe.

Procedure

Participants were assigned a code number by their teacher to ensure confidentiality on the probes. Only the teachers know the identity of the participants and their code numbers. The participants used their same code on each probe instead of their name. Each participant was asked to complete a thirty question probe on Monday, Wednesday, and Friday. Participants were permitted to use a calculator. The decision to use calculators was based upon the current practice to solve two-step algebra problems in the algebra one classrooms at this high school. Given sixty seconds, participants were asked to complete as many questions as possible. While the problems are numbered one through thirty, participants were told they could skip problems if they could not solve it and continue with the next problem. Participants were given credit only for problems that are entirely correct (not correct by digit) and the work was not corrected. Participants were not compensated with bonus points, rewards, or financial compensation. The outcomes of the assessments were not attached to participant grades.

Using the codes to maintain participant confidentiality, the teachers provided the age, grade, race, gender, disability, participant disability, recent PSSA mathematics score, math grades, and the current level of algebra one the participant is currently taking and the information was merged on the spreadsheet. The probes were scored by a research team trained by the lead investigator. To ensure reliability a twenty five percent sample of the probes were scored by two researchers who received training on scoring procedures. The reliability resulted in one hundred percent interobserver agreement. The results of the probes were transferred to the study spreadsheet which includes the participant's code number by the primary researcher.

Results

The data in Tables 1-3 and Figure 2 report the difference in the mean probe scores among the four grades (eight, nine, ten, and eleven) of students involved in this study. The average mean across grades was 4.4564 with a range from 3.1667 to 4.9570. The lowest fluency average was the eleventh graders and the highest average fluency average was the eighth graders. The data indicate the standard deviation for the average probe is 1.78381. The outcome of this data answer part of the second research question indicating that there is no significant difference in the performance across mean probes from the sample of eighth, ninth, tenth, and eleventh grade students.

Table 4 and 5 reports the fluency rates of students with and without disabilities. There were seven of one hundred-thirty students in this study identified with a disability. The average fluency rate of a student without a disability was 4.5772 and those with a disability performed at an average fluency rate of 2.3333. The data indicate standard deviation is 1.78381. Therefore, the mean fluency rate in solving two-step algebraic equations is significantly different between students with a disability than those students without a disability.

The data in Tables 6 and 7 and Figures 3 and 4 report on the differences between the mean probe scores between the advanced algebra one, standard algebra one, academic algebra one, and the co-taught algebra one courses. There is a significant difference between the mean scores (2.75) of students in the co-taught algebra one class and the mean scores from the other courses: advanced algebra one (4.9570), standard algebra one (4.2593), academic algebra one (5.1053). There was not a significant difference between the performance of the advanced, standard, and academic algebra one courses.

According to the data in Table 8 and 9 divides students by achievement criteria on the PSSA mathematics exam and compares them with the mean of the fluency probes. The achievement group criteria for the PSSA in mathematics below basic is 700-1170, basic is 1171-1283, proficient 1284-1445, advanced 1446 and higher. The data indicate the mean probe scores are not significantly different among the PSSA mathematics achievement group in eighth, ninth, tenth, and eleventh grades.

Discussion

In review of the student's performance on the PSSA mathematics exam in connection with their mean score on the algebra probe there is no significant difference. The intention of the Mathematics Keystone Exams is to measure the student's progress toward meeting the CCSS-M. The CCSS-M is recommending more algebra fluency. If schools are using student's performance on the eighth grade PSSA mathematics exam as a predictor of how students will perform on the Algebra Keystone Exam, the results of this study indicate that there is not a significant correlation and neither assessment should be used as a forecaster the other exam.

In alignment with the findings of Qin's 2014 study, *Hippocampal-Neocortical Functional Reorganization*, that the memory increases during child development and stabilize through adolescence and adulthood. The statistics in this study demonstrate that the students in the advanced class, who are entirely eighth graders, is higher than students in the other courses in ninth, tenth, and eleventh. Given the eighth grade students were the primary participants in the advance algebra one class, this further gives leverage to the need for early interventions for fluency in algebra as cited by the CCSS-M and the considerable body of research mentioned in this review. One can conclude that by providing earlier algebra fluency interventions and algebra target courses in the elementary and middle school years will prepare students for the outcomes identified in the CCSS-M and PARCC- Mathematics frameworks for students related to graduation, career, and post-secondary education. Further, given these eighth grade students are already taking advanced algebra, they will have the mathematical skill set and working memory to take higher level mathematics throughout their secondary education.

The research cited in this review by Foegen and Geary discusses students with specific learning disabilities (SLD) and the implications of how their deficiencies may impair a student's ability to become fluent in algebra. Geary notes that deficiencies in long-term memory, memorization of basic facts, inability to attend to task, processing issues, and procedural errors can impede algebra fluency from developing. Foegen notes that a student with a SLD may not be able to reach fluency in solving two-step algebra equations, since memory is relevant to remember the steps of algorithms. Without automaticity in algorithms and basic skills, fluency will not be achieved. Students with language deficiencies impede fluency in two-step algebraic equations as it leads to impairments in recall arithmetic facts, calculations, and solving multi-step problems; all skills necessary to become fluent in two-step algebra equations. This study supports the research done by both Foegen and Geary as it relates to students with SLD. The statistics in this study demonstrate that the students with a disability had a significantly lower probe average than students without a disability. Interesting, students had not accessed algebra curriculum until later in their secondary curriculum. The current CCSS-M standards recommend access to algebra curriculum by the middle school grades in order to reach fluency and access higher level mathematics at the secondary level. The findings in this study further support the body of research stressing the need for fluency in basic facts, access to algebra and multi-step algorithms by the middle grades, and early and rigorous math fluency interventions for students identified with SLD in mathematics.

The PARCC Frameworks in Mathematics address the importance of algebra fluency to a student's success in career, post-secondary education, and compete in the growing STEM fields. Considering the average rate of algebra fluency by the students with SLD in this study, one questions whether or not these students will reach the recommended graduation requirement of Algebra II prior to their high school graduation. The body of research in this review supports that failure for students to access Algebra II curriculum can have negative long-term effects on their careers and post-secondary education. More importantly, these students will be at a disadvantage in acquiring employment in the STEM fields where employment will be most available. Given President Obama's one hundred seventy million dollar proposed 2015 fiscal budget to support rigor for STEM education in the nation's public school classrooms, researchers must embrace this opportunity for funding and further study in the area of algebra fluency as it impacts a student's ability to progress in higher level mathematics and the STEM fields of study. In particular, Obama wants to increase the participation of underrepresented populations in the STEM fields such as persons with disabilities. Recruitment of underrepresented populations in the STEM fields, such as persons with disabilities, provides further rationale for research and interventions in algebra fluency. This same population demonstrated the lowest average algebra fluency rate in this study.

Limitations

There are at least two limitations to this study. First, participants were permitted to use calculators, but some chose not use a calculator. Therefore, the inconsistency of calculator use could be seen as a limitation to the results. The conditions were not the same for each participant. Second, the intention of the probes is to measure fluency, but no data will be collected on behavioral or environmental factors that may influence the participant's performance. For example, knowing the probe is not attached to his grade, a participant may not be motivated to complete the probe or do his best work on the probe. There will not be data collected on distractions in the environment that may impact the participant outcomes. In Fuchs et al. (2006) research on elementary grades, it was found that arithmetic and attentive behavior were the only two significant predictors of success in algorithmic computation. Russel and Ginsburg (1984) and Ackerman and Dykman (1995) found that inattentive behavior can cause low achievement in computational fluency.

Implications for Practice

Based upon the results of the data in this study, one can conclude that remedial interventions for basic math facts are not only needed at the elementary level for students, but need to continue for students at the secondary level in algebra with particular attention to students with a learning disability in mathematics. Considering the research by Qin, et. al. (2014), the part of the brain believed to be utilized for basic math fact fluency and memory may reach its peak development by adolescence, therefore, more rigors should be given to developing algebraic fluency early in a secondary student's education. One should consider that students with disabilities may have brain damage or impairment to the hippocampus portion of the medial temporal lobe and develop interventions that will assist students in compensating for these deficits. With the development of interventions, Foegen's research of assessments and progress monitoring discusses the need for effective assessments and methods of progress monitoring in algebra at the secondary level to measure the success of the interventions.

A transition plan is a necessary component of an individualized education plan (IEP) for students with a disability over the age of fourteen. Algebraic fluency should be a part of the transition plan since it can lead students to success in both their secondary and post-secondary pursuits. If students are able to meet the CCSS-M that stress fluency in algebra, students meet proficiency on the Algebra Keystone Exam. Automaticity in algebra will facilitate the ability to advance to higher levels mathematics such as Algebra II, which is a graduation requirement. With the completion of Algebra II, students will double their chances of graduating from college.

Conclusion

Basic math facts (one-digit by one-digit addition, subtraction, multiplication, and division computations) are the most fundamental computational skill for all higher math tasks. Research has historically focused on math fact acquisition, or teaching strategies and procedures for correctly solving these basic math computations (Poncy, Skinner, & Jaspers, 2007). Typically, teachers encourage the use finger counting, number lines, and manipulatives to acquire basic math facts. However, the disadvantages of these techniques are that they allow for counting errors and are laborious and time-consuming in nature. All of these factors interfere with a participant's basic math fact fluency (McCullum & Schmitt, 2011).

While most of the discussion thus far has addressed the cognitive processing theory and freeing working memory in order to increase the ability to ascertain higher order problem solving, math fact fluency is also important from the standpoint of behavioral learning theory. Behavioral research supports opportunities for active class participation and responses will lead to an increase in academic performance. Participants who are more fluent will likely be those who complete more math problems within a time period. Such fluency will increase participants' opportunities to respond and receive reinforcement for correct responses

(McCullum & Schmitt, 2011). Additionally, research has demonstrated that participants who are fluent with math facts are less likely to suffer from math related anxiety (Cates & Rhymer, 2003). Finally, data is present to suggest that fluent participants are more likely to choose to engage in assigned math tasks than dysfluent participants (Billington, Skinner, & Cruchon, 2004).

Although, transition planning for post-secondary education or careers for participants with and without disabilities does not begin until high school, educators need to consider the long-term impact of basic math fact fluency at the elementary school level for all participants. Math fact accuracy and fluency are typically part of the elementary school curriculum; the overwhelming majority of middle- and high-school participants with disabilities have not mastered these skills (NCES, 2009). This is problematic as individuals with math fact deficiencies may be excluded from certain vocational and career paths (Sante, McLaughlin, & Weber, 2001). All participants should have access to the skills needed to complete daily task such as money and time, college graduation, and retaining gainful employment. The results of this study would suggest that some participants are not fluent in two-step quadratic equations at mid-term of their algebra one course work, not passing their algebra one course, not proficient on the PSSA exam and not on track to complete the high school graduation requirements of completing algebra two. Furthermore, many of the participants who are in one of these categories are participants with disabilities and fall in several of the categories. Should the participants' progress in algebra continue at this rate, one could conclude the lack of algebraic fluency could impede post-secondary success in life skills, education, and the work-force. Considering the stress on algebraic fluency in the CCSS-M and PARCC Frameworks in

Mathematics, importance of fluency to academic and behavioral performance, and participants' ability to succeed in their post-secondary life, this study establishes the need for further research.

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Appendix

Figure 1. Sample of Probe One

Code:
Mrs. Tracey Ambuka

Prompt One
Date:__________________________

1.) $4x - 5 = -29$

2.) $-2x + 7 = -3$

3.) $2x - 1 = -1$

4.) $-1 + 3x = 17$

5.) $3 + 6x = 15$

6.) $-5 - 7x = -5$

7.) $5x + 6 = 46$

8.) $-3x + 4 = -29$

9.) $7x + 4 = 53$

10.) $5x - 4 = 46$

11.) $-5x + 5 = -50$

12.) $-3x + 2 = 5$

13.) $-10 - 7x = 11$

14.) $6x - 8 = 4$

15.) $4x + 9 = 53$

16.) $3x + 9 = 3$

17.) $-3x + 10 = 34$

18.) $6x + 1 = -53$

19.) $-6 + 2x = 0$

20.) $-6 + 2x = 0$

21.) $4 - 3x = 16$

22.) $-4 + 6x = -46$

23.) $7 + 4x = 43$

24.) $-3x + 4 = 40$

25.) $6 + 6x = -48$

26.) $4x + 9 = 41$

27.) $6 - 3x = -27$

28.) $7x - 2 = -30$

29) $5x + 4 = 19$

30) $-5x + 3 = -42$

Table 1. Number of Cases by Grade

	Valid		Missing		Total	
Grade	N	Percent	N	Percent	N	Percent
8	27	87.1%	4	12.9%	31	100.0%
9	62	96.9%	2	3.1%	64	100.0%
10	28	96.6%	1	3.4%	29	100.0%
11	5	83.3%	1	16.7%	6	100.0%

Note. N=Number of Cases. The valid cases demonstrate cases who participated in all Three probes. The Missing cases demonstrate students who missed one or more probes. The total cases are the number of students who took at least one or more probes.

Table 2. Average Probe Score Statistics across Grade

Grade	Mean	Standard Deviation	N
8	4.9570	1.40038	31
9	4.5208	1.86859	64
10	4.0460	1.85533	29
11	4.4564	1.57410	1
Total	4.4564	1.78381	130

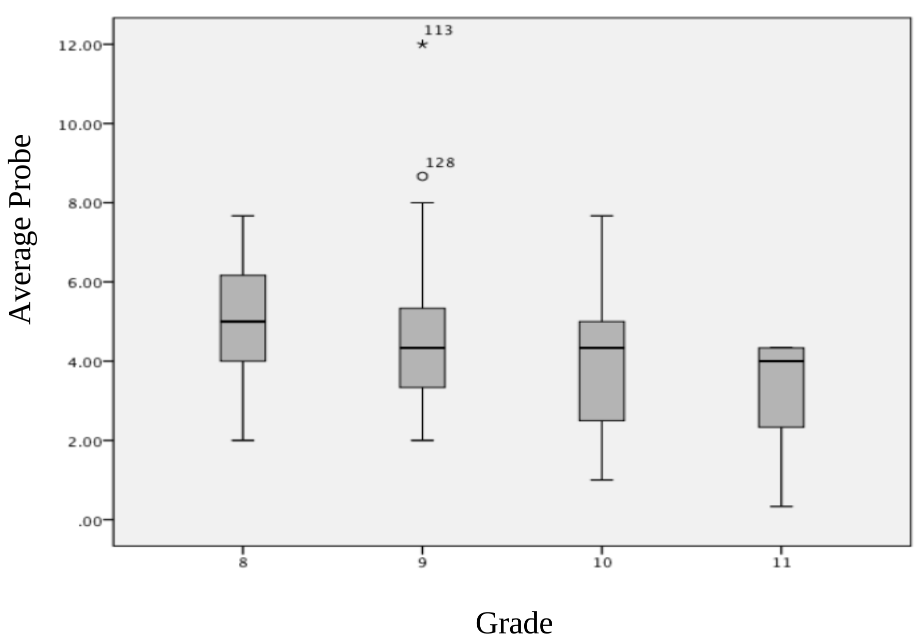
Note. N= Number of Cases. Probe score means and standard deviation by grade.

Table 3. Tests for Statistical Significance of Average Probe Scores between Grades

Source	Sum of Squares	df	Mean Square	F	Significance
Corrected Model	22.899	3	7.633	2.482	.064
Intercept	1118.616	1	1118.616	363.659	.000
Grade	22.899	3	7.633	2.482	.064
Error	387.576	126	3.076		
Total	2992.222	130			
Corrected Total	410.475	129			

Note: df = degrees of freedom, F= frequency. There is no significant difference between the average probe scores of eighth, ninth, tenth, and eleventh graders.

Figure 2. Average Probe Scores by Grade (8-11)



Note: There is no significant difference of average probe scores between grade levels. The lowest performing students are the eleventh graders taking Algebra I.

Table 4. Average Probe Scores of Students with and without Disabilities

Grade	Mean	Standard Deviation	N
No Disability	4.5772	1.73734	123
Disability	2.3333	1.21716	7
Total	4.4564	1.78381	130

Note. N= Number of Cases. The mean and standard deviation of students with and without a disability.

Table 5. Tests for Statistical Significance between Average Probe Scores of Students with and without Disabilities

Source	Sum of Squares	df	Mean Square	F	Significance
Corrected Model	33.348	1	33.348	11.319	.001
Intercept	1302.343	1	316.291	107.352	.000
Disability Status	41.860	1	33.348	11.319	.001
Error	368.615	128	2.946		
Total	2992.222	130			
Corrected Total	410.475	129			

Note: R Squared = .081 (Adjusted R Squared = .074), df= degrees of freedom, F= frequency. There is a significant difference between average probe scores of students with and without disabilities.

Table 6. Average Probe Scores of Algebra One Courses

Course	Mean	Standard Deviation	N
Advanced	4.9570	1.40038	31
Standard	4.2593	1.54760	72
Academic	5.1053	2.38579	19
Co-taught	2.7500	2.30768	8
Total	4.4564	1.78381	130

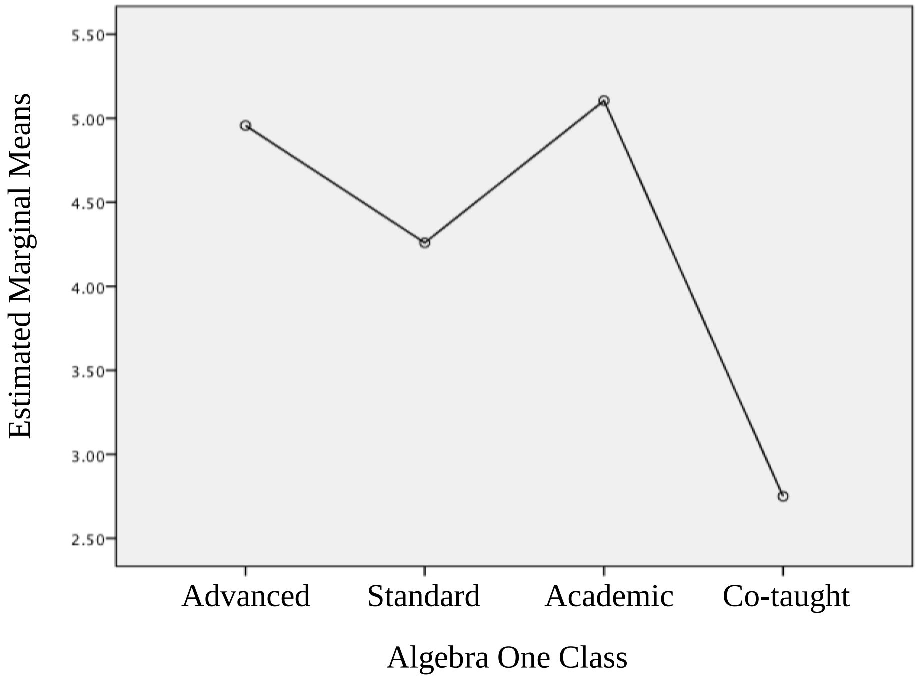
Note. N= Number of Cases. The mean of probe scores and standard deviations of the four Algebra 1 courses. The Academic course is comprised the highest track and are all eighth graders, the Advanced course is primarily ninth graders and is the second highest track, the Standard track has both ninth and tenth graders and is the third track, and Co-taught is eleventh graders and students with a SLD.

Table 7. Tests for Statistical Significance of Average Probe Scores between Algebra One Courses

Source	Sum of Squares	df	Mean Square	F	Significance
Corrected Model	41.860	3	13.953	4.770	.003
Intercept	1302.343	1	1302.343	445.167	.000
Alg1ClassType	41.860	3	13.953	4.770	.003
Error	368.615	126	2.926		
Total	2992.222	130			
Corrected Total	410.475	129			

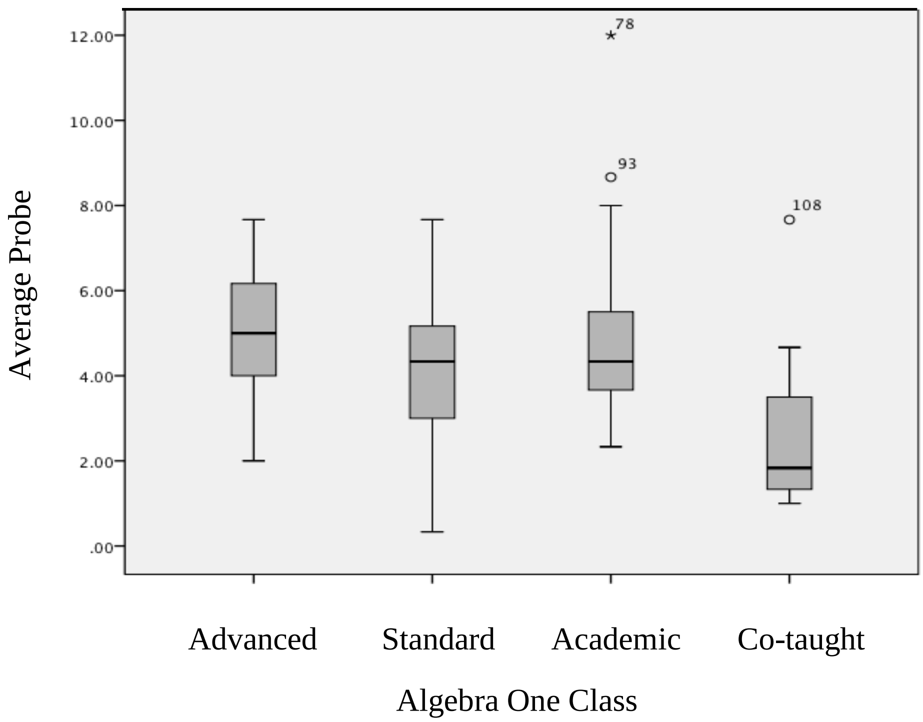
Note. df= Degrees of freedom, F= Frequency. There is a significant difference between the average probe scores of co-taught Algebra I class and the other Algebra I classes. The students in co-taught class performed at a significantly lower average fluency rate than the other classes.

Figure 3. Estimated Marginal Means of Average Probe between Algebra One Classes



Note: The co-taught Algebra One average probes were significantly below the average probe scores of the advanced, standard, and academic classes. The co-taught class is populated by the eleventh grade students and students with disabilities.

Figure 4. Average probe scores of Algebra One Courses



Note: Students average probe scores in the Co-taught class are significantly below the scores in the Advanced, Standard, and Academic Algebra I classes.

Table 8. Average Probe Scores of Eighth Grade Students compared to PSSA Mathematics Scores

Course	Mean	Standard Deviation	N
Below Basic	5.0833	1.25831	4
Proficient	4.8333	1.64992	2
Advanced	4.9467	1.46148	25
Total	4.4564	1.40038	31

Note. N= total number of cases. A comparison between student mean probe scores, standard deviations, and proficiency on the PSSA Mathematics. The Basic category is not represented since no students in this study performed in the Basic range on the PSSA Mathematics.

Table 9. Average Probe Scores of Ninth-Eleventh Grade Students compared to PSSA Mathematics Scores

Source	Sum of Squares	df	Mean Square	F	Significance
Corrected Model	.097	2	.049	.023	.977
Intercept	279.644	1	279.644	133.312	.000
Grade 8 PSSA	.097	2	.049	.023	.977
Error	58.734	28	2.098		
Total	820.556	31			
Corrected Total	58.832	30			

Note. R Squared =.002 (Adjusted R Squared = -.070), df= Degrees of Freedom, F=Frequency. The table indicates that there is not a significant correlation between performance on the PSSA and the average algebra probe.