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MULTICRITERIA APPROACH TO SPARE PARTS INVENTORY PROBLEM WITH OBSOLESCENCE

A Thesis in

Industrial Engineering

by

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ABSTRACT

This thesis considers multi-criteria optimization models to solve the spare parts inventory problem which are prone to obsolescence. In addition to the obsolescence, the criticality and carry over potential of the spare parts are also taken into consideration. A centralized supply chain of two stages is considered, which includes one company and multiple suppliers providing the spare parts to the company. The demand for spare parts and number of obsolete spare parts are considered to be stochastic and assumed to follow Poisson distributions. They are used to derive the distribution of spare parts consumption through convolution of two random variables. The Lead Time Demand (LTD) and Lead Time Obsolescence (LTO) are assumed to follow normal distributions. Next, a bi-criteria optimization problem is formulated on the basis of a continuous review inventory policy with two criteria: minimizing the Weighted Expected Total Cost (ETC) and maximizing the Weighted Cycle Service Level (CSL). The weights for the spare parts are calculated using the Analytic Hierarchy Process (AHP) with three main criteria: Stockout Implication, Type of Spare Part and Carryover Potential. The efficient frontier of the bicriteria optimization problem is generated using the "constraint" approach. Finally, an example is presented to illustrate the bi-criteria problem, solution by the constraint method and generation of the efficient frontier.

TABLE OF CONTENTS

LIST OF FIGURES	vi
LIST OF TABLES	vii
ACKNOWLEDGEMENTSi	X
Chapter 1 Introduction	l
1.1 Spare Parts Problem1	1
1.2 Inventory Policies	2
1.3 Multi Criteria Inventory Models	3
1.4 Overview of the thesis	1
1.5 Thesis Outline	5
Chapter 2 Literature Review	5
2.1 Supply Chain6	5
2.2 Inventory Control of Spare Parts7	7
2.2.1 General Papers8	3
2.2.2 Management Issues9)
2.2.3 Age-based replacement1	11
2.2.4. Obsolescence	12
2.3 Multi-criteria Optimization1	13
2.3.1 Problem Description	13
2.3.2 Definitions	14
2.3.3 Solution Methods1	14
2.4 Multiple Criteria Inventory Control (Jin, 2013)2	20
Chapter 3 Problem Description	28
3.1 Assumptions	28
3.2 Notations	30
3.3 Methodology and Approach	31
Step-1: Determination of the distribution of spare parts consumption	32
Step 2: Determination of the distribution of Lead Time Consumption	32
Step 3: Formulation of the bi-criteria optimization problem	34
Step 4: Determining the criticality of spare parts using AHP	37
3.4 Formulation of the Bi-criteria spare parts problem	40
3.5 Solution of the bi-criteria optimization problem4	13
Chapter 4 Illustration and Solution Methods	45

4.1 Convolution of Two Random Processes (Demand and Obsolescence)	45
4.2 Illustrative Example	45
Characteristics of the spare parts	46
4.3 Calculations for Spare Part 1	47
Mean and Variance of Lead Time Demand (LTD)	47
Mean and Variance of Lead Time Obsolescence (LTO)	
Mean and Variance of Lead Time Consumption (LTC)	
4.4 Calculation of Weights Using Analytical Hierarchy Process (AHP)	52
4.5 Solution Procedure	56
4.6 Solution of the Bi-Criteria Problem using Constraint Approach	58
Step 1: Determine the bounds on f1 and f2 in the efficient set	58
Step 2: Obtain 30 different values between the bounds of f1	60
Step 3: Generate the 30 efficient points by solving 30 single objective	
optimization problems	61
Chapter 5 Conclusion and Future Extension	66
5.1 Conclusion	66
5.2 Limitations and Future Extension	67
REFERENCES	68

LIST OF FIGURES

Figure 2.1 Five Stage serial Supply Chain	.6
Figure 2.2 Five Stage Supply Chain Network	.6
Figure 2.3 An example of AHP hierarchy for criticality of spare parts	. 18
Figure 3.1 Poisson demand with exponential obsolescence rate	. 30
Figure 3.2 Criteria and sub-criteria considered in AHP	.40
Figure 4.1 Efficient Frontier with two objectives as axes	. 64

LIST OF TABLES

Table 2.1 Degree of importance scale in AHP 18
Table 2.2 Experimentally derived RI values for AHP
Table 2.3 Comparison of Research in Multi-criteria Inventory Policies
Table 4.1Summary of spare parts characteristics 47
Table 4.2 Data for spare part 1
Table 4.3 Data for spare part 2
Table 4.4 Data for spare part 3
Table 4.5 Pairwise comparison for main criteria 52
Table 4.6 Weights for main criterion 52
Table 4.7 Consistency check for main criteria
Table 4.8 Pairwise comparison for sub-criteria for criterion 1 53
Table 4.9 Weights for sub-criteria of main criterion 1 53
Table 4.10 Consistency check for sub-criteria for main criterion 1 53
Table 4.11 Pairwise comparison for sub-criteria for criterion 2
Table 4.12 Weights for sub-criteria of main criterion 2 54
Table 4.13 Consistency check for sub-criteria for main criterion 2 54
Table 4.14 Pairwise comparison for sub-criteria for criterion 3
Table 4.15 Pairwise comparison for levels of criterion 3 55
Table 4.16 Pairwise comparison for levels of criterion 3 55
Table 4.17 List of all weights for all criteria and sub-criteria 55
Table 4.18 Weights for each of the spare parts

Table 4.19 Thirty points between the bounds of Criterion 1(Weighted Expected	
Total Cost)	60
Table 4.20 Results with 30 efficient points	62

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Chapter 1 Introduction

1.1 Spare Parts Problem

In the fast paced consumer electronics industry, where the demand of the product is high but the product life cycle is short, the equipment used for manufacturing the product may be changed every year depending on the new product. For high demand, high margin electronic products, the companies cannot afford high downtime of the line and often have spare equipment to take care of any equipment failure. In addition, the spare equipment is typically supplemented with spare parts to shorten the repair time of the faulty equipment.

What makes the spare parts so special is that it is very different from other types of inventory problems dealing with finished goods, raw materials or Work-In-Progress. The reason we hold the spares is to minimize the downtime of the assembly line which can cause significant loss to the company, while WIP and finished goods inventory are used to smoothen the customer demand. This means that the cost of under stocking the spare parts could be very high (based on the criticality of the spare parts) regardless of the actual cost of the spare part. Another reason why spare parts are different from WIP/finished goods inventory is that the demand is not based on the customer but is dependent on other factors such as obsolescence rate, equipment failures, replenishment lead times and carry over potential etc.

Two of the unique issues we need to address of while managing the spare parts are their obsolescence and criticality. Obsolescence is the state in which an item is no longer needed even though it may be in a perfect working condition. In the manufacturing setting, this happens very often with the spare parts since companies come up with new products every year. If obsolescence is not accounted for, the company might end up holding a lot of obsolete spare parts, which in turn drive up the inventory holding costs. It is also important to take the criticality of spare parts into consideration while formulating the inventory policy since all the spare parts are not of the same type and importance. While some of them are readily available, such as nuts and bolts, others could be highly customized and single sourced.

The objective of the spare strategy should be to minimize the total cost of spare equipment and spare parts along with maximizing the availability of spare parts and spare equipment. The cost of the spare parts will be borne by the vendor (equipment supplier) or the company depending on whether the equipment in under warranty or not. This thesis addresses this problem to develop an optimal policy.

1.2 Inventory Policies

The most basic inventory model is the Economic Order Quantity (EOQ) model. The decision variable of the EOQ model is the order quantity and the objective is to minimize the total cost for a company where the demand is deterministic and shortages are not allowed. Based on the basic EOQ model, other inventory models considering shortages or production period have also been proposed and studied.

When the demand is stochastic, there are two main types of inventory control policies: *continuous review and periodic review policies*. In a continuous review policy, known as the (Q, r) policy, an order of fixed quantity "Q" units is placed whenever the inventory position drops to the reorder point "r" or lower. Note that the inventory position, and not the net stock, is used to trigger an order. The inventory position, because it includes the on-order stock, takes proper account of the material requested but not yet received from the supplier. In contrast, if net stock was used for ordering purposed, we might unnecessarily place another order today even though a large shipment was due in tomorrow.

The advantages of the fixed order quantity (Q, r) policy are that it is quite simple, particularly in the two-bin form, for the stock clerk to understand, that errors are less likely to occur, and that the production requirements for the supplier are predictable. The primary disadvantage of a (Q, r) system, is that in its unmodified form it may not be able to effectively cope with the situation where individual transactions are large; in particular, if the transaction that triggers the replenishment in a (Q, r) system is large enough, then a replenishment of size Q won't even raise the inventory position above the reorder point. Of course, in such a situation one could instead order an integer multiple of Q where the integer was large enough to raise the inventory position above r (Silver et al., 1998).

In a periodic review policy, an (s, S, T) policy is widely used in practice. It is a reorder point (s) – order-up-to (S) system under periodic review with period length T. Every T time units, review the inventory position x, and if x < s, order a sufficient amount to bring the inventory level back to the order-up-to level S. The order quantity in this system will be Q= S-x. One variation of the periodic review policy is called the (T, S) policy. This policy considers the case where the ordering cost is sufficiently small that we can essentially be assured that the optimal behaviour is to order at each review period (Ravindran and Warsing Jr, 2013). The orders are made at a fixed time interval and are order-up-to level. Therefore, in the periodic review policy, the time interval between orders is always the same, while the order quantity can vary every time based on the on-hand inventory level at the time of review. In this thesis, considering the criticality of the spare parts the (Q, r) policy will be used.

1.3 Multi Criteria Inventory Models

Multi-criteria optimization problems are very common in industries. In practice, often the objectives that we are trying to achieve are conflicting with one another. The efficiency and the responsiveness of a supply chain are two conflicting objectives. Having larger inventory of several different products will result in the increase of customer responsiveness, but this will also increase the inventory cost and reduce the efficiency. Similarly, fewer warehouses can reduce the facility costs and the overall inventory level while the responsiveness of the supply chain decreases due to the longer transportation time. (Ravindran and Warsing Jr, 2013) point out that supply chain optimization problems are generally multiple criteria optimization models. There are several approaches to solve Multi Criteria Optimization problems.

There are three types of approaches that can be used to solve the multi-criteria mathematical programming problem. The first approach does not need to use the preference information from the decision maker. The second approach needs the input of preferences from the decision maker. The third approach is known as the interactive method. Here, the preference information from the decision maker is progressively used and the decision maker is involved in every step. This approach is more preferred in practice. A survey of multi-criteria methods is given by Masud and Ravindran (2008).

1.4 Overview of the thesis

The objective of this research is to help companies with their spare parts strategy by building a multi criteria optimization model, which minimizes the spare parts/equipment cost and maximizes the availability or service level simultaneously.

In supply chain inventory control, most research has focused on single objective optimization problem of minimizing total cost. In practice, the companies would want to focus on not only cost, but also service level/fill rate. In the technology industry, the obsolescence rate of the spare parts cannot be ignored since every year the companies come

5

up with new products which may render a lot of spare parts obsolete. Furthermore, the stochastic demand is closer to real-world scenario compared with deterministic demand.

We will consider a supply chain problem in series with one company and one or more suppliers of spare parts. The company's demand of spare parts is stochastic and stationary. A stochastic process is called stationary if its mean and variance does not change with time. There are two conflicting objectives: 1) capital invested in inventory, 2) customer service level. Our methodology is to first model the overall consumption of the spare parts by convoluting the probability mass functions for spare parts demand and number of obsolete parts. For the purpose of this thesis we have considered the probability distributions of the spare parts demand and obsolescence to be Poisson distributions. Next, we apply the Analytic Hierarchy Process (AHP) to get the weights for spare parts and use them to formulate our Multi Criteria Mathematical Programming (MCMP) problem with two conflicting criteria. The weights represent the criticality of the spare parts. We will then generate the efficient frontier. A continuous review (Q, r) inventory policy will be used in this thesis and a multicriteria optimization model will be presented to determine the optimal order quantity and reorder point for all the spare parts.

1.5 Thesis Outline

Chapter 2 is a literature review of inventory control, multi-criteria inventory control and multi-criteria optimization methods. Chapter 3 introduces the general multi-criteria optimization model for the spare parts problem. Chapter 4 presents a specific case study illustrating the optimization model and the solution procedure. Chapter 5 presents conclusions and future research.

Chapter 2 Literature Review

2.1 Supply Chain

A typical supply chain consists of suppliers, manufacturers, distributors, retailers and customers. An example of a simple five stage serial supply chain is given in Figure 2.1.



Figure 2.1 Five Stage serial Supply Chain

A slightly more complex example of a supply chain network is given in Figure 2.2:



Figure 2.2 Five Stage Supply Chain Network

As shown in Figure 2.2 the flow of raw materials/products follow the direction of the arrows and move from the supplies to the customers. We will be focussing on the area enclosed in the box, i.e., manufacturers and suppliers.

Supply chains are classified on the basis of its organization. In centralized supply chains, one company controls all the stages of the supply chain and all the major decisions are made by this company. One company alone is responsible for the entire optimization of the supply chain. Centralized supply chains are very rare in the real world and have its own advantages and disadvantages. On the one hand, it gives more control to a company on its

supply chain and hence easier to make changes and do process improvements across the supply chain; on the other hand, the company has to invest resources and time into all the supply chain stages, which may not be its core competency, and thereby increasing the supply chain cost.

In a de-centralized Supply Chain, which is common in practice, the whole supply chain is not owned by a single company and each stage may be owned by a different company, which is run independently and may have its own goals. Each company makes its own decision that suits its own interest and often the objectives of the companies conflict with one another. As a result of this, different companies in the supply chain only optimize their own objectives and the optimization of the entire supply chain becomes difficult.

2.2 Inventory Control of Spare Parts

Kennedy et al., 2002 did an extensive literature review of spare parts inventory in their paper, in which they discussed all the research done in the spare parts inventory domain till 2002 and segregated the research on the basis of management issues, age-based replacement, multi-echelon problems, problems involving obsolescence, repairable spare parts and special applications. We have modeled our literature review on similar lines and discuss some of the papers which they discuss and then some more which are relevant to the problem at hand. We will also discuss some relevant research done after 2002. The literature in inventory control can be categorized as follows:

• General Papers: The papers in this section mainly discuss the maintenance function of the supply chain and the various problems one needs to solve to determine the spare parts inventory.

- Management issues: The papers in this section discuss the various ways for organizing and reducing the spare parts held in the inventory
- Age-based replacement: The papers discussed in this section discuss the different age-based replacement policies for the spare parts
- Problems involving obsolescence: These papers discuss the obsolescence of the spare parts.

2.2.1 General Papers

Mann (1966) presents formulae to calculate optimum reorder points and reorder quantities for maintenance stores. He develops a simple formula which does not require any optimization model and is very practical and relevant to the industries. However, one of the limitations of the model is that it assumes the demand to be deterministic, which in real life scenario rarely happens. Hegde and Karmarkar (1993) studied the implications of incorporating customer cost while designing the service packages of spare parts. They solve an optimization model to support the selection of appropriate level of service package to offer to the customers and conclude that the results for the spare parts inventory are different if the objective is to minimize the life cycle engineering cost instead of maximizing the availability. Hollier (1980) offers a method to optimize the initial provisioning problem of spare parts in the case of new capital equipment by ranking the suppliers according to the ratio of its expected usage and a figure incorporating the total acquisition costs of the spare parts and the savings made by having close access to the spare parts. For the field repair kits, Mamer and Smith (1982) suggest considering a kit to include both spare parts and tools used for repair. Optimal spare part kits are computed using one period network-based inventory model, where restocking is permissible between jobs.

2.2.2 Management Issues

There is a lot of literature which deals with the spare parts inventory problem with a non-technical approach, with a systematic view of spare parts management, as well as those which deal with some technical approach. Moore (1996), in a non-technical paper suggests an approach based on segregation of spare parts into manageable categories like obsolete, project and surplus and then developing a set of rules for dealing with each category of spare parts economically.

Moore's study also talks about the need to consider other factors like lead times, machine failure, part use history, supplier reliability, stock-out objectives and inventory turn goals. Some papers stress on an intensive study of spare parts for the great cost benefits and savings which come from it. In his paper, Donoghue (1996) suggests that the cost of spare engines for major airlines is over \$11 billion and the estimated demand of the spare parts in the next two decades is 6600. He concludes that from the cost stand point, leasing the spare engines will be the best alternative. As per Flint's (1995) calculations, the airline industry stores spare parts worth \$45 billion and suggest reducing spares inventories by taking various steps such as developing leasing and pooling inventories, supplier partnerships, eliminating non-productive inventory and reducing cycle times. He also concludes that the airline industry is slow and lagging behind other industries in its initiatives to effectively reduce the inventory. The need to take action on the spare parts inventory is even more important in this industry considering the staggering costs of spare parts in airline industry. On the other hand, there are other studies, which take a more technical approach and discuss the need for designing and testing the various parts of the inventory models to show how we can improve the existing system.

Foote (1995) carried out a case study at Aviation Supply Office, Philadelphia, Pennsylvania to put in place a new control based forecasting system. The study is concentrated on the experience, mathematical principles, formulae and overall philosophy of a new forecasting system. He points that the (Q, r) model need not be accurate every month, but it must forecast the cumulative demand over the lead time. In the end, the results from three implementations are presented. Gupta and Rao (1996) provide a queuing system performance for various repair time distributions and present a recursive method to get the steady state probability distribution of the number of down machines at arbitrary time epoch of a machine interference problem with spares.

Luxhoj and Rizzo (1988) group similar spare parts in homogenous populations based on their failure characteristics by employing a population model concept (Kaio and Osaki, 1978). First a derivation is done to forecast the demand of the spare parts in each of the homogenous populations and then a mathematical procedure is presented for measuring the lead-time distributions for spares. A number of researchers have developed optimization methods for managing the spare parts inventory. Petrović et. al. (1990) suggested that for dealing with the spare parts problem, we must rely on knowledge and conventional data processing. As a part of their study they developed a microcomputer based expert system known as Sparta (spare parts advisor). Sparta consists of three parts. The first part forecasts the demand. This is done using the set of rules which use the if-then conjunction. The second part uses heuristic rules to select recommended stock of the spare part in consideration and the third part uses systems approach to determine the system consequences for the recommended stock for each of the spare parts.

Haneveld and Teunter (1997) solve an optimization model to find a near optimal solution to the inventory problem for slow moving expensive spare parts. In the model, the total purchase is divided into initial purchase and the purchase during the lifetime. Another interesting part is that the time is modeled as a continuous variable instead of dividing it into equal discrete parts. The total expected cost is a summation of purchasing cost, holding cost

and back-order cost. An approximate initial procurement is calculated, which is then coupled with the optimal strategy after the initial provisioning to get the total optimal strategy for the whole lifetime of the equipment. Cohen et al. (1986) use a branch and bound algorithm for low usage items in multi-echelon inventory systems to come up with a formula to find the quantity of spare parts such that the total cost is minimized at a specific service level.

Cohen et al. (1992) offer a near optimal (S, s) policy for customer's emergency demand and normal demand in a multi-echelon system. They derive an approximate cost function and service level constraint and then use a greedy algorithm and simulation to gauge the performance of the spares. Gajpal et al., (1994) categorized the spare parts into Vital Essential and Desirable (VED) and then used Analytical Hierarchy Process (AHP) (Saaty, 1980) to find the criticality of the spare parts by quantifying them as weights. The total weight of a spare part is a function of its type, its lead-time, and its stock out implication.

2.2.3 Age-based replacement

The Age-based replacement policy is based on the simple idea of replacing the spare parts after a pre-determined time interval. Armstrong and Atkins (1996) studied the joint optimization of the age replacement decision and the ordering decision for a system with one component subject to random failure with the storage space constraint of holding one spare in stock and concluded that joint optimization can lead to significant savings over sequential optimization. As an extension to the work done by Armstrong and Atkins, Kabir and Al-Olayan (1996) suggest a jointly optimized age-based replacement and ordering policy with many identical units and solve it using a simulation model. Kabir and Al-Olayan (1996) also studied the joint stocking policy and age-replacement policy in which they consider the time to replace a component instead of regular replenishment time along with the optimum reorder point and the maximum stock level S.

2.2.4. Obsolescence

Spare parts are different from standard items, such as valves and bolts, in a way that they are parts of the equipment and hence they are relatively expensive compared to standard maintenance parts. Since the spare parts are parts of the equipment and have specific usage if the equipment is changed or becomes obsolete due to change of product, the spare parts in the inventory become obsolete too.

The need to store the spare parts and incur the danger of obsolescence was mentioned in the survey done by Ikhwan and Burney (1994) who noted that over 34% of the companies in Saudi Arabia found that their most severe problem was the delays in obtaining spare parts. Also, the spare parts obsolescence is a large factor in some industries like consumer electronics and computers, where the products keep changing every year making some of the spare parts obsolete. Many inventory models do not explicitly consider the costs of obsolete inventory. This is evident from the fact that in their review of maintenance models, Cho and Parlar (1991) did not identify any research that considered obsolescence. There were some models which considered the cost of obsolete spare parts as a part of inventory holding cost but the overall risk distribution of the obsolete spare parts or the obsolescence rate was not considered. (D.P. Hayman, 1978) incorporated the disposal costs of spare parts which could not be repaired in their inventory model. Kabir and Al-Olayan (1996) consider the case where procurement lead time can increase the cost and use a simulation model to calculate the optimal age replacement policy. In their model they assume that the holding cost of the spare parts is much lower than ordering and shortage costs which they suggest is the case in most of the manufacturing facilities. Similar approach is taken by Kim et al. (1996) who consider a multi-echelon system in which they incorporate the cost of obsolescence into the inventory holding cost. They further go on to develop an optimal algorithm to calculate the spare parts level that minimizes the total expected cost while meeting a specified service level.

According to Walker (1996), the obsolescence is considered only for 'insurance' type inventories which he defines as the spare part inventories which have a high probability of not being used or needed during the entire lifetime of the equipment. Walker further develops an inventory model which recommends the minimum number of spare parts to sustain a specified service level requirement. Cobbaert and Van Oudheusden (1996) incorporate the risk of immediate and unexpected obsolescence of spare parts in the basic EOQ model, where the shortages are not allowed/ allowed, but the risk of obsolescence varies. They conclude that not taking obsolescence risk of around 20% into consideration can result in the increase in the average cost of the system by 15%. Bridgman and Mount-Campbell (1993) consider a situation in space shuttle, where some spare parts can become obsolete because a sub part of the shuttle is upgraded.

2.3 Multi-criteria Optimization

There are many decision problems in the practical world which involve multiple conflicting criteria. Multi-criteria optimization methods include ways to solve these type of problems by finding a compromised solution which maximizes the decision maker's preferences. A brief overview of multi-criteria optimization problem, important definitions and solution methods is presented below:

2.3.1 Problem Description

A generalized multi-criteria mathematical programming problem (MCMP) can be written as follows (Masud and Ravindran, 2008):

Maximize {
$$C_1(x), C_2(x) \dots C_k(x)$$
} (2.1)

Subject to
$$x \in X$$
 (2.2)

where,

x is any specific alternative,

X is the set of all feasible alternatives, and

C_i is the ith evaluation criterion.

2.3.2 Definitions

Ideal Solution

When we are working with multiple objectives with conflicting criteria, it is impossible to find a solution which simultaneously optimizes all the objectives. The ideal solution is that artificial solution H* defined in the criterion space, such that each of its elements is the optimum values of the respective criterion.

Mathematically, the ideal solution for the following MCMP problem defined by Equation (2.1) and (2.2) is obtained by solving:

$$H^* = \{H_i^* = MaxC_i(x) | x \in X, i = 1, 2, ..., k\}$$
(2.3)

Efficient/Non-dominated Solution

A feasible solution x^1 dominates another feasible solution x^2 if it is better than x^2 in at least one objective and no worse in any of the other objectives. On the other hand, a nondominated or efficient solution is one which cannot be bettered in one or more objectives without worsening at least one objective. Mathematically, a solution $x^1 \in X$ is efficient if there is no other $x \in X$ such that $C_i(x) \ge C_i(x^1)$, i = 1, 2, ..., k, and $C_i(x) \ne C_i(x^1)$.

2.3.3 Solution Methods

There has been a considerable amount of research in the solution methods domain of multi-criteria optimization problems. A multi-criteria decision making (MCDM) problem has many efficient solutions and the best compromise solution is chosen by the decision maker out of these efficient solutions. All these efficient solutions together form what is called as

the efficient frontier which is can be plotted in a two dimensional space in case of a bi-criteria problem. In this section, we will discuss some important definitions and solution methods which are relevant to our research.

Best Compromise Solution

Let us consider the decision maker's preference function to be U. Then the multicriteria optimization problem can be written as,

$$Maximize U(f(x))$$
(2.4)

Subject to
$$x \in X$$
 (2.5)

The best compromise solution can be defined as the efficient solution that maximizes the decision maker's utility function U. Here it is important to note that the utility function U is not explicitly known in practical settings.

Although Keeney and Raiffa (1993) present methods to estimate the utility functions, it is difficult to define and map the utility function accurately. This problem is solved by asking questions to the decision maker about various efficient solutions. By doing this, we are able to help the decision maker reach the best compromise solution. Below we discuss three different types of approaches to solve the MCMP problem which are characterized by what and when the preference information is asked to the decision maker.

Prior Articulation of Preferences

In this type of approach the preferences of the decision maker are taken before the problem is solved and the priorities of the various criteria are set according to this preference information. The problem when solved with this information leads to an efficient solution which incorporates all the preference information from the decision maker and will be the decision maker's most preferred solution.

Lee (1972) in his paper provides one such solution method called Goal Programming, where he presented the formulation and solution of Goal Programming problems. In this method, the decision maker will be asked two types of information. One, goals for each of the objectives and two, the preference or priority information of the goals. The optimization problem is solved with an objective of finding an optimal solution that comes as close as possible to achieving the goals with given preference information.

Post Articulation of Preferences

In this type of method the decision makers chooses the best preferred alternative from the set of all efficient points from the efficient frontier. This type of methods are generally used where one can generate the efficient frontier easily like in the case of bi-criteria problems.

Interactive Methods

In Interactive Methods the best compromise solution is reached by progressive articulation of preference from the decision maker. The decision maker is asked questions progressively, thus reducing the decision space after every interaction until the best compromise solution is reached. A comprehensive survey of these type of methods for MCMP problems was done by Shin and Ravindran (1991). Most of the literature surveyed fall in one of the following categories: feasible region reduction methods; feasible direction methods; criterion weight space methods; trade-off cutting plane methods; Lagrange multiplier methods; visual interactive methods; branch and bound methods.; relaxation methods; sequential methods; and scalarizing function methods. According to the paper, the research done typically includes four interaction styles: binary pairwise comparison; pairwise comparison; precise local trade-off ratio.

Sadagopan and Ravindran (1982) presents an interactive method called Paired Comparison Method for bi-criteria MCMP problems in which the decision maker (DM) is asked to choose between two efficient solutions. The decision space is reduced on the basis of the response and the decision maker (DM) is again asked to choose between two efficient solutions. This cycle is repeated until the compromise solution is reached.

Analytic Hierarchy Process (Masud and Ravindran, 2008; Ravindran and Warsing, 2013)

The Analytic Hierarchy Process (AHP) is a multiple criteria decision making tool, developed by Saaty (1980) which is used to rank the alternatives. In this method the decision maker is asked how much he prefers one criterion (sub-criterion) to the other criterion (subcriterion) on a scale of 1 to 9 by pairwise comparison and weights are calculated on the basis of this feedback. The feature that makes AHP so useful is that it can be used to consider not only quantitative but qualitative factors too. For example, in case of supplier selection the decision maker can consider qualitative aspects such as financial stability, feeling of trust into consideration of criteria. On the basis of decision maker's preferences, Total Scores (TS) are calculated for each of the alternatives and then alternatives are ranked according to these scores.

Basic Principles of AHP

• Design a hierarchy: The hierarchy of the AHP is the overview of how the objective, criteria, sub-criteria and the alternatives are related to each other. The top vertex of the hierarchy is the objective of the AHP problem and the bottom most ones are the alternatives. All the intermediate vertices are criteria or sub-criteria which are more and more aggregated as you go up in the hierarchy. A hierarchy for the spare parts problem is given in Fig 2.3.

• A paired comparison of the criteria/sub-criteria is done with respect to its contribution or weights to the higher level vertices it is linked to.

• AHP uses both rating method and comparison method to find the weights. A numerical scale of 1–9 is generally used where 1 means equal importance and 9 means most important (See Table 2.1).



Figure 2.3 An example of AHP hierarchy for criticality of spare parts

- Weights in the form of numerical values are calculated for each of the alternatives with respect to each criterion or sub-criterion by using pairwise comparison of alternatives.
- The alternatives are ranked according to the weights calculated.

The hierarchy for spare parts criticality given in figure 2.3 is used to solve the illustrative problem used in this thesis. More about it will be discussed in Chapter-4.

Steps of the AHP Model

Step 1: The first step is to carry out a pair-wise comparison of criteria using the 1–9 degree of importance scale shown in Table 2.1.

Degree of Importance	Definition
1	Equal importance
3	Weak importance of one over other
5	Essential or strong importance
7	Demonstrated importance
9	Absolute importance
2,4,6,8	Intermediate values between two adjacent judgments

Table 2.1 Degree of importance scale in AHP

It should be noted that if we have n criteria to evaluate, then the pair-wise comparison matrix for the criteria will be a matrix given by, $A_{(n \times n)} = [a_{ij}]$, where a_{ij} represents the relative importance or preference of criterion i with respect to criterion j. Also, we assume that $a_{ii} = 1$ and $a_{ji} = \frac{1}{a_{ij}}$.

Step 2: The next step is to calculate the normalized weights for the main criteria. We compute the normalized weights for the main criteria by using the L_1 norm as shown below:

• Normalizing each of the columns of matrix A using L₁ norm:

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}$$
(2.5)

• Averaging the normalized values found in the above step across each row to get the criteria weights: `

$$w_i = \frac{\sum_{j=1}^{n} r_{ij}}{n}$$
 (2.6)

Step 3: Since the decision maker is a human and can make errors when asked to do a lot of pairwise comparisons, it is important to check the consistency of the decision maker. In this step we check the consistency of the pairwise comparison matrix using Eigen Value theory (Saaty, 1980):

1. Let the matrix consisting of weights calculated in the above step be W. Now we compute the vector $X = (X_1, X_2, X_3..., X_n)$ such that the vector denote the values of AxW.

2. Compute the following:

$$\lambda_{max} = Average\left[\frac{X_1}{W_1}, \frac{X_2}{W_2}, \frac{X_3}{W_3} \dots \frac{X_4}{X_4}\right]$$
(2.7)

3. Now the Consistency index (CI) is given by:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{2.8}$$

The following table gives the average of a number of CI values (denoted by RI) computed from reciprocal matrices $a_{ij} \in (1, 9)$ for different sizes. The values were first computed and presented by Saaty (1980)

Table 2.2 Experimentally derived RI values for AHP

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Saaty mentions in the paper that if consistency ratio (CR), where $CR = \frac{CI}{RI}$ is less than 0.15, then we can accept the pair-wise comparison matrix as consistent.

Step 4: In this step, the weights and CR values are calculated for the sub-criteria with respect to main criteria just as they are calculated for the main criteria in the above Step 2 and 3. The final weights are the products of the sub-criteria weight and the corresponding criteria weight. **Step 5:** In this step, we go a hierarchy down and repeat steps 1, 2, and 3 for alternatives and obtain the pairwise comparison matrix with respect to each of the criterion using the ratio scale (1-9) and normalized scores of all alternatives with respect to each criterion. In this step, we get an (mxm) matrix S, where S_{ij} = normalized score for alternative i with respect to criterion j and m is the number of alternatives.

Step 6: Finally the total score (TS) for each alternative can be calculated by using the following: $TS_{(mx1)} = S_{(mxn)}W_{(nx1)}$, where W is the weight vector obtained after step 4. Now, we can rank the alternatives using the Total Scores (TS).

2.4 Multiple Criteria Inventory Control (Jin, 2013)

Starr and Miller (1962) solved the multi criteria problem for spare parts inventory using bi-criteria method and Lagrangian relaxation technique. The objective was to minimize the total cycle stock investment and number of orders. They were the first to plot the efficient curve and noted that the inventory policy should be chosen on the efficient curve.

The concept of exchange curves was first introduced by Brown (1967). He considered two cases. In the deterministic demand case, the criteria were capital invested and the number of orders. In the stochastic demand case, the criteria were capital invested and the customer service level. He concluded that the preferred solution should be chosen by the decision maker on the basis of the exchange curves.

Gardner Jr and Dannenbring (1979) suggest that in practice and traditional inventory models, it is very difficult to estimate the marginal ordering cost, inventory holding cost and inventory shortage cost. In case of ordering cost, they point out that the approaches used in the literature to determine the ordering cost result in average cost instead of marginal costs. Similarly, the inventory holding cost is also difficult to estimate because it is usually related to the cost of capital and it is highly subjective. Shortage costs are also difficult to estimate because they involve "goodwill loss". They also note that the traditional inventory models have been focussing on single item inventory systems and there has been no significant research in the area of multi-item inventory systems. Hence, in their discussion they incorporate policy tradeoffs based on three objectives in their inventory policy namely, the aggregate customer service, workload and investment in cycle and the safety stock. They further go on to generate a response surface with the help of these objectives, which is also the efficient frontier. They also state that any optimal decision or non-dominated solution must be a point located on this efficient frontier. The computational results go on to prove that the models considered in the paper are efficient and can make improvements to the inventory system. Buffa and Munn (1989) note that a firm's logistics costs are a substantial portion of its sales and that the typical inventory models ignore the shipping costs. They present a recursive algorithm that minimizes the total logistics cost by determining the reorder cycle time. A relaxation procedure is used as a part of the algorithm to find a suitable initial approximation to the optimal order intervals and then the optimal solution is reached through a finite number of recursive steps.

Bookbinder and Chen (1992) applies the Multiple Criteria Decision Making (MCDM) methodology in a two-echelon serial inventory/ distribution system consisting of a warehouse, retailer and discusses various situations like deterministic/ probabilistic demands and cases where the marginal costs are known vs. unknown. They go on to present three MCDM models to get the efficient inventory policies considering the trade-offs between objectives like customer service, inventory investment and transportation cost under both deterministic and probabilistic demands. The results of this paper are MCDM generalizations of Brown's exchange curve, Starr and Miller's optimal policy curve and Gardner and Dannenbring's optimal policy surface.

Crowston et al. (1973) discusses the algorithm to find the optimal lot size in multistage assembly systems. In the system, each facility or stage can have inputs from multiple predecessor stages but supplies in turn to only a single successor. The objective is to find the lot size for each of the facility which minimizes average per period production and inventory holding cost. They go on to prove that the optimal lot size exist such that the lot size is an integer multiple of the lot size at its successor facility. Next, on the basis of this information they construct a dynamic programming algorithm to obtain the optimal lot sizes. Lenard and Roy (1995) propose another approach to determine inventory policies based on the notion of efficient policy surfaces. In the discussion, multi-criteria and multi-item inventory problem are considered with three criteria namely, inventory level, customer service and workload. Next by using grouping techniques, they group the items into coherent families and then go on to propose to have one aggregate item to represent all the items in a family in order to make it easier for the decision maker, since the decision now has to be made on the basis of a multi-criteria single-item model.

Agrell (1995) presents an interactive multi-criteria framework for an inventory control decision system. In this approach the preference structure of the decision maker is monitored progressively in the exploration of the solution space. The study considers a multi-criteria problem for a single item with stochastic demand and three criteria namely, total annual cost, expected number of annual stock out occurrences and the expected number of items stocked out annually. It is implemented as FORTRAN modules and Excel spreadsheet macros such that the decision maker can use the interface to interact with the models and find the preferred efficient solution.

Ettl et al. (2000) presented an optimization model of the supply network which provides the base stock level at each of the store by taking bill of materials (BOM), the nominal lead times, the cost and demand data and the customer service level requirement as inputs. The objective of the optimization model is to minimize the total inventory capital in the supply network to meet the specified service level requirement. What sets this model apart from the other ones is its inventory queue model that uses detailed but approximate analysis of the actual lead times at the stores and the demand over the lead time along with the type of store operations. They also illustrate their model using various numerical examples.

Puerto and Fernández (1998) provided pareto-optimal solutions for the inventory problem with backorder for both deterministic and stochastic demands. In addition to that, they also provided a new way to use the pareto-optimal solutions in case of classic inventory problems and analysed these models using trade-off analysis in the context of vector optimization theory to come up with new solutions with error bounds. The criteria considered are the ordering cost and the summation of holding and backorder cost. In his dissertation Thirumalai (2001) considers a multi-criteria inventory problem in a serial supply chain with three companies (manufacturer, warehouse and retailer) and thus multiple independent decision makers. He considers the case of both deterministic and stochastic demands and the number of criteria used varies from two in case of deterministic demand to three in case of stochastic demand. The two criteria used in case of deterministic demand are the average inventory capital and number of orders per year. He goes on to propose an interactive method to solve this bi-criteria problem with single decision maker. In case of stochastic demand another criterion of risk of stockout is added to the problem. He then extends the problem to whole supply chain system with three companies in series, multiple decision makers and provides a collaborative and interactive optimization algorithm to solve this problem. He also shows that the compromise solution is efficient on both, supply chain and individual company level.

DiFilippo (2003) presents a multi-criteria, two-echelon supply chain inventory problem with deterministic demand. Both centralized and de-centralized systems are considered and the two stages are warehouse and retailers. There are three criteria used in the problem- capital invested in the inventory, number of orders and transportation cost which is calculated using the freight rates. In the de-centralized system, the multi-criteria problem is solved first for the retailer and then the solution is used to solve the problem for warehouse. In case of warehouse, two types of policies are considered, in stationary policy the order placed is stationary which means the quantity ordered is fixed every time and the nonstationary policy in which the order placed can vary in quantity each time. Both the policies are compared and the results are presented. In case of centralized supply chain, two cases are discussed. One in which the retailers place the order at the same time which essentially becomes the case of one warehouse, one retailer problem and the other in which the retailers may place the order at different times. The two policies are discussed and the results show that the policy with different order placement time performs better in the numerical example.

Natarajan (2007) presents an inventory model with two stages where one warehouse is supplying to various retailers under a de-centralized control which means there is an independent decision maker at each company. Three models are presented. The first one is the conventional single cost objective framework and the other two are based on multiple criteria which are more practical cases. The number of criteria depends on whether the demand is deterministic or stochastic. The first two models are based on deterministic demand and constant lead time and have three criteria- capital invested in the inventory, number of orders in a year and annual transportation cost which is calculated using the actual freight rates. In the case of stochastic demand, a fourth criterion of fill rate is introduced in addition to the above mentioned three criteria. Finally these multi-criteria models are solved and the solutions are compared using value path analysis. A real life case study is presented using the data from a Fortune 500 consumer products company.

From our literature review it is evident the field of inventory control is widely studied and that most of the research done in the space of inventory management policies is focussed on centralized supply chain. Table 2.3 gives the comparison of the literature in multiple criteria inventory control and shows that the field has been widely studied over the past few decades. However, there is some research done in the de-centralized supply chains too in the recent past. There are models developed for both deterministic and stochastic demand in multi-echelon supply chain system. It is also worth nothing that most of the models use a continuous review policy for inventory management with a few opting for periodic review. However, not many discuss the problem of criticality of spare parts, their obsolescence rate and carry over potential. Therefore, the focus of this thesis will be to present a multi-criteria <u>continuous review model for stochastic demand for a single facility considering the criticality</u> <u>of the spare parts, obsolescence rate and carry over potential.</u> .

Paper	Year	Demand	Review Policy	Facility	Туре	Criteria					
						Capital Invested	Customer Service	Criticality	Obsolescence		
Starr & Miller	1962	Deterministic	-	Single	Multi Criteria	\checkmark					
Gardner & Dannerbring	1979	Stochastic	Continuous	Single	Multi Criteria	\checkmark	\checkmark				
Bookbinder & Chen	1992	Stochastic	Continuous	Multiple	Multi Criteria	\checkmark	\checkmark				
Agrell	1995	Stochastic	Continuous	Single	Multi Criteria	\checkmark	\checkmark				
Cobbaert& Van Oudheusden	1996	Deterministic	Periodic	Single	Single Objective	~			\checkmark		
Thirumalai	2001	Stochastic	Continuous	Multiple	Multi Criteria	✓	\checkmark				
DiFilippo	2003	Deterministic		Multiple	Multi Criteria	✓					
Natarajan	2007	Stochastic	Continuous	Multiple	Multi Criteria	\checkmark	\checkmark				
Yang Jin	2013	Stochastic	Periodic	Multiple	Multi Criteria	\checkmark	\checkmark				
This Thesis	2015	Stochastic	Continuous	Single	Multi Criteria	\checkmark	\checkmark	\checkmark	\checkmark		

Table 2.3 Comparison of Research in Multi-criteria Inventory Policies
Chapter 3 Problem Description

The multi-criteria spare part inventory problem we are trying to solve can be defined as a problem with two objectives: minimize the total cost per year and maximize cycle service level (CSL). The problem will be constrained by the budget of the company and the minimum CSL it wants to ensure. We will also consider the following in the optimization model:

- 1. The obsolescence of spare parts
- 2. The criticality of the spare part
- 3. Carry over potential of the spare parts (the probability that the spare part can be used for the next year's program)

3.1 Assumptions

- From the review of literature of the spare parts inventory it was noted that all the research done in this area have assumed that failures of the spare parts follow a Poisson distribution and the time-to –failure to be an exponential distribution. The major reason for this is because the exponential distribution is memory less which means that the probability of a spare part to fail in a particular time period does not depend on the number of spare parts failed already.
- 2. The obsolescence time is an exponential distribution.
- 3. A continuous review policy (Q, r) under uncertain demand is used.
- 4. Backorders are allowed.

- 5. The lead time demand and number of obsolete spare parts during lead time follow a normal distribution.
- 6. The replenishment rate is infinite. This means there is no restriction on the number of units that can be ordered at any one time to replace the depleted stock.
- 7. Until the moment of obsolescence, demand of the spare parts follows a Poisson distribution.
- 8. The unit holding cost is a constant value to be paid per unit of stock per annum.
- 9. The replenishing order cost is a constant value to be paid per order
- 10. The unit shortage cost of the spare parts is a constant value paid per unit
- 11. The unit cost of the spare parts is a constant value (No quantity discounts)
- 12. The unit obsolescence cost is a constant value to be paid per unit of stock if obsolescence occurs (e.g. the price of the item minus the salvage value).

Figure 3.1 illustrates the situation when the demand of the spare parts follows a Poisson distribution and at the same time the spare parts obsolescence rate follows an exponential distribution. It shows that if the obsolescence is not taken into consideration, the physical quantity of spare parts might appear adequate to meet the consumption, but a significant number in that stock could be obsolete.



Figure 3.1 Poisson demand with exponential obsolescence rate

3.2 Notations

- X is a random variable denoting the demand of the spare parts per annum and is given by X~ Poisson(β)
- Y is a random variable denoting the number of obsolete spare parts per annum and is given by Y~ Poisson(λ)
- 3. L is assumed to be a random variable denoting the lead time of the spare parts and is given by L~ N(μ_L , σ^2_L).
- 4. LTD is assumed to be normal random variable denoting the lead time demand of the spare parts and is given by LTD~ $N(\mu_{LTD}, \sigma^2_{LTD})$.
- 5. LTO is assumed to be a normal random variable denoting the lead time obsolescence of the spare parts and is given by LTO~ $N(\mu_{LTO}, \sigma^2_{LTO})$.
- 6. The consumption of spare parts per annum is given by a random variable C such that: C = X + Y with mean μ_C and standard deviation σ_C .

- 7. The Lead Time Consumption (LTC) is given by a random variable LTC such that: LTC =LTD + LTO with mean μ_{LTC} and standard deviation σ_{LTC} in units.
- 8. ρ_i is the correlation coefficient between the Lead Time Demand (LTD) and Lead Time Obsolescence (LTO) for the ith spare part
- 9. c_P is the fixed ordering cost for the spare parts per order in \$.
- 10. c_h is the inventory holding cost per unit in \$/year
- 11. f_i is the fraction of the Expected Inventory Holding Cost (EIHC) which is real for the ith spare part
- 12. c_s is shortage cost of the spare parts per unit in \$
- 13. c_u is the cost of the spare part per unit in \$
- 14. Q and r are the decision variables for the (Q, r) policy, where Q is the quantity of spare parts ordered in units and r is the reorder point.
- 15. k_{SL} is the safety factor of spare parts for a given service level CSL
- 16. CSL is the cycle service level for the spare parts

3.3 Methodology and Approach

We will approach this problem in four steps:

- Step-1. Determination of the distribution of spare parts consumption
- Step-2. Determination of the distribution of Lead Time Consumption (LTC)
- Step-3. Formulation of the bi-criteria optimization problem
- Step-4. Determination of the criticality of spare parts using AHP

The four steps are explained below.

Step-1: Determination of the distribution of spare parts consumption

In this problem, there are two factors which are contributing to the consumption of the spare parts inventory. One is the demand due to failed spare parts in manufacturing and the other is due to the obsolescence of the spare parts. Our methodology is to convolute the two distributions to get a resulting distribution which gives the overall consumption of spare parts. The expressions for the mean and variance of the overall spare parts consumption is given by:

$$\mu_C = \mu_X + \mu_Y = \beta + \lambda \tag{3.1}$$

$$\sigma_c^2 = \sigma_X^2 + \sigma_Y^2 + 2Cov(X, Y) \tag{3.2}$$

However, since the spare parts demand and obsolescence are no way related to each other, they can be assumed to be independent. So, Equation 3.2 can be written as:

$$\sigma_C^2 = \sigma_X^2 + \sigma_Y^2 = \beta + \lambda \tag{3.3}$$

In Equations 3.1 and 3.3, we have used the fact that the mean and variance of a Poisson random variable are equal to its rate.

Step 2: Determination of the distribution of Lead Time Consumption

In this step we will derive an expression for the total consumption of spare parts during the lead time (LTC). There are two factors contributing towards the total consumption of spare parts during lead time. First, the Lead Time Demand (LTD) and the second is the Lead Time Obsolescence (LTO). We have assumed that LTD and LTO are normal distributions and hence, the resulting total consumption of the spare parts will be the sum of the two random variables, which is also normal.

The expressions for mean and variance of Lead Time Demand (LTD), i.e., μ_{LTD} and σ^2_{LTD} are given by the following equations:

$$\mu_{LTD} = \mu_L.\,\mu_X = \mu_L.\,\beta \tag{3.4}$$

$$\sigma_{LTD}^2 = \mu_L \cdot \sigma_X^2 + \mu_X^2 \cdot \sigma_L^2 = \mu_L \cdot \beta + \beta^2 \cdot \sigma_L^2$$
(3.5)

Since, the random variable X follows a Poisson distribution, both mean and variance of X will be β .

Similarly, the expressions for mean and variance of Lead Time Obsolescence (LTO), i.e., μ_{LTO} and σ^2_{LTO} are given by the following equations:

$$\mu_{LTO} = \mu_L \cdot \mu_Y = \mu_L \cdot \lambda \tag{3.6}$$

$$\sigma_{LTO}^{2} = \mu_{L} \cdot \sigma_{Y}^{2} + \mu_{Y}^{2} \cdot \sigma_{L}^{2} = \mu_{L} \cdot \lambda + \lambda^{2} \cdot \sigma_{L}^{2}$$
(3.7)

Since Y is also Poisson, its mean and variance will be λ .

Since we have assumed the Lead Time Demand (LTD) and Lead Time Obsolescence (LTO) to be normal, the convolution of both these random variables, which is Lead Time Consumption (LTC), will also be normal. The mean and variance of LTC are given by Equations 3.8 and 3.9.

$$\mu_{LTC} = \mu_{LTD} + \mu_{LTO} = \mu_L(\beta + \lambda) \tag{3.8}$$

$$\sigma_{LTC}^2 = \sigma_{LTD}^2 + \sigma_{LTO}^2 + 2Cov(LTD, LTO) \qquad (3.9)$$

It should be noted that both LTD and LTO are positively correlated. Equation 3.9 can thus be rewritten as:

$$\sigma_{LTC}^2 = \sigma_{LTD}^2 + \sigma_{LTO}^2 + 2.\,\rho.\,\sigma_{LTD}.\,\sigma_{LTO} \qquad (3.10)$$

Where, ρ is the positive correlation coefficient between LTC and LTO.

Next we model the problem as a continuous review (Q, r) model under uncertain demand, where the demand is given by this 'consumption' random variable and derive an expression for Expected Total Cost (ETC) per annum. Also, since we have assumed the distribution for Lead Time Demand (LTD) and Lead Time Obsolete (LTO) spare parts to be normal, the resulting distribution is going to be normal and hence we can easily derive the expression for Cycle Service Level (CSL). These two expressions (ETC and CSL) will serve as the two objective functions for the optimization problem when we solve it in Step-3.

Step 3: Formulation of the bi-criteria optimization problem

The optimization problem has two objective functions namely: Minimizing the Expected Total Cost (ETC) per annum and the maximizing the Cycle Service Level (CSL) for each spare part. Below we derive the expression for the Expected Total Cost:

Expected Total Cost (ETC) per annum

The Expected Total Cost (ETC) per annum is a summation of Expected Ordering Cost (EOC) per annum, Expected Inventory Holding Cost (EIHC) per annum and Expected Stockout Cost (ESC) per annum. The derivation for each one of these is discussed below.

Expected Ordering Cost (EOC) per annum

The expected ordering cost per unit time is given by:

$$EOC = c_p \frac{\mu_C}{\rho} \tag{3.11}$$

Where,

c_p= Ordering cost per order

 μ_{C} = Mean consumption of the spare parts per annum

Q =Order quantity (decision variable)

Expected Inventory Holding Cost

For calculating the expected inventory holding cost per annum, we need the average inventory per annum. In our case (backorders are allowed), the average inventory is given by:

$$\frac{Q}{2} + r - \mu_{LTC} \tag{3.12}$$

Where,

Q = order quantity (decision variable)

r = reorder point (decision variable)

 μ_{LTC} = expected consumption during lead time (Equation 3.8)

Therefore, the Expected Inventory Holding Cost is given by:

$$EIHC = c_h\{\frac{Q}{2} + r - \mu_{LTC}\}$$
(3.13)

Where,

 c_h = holding cost per item per annum

Expected Stockout Cost

For calculating the stockout cost, we need to calculate the number of shortages per annum. The expression for expected number of shortages per cycle is given by:

$$B(r) = \int_{r}^{\infty} (x - r) f(x) dx$$
 (3.14)

Where,

B(r) is the expected number of stockouts per cycle

f(x) is the pdf of the Lead Time Consumption (LTC)

x is the Lead Time Consumption (LTC).

Then the Expected Stockout Cost (ESC) per annum is given by:

$$ESC = c_s \left(\frac{\mu_c}{Q}\right) B(r) \tag{3.15}$$

Where,

 $c_s =$ Stockout cost per unit

 μ_{C} = The mean consumption of the spare parts per annum (Equation 3.1)

Q = Order Quantity (decision variable)

Expected Total Cost per annum

Thus, the expected total cost per annum is given by

$$ETC = EOC + EIHC + ESC \tag{3.16}$$

$$ETC = c_p \frac{\mu_c}{Q} + c_h \{ \frac{Q}{2} + r - \mu_L \} + c_s \left(\frac{\mu_c}{Q} \right) B(r)$$
(3.17)

Cycle Service Level

The probability of stockout during lead time is given by:

$$\int_{r}^{\infty} f(x) \qquad (3.18)$$

Where,

f(x) is the pdf of Lead Time Consumption (LTC)

r is the reorder point

Then cycle service level is given by:

$$CSL = 1 - \int_{r}^{\infty} f(x)$$
 (3.19)

Safety Stock

The safety stock is given by:

$$SS = k_{SL}\sigma_{LTC} \tag{3.20}$$

Where,

 k_{SL} is the safety factor for achieving a particular service level, given by $\emptyset^{-1}(SL)$ where $\emptyset \sim N(0,1)$ and σ_{LTC} is the standard deviation of the consumption during lead time, obtained from Equation 3.10.

Step 4: Determining the criticality of spare parts using AHP

In step 4, we determine the criticality of the spare parts using the Analytical Hierarchy Process $(AHP)^1$ The criticality of the spare part can be measured by finding the weights using AHP. The AHP hierarchy is represented by three levels as shown in Figure 3.2. The focus at level 1 is the evaluation of the weights of the main criteria, which define the criticality of the spare parts. At level 2, we identify and calculate the weights of the

¹ For a detailed discussion of AHP with an illustrative example, the reader is referred to Chapter 6 (Section 6.3.7) of Ravindran and Warsing (2013).

sub-criteria for each main criterion. The main criteria used to measure the criticality of the spare parts are given below:

Criterion 1 (C₁): Stockout Implication: It refers to the status of availability of a spare part (when an original part fails and the spare part is required)

Criterion 2 (C₂): Type of spare parts required (standard or nonstandard)

- Criterion 3 (C₃): Carry over potential of the spare part. (The probability that the same spare part could be used for next year's program.
- At level 2, we present the sub-criteria characterizing each of the main criterion. For criterion 1 (stockout implication) the sub-criteria are:

C₁₁: Spare part is available

C₁₂: Spare part is available if suitable changes are made to some other equipment

C₁₃: No spare part is available

For criterion 2, (type of spare part) the sub-criteria are:

C₂₁: Standard spare part (available off the shelf)

C₂₂: Standard spare part, but availability not certain.

C₂₃: Non-standard spare part, to be fabricated according to specifications

For criterion 3 (carryover potential) the sub-criteria are:

C₃₁: High C₃₂: Medium

C₃₃: Low

All the sub-criteria values will be either 1 or 0. It should be noted that each spare part will come under exactly one of the sub-criteria for each of the main criterion. For example, if spare part 1 is readily available, is a standard spare part and has a high carryover potential, then the sub-criteria values for the spare part will be $C_{11}=1$, $C_{21}=1$, $C_{31}=1$ and all other $C_{ij}=0$ for i,j=1,2,3. The methodology of the AHP is discussed below:

- First, we get the weights of the criteria by using the pairwise comparison of the main criteria with respect to the criticality of the spare parts. Input of the decision maker (DM) is required here.
- 2. Next, we get the weights for each of the sub-criteria with respect to each main criterion by using the pairwise comparison matrix from the decision maker.
- 3. For each spare part, we assign the value of 1 for the sub-criteria it belongs to, under each of the criteria and assign 0 for all other values. For any spare part, the values for all C_{ij} are either 1 or 0. Depending on the spare part's characteristics, C_{ij} values will be assigned.
- 4. Next we calculate the weights for each spare part at the sub-criteria level by multiplying the value assigned (0 or 1) with the weights calculated in point 2 and then adding them.
- 5. Then we calculate the weights of the spare parts at the criteria level by multiplying the weights calculated in point 4 with the weights calculated in point 1.
- Next we add up the weights of each criterion for a spare part to get the total weight of the spare part.

7. Finally we normalize the weights of each of the spare parts such that the weights of the spare parts add up to 1.

This weight will be used as the measure of criticality of each spare part, higher weights implying that the spare part is more critical.



Figure 3.2 Criteria and sub-criteria considered in AHP

3.4 Formulation of the Bi-criteria spare parts problem

Objective 1: Minimize the Expected Total Cost (ETC)

The expression for the total cost may be written as summation of the weighted cost for all the spare parts where the weights are given by the criticality analysis from the AHP.

$$Min Z_1 = \sum_{i=1}^n w_i ETC_i \tag{3.21}$$

Where,

 ETC_i is the Expected Total Cost of the ith spare part and w_i is its weight calculated from AHP.

n is the total number of spare parts

Objective 2: Maximize the Cycle Service Level

The objective function for maximizing the cycle service level is given below:

$$Max Z_2 = \sum_{i=1}^n w_i CSL_i \tag{3.22}$$

Where,

$$CSL_i = 1 - \int_{r_i}^{\infty} f(x)$$
 (3.23)

is the cycle service level of spare part 'i and weight w_i is its weight

Constraint 1: Budget

The first constraint will be the budget for the spare parts in \$ per annum. The real cost paid by the company on a spare part will be the cost of the spare parts, the ordering cost of the spare parts and a part of the Expected Inventory Holding Cost.

$$C_{Total}^{i} = Q_{i}c_{u_{i}} + c_{p_{i}}\frac{\mu_{c_{i}}}{Q_{i}} + f_{i}\left[c_{h_{i}}\left\{\frac{Q_{i}}{2} + r_{i} - \mu_{LTC_{i}}\right\}\right] \le B_{i} \qquad (3.24)$$

Where,

 $B_i \mbox{ is the total budget of the company per unit time for the <math display="inline">i^{th} \mbox{ spare part}$

 Q_i is the quantity of the ith spare part ordered per cycle

 c_{u_i} is the unit cost of the ith spare part in \$

 c_{p_i} is the unit ordering cost of the ith spare part in \$

 μ_{c_i} is the expected total consumption of the ith spare part

 c_{h_i} is the unit holding cost the ith spare part in \$ per annum

 r_i is the reorder point of the ith spare part

 μ_{LTC_i} is the expected total lead time consumption of the ith spare part

 f_i is the fraction of holding cost of the i^{th} spare part which is real (This represents

warehousing cost, taxes and interest on borrowed inventory capital)

n is the total number of spare parts

Constraint 2: Minimum Cycle Service Level

If the company wants to ensure a certain service level, then that can be inserted as one of the constraints:

$$CSL_i \ge CSL_{i_0} \text{ for } i = 1 \text{ to } n \tag{3. 25}$$

Where,

 CSL_{i_0} is the minimum cycle service level to be achieved for the ith spare part and CSL_i is given by equation 3.13

Constraint 3: Non-negativity constraints and upper bounds

$$Q_i, r_i \ge 0 \text{ and integers}$$
 (3. 26)

$$0 \leq CSL_i \leq l \tag{3.27}$$

3.5 Solution of the bi-criteria optimization problem

The bi-criteria problem may be solved by generating the entire efficient frontier. There are two possible approaches to solve a bi-criteria problem to generate the efficient frontier namely, the P_{λ} Method (weighted objective) and the Constraint Method. We will be using the constraint method in this thesis to generate the efficient frontier.

3.5.1 Generating the efficient frontier using the Constraint Method

We shall describe the constraint method using a general bi-criteria problem given below:

$$Min f_1(x)$$
 (3.28)

$$Max f_2(x)$$
 (3.29)

Subject to
$$X \in S$$
 (3.30)

Where, S is the feasible region.

Step 1: Determine the bounds on f_1 and f_2 in the efficient set.

 $Min f_1(x)$ (3.31)

Subject to
$$X \in S$$
 (3.32)

Let the optimal solution be X_1^* . Then $f_1(X_1^*)$ is the minimum (optimal) value for f_1 and $f_2(X_1^*)$ is the lower bound on f_2 in the efficient set. Similarly you can compute $f_1(X_2^*)$ and $f_2(X_2^*)$ by solving:

$$Max f_2(x)$$
 (3.33)

Subject to
$$X \in S$$
 (3.34)

Here $f_2(X_2^*)$ is the maximum value for $f_2(x)$ and $f_1(X_2^*)$ is the upper bound on $f_1(x)$. The bounds on f_1 and f_2 in the efficient set is given by:

For
$$f_1 = [f_1(X_1^*), f_1(X_2^*)]$$

For $f_2 = [f_2(X_1^*), f_2(X_2^*)]$

Step 2: Obtain '*n*' different values between bounds of f_1

Take either f_1 or f_2 bounds and divide that into *n* equal intervals. For illustration, we use f_1 bounds and let the 'n' different values between $f_1(X_1^*)$ and $f_1(X_2^*)$ be f_{1j} for j = 1, 2, ... n with $f_{11} = f_1(X_1^*)$ and $f_{1n} = f_1(X_2^*)$.

Step 3: Generate the '*n*' efficient points

Generate the 'n' efficient points by solving 'n' single objective optimization problems as follows:

Max
$$f_2(x)$$

Subject to,

$$f_1(x) \le f_{1j} \text{ for } j=1, 2, \dots, n$$
$$X \in S$$

Let the max $f_2(x)$ be equal to f_{2j}^* . Then the 'n' efficient points are given by

 (f_{1j}, f_{2j}^*) for j= 1, 2,..., n, in the objective space of the bi-criteria problem.

Chapter 4 Illustration and Solution Methods

4.1 Convolution of Two Random Processes (Demand and Obsolescence)

As discussed earlier, we have assumed the demand for the spare parts and the number of spare parts becoming obsolete to be Poisson distributions. Therefore, the total consumption of the spare parts follows a distribution which is a convolution of these two distributions. Since the demand in no way related to the number of spare parts becoming obsolete, it can be safely assumed that the two distributions are independent. From Chapter-3, the expressions for the mean and variance were given as:

$$\mu_C = \mu_X + \mu_Y = \beta + \lambda \tag{4.1}$$

$$\sigma_C^2 = \sigma_X^2 + \sigma_Y^2 = \beta + \lambda \tag{4.2}$$

Where,

 μ_c and σ_c^2 are the mean and variance of consumption of spare parts per annum μ_X and σ_X^2 are the mean and variance of demand of spare parts per annum which is equal to the rate β

 μ_Y and σ_Y^2 are the mean and variance of the number of obsolete spare parts per annum which is equal to the rate λ

4.2 Illustrative Example

For the illustrative example, we will assume that an electronics company sells microprocessors under its own brand name. The manufacturing of the microprocessors

need three spare parts from three different suppliers. Each of the spare parts has its own demand and the number of obsolescent spare parts per annum. Each of the spare parts is also of different type and the criteria and sub-criteria considered for evaluating the criticality of the spare parts are given below:

Characteristics of the spare parts

Spare part 1:

- 1. The spare equipment is available
- 2. It is a non-standard spare part, which needs to be fabricated according to the specifications
- 3. It has a high carry over potential

Spare part 2:

- 1. Spare equipment is available, if suitable changes are made in some other equipment
- 2. Standard spare part (available off the shelf)
- 3. It has a low carry over potential

Spare part 3:

- 1. No spare equipment is available
- 2. Standard spare part, but availability not certain.
- 3. It has a high carry over potential

The summary of the spare parts characteristics is given in the following table

Spare Part 1		Spare Part 2	Spare Part 2			Spare Part 3	
Criteria 1	$C_{11} = 1, C_{12} = C_{13} = 0$	Criteria 1	$C_{12}=1, C_{11}=C_{13}=0$		Criteria 1	$C_{13}=1, C_{11}=C_{12}=0$	
Criteria 2	$C_{23}=1, C_{21}=C_{22}=0$	Criteria 2	$C_{21}=1, C_{22}=C_{23}=0$		Criteria 2	$C_{22}=1, C_{21}=C_{23}=0$	
Criteria 3	$C_{31}=1, C_{32}=C_{33}=0$	Criteria 3	$C_{33}=1, C_{31}=C_{32}=0$		Criteria 3	$C_{31}=1, C_{32}=C_{33}=0$	

All the data for each of the Spare Parts 1, 2 and 3 are given in Table 4.2, 4.3 and 4.4 respectively. It should be noted that the calculated inputs are highlighted in grey. Please refer to Chapter 3, Section 3.2 for explanation of the various notations in the tables. It should be noted that we have considered the lead time of all spare parts to follow a normal distribution with mean of 4 days and variance of 1 day. The expressions for the mean and variance of LTD, LTO and LTC and the convolution of random variables of Lead Time Demand (LTD) and Lead Time Obsolescence (LTO) are given in Section 3.3, Chapter 3.

4.3 Calculations for Spare Part 1

In this section, we illustrate the calculations for Spare part 1 data, given in Table 4.2. The mean and variance of LTD, LTO and LTC need to be calculated before the optimization problem is solved. The calculated values are highlighted in Table 4.2.

Mean and Variance of Lead Time Demand (LTD)

Mean and variance of LTD is given by Equations 3.4 and 3.5. Substituting the values of mean and variance of lead time (L) and demand of spare parts (X), we get:

$$\mu_{LTD} = \mu_L \cdot \mu_X = \mu_L \cdot \beta = \frac{4}{365} \cdot 2000 = 21.92$$
 (4.3)

$$\sigma_{LTD}^2 = \mu_L \cdot \sigma_X^2 + \mu_X^2 \cdot \sigma_L^2 = \mu_L \cdot \beta + \beta^2 \cdot \sigma_L^2 = \frac{4}{365} \cdot 2000 + 2000^2 \cdot \frac{1}{365} = 10980.82 \quad (4.4)$$

Mean and Variance of Lead Time Obsolescence (LTO)

Mean and variance of LTO is given by Equations 3.6 and 3.7. Substituting the values of mean and variance of lead time (L) and number of obsolete spare parts per annum (O):

$$\mu_{LTO} = \mu_L \cdot \mu_Y = \mu_L \cdot \lambda = \frac{4}{365} \cdot 500 = 5.48 \tag{4.5}$$

$$\sigma_{LTO}^2 = \mu_L \cdot \sigma_Y^2 + \mu_Y^2 \cdot \sigma_L^2 = \mu_L \cdot \lambda + \lambda^2 \cdot \sigma_L^2 = \frac{4}{365} \cdot 500 + 500^2 \cdot \frac{1}{365} = 690.41$$
(4.6)

Mean and Variance of Lead Time Consumption (LTC)

Mean and variance of LTC is given by equation 3.8 and 3.10. Substituting the values of mean and variance of LTD, LTO and correlation coefficient ρ_1 :

$$\mu_{LTC} = \mu_{LTD} + \mu_{LTO} = \mu_L(\beta + \lambda) = \frac{4}{365} (2000 + 500) = 27.40$$
(4.7)

$$\sigma_{LTC}^{2} = \sigma_{LTD}^{2} + \sigma_{LTO}^{2} + 2. \rho. \sigma_{LTD} \cdot \sigma_{LTO}$$

$$= 10980.82 + 690.41 + 2. (0.20) \cdot \sqrt{10980.82} \cdot \sqrt{690.41}$$

$$= 12772.60$$
(4.8)

Similar calculations are done for spare parts 2 and 3. The calculations and other input data for spare parts 2 and 3 are given in Tables 4.3 and 4.4 respectively.

48

Spare Part-1

μ_L in days	4.00
σ^{2}_{L} in days	1.00
μ_{LTD} in units	21.92
σ^2_{LTD} in units	10980.82
σ_{LTD} in units	104.79
μ_{LTO} in units	5.48
σ^{2}_{LTO} in units	690.41
σ_{LTO} in units	26.28
μ _{LTC} in units	27.40
σ^{2}_{LTC} in units	12772.60
σ_{LTC} in units	113.02
c _P in \$	130.00
μ_x in units (β)	2000.00
σ^2_X in units (β)	2000.00
μ_y in units (λ)	500.00
$\sigma^2_{\rm Y}$ in units (β)	500.00
$\mu_{\rm C}$ in units $(\beta + \lambda)$	2500.00
f_1	0.60
c _h in \$	0.25
c _u in \$	10.00
c _s in \$	0.30
ρ1	0.20
Minimum CSL to achieve	0.90
Annual budget for Spare Part in \$	27000.00

Table 4.2 Data for spare part 1

Spare Part-2

μ_L in days	4.00
σ^{2}_{L} in days	1.00
μ_{LTD} in units	87.67
σ^2_{LTD} in units	175430.14
σ_{LTD} in units	418.84
μ _{LTO} in units	21.92
σ^{2}_{LTO} in units	10980.82
σ _{LTO} in units	104.79
μ_{LTC} in units	109.59
σ^{2}_{LTC} in units	203967.12
σ_{LTC} in units	451.63
c _P in \$	130.00
μ_x in units (β)	8000.00
σ^2_X in units (β)	8000.00
μ_y in units (λ)	2000.00
$\sigma^2_{\rm Y}$ in units (β)	2000.00
$\mu_{\rm C}$ in units $(\beta + \lambda)$	10000.00
f_1	0.60
c _h in \$	0.25
c _u in \$	15.00
c _s in \$	0.30
ρ ₂	0.20
Minimum CSL to achieve	0.90
Annual budget for Spare Part in \$	5000.00

Table 4.3 Data for spare part 2

Spare Part-3

μ_L in days	4.00
σ^{2}_{L} in days	1.00
μ_{LTD} in units	43.84
σ^2_{LTD} in units	43879.45
σ_{LTD} in units	209.47
μ_{LTO} in units	10.96
σ^2_{LTO} in units	2750.68
σ_{LTO} in units	52.45
μ_{LTC} in units	54.79
σ^{2}_{LTC} in units	51024.65
σ_{LTC} in units	225.89
C _P in \$	130.00
μ_x in units (β)	4000.00
σ^2_X in units (β)	4000.00
μ_y in units (λ)	1000.00
$\sigma^2_{\rm Y}$ in units (β)	1000.00
$\mu_{\rm C}$ in units $(\beta + \lambda)$	5000.00
\mathbf{f}_1	0.60
c _h in \$	0.25
c _u in \$	3.00
c _s in \$	0.30
ρ₃	0.20
Minimum CSL to achieve	0.90
Annual budget for Spare Part in \$	3000.00

Table 4.4 Data for spare part 3

4.4 Calculation of Weights Using Analytical Hierarchy Process (AHP)

In this thesis, we have assumed the pairwise comparison matrix which in practice would come from the decision maker (DM). It is given in Table 4.5. The weights for the three main criteria are calculated in the following steps:

Step 1: Pairwise comparison for main criteria

The cells filled in gray are the inputs from the DM.

Criteria Matrix A	1	2	3
1	1	2	4
2	1/2	1	1 1/6
3	1/4	6/7	1

Table 4.5 Pairwise comparison for main criteria

Step 2: Normalizing each column of matrix A using L1 norm

In this step we normalize each of the columns of matrix A using L₁ norm and find the

weights for the main criteria.



Criteria	1	2	3	Weights
1	0.571	0.519	0.649	0.5795
2	0.286	0.259	0.189	0.2447
3	0.143	0.222	0.162	0.1757

Step 3: Checking the consistency of the Decision Maker (DM)

The consistency of the Decision Maker can be checked calculating the CR value.

For the decision maker to be consistent the CR/RI value should be less than 0.15 in the

case of 3 criteria.





Next, we need to find the weights of the sub-criteria with respect to main criteria.

The weights of sub-criteria with respect to main criterion 1 are calculated below:

Step 4: Pairwise comparison for sub-criteria with respect to main criterion 1

Table 4.8 Pairwise comparison for sub-criteria for criterion 1

Criteria Matrix A	A1	A2	A3
A1	1	1/2	1/3
A2	2	1	4/5
A3	3	1 1/4	1

Step 5: Normalizing each column of matrix A using L1 norm

Table 4.9 Weights for sub-criteria of main criterion 1

	A1	A2	A3	Weights
A1	0.167	0.182	0.156	0.1682
A2	0.333	0.364	0.375	0.3573
A3	0.500	0.455	0.469	0.4744

Step 6: Checking the consistency of the Decision Maker (DM)

Table 4.10 Consistency check for sub-criteria for main criterion 1



The weights of sub-criteria with respect to main criterion 2 are calculated below:

Step 7: Pairwise comparison for sub-criteria with respect to main criterion 2

Criteria Matrix A	A1	A2	A3
A1	1	1/2	1/3
A2	2	1	4/5
A3	3	1 1/4	1

Table 4.11 Pairwise comparison for sub-criteria for criterion 2

Step 8: Normalizing each column of matrix A using L1 norm

Table 4.12 Weights for sub-criteria of main criterion 2

	A1	A2	A3	Weights
A1	0.091	0.053	0.106	0.0833
A2	0.273	0.158	0.149	0.1932
A3	0.636	0.789	0.745	0.7235

Step 9: Checking the consistency of the Decision Maker (DM)

Table 4.13 Consistency check for sub-criteria for main criterion 2



The weights of sub-criteria with respect to main criterion 3 are calculated below:

Step 10: Pairwise comparison for sub-criteria with respect to main criterion 3

Table 4.14 Pairwise comparison for sub-criteria for criterion 3

Criteria Matrix A	A1	A2	A3
A1	1	1/2	1/3
A2	2	1	4/5
A3	3	1 1/4	1

Step 11: Normalizing each column of matrix A using L₁ norm

Table 4.15 Pairwise comparison for levels of criterion 3

	A1	A2	A3	Weights
A1	0.545	0.545	0.545	0.5455
A2	0.273	0.273	0.273	0.2727
A3	0.182	0.182	0.182	0.1818

Step 12: Checking the consistency of the Decision Maker (DM)

Table 4.16 Pairwise comparison for levels of criterion 3



Therefore, the final weights for each of the spare parts for each criterion can be calculated by multiplying criteria weight and corresponding level weights. The total list of all the weights is given as follows:

Criterion	Sub-criterion	Weights	Rank
1	1	0.098	4
1	2	0.207	2
1	3	0.275	1
2	1	0.020	9
2	2	0.047	7
2	3	0.177	3
3	1	0.096	5
3	2	0.048	6
3	3	0.032	8
	Total	1.000	

Table 4.17 List of all weights for all criteria and sub-criteria

The final weights for each of the spare parts are given by the following table:

Table 4.18 Weights for each of the spare parts

Spare Part 1		Spare Part 2		Spare Part 3	
Criteria 1 A1	0.098	Criteria 1 A2	0.207	Criteria 1 A3	0.275
Criteria 2 A3	0.177	Criteria 2 A1	0.020	Criteria 2 A2	0.047
Criteria 3 A1	0.096	Criteria 3 A3	0.032	Criteria 3 A1	0.096
Total Weight	0.370	Total Weight	0.259	Total Weight	0.418
Normalized Weights	0.353	Normalized Weights	0.248	Normalized Weights	0.399

4.5 Solution Procedure

The bi-criteria optimization problem defined in Section 3.4, Chapter 3, can be solved by generating the entire efficient frontier using the constraint approach. The general problem is restated as follows:

$$f_1 = Min \, Z_1 = \sum_{i=1}^n w_i ETC_i \tag{4.9}$$

$$f_2 = Max \, Z_2 \,=\, \sum_{i=1}^n w_i CSL_i \tag{4.10}$$

Subject to,

$$C_{Total}^{i} \le B_{i} \tag{4.11}$$

$$CSL_i \ge CSL_{i_0} \tag{4.12}$$

$$Q_i, r_i \ge 0 \tag{4.13}$$

$$0 \leq CSL_i \leq l \tag{4.14}$$

Where,

 w_i is the weight of the ith spare part as calculated using the AHP

 ETC_i is the Expected Total Cost of the ith spare part

 $CSL_i = 1 - \int_{r_i}^{\infty} f_i(x)$ and $f_i(x)$ is the pdf of lead-time consumption for spare part 'i',

which in this case is normal

 C_{Total}^{i} is defined in Equation 3.24 (Chapter 3)

Numerical Example after Substituting Data

By substituting the data from the illustrative example, the bi-problem becomes:

Objective 1: Minimize the Expected Total Cost ETC

$$Min Z_{1} = w_{1} \left\{ c_{p} \frac{\mu_{C}^{1}}{Q_{1}} + c_{h} \left\{ \frac{Q_{1}}{2} + r_{1} - \mu_{L}^{1} \right\} + c_{s} \left(\frac{\mu_{C}^{1}}{Q} \right) B(r_{1}) \right\} + w_{2} \left\{ c_{p} \frac{\mu_{C}^{2}}{Q_{2}} + c_{h} \left\{ \frac{Q_{2}}{2} + r_{2} - \mu_{L}^{2} \right\} + c_{s} \left(\frac{\mu_{C}^{2}}{Q_{2}} \right) B(r_{2}) \right\} + w_{3} \left\{ c_{p} \frac{\mu_{C}^{3}}{Q_{3}} + c_{h} \left\{ \frac{Q_{3}}{2} + r_{3} - \mu_{L}^{3} \right\} + c_{s} \left(\frac{\mu_{C}^{3}}{Q_{3}} \right) B(r_{3}) \right\}$$

$$(4.15)$$

After substituting the values the above formula becomes:

$$Min Z_{1} = 0.353. \left\{ 130. \frac{2500}{Q_{1}} + 0.25. \left\{ \frac{Q_{1}}{2} + r_{1} - 27 \right\} + 0.3. \left(\frac{2500}{Q_{1}} \right). \int_{r_{1}}^{\infty} (x - r_{1}) f_{1}(x) dx \right\} + 0.248. \left\{ 130. \frac{10000}{Q_{2}} + 0.25. \left\{ \frac{Q_{2}}{2} + r_{2} - 110 \right\} + 0.3. \left(\frac{10000}{Q_{2}} \right) \int_{r_{2}}^{\infty} (x - r_{2}) f_{2}(x) dx \right\} + 0.399. \left\{ 130. \frac{5000}{Q_{3}} + 0.25. \left\{ \frac{Q_{3}}{2} + r_{3} - 55 \right\} + 0.3. \left(\frac{5000}{Q_{3}} \right) \int_{r_{3}}^{\infty} (x - r_{3}) f_{3}(x) dx \right\}$$

$$(4.16)$$

Objective 2: Maximize the Cycle Service Level

$$MaxZ_{2} = \sum_{i=1}^{n} w_{i}CSL_{i}$$

$$MaxZ_{2} = 0.353.CSL_{1} + 0.248.CSL_{2} + 0.399.CSL_{3}$$
(4.17)

Constraint 1: Budget

$$C_{Total}^i \leq B_i$$

$$\left\{130.\frac{2500}{Q_1} + 0.25.\left\{\frac{Q_1}{2} + r_1 - 27\right\} + 0.6.\left[0.3.\left(\frac{2500}{Q_1}\right).\int_{r_1}^{\infty} (x - r_1)f_1(x)dx\right]\right\} \le 27000 \quad (4.18)$$

$$\left\{130.\frac{10000}{Q_2} + 0.25.\left\{\frac{Q_2}{2} + r_2 - 110\right\} + 0.6.\left[0.3.\left(\frac{10000}{Q_2}\right)\int_{r_2}^{\infty} (x - r_2)f_2(x)dx\right]\right\} \le 5000 \quad (4.19)$$

$$\left\{130.\frac{5000}{Q_3} + 0.25 \cdot \left\{\frac{Q_3}{2} + r_3 - 55\right\} + 0.6 \cdot \left[0.3 \cdot \left(\frac{5000}{Q_3}\right) \int_{r_3}^{\infty} (x - r_3) f_3(x) dx\right]\right\} \le 3000$$
(4.20)

Constraint 2: Minimum Cycle Service Level

$$CSL_i \geq CSL_{i_0}$$

$$CSL_i \ge 0.90 \text{ for } i = 1,2,3$$
 (4.21)

$$1 - \int_{r_i}^{\infty} f(x_i) \ge 0.90 \, for \, i = 1,2,3 \tag{4.22}$$

Constraint 3: Non-negativity Constraints and Upper bounds

$$Q_i, r_i \ge 0 \tag{4.23}$$

$$0 \le CSL_i \le l \text{ for } i = 1,2,3$$
 (4.24)

4.6 Solution of the Bi-Criteria Problem using Constraint Approach

The optimization problem is solved in MS Excel 2013 Solver add-in with multi-start option to get globally optimized solution. The above bi-criteria is solved using the following steps:

Step 1: Determine the bounds on f_1 and f_2 in the efficient set.

$$f_1 = Min \, Z_1 = \sum_{i=1}^n w_i ETC_i \tag{4.25}$$

Subject to,

$$C_{Total}^{i} \le B_{i} \tag{4.26}$$

$$CSL_i \ge CSL_{i_0} \tag{4.27}$$

58

$$Q_i, r_i \ge 0 \tag{4.28}$$

59

Let the optimal solution be X_1^* . Then $f_1(X_1^*)$ is the minimum (optimal) value for f_1 and $f_2(X_1^*)$ is the lower bound on $f_2(x)$ in the efficient set. Similarly you can compute $f_1(X_2^*)$ and $f_2(X_2^*)$ by solving:

$$f_2 = Max \, Z_2 = \sum_{i=1}^n w_i CSL_i \tag{4.29}$$

Subject to,

$$\sum_{i}^{n} C_{Total}^{i} \le B \tag{4.30}$$

$$CSL_i \ge CSL_{i_0} \tag{4.31}$$

$$Q, r \ge 0 \tag{4.32}$$

Here $f_2(X_2^*)$ is the maximum value for $f_2(x)$ and $f_1(X_2^*)$ is the upper bound on $f_1(x)$. The bounds on f_1 and f_2 in the efficient set is given by:

For
$$f_1 = [f_1(X_1^*), f_1(X_2^*)]$$

For $f_2 = [f_2(X_1^*), f_2(X_2^*)]$

The two efficient points that we get after solving the bi-criteria problem are (1082.67, 0.90) and (1192.80, 1). This gives us the bounds for both the criteria.

Criterion 1: Minimize the Expected Total Cost (ETC): (1082.67, 1192.80)

Criterion 2: Maximize the Cycle Service Level (CSL): (0.90, 1.00)

Step 2: Obtain 30 different values between the bounds of f_1

Take the f_1 bounds and divide that into 30 equal intervals between $f_1(X_1^*)$ and $f_1(X_2^*)$. Let the 30 equal intervals between 1082.67 and 1192.80 be $f_{1,j}$ for j = 1, 2, ...30with $f_{1,1}$ = 1082.67 and $f_{1,30}$ = 1192.80. The 30 values are given in Table 4.19.

Sr. No.	Weighted Total Cost
1	1082.67 (Min Cost)
2	1086.47
3	1090.27
4	1094.07
5	1097.86
6	1101.66
7	1105.46
8	1109.26
9	1113.05
10	1116.85
11	1120.65
12	1124.45
13	1128.24
14	1132.04
15	1135.84
16	1139.63
17	1143.43
18	1147.23
19	1151.03
20	1154.82
21	1158.62
22	1162.42
23	1166.22
24	1170.01
25	1173.81
26	1177.61

Table 4.19 Thirty points between the bounds of Criterion 1(Weighted Expected Total Cost)

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27	1181.41
28	1185.20
29	1189.00
30	1192.80

Step 3: Generate the 30 efficient points by solving 30 single objective optimization problems

Next, we generate the 30 efficient points by solving 30 single objective

optimization problems as follows:

Max
$$f_2(x)$$

Subject to,

$$\sum_{i}^{n} C_{Total}^{i} \le B \tag{4.33}$$

$$CSL_i \ge CSL_{i_0} \tag{4.34}$$

$$f_1(x) \le f_{1,j}$$
 for j= 1, 2,..., 30 (4.35)

$$Q, r \ge 0 \tag{4.36}$$

Let the max $f_2(x)$ be equal to f_{2j}^* . Then the 30 efficient points are given by $(f_{1,j}, f_{2,j})$ for j= 1, 2,..., 30, in the objective space of the bi-criteria problem. The 30 efficient points and the respective optimal policies (Q_i, r_i) , i=1,2,3 are given in Table 4.20.

Table 4.20 Results with 30 efficient points

Cr. No.	Spare Part 1				Spare Part 2				Spare Part 3				Weighted	Weighted
Sr. NO.	Q	r	Total Cost	CSL	Q	r	Total Cost	CSL	Q	r	Total Cost	CSL	Total Cost	CSL
1	416	173	879.16	0.9012	1735	689	1147.87	0.9002	621	345	1222.53	0.9006	1082.67	0.90
2	416	211	883.62	0.9479	1735	722	1150.78	0.9125	619	364	1226.25	0.9145	1086.45	0.92
3	415	245	891.52	0.9729	1735	729	1151.49	0.9149	618	373	1228.30	0.9205	1090.24	0.93
4	415	253	893.16	0.9770	1734	775	1157.14	0.9297	616	393	1232.97	0.9328	1094.08	0.94
5	415	263	895.29	0.9815	1734	806	1161.36	0.9385	614	410	1237.86	0.9421	1097.83	0.95
6	415	265	895.72	0.9822	1733	844	1167.45	0.9480	612	428	1243.26	0.9508	1101.64	0.96
7	416	265	893.96	0.9822	1733	882	1173.82	0.9564	610	445	1248.87	0.9580	1104.84	0.96
8	415	265	895.72	0.9822	1733	912	1179.20	0.9622	608	463	1254.94	0.9646	1109.22	0.97
9	414	294	904.05	0.9908	1733	930	1182.58	0.9654	608	463	1254.94	0.9646	1113.00	0.97
10	414	304	906.41	0.9928	1733	940	1184.49	0.9670	606	481	1261.30	0.9704	1116.84	0.97
11	414	309	907.60	0.9936	1732	981	1192.94	0.9732	605	489	1264.40	0.9727	1120.59	0.98
12	414	312	908.31	0.9941	1732	999	1196.64	0.9755	603	507	1271.12	0.9774	1124.44	0.98
13	414	317	909.51	0.9948	1732	1038	1204.89	0.9801	602	515	1274.36	0.9792	1128.20	0.98
14	414	318	909.75	0.9949	1731	1055	1208.90	0.9818	600	533	1281.37	0.9829	1132.07	0.98
15	414	320	910.24	0.9952	1731	1073	1212.86	0.9835	598	548	1287.91	0.9855	1135.83	0.98
16	414	320	910.24	0.9952	1731	1115	1222.29	0.9870	597	557	1291.55	0.9869	1139.62	0.98
17	414	320	910.24	0.9952	1731	1131	1225.94	0.9881	595	574	1298.71	0.9892	1143.38	0.99
18	413	344	917.86	0.9975	1731	1137	1227.32	0.9885	594	583	1302.44	0.9903	1147.90	0.99
19	413	348	918.84	0.9977	1729	1159	1233.03	0.9899	593	590	1305.75	0.9911	1150.98	0.99
20	413	358	921.30	0.9983	1730	1210	1244.71	0.9926	593	590	1305.75	0.9911	1154.74	0.99
21	413	368	923.78	0.9987	1729	1258	1256.48	0.9945	593	590	1305.75	0.9911	1158.53	0.99
22	413	372	924.77	0.9989	1729	1316	1270.52	0.9962	593	590	1305.75	0.9911	1162.36	0.99

Sr. No		9	Spare Part 1		Spare Part 2				Spare Part 3				Weighted	Weighted
51. NO.	Q	r	Total Cost	CSL	Q	r	Total Cost	CSL	Q	r	Total Cost	CSL	Total Cost	CSL
23	413	374	925.26	0.9989	1728	1374	1285.01	0.9974	593	590	1305.75	0.9911	1166.12	0.99
24	412	399	933.26	0.9995	1728	1390	1288.95	0.9977	593	590	1305.75	0.9911	1169.92	0.99
25	412	406	935.01	0.9996	1728	1429	1298.56	0.9983	592	592	1307.94	0.9913	1173.79	0.99
26	412	412	936.50	0.9997	1727	1481	1311.74	0.9988	592	592	1307.94	0.9913	1177.58	0.99
27	412	412	936.50	0.9997	1727	1527	1323.16	0.9992	592	592	1307.94	0.9913	1180.41	0.99
28	412	412	936.50	0.9997	1726	1603	1342.39	0.9995	592	592	1307.94	0.9913	1185.17	0.99
29	412	412	936.50	0.9997	1725	1663	1357.66	0.9997	592	592	1307.94	0.9913	1188.95	0.99
30	412	412	936.50	1.0000	1724	1724	1373.20	1.0000	592	592	1307.94	1.0000	1192.80	1.00
Finally, we plot the 30 efficient points on a graph such that the two objectives form the X and Y axis of the graph. The curve joining the efficient points gives us the complete efficient frontier.



Figure 4.1 Efficient Frontier with two objectives as axes

From the various efficient solutions in Table 4.20, it can be seen that the Weighted Expected Total Cost per annum varies from \$1082.67 to \$1192.80 and the weighted Cycle Service Level (CSL) varies from 0.90 to 1.00. This gives the decision maker a variety of efficient solutions to choose from in order to maximize his utility function. From the Efficient Frontier, it can be seen that the weighted CSL rapidly increases initially from 0.90 with the increase in weighted ETC, but after 0.99, a large change in weighted ETC does not affect the weighted CSL which makes perfect sense since the weighted CSL cannot be more than 1. Any further increase in weighted ETC cannot result in higher weighted CSL.

The decision maker (DM) will decide on the policy to choose on the basis of cost vs. service level tradeoff. The DM can also note the ETC per annum and CSL of different spare

parts, while taking a decision, in case the individual spare part ETC or CSL are important criteria. For example, the DM may have a preference for a solution which makes sure that spare part 2 has a CSL of at least 0.98 with minimum weighted ETC of all spare parts. In that case, the DM can go with solution number 13 in Table 4.20.

From Table 4.20 it can also be noted that for all spare parts, an increase in weighted CSL results in decrease in the quantity ordered (Q), but an increase in the reorder point (r). It should also be noted that the rate of increase in reorder point is higher than the rate of decrease in the quantity ordered (Q). This makes sense because higher CSL warrants a higher reorder point to make sure that there are no stockouts.

Chapter 5 Conclusion and Future Extension

5.1 Conclusion

In this thesis our focus was to incorporate criticality, obsolescence and carry over potential of the spare parts into the spare parts inventory model and solve it as a multi-criteria mathematical programming (MCMP) problem with two objectives: minimize the weighted Expected Total Cost per annum and maximize the weighted Cycle Service Level, to help the decision maker arrive at a compromise solution which suits his/her preferences.

We have considered a case of centralized supply chain with one company and multiple spare parts suppliers. The demand of the spare parts and the number of spare parts becoming obsolete are assumed to be stochastic random variables with a Poisson distribution. The overall consumption of the spare parts is derived through convolution of two random variables. The Lead Time Demand (LTD) and Lead Time Obsolescence (LTO) are assumed to be normal distributions. A bi-criteria optimization problem is formulated with two criteria: minimizing the Weighted Expected Total Cost (ETC) and maximizing the Weighted Cycle Service Level (CSL) using the continuous review policy and a solution method ("constraint" approach) is presented which generates the entire efficient frontier. The weights for the spare parts are calculated using the Analytic Hierarchy Process (AHP) with three main criteria: Stockout Implication, Type of Spare Part and Carryover Potential. The efficient frontier of the bi-criteria optimization problem is generated using the "constraint" approach. Finally, an example is presented to illustrate the bicriteria problem, solution by the constraint method and generation of the efficient frontier.

5.2 Limitations and Future Extension

One of the limitations of the current model is that we have assumed the Lead Time Demand and Lead Time Obsolescence to follow a normal distribution. It is possible that the lead time demand and obsolescence follow a different distribution in which case the convolution to get the Lead Time Consumption (LTC) is difficult and the problem can no longer be solved using Solver add-in of MS Excel. The other limitation is that we have considered a centralized supply chain in which the there is only one decision maker taking all the decisions for the supply chain optimization. In practice, supply chains are often de-centralized. Also, in our model we have considered only two stages, the company and tier-1 suppliers. Hence, this research can be extended to different LTD and LTO distributions for de-centralized, multi-echelon supply chains. Similar models for periodic review policy can also be developed for B and C class spare parts, which do not require continuous review and hence result in lower inventory holding cost. The model can be extended to incorporate more number of criteria in the AHP. For example, qualitative aspects like supplier relationship, risk and quantitative aspects like distance from the factory can be added as criteria.

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