GEOTHERMAL ENERGY HARVESTING
THROUGH PILE FOUNDATIONS – ANALYSIS-
BASED PREDICTION AND PERFORMANCE
ASSESSMENT

A Dissertation in

Civil Engineering

by

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ABSTRACT

Seasonal variation of ground temperature is insignificant below a shallow depth, usually couple of meters, from the ground surface and thus pile foundations are good candidates for harvesting geothermal energy through heat exchange with ground. Such piles are commonly known as geothermal piles, heat exchanger piles or energy piles. The great potential of environmental, social and economic benefits of utilizing shallow geothermal energy has made the use of geothermal piles quite popular in different parts of the world. The aim of this study is to assess and quantify the potential of heat exchange through geothermal piles with a view to promote efficient design of pile-anchored geothermal energy harvesting systems. Research objective is achieved through development of numerical models that employ finite difference solution scheme and simulate pile-soil heat exchange with different levels of accuracy. Developed models are validated through comparison of model predictions using available analytical solutions under idealized conditions, field test data reported in literature and data recorded during thermomechanical tests on model geothermal pile installed in dry and saturated sand.

An annular cylinder heat source model, which simulates heat transport by the fluid circulating through tubes embedded in heat exchanger piles, is developed as a first modeling attempt. Results obtained from analyses using this model demonstrate that the use of a constant heat flux along the entire length of a heat exchanger pile may significantly misinterpret thermal response of the pile-soil system. The annular cylinder model considers one limb of the embedded heat exchanger element (i.e., circulation tube) and thus, can provide only approximate solution for real-life scenarios. Simultaneous heat transfer from both branches of an embedded U-shaped circulation tube is modeled next. Finite difference analysis (FDA) results are used to develop closed-form equations that can be used in calculation of power output from geothermal piles with
a single U-shaped circulation tube. Parameter sensitivity study and advanced first order second moment (AFOSM) reliability analysis are performed to determine the hierarchy of different input variables in order of their relative impacts on heat transfer performance.

The first two generations of models developed as part of this research consider heat transport (advection) by the circulation fluid and heat conduction in pile and soil surrounding it. While the model considering both branches of U-shaped circulation tube can predict field and laboratory test data with reasonable accuracy, some discrepancies were observed for predictions of heat transfer in saturated soil. Comparison of data recorded during instrumented laboratory tests on model geothermal pile installed in dry and saturated sand also indicated that heat convection through thermally-induced pore fluid flow within a saturated medium may further facilitate heat exchange through geothermal piles. This feature is incorporated in the developed pile-soil heat exchange model by coupling heat energy balance and Navier Stokes equations that considers a Boussinesq buoyancy term. Results indicate that thermal operation of geothermal piles alters pore fluid density, buoyant flow occurs (even under hydrostatic condition) within saturated soil in the vicinity of heat exchanger piles, and thermally induced pore water flow (under saturated condition) facilitates pile-soil heat exchange.
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<tbody>
<tr>
<td>A</td>
<td>Area (m²)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat (J/kg·K)</td>
</tr>
<tr>
<td>$C_F$</td>
<td>Drag constant (-)</td>
</tr>
<tr>
<td>D</td>
<td>Center to Center distance between downward and upward tubes (m)</td>
</tr>
<tr>
<td>d</td>
<td>Particle diameter (m)</td>
</tr>
<tr>
<td>$D_e$</td>
<td>Equivalent diameter of the bed (m)</td>
</tr>
<tr>
<td>E</td>
<td>Energy (J)</td>
</tr>
<tr>
<td>F</td>
<td>Body force (N)</td>
</tr>
<tr>
<td>$F_o$</td>
<td>Fourier number (-)</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity (m·s⁻²)</td>
</tr>
<tr>
<td>$h$</td>
<td>Convective heat transfer coefficient (W/m²·K)</td>
</tr>
<tr>
<td>i, j</td>
<td>Node index (-)</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity (W/m²·K)</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Absolute permeability (m²)</td>
</tr>
<tr>
<td>L</td>
<td>Pile length (m)</td>
</tr>
<tr>
<td>n</td>
<td>Porosity</td>
</tr>
<tr>
<td>N</td>
<td>Input parameter index</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate (kg·s⁻¹)</td>
</tr>
<tr>
<td>P</td>
<td>Power output (W)</td>
</tr>
<tr>
<td>$P_u$</td>
<td>Pressure (Pa)</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (-)</td>
</tr>
<tr>
<td>Q</td>
<td>Heat energy (J)</td>
</tr>
<tr>
<td>q</td>
<td>Heat flow rate (W)</td>
</tr>
<tr>
<td>$T_g$</td>
<td>Ground temperature (°C)</td>
</tr>
<tr>
<td>$T_{inlet}$</td>
<td>Temperature at circulation tube inlet (°C)</td>
</tr>
<tr>
<td>$T_{initial}$</td>
<td>Initial temperature of the analysis domain (°C)</td>
</tr>
<tr>
<td>$T_{outlet}$</td>
<td>Temperature at circulation tube outlet (°C)</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Average pile temperature (°C)</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Difference in fluid temperature between inlet and outlet points of the circulation tube (°C)</td>
</tr>
<tr>
<td>$\Delta T_b$</td>
<td>Temperature difference between inlet and outlet points for base analysis (analysis using a set of expected values of input parameters) (°C)</td>
</tr>
<tr>
<td>u</td>
<td>Groundwater velocity vector (m·s⁻¹)</td>
</tr>
<tr>
<td>U</td>
<td>Dimensionless groundwater velocity vector (-)</td>
</tr>
<tr>
<td>v</td>
<td>Circulation velocity of the heat carrier fluid (m·s⁻¹)</td>
</tr>
<tr>
<td>z</td>
<td>Depth (m)</td>
</tr>
</tbody>
</table>

### Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity (m²·s⁻¹)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Coefficient of thermal expansion (K⁻¹)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Forschheimer resistance (-)</td>
</tr>
<tr>
<td>$\theta_g$</td>
<td>Normalized ground temperature</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>Normalized fluid temperature</td>
</tr>
<tr>
<td>$\Delta \theta$</td>
<td>Initial temperature difference between fluid inlet point and ground (°C)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mass density (kg/m³)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>q&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Fluid circulation rate (m&lt;sup&gt;3&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt; or liter/min)</td>
</tr>
<tr>
<td>q&lt;sub&gt;l&lt;/sub&gt;</td>
<td>Heat flux per unit length (Wm&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
<tr>
<td>q&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Heat flux per area(Wm&lt;sup&gt;-2&lt;/sup&gt;)</td>
</tr>
<tr>
<td>R</td>
<td>Radius of the domain (m)</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number for thermal diffusion (-)</td>
</tr>
<tr>
<td>r</td>
<td>Radial coordinate (m)</td>
</tr>
<tr>
<td>r&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Radius of circulation tube (m)</td>
</tr>
<tr>
<td>r&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Pile radius (m)</td>
</tr>
<tr>
<td>s&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Shank spacing (wall-to-wall distance between two branches of the circulation tube (m)</td>
</tr>
<tr>
<td>t</td>
<td>Time (s)</td>
</tr>
<tr>
<td>t&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Thickness of the circulation tube (m)</td>
</tr>
<tr>
<td>T</td>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>T&lt;sub&gt;fw&lt;/sub&gt;</td>
<td>Fluid temperature at a point adjacent to the tube wall (°C)</td>
</tr>
<tr>
<td>T&lt;sub&gt;ag&lt;/sub&gt;</td>
<td>Temperature in the medium above ground surface (°C)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>Dynamic viscosity (Pa-s)</td>
</tr>
<tr>
<td>ν</td>
<td>Kinematic viscosity (m&lt;sup&gt;2&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
<tr>
<td>τ</td>
<td>Dimensionless time (-)</td>
</tr>
<tr>
<td>ψ</td>
<td>Stream function (m&lt;sup&gt;2&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
<tr>
<td>Ψ</td>
<td>Dimensionless stream function (-)</td>
</tr>
<tr>
<td>ω</td>
<td>Vorticity function (s&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
<tr>
<td>Ω</td>
<td>Dimensionless vorticity function (-)</td>
</tr>
<tr>
<td>γ</td>
<td>Porosity parameter (-)</td>
</tr>
</tbody>
</table>

*Subscripts*
- c Concrete
- f Fluid
- g Ground
- gw Groundwater
- m Medium
- p Pile
- s Soil
- t Circulation tube
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TO MY BELOVED

MOTHER AND FATHER
Chapter 1: Introduction

1.1 Background

Fossil fuels (e.g., coal, petroleum, natural gas) have long been used as the primary sources of energy in the U.S. and other parts of the world. Steady depletion of such non-renewable energy sources is among the top concerns that our civilization is facing today. Additionally, emission of greenhouse gases (that are predominantly responsible for global warming) resulting from the production and use of conventional energy, ever increasing cost of energy production, and low efficiency of some form of traditional energy sources (e.g., many traditional coal power plants operate only at around 32-36% efficiency; EIA 2010) have added to the worries of scientists, engineers and law makers around the world. This justifies the tremendous worldwide thrust for finding sustainable forms of alternate energy sources that will have reduced environmental, societal and economic impacts. Several renewable forms of energy, e.g., solar, wind, biomass, hydropower, geothermal, have demonstrated significant promise to provide suitable replacement for conventional energy sources used in several applications including electricity generation, water and space heating, transport fuels, to name a few. A recent report from Renewable Energy Policy Network for the 21st Century (REN21 2014) suggests that 19% of total energy consumption and 22% of electricity generation, respectively, in 2012 and 2013 were supplied by some form of renewable energy.

This research focuses on harvesting and use of shallow geothermal energy that relies exclusively on heat exchange with ground, as opposed to deep geothermal technologies that utilize extremely high temperature (~ 5,000°C) at the earth’s core for producing electricity. The thermal potential of ground can be utilized as a heat source or sink because temperature beyond a
certain depth (usually 6 to 8 meters) below the ground surface remains constant throughout the year (Brandl 2006, Florides and Kalogirou 2007, and Bouaza et al. 2012). Cast-in-place concrete piles with closed circulation loop are increasingly being used in different parts of the world to harvest shallow geothermal energy through pile-soil heat exchange. Heat is transported by circulating heat carrier fluid through the closed circulation loop (or loops) embedded within concrete piles. Such piles, commonly known as geothermal piles, heat exchanger piles or energy piles®, serve dual purposes – supporting the structural load and acting as a vertical heat exchanger element. Harvesting shallow geothermal energy through geothermal piles is an innovative way of using a sustainable form of energy and thus reducing conventional energy (e.g., electricity) consumption in residential and office buildings.

1.2 Motivation

A recent study by the U.S. Department of Energy (DOE) indicates that about 54% and 65% of the total cost of the residential and commercial buildings belongs, respectively, to heating and cooling (EIA 2010). In an earlier study, the U.S. Environmental Protection Agency (EPA) identified GSHP as the most efficient, economical and environmentally friendly space heating and cooling systems (EPA 1993). The use of GSHP systems results in a higher coefficient of performance (COP) compared with the use of air source heat pump (ASHP) systems because the temperature of the ground (which can be considered as a heat source) is relatively stable compared with air temperature. Several research articles (Boyd and Lieneue 1995, Brandl 1998, Swardt and Meyer 2001, Takasugi et al. 2001, Bloomfield et al. 2003, O’Conell et al. 2003, Sanner et al. 2005, Brandl 2006, Banks 2008, and Clarke et al. 2008) indicate that the use of GSHP and ground-water heat pump (GWHP) can result in a cost savings of 18-56% and in a
reduction in CO₂ emission by 45-80% as compared to ASHP and conventional sources of energy (e.g., coal, petroleum and natural gas) in residential and commercial buildings which is one of the major energy consumption sectors in the U.S. In fact, the use of geothermal piles as heat exchangers in GSHP systems further reduces the initial cost of GSHP system installation (Geotechnics Arup 2002, Presetschnik and Huber 2005, Hamada et al. 2007, Hwang et al. 2010).

In spite of the growing recognition of the benefits of harvesting shallow geothermal energy through heat exchanger piles, present practice heavily relies on experience and empirical rules which are purely based on ad-hoc approximations (Brandl 2006, Preene and Powrie 2009). The ability of geothermal piles to reject and extract heat to and from the ground directly affects both near- and long-term performance of ground source heat pump (GSHP) systems. Heat extraction from the ground in winter followed by heat rejection in summer can be a sustainable way of harvesting an alternative form of energy if pile-anchored GSHP systems are designed at an optimized thermal performance level. Therefore, for cost-efficient design of GSHP systems with heat exchanger piles, accurate quantification of heat transfer through these piles should be an integral part of the design. Moreover, it is important to identify and characterize different design, operational and site-specific variables (e.g., radii of pile and circulation tube, fluid circulation rate, thermal conductivity of soil and pile material) that may significantly affect heat transfer through geothermal piles.

1.3 Objectives and Hypothesis

The key objective of this study is to facilitate the development of precise yet easy-to-use analysis-based prediction tools that can be used in thermal performance assessment of geothermal piles. It is anticipated that research performed within the scope of this dissertation
will eventually help in promoting sustainable and efficient design practice for geothermal heat exchangers. A hypothesis – *thermally-induced fluid flow in saturated soil affects long term heat transfer efficiency and ground temperature increments in region surrounding geothermal piles* – is also formed and tested during the course of this research.

### 1.4 Methodology

Research objective is accomplished through (i) careful investigation and assessment of available tools to predict thermal performance of geothermal piles, (i) identification of areas of improvement to minimize approximations in quantification of thermal performance of geothermal heat exchangers, (iii) hierarchical development of numerical models with different levels of sophistications, and (iv) comparison of model predictions with analytical solutions, field test results and data recorded during large-scale physical model tests.

The governing differential equations describing energy and momentum balance equations are solved employing finite difference method. While explicit finite difference solution scheme is successfully used in the first two generations of the developed models, the third generation of model uses a combination of implicit and explicit solution schemes to solve Navier-Stokes and heat energy balance equations, respectively. Sensitivity of different design, operational and field parameters in affecting pile-soil heat exchange is identified using Advanced First Order Second Moment (AFOSM) reliability analysis. Moreover, a series of thermal tests on a model geothermal pile were performed to gather data that helped in validating performance of second and third generations of numerical models. Based on the results from a series of heat transfer analyses, regression-based closed form analytical expressions are proposed that can be used to predict power output from a single heat exchanger pile.
1.5 Organization

This dissertation consists of eight chapters. Following the motivation, objective and hypothesis presented in this chapter, Chapter 2 presents a synthesis of published analytical and numerical studies on heat transfer through vertical geothermal heat exchangers. Field and laboratory tests that investigated thermal performance of geothermal piles are also included Chapter 2.

Chapter 3 presents an annular cylinder model that approximately simulates heat transfer through a geothermal heat exchanger pile. Predictions using the developed model are compared with that using an idealized analytical heat transfer model (finite line source model; Zeng et al. 2002) and limitations of the idealized analytical model in predicting pile-soil heat exchange are highlighted in this chapter. A rigorous pile-soil heat exchange model that accounts for simultaneous heat transfer through both branches of an embedded U-shaped circulation tube, convective resistance of circulation fluid and thermal resistance of the circulation tube is presented in Chapter 4. Prediction accuracy of the developed model is investigated through comparison with available analytical solutions for finite line source model (Zeng et al. 2002) and thermal resistivity based model of pile-soil heat exchange (Zeng et al. 2003a and 2003b, Marcotte and Pasquier 2008). Performance of the developed finite difference model under different thermal operational conditions is also compared with a 3D finite element model developed using COMSOL Multiphysics (Ozudogru et al. 2014). Moreover, the developed model is used to predict published field test data recorded during thermal tests on geothermal piles.

Chapter 5 presents thermal performance analysis of a geothermal pile using the model described in Chapter 4. Effects of different design, operational, and site specific parameters on ground temperature response and heat exchange efficiency of geothermal piles with a single U-
shaped circulation tube are investigated in this chapter. A regression-based equation is proposed to predict expected power output from a geothermal pile with arbitrary input parameters. Heat exchange performance of the geothermal piles under uncertain field and operational conditions is assessed through AFOSM reliability analysis.

Results from a series of laboratory thermal tests on a instrumented model geothermal pile installed in saturated sand are presented in Chapter 6. Element thermal conductivity tests performed to obtain (using finite line source model and Fourier’s law) values of soil and concrete thermal conductivity that are to be used as input parameters for the numerical model described in Chapter 4 are also discussed in this chapter. The developed numerical platform is used to model the laboratory-scale experiment and thermal performance of the model geothermal pile under dry and saturated conditions are compared with model predictions.

Chapter 7 describes numerical verification of research hypothesis developed based on performance comparison presented in Chapter 6. The model described in Chapter 4 is modified to additionally account for convective heat transfer through buoyant flow and in saturated soil. A hybrid solution scheme that can simultaneously solve energy and momentum equations is described in this chapter. Furthermore, model performance is verified through comparison with published data for a cavity flow problem described in (Hossain et al. 2013). Chapter 8 of this dissertation presents the summary and conclusions derived from this study; this chapter also provides some recommendations for future research.
1.6 Cited Reference


Chapter 2: Synthesis of Literature Focusing on Heat Exchange through Vertical Geothermal Heat Exchangers

This chapter presents a brief synthesis of past studies aimed at quantification and thermal performance assessment of vertical geothermal heat exchangers, i.e., geothermal piles and boreholes. Although the focus of this dissertation is on geothermal piles, some analytical techniques and numerical studies mentioned in this chapter apply equally for both geothermal piles and boreholes. Analytical solutions based on heat transfer (conduction) and thermal resistance frameworks are discussed separately; numerical and physical modeling approaches reported in literature are also included in this chapter. Moreover, key findings from some well documented thermal tests on full-scale and model geothermal piles are reported.

2.1 Analytical Heat Conduction Models

Researchers from Petroleum Engineering analyzed heat transfer through geothermal boreholes as early as in 1947. Since then several researchers have developed and modified idealized heat transfer models for predicting heat exchange between vertical geothermal heat exchangers and ground. Different idealized heat source models were analyzed over the last two decades to estimate variation of ground temperature due to the presence of a finite or infinite heat source within the ground. Available heat source models can broadly be divided into four main categories: (i) infinite and finite line sources, (ii) hollow cylinder source, (iii) one- and two-dimensional solid cylinder sources, and (iv) spiral source model. Simplified assumptions were made for each of these categories in order for the analytical solutions to be possible.
Vertical heat exchanger boreholes were first modeled as infinite line and infinite hollow cylinder heat sources with a constant value of heat flux along the length. Carslaw and Jaeger (1947) and Ingersoll et al. (1954) provided analytical solutions for heat transfer through infinite hollow cylinder and infinite line sources, respectively. Analytical solutions for heat conduction in soil surrounding a finite line heat source with constant heat flux were proposed by Eskilson (1987) for steady state condition and by Zeng et al. (2002) and Lamarche and Beauchamp (2007a) for transient condition. Cui et al. (2006) and Lamarche (2011) provided analytical solutions for transient ground temperature response caused by a single inclined finite line source. Lamarche and Beauchamp (2007b) studied ground temperature distribution around a heat exchanger borehole using infinite solid cylinder heat source model with two different boundary conditions: (i) constant heat flux and (ii) constant mean temperature for the heat carrier fluid (or grout). Man et al. (2010) developed analytical solutions for heat conduction through one- and two-dimensional solid cylinder sources using Green’s function. Researchers have also suggested the use of spiral heat source model for heat exchanger elements with spiral heat sources (Cui et al. 2011, and Li and Lai 2012). Table 2-1 summarizes available solutions for different idealized heat source models.

Note that previous analytical studies used only idealized heat source models to quantify temperature distribution in the medium surrounding the heat source. Circulation of heat transporting fluid within a geothermal heat exchanger element (such as borehole heat exchanger or geothermal pile) is not captured through any idealized analytical model. Consequently, a constant value of either temperature or heat flux along the entire length of a geothermal heat exchanger has to be assumed in order to use an analytical model to quantify heat exchange through geothermal piles. However, such assumptions are far from being practical since heat flux
(and temperature) varies not only along the length of the pile but also over the duration of heat transfer operation (Lamarche et al. 2010). Moreover, the effect of fluid circulation rate on heat transfer efficiency of a geothermal pile cannot be quantified using solutions obtained from available idealized heat source models. Such solutions cannot also predict the time-dependent evolution of heat flux and thus would lead to an inaccurate estimation of temperature distribution in soil surrounding geothermal piles.
<table>
<thead>
<tr>
<th>Model description</th>
<th>Analytical solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite line (Ingersoll et al., 1954)</td>
<td>$\Delta T(r,t) = -\frac{q_l}{4\pi k} Ei\left(\frac{-r^2}{4\pi t}\right)$</td>
</tr>
<tr>
<td>Finite line (Zang et al., 2002)</td>
<td>$\Delta T(r,z,t) = \frac{q_l}{4\pi k} \frac{1}{2} \int_0^\infty \frac{1}{2\sqrt{\alpha t}} I_0 \left(\frac{rr}{2\alpha(t-t')}\right) \left{ \exp \left[ -\frac{</td>
</tr>
<tr>
<td>Infinite hollow cylinder (Carstew and Langer, 1947)</td>
<td>$\Delta T(r,z,t) = \frac{q_l}{\pi^2 k r_e^2} \int_0^\infty \left[ (e^{-\frac{r^2}{4\alpha t}} - 1) J_0(2r_0 u_r) - J_0(2r_0 u_r) J_0(2r_0 u_r) \right] du$</td>
</tr>
<tr>
<td>Infinite solid cylinder (Man et al., 2010)</td>
<td>$\Delta T(r,t) = -\frac{q_l}{4\pi k} \int_0^\infty \frac{1}{\pi} Ei\left(\frac{-r^2}{4\pi t}\right) d\phi$</td>
</tr>
<tr>
<td>Finite solid cylinder (Man et al., 2010)</td>
<td>$\Delta T(r,z,t) = -\frac{q_l}{\rho c_v} \frac{1}{8} \int_0^\infty \frac{1}{\sqrt{\pi \alpha(t-t')}} I_0 \left(\frac{rr}{2\alpha(t-t')}\right) \left{ \exp \left[ -\frac{</td>
</tr>
<tr>
<td>Spiral line model (Li and Lai 2012)</td>
<td>$\Delta T(r,\varphi,z,t) = -\frac{q_b}{4\pi \sqrt{\alpha \alpha_t}} \frac{1}{d} \frac{1}{2} \text{erfc} \left( \frac{d}{2\sqrt{\alpha_t}} \right) - \frac{1}{d'} \text{erfc} \left( \frac{d'}{2\sqrt{\alpha_t}} \right) d\beta$</td>
</tr>
</tbody>
</table>

- $q$: heat flux per unit length; $C_p$: specific heat; $k$: thermal conductivity; $L$: length of the heat source; $r_0$: cylinder radius; $r, z,$ and $\varphi$: Cylindrical coordinate system; $\beta$: integration parameter; $t$: time; $T$: temperature; $\alpha$: thermal diffusivity; $\rho$: mass density; $J_0, Y_0,$ and $I_0$: Bessel’s functions; $x, y,$ and $z$: Cartesian coordinate system

\[ Ei(x) = \int_{-\infty}^x \frac{e^u}{u} du; \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du; I_0(x) = \frac{1}{\pi} \int_0^\infty \exp(x\cos\beta) d\beta \]
2.2 Analytical Models Based on Thermal Resistance Framework

Several past studies adopted decoupled approaches for solving heat transfer within the heat exchanger using thermal resistance approach (Eskilson 1987, Hellstrom 1991, Zeng et al. 2003a, 2003b, Marcotte and Pasquier 2008, Sharqawy et al. 2009, Lamarche et al. 2010, Bauer et al. 2011). Following the research performed by Eskilson (1987) and Hellstrom (1991), Zeng et al. (2003a and 2003b) focused on heat transfer inside the borehole for two different cases (i) geothermal heat exchanger with a single U-shaped circulation tube and (ii) geothermal heat exchanger with two separate U-shaped circulation tubes for both parallel and series configuration (Figure 2-1). An analytical solution was proposed based on thermal resistance framework to predict normalized fluid temperature along the U-tube embedded within geothermal pile or borehole. Three simplified assumptions were adopted in order to make an analytical solution possible: (1) thermal resistance of the tube material (i.e., the effect of tube thickness) and convective thermal resistance of the circulation fluid were neglected, (2) temperature of the heat exchanger element (pile or borehole) was assumed to be constant along the depth for any time step, and (3) the ground and the grout materials (fill material for boreholes) were treated as homogenous materials.
2.2.1 Single U-tube

Considering a constant value of heat flux per unit length $q_{l,i}$ ($i = 1, 2$) for each branch of the circulation tube, fluid temperature circulated though the pipes can be expressed as:

$$T_{l,1} - T_p = R_{i1}q_{l,1} + R_{i2}q_{l,2}$$  \hspace{1cm} (2-1a)

$$T_{l,2} - T_p = R_{22}q_{l,2} + R_{21}q_{l,1}$$  \hspace{1cm} (2-1b)

where $R_{ii}$ ($i=1, 2$) is the thermal resistance between the circulation fluid and the borehole wall, and $R_{ij}$ ($i\neq j$) is the thermal resistance between two branches of circulation tube and $T_p$ is pile wall temperature.

Since the material is homogenous and the geometry is symmetric, $R_{11}$ equals to $R_{22}$, and $R_{12}$ is equal to $R_{21}$ due to geometric symmetry. Based on the study performed by Hellstrom (1991) the thermal resistance between the pipes and the thermal resistance between the circulation fluid and borehole wall can be expressed using the line source assumption as:
\[ R_{11} = \frac{1}{2\pi k_c} \left[ \ln \left( \frac{r_p}{r_i} \right) - \frac{k_c - k_s}{k_c + k_s} \ln \left( \frac{r_p^2 - D^2}{r_p^2} \right) \right] + R_{\text{tube}} \] (2-2a)

\[ R_{12} = \frac{1}{2\pi k_c} \left[ \ln \left( \frac{r_p}{2D} \right) - \frac{k_c - k_s}{k_c + k_s} \ln \left( \frac{r_p^2 + D^2}{r_p^2} \right) \right] \] (2-2b)

where thermal resistance \( R_{\text{tube}} \) of the circulation tube.

Zeng et al. (2003a and 2003b) calculated the variation of the dimensionless fluid temperature along the downward and upward branches by considering Equations (2-2a) and (2-2b) which represent respectively, thermal resistance between circulation fluid and the borehole/pile wall and that between the two branches of circulation tube. The difference between pile (or borehole) wall temperature \( T_p \) and fluid temperature along the downward and upward tube branches \( (T_{fd} \text{ and } T_{fu}) \) can be normalized with respect to the difference between inlet fluid temperature \( T_{\text{inlet}} \) and the pile wall temperature. Normalized fluid temperature \( \theta_d(z) \) and \( \theta_u(z) \) along the downward and upward branches of the circulation tube are expressed as:

\[ \theta_d(z) = \frac{T_{fd}(z) - T_p}{T_{\text{inlet}} - T_p} \] (2-3a)

\[ \theta_u(z) = \frac{T_{fu}(z) - T_p}{T_{\text{inlet}} - T_p} \] (2-3b)

According to Zeng et al. (2003a and 2003b) \( \theta_d(z) \) and \( \theta_u(z) \) can be calculated as:
\[
\theta_a(z) = \cosh(\beta Z) - \frac{1}{\sqrt{1 - p^2}} \left[ 1 - p \frac{\cosh(\beta) - \frac{1-p}{\sqrt{1+p}} \sinh(\beta Z)}{\cosh(\beta) + \frac{1-p}{\sqrt{1+p}} \sinh(\beta)} \right] \sinh(\beta Z) \tag{2-4a}
\]

\[
\theta_s(z) = \frac{\cosh(\beta) - \frac{1-p}{\sqrt{1+p}} \sinh(\beta)}{\cosh(\beta) + \frac{1-p}{\sqrt{1+p}} \sinh(\beta)} \cosh(\beta Z) + \\
\frac{1}{\sqrt{1 - p^2}} \left[ \cosh(\beta) - \frac{1-p}{\sqrt{1+p}} \sinh(\beta) \right] \sinh(\beta Z) \tag{2-4b}
\]

where,

\[
p = \frac{R_{11}}{R_{12}} \tag{2-5}
\]

\[
\beta = \frac{L}{\rho_i v \pi r_i^2 C_{pf} \sqrt{(R_{11} + R_{12})(R_{11} - R_{12})}} \tag{2-6}
\]

\[
Z = \frac{z}{L} \tag{2-7}
\]

Bernier (2001) and Marcotte and Pasquier (2008) recommended that tube thickness \(t_i\) and fluid convection resistance should be considered in calculation of \(\theta_a(z)\) and \(\theta_s(z)\). Equation (2-2a) can be modified as follows considering the tube thickness. Note, Equation (2-8b) is same as Equation (2-2b).
\[
R_{11} = \frac{1}{2\pi k_c} \left[ \ln \left( \frac{r_p}{r_i + t_i} \right) - \frac{k_c - k_s}{k_c + k_s} \ln \left( \frac{r_p^2 - D^2}{r_p^2} \right) \right] + R_{\text{tube}} \quad (2-8a)
\]

\[
R_{12} = \frac{1}{2\pi k_c} \left[ \ln \left( \frac{r_p}{2D} \right) - \frac{k_c - k_s}{k_c + k_s} \ln \left( \frac{r_p^2 + D^2}{r_p^2} \right) \right] \quad (2-8b)
\]

where thermal resistance \( R_{\text{tube}} \) is Equation (2-8b) can be expressed as:

\[
R_{\text{tube}} = R_{\text{cond}} + R_{\text{conv}} \quad (2-9)
\]

\( R_{\text{cond}} \) and \( R_{\text{conv}} \) are, respectively, thermal resistance of tube material and fluid convection resistance.

\[
R_{\text{cond}} = \frac{\ln \left( \frac{r_i + t_i}{r_i} \right)}{4\pi k_i} \quad (2-10a)
\]

\[
R_{\text{conv}} = \frac{1}{4\pi r_i h_f} \quad (2-10b)
\]

By knowing the tube thermal resistance, the pile or borehole thermal resistance can be predicted as a summation of thermal resistance \( R_{\text{concrete (or grout)}} \) of concrete (for geothermal piles) (or grout for geothermal boreholes) and that of the circulation tube \( R_{\text{tube}} \).

\[
R_{\text{pile (or borehole)}} = R_{\text{tube}} + R_{\text{concrete (or grout)}} \quad (2-11)
\]

where:
\[ R_{\text{concrete}} = \frac{1}{S_c k_c} \] (2-12)

\[ S_c \] is the shape factor which depends on the geometry of the circulation tubes.

\[ S_c = \beta_0 \left( \frac{r_i}{r_i + t_i} \right)^{\beta_i} \] (2-13)

The pile or borehole thermal resistance can also be analytically predicted (Zeng et al. 2003a and 2003b). Since \( R_{11} \) equals to \( R_{22} \), and \( R_{12} \) is equal to \( R_{21} \), and by considering equal heat flux per unit length of both downward and upward branches of circulation tube, Equation (2-14) can be obtained by summation of Equations (2-1a) and (2-1b).

\[ T_{e,1} + T_{e,2} - 2T_p = 2R_{11}q_{l,1} + 2R_{12}q_{l,1} \] (2-14)

Rearranging Equation (2-14) and using total heat flux per unit length for the pile (or borehole) Equation (2-14) can be rewritten as:

\[ T_{e,\text{mean}} - T_p = q_{l,1} \left( R_{11} + R_{12} \right) = \frac{1}{2} q_{l} \left( R_{11} + R_{12} \right) \] (2-15)

where \( T_{e,\text{mean}} = \frac{T_{e,1} + T_{e,2}}{2} \) is mean fluid temperature (\( T_{e,1} \) and \( T_{e,2} \) are the fluid temperatures respectively in downward (1) and upward (2) branches of the circulation tube), and heat flux per unit length of the U-tube is the half of the heat flux per unit length of the pile (i.e., \( q_{l,1} = \frac{q_l}{2} \)).

Considering the tube resistance and fluid convection resistance the dimensionless fluid temperature can be calculated through Equations (2-4) to (2-10). Following the calculation of
the normalized fluid temperature, pile thermal resistance can also be analytically predicted using Equation (16a) or Equation (16b).

\[ R_{\text{pile}} = \frac{R_{11} + R_{12}}{2} \]  

\[ R_{\text{pile}} = \frac{L}{2 \rho_f v \pi r_i^2 C_{pf}} \times \left( 1 + \theta_u^w \right) \left( 1 + \theta_u^w \right) \]  

where \( \theta_u^w \) is the normalized fluid temperature at the outlet point.

### 2.2.2 Double U-tube

Based on the line source assumption, H Winsl öm (1991) proposed expressions for thermal resistance between circulation tube branches and the thermal resistance between circulation fluid and borehole wall for double U-tube configuration (Figure 2-1).

\[ R_{11} = \frac{1}{2 \pi k_c} \left[ \ln \left( \frac{r_p}{r_i + t_i} \right) - \frac{k_c - k_i}{k_c + k_i} \ln \left( \frac{r_i^2 - D_c^2}{r_p^2} \right) \right] + R_{\text{tube}} \]  

\[ R_{13} = \frac{1}{2 \pi k_c} \left[ \ln \left( \frac{r_p}{\sqrt{2} D_c} \right) - \frac{k_c - k_i}{2 (k_c + k_i)} \ln \left( \frac{r_i^4 + D_c^4}{r_p^4} \right) \right] \]  

\[ R_{12} = \frac{1}{2 \pi k_c} \left[ \ln \left( \frac{r_p}{2 D_c} \right) - \frac{k_c - k_i}{k_c + k_i} \ln \left( \frac{r_i^2 + D_c^2}{r_p^2} \right) \right] \]  

Temperature of fluid circulated though the double U-tube can be expressed as (Zeng et al. 2003a and 2003b):
\[ T_{t,1} - T_p = R_{11} q_{11} + R_{12} q_{12} + R_{13} q_{13} + R_{14} q_{14} \] (2-18a)

\[ T_{t,2} - T_p = R_{21} q_{21} + R_{22} q_{22} + R_{23} q_{23} + R_{24} q_{24} \] (2-18b)

\[ T_{t,3} - T_p = R_{31} q_{31} + R_{32} q_{32} + R_{33} q_{33} + R_{34} q_{34} \] (2-18c)

\[ T_{t,4} - T_p = R_{41} q_{41} + R_{42} q_{42} + R_{43} q_{43} + R_{44} q_{44} \] (2-18d)

Pile thermal resistance for geothermal piles with double U-tubes is finally expressed as:

\[ R_{\text{pile}} = \frac{R_{11} + R_{12} + R_{13} + R_{14}}{2} \] (2-19)

### 2.3 Numerical Studies

Several numerical studies on vertical geothermal heat exchangers are also reported in literature (Yavusturk 1999, Nam et al. 2008, Abdelaziz et al. 2011, Suryatriyastuti et al. 2012, Gao et al. 2008a, Lamarche et al. 2010, Rouissi et al. 2012, Ozudogro et al. 2014); however, only a few studies focused purely on geothermal piles and considered the temperature variation of heat carrier fluid as it circulates through circulation tube embedded within geothermal piles (Gao et al. 2008a, Lamarche et al. 2010, Rouissi et al. 2012, Ozudogro et al. 2014).

Yavusturk (1999) simulated (using finite volume method) ground temperature response in a horizontal 2D cross section assuming constant fluid temperature. Nam et al. (2008) studied (using finite element model) different heat exchange rates per unit length of a geothermal pile for both heating and cooling and compared the coefficient of performance (COP) for the heating (heat extraction from the ground) and cooling (heat rejection to ground) modes of operation. Gao
et al. (2008a, 2008b) and Lamarche et al. (2010) numerically computed (using finite element method) a variation of fluid temperature along the length of a geothermal pile. Through two-dimensional finite difference analysis of a geothermal pile foundation, Rouissi et al. (2012) showed that the temperature gained from the ground is higher for lower fluid circulation velocity. Thermal response of geothermal piles was also numerically studied by Abdelaziz et al. (2011) using COMSOL Multiphysics® (a commercially available finite element software) platform. Through three dimensional finite element analyses in COMSOL Multiphysics® Ozudogro et al. (2014) simulated thermal response tests of geothermal piles and boreholes under constant heat injection scenario and compared the ground temperature response, fluid temperature variation along the circulation tube, and pile and borehole thermal resistivity with analytical solutions. Apart from studies that focus on geothermal piles, numerical and analytical studies have also been performed to predict variation of fluid temperature and thermal resistance of borehole heat exchangers (Diersch et al. 2011a, and 2011b, Bauer et al. 2011, Mottaghy et al. 2012, Al-Khoury et al. 2005, and Al-Khoury and Bonnier 2006).

Although some recent studies dealt with realistic heat exchange behavior of geothermal piles, the computational cost for those analyses are relatively high. Therefore, no studies have yet comprehensively quantified the expected variance in heat exchange efficiency of geothermal piles due to individual and combined statistical variation of different design, operational and field parameters. Discrete attempts have been made in the past to study the effects of some individual parameters on few aspects of heat exchange through geothermal piles; however, such attempts fall short of finding practical applications due to the lack of easy-to-use design solutions that can be implemented by practitioners. Development of a numerical model with low computational cost to analyze heat exchange through geothermal piles would be a welcome
approach in this regard. Such a model should realistically capture all key features of heat transfer through energy piles to predict energy efficiency of a pile-anchored geothermal energy harvesting system for both short- and long-term thermal operations under any combinations of climatic, geologic and project-specific conditions will be a step forward in promoting efficient design of geothermal piles. Note all that existing models only consider a sinusoidal function for climatic temperature instead of a real annual hourly temperature variation.

Moreover, previous numerical studies overlooked the possibility of thermally-induced pore fluid flow around geothermal piles installed in saturated soil; consequently, the effect of heat convection through pore fluid flow (vortex) on ground temperature response and heat exchange efficacy (power output) of a geothermal pile is missing in literature.

2.4 Laboratory and Field Tests

Field investigations in China and Japan revealed the effects of shape of the circulation pipes and flow rate of heat transporting fluid on thermal performance of energy pile systems. Hamada et al. (2007) performed field tests and compared the workability and cost-efficiency of geothermal piles with different geometries of embedded circulation tubes and concluded that U-tube has the highest workability and cost efficiency. Gao et al. (2008a, 2008b) concluded that an increase in the flow rate increases thermal efficiency of geothermal piles and that W-shaped circulation tubes provide higher efficiency when compared with single, double and triple U-tubes. Double U-tube system has two separate tubes with two inlet points and two outlet points (a parallel system) however W-shaped system has one inlet point and one outlet point (a serial system). Jalaluddin et al. (2011) showed that heat exchange rates increases with increasing fluid circulation rate. Laboratory tests performed on model geothermal piles are limited in number:
(a) a series of centrifuge tests were performed at the University of Colorado at Boulder (McCartney and Rosenberg 2011, and Stewart and McCartney 2012), (b) a small-scale laboratory test performed at Monash University, Australia (Wang et al. 2011, and 2012), (c) thermal performance tests performed at Darmstadt University of Technology, Germany (Ennigkeit and Katzenbach 2001), and (d) a series of thermal performance tests on an instrumented heat exchanger pile with an embedded U-shaped circulation tube at the Pennsylvania State University (Kramer 2013, Kramer and Basu 2014a and 2014b, and Kramer et al. 2014). The large-scale laboratory test setup reported in Kramer et al. (2014) along with saturated tests performed in this study are also used during the course of this study to analyze thermal performance of a model geothermal heat exchanger pile under saturated condition and investigate the effect of convective boundary conditions on ground and pile temperature response. Details of these tests and recorded data are presented in Chapter 6.

2.5 Summary

Several previous studies have adopted decoupled approaches for either solving heat transfer within the heat exchanger using thermal resistance approach (e.g., Eskilson 1987, Hellstorm 1991, Zeng et al. 2003a, 2003b, Marcotte and Pasquier 2008, Sharqawy et al. 2009, Lamarche et al. 2010, Bauer et al. 2011) or quantifying ground temperature increments based on idealized heat source assumptions (e.g., Zeng et al. 2002, Cui et al. 2006, Lamarche and Beauchamp 2007a and 2007b, Man et al. 2010, Lamarche 2011, Cui et al. 2011, Li and Lai 2012). However, such solutions do not simultaneously provide temperature increments within the heat exchanger and in the medium surrounding it. In most cases such solutions are also based on idealized assumptions such as, constant heat flux (or temperature) along the heat exchanger length, neglecting fluid
convection resistance within the circulation tube, homogenization of media surrounding the heat sources, etc. None of the previous studies quantified the relative variance of heat exchange efficiency and pile-soil interface temperature increments due to individual or combined variation of different operation, design and in situ parameters. Moreover, the potential for convective heat transfer due to thermally induced pore fluid flow in saturated ground was overlooked in the past studies.

2.6 Cited References

Abdelaziz SL, Olgun CG, Martin JR. (2011). Design and operational considerations of geothermal energy piles, Geo-Frontiers, American society of Civil Engineers Conference; 450-459.


Chapter 3: Annular Cylinder Model for Heat Exchanger Piles

Comparison of existing numerical and analytical approaches to analyze heat transfer through geothermal piles reveals that the available analytical models are comparatively easy and fast tool to predict ground temperature response. These models can also be used for indirect estimation of thermal efficiency for pile-soil heat exchange system. However, as discussed in the Chapter 2, analytical models suffer from several simplified assumptions that deviate from real-life scenario. Development of an approximate numerical model, which can capture some essential features of pile-soil heat exchange, is discussed in this chapter. A finite difference code is developed for solving a system of partial differential equations which describe heat flow through heat carrier fluid and heat conduction in soil and concrete. The developed model can capture the effects of different design, operational and site-specific variables on time-dependent variation of ground and circulation fluid temperature. A sensitivity study is performed to identify important parameters that may significantly affect heat transfer efficiency of geothermal piles.

3.1 Model Development

Heat transfer through a concrete geothermal pile with an embedded U-shaped circulation tube is modeled using an annular cylinder approximation. Half of the pile is modeled exploiting the approximately axisymmetric heat flow condition in the medium surrounding the pile (Figure 3-1a). Note that the location and arrangement of the circulation tubes within a geothermal pile does not strictly satisfy the condition of an axisymmetric geometry. However, the diameter of the circulation tube (heat source) is two orders of magnitude smaller than the expected thermal
influence zone surrounding the pile. Therefore, the assumption of axisymmetric heat conduction in the media (i.e., concrete and soil) surrounding the heat source is not far from reality.

![Figure 3-1 Annular cylinder heat source model (a) isometric and plan view and (b) finite difference grid and boundary conditions](image)

Interaction between two vertical limbs of the U-tube is not considered at this stage. The wall thickness of circulation tube is also assumed to be zero; thus, possible heat loss within the tube wall is neglected. For the first generation model, this seemed to be a reasonable assumption because the thickness of circulation tubes used in practice is often in the order of couple of millimeters only. However, subsequent refinement achieved in the second generation model discussed in the following chapters revealed that omission of thermal resistance of the tube wall introduces non-negligible overprediction of ground temperature at the pile-soil interface. Heat transfer from the heat carrier fluid to the surrounding media is analyzed by coupling heat
conduction and heat balance equations. Heat energy balance for an arbitrary element within the domain is considered to obtain the time-dependent evolution of temperature $T$ within the analysis domain (concrete, and soil). Equation (3-1) shows the heat energy balance for an arbitrary element.

$$
\left[ \rho C_p \pi \left( \left( r'_{i+1} \right)^2 - \left( r'_{i} \right)^2 \right) \Delta z \right] \Delta T^y = Q_{in} - Q_{out}
$$

Where $r' (= r - r_i)$ is a radial distance measured from the center of the tube, and $r$ is the measured radial distance from the origin, $\Delta z$ is the distance between the top and bottom surfaces of the FD stencil encompassed by computational points $(i, j)$, $(i+1, j)$, $(i+1, j+1)$, and $(i, j+1)$, $\rho$ is mass density and $C_p$ is specific heat capacity of the medium where the stencil is located. $Q_{in} - Q_{out}$ is the net heat flow into the stencil due to the circulation tube. The amount of heat injection and rejection from each side of the desired element can be calculated using the heat flux at each side.

$$
\left[ \rho C_p \pi \left( \left( r'_{i+1} \right)^2 - \left( r'_{i} \right)^2 \right) \Delta z \right] \frac{\Delta T^y}{\Delta t} = \\
-2\pi r'_{i,j} \Delta z k_{i,j} \frac{\Delta T}{\Delta r} \bigg|_{r=r'_{i+1}} - \pi \left( \left( r'_{i+1} \right)^2 - \left( r'_{i} \right)^2 \right) k_{i,j} \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j}}
$$

$$
+2\pi r'_{i+1,j} \Delta z k_{i,j} \frac{\Delta T}{\Delta r} \bigg|_{r=r'_{i+1}} + \pi \left( \left( r'_{i+1} \right)^2 - \left( r'_{i} \right)^2 \right) k_{i,j} \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j+1}}
$$

Simplifying Equation (3-2), heat balance equation can be expressed as:
\[
\left[ \left( (r_{i+1}^r)^2 - (r_i^r)^2 \right) + \frac{\Delta T_{ij}^y}{\Delta t} \right] = \alpha_{i,j} \left[ -2r_{i,j} \frac{\partial T}{\partial r} \bigg|_{r=r_{i,j}} - \frac{\left( (r_{i+1}^r)^2 - (r_i^r)^2 \right)}{\Delta z} \bigg|_{z=z_{i,j}} \Delta T \bigg] + 2r_{i+1,j}^r \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1,j}}^r + \frac{\left( (r_{i+1}^r)^2 - (r_i^r)^2 \right)}{\Delta z} \bigg|_{z=z_{i,j}} \Delta T \bigg]
\] (3-3)

Further rearrangement of Equation (3-3) yields:

\[
\frac{\Delta T_{ij}^y}{\alpha_{i,j} \Delta t} = \left[ \frac{-2r_{i,j} \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i,j}} + 2r_{i+1,j}^r \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1,j}}}{\left( (r_{i+1}^r)^2 - (r_i^r)^2 \right)} \right] + \left[ -\frac{1}{\Delta z} \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j}} + \frac{1}{\Delta z} \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j+1}} \right]
\] (3-4)

\[
\frac{\Delta T_{ij}^y}{\alpha_{i,j} \Delta t} = \left[ \frac{-2r_{i,j}’ \frac{\partial T}{\partial r} \bigg|_{r=r_{i,j}} + 2r_{i+1,j}^r \frac{\partial T}{\partial r} \bigg|_{r=r_{i+1,j}}}{\left( (r_{i+1}^r)^2 - (r_i^r)^2 \right)} \right] + \left( \frac{\partial^2 T}{\partial z^2} \bigg|_{(i,j)} \right)
\] (3-5)

where \( r_m’ \) is the mean radius (distance of the midpoint of the stencil) from the center of the circulation tube. Equation (3-5) results in the partial differential equations which has derivatives over \( r \) and \( z \).
Thus time-dependent evolution of temperature \( T(r, z, t) \) due to heat conduction within the analysis domain using Equation (3-7) can be expressed as:

\[
\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r_m} \frac{\partial T}{\partial r}
\]

(3-8a)

\[
\alpha = \frac{k}{\rho C_p}
\]

(3-8b)

where \( \alpha, k, \rho \) and \( C_p \) are, respectively, thermal diffusivity, thermal conductivity, mass density and specific heat capacity of the heat conduction medium and \( t \) is time.

Equation (3-8) alone cannot describe heat transfer through a geothermal pile because it does not capture heat flow within the circulation tube. Considering that average temperature of an element A (Figure 3-1a) within the circulation tube increases by an amount \( dT \) over time \( dt \) and assuming an average heat flow rate \( q \) (from element A to concrete pile) over the length \( dz \), the heat balance equation for element A can be written as:
\[ \dot{m}C_{pf} \, dt dT^e = q \, dt + \rho_f \pi r_t^2 dz C_{pf} \, dT \]  

(3-9)

where \( dT^e \) is the temperature difference (over the length \( dz \)) between top and bottom of element \( A \), \( \dot{m} \) and \( C_{pf} \) are, respectively, mass flow rate and specific heat capacity of heat carrier fluid circulating through the tube, and \( r_t \) is radius of the circulation tube. Radial variation of fluid temperature (expected in laminar flow condition due to fluid convective resistance) at a particular depth is neglected in Equation (3-9). Heat flow rate \( q \) can be related to heat flux \( \dot{q} \) as:

\[ q = \dot{q} \, dA \]  

(3-10)

where \( dA = 2\pi r_t \, dz \) is the surface area available for heat transfer from element \( A \) to the concrete pile. It is important to accurately account for the total amount of heat energy input to the pile-soil system and thus the three-dimensional (cylindrical) geometry of the circulation tube branch is considered in Equation (3-10). Heat flux \( \dot{q} \) is further defined as:

\[ \dot{q}_r(z, t) = -k_c \frac{\partial T}{\partial r} \]  

(3-11)

where \( k_c \) is thermal conductivity of concrete. Using the definition of mass transfer (flow) rate \( \dot{m} \) and replacing Equations (3-10) and (3-11) in Equation (3-9), the heat balance equation expressed through Equation (3-9) can be written as:

\[ \rho_f \pi r_t^2 C_{pf} \, dT dT^e = \rho_f \pi r_t^2 C_{pf} \, dz dT - 2k_c \pi r_t \, dz \, dt \left( \frac{\partial T}{\partial r} \right) \]  

(3-12)

where \( \rho_f \) is mass density of circulation fluid and \( \nu \) is fluid circulation velocity. Rearrangement of Equation (3-12) yields the partial differential equation (PDE) of heat transport by the heat carrier fluid flowing through circulation tubes embedded in a geothermal pile.
Simultaneous solution of Equations (3-8) and (3-13) under different boundary and initial conditions provides time-dependent evolution of temperature within a geothermal pile and that in the soil surrounding the pile.

### 3.2 Finite Difference Formulation

The developed finite difference (FD) code uses an explicit solution scheme. A schematic FD grid and boundary conditions used for the analyses presented in this chapter are shown in Figure 3-1(b). Using explicit FD definitions, the following expressions are obtained, respectively, for Equations (3-8) and (3-13):

\[
\frac{\partial T}{\partial t} = \frac{\partial T}{\partial z} + \frac{2k_t}{\rho_t C_p r_t} \frac{\partial T}{\partial r} \quad (3-13)
\]

\[
\frac{\partial T}{\partial t} + \frac{\partial T}{\partial z} = \frac{1}{2} \left( \frac{\Delta r_i}{\Delta r_{i+1}} \right)^2 \left( T_{i,j+1} - (\Delta r_i + \Delta r_{i+1}) T_{i,j} + \Delta r_i T_{i+1,j} \right) + \frac{1}{\Delta r_i} \left( T_{i+1,j} - T_{i,j} \right)
\]

\[
T_{i+1,j} - T_{i,j} = \alpha \Delta t \left[ \frac{\Delta r_i}{\Delta r_{i+1}} \left( T_{i,j+1} - (\Delta r_i + \Delta r_{i+1}) T_{i,j} + \Delta r_i T_{i+1,j} \right) \right. \left. + \frac{1}{\Delta r_i} \left( T_{i+1,j} - T_{i,j} \right) \right]
\]

\[
T_{i,j+1} - T_{i,j} = \frac{\Delta t}{\Delta z_j} \left( T_{i,j+1} - T_{i,j} \right) + \frac{2k_t \Delta t}{\rho_t C_p r_t \Delta r_i} \left( T_{i+1,j} - T_{i,j} \right)
\]

Stability of FD solutions presented in this chapter is ensured by selecting a time step \( \Delta t \) that is small enough to satisfy the Courant-Friedrichs-Lewy condition (Courant et al. 1967). For simultaneous solution of Equations (3-14) and (3-15), the time-step stability criterion is expressed as:
\[
\Delta t \leq \min \left\{ \frac{1}{2\alpha + \frac{2\alpha}{\Delta r^2} + \frac{\alpha}{r_i \Delta r}}, \frac{1}{\frac{\nu}{\Delta z} + \frac{2k_e}{\rho_i C_p r_i \Delta r}} \right\}
\]

(3-16)

The boundary conditions shown in Figure 3-1(b) and the initial condition used in the analyses are:

\[
T = T_{\text{initial}} \quad \text{for } r = R, \; z \geq 0; \; r \geq 2r_i, \; z = 0 \text{ and } r \geq 0, \; z
\]

(3-17a)

\[
\frac{\partial T}{\partial r} = 0 \quad \text{for } r = 0, \; z \geq 0
\]

(3-17b)

\[
T = T_{\text{initial}} \quad \text{for } t = 0; \; 0 \leq r \leq R \text{ and } 0 \leq z \leq
\]

(3-18)

In addition to the boundary and initial conditions specified by Equations (3-17) and (3-18), a heat flow continuity condition is used at the pile-soil interface (i.e., at \( r = r_p \)).

\[
\left( \frac{T_{i+1}^t - T_i^t}{\Delta t} \right) \left[ \frac{\left( r_{i+2}^2 - r_{i+1}^2 \right) \rho_i C_p + \left( r_{i+2}^2 - r_{i+1}^2 \right) \rho_i C_p}{r_{i+1}^2 - r_{i+1}^2} \right] =
\]

\[
\frac{1}{r_i} \left[ k_i r_{i+1} \left( \frac{T_{i+2}^t - T_{i+1}^t}{r_{i+2} - r_{i+1}} \right) - k_i r_{i-1} \left( \frac{T_i^t - T_{i-1}^t}{r_i - r_{i-1}} \right) \right] + \frac{\left( r_{i+1}^2 - r_{i+1}^2 \right) k_e \left( T_{i,j-1}^t - 2T_{i,j}^t + T_{i,j+1}^t \right)}{\Delta z_i \Delta z_{j+1}}
\]

(3-19)
The continuity condition expressed through Equation (3-19) is required to obtain realistic solution for heat transfer from a heat exchanger pile to the surrounding soil because the values of thermal diffusivity for concrete and soil are likely to be different for practical purposes. As discussed in Chapter 2, most of the available idealized heat transfer models, except the ones developed by Hellstrom (1991) and Lamarche and Beauchamp (2007), assume a single homogeneous medium surrounding a heat source. Therefore, such idealized models cannot accurately quantify the variation of temperature in two different media (concrete and soil) surrounding a heat source. Moreover, none of the idealized models can capture variations of heat flux and fluid temperature along the length of a circulation tube; hence, cannot quantify the effects of these variations on heat transfer efficiency of geothermal piles.

### 3.3 Validation of the Developed Finite Difference Code

The FD code is developed for solving PDEs associated with the proposed annular cylinder heat source model; however, with certain adjustments in boundary and initial conditions, this code can also produce solutions for idealized heat source models available in literature. The FD code was verified by comparing available analytical solutions for finite line and infinite hollow cylinder heat sources (Zeng et al. 2002, and Carslaw and Jaeger 1947) with the respective solutions obtained using the developed code. Note that both finite line and infinite hollow cylinder heat source models use constant heat flux (an input parameter for these models) along the entire length of the heat source. Additionally, both of these models consider a single value of thermal conductivity $k$ for the homogeneous medium around the heat source. Hence, the following modifications are required in order for the developed FD code to capture the constant-heat-flux condition at $r = 2r_i$:

35
\[ \frac{\partial T}{\partial r}
|_{r=2} = - \frac{q_l}{2\pi r_k} \]  

(3-20)

\[ \frac{\partial^2 T}{\partial r^2}
|_{r=2} = \frac{\left( \frac{\partial T}{\partial r} \right)_{r=1} - \frac{\partial T}{\partial r}
|_{r=2}}{\Delta r_{i+1}} = \frac{T_{i+1,j} - T_{i,j}}{\Delta r_{i+1}} + \frac{q_l}{2\pi r_k} \]  

(3-21)

where \( q_l \) is the constant heat flux per unit length of the heat exchanger. Figure 3-2 shows that the developed FD code can successfully predict analytical heat transfer solutions for finite line source (\( q_l = 100 \text{ Wm}^{-1}, \ r_t = 0.025 \text{ m}, \ L_t = 5 \text{ m} \) and \( R_b = r_t/L_t = 0.005; \) \( r_t \) and \( L_t \) are, respectively, radius and length of the idealized heat source) and infinite hollow cylinder source (\( q_l = 100 \text{ Wm}^{-1}, \ r_t = 0.3 \text{ m} \) and \( L_t/r_t = 100 \)).
Figure 3-2 Comparison between analytical solutions and results obtained using the developed Finite Difference code (with appropriate modifications) for (a) finite line heat source (steady-state solution) and (b) infinite hollow cylinder heat source (transient solution)
3.4 Analysis Results

Analyses are performed using the developed FD code to quantify heat transfer through a 30-m-long, 0.6-m-diameter ($r_p=0.3$ m) geothermal pile under different thermal loading conditions. A soil domain with radius $R = 10$ m and height $Z = 35$ m is considered around the pile. Thermal properties for concrete and soil, as used in the analyses, are given in Table 3-1; specific heat of the heat carrier fluid $C_{pf}$ is assumed to be equal to 4190 $\text{Jkg}^{-1}\text{C}^{-1}$. Few additional analyses are performed to identify the effects of some important input variables on thermal efficiency of heat exchanger piles and on time-dependent evolution of ground temperature $T_g$.

Table 3-1 Thermal properties of concrete and soil used in the analyses

<table>
<thead>
<tr>
<th>Thermal Properties</th>
<th>Concrete</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusivity $\alpha$ ($\text{m}^2\text{s}^{-1}$)</td>
<td>$\alpha_c = 0.66\times10^{-6}$</td>
<td>$\alpha_s = 1.02\times10^{-6}$</td>
</tr>
<tr>
<td>Conductivity $k$ ($\text{Wm}^{-1}\text{K}^{-1}$)</td>
<td>$k_c = 1.5$</td>
<td>$k_s = 2.3$</td>
</tr>
</tbody>
</table>

3.4.1 Thermal influence zone

Figure 3-3 shows that the thermal influence zone around the heat exchanger pile extends approximately up to a radius of $160r_t$ (= 3.2 m $\approx 11r_p$) after 60 days of heat rejection from the pile to the ground (a thermal loading condition that simulates operation of a geothermal pile during summer). Note that the thermal influence zone continuously grows with time after heat rejection starts. Nevertheless, two months of continuous heat rejection from a geothermal pile to the ground (as simulated in this analysis) can be considered as an extreme scenario for thermal operation of such a pile during summer in most part of the world and thus $160r_t$ ($\approx 11r_p$) would practically be an upper bound of thermal influence zone around a heat exchanger pile. Except in
the vicinity of pile head and base, radial heat transfer is observed for the entire length of the pile. Such radial heat transfer is also observed in previous numerical studies of heat exchanger piles (Laloui et al. 2006, Abdelaziz et al. 2001). Even after 60 days of heat exchange operation, change in ground temperature is negligible (less than 1°C) beyond a depth of $6r_p$ below the pile base (Figure 3-3).

![Figure 3-3 Temperature profile in homogeneous ground surrounding a geothermal pile after 60 days of heat rejection](image)

Thermal conductivity of soil $k_s$ (and consequently, thermal diffusivity $\alpha_s$) depends on various factors such as dry density, water content, and soil texture. For coarse- and fine-grained soils, the range of $k_s$ varies, respectively, from 0.9 to 4.2 Wm$^{-1}$K$^{-1}$ and from 0.3 to 2.1 Wm$^{-1}$K$^{-1}$ (Brandl 2006). The value of $k_s$ reduces with decrease in soil water content; $k_s$ is minimum for dry soil (usually 0.2-0.4 W/m°C; Tarnawski et al. 2011). Soil near the ground surface is often not fully saturated and a low value of $k_s$ (and thus $\alpha_s$) is expected within this desiccated zone. Heat
transfer performance of a geothermal pile is investigated in the presence of a 5 m desiccated zone of soil (with $k_s=0.38 \text{ Wm}^{-1}\text{K}^{-1}$ and $\alpha_s = 1.7\times10^{-7} \text{ m}^2\text{s}^{-1}$) just below the ground surface. Figure 3-4 shows that the thermal influence zone is smaller within the top desiccated soil layer; however, increase in ground temperature $T_g$ adjacent to the pile is greater in the desiccated soil layer with lower value of $\alpha_s$ than that in the soil layer with higher value of $\alpha_s$.

![Figure 3-4 Temperature (°C) profile (after 60 days of heat rejection) around a geothermal pile installed in ground with a top 5 m desiccated zone](image)

3.4.2 Effect of operational parameters

Velocity of fluid circulation through the embedded circulation tube and fluid temperature at the inlet point are expected to play key roles on ground temperature response and energy exchange efficiency of the pile-soil system. Hence, the effects of inlet fluid temperature $T_{inlet}$ (which
implies thermal gradient based on the difference $\Delta \theta$ between inlet and initial ground temperature) and fluid circulation velocity $v$ on ground temperature response are investigated.

The effects of initial temperature difference $\Delta \theta (= T_{inlet} - T_{initial})$ and fluid circulation velocity $v$ on ground temperature $T_g$ is shown in Figure 3-5. It is observed that at any given time $t$ after the start of the heat transfer operation, the thermal influence zone is independent of $\Delta \theta$ and $v$. Ground temperature $T_g$ within the thermal influence zone increases with increase in both $\Delta \theta$ and $v$. Figure 3-6 shows (for $v = 0.02$ and $0.1 \text{ ms}^{-1}$) the variation of temperature $T$ along depth $z$ at different radial distances; temperature gradient along depth ($dT/dz$) increases as $v$ decreases.

**Figure 3-5 (a)**

- $T_{inlet}=37^\circ\text{C} \ (\Delta\theta=19^\circ\text{C})$, $t=12$ days
- $T_{inlet}=37^\circ\text{C} \ (\Delta\theta=19^\circ\text{C})$, $t=60$ days
- $T_{inlet}=27^\circ\text{C} \ (\Delta\theta=9^\circ\text{C})$, $t=12$ days
- $T_{inlet}=27^\circ\text{C} \ (\Delta\theta=9^\circ\text{C})$, $t=60$ days

$T_{initial}=18^\circ\text{C}$

$r_i=0.02 \text{ m}$

$r_p=0.3 \text{ m}$

$L=30 \text{ m}$

$z=L/2$

$k_s=2.3 \text{ Wm}^{-1}\text{K}^{-1}$

$k_c=1.5 \text{ Wm}^{-1}\text{K}^{-1}$

$v=0.1 \text{ ms}^{-1}$
Figure 3-5 Variation of ground temperature $T_g$ for different values of (a) initial temperature difference $\Delta \theta (= T_{\text{inlet}} - T_{\text{initial}})$ and (b) fluid circulation velocity $v$

Figure 3-6 Effect of fluid circulation velocity $v$ on temperature $T$ along depth $z$
3.4.3 Variation of heat flux and fluid temperature

The time-dependent evolution of heat flux (per unit length) $q_l$ along the length of the circulation tube is shown in Figure 3-7. Heat flux $q_l$ decreases linearly along the length of the circulation tube. Over a heat rejection period of 60 days, $q_l$ at the middle of the pile (i.e., at $z = 15$ m) reduces by almost 30% from its value at the end of the first day of operation. Therefore, the use of idealized heat transfer models with constant values of $q_l$ along the entire length of the heat source would introduce significant errors in the quantification of heat transfer through a geothermal pile.

![Figure 3-7 Variation of heat flux $q_l$ (per unit length) with depth $z$ at different instants of heat rejection operation](image)

Figure 3-7 Variation of heat flux $q_l$ (per unit length) with depth $z$ at different instants of heat rejection operation

Transient variation of fluid temperature $T_f$ along the length of the circulation tube is shown in Figure 3-8. Only few minutes after the heat transfer starts, $T_f$ varies linearly with depth $z$.  

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From in-situ performance tests on geothermal piles, Gao et al. (2008a and 2008b) observed similar linear distribution of fluid temperature along the length of circulation tubes. The distribution of $T_f$ along the length of the circulation tube stabilizes (i.e., reaches steady state) after 12 days of heat exchange operation.

![Figure 3-8 Variation of fluid temperature $T_f$ (°C) along the length of the circulation tube](image)

In order to investigate the effect of variable heat flux on evolution of temperature within the heat exchanger pile and that in soil surrounding the pile, result obtained using the proposed annular cylinder heat source model is compared with finite line source solution (Figure 3-9). For such a comparison, a constant value of heat flux $q_l$ needs to be assigned for the finite line source. However, the choice of $q_l$ for use in the finite line source model introduces significant uncertainty in the prediction because $q_l$ varies along the length of a real geothermal pile and such variation of $q_l$ changes with time during heat exchange operation (Figure 3-7). The values
of $q_l$ used for finite line source solutions plotted in Figure 3-9 are the maximum and minimum heat flux values (i.e., $q_{l,\text{max}}$ and $q_{l,\text{min}}$, respectively at points near the top and bottom of the circulation tube) obtained from simulations of one hour, one day, and one week of heat exchange operation using the proposed annular cylinder model. It is observed that finite line source solutions (i.e., the use of a constant value of $q_l$ along the entire length of the heat source) can significantly misinterpret the increase in temperature within both pile and soil. The maximum difference between predictions using the proposed annular cylinder model and the idealized finite line source model can be as high as 17°C at a point adjacent to the heat source and 12°C at the pile-soil interface. While the use of finite line source model with high values of constant $q_l$ would result in significant overprediction for pile and soil temperature, the use of low values of $q_l$ in finite line source model may consistently underpredict such temperature (Figure 3-9).
Figure 3-9 Effect of variable heat flux on temperature within pile and soil at different times after the start of heat exchange operation for (a) \( t = 4 \) days, (b) \( t = 12 \) days, (c) \( t = 35 \) days and (d) \( t = 60 \) days.
3.4.4 Cyclic thermal loading

The effect of thermal cycles (i.e., successive heat injection and extraction) on thermal efficiency of a heat exchanger pile is also investigated using the developed model (Figure 3-10). Power output $P$ per unit length of the heat exchanger pile, as plotted in Figure 3-10 (a), is calculated as:

$$ P = \frac{mC_{pf} \Delta T}{L} = \frac{\rho_L v \pi r_f^2 C_{pf} (T_{\text{inlet}} - T_{\text{outlet}})}{L} \quad (3-22) $$

For the same values of analysis parameters, power output (a measure of thermal efficiency) of the heat exchanger pile does not change due to individual equivalent cycles (with same $\Delta \theta$) of heat injection and extraction (Figure 3-10 a). However, if a heat extraction cycle follows a heat injection cycle, thermal efficiency of the heat exchanger pile increases during heat extraction. This is because heat energy injected into the ground during the preceding heat injection operation creates a higher temperature gradient between pile and soil as soon as the following heat extraction operation starts. Figure 3-10 (b) shows ground temperature response due to individual 60 days cycles of heat injection and extraction and a combined 120 days injection-extraction cycle (heat extraction follows heat injection). It is observed that compared to a sole heat extraction cycle, ground temperature $T_g$ is always higher at any time during a heat extraction cycle that follows a heat injection cycle. Therefore, elevated ground temperature caused by the operation of a heat exchanger pile in summer would help in increasing thermal efficiency of a geothermal pile operation during winter season.
Figure 3-10 Effect of thermal loading cycles on (a) power output (heat transfer efficiency) of a geothermal pile and (b) ground temperature response
3.5 Sensitivity Study

Sensitivity analysis is performed to investigate the effects of important analysis parameters on thermal efficiency (expressed in terms of power output) of geothermal piles and on ground temperature increment at pile-soil interface. Results from this sensitivity study is presented in the form of Tornado diagrams (Figure 3-11), which show the relative influences of important model parameters on power output from a heat exchanger pile and on ground temperature increment. The vertical dashed lines in Figure 3-11 show the values of desired output (i.e., power output and ground temperature increment) for an expected set of input parameters. The horizontal bars, known as swings of Tornado diagram, represent the variation of a desired output due to the expected variation of individual input parameters considered one at a time. Longer the swing, higher the influence of the corresponding input parameter on an output parameter.

From analysis with expected values of parameters; 
\[ T_{\text{inlet}} = 37°C, \ T_{\text{initial}} = 18°C, \]
\[ r_t = 0.02 \text{ m}, \ r_p = 0.3 \text{ m}, \ v = 0.1 \text{ ms}^{-1} \]
\[ k_s = 2.3 \text{ Wm}^{-1}\text{K}^{-1}, \ k_c = 1.5 \text{ Wm}^{-1}\text{K}^{-1} \]

\[ \Delta \theta = 9°C \]
\[ k_c = 0.35 \text{ Wm}^{-1}\text{K}^{-1} \]
\[ r_t = 0.01 \text{ m} \]
\[ k_c = 1.3 \text{ Wm}^{-1}\text{K}^{-1} \]
\[ v = 0.02 \text{ ms}^{-1} \]
\[ r_p = 0.6 \text{ m} \]

\[ \Delta \theta = 27°C \]
\[ k_s = 4.0 \text{ Wm}^{-1}\text{K}^{-1} \]
\[ r_t = 0.04 \text{ m} \]
\[ k_c = 2.0 \text{ Wm}^{-1}\text{K}^{-1} \]
\[ v = 0.5 \text{ ms}^{-1} \]
\[ r_p = 0.15 \text{ m} \]
The initial temperature difference $\Delta \theta (= T_{\text{inlet}} - T_{\text{initial}})$, soil thermal conductivity $k_s$, and radius of circulation tube $r_t$ are, sequentially, the three most important parameters affecting thermal efficiency of a heat exchanger pile (Figure 3-11). $k_s$ is the most sensitive parameter for ground temperature increment (Figure 3-11 b); it has reverse effects on thermal efficiency of a heat exchanger pile and on the ground temperature increment.

### 3.6 Summary

An annular cylinder model is proposed that approximates heat transfer through a geothermal pile. A finite difference code is developed for simultaneous solution of PDEs describing heat conduction within soil and concrete and heat flow through heat carrier fluid. Results from analyses using the proposed model confirm that heat transfer through a geothermal pile is mostly...
a radial phenomenon. Temperature of the heat carrier fluid decreases linearly along the length of the circulation tube and reaches to a steady state within a few days after the beginning of heat exchange operation. Based on a comparison of result obtained using the proposed model and prediction using the idealized finite line source model, it can be concluded that the use of a constant heat flux along the entire length of a geothermal pile may significantly misinterpret time-dependent evolution of temperature. Sensitivity analysis performed with important analysis parameters reveals that the initial temperature difference between ground and circulation fluid, thermal conductivity of soil, and circulation tube radius are, sequentially, the most important parameters affecting thermal efficiency of a heat exchanger pile.

3.7 Cited References


Chapter 4: Geothermal Piles with Single U-shaped Circulation Tube – Model Development and Validation

This chapter presents the development and validation of a finite difference model that can imitate thermal operation of a geothermal heat pile with an embedded single U-shaped circulation tube. The FD model discussed in this chapter features significant improvements over the first generation annular cylinder model discussed in Chapter 3 and provides a reliable and fast alternative for precise estimation of fluid, pile and ground temperature during thermal operation of a heat exchanger pile (or borehole). The accuracy of the developed model is investigated through comparisons with analytical solutions, results from three-dimensional (3D) finite element simulations and published field test data.

4.1 Problem Geometry and Heat Transfer Equation

A concrete geothermal pile with an embedded U-shaped PVC (polyvinyl chloride) circulation tube is modeled using a two-dimensional finite difference framework (Figure 4-1). Fluid-concrete-soil heat exchange is analyzed by coupling heat transport through circulation fluid and heat conduction in the media surrounding two branches of the circulation tube (heat source). Such coupling is achieved by enforcing heat flux continuity at material boundaries (i.e., fluid–tube, tube–concrete and concrete–soil). Heat transfer from the heat carrier fluid to the surrounding media (PVC pipe, concrete and soil) is assumed to be axisymmetric around both heat sources. Such an assumption of heat flow in soil surrounding a geothermal pile would produce practical solutions because (i) the radial extent $R$ of the soil domain that would potentially be affected by pile-soil heat exchange is much greater than heat source (circulation
tube) radius \( r \) (i.e., \( R >> r_p > r_t \); \( r_p \) is the pile radius) and (ii) results from filed and laboratory tests on geothermal piles with single U-shaped circulation tube show that, beyond a very short period of time after the start of heat exchange operation, ground temperature profile is nearly symmetric around the pile (Bourne-webb et al. 2009, Kramer and Basu 2014, and Kramer et al. 2014). Time-dependent evolution of temperature \( T \) in a medium due to axisymmetric heat conduction can be expressed as (Carslaw and Jaeger 1947, and Mills 1999):

\[
\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}
\]

(4-1a)

\[
\alpha = \frac{k}{\rho C_p}
\]

(4-1b)

where \( \alpha \) is thermal diffusivity, \( k \) is thermal conductivity, \( \rho \) is mass density, and \( C_p \) is specific heat capacity of the medium, \( t \) is time, and \( r \) and \( z \) are the radial and vertical coordinates, respectively. Idealized heat transfer models, commonly used to analyze heat transfer through geothermal piles, assume either constant temperature or constant heat flux boundary condition to solve Equation (4-1). However, as demonstrated later in this paper, such assumptions cannot truly represent heat exchange through a geothermal pile because Equation (4-1) only describes heat conduction in the media surrounding the heat carrier fluid but cannot capture heat transport within the circulating fluid. Moreover, the use of Equation (4-1) would require further simplification that would approximate the effects of two branches of the circulation tube through consideration of a single heat source placed at the center of the pile, as often done in present practice while using analytical solutions.
He expressed $m\Delta T_{avg}$ is an average specific heat of circulation fluid, $\Delta T^e$ is temperature difference across the length $dz$ of element A, $\Delta T_{avg}$ is an average increment in temperature of element A within a small time interval $\Delta t$, and

$$\dot{m}C_{pf}\Delta t\Delta T^e = q_{wall}\Delta t + \rho_f\pi r_i^2 dz C_{pf}\Delta T_{avg}$$  \hspace{1cm} (4-2)
\( q_{\text{wall}} \) is an average heat flow rate (from element A to the surrounding media) over the length \( dz \).

Heat flow rate \( q_{\text{wall}} \) can further be related to radial heat flux \( \dot{q}_r \) as:

\[
q_{\text{wall}} = \dot{q}_r dA
\]

\[
\dot{q}_r (z,t) = h_f (T_{\text{fm}} - T_{\text{fw}})
\]

where \( dA (= 2\pi r dz) \) is the element (cylindrical) surface area available for heat exchange between the circulation fluid and surrounding media, \( h_f \) is the coefficient of convective heat transfer within circulation fluid, \( T_{\text{fm}} \) is the fluid temperature at the middle of the tube cross section, and \( T_{\text{fw}} \) is the fluid temperature adjacent to the tube wall. Equation (4-2), which describes heat balance (transport) within circulating fluid, can now be rewritten as:

\[
\rho_f \pi r_i^2 C_{pf} \Delta \Delta T = h_f \left( T_{\text{fm}} - T_{\text{fw}} \right) \left( 2\pi r d z \right) \Delta t + \rho_f \pi r_i^2 C_{pf} d z \Delta T_{\text{avg}}
\]

Rearrangement of Equation (4-4) yields the partial differential equation that describes heat flow within and heat transfer from circulation fluid.

\[
\frac{\partial T}{\partial z} = \frac{2h_f}{\rho_f C_{pf} r_i^2} (T_{\text{fm}} - T_{\text{fw}}) + \frac{\partial T_{\text{avg}}}{\partial t}
\]

where average fluid temperature \( T_{\text{avg}} \) at any cross section can be calculated as

\[
T_{\text{avg}} = \frac{2}{v_m r_f^2} \int_0^r \nu T d\nu
\]

\( v_m \) is the mean fluid velocity at the cross section under consideration. The variation of flow velocity at any depth within the circulation tube is not considered in this study, therefore, \( v = v_m \). The convective heat transfer coefficient \( h_f \) in Equation (4-5) is expressed as:
where \( k_f \) is the thermal conductivity of circulation fluid and \( Nu \) is the Nusselt number (i.e., the ratio of convective to conductive heat transfer across the tube boundary). Mathematically,

\[
Nu = 0.065 \frac{Re Pr 2r_i}{L_i} \left( 1 + 0.04 \left( \frac{Re Pr 2r_i}{L_i} \right)^{2/3} \right)
\] (4-7)

for laminar flow (Edwards et al. 1979), and

\[
Nu = \frac{f}{8} \left( Re - 1000 \right) \frac{Pr}{1 + 12.7 \left( \frac{f}{8} \right)^{1/2} \left( Pr^{2/3} - 1 \right)}
\] (4-8)

for turbulent flow (Gnielinski 1976). \( Re = 2 \rho vr/\mu_f \); \( \mu_f \) is dynamic viscosity of fluid) is the Reynolds number, \( Pr \) is the Prandtl number (i.e., the ratio of kinematic viscosity to thermal diffusivity; \( Pr = \mu (C_p/k_f) \)), and \( f \) is the Darcy friction factor. For smooth pipes, like the ones commonly used as circulation tubes in geothermal piles, \( f \) can be calculated as:

\[
f = \begin{cases} 
64/Re & \text{for laminar flow} \\
\left[ 0.79 \ln(Re) - 1.64 \right]^2 & \text{for turbulent flow; Petukhov (1970)} 
\end{cases}
\] (4-9)
4.2 Finite Difference Formulation

Figure 4-2 shows a schematic FD grid and boundary conditions used for numerical solution. Equation (4-1a) cannot be used directly in the present formulation which considers simultaneous heat transfer from both branches (downward and upward) of the circulation tube to the surrounding media. Therefore, an equivalent form of heat conduction equation is derived by considering heat energy balance in a FD stencil (Figure 4-2) encompassed by computational points \((i, j), (i+1, j), (i+1, j+1),\) and \((i, j+1)\). The temperature increment \(\Delta T^i\) at node \((i, j)\) over any given time segment \(\Delta t\) depends on the difference between heat entering and exiting the FD stencil during time step \(\Delta t\). Thus heat balance equation for node \((i, j)\) can be written as:

\[
m_{i,j} C_p \Delta T = Q_{i,j}^{in} - Q_{i,j}^{out}
\]

where \(m_{i,j}\) is the mass corresponding to the FD stencil in consideration and \(Q_{i,j}^{in}\) and \(Q_{i,j}^{out}\) are, respectively, heat inflow into and outflow from the stencil. Now assuming axisymmetric form of heat conduction and accounting for simultaneous contributions of both downward and upward branches of the circulation tube in changing temperature of the surrounding media Equation (4-10) can be expanded as

\[
\rho \pi \Delta z \left[ \left( r_t - r_{i+1} \right)^2 - \left( r_t - r_i \right)^2 \right] \left( r_t - r_{i+1} \right)^2 - \left( r_t - r_i \right)^2 \right] C_p \Delta T^i = \left( Q_{i,j}^{in} - Q_{i,j}^{out} \right)^L + \left( Q_{i,j}^{in} - Q_{i,j}^{out} \right)^R
\]

where \(r_t^{R} = \{r - 0.5s_t - (r_1 + t_1)\}\) and \(r_t^{L} = \{r + 0.5s_t + (r_1 + t_1)\}\) are radial distances measured, respectively, from the center of the downward (left) and upward (right) branches of the circulation tube, \(\Delta z\) is the distance between the top and bottom surfaces of the FD stencil, \(\rho\) and \(C_p\) are, respectively, mass density and specific heat capacity of the medium in which the FD
stencil is located. \((Q_{i,j}^{in} - Q_{i,j}^{out})^L\) and \((Q_{i,j}^{in} - Q_{i,j}^{out})^R\) are net heat flow into the stencil, respectively, due to the left (designated by the letter \(L\)) and right (designated by the letter \(R\)) branches of the circulation tube. Equation (4-10) is valid for a general heat transfer problem because it explains the physics of heat energy transfer. However, Equation (4-11) which considers interaction between both downward and upward branches of the circulation tube is particularly derived for the problem discussed in this chapter. Thus \(\Delta T_{ij}\) calculated from Equation (4-11) in any time step is a “net” temperature change at a point within the solid domain (i.e., PVC, concrete, soil) due to combined effects of two branches of circulation tubes (heat sources). It is important to point out that simple superposition (or summation) of temperature increments calculated considering each branch of the circulation tube separately (as individual heat source) will not yield a practical solution and such a solution may significantly differ from \(\Delta T_{ij}\) calculated from Equation (4-11) because of the nonlinear nature of the problem.

![Figure 4-2 Schematic finite difference grid and boundary conditions](image)
The amount of injection and rejection of heat into the desired element can be calculated using the heat flux at each side of the element. Further expansion of (4-11) yields:

\[
\rho \pi \Delta z \left[ \left( \left( r_{i+1}^R \right)^2 - \left( r_i^L \right)^2 \right) + \left( \left( r_i^R \right)^2 - \left( r_{i+1}^R \right)^2 \right) \right] C\rho \Delta T^{ij} =
\]

\[
\left[ -\left( 2\pi r_i^L \Delta z \right) k \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i-1}^L} - \pi \left( \left( r_i^L \right)^2 - \left( r_{i+1}^L \right)^2 \right) k \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j}} \right. \\
\left. -\left( 2\pi r_i^R \Delta z \right) k \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1}^R} - \pi \left( \left( r_i^R \right)^2 - \left( r_{i+1}^R \right)^2 \right) k \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j}} \right. \\
\left. +\left( 2\pi r_{i+1}^L \Delta z \right) k \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1}^L} + \pi \left( \left( r_{i+1}^L \right)^2 - \left( r_i^L \right)^2 \right) k \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i+1,j}} \right. \\
\left. +\left( 2\pi r_{i+1}^R \Delta z \right) k \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1}^R} + \pi \left( \left( r_{i+1}^R \right)^2 - \left( r_i^R \right)^2 \right) k \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i+1,j}} \right] \times \Delta t
\]

where \( \Delta r \) is the distance between two vertical sides of the FD stencil and \( k \) is thermal conductivity of the medium in which the FD stencil is located. Substituting \( k \) and \( C\rho \) with thermal diffusivity \( \alpha \) in Equation (4-12) the heat balance energy can be expressed as:
\[ \Delta T^y = \alpha_{i,j} \left[ -2r_{i,j}^L \frac{\partial T}{\partial r} \bigg|_{r=r_{i,j}} - \frac{\left( (r_{i+1}^R)^2 - (r_i^L)^2 \right) \Delta T}{\Delta z} \bigg|_{z=z_{i,j}} - 2r_{i,j}^R \frac{\partial T}{\partial r} \bigg|_{r=r_{i,j}} + \frac{\left( (r_{i+1}^L)^2 - (r_i^R)^2 \right) \Delta T}{\Delta z} \bigg|_{z=z_{i,j}} \right] \Delta t \]

Rearrangement of Equation (4-13) yields the governing differential equation for heat conduction as follows

\[ \frac{\Delta T^y}{\alpha_{i,j} \Delta t} = \left[ -2r_{i,j}^L \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i,j}} + 2r_{i+1,j}^L \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1,j}} \right] + \left[ -2r_{i,j}^R \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i,j}} + 2r_{i+1,j}^R \frac{\Delta T}{\Delta r} \bigg|_{r=r_{i+1,j}} \right] \]

\[ + \left[ - \frac{1}{\Delta z} \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j}} + \frac{1}{\Delta z} \frac{\Delta T}{\Delta z} \bigg|_{z=z_{i,j+1}} \right] \times \left[ \frac{\left( (r_{i+1}^R)^2 - (r_i^R)^2 \right)}{\left( (r_{i+1}^L)^2 - (r_i^L)^2 \right) + \left( (r_{i+1}^R)^2 - (r_i^R)^2 \right)} \right] \]

(4-14)

Simplification of Equation (4-14) yields the heat energy balance equation:
\[
\frac{\Delta T^i_{i,j}}{\alpha_{i,j} \Delta t} = \left[ \frac{-2r^l_{i,j} \frac{\Delta T}{\Delta r}_{r=r^L_{i,j}} + 2r^L_{i+1,j} \frac{\Delta T}{\Delta r}_{r=r^L_{i+1,j}}}{\left\{(r^L_{i+1})^2 - (r^L_{i,j})^2\right\}} + \frac{-2r^R_{i,j} \frac{\Delta T}{\Delta r}_{r=r^R_{i,j}} + 2r^R_{i+1,j} \frac{\Delta T}{\Delta r}_{r=r^R_{i+1,j}}}{\left\{(r^R_{i+1})^2 - (r^R_{i,j})^2\right\}} \right]
\left[ \frac{\left\{(r^R_{i+1})^2 - (r^R_{i,j})^2\right\}}{\left\{(r^L_{i+1})^2 - (r^L_{i,j})^2\right\}} \right]
\]

+ \left[ \frac{\partial^2 T}{\partial z^2} \right] \times \left[ \frac{\left\{(r^R_{i+1})^2 - (r^R_{i,j})^2\right\}}{\left\{(r^L_{i+1})^2 - (r^L_{i,j})^2\right\}} \right]
\]

\[
\left[ \frac{\left\{(r^R_{i+1})^2 - (r^R_{i,j})^2\right\}}{\left\{(r^L_{i+1})^2 - (r^L_{i,j})^2\right\}} \right]
\]

Rearrangement of Equation (4-15) yields the governing differential equation for heat conduction as follows:

\[
\frac{1}{\alpha} \left( \frac{dT}{dt} \right) = \left[ \frac{-2r^l_{i,j} \frac{\partial T}{\partial r}_{r=r^L_{i,j}} + 2r^L_{i+1,j} \frac{\partial T}{\partial r}_{r=r^L_{i+1,j}}}{\left\{(r^L_{i+1})^2 - (r^L_{i,j})^2\right\}} + \frac{-2r^R_{i,j} \frac{\partial T}{\partial r}_{r=r^R_{i,j}} + 2r^R_{i+1,j} \frac{\partial T}{\partial r}_{r=r^R_{i+1,j}}}{\left\{(r^R_{i+1})^2 - (r^R_{i,j})^2\right\}} \right]
\]

\[
+ \left( \frac{\partial^2 T}{\partial z^2} \right)_{(i,j)}
\]

(4-16)

Now considering two consecutive time steps (\(t\) and \(t+1\)) with interval \(\Delta t\), Equation (4-16) can be rewritten as:

\[
\frac{\Delta T^i_{i,j}}{\Delta t} = \frac{T^i_{i+1,j} - T^i_{i,j}}{\Delta t} = \left[ \frac{-2r^l_{i,j} \frac{\partial T}{\partial r}_{r=r^L_{i,j}} + 2r^L_{i+1,j} \frac{\partial T}{\partial r}_{r=r^L_{i+1,j}}}{\left\{(r^L_{i+1})^2 - (r^L_{i,j})^2\right\}} + \frac{-2r^R_{i,j} \frac{\partial T}{\partial r}_{r=r^R_{i,j}} + 2r^R_{i+1,j} \frac{\partial T}{\partial r}_{r=r^R_{i+1,j}}}{\left\{(r^R_{i+1})^2 - (r^R_{i,j})^2\right\}} \right]
\]

\[
+ \left( \frac{\partial^2 T}{\partial z^2} \right)_{(i,j)}
\]

(4-17)
For the pile-soil heat exchange model presented in this paper, Equation (4-17) governs heat conduction in the solid media (i.e., pipe material, concrete and soil). Coupling of Equations (4-5) and (4-17) at the fluid–tube (PVC) interface [i.e. at \( r = \pm(0.5s_t+t_t), \pm(0.5s_t+2r_t+t_t) \); \( t_t \) is the thickness of the PVC tube] is achieved through heat flux continuity condition imposed at the fluid-PVC interface.

\[
h_i \left( T_{\text{fm}} - T_{\text{fw}} \right) = -k_t \left( \frac{\partial T}{\partial r} \right)_{\text{fluid-PVC interface}}
\]

where \( k_t \) is thermal conductivity of the circulation tube material (PVC). Heat flux continuity is also enforced at tube–concrete boundary [i.e., at \( r = \pm0.5s_t \) and \( r = \pm(0.5s_t+2r_t+2t_t) \)] and concrete (pile)–soil boundary (i.e., at \( r = \pm r_p \)). Combination of Equations (4-5) and (4-18) provides fluid temperature variation within a cross section of the circulation tube at any depth. FD forms of the resulting equations are:

\[
\Delta T_{\text{fw},j}^t = \left( T_{\text{fw},j}^{t+1} - T_{\text{fw},j}^t \right) = \left[ v \frac{\Delta z}{\Delta x} \left( 0.5(T_{\text{fm},j-1}^t - T_{\text{fm},j}^t) + 0.5(T_{\text{fm},j-1}^t - T_{\text{fm},j}^t) \right) \right]
\]

\[
+ \frac{2k_t}{\rho_t C_{\text{pr},t}} \left( \frac{\partial T}{\partial r} \right)_{\text{fluid-PVC interface}} + 0.5(T_{\text{fm},j}^t + T_{\text{fw},j}^t) + \frac{0.5k_t}{h_t} \left( \frac{\partial T}{\partial r} \right)_{\text{fluid-PVC interface}}
\]

(4-19)

\[
\Delta T_{\text{fm},j}^t = T_{\text{fw},j}^{t+1} - T_{\text{fm},j}^t = \frac{k_t}{h_t} \left( \frac{\partial T}{\partial r} \right)_{\text{fluid-PVC interface}}
\]

(4-20)

FD form of heat flux continuity conditions at nodes on the pipe–concrete and concrete–soil boundaries is expressed as:
Equations (4-17), (4-19) and (4-20), subjected to the heat flow continuity condition expressed by Equation (4-21), are solved simultaneously using an explicit FD solution scheme.

Explicit scheme has been widely used in literature to predict transient or steady state heat conduction, diffusion, and convection (Bhattacharya 1985, Tavakoli and Davami 2007, and Shen et al. 2011). In comparison with an implicit solution scheme, a smaller time step is needed for solution convergence in an explicit solution scheme. However, an implicit solution scheme may incur a considerably high computational time, particularly for a problem like the one presented in the paper which involves nearly 100,000 computation points (or nodes). It is demonstrated later in this chapter that the use of an explicit solution scheme for the present problem can produce reasonably accurate solution which is in good agreement with analytical solution for a finite line source model (Zeng et al. 2002), analytical solution based on thermal resistivity framework (Zeng et al. 2003a and 2003b, Marcotte and Pasquier 2008) and can reasonably predict field test results reported in literature.
4.3 Initial and Boundary Conditions

An initial temperature condition \([T = T_{\text{initial}}\) at time \(t = 0\)] is designated for the entire analysis domain (i.e., \(-R \leq r \leq R\) and \(0 \leq z \leq Z\); \(R\) is the distance from the center of the pile to the far boundaries of the analysis domain). Constant temperature boundary condition is used for the far left, right and bottom boundaries of the analysis domain and convective boundary condition is used for the top boundary to accommodate for potential heat convection through this boundary. Finite difference expressions for constant and convective boundary conditions used in the analysis are expressed as:

\[
T = T_{\text{initial}} \quad \text{for} \quad r = \pm R, \ z \geq 0 \ \text{and} \quad -R \leq r \leq R, \ z = Z
\]  

\[
\Delta T_{\text{boundary}} = T_{i+1,j}^{t'} - T_{i,j}^{t'} = \alpha_{i,j} \Delta t \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r_i} \frac{(T_{i+1,j}^{t'} - T_{i,j}^{t'})}{\Delta r} \right. \\
\left. - \frac{1}{\Delta z} \left( \frac{T_{i,j+1}^{t'} - T_{i,j}^{t'}}{\Delta z} + h_{tg} (T_{i,j}^{t'} - T_{ag}) \right) \right] \quad \text{for} \quad -R < r < R, \ z = 0
\]  

where \(h_{tg}\) is the coefficient of convective heat transfer through the top boundary of the analysis domain and \(T_{ag}\) is the temperature of the medium above ground surface. Figure 4-3 describes, in the form of a flow chart, the sequential solution algorithm used for the heat transfer model presented herein. For non-uniform FD grid, the second derivatives of temperature over \(r\) and \(z\) are evaluated as:

\[
\frac{\partial^2 T}{\partial r^2} = \frac{\Delta r_{i+1} \times T_{i+1,j}^{t'} - (\Delta r_i + \Delta r_{i+1}) T_{i,j}^{t'} + \Delta r T_{i+1,j}^{t'}}{\frac{1}{2} (\Delta r_{i+1}^2 \Delta r_i + \Delta r_{i+1} \Delta r_i)} \quad \text{(4-23a)}
\]

\[
\frac{\partial^2 T}{\partial z^2} = \frac{\Delta z_{j+1} \times T_{i,j+1}^{t'} - (\Delta z_j + \Delta z_{j+1}) T_{i,j}^{t'} + \Delta z_j T_{i,j+1}^{t'}}{\frac{1}{2} (\Delta z_{j+1}^2 \Delta z_j + \Delta z_{j+1} \Delta z_j)} \quad \text{(4-23b)}
\]
START

Select input parameters (as in Table 1)

Generate FD grid and store grid coordinates (i, j)

Initialize temperature (at time t = 0) for the entire domain and assign material properties for every grid point (node)

Calculate maximum time step increment \( \Delta t \) following CFL stability criterion expressed in Eq. (18)

START

Select a node (i, j)

March forward in time

Laminar flow

Calculate \( h_f \) using Eqs. (6) and (7a)

Turbulent flow

Identify flow characteristics, calculate \( Re, Pr \)

Calculate \( h_f \) using Eqs. (6) and (7b)

Select another node in the domain (i, j)

Grid points on the boundaries

Assign constant boundary temperature (Eq. 19a) or calculate temperature change \( \Delta T_{\text{boundary}} \) from Eq. (19b)

Grid points on material interfaces

Calculate temperature change \( \Delta T_{\text{fw},j} \) from Eq. (15) and \( \Delta T_{\text{fm},j} \) from Eq. (16)

Grid point is within fluid (circulation fluid) domain

Calculate temperature change \( \Delta T_{\text{boundary}} \) from Eq. (17)

Grid point is within any solid domain (i.e., concrete, soil or PVC)

Calculate temperature change \( \Delta T_{ij} \) from Eq. (14)

Update temperature at node (i, j)

\[ T_{i,j}^{t+1} = T_{i,j}^t + \Delta T_{ij} \]

No

No

\( \Delta T \) calculated for all nodes

Yes

Desired total time of operation reached

Yes

Merge on time step \( t+1 \)

END

Figure 4-3 Solution algorithm flow chart
4.4 Stability Criterion

A time-step interval $\Delta t$ used in finite difference analyses should be small enough to satisfy the solution stability specified by Courant-Friedrichs-Lewy condition (Courant et al. 1976). The time-step stability criterion for Equations (4-17) and (4-19) is derived to be equal to:

$$\Delta t \leq \min \left\{ \frac{1}{\frac{2\alpha}{\Delta r^2} + \frac{2\alpha}{\Delta z^2} + \frac{\alpha}{r_i \Delta r}} - \frac{1}{\frac{\nu}{\Delta z} + \frac{2k_p}{\rho_f C_p r_p \Delta r}} \right\}$$

(4-24)

4.5 Model Validation

The developed FD model is validated through comparison with (1) idealized finite line source model prediction for ground temperature response, (2) analytical solution for nondimensional fluid and pile temperature and pile thermal resistance obtained using thermal resistivity framework, (3) fluid and ground temperature predicted using 3D finite element model, and (4) published field test data for homogeneous and layered subsurface conditions.

4.5.1 Comparison with theoretical models

4.5.1.1 Comparison with classical analytical solution for Finite Line Source (FLS) model

Accuracy of the developed FD model is verified by comparing model predictions with available analytical solution for a finite-length heat source embedded within a homogeneous medium, commonly known as finite line source (FLS) model (Zeng et al. 2002). The FD model is used to
perform a set of trial finite difference analyses (FDAs) of short- and long-term heat transfer from a 25-m-long heat exchanger pile with a U-shaped circulation tube. Two different thermal input conditions are considered in the trial FDAs: (a) a constant rate of power injection (as often done in practice for thermal response tests on geothermal piles) at the inlet point, and (b) a constant temperature at the inlet point. In order to make the comparison with FLS model possible, the trial FDAs use a single value of thermal conductivity $k$ for all materials (PVC tube, concrete, and soil) surrounding the heat sources (vertical branches of the circulation tube).

Thermal response tests on geothermal piles are often analyzed using the FLS model. For such analyses, the heat exchanger pile is idealized as a line source (with length equal to that of the pile) placed along the center of the pile. An equivalent amount of constant power per unit length $q_l (= q/L)$; where $q$ is constant power injected at the inlet point and $L$ is pile length) is used in the FLS model. The fluid inlet temperature of the model depends on the output temperature and the applied temperature differences calculated from the heat injection power.

$$T_{in} (t) = T_{out} (t) + \frac{q}{\left( \rho_f \nu \pi r_i^2 \cdot c_{pf} \right)}$$

(4-25)

Results from a trial analysis (for $q = 2500$ W and $L = 25$ m) using the developed FD model show that, even for constant rate of power injection, heat rate varies from 55 Wm$^{-1}$ at the inlet point to 45 Wm$^{-1}$ at the outlet point of the circulation tube (Figure 4-4 a). Nevertheless, both short- and long-term ground temperature response obtained from analysis using the FD model agrees well with that predicted by the FLS model (Figure 4-4 b and Figure 4-4 c).
Figure 4-4 (a)

Heat rate per unit length $q$ (W/m)

Depth $z$ (m)

Figure 4-4 (b)

Ground temperature $T_g$ (°C)

Radial distance $r$ (m)
The comparison shown in Figure 4-4 does not substantiate the need for use of a detailed numerical model in analyzing heat exchange through a geothermal pile. However, for real-life operations of geothermal piles, heat flux (per unit length) varies with both time and depth along the circulation tube. Consequently, the use of FLS model does not warrant a precise estimation of ground temperature response (and consequently, power output from the pile-soil system). Figure 4-5 shows results from a set of FDAs with constant inlet temperature $T_{\text{inlet}}$, for which non-negligible variations of heat rate with time is observed along the length of the circulation tube (Figure 4-5 a). Also plotted in this figure are ground temperature predictions using the FLS model with different constant values of average heat rate $q_i^{\text{FLS}}$ (per unit length). The values of $q_i^{\text{FLS}}$ (at different time $t$) used in the FLS model are summation of average $q_i$ values (obtained
using the FD model) along the inlet and outlet branches of the circulation tube, i.e.,

\[ q_1^{FLS} = q_{1,\text{avg, inlet}} + q_{1,\text{avg, outlet}} \]

Figure 4-5(b) through Figure 4-5(e) demonstrate that idealization of thermal performance of heat exchanger piles using the FLS model can significantly misinterpret thermal response and power output (i.e., energy exchange rate) of the pile-soil system. The error in ground temperature estimation using FLS model depends on the constant value of \( q_l \) used; for different constant values used in the comparison presented in Figure 4-5(b) through Figure 4-5(e), the maximum differences between the predictions using FD and FLS model vary between +59% to −17% of average ground temperature increment (i.e., average of inlet and outlet side) predicted using the FD model.
Figure 4-5 (b) Comparison between solutions from FLS model and FDA results for constant inlet temperature: (a) variation of $q_l$ with depth, (b) ground temperature profile at $t = 1$ days, (c) ground temperature profile at $t = 5$ days and (d) ground temperature profile at $t = 15$ days, and (e) ground temperature profile at $t = 60$ days.
4.5.1.2 Comparison with theoretical solutions based on thermal resistance

FDA results are also compared with a thermal resistivity based analytical solution that predicts thermal resistance of geothermal piles (or boreholes) and normalized fluid temperature along the embedded circulation tubes (Zeng et al. 2003a and 2003b, Marcotte and Pasquier 2008). Building on the previous work by Eskilson (1987) and Hellstrom (1991), the solution proposed by Zeng et al. (2003a and 2003b) focuses on heat transfer inside vertical geothermal heat exchangers. Assumptions and simplifications adopted in the Zeng et al. (2003a and 2003b) model are: (i) at any specific instant of heat exchange operation, temperature of the heat exchanger element (pile or borehole) is constant along the depth, (ii) material within the heat exchanger element (grout or concrete) and surrounding ground are homogenous, and (iii) thermal resistance of the tube material (i.e., the effect of tube thickness) and convective resistance of circulation fluid are ignored. Following the recommendation by Bernier (2001), recent research incorporates thickness of circulation tube and fluid convection resistance in the calculation of thermal resistance for geothermal heat exchangers (Marcotte and Pasquier 2008, Sharqawy et al. 2009, Lamarche et al. 2007 and 2010).

A 120 hour thermal response test on a geothermal heat exchanger pile is simulated [with input parameters \( q = 2500 \text{ W}, L = 25 \text{ m}, T_{\text{initial}} = 18^\circ \text{C}, r_t = 0.02 \text{ m}, r_p = 0.3 \text{ m}, k_s = 2.5 \text{ Wm}^{-1} \text{K}^{-1}, k_c = 1.7 \text{ Wm}^{-1} \text{K}^{-1}, S_f = 0.37 \text{ m}, t_t = 0.006 \text{ m}, \rho = 1000 \text{ kgm}^{-3}, v = 0.17 \text{ ms}^{-1}, C_{pf} = 4187 \text{ Jkg}^{-1} \text{K}^{-1}, \mu_f = 0.725 \times 10^{-3} \text{Pa.s}, k_f = 0.58 \text{ Wm}^{-1} \text{K}^{-1}]\) using the developed FD model. Figure 4-6(a) shows that the normalized fluid temperature \( \theta_z = \left[ T_t(z) - T_{\text{p,avg}}(z) \right] / \left[ T_{\text{inlet}} - T_{\text{p,avg}}(z) \right] \) derived from FDA results are in excellent agreement with the analytical expressions proposed by Zeng et al. (2003a and 2003b) when thickness of the circulation tube and fluid convective resistance are considered.
as cited in Marcotte and Pasquier (2008); $T_f(z)$ and $T_p(z)$ are, respectively, fluid and average pile temperature at depth $z$ and $T_{inlet}$ is the fluid temperature at the inlet point. The comparison shown in Figure 4-6(b) confirms that except for a short initial transition period, when the pile temperature is on its way to reaching a nearly steady state value (the analytical solution fails to capture this phase), the pile thermal resistance $R_{pile}$ predicted by the developed model is in complete agreement with the analytical solution. Based on the results obtained from 3D finite element analyses of geothermal heat exchangers, Ozudogru et al. (2014b) predicted similar trend of pile thermal resistance with time.

The dimensionless fluid temperature can be calculated using Equations (4-26a), and (4-26b), respectively, for downward and upward branches of circulation tube embedded within the geothermal pile. Using the FD results (i.e., mean fluid temperature, average pile temperature, and heat flux per unit length of the pile) pile thermal resistance can be predicted through Equation (4-27). Since the pile wall temperature varies along the depth (Marcotte and Pasquier 2008) the average pile temperature is used in Equations (4-26) and (4-27).

$$\theta_d (z) = \frac{T_{fd} (z) - T_{p,avg}}{T_{in} - T_{p,avg}}$$ (4-26a)

$$\theta_u (z) = \frac{T_{fu} (z) - T_{p,avg}}{T_{in} - T_{p,avg}}$$ (4-26b)

$$R_{pile} = \frac{T_{f,mean} - T_{p,avg}}{q_1}$$ (4-27)
Figure 4-6 Comparison of results obtained from FDA with analytical solutions presented by (Zeng et al. 2003a and 2003b, Marcotte and Pasquier 2008): (a) variation of normalized fluid along the pile length and (b) pile thermal resistance
4.5.2 Comparison with 3D finite element simulations

Prediction using the developed FD model is further compared with results obtained using a 3D finite element (FE) model (developed in COMSOL Multiphysics®, a well-known software platform for solving multiphysics problems) presented by Ozudogru et al. (2014a and 2014b). Two different thermal loading cases for a 100-m long borehole with the diameter of 0.15 m are investigated in this section. A thermal response test with constant rate heat injection and thermal load test using a constant temperature injection are simulated using both FE and FD model. Fluid temperature, ground temperature, heat rejection flux into the ground from both limbs of U-tube and from both inlet and outlet sides of the borehole obtained from FD simulations are compared with results obtained using the FE model. The thermal load is applied using either a constant temperature fluid (35°C) or a constant heat (5000W) for the first 100 hours of the thermal operation. Following the thermal injection period, 150 hours of recovery phase are applied for both cases. Initial fluid temperature is assumed to be same as the initial ground temperature (15°C). Fluid properties of a 20% mixture of glycol and water are adopted from the ASHRAE (American Society of Heating, Refrigerating, and Air-Conditioning Engineers) Handbook Fundamentals (ASHRAE, 2009). The thermal properties of the ground and borehole are adopted from Farouki (1981) and Salomone et al. (1989). Figure 4-7 shows the schematic view of the FD analysis domain, boundary conditions, and the grid points used in finite difference simulation.
Figure 4-7 Finite difference model used for comparison with 3D FE analysis: (a) schematic view of the analysis domain and boundary conditions (b) Grid points used in the FD simulation
Figure 4-8 shows the finite element mesh in COMSOL Multiphysics® (adopted from Ozudogru et al. 2014a). The details of the finite element model are presented in Ozudogru et al. (2014a and 2014b). All input parameters used in these analyses are presented in Error!

Reference source not found..
### Table 4-1 Input parameters for comparison analyses using FD and FE models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial ground temperature</td>
<td>15</td>
<td>°C</td>
</tr>
<tr>
<td>Heat rate per depth (constant heating case)</td>
<td>50</td>
<td>W m⁻¹</td>
</tr>
<tr>
<td>Injection temperature (constant temperature case)</td>
<td>35</td>
<td>°C</td>
</tr>
<tr>
<td>Flow rate</td>
<td>20</td>
<td>dm³ min⁻¹</td>
</tr>
<tr>
<td>Dynamic viscosity of fluid</td>
<td>2.02</td>
<td>mPa s</td>
</tr>
<tr>
<td>Thermal conductivity of fluid</td>
<td>0.48</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity fluid</td>
<td>3962</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of fluid</td>
<td>1020.91</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Thermal conductivity of PVC tube</td>
<td>0.39</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of PVC tube</td>
<td>2300</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of PVC tube</td>
<td>960</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Pipe inner diameter</td>
<td>21.5</td>
<td>Mm</td>
</tr>
<tr>
<td>Pipe wall thickness</td>
<td>2.4</td>
<td>Mm</td>
</tr>
<tr>
<td>Center to center spacing</td>
<td>50</td>
<td>Mm</td>
</tr>
<tr>
<td>Thermal conductivity of borehole</td>
<td>1.00</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of borehole</td>
<td>1600</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of the grout</td>
<td>1500</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Diameter of the borehole</td>
<td>0.15</td>
<td>M</td>
</tr>
<tr>
<td>Length of the borehole</td>
<td>100</td>
<td>M</td>
</tr>
<tr>
<td>Thermal conductivity of soil</td>
<td>2.00</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity soil</td>
<td>1500</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of soil</td>
<td>2000</td>
<td>kg m⁻³</td>
</tr>
</tbody>
</table>

#### 4.5.2.1 Constant rate heat injection scenario (field thermal response test)

Circulation fluid with a constant heat (5000 W) is injected at the inlet point of a geothermal heat exchanger borehole. The fluid inlet temperature is updated during the operational time using Equation (4-25). $q$ is set to be equal to 5000 W for the first 100 hours of thermal operation and for the recovery phase (i.e, the next 150 hours) $q$ is assumed to be equal to zero. Figure 4-9 shows comparison of heat flux calculated (from FD and FE results) along both upward and downward branches of U-tube (closed-loop circulation tube within the geothermal borehole heat exchanger). As expected, the heat flux at any depth remains constant with time and
its value decreases linearly from the inlet point to the outlet point. Mean fluid temperature \((T_{f,in}+T_{f,out})/2\) and fluid temperature along the U-tube length at both upward and downward sides are presented in Figure 4-10; fluid temperature obtained from FD and FE models are in good agreement (Figure 4-10). The maximum difference between the FD and FE predictions is 0.5°C during the heat injection phase and fluid temperature obtained using the FD model during the recovery phase (from \(t=100\) hours to 250 hours) is in complete agreement with FE results.

![Figure 4-9 Heat flux, as obtained from FD and FE analyses, along both upward and downward tubes](image-url)
Figure 4-10 Comparison of results obtained from FD and FE analyses for a constant heat rate injection ($q=5000$ W) scenario (a) average fluid temperature (b) variation of fluid temperature along the U-tube length at different time steps.
Heat flux along the borehole wall at both inlet and outlet sides are computed from results (temperature gradient at borehole wall) obtained from both FD and FE models. Figure 4-11 shows that heat flux values computed from FD and FE simulations are in good agreement. Heat flux at the inlet side is expectedly higher than that at the outlet side. Hence, ground temperature increment at the inlet side is expected to be higher than the outlet side. This difference in ground temperature is expected to be higher within a region at close proximity of the geothermal heat exchanger (i.e., borehole or pile) and such difference gradually diminishes at distant points. Note that heat flux computed even after 5 days of thermal operation does not reach a nearly steady-state condition; amount of heat received (by the heat exchanger) within this period is partially consumed in raising the borehole temperature. Heat flux should overcome the thermal resistance of the borehole (or pile) material before part of it can raise ground temperature; this process depends on time and amount of heat injection. For the initial part of heat injection the actual amount of heat rejected to the ground is lower than 5000 W and this value increases with time. Since heat is injected at a constant rate for 100 hours, the summation of heat flux values computed at the inlet and outlet sides after a couple of hours of thermal operation will be approximately equal to 5000W. Average of ground temperature at the inlet and outlet sides, as obtained from FD and FE simulations, are also compared at different time steps. The maximum difference between ground temperature obtained from FD and FE simulations is less than 0.5°C (Figure 4-12).
Figure 4-11 Variation of calculated heat flux along the borehole length from FD and FE analyses

Figure 4-12 Comparison of average ground temperature obtained from FD and FE simulations
It is interesting to note that results (e.g., heat flux, ground temperature response, and fluid temperature) obtained using the developed FD model (that uses a 2D framework) are in good agreement with those obtained through 3D FE simulations, in spite of the fact that these two models utilize two completely different computational platforms and model idealization approaches. The observed minimal differences between predictions using the FD and FE models can be attributed to different modeling assumptions adopted during model development.

4.5.2.2 Constant-temperature fluid injection scenario (heat exchanger operation)

FD model prediction for a constant temperature injection scenario is also compared with FE simulations using COMSOL Multiphysics®. All input parameters used for this comparison is kept the same as used for constant heat rate comparison, except that circulation fluid is injected at a constant temperature for the first 100 hours of thermal operation. A thermal injection period is followed by a 150 hours of recovery step. As suggested by the mean fluid temperature plotted in Figure 4-13, fluid temperature at the outlet point increases rapidly to reach a nearly steady-state value within a very short time after the start of thermal operation; note that fluid temperature at the inlet point is kept constant throughout the duration of heat injection. During the recovery phase the mean fluid temperature drastically decreases and reaches its initial value. Fluid temperature obtained using the FD model is in complete agreement with COMSOL results. Figure 4-14 shows that fluid temperature variations along both branches of the U-tube, predicted using the FD and FE models, are in complete agreement.
Figure 4-13 Mean fluid temperature comparison between FD and COMSOL Multiphysics™ for a constant temperature fluid injection ($T_{\text{inlet}}=35^\circ \text{C}$) scenario.

Figure 4-14 Comparison of variation of fluid temperature along the circulation tube (for constant temperature injection with $T_{\text{inlet}}=35^\circ \text{C}$)
Figure 4-15 shows heat flux calculated based on the temperature gradient around the downward and upward tubes. In contrast to what observed for a constant heat rate injection scenario, heat flux decreases by 25% from 5 hours to 100 hours after the heat injection with a constant inlet temperature. This implies that the amount of heat rejected to the ground and consequently the difference between inlet and outlet fluid temperatures decreases in time to reach a nearly steady-state condition. Note that the typical heat injection period for buildings is around 6 to 8 hours, and the system may be turned off for a recovery phase. Therefore, the loss in system efficiency will be relatively lower than what is observed in this analysis. Ground temperature response for the constant inlet fluid temperature scenario obtained from FD model is also compared with results obtained from COMSOL analysis. The maximum difference in ground temperature predicted using the FE and FD models is 0.5°C.
4.5.2.3 Back-calculation of soil thermal conductivity

Results obtained from FD and FE analyses with constant heat rate injection scenario are further analyzed using FLS model (Carslaw and Jaeger 1959, Mogensen 1983, Gehlin 1998, Austin 1998, Gehlin et al. 2003, Lim et al. 2007) to back-calculate the values of effective ground thermal conductivity. Following a transient method presented by Gehlin (1998) and Austin (1998), which considers the FLS solution method, the value of soil thermal conductivity \( k_s \) can be obtained as:

\[
k_s = \frac{\dot{m}C_p(T_{in} - T_{out})}{4\pi L \lambda} = \frac{Q}{4\pi L \lambda}
\]  

(4-28)
where \( \lambda \) is the slope of the mean fluid temperature \( T_m \) \( [= (T_{in} + T_{out})/2] \) versus \( \ln(t) \) plot and \( t \) is real time. Ten (10) different time intervals are selected to calculate the slope of the mean fluid temperature.

Typical temperature data obtained from FD and FE simulations are shown in Figure 4-17. Note that the use of Equation (4-28) requires a constant power to be injected to the system. As Figure 4-18 shows, estimated effective soil thermal conductivity back-calculated using both FE and FD analyses results are close to the soil thermal conductivity used in these analyses \( (k_s=2 \text{ Wm}^{-1}\text{K}^{-1}) \). The relative difference between effective thermal conductivity values calculated from FD results and the actual soil thermal conductivity used as input parameter is less than 9%. Although effective thermal conductivity values obtained from FE analyses are closer to the soil thermal conductivity (within 1.5%), grout thermal conductivity and thermal conductivity of the PVC tube are lower than soil thermal conductivity.
Figure 4-17 Mean fluid temperature versus time for the time intervals between (a) 5 hours to 100 hours and (b) 20 hours to 100 hours
4.5.2.4 Computational cost comparison

It is demonstrated in this chapter that the developed FD model is capable to produce good quality results which are in agreement with those obtained from 3D FE analyses. Note that the FD model seems to have much lower computational cost compared to the 3D FE model used for comparison. For the constant heat rate and constant fluid temperature analyses, computational time for FD analyses are around one-third to one-fourth of that needed for the FE analyses (Ozudogru *et al.* 2014a). To compare computational cost associated with the developed 2D model with that for the 3D FE model, the hardware characteristics as well as the CPU usage and the computational time are compared for both models. Table 4-2 shows that the FD model only uses 15% of the CPU while it has a comparatively lower computational time (just 35 % of the 3D
model). Moreover, four (4) to five (5) different FD analyses can run simultaneously in one computer with specifications shown in Table 4-2.

Table 4-2 Computational cost comparison between FEM and FDA

<table>
<thead>
<tr>
<th>100 hours of heating + 150 hours of recovery phases using constant Q</th>
<th>FE Model described in Ozudogru et al. 2014a</th>
<th>Developed FD Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>intel® core™ i7-930 processor (8m cache, 2.80 GHz, 4.80 gt/s)</td>
<td>intel® core™ i7-2600 processor (8m cache, 3.40 GHz, 5.00 gt/s)</td>
</tr>
<tr>
<td>Memory</td>
<td>6 x 4.0 GB</td>
<td>2 x 4.0 GB</td>
</tr>
<tr>
<td>Analysis info</td>
<td>41444 domain elements, 10086 boundary elements, and 1080 edge elements.</td>
<td>17850 grid points, 675 grid points at the boundaries</td>
</tr>
<tr>
<td>Environment</td>
<td>COMSOL 4.4.0.248</td>
<td>Fortran (developed in this study)</td>
</tr>
<tr>
<td>Number of degrees of freedom</td>
<td>172007</td>
<td>17850</td>
</tr>
<tr>
<td>Average ram usage</td>
<td>3.55 GB</td>
<td>3.2 GB</td>
</tr>
<tr>
<td>Average CPU usage</td>
<td>58%</td>
<td>15% (5 models can be run simultaneously)</td>
</tr>
<tr>
<td>Solution time</td>
<td>31149 s. (8 hours, 39 minutes, 9 seconds)</td>
<td>10943 s (3 hours, 2 minutes, 23 seconds)</td>
</tr>
</tbody>
</table>

4.5.3 Comparison with published field test data

The developed FD model is also verified by comparing model predictions with recorded field test data reported by Geo et al. (2008a and 2008b), Jalaluddin et al. (2011), Javed and Fahlen (2011), and Abdelaziz (2013). The short-term thermal test reported by Gao et al. (2008a, and 2008b) on a 25-m-long geothermal pile with an embedded circulation tube (with radius \( r_t = 1\mathrm{cm} \)) is simulated using the FD model. Boundary conditions and values of different input parameters \((k_s = 1.3 \ \text{Wm}^{-1}\text{K}^{-1}, \ \alpha_s = 5.86 \times 10^{-7} \ \text{m}^2\text{s}^{-1}, \ k_c = 1.63 \ \text{Wm}^{-1}\text{K}^{-1} \text{and} \ a_c = 7.78 \times 10^{-7} \ \text{m}^2\text{s}^{-1}, \ \mu_f = 0.725 \times 10^{-3} \ \text{Pa-s}, \ T_{\text{initial}} = 18.2 \ \text{°C}, \ T_{\text{inlet}} = 35.13 \ \text{°C} \text{and} \ t = 3 \ \text{hours})\) are adopted from Gao et al. (2008a, and 2008b). Figure 4-19 shows that the outlet temperature \( T_{\text{outlet}} \) predicted using the developed FD model compares very well with the recorded field data. Difference between
recorded field data and predicted value of $T_{\text{outlet}}$ is only 0.06 °C (0.2 % relative error). Figure 4-19 also shows that the annular cylinder model described in Chapter 3 underpredicts fluid temperature along the length of the circulation tube because the interaction between two branches of the tube is not considered in the annular cylinder model. In absence of any recorded temperature data along the length of the circulation tube, only the predicted distribution of fluid temperature along the circulation tube is shown in Figure 4-19. The predicted linear variation of fluid temperature along the length of the circulation tube is consistent with fluid temperature variation reported by Gao et al. (2008a, and 2008b).

Jalaluddin et al. (2011) conducted field thermal performance tests on a 20-m-long steel geothermal pile filled with silica sand. Different configurations of circulation tubes (single U-tube, double-tube and multi-tube) were used in the field study. Results obtained for the single U-shaped circulation tube (with inner radius $r_t = 1.3$ cm) is used to gage the prediction capability of the FD model. Field tests with two different values of fluid circulation (flow) rate $q_f = 6.67 \times 10^{-5}$ and $1.33 \times 10^{-4}$ m$^3$s$^{-1}$ (corresponding to fluid circulation velocity $v = 0.126$ and 0.251 ms$^{-1}$, respectively) are simulated using the FD model. All input parameters ($r_t = 0.013$ m, $k_s = 1.2$ Wm$^{-1}$K$^{-1}$, $k_f = 0.58$ Wm$^{-1}$K$^{-1}$, $\mu_f = 0.798 \times 10^{-3}$ Pa-s, $T_{\text{initial}} = 18$ °C, $T_{\text{inlet}} = 26.5$ °C for $q_f = 3.33 \times 10^{-5}$ m$^3$s$^{-1}$, and $T_{\text{inlet}} = 26.3$ °C for $q_f = 1.33 \times 10^{-4}$ m$^3$s$^{-1}$) are adopted based on the information provided in Jalaluddin et al. (2011). The thermal conductivity of the grout (silica sand), as reported by Jalaluddin et al. (2011), is used as the value of $k_c (= 1.4$ Wm$^{-1}$K$^{-1}$). Figure 4-20 shows that values of outlet fluid temperature $T_{\text{outlet}}$ obtained from FDAs at both transient and steady state are in good agreement with field data reported by Jalaluddin et al. (2011). The maximum difference observed between recorded and predicted values of $T_{\text{outlet}}$ at steady state condition is within 0.5% (0.08 °C, and 0.12 °C temperature differences, respectively, for $q_f = 6.67 \times 10^{-5}$ and $1.33 \times 10^{-4}$
m$^3$s$^{-1}$). Mean percentage error (MPE) calculated for the comparison of FDA results with field test data (as presented in Figure 4-20) are 1.13% and 0.6%, respectively, for $q_f = 6.67 \times 10^{-5}$ and $1.33 \times 10^{-4}$ m$^3$s$^{-1}$.

Figure 4-19 Comparison of FD model prediction with circulation fluid temperature reported by Gao et al. (2008a, and 2008b)
Javed and Fahlen (2011) conducted field thermal performance tests on an 80-m-long geothermal borehole filled with groundwater; diameter of the borehole was 110 mm. The inside and outside diameter of the U-tube were, respectively, 35.4 and 40 mm. Different configurations of circulation tubes were used in this field study. Result obtained for the single U-shaped circulation tube (BH3) is used to investigate the prediction capability of the developed FD model. Thermal input parameters used in the analysis are presented in Table 4-3 (Javed and Fahlen 2011). Constant heat rate injection scenario \( q = 4496 \text{ W} \) was used in field test. The missing parameters for the circulation fluid were taken from Ethyl Alcohol Handbook (Equistar Chemicals, 2003). Shank spacing was reported between 0 to 3 cm. In this analysis an average of 1.5 cm is used for the shank spacing between the downward and upward branches of the circulation tube. The initial fluid temperature was same as the ground initial temperature (8.9
°C). Figure 4-21 confirms that the developed model can predict well the mean fluid temperature measured during the thermal response test. The maximum difference between the model and field measurement is 0.6 °C.

Table 4-3 Parameters for BH3 in Javed and Fahlem (2011).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial ground temperature</td>
<td>8.9</td>
<td>°C</td>
</tr>
<tr>
<td>Heat rate per depth</td>
<td>56.2</td>
<td>W m⁻¹</td>
</tr>
<tr>
<td>Flow rate</td>
<td>23.33</td>
<td>dm³ min⁻¹</td>
</tr>
<tr>
<td>Dynamic viscosity of fluid</td>
<td>2.15</td>
<td>mPa s</td>
</tr>
<tr>
<td>Thermal conductivity of fluid</td>
<td>0.401</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of fluid</td>
<td>4180</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of fluid</td>
<td>946.63</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Thermal conductivity of PVC tube</td>
<td>0.42</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of PVC tube</td>
<td>2300</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of PVC tube</td>
<td>960</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Pipe inner diameter</td>
<td>35.4</td>
<td>mm</td>
</tr>
<tr>
<td>Pipe wall thickness</td>
<td>2.3</td>
<td>mm</td>
</tr>
<tr>
<td>Borehole thermal conductivity</td>
<td>0.59</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of borehole</td>
<td>4187</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of borehole (Groundwater)</td>
<td>1000</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Diameter of borehole</td>
<td>0.11</td>
<td>m</td>
</tr>
<tr>
<td>Length of the borehole</td>
<td>80</td>
<td>m</td>
</tr>
<tr>
<td>Ground thermal conductivity</td>
<td>3.04</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of the ground</td>
<td>956.52</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Ground density</td>
<td>2300</td>
<td>kg m⁻³</td>
</tr>
</tbody>
</table>
Finally the field thermal conductivity test presented by Abdelaziz (2013) is analyzed. The geometry of the analysis domain, geothermal borehole, thermal input parameters and other site specific parameters are presented in Figure 4-22 and Table 4-4. The initial ground and fluid temperatures were 14.7°C. Water was used as a circulation fluid and the constant heat rate was injected to the ground. The temperature difference between the inlet and outlet point during the field test was kept (using a heat tank) at around 5.56°C. Considering the fluid properties and circulation flow rate, this temperature difference implies a constant heat rate of 72 Wm⁻¹ along the length of the pile.
Figure 4-22 Subsurface profile and dimensions of the field test used in FD model (a) geometry of the analysis domain (b) plan view of the geothermal borehole
Table 4-4 Input parameters for numerical analysis to simulate field test reported by Abdelaziz, (2013)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial ground temperature</td>
<td>14.7</td>
<td>°C</td>
</tr>
<tr>
<td>Heat rate per depth</td>
<td>72</td>
<td>W m⁻¹</td>
</tr>
<tr>
<td>Test duration</td>
<td>50</td>
<td>h</td>
</tr>
<tr>
<td>Flow rate</td>
<td>5.68</td>
<td>dm³ min⁻¹</td>
</tr>
<tr>
<td>Dynamic viscosity of fluid</td>
<td>0.9772</td>
<td>mPa s</td>
</tr>
<tr>
<td>Thermal conductivity of fluid</td>
<td>0.6048</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of fluid</td>
<td>4180</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of fluid</td>
<td>997.8</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Pipe thermal conductivity</td>
<td>0.40</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Pipe specific heat capacity</td>
<td>2300</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Pipe density</td>
<td>940</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Pipe inner diameter</td>
<td>21.844</td>
<td>mm</td>
</tr>
<tr>
<td>Pipe wall thickness</td>
<td>2.413</td>
<td>mm</td>
</tr>
<tr>
<td>center-to-center spacing between two branches</td>
<td>7.5</td>
<td>cm</td>
</tr>
<tr>
<td>Thermal conductivity of borehole</td>
<td>1.28</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of borehole</td>
<td>880</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of borehole</td>
<td>1600</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Diameter of borehole</td>
<td>25.4</td>
<td>cm</td>
</tr>
<tr>
<td>Length of borehole</td>
<td>30.48</td>
<td>m</td>
</tr>
<tr>
<td>Thermal conductivity of ground layer 1</td>
<td>1.0</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of ground layer 1</td>
<td>1500</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of ground layer 1</td>
<td>1900</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Layer thickness of ground layer 1</td>
<td>12.80</td>
<td>m</td>
</tr>
<tr>
<td>Thermal conductivity of ground layer 2</td>
<td>2.90</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity of ground layer 2</td>
<td>1200</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>Density of ground layer 2</td>
<td>2400</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Layer thickness of ground layer 2</td>
<td>22.68</td>
<td>m</td>
</tr>
</tbody>
</table>

Mean fluid temperature predicted using the input parameters presented in Table 4-4 is in a very good agreement with field measurements reported by Abdelaziz (2013). The FD model can predict field test measurement with the coefficient of determination ($R^2$) of 0.950.
4.6 Chapter Summary

A rigorous finite difference model that can predict thermal performance of a geothermal heat exchanger pile (or borehole) with an embedded single U-shaped circulation tube is presented in this chapter. The developed model accounts for heat transport by the heat carrier fluid and heat conduction in soil and concrete; convective heat loss within circulation fluid and thermal resistance of the PVC circulation tube are also considered in model formulation. Such features are missing in the available analytical solutions which are frequently used in practice to quantify ground temperature response and heat exchange capabilities of vertical geothermal heat exchangers. Prediction capability of the developed model is investigated extensively through comparison with analytical solutions based on heat conduction (FLS model) and thermal resistivity framework. Model predictions are also compared with predictions using a 3D FE
model developed in in COMSOL and with published field test data. The developed FD model seems to have higher computational efficiency (with comparable level of accuracy) compared to 3D FE model; part of the lower computational cost compared to 3D model originates from the lower number of elements required to model the 2D domain.

4.7 Cited References


Farouki, O.T., (1981). Thermal properties of soils. United States Army Corps of Engineers Cold Regions Research and Engineering Laboratory, Hanover, NH.


Chapter 5: Quantification of Heat Exchange Performance for Geothermal Piles with Single U-shaped Circulation Tube

Heat exchange through geothermal piles is analyzed with particular emphasis on quantifying the effects of design, operational and site-specific variables on energy output from these piles. The finite difference model described in Chapter 4 is used for the analyses presented in this chapter. Analyses are performed for a range of different input parameters to quantify their influences on thermal performance of geothermal pile and ground temperature response. Based on the results of finite difference analyses (FDAs), a set of equations are proposed that can be used to quantify energy output from geothermal piles under varying operational and design conditions. FDA results are also used to determine the hierarchy of different input variables based on their relative impacts on heat exchange efficiency of the pile. Moreover, advanced first order second moment (AFOSM) reliability analysis is performed to assess expected variation of geothermal energy exchange rate (or power output) due to practical variations of several input parameters.

5.1 Heat Exchange Analysis

Heat exchange through a 25-m-long, 0.6-m-diameter concrete geothermal pile with an embedded U-shaped circulation tube is analyzed using the FD model described Chapter 4. A cylindrical soil domain (radius \( R = 12 \text{ m} \) and depth \( Z = 30 \text{ m} \)) with the pile in its center is considered for all analyses. Few trial analyses are performed with different boundary distances to ensure that the constant temperature boundary condition at the far boundaries is valid. Figure 5-1 shows the geometry and the finite difference grid generated by the grid generation module of the developed FD code. This module needs discretization length, first spacing between nodes, and the ratio of
increment. Following a geometric progression series, nonuniform mesh (that would reduce the computational cost) can be developed by this module. Note that several commercially available software also use similar algorithms to generate and update finite element mesh or finite difference grid points. An initial simulation, hereafter referred to as *base analysis*, is performed using a set of expected values for model input parameters (Table 5-1).

Figure 5-2 shows temperature contour around the heat exchanger pile after 60 days of heat rejection from the pile to the surrounding ground (a typical case for geothermal pile operation during summer months). The thermal influence zone, which is defined as the region for which minimum change in initial temperature $T_{\text{initial}}$ is 1°C, after 60 days of heat rejection extends approximately up to a distance of $13.5r_p (= 200r_i \approx 4 \text{ m})$ from the center of the pile. It is observed that change in ground temperature is negligible (even after 60 days of heat exchange operation) beyond a distance $6r_p$ below the geothermal pile (Figure 5-2). Radial heat transfer is observed for most part of the pile length except in the vicinity of the ground surface, which is a convective boundary (i.e., heat can dissipate through this boundary). A convective boundary condition at the ground surface affects heat transfer within approximately top 4 m of the ground. Radial heat transfer through geothermal pile has also been reported in other numerical studies on geothermal piles (Laloui *et al*. 2006, Gao *et al*. 2008a, and Abdelaziz *et al*. 2011); however, the thermal influence zone and the effect of a convective ground surface on ground temperature response have not been quantified before.
Figure 5-1 Finite difference grid points and problem geometry for analyses presented in this chapter
### Table 5-1 Input parameters used for base analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid circulation velocity</td>
<td>( v = 0.1 \text{ m/s} )</td>
</tr>
<tr>
<td>Convective heat transfer coefficient (from ground surface)</td>
<td>( h_g = 5 \text{ W/(m}^2\text{K)} )</td>
</tr>
<tr>
<td>Convective heat transfer coefficient (from top of the pile)</td>
<td>( h_p = 0.01 \text{ W/(m}^2\text{K)} )</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>( T_{\text{initial}} = 18^\circ \text{C} )</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>( T_{\text{inlet}} = 35^\circ \text{C} )</td>
</tr>
<tr>
<td>Length of each vertical branch of the circulation tube</td>
<td>( L_t = 24.8 \text{ m} )</td>
</tr>
<tr>
<td>Mass density of heat carrier fluid</td>
<td>( \rho_f = 1000 \text{ kg/m}^3 )</td>
</tr>
<tr>
<td>Pile length</td>
<td>( L = 25 \text{ m} )</td>
</tr>
<tr>
<td>Pile radius</td>
<td>( r_p = 0.30 \text{ m} )</td>
</tr>
<tr>
<td>Radius of circulation tube</td>
<td>( r_t = 0.02 \text{ m} )</td>
</tr>
<tr>
<td>PVC tube thickness</td>
<td>( t_t = 6 \text{ mm} )</td>
</tr>
<tr>
<td>Shank distance (i.e., center-to-center distance between two branches of the circulation tube)</td>
<td>( s_t = 40 \text{ cm} )</td>
</tr>
<tr>
<td>Specific heat of heat carrier fluid</td>
<td>( C_{pf} = 4190 \text{ J/(kg}^\circ \text{C)} )</td>
</tr>
<tr>
<td>Temperature in the medium above ground surface</td>
<td>( T_{\text{ag}} = 23^\circ \text{C} )</td>
</tr>
<tr>
<td>Thermal conductivity of concrete</td>
<td>( k_c = 1.7 \text{ W/(mK)} )</td>
</tr>
<tr>
<td>Thermal conductivity of soil</td>
<td>( k_s = 2.5 \text{ W/(mK)} )</td>
</tr>
<tr>
<td>Thermal conductivity of PVC tube</td>
<td>( k_t = 0.41 \text{ W/(mK)} )</td>
</tr>
<tr>
<td>Thermal diffusivity of concrete</td>
<td>( \alpha_c = 0.9 \times 10^{-6} \text{ (m}^2/\text{s)} )</td>
</tr>
<tr>
<td>Thermal diffusivity of soil</td>
<td>( \alpha_s = 1.4 \times 10^{-6} \text{ (m}^2/\text{s)} )</td>
</tr>
<tr>
<td>Thermal diffusivity of PVC tube</td>
<td>( \alpha_t = 0.284 \times 10^{-6} \text{ (m}^2/\text{s)} )</td>
</tr>
<tr>
<td>Dynamic viscosity of fluid</td>
<td>( \mu_f = 0.725 \text{ m Pa. s} )</td>
</tr>
<tr>
<td>Thermal conductivity of fluid</td>
<td>( k_i = 0.58 \text{ W/(mK)} )</td>
</tr>
</tbody>
</table>
Figure 5-2  Temperature contour (in °C) after 60 days of heat rejection from a geothermal pile

5.2 Temperature Difference $\Delta T$ between Inlet and Outlet Points

The amount of heat transfer through a geothermal pile is related to the temperature difference $\Delta T$ ($= T_{\text{inlet}} - T_{\text{outlet}}$) between inlet and outlet points of the circulation tube. Figure 5-3 shows the variation of $\Delta T$ with both real and normalized time expressed by Fourier number $Fo$ ($= \alpha t / r_t^2$). Temperature difference $\Delta T$, and thus the heat transfer rate, reduces sharply within a very short period of time (approximately 1 day) after the heat transfer starts and reaches nearly a constant value after 20 days of continuous heat rejection. Pile (concrete) and soil temperature increases with time as heat is rejected from heat carrier fluid to the surrounding media. Such temperature increase in the medium surrounding the heat source causes reduction in temperature gradient.
The reduction in temperature gradient deters the rate of heat transfer from the heat carrier fluid and thus, temperature \( T_{\text{outlet}} \) at the fluid outlet point increases. Consequently, for a constant value of inlet temperature \( T_{\text{inlet}} \), \( \Delta T \) reduces with time and reaches a nearly constant value at a certain time (\( \approx 20 \) days for base analysis) after heat rejection starts.

Regression analysis is performed to describe a simple mathematical form for calculation of \( \Delta T_b \), i.e., the value of \( \Delta T \) (in °C) as obtained from the base analysis, as a function of time \( t \) (in hours). Figure 5-3 shows that Equation (5-1) can successfully predict result from the base analysis with a coefficient of determination \( R^2 = 0.99 \).

\[
\Delta T_b(t) = 4.9(t)^{-0.12}
\]

(5-1)

Note that Equation (5-1) is valid only for the set of input parameters used in the base analysis. The value of \( \Delta T \) is expected to vary for any other combination of design, operational and site-specific parameters such as fluid circulation velocity \( v \), radius of circulation tube \( r_t \), pile radius \( r_p \), initial temperature difference \( \Delta \theta (= T_{\text{inlet}} - T_{\text{initial}}) \), thermal conductivity of concrete \( k_c \), and thermal conductivity of soil \( k_s \). Considering individual effects of the above parameters on \( \Delta T \), the following general expression is proposed for \( \Delta T \):

\[
\Delta T(t) = \Delta T_b(t) \prod_{N} \left( \frac{x_N}{x_{Nb}} \right)^{a_N}
\]

(5-2)

where \( N \) is an index that indicates total number of important input parameters (for this study \( N = 6 \)), \( x_N \) takes the value of \( N^{th} \) input parameter (e.g., \( v \), \( \Delta \theta \), \( k_s \), \( k_c \), \( r_t \), and \( r_p \)), \( x_{Nb} \) is the base value of the \( N^{th} \) input parameter (as reported in Table 5-1), and \( a_N \) is a regression coefficient for the \( N^{th} \)
input parameter. In order to obtain the unknown coefficients \( a_N \) in Equation (5-2), additional FDAs are performed by varying one input parameter at a time while all other input parameters are kept constant at their base values reported in Table 5-1. The expression for \( a_N \) for each input variable \( x_N \) is determined through regression analyses of results obtained from these additional FDAs.

### 5.2.1 Effect of fluid circulation velocity \( v \) on \( \Delta T \)

Separating the velocity term from all other input parameters, Equation (5-2) can be written as:

\[
\Delta T(t) = \Delta T_b(t) \left( \frac{v}{0.1} \right)^{a_1} \prod_{N=1}^{n} \left( \frac{x_{(N-1)}}{x_{(N-1)b}} \right)^{a_{N-1}} 
\]

(5-3)

Given that all other input parameters are kept at their base values, i.e., \( x_{(N-1)} = x_{(N-1)b} \), Equation (5-3) yields:

\[
\Delta T(t) = \Delta T_b(t) \left( \frac{v}{0.1} \right)^{a_1} \quad [v \text{ is in m/s}] 
\]

(5-4a)

\[
\Rightarrow a_1 = \frac{\log \left( \frac{\Delta T(t)}{\Delta T_b(t)} \right)}{\log \left( \frac{v}{0.1} \right)} \quad ; \quad a_1 = \begin{cases} 
-0.7 & \text{for laminar flow} \\
-0.9 & \text{for turbulent flow}
\end{cases}
\]

(5-4b)

Based on regression analysis of results from FDAs with different values of circulation velocity \( v (= 0.07, 0.3 \text{ and } 0.5 \text{ ms}^{-1}) \), the value of \( a_1 \) in Equation (5-4a) is found to be equal to \(-0.9\) and \(-0.7\) for turbulent and laminar flow conditions, respectively. Figure 5-3 shows that the use of Equations (5-3) and (5-4) can successfully predict \( \Delta T(t) \) for different values of \( v \).
5.2.2 Effect of initial temperature difference $\Delta \theta$ on $\Delta T$

The effect of initial temperature difference $\Delta \theta (= T_{\text{inlet}} - T_{\text{initial}})$ on $\Delta T$ is investigated through FDAs for $\Delta \theta = 9$, 22, and $-17 \, ^\circ\text{C}$, while all other input parameters are kept at their base values. Based on FDA results, it is concluded that $\Delta \theta$ linearly affects $\Delta T$ (Figure 5-4) for thermal operation of a geothermal pile with constant $T_{\text{inlet}}$ and uniform initial temperature $T_{\text{initial}}$.

\[
\Delta T(t) = \Delta T_b(t) \left( \frac{\Delta \theta}{17} \right)^{a_2} \quad [a_2 = 1 \text{ from regression analysis; } \Delta \theta \text{ is in } ^\circ\text{C}] 
\]  

\[ (5-5) \]
5.2.3 Effect of thermal conductivity of concrete $k_c$ on $\Delta T$

A previous research suggests that the use of concrete with higher thermal conductivity may result in an increase in heat exchange efficiency (i.e. an increase in $\Delta T$) of geothermal piles (Gao et al. 2008a and 2008b). To explore the potential for such an effect of $k_c$ on $\Delta T$, FDAs are performed by varying the value of $k_c$ (while keeping other values of input parameters constant as used in base analysis) within a range of 1.1–2.5 Wm$^{-1}$K$^{-1}$, which is a typical range of $k_c$ used in the piling industry (Choktaweekarn et al. 2009, and Lixia et al. 2011). Figure 5-5 shows that the effect of $k_c$ on $\Delta T$ lasts only for a very short period of time (few hours) after the heat exchange operation starts. Once pile temperature reaches a nearly steady-state value, $k_c$ does not have a significant influence on $\Delta T$. Therefore, it does not seem feasible to effectively increase thermal efficiency of
concrete geothermal piles by just using a special concrete with high value of $k_c$. Following equations are proposed to quantify both short- and long-term effects of $k_c$ on $\Delta T$:

$$\Delta T(t) = \Delta T_b(t) \left( \frac{k_c}{1.7} \right)^{a_5}$$  \hspace{1cm} [\text{$k_c$ is in Wm}^{-1}\text{K}^{-1}] \hspace{1cm} (5-6a)

$$a_5 = 0.35 - 0.02 \ln t$$  \hspace{1cm} [\text{$t$ is in hours}] \hspace{1cm} (5-6b)

Figure 5-5 Effects of thermal conductivity of concrete $k_c$ on evolution of temperature difference $\Delta T$

5.2.4 Effect of thermal conductivity of soil $k_s$ on $\Delta T$

Thermal conductivity of soil $k_s$ depends on various factors such as dry density, water content, and soil texture. Typical values of $k_s$ in coarse grained soil vary within 0.9–4.2Wm\(^{-1}\)K\(^{-1}\); for fine grained soil this variation is from 0.3 to 2.1 Wm\(^{-1}\)K\(^{-1}\) (Brandl 2006). $k_s$ reduces with decrease in
soil water content and the value of $k_s$ is minimum for dry soil, usually 0.2–0.4 m$^{-1}$K$^{-1}$ (Tarnawski et al. 2011). It is observed that within a short period of time after the heat exchange operation starts an increase $k_s$ does not have a significant effect on $\Delta T$; however, such effect increases with time as the thermal operation of a geothermal pile continues (Figure 5-6). Mathematically:

$$\Delta T(t) = \Delta T_b(t) \left( \frac{k_s}{2.5} \right)^{a_s}$$

$[k_s$ is in Wm$^{-1}$K$^{-1}]$ 

(5-7a)

$$a_s = (0.07 + 0.07 \ln t)$$

$[t$ is in hours] 

(5-7b)

Figure 5-6 Influence of soil thermal conductivity $k_s$ on evolution of temperature difference $\Delta T$
5.2.5 Effect of pile radius \( r_p \) on \( \Delta T \)

The effect of pile radius \( r_p \) on \( \Delta T \) depends on the ratio of soil thermal conductivity \( k_s \) over thermal conductivity of concrete \( k_c \) (Figure 5-7). For the base analysis presented in this chapter, \( k_s \) is greater than \( k_c \) (\( k_s/k_c = 1.47 \)), and for such a condition, \( \Delta T \) decreases as \( r_p \) increases. The reverse is true for \( k_s/k_c < 1 \), i.e., increase in \( r_p \) causes increase in \( \Delta T \).

For \( k_s/k_c < 1 \), heat conduction within concrete pile is faster than that in soil. Therefore, the fluid-to-concrete heat transfer rate is greater than the rate of heat transfer from the concrete pile to the soil surrounding it and concrete temperature increases faster than soil temperature (i.e., the pile acts as a heat storage). In such a case, a bigger pile diameter helps in reducing the rate of average temperature increase within the pile and slows down the rate of heat flux reduction in the vicinity of fluid-concrete interface. This results in an increase in \( \Delta T \). Such dependence of \( \Delta T \) on pile radius \( r_p \) can be expressed as:

\[
\Delta T(t) = \Delta T_b(t) \left( \frac{r_p}{0.3} \right)^{a_5} \quad [r_p \text{ is in m}] \tag{5-8a}
\]

\[
a_s = -(0.05 + 0.01 \ln t) \left( \frac{k_s/k_c}{1.47} \right) \left( \frac{r_p}{0.3} \right) \tag{5-8b}
\]
5.2.6 Effect of radius of circulation tube $r_t$ on $\Delta T$

Results from FDAs with different values of $r_t = 1, 2, \text{and } 4 \text{ cm}$ shows that an increase in radius $r_t$ of the circulation tube decreases $\Delta T$ (Figure 5-8). As tube radius $r_t$ increases, the surface area available for heat transfer and the volume of heat carrier fluid passing through any pipe cross section increases. Consequently, higher amount of heat energy is transferred to the ground. Such heat transfer increases ground temperature surrounding the pile and reduces heat flux at the heat source boundary. The reduction in heat flux hinders any further reduction in temperature of the heat carrier fluid. Therefore, except for a time immediately after the heat exchange operation starts, $\Delta T$ reduces with increase in $r_t$. Based on regression analysis of FDA results the following equation is proposed to quantify the effect of $r_t$ on $\Delta T$:

\[
\Delta T = f(r_t) = \alpha_{st} t r_t^2
\]
\[ \Delta T(t) = \Delta T_b(t) \left( \frac{r_t}{0.02} \right)^{a_0} \quad [r_t \text{ is in m}] \]  \hspace{1cm} (5-9a)

\[ a_0 = \begin{cases} 
-(0.95 + 0.05 \ln t) & \text{for laminar flow} \\
-(1.15 + 0.05 \ln t) & \text{for turbulent flow} 
\end{cases} \]  \hspace{1cm} (5-9b)

Figure 5-8 Effects of circulation tube radius \( r_t \) on \( \Delta T \)

5.2.7 Effect of shank distance \( s_t \) on \( \Delta T \)

The wall-to-wall spacing \( s_t \) between two limbs of the circulation tube (commonly referred to as shank distance) usually changes based on pile radius. Figure 5-9 shows that a decrease in spacing \( s_t \) decreases \( \Delta T \). This reduction is very significant if two branches of U-tube are close enough so that they will have a significant interaction in changing circulation fluid temperature. However,
increment in $\Delta T$ becomes insignificant when $s_t$ increases beyond a critical value ($\approx 30 \text{ cm}$ for the cases presented in Figure 5-9).

Combining Equations (5-1) through (5-9), the time-dependent variation of $\Delta T$ can be written as:

$$\Delta T(t) = A(t)^{a_0} \left( \frac{v}{0.1} \right)^{a_1} \left( \frac{\Delta \theta}{17} \right)^{a_2} \left( \frac{k_c}{2.5} \right)^{a_3} \left( \frac{k_p}{0.3} \right)^{a_4} \left( \frac{r_i}{0.02} \right)^{a_5}$$

(5-10)

The expressions for regression coefficients $A$, $a_0$, $a_1$, $a_2$, $a_3$, $a_4$, $a_5$ and $a_6$ in Equation (5-10) are summarized in Table 5-2.
Table 5-2 Regression coefficients for different input variables

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Turbulent ($t$ is in hours)</th>
<th>Laminar ($t$ is in hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>4.9</td>
<td>4.9</td>
</tr>
<tr>
<td>$a_0$</td>
<td>$-0.12$</td>
<td>$-0.12$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$-0.9$</td>
<td>$-0.7$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$0.35 - 0.02 \ln t$</td>
<td>$0.35 - 0.02 \ln t$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$(0.07 + 0.07 \ln t)$</td>
<td>$(0.07 + 0.07 \ln t)$</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$-(0.05 + 0.01 \ln t) \frac{k_s/k_c}{1.47} \frac{r_p}{0.3}$</td>
<td>$-(0.05 + 0.01 \ln t) \frac{k_s/k_c}{1.47} \frac{r_p}{0.3}$</td>
</tr>
<tr>
<td>$a_6$</td>
<td>$-(1.15 + 0.05 \ln t)$</td>
<td>$-(0.95 + 0.05 \ln t)$</td>
</tr>
</tbody>
</table>

5.3 Ground Temperature Response

Figure 5-10 shows the effect of soil thermal conductivity $k_s$ on temperature distribution within the pile and in the soil surrounding it. The thermal influence zone increases with an increase in $k_s$. Ground temperature in the vicinity of the pile is considerably higher (as much as 3°C) for $k_s = 1.1$ Wm$^{-1}$K$^{-1}$ compared to that for $k_s = 4$ Wm$^{-1}$K$^{-1}$ (Figure 5-10). Also, ground temperature distribution depends on pile radius $r_p$. For a constant value of $k_s$, ground temperature near a pile with $r_p = 0.3$ m is higher compared that near a pile with $r_p = 0.6$ m (Figure 5-10b). At radial distances beyond 5~8$r_p$, temperature rise in ground is always greater for higher values of soil thermal conductivity.
Figure 5-10 Temperature distributions within pile and soil after 60 days of heat rejection (a) constant pile radius (b) different pile radius

All other input parameters are same as used in the base analysis (Table 5-1)
5.4 Geothermal Power Output

The total amount of heat exchange between a geothermal pile and ground over a certain period of thermal operation of the pile is defined as the energy output. Mathematically, the rate of energy extraction or rejection $P(t)$ (or power output) can be expressed as:

$$P(t) = \dot{m}C_pf\Delta T(t) = \rho v \pi r_t^2 C_pf \Delta T(t)$$

(5-11)

Note that fluid circulation velocity $v$ and radius of circulation tube $r_t$ affect both $\dot{m}$ and $\Delta T(t)$. Therefore, the individual effects of $v$ and $r_t$ on $P(t)$ should be investigated separately. Although some field studies quantified the effect of circulation flow rate $q_f$ (Gao et al. 2008a and 2008b, and Jalaluddin et al. 2011) on energy extraction (or rejection) rate $P(t)$, none of those studies could conclusively differentiate individual effects of $v$ and $r_t$ on $P(t)$. The circulation velocity $v$ reversely affects $\dot{m}$ and $\Delta T(t)$ in Equation (5-11); an increase in $v$ increases $\dot{m}$ but reduces $\Delta T(t)$. Figure 5-11(a) shows, for a set of input parameters, that power output (at time $t=7$ days) per unit length of the pile increases with an increase in $v$ and reaches to a constant value beyond a threshold value of $v$ ($= 0.3$ ms$^{-1}$) that can be designated as an efficient circulation velocity for geothermal pile operation. $P$ increases with increase in circulation tube radius $r_t$, and it is interesting to note that the value of efficient circulation velocity is independent of the value of $r_t$. Figure 5-11(b) shows power output (at time $t=7$ days) per unit length of a geothermal pile as obtained from FDAs with a constant circulation flow rate $q_f = 6.28 \times 10^{-4}$ m$^3$s$^{-1}$ ($= 37.7$ liter/minute). For a constant circulation flow rate $q_f$, different combinations of $v$ and $r_t$ can result in an increase (or decrease) in energy extraction (or rejection) rate. It is evident from Figure 5-11(b) that for a constant flow rate $q_f$, $P$ increases as $r_t$ increases, and thus, the effect of $r_t$ on $P$ is higher than that of $v$. 

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Figure 5-11 Effects of circulation tube radius $r_t$ and fluid circulation velocity $v$ on power output: (a) variable circulation flow rate $q_f$ and (b) constant circulation flow rate $q_f$. All other input parameters are same as used in the base analysis (Table 5-1). $t = 7$ days.
The effect of flow characteristic (laminar or turbulent) on the amount of heat loss within the circulation fluid is also studied in this research (Figure 5-12). Figure 5-12 shows fluid temperature variation within a cross section (at the middle of the geothermal pile) of the inlet branch of the circulation tube. The high value of convective heat transfer coefficient in case of a turbulent flow results in minimal heat loss within the circulation tube; however, this is not the case for a laminar flow condition, which exhibits as much as a 10% (3.25°C) reduction of fluid temperature between the centerline and inside wall of the circulation tube.

![Fluid temperature variations](image)

Figure 5-12 Fluid temperature variations (within a cross section of circulation tube) with flow characteristics

5.4.1 Closed-form expression to predict power output

The equations proposed for calculation of $\Delta T(t)$ [i.e., Equation (5-10) in combination with Table 5-2] can be used in conjunction with Equation (5-11) to predict energy output over a certain
period of time of operation of a geothermal pile. Figure 5-13 compares power output (per unit length of the pile) values obtained from several FDAs (performed with different combinations of input parameters) and those predicted using proposed Equations (5-10), (5-11) and Table 5-2. The proposed equations can successfully predict (with a maximum difference of less than 10%) power output values for both short- and long-term operation (respectively, for $t = 12$ h and 60 days) of a geothermal pile with U-shaped circulation tube.

![Figure 5-13 Comparison between power output obtained from FDAs and that predicted using proposed equations [Equations (5-10), (5-11) and Table 5-2]](image)

5.5 Identification of Sensitive Parameters

The hierarchical effects of important design, operational and site specific parameters on increase in ground temperature at the pile-soil interface and on heat exchange efficiency of a geothermal pile at short- and long-term after the start of the thermal (heat rejection) operations are
investigated through sensitivity analysis. Results from the sensitivity study are presented in the form of Tornado diagrams, which show relative influences of important model parameters on power output from a geothermal pile and on ground temperature increment (Figure 5-14 and Figure 5-15). Table 5-1 presents a set of expected input parameters. The maximum and minimum values considered for different input variables are shown in Figure 5-14 and Figure 5-15.

Figure 5-14(a) shows that the radius of circulation tube $r_t$, initial temperature difference $\Delta \theta$ and soil thermal conductivity $k_s$ are, sequentially, the three most important parameters affecting the short-term thermal efficiency of a heat exchanger pile. The same three parameters, with a reverse hierarchy, are also found to be the most important parameters that affect long-term heat exchange efficiency of the system (Figure 5-14b). While the long-term heat exchange efficiency is most sensitive to $k_s$, this parameter has least influence among three most sensitive parameters on the short-term thermal efficiency of a geothermal pile. As can be seen from Figure 5-15, initial temperature difference $\Delta \theta$, soil thermal conductivity $k_s$, and radius of circulation tube $r_t$ are the most sensitive parameters that affect ground temperature increment at the pile-soil interface. Note that $k_s$ have opposite effects on energy output and ground temperature increment.
Figure 5-14 Hierarchy of model parameters in affecting thermal efficiency of a geothermal pile:
(a) after 12 hours of operation (short-term) and (b) after 60 days of operation (long-term)
5.6 Reliability-based Assessment of Heat Exchange Performance

Although the hierarchy of different parameters can be established through a sensitivity study, the tornado diagrams shown in Figure 5-14 and Figure 5-15 cannot be used when two or more parameters vary simultaneously within their acceptable ranges. The potential variation of heat exchange efficiency of geothermal piles due to combined variation of any two or more input parameters is estimated using AFOSM reliability method (also known as Hasofer-Lind method, Hasofer and Lind 1974). The input parameters are considered to be statistically independent; i.e., an arbitrarily chosen statistical value of an input parameter does not depend on the value of another input parameter. Mean values of six input parameters ($v$, $r_t$, $r_p$, $k_s$, $k_c$, and $\Delta \theta$) are kept same at their base values used in the sensitivity study (Tornado diagrams; Figure 5-14 and Figure 5-15). 30% coefficient of variation (COV) is assumed for input parameters $\Delta \theta$ and $k_c$. COV for
all other important input parameters (i.e., \( k_s, v, r_p \) and \( r_t \)) is assumed to be equal to 50%. These COV values are selected based on a realistic judgment to encompass almost all practical values of input parameters. The possible range of input parameters is shown in Table 5-3. The expression of power output in Equation (5-11) is used to establish the performance function \( Y(t) \), which is a function of time and random input variables \( X_m \) (where \( m = 1, 2, 3, 4, 5 \) and 6):

\[
Y(t) = \frac{P(t)}{L} = g \left( t, X_1, X_2, \ldots, X_6 \right) \quad (5-12)
\]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (circulation rate)</td>
<td>( v = 0.05 \text{ m/s} )</td>
<td>( v = 0.15 \text{ m/s} )</td>
</tr>
<tr>
<td>Radius of circulation tube</td>
<td>( r_b = 1 \text{ cm} )</td>
<td>( r_b = 3 \text{ cm} )</td>
</tr>
<tr>
<td>Pile radius</td>
<td>( r_p = 15 \text{ cm} )</td>
<td>( r_p = 45 \text{ cm} )</td>
</tr>
<tr>
<td>Thermal conductivity of soil</td>
<td>( k_s = 1.25 \text{ W/mC} )</td>
<td>( k_s = 3.75 \text{ W/mC} )</td>
</tr>
<tr>
<td>Thermal conductivity of concrete</td>
<td>( k_c = 1.19 \text{ W/mC} )</td>
<td>( k_c = 2.21 \text{ W/mC} )</td>
</tr>
</tbody>
</table>

Substituting \( t = 60 \times 24 \) hour in Equation (5-10) and in the equations presented in Table 5-2, the performance function \( Y \) for two months of heat rejection can be expressed as:

\[
Y_{60\text{days}} = \frac{P_{60\text{days}}}{L} = 9.75 \left( v^{0.1} \Delta \theta k_c^{0.2} k_s^{0.58} k_p^{-0.12} \right) \left( \frac{k_s}{k_c} \right)^{0.3} \left( \frac{r_p}{r_t} \right)^{0.49} \quad (5-13)
\]

The Taylor series expansion of \( Y(t) \) as a function of mean \( \mu_x \) and variance \( \sigma_x^2 \) of \( X_m \), can be written as (Halder et al. 2000):

\[
Y(t) = g \left( t, \mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_6} \right) + \sum_{m=1}^{6} \left( X_m - \mu_{X_m} \right) \frac{\partial g}{\partial X_m} + \sum_{l=1}^{6} \sum_{m=1}^{6} \left( X_l - \mu_{X_l} \right) \left( X_m - \mu_{X_m} \right) \frac{\partial^2 g}{\partial X_l \partial X_m} + \cdots \quad (5-14)
\]
AFOSM method considers only the first order terms of a performance function (Hasofer and Lind 1974, and Halder et al. 2000) and thus, truncation of Equation (5-14) at the linear terms yields:

\[ Y(t) \approx g\left(t, \mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_6}\right) + \sum_{m=1}^{6} \left( X_m - \mu_{X_m} \right) \frac{\delta g}{\delta X_m} \]  

(5-15)

Now taking expectations from both sides of Equation (5-15), the mean value of \( Y \), i.e., the mean value of power output per unit pile length is obtained as:

\[ \mu_Y (t) = g\left(t, \mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_6}\right) \]  

(5-16)

Variance of \( Y(t) \) can be obtained by calculating a second moment of \( Y(t) \) from Equation (5-16). Since input parameters are statistically independent, the off-diagonal terms (i.e., \( \text{cov}[X_l, X_m]; l \neq m \)) of the covariance matrix become equal to zero and the diagonal terms (i.e., \( \text{cov}[X_m, X_m] \)) are the variance of \( X_m \). Therefore, total variance of \( Y(t) \) can be calculated as expressed in Equation (5-17).

\[ \sigma_Y^2 \approx \sum_{l=1}^{6} \sum_{m=1}^{6} \text{cov}[X_l, X_m] \left( \frac{\delta g(X_1, X_2, \ldots, X_N)}{\delta X_l} \right) \left( \frac{\delta g(X_1, X_2, \ldots, X_N)}{\delta X_m} \right) \] 

\[ \sigma_Y^2 \approx \sum_{m=1}^{6} \sigma_{X_m}^2 \left( \frac{\delta g(X_1, X_2, \ldots, X_N)}{\delta X_m} \right)^2 \]  

(5-17)

A ratio of relative variance \( \frac{\sigma_{X_m}^2}{\sigma_Y^2} \) of parameter \( X_m \) with the total variance \( \sigma_Y^2 \) of \( Y(t) \) is obtained for each random input variable at time \( t = 60 \) days. Such a ratio represents relative variance contribution of an input variable on energy output (Figure 5-16). For the practical variations of input variables considered in this study, it is observed that the same three parameters \( (k_s, \Delta \theta \text{ and } r_c) \) that showed most significant effects on long-term power output
from a geothermal pile (Figure 5-14b) also produce (following the same hierarchy) maximum contributions on the total variance of heat exchange through a geothermal pile. It is also evident that uncertainties in $v$, $k_c$, and $r_p$ do not have significant effects on power output of geothermal piles (Figure 5-16). The variability in power output from heat exchanger piles is primarily introduced by the uncertainties in $k_s$ and circulation tube radius $r_t$.

![Graph showing relative variance contributions of random input parameters](image)

**Figure 5-16 Result from AFOSM reliability analysis: relative variance contributions of random input parameters**

Mean power output $\mu_y(t)$ per unit length of the geothermal pile and total variance $\sigma_y^2$ are plotted in Figure 5-17. Both $\mu_y(t)$ and $\sigma_y^2$ decrease with time. While the mean value of $P/L$ decreases from 103 Wm$^{-1}$ (at $t = 1$ hour) to 43.1 Wm$^{-1}$ after two months of continuous heat rejection to the ground, total variance of energy output decreases from 55.1 Wm$^{-1}$ to 23.0 Wm$^{-1}$ within the same timeframe. Coefficient of variation of power output for the case presented in this

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chapter can be as high as 53% during two months of operation. Therefore, an appropriate set of parameters should be selected for thermal design of a geothermal pile.

Figure 5-17 Variations of mean power output and total variance per unit length of the geothermal pile during two months of thermal operation

5.7 Energy Output under 15-year Variation of Inlet Temperature

The heat exchange analysis presented so far in this chapter considers a constant value of inlet fluid temperature $T_{inlet}$. However, $T_{inlet}$ is expected to vary with time for real-life operation of geothermal piles. The developed FD model is used to analyze performance (energy output; $E = Pt$) of a 25-m-long geothermal pile under 15 years (from 1999 to 20013) of the recorded hourly temperature variation of State College, PA (NOAA’s National Climatic Data Center; http://www.weather.gov/climate/). Soil temperature was adopted from National Resources Conservation Service (NRCS), United States Department of Agriculture (USDA)
thermal soil conductivity, and soil parameters are adopted from Web Soil Survey of USDA.

Input parameters are mentioned in Figure 5-18. Figure 5-18(a) shows the daily average values of $T_{\text{inlet}}$ used in this analysis. The energy output obtained over one year of operation of the geothermal pile is predicted from both FD analysis with an hourly variation of input temperature, and proposed simple equation. The exact energy output predicted by rigorous analysis is obtained to be equal to 70,000 kWh (Figure 5-18b). However, the predicted power output using proposed equation (Equations (5-10, and (5-11) is calculated to be 80,000 kW-h which is only 15% more than the exact value. Note that the time-dependent variation of $\Delta T$ presented in Equation (5-10) is only valid for the 25-m-geothermal pile/borehole. For any other geothermal pile/borehole with different length, the energy output should be normalized to 25-m, $(\Delta T_d = \Delta T \times (L_d/25))$, where $\Delta T_d$ is a desirable time-dependent variation and $L_d$ is the length of the desirable pile. For example the power output which is shown in Figure 5-18 was multiplied by 1.2 (=30/25).
Figure 5-18 Performance of a 25-m-long geothermal pile in State College, PA: (a) recorded daily average of hourly temperature data and (b) weekly average of energy output.
5.8 Chapter Summary

Results from finite difference analyses presented in this chapter show that fluid circulation velocity and radius of circulation tube independently affect energy output from a geothermal pile. Beyond a short time after the start of heat transfer, thermal conductivity of concrete does not have any significant effect on heat exchange through geothermal piles; however, the effect of soil thermal conductivity on heat transfer efficiency increases with time. The effect of pile radius on energy output depends on the ratio \( k_s/k_c \); increase in pile radius increases heat exchange efficiency for \( k_s/k_c < 1 \); the reverse is true when \( k_s/k_c > 1 \). A set of equations is proposed which can be used to calculate power output from a geothermal pile for any practical set of input variables. Comparison of heat loss within the fluid for laminar and turbulent flow shows that heat loss within the circulation tube is relatively higher in laminar flow and consequently thermal resistivity of fluid convection is higher in laminar flow. It is also evident from the results presented in this chapter that an increase in circulation velocity results in an increase in the harvested geothermal energy. Nevertheless, fluid circulation at higher velocity would require more electric power; therefore, optimum circulation fluid velocity would eventually depend on power efficiency of the circulation heat pump.

Effects of different design, operational, and site-specific variables on heat exchange efficiency of geothermal piles must be quantified for reliable and efficient design of geothermal piles. Sensitivity and reliability analyses show that thermal conductivity of soil, initial temperature difference between ground and circulation fluid, and radius of circulation tube are sequentially three most important parameters that affect long-term heat exchange performance of a geothermal pile.
The developed FD model and the proposed equation were used to predict the amount of geothermal energy that can be harvested in 15 years through thermal operation of a 25-m-long geothermal pile installed in State College, PA. Using real air temperature data recorded at the University Park Airport and for typical soil thermal properties for this area, the total geothermal energy harvested through a single geothermal pile in 15 years is predicted to be approximately equal to 70,000 kW-h and 80,000 kW-h, respectively, using FD analyses and the proposed equations.

### 5.9 Cited References

Abdelaziz SL, Olgun CG, Martin JR. (2011). Design and operational considerations of geothermal energy piles, Geo-Frontiers, American society of Civil Engineers Conference; 450-459.


National Oceanic and Atmospheric Administration, National Climatic Data Center
http://www.ncdc.noaa.gov/

National Resources Conservation Service, United Staezt Department of Agriculture (NRCS)
http://www.wcc.nrcs.usda.gov/nwcc/sensors?sitenum=1231&timeseries=hourly


Chapter 6: Thermal Performance Tests on a Model Geothermal Pile and Numerical Simulations of Model Tests

This chapter presents data from a series of large-scale laboratory experiments on a model geothermal pile installed in dry and saturated sand. Ground temperature response obtained from thermal operation of the model pile in saturated condition is compared with thermal performance tests performed under dry condition (Kramer 2013, Kramer et al. 2014). The effect of different boundary conditions (convective versus insulated boundary) on heat exchange efficiency and ground temperature response is investigated for saturated ground condition. Response obtained from physical model tests are compared with those obtained from numerical simulations that replicate laboratory-controlled test conditions. Independent element thermal conductivity tests performed to obtain soil and concrete thermal conductivity values that are to be used as input parameters in the numerical analyses are also discussed.

6.1 Laboratory-scale Tests on a Model Heat Exchanger Pile

The experiment setup used to obtain the test results described in this chapter was developed in a parallel research study (Kramer 2013). Details of the soil tank test facility, model geothermal pile, instrumentation and data acquisition system are presented in Kramer (2013) and Kramer et al. (2014). A brief description of the test setup is included herein for completeness.

A 1.38-m-long concrete model pile with diameter equal to 0.1 m was embedded 1.22 m into the sand bed prepared within a custom-designed steel tank (1.83 m × 1.83 m × 2.13). To achieve a uniform test bed with desirable relative density a sand raining technique was designed and used for preparing the sand bed. A constant-temperature water bath was used to circulate
heat carrier fluid (1:1 laboratory standard mixture of ethylene glycol and distilled water) through a U-shaped circulation tube (PVC tube) embedded within the geothermal model pile. Inner and outer diameters of the PVC tube are, respectively, 12.4 and 15.8 mm. The shank spacing between the two branches of the U-tube was designed to be equal to 46 mm. Temperature measurements were monitored and obtained at 96 different locations within the soil bed, at the pile-soil interface, on the tank boundaries, and at inlet and outlet points.

6.2 Temperature Measurements

Temperature increments were monitored at 94 different points using type T thermocouples which were embedded at different depths in soil to investigate pile-soil heat exchange. Inlet and Outlet fluid temperatures were obtained using two thermocouples placed within the circulation tube at the inlet and outlet points. Measurements of the circulation fluid at the inlet and outlet points enabled quantification of heat exchange efficiency of the system. A vertical plane passing through the U-tube (inlet-outlet plane, or XZ plane) was heavily instrumented using thermocouples placed at 71 locations. Temperature increments were also monitored on a plane perpendicular to the inlet-outlet plane (YZ plane; hereafter referred to out of plane). Soil temperature variations at the soil tank boundaries were monitored through 6 thermocouples placed on the inner boundaries of the tank. Temperature measurements at the tank boundaries helped to determine when the thermal tests should be stopped to avoid any thermal effects of the side boundaries; tests were stopped before any change in tank boundary temperature was recorded. Figure 6-1 schematically shows the exact locations of all 94 thermocouples. Temperature records were monitored and recorded using a National Instruments data acquisition system (Kramer 2013).
Figure 6-1 Experiment setup and temperature measurement locations: (a) on XZ plane and (b) on YZ plane (Kramer et al. 2014)
6.3 Element Thermal Conductivity Tests

Element thermal conductivity (TC) test setups were designed and used to determine values of thermal conductivity for concrete, dry sand and saturated sand. Thermal properties of soil and pile material (concrete) play key roles in thermal performance of geothermal piles. Figure 6-2 shows the custom-built setup used for thermal conductivity measurements for dry sand and concrete. The element test setup used for saturated sand had impermeable boundaries and diameter and height equal to 20 and 35 cm, respectively. The radial and vertical spacing’s between thermocouple locations were 4 and 6 cm, respectively. These setups are very similar to the one specified by ASTM D5334, except that heat carrying fluid is circulated through the specimen instead of using a heating probe as a heat source. Heat carrying fluid was circulated through a PVC tube (with 12.4 mm inner and 15.8 mm outer diameters) embedded at the middle of the cylinder. A nearly constant fluid temperature was maintained (and recorded) at the inlet point at the bottom of the specimens. Temperature increments at 9 different locations (arranged in a 3×3 grid) inside the TC test setup were monitored using type T thermocouples (Figure 6-2b). Using a 3×3 grid pattern for the thermocouple arrangement allowed for material thermal conductivity calculation using different pairs of thermocouple readings. In order to calculate the amount of heat energy rejected to (or extracted from) the specimen, fluid temperatures at the inlet and outlet points were also measured and recorded during the TC tests. Top and bottom surfaces of the cylindrical specimens were insulated using rock wool to minimize heat loss from these surfaces. Thermal conductivity values are calculated using (1) the Fourier’s law and (2) using the finite line source model for the transient part of the thermal response test (Gehlin 1998, Austin 1998).
Figure 6-2 Element test setup for thermal conductivity measurement of dry sand and concrete: (a) custom-built test apparatus and (b) temperature measurement locations
Based on Fourier’s law, thermal conductivity of sand \( k_{s, \text{dry}}^{\text{ref}} \) for dry sand and \( k_{s, \text{sat}}^{\text{ref}} \) for saturated sand) and concrete \( k_c^{\text{ref}} \) at steady state heat flow condition can be calculated using the following equation:

\[
-kA \frac{\partial T}{\partial r} = \dot{m} C_p (T_{\text{inlet}} - T_{\text{outlet}})
\]  

(6-1)

where \( k \) is the thermal conductivity of the material, \( A \) is the area of the specimen, \( \dot{m} \) and \( C_p \) are, respectively, mass flow rate and specific heat capacity of the circulating fluid, \( T_{\text{inlet}} \) and \( T_{\text{outlet}} \) are fluid temperatures at the inlet and outlet points. Rearranging Equation (6-1) and integrating both sides, material thermal conductivity can be calculated as:

\[
k_{\text{ref}} = \frac{\dot{m} C_p (T_{\text{inlet}} - T_{\text{outlet}}) \ln \left( \frac{r_2}{r_1} \right)}{2\pi L (T_1 - T_2)}
\]  

(6-2)

where \( T_1 \) and \( T_2 \) are temperatures recorded at two different points at same depth and radial distances \( r_1 \) and \( r_2 \) from the heat source (circulation tube) center, and \( L \) is the length of the heat source. Since temperature increments were collected at 9 locations, three combinations of temperature measurements \( T_1 \) and \( T_2 \) are possible at each depth; thus 9 different values of thermal conductivity can be calculated from a single TC test. Figure 6-3 shows fluid temperature measurements and power output (i.e., a measure of heat energy injected to the specimen) for three different TC tests. After a few hours (\( \approx 15 \) hours) temperature increments at grid points for dry soil and concrete are negligible and therefore Fourier’s law can be applied to calculate material thermal conductivity. Figure 6-3(c) shows mean fluid temperature \( T_m \) \( = (T_{\text{in}} + T_{\text{out}})/2 \), power output and temperature increments at three thermocouple locations during a thermal conductivity test for saturated sand specimen.
Transition part used in line source model

Figure 6-3 (a)

TC1, $T_m = +40 \, ^\circ C$

$v = 0.66 \, m/s$

$T_{in} = 40 \, ^\circ C$

$T_{out} = +40 \, ^\circ C$

$v = 0.22 \, m/s$

Data shown for circled thermocouple locations

Figure 6-3 (b)
Soil thermal conductivity was also calculated using the finite line source model (Carslaw and Jaeger 1959, Mogensen 1983, Gehlin 1998, Austin 1998, Gehlin et al. 2003, Lim et al. 2007). Thermal response tests which have been extensively used in literature and field to predict ground thermal conductivity is based on the finite line source (FLS) model. Constant heat flux per unit length of the pile is assumed in this method. A short period of time after the start of a test, a constant power output was achieved for the dry tests (Figure 6-3a) and, therefore, the transient line source model can be used to predict soil thermal conductivity. Following this method, soil thermal conductivity can be calculated as:

$$k_s = \frac{\dot{m} C_p (T_{\text{inlet}} - T_{\text{outlet}})}{4 \pi L \lambda}$$

(6-3)
where \( \lambda \) is the slope of the mean fluid temperature \( T_m \) on the temperature-ln(\( t \)) plot, and \( t \) is real time.

Figure 6-4 shows the variation of dry soil thermal conductivity obtained from both Fourier’s law and finite line source methods. Using the results calculated by the Fourier’s law from three different thermal conductivity tests the statistical mean \( \bar{k}_{s,\text{dry}}^{\text{ref}} \) and standard deviation \( \sigma_k \) of dry soil thermal conductivity are, respectively, equal to 0.25 and 0.04 Wm\(^{-1}\)K\(^{-1}\). Soil thermal conductivity calculated using the line source model are equal to 0.18, 0.22 and 0.30 Wm\(^{-1}\)K\(^{-1}\), for three TC tests. Average thermal conductivity for the concrete \( \bar{k}_c^{\text{ref}} \) and saturated sand \( \bar{k}_{s,\text{sat}}^{\text{ref}} \) are calculated to be equal to 1.9 and 3.2 Wm\(^{-1}\)K\(^{-1}\), respectively.

![Figure 6-4 Variation of thermal conductivity values for dry sand calculated using Fourier’s law and finite line source model](image)

Thermal diffusivity \( \alpha \) (ms\(^{-2}\)) can also be calculated based on the measured thermal conductivity \( k \) (Wm\(^{-1}\)K\(^{-1}\)) and Equation (6-4).
\[ \alpha = \frac{k}{\rho C_p} \]  

(6-4)

where \( \rho \) is material mass density (kgm\(^{-3}\)) and \( C_p \) is specific heat capacity (Jkg\(^{-1}\)K\(^{-1}\)). Specific heat capacity of sand was calculated using specific heat capacity values for the solid phase (sand particles), air and water (Rees et al. 2000 and Brandl 2006) using the following equation.

\[ C_p = (1-n)C_{ps} + nS_r C_{pw} + n(1-S_r) C_{pa} \]  

(6-5)

where \( C_{ps}, C_{pw}, \) and \( C_{pa} \) are specific heat capacities of the solid phase, water, and air, respectively; \( n \) is soil porosity and \( S_r \) is degree of saturation.

For the laboratory determined relative density of the sand bed \( n = 0.41 \) and the specific heat capacity for dry sand particles \( C_{ps} = 850 \) Jkg\(^{-1}\)K\(^{-1}\) (http://www.engineeringtoolbox.com/specific-heat-capacity-d_391.html). The specific heat capacity for water \( C_{pw} = 4190 \) Jkg\(^{-1}\)K\(^{-1}\) (www.engineeringtoolbox.com). Thus the specific heat capacity for saturated sand can be calculated following (6-5) as:

\[ C_p = (1-0.41)\times850 + 0.41\times4190 = 2220 \text{ J.kg}^{-1}.\text{K}^{-1} \]  

(6-6)

Calculated values of thermal conductivity and diffusivity for dry and saturated sand and concrete are presented in Table 6-1. This table also includes these values for the circulation tube material (PVC), as obtained from Vinidex (a PVC producing company http://www.vinidx.com.au/technical/material-properties/pvc-properties/) and Engineering Toolbox (http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html).
Table 6-1 Thermal properties of different materials used in

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity (Wm⁻¹K⁻¹)</th>
<th>Thermal diffusivity (ms⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>0.25</td>
<td>1.5×10⁻⁷</td>
</tr>
<tr>
<td>Saturated sand</td>
<td>3.20</td>
<td>7.0×10⁻⁷</td>
</tr>
<tr>
<td>Concrete</td>
<td>1.7</td>
<td>4.8×10⁻⁷</td>
</tr>
<tr>
<td>PVC</td>
<td>0.15</td>
<td>0.72×10⁻⁷</td>
</tr>
</tbody>
</table>

6.4 Results from Thermal Performance Tests

A series of thermal performance tests on model geothermal pile installed in saturated sand were performed as part of the present research. Results obtained from these tests are compared with results of similar tests performed on model pile installed in dry sand (reported by Kramer 2013 and Kramer et al. 2014). Test conditions for all thermal performance tests conducted as part of this study and the ones performed by Kramer (2013) are presented in Table 6-2. For the tests under dry condition, the ratio $k_{s,dry}/k_c$ of soil and concrete thermal conductivity was equal to 0.15; the ratio $k_{s,sat}/k_c$ was equal to 1.9 for the tests under saturated test bed condition. Two thermal performance tests (TPCS1 and TPHS3) were conducted using a layer of rock wool insulator (with R-value = 50; R-value is defined as the ratio of temperature difference across an insulator and heat flux) placed at the top boundary to reduce heat rejection from the soil surface. The effects of convective (or insulated) boundary condition at the ground surface and $k_s/k_c$ on thermal performance of geothermal pile can be analyzed using the data gathered during the aforementioned tests. Before the start of a test, the soil and pile were at room temperature ($\approx 19^°C$) with a minor temperature gradient within the test bed due to the variation in room temperature (Figure 6-5). However, such initial temperature gradient is negligible when compared to the difference between inlet fluid temperature and mean initial soil temperature. Such a small temperature gradient may also be present in real ground.
Table 6-2 Thermal performance test matrix

<table>
<thead>
<tr>
<th>Test Tag</th>
<th>Type of test</th>
<th>Initial temperature gradient $\Delta \theta$ ($^\circ$C)</th>
<th>Circulation velocity $v$ (m s$^{-1}$)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPHD1</td>
<td>Heating</td>
<td>+20</td>
<td>0.11</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPHD2</td>
<td>Heating</td>
<td>+20</td>
<td>0.33</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPHD3</td>
<td>Heating</td>
<td>+20</td>
<td>0.66</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPHD4</td>
<td>Heating</td>
<td>+20</td>
<td>0.11</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPCD1</td>
<td>Cooling</td>
<td>−20</td>
<td>0.11</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPCD2</td>
<td>Cooling</td>
<td>−20</td>
<td>0.66</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPSCHD</td>
<td>Cooling followed by heating</td>
<td>−20 and +40</td>
<td>0.66</td>
<td>Kramer (2013)</td>
</tr>
<tr>
<td>TPHS1</td>
<td>Heating</td>
<td>+20</td>
<td>0.11</td>
<td>Present research</td>
</tr>
<tr>
<td>TPHS2</td>
<td>Heating</td>
<td>+20</td>
<td>0.66</td>
<td>Present research</td>
</tr>
<tr>
<td>TPHS3</td>
<td>Heating - Top surface is insulated</td>
<td>+20</td>
<td>0.66</td>
<td>Present research</td>
</tr>
<tr>
<td>TPCS1</td>
<td>Cooling - Top surface is insulated</td>
<td>−20</td>
<td>0.66</td>
<td>Present research</td>
</tr>
</tbody>
</table>

‘Heating’ and ‘Cooling’ signifies, respectively, heat rejection to and heat extraction from the test bed.

Figure 6-5 Initial temperature profile within the soil tank before the start of test TPHS1
6.4.1 Comparison of Results Obtained from Dry and Saturated Tests

The effect of $k_s/k_c$ on ground temperature response is investigated using data gathered from dry (Kramer 2013) and saturated tests (performed as part of this study). Radial variation of soil temperature increment for saturated heating and cooling tests are compared, respectively, with heating and cooling tests performed in dry soil. Soil temperature increments $\Delta T_s$ at two different time steps are shown in Figure 6-6. In case of a thermal test under saturated soil condition, heat quickly dissipates from the pile to the soil surrounding it and, therefore, $\Delta T_s$ near the heat source (geothermal pile) is lower in saturated soil when compared to that under dry condition. However, saturated soil shows a relatively higher temperature increment at farther measurement points (e.g., at distances 0.50 and 0.94 m). Therefore, higher heat exchange efficiency (reflected through higher fluid temperature difference between inlet and outlet points) is expected for saturated ground condition. Figure 6-6 also shows that $\Delta T_s$ at all points on both XZ and YZ planes (at depth $z = 0.6$ m from the top of the soil bed) are almost identical with a maximum difference of 0.3°C (even at $t = 4$ days). Such an observation supports the assumption of axisymmetric heat conduction in the solid media, as discussed in Chapters 3 and 4. Nonetheless, the measurement points just adjacent to the pile (at $r = 0.05$ m) consistently show higher temperature at the inlet side compared to that at the outlet side. This aspect of heat transfer is captured in the second generation FD model discussed in Chapter 4 through simultaneous consideration of the effects of two branches of the circulation tube on temperature increment of the surrounding media.
Figure 6-6 Comparison of soil temperature increments at different radial distances for dry and saturated tests for (a) $t = 1$ day and (b) $t = 4$ days.
The time-dependent evolutions of temperature increment in dry and saturated sand are also compared at two different radial distances (Figure 6-7). $\Delta T_s$ (measured at $r = 0.5$ m) at any time for both heating and cooling tests is higher under saturated condition compared to dry condition. However, as mentioned earlier, $\Delta T_s$ is lower at $r = 0.1$ m for saturated condition than the corresponding value in dry ground condition.
Figure 6-7 Variations of soil temperature increment $\Delta T_s$ with normalized time $F_o$ for dry and saturated tests: (a) heating tests TPHD3 and TPHS3 at $r = 0.1$ m, and $r = 0.5$ m and (b) cooling tests at $r = 0.10$ m, and $r=0.50$ m

6.4.2 Effect of Convective Boundary Condition on Heat Exchange Performance

In contrast to the free ground surface condition near geothermal boreholes, concrete floor slab and pile cap are expected to reduce convective heat transfer from the boundary near the geothermal pile head. The presence of convective boundary condition near the pile head has an important effect on soil temperature increment within a zone near the ground surface. Therefore, the effect of convective boundary conditions on ground temperature response was also investigated during thermal tests on the model geothermal pile. A thermal insulator layer (rock wool) with R value of 50 (i.e., convective heat transfer coefficient $h = 0.02$ W/m$^2$C) was placed on top of the saturated sand bed for tests TPHS3 and TPCS1. Soil temperature increment and heat exchange efficiency recorded during two different tests (TPHS2 and TPHS3) with different
surface boundary conditions are plotted in Figure 6-8. Soil temperature increment $\Delta T_s$ within a zone near the top surface is lower for test TPHS2 which did not have any ground surface insulation when compared to that for test TPHS3 with insulated top surface because heat can dissipate easily through the free ground surface (i.e., uninsulated top boundary of the test bed). Such convective heat loss, however, does not have any any significant effect on the temperature difference between fluid inlet and outlet points $\Delta T_f$, which is a measure of heat exchange efficiency of the geothermal pile (Figure 6-8b).

For the laboratory determined soil thermal conductivities ($k_{s,dry} = 0.25 \text{ Wm}^{-1}\text{K}^{-1}$ and $k_{s,sat} = 3.2 \text{ Wm}^{-1}\text{K}^{-1}$) and for reasonable values of specific heat capacity for dry and saturated sand [$C_{ps} = 850 \text{ Jkg}^{-1}\text{K}^{-1}$ (engineeringtoolbox.com 2013) and $C_p = 2220 \text{ Jkg}^{-1}\text{K}^{-1}$, calculated using Equation (6-5)] the normalized time $Fo$ at time $t = 7$ days for dry sand and $t = 4$ days for saturated sand can be calculated as:

$$Fo = \frac{\alpha t}{r_p^2} = \frac{k t}{\rho C_{ps} r_p^2} = \frac{(0.25)(604800)}{(1900)(850)(0.05)^2} = 37.4$$

$$Fo = \frac{(3.20)(345600)}{(2100)(2200)(0.05)^2} = 95.75$$
Figure 6-8 Effect of insulation at the surface boundary on (a) soil temperature increment $\Delta T_s$ and (b) difference in fluid temperature $\Delta T_f$ between inlet and outlet points.
6.5 Comparison between Model Test Results and Numerical Predictions

Four thermal performance tests (TPHD3, TPCD2, TPHS3 and TPCS1) on dry and saturated sand are simulated using the second generation finite difference model described in Chapter 4. Figure 6-9 shows the FD grid and boundary conditions used in the numerical modeling of the laboratory experiments. All dimensions used in the numerical model (i.e., pile length and radius, tube radius, dimensions of the soil domain) are exactly same as those in the laboratory experiment (Table 6-3). Values of other input parameters used in FDAs are either obtained from material characterization tests or adopted from the literature for standard cases. The left, right and top boundaries are considered as convective boundaries because heat loss can occur through these boundaries and a constant-temperature boundary condition is used for the bottom boundary of the analysis domain.

Figure 6-9  Computation grid and boundary conditions used in the FD model
Table 6-3 Input parameters used in the numerical model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convective heat transfer coefficient for the ground surface</td>
<td>( h_g = 1 \text{ W.m}^{-2}.\text{K}^{-1} )</td>
<td>Rouissi et al. 2012</td>
</tr>
<tr>
<td>Fluid circulation velocity</td>
<td>( v = 0.66 \text{ m.s}^{-1} )</td>
<td></td>
</tr>
<tr>
<td>Mass density of heat carrier fluid</td>
<td>( \rho_f = 1060 \text{ kg.m}^{-3} )</td>
<td>Kramer et al. 2014</td>
</tr>
<tr>
<td>Length of the model pile</td>
<td>( L = 1.2 \text{ m} )</td>
<td></td>
</tr>
<tr>
<td>Radius of the model pile</td>
<td>( r_p = 0.05 \text{ m} )</td>
<td></td>
</tr>
<tr>
<td>Inner radius of circulation tube</td>
<td>( r_t = 0.006 \text{ m} )</td>
<td></td>
</tr>
<tr>
<td>Specific heat of heat carrier fluid</td>
<td>( C_{pf} = 3455 \text{ J.kg}^{-1}.\text{K}^{-1} )</td>
<td>Measurement in this study and <a href="http://www.engineeringtoolbox.com">www.engineeringtoolbox.com</a></td>
</tr>
<tr>
<td>Thermal conductivity of concrete</td>
<td>( k_c = 1.5 \text{ (W.m}^{-1}.\text{K}^{-1}) )</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity of dry soil</td>
<td>( k_{sdry} = 0.25 \text{ (Wm}^{-1}.\text{K}^{-1}) )</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity of saturated soil</td>
<td>( k_{ssat} = 3.2 \text{ (Wm}^{-1}.\text{K}^{-1}) )</td>
<td></td>
</tr>
<tr>
<td>Thermal diffusivity of concrete</td>
<td>( \alpha_c = 4.8 \times 10^{-6} \text{ (m}^2.\text{s}^{-1}) )</td>
<td><a href="http://www.engineeringtoolbox.com">www.engineeringtoolbox.com</a></td>
</tr>
<tr>
<td>Thermal diffusivity of dry soil</td>
<td>( \alpha_{sdry} = 1.5 \times 10^{-7} \text{ (m}^2.\text{s}^{-1}) )</td>
<td></td>
</tr>
<tr>
<td>Thermal diffusivity of saturated soil</td>
<td>( \alpha_{ssat} = 7.6 \times 10^{-6} \text{ (m}^2.\text{s}^{-1}) )</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity of PVC tube</td>
<td>( k_t = 0.15 \text{ (W.m}^{-1}.\text{K}^{-1}) )</td>
<td><a href="http://www.engineeringtoolbox.com">www.engineeringtoolbox.com</a></td>
</tr>
<tr>
<td>Thermal diffusivity of PVC tube</td>
<td>( \alpha_t = 0.72 \times 10^{-7} \text{ (m}^2.\text{s}^{-1}) )</td>
<td></td>
</tr>
<tr>
<td>Dynamic viscosity of fluid for heating</td>
<td>( \mu_f = 2.2 \times 10^{-3} \text{ Pa}.\text{s} )</td>
<td></td>
</tr>
<tr>
<td>Dynamic viscosity of fluid for cooling</td>
<td>( \mu_c = 6.5 \times 10^{-3} \text{ Pa}.\text{s} )</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity of fluid</td>
<td>( k_f = 0.41 \text{ W.m}^{-1}.\text{K}^{-1} )</td>
<td>ASHRAE 2009 and <a href="http://www.engineeringtoolbox.com">www.engineeringtoolbox.com</a></td>
</tr>
</tbody>
</table>

6.5.1 **Comparison for Dry Tests**

Initial test bed temperature \( T_{s0} \) and fluid inlet temperature \( T_{in} \) are different for each test. Initial temperature distribution in the test bed and fluid inlet temperature used in the numerical analyses are directly obtained from the data recorded during the specific test under consideration. Figure 6-10 shows an initial temperature contour recorded in the test bed just prior to test TPCD2. \( T_{s0} \) values at all computation (grid) points are obtained using a MATLAB code that maps the FD grid on recorded initial soil temperature contour and calculates temperature values at all FD nodes using a triangulation-based linear interpolation command. Data recorded during one heating (TPHD3) and one cooling test (TPCD2) under dry test bed condition are compared with FDA predictions. Since all test conditions between TPHD3 and TPCD2 were kept the same and
the initial temperature difference \( \Delta \theta (= T_{\text{inlet}} - T_{s0}) \) were approximately equal with opposite signs, these thermal loadings (i.e., heating and cooling) had equal but opposite effects on soil temperature increments and power output (Kramer et al. 2014). Expectedly, equal absolute soil temperature increments were observed during the heating and cooling tests. Figure 6-11 shows that FDA results for both heating and cooling tests are in complete agreement with recorded data.

Figure 6-10 Initial soil temperature profile used in numerical simulation of test TPC2 (Kramer et al. 2014)
Figure 6-11 Comparison between soil temperature increment $\Delta T_s$ recorded during model tests and that obtained from FDAs: (a) $t = 0.5$ day (c) $t = 7.0$ days
6.5.2 Comparison for Saturated Tests

Two heating and cooling thermal performance tests (TPHS3 and TPCS1) on the geothermal model pile installed in saturated sand are also simulated using the FD model described in Chapter 4. Radial variations of $\Delta T_s$ at three different time steps ($t=0.5$, 1 and 4 days) are presented in Figure 6-12. The FD model can predict recorded soil temperature increments at $t = 0.5$ day and $t = 1$ day very well; however, prediction from the FD model is slightly higher than recorded $\Delta T_s$ values at $t = 4$ days.

![Figure 6-12 (a)](image-url)

**Figure 6-12 (a)**

- **TPHS3**
  - $\Delta \theta = +20 \degree C$, $v=0.66$ m/s

- **TPCS1**
  - $\Delta \theta = -20 \degree C$, $v=0.66$ m/s

*Experimental observations*
- Inlet side (XZ plane)
- Outlet side (XZ plane)
- Out of plane (YZ plane)

*FD - Average between inlet and outlet sides*
Figure 6-12 Comparison between soil temperature increments obtained by numerical results and experimental observation at (a) $t = 0.5$ day (b) $t = 1.0$ day (c) $t = 4.0$ days
Although the difference between recorded and predicted values of $\Delta T_s$ at $t = 4$ days is less than 0.3°C in the cooling phase and 0.9°C in the heating phase, this difference may eventually be significant for long term thermal operation of a geothermal pile. It is hypothesized that a possible reason for such over prediction of soil temperature increment near the geothermal model pile is temperature-induced pore fluid flow in saturated soil. Temperature gradient induced in the ground due to heat rejection through a geothermal pile may induce significant pore fluid flow, and as a result, heat convection through pore fluid flow might have further facilitated heat transport from the model geothermal pile towards the tank boundaries. Some previous research have studied heat and moisture transport in saturated and unsaturated medium (Abdel-Hadi and Mitchell 1981, Krishnaiah and Singh 2003), however, the potential effect of temperature-induced pore fluid flow on heat transport in saturated ground surrounding geothermal piles has not been investigated so far. The hypothesis described herein is tested in Chapter 7 through further refinement of the second generation FD model (discussed in Chapter 4) to additionally account for temperature-induced pore fluid flow and convective heat transport in saturated ground surrounding geothermal piles.

6.5.3 Comparison between Recorded and Predicted Fluid Temperature

The developed FD model (second generation) can also predict variations of fluid temperature along the length of the pile and over time. Variations of fluid temperature differences between inlet and outlet points recorded during the heating thermal performance test in saturated soil (TPHS3) and the cooling performance test under dry condition (TPCD2) are compared with FDA predictions. Figure 6-13 shows that predicted temperature difference $\Delta T_f$ between inlet and outlet points are consistent with recorded data. It can also be concluded from data plotted in
Figure 6-13 that a higher heat exchange efficiency can be expected under thermal operation of geothermal piles installed in saturated soil compared to those in dry soil.

![Figure 6-13 Comparison of recorded fluid temperature difference $\Delta T_f$ between inlet and outlet points with that predicted using FD model](image)

**6.6 Comparison of Recorded Data with Predictions using Finite Line Source Model**

Prediction capability of a widely-used idealized heat source model (FLS model; Zeng et al. 2002) is investigated through comparison of model prediction with data recorded during test TPCD2. The radius-to-length ratio $r_p/L$ for the model geothermal pile (heat source) is equal to 0.005; therefore, the application of FLS model to predict data recorded during the model-scale thermal test is theoretically valid. FDA results and recorded test data reveal that heat extraction rate $q_l$ (Wm$^{-1}$) changes with time of thermal operation, and at any time instant $q_l$ decreases linearly along the pile length (on both inlet and outlet sides of the pile). Nonetheless, a constant value of heat extraction rate has to be decided for use in the FLS solution. For this purpose, a net
heat extraction rate $q_1^{FLS}$ is calculated at different depths by adding the $q_1$ values (obtained from FDA) at the inlet and outlet sides of the pile (i.e., $q_1^{FLS} = q_{1\text{inlet}} + q_{1\text{outlet}}$). Figure 6-14 shows $q_1^{FLS}$ calculated at different depths of the pile for two different instants of time ($t = 30\text{ minutes}, Fo = 7.5; t = 1\text{ hour}, Fo = 15$) after the start of thermal operation. It is evident that for a particular time $q_1^{FLS}$ is nearly constant along the pile length.

Comparison between $\Delta T_i$ recorded during the test TPCD2 and that predicted using the FLS model with two different values of $q_1^{FLS}$ (= 7.5 and 16.5 W/m) demonstrates that the quality of prediction using the FLS model depends on the value of $q_1^{FLS}$ used in the analytical solution (Figure 6-15). In other words, the model bias (i.e., the difference between predicted and recorded values) for the FLS model depends greatly on the value of constant $q_1^{FLS}$ to be used, which is not known at a priori for a constant (or variable) temperature injection condition.

![Figure 6-14 Variation of net heat extraction rate along the pile length (as obtained from FDA)](image_url)
Figure 6-15 Comparison between soil temperature change $\Delta T_s$ recorded during test TPC2 and that predicted using the FLS model

6.7 Chapter Summary

Thermal performance tests on a model geothermal pile installed in saturated sand bed were performed under laboratory-controlled conditions. Data recorded during these tests are compared with data reported by Kramer (2013) on similar tests performed with dry test bed condition. The effects of the ratio of soil to concrete thermal conductivity, inlet fluid temperature, and boundary insulation on ground temperature response and heat exchange efficiency of the model geothermal pile are discussed. For tests under dry test bed condition, temperature increments are mostly localized within and near the geothermal pile (a direct effect of low ratio of soil-to-concrete thermal conductivity) resulting in a low heat exchange efficiency of the pile-soil system. A considerably higher value of the ratio $k_{s,sat}/k_c$ for the saturated tests facilitated radial heat dissipation away from the pile and consequently, soil temperature increment near the geothermal
model pile was much lower (≈ 7°C) compared to that under dry condition. In contrast, at points far from the model pile, soil temperature increment was higher for the saturated case. At any rate, higher pile-soil heat exchange efficiency can be expected when geothermal piles are installed in saturated ground compared to thermal operation of these piles installed in dry soil. Tests performed with and without insulation placed on top soil boundary confirmed that surface insulation does not have any significant effect on heat exchange efficiency of pile-soil system but such insulation may significantly reduce convective heat transfer through ground surface. Consequently, soil temperature increments near the ground surface will be significantly higher for insulated (or partially insulated) top boundary condition when compared to that in case for non-insulated top boundary. Recorded model test data are compared with predictions using the finite difference model described in Chapter 4. For most cases, except for long-term thermal performance under saturated soil condition, FDA results are in excellent agreement with recorded data and such comparisons further validate the developed model. A hypothesis is formed to explain the discrepancies between numerical model predictions and data recorded at \( t = 4 \) days after the start of thermal tests on model pile installed in saturated sand bed.

### 6.8 Cited References


Chapter 7:  Modeling of Thermally Induced Pore Fluid Transport in Saturated Ground

The hypothesis on the effect of heat induced pore fluid flow on pile-soil heat exchange in saturated ground condition is tested in this chapter. In addition to heat conduction in ground, the effect of convective heat transfer through thermally induced pore fluid flow on temperature increment in the soil surrounding geothermal piles is investigated. A numerical modeling technique that can analyze thermally induced pore fluid flow in saturated porous medium is discussed. The developed finite difference code discussed in Chapter 4 is further refined to account for the coupled effects of both conduction and convection on heat flow in saturated ground. Heat transfer energy equation and Brinkman’s momentum equation that considers a Boussinesq buoyancy term are coupled to analyze convective heat transfer through pore fluid flow. Results demonstrate that thermally induced pore water flow in saturated soil results in a considerable reduction in ground temperature increment within a zone surrounding the pile. Such a reduction in ground temperature increment for most part of the pile length is expected to increase heat exchange efficiency of geothermal piles.

7.1 Theoretical Background

Among several parameters that affect pile-soil heat exchange, ground thermal properties play a major role. Although heat conduction is commonly considered to be the key heat transfer mechanism within the ground, heat convection in the presence of ground water flow can increase heat rejection and extraction to and from the ground. On the other hand, temperature gradient induced by thermal operations of geothermal piles may alter pore fluid density; consequently,
buoyant flow can occur in the vicinity of heat exchanger piles even under hydrostatic groundwater condition within saturated soil. Potential effects of thermally induced pore water flow in saturated ground on heat transfer behavior of geothermal piles have not been explored so far. Kaviany (1986) investigated the effect of non-darcian flow on free convection inside two isothermal horizontal cylinders. Eskilson (1987) analytically examined the effect of ground water flow on ground temperature response in the vicinity of idealized heat exchangers and concluded that the presence of ground water flow does not have any significant effect on soil temperature increments. On the contrary, Jalaluddin et al. (2011) reported changes in ground temperature increments due to groundwater flow near a geothermal pile installed in field. Change in soil moisture content during laboratory measurement of soil thermal conductivity using thermal needle probe has been reported in literature (Low et al. 2013). Through laboratory experiments, Krishnaiah and Singh (2003, 2004) studied thermally induced moisture movement in soil. Hossain and Wilson (2002) studied natural convection flow induced by non-isothermal boundaries using upwind finite difference scheme; Hossain et al. (2013) further modified this model to examine the effect of conduction-convection-radiation phenomena on natural convection flow.

7.2 Heat Transport and Navier Stokes Equations

The heat transport equation considering both conduction through the solid particles and convection due to pore fluid flow can be expressed as (Nield and Bejan 2013):

\[
(\rho C_p)_m \frac{\partial T}{\partial t} + \frac{1}{\rho C_p} \nabla \cdot (\mathbf{u} T) = \nabla \cdot (k_m \nabla T)
\]  

(7-1)
where $\rho$ (kg.m$^{-3}$) is mass density, $C_p$ (J.kg$^{-1}$.K$^{-1}$) is specific heat capacity and $k$ (W.m$^{-1}$.K$^{-1}$) is thermal conductivity of the medium, $T$ is temperature, $t$ is time, and $u$ (m.s$^{-1}$) is the velocity of pore fluid; the subscripts $m$ and $f$ denote porous medium (soil mass) and pore fluid (water), respectively. Three main assumptions pertaining to Equation (7-1) are: (i) the viscosity dissipation is negligible, (ii) solid and fluid temperature at a point within the porous medium are identical (i.e., local thermal equilibrium exists), and (iii) heat conduction in solid and fluid phases occurs simultaneously; therefore, the amount of heat transfer from conduction mode to convection mode (and vice versa) is zero.

The term $(\rho C_p)_f (\mathbf{u} \nabla T)$ in Equation (7-1) is the convective part that accounts for thermally induced pore fluid velocity and the effect of pore fluid flow on heat transport in a porous medium. Thermal properties of saturated soil depend on individual properties of both constituent phases, i.e., soil particles and water. The medium properties can be calculated as:

$$
\begin{align*}
((\rho C_p)_m) &= (1-n)((\rho C_p)_s + n(\rho C_p)_f) \\
((k_m)) &= (1-n)((k)_s + n(k)_f)
\end{align*}
$$

where $n$ is the porosity of the medium and the subscripts $s$ and $f$, respectively, denote the solid and fluid phases.

Considering axisymmetric condition, the heat energy transport equation can be expressed as:

$$
\frac{1}{\alpha_m} \frac{\partial T}{\partial t} + \frac{(\rho C_p)_f}{(\rho C_p)_m} \left[ u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} \right] = \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}
$$

(7-3)

For an extreme case in which a porous media is idealized to be constituted of only the fluid phase (commonly referred to as cavity flow problems), the material properties for the medium and fluid phase are identical and then heat energy transport only depends on the velocity and
thermal diffusivity of the fluid. The numerical solution technique described in this chapter is validated through comparison with published data for a cavity flow problem that considered a rectangular cavity filled with fluid (Hossain et al. 2013).

The Navier Stokes equation with Brinkman term can be used to calculate pore fluid velocity considering volumetric forces (e.g. body forces), pressure, and fluid viscosity.

\[
\rho_{gw} \frac{1}{n} \left[ \frac{\partial \mathbf{u}}{\partial t} + \frac{1}{n} (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p_u + \frac{\mu}{n} \nabla^2 \mathbf{u} + F - \frac{\mu}{k_a} \mathbf{u} - \frac{C_F \rho_{gw}}{\sqrt{k_a}} |\mathbf{u}| \mathbf{u} 
\]  

(7-4)

where \( n \) is porosity of soil (porous medium) surrounding a geothermal pile, \( \rho_{gw} \) (kg.m\(^{-3}\)) is groundwater mass density which depends on temperature, \( \mu \) (Pa.s) is dynamic viscosity of the pore fluid, \( p_u \) (Pa) is pore pressure, \( k_a \) is absolute permeability, \( C_F \) is a dimensionless form drag (or pressure drag) constant, and \( F \) is the body force (N). The first term in the Navier Stokes equation (7-4) shows the variation of velocity vector with time, the second term \( (\mathbf{u} \cdot \nabla) \mathbf{u} \) represent the inertia term, and \( \frac{\mu}{n} \nabla^2 \mathbf{u} \) is defined as Brinkman’s term (Brinkman 1947a, and 1947b), and the last term is the drag term known as Forchheimer term (Forchheimer 1991). The drag term depends on porous medium. Equation (7-5) shows the variation of \( C_F \) based on the particle diameters and equivalent diameter of the analysis (Beavers et al. 1973).

\[
C_F = 0.55 \left( 1 - 5.5 \frac{d_p}{D_e} \right) 
\]  

(7-5)

where \( d \) is the diameter of the spheres particles and \( D_e \) is the equivalent diameter of the analysis domain; e.g., for a rectangular domain with dimensions \( w \) and \( h \), \( D_e \) can be expresses as:

\[
D_e = (2wh)/(w+h) 
\]  

(7-6)
The body force in Equation (7-4) that can be substituted by Equation (7-7) for the present problem:

\[
F = \begin{bmatrix} 0 \\ \rho_f g \end{bmatrix}
\]  

(7-7)

The spatial variation of fluid mass density induced by temperature gradient can be estimated using the Boussinesq approximation (Oberbeck 1879, Boussinesq 1903). Such a variation of fluid mass density results in buoyancy-driven flow and heat convection mechanism.

\[
\rho_t = \rho_{t0} \left[ 1 - \beta(T - T_0) \right]
\]  

(7-8)

where \( \beta \) (K\(^{-1}\)) is volumetric coefficient of thermal expansion and \( T_0 \) (°C) is initial temperature.

A continuity equation is needed to satisfy the momentum equation expressed in Equation (7-4) and to calculate the fluid velocity vector \( \mathbf{u} \). The continuity equation can be expressed as (Nield and Bejan 2013):

\[
\frac{\partial \rho_t}{\partial t} + \nabla (\rho_t \mathbf{u}) = 0
\]  

(7-9)

Following Boussinesq approximation, all properties of a porous medium are considered to be constant except the one multiplying to thermal expansion (buoyancy term). Therefore, the continuity equation reduces to Equation (7-10) because of very small variation of fluid mass density compared to its initial value (Nield and Bejan 2013)

\[
\nabla (\mathbf{u}) = 0
\]  

(7-10)

Temperature increments of the ground and pore fluid velocity can be obtained by coupling of Equations (7-3), (7-4), and (7-10).
For simultaneous solution of heat energy balance, Navier Stokes and continuity equations a finite difference model is developed as part of this study. Equation (7-4) can be expressed in both $r$ and $z$ direction as follows:

\[
\frac{1}{n} \left[ \frac{\partial u_r}{\partial t} + \frac{1}{n} \left( u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} \right) \right] = -\frac{\partial p_u}{\rho_g \partial r} + \frac{\mu}{\rho_g} \left( \nabla^2 u_r \right) - \frac{\mu}{\rho_g k_a} u_r - \frac{C_f}{\sqrt{k_a}} |u| u_r
\]

(7-11a)

\[
\frac{1}{n} \left[ \frac{\partial u_z}{\partial t} + \frac{1}{n} \left( u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) \right] = -\frac{\partial p_u}{\rho_g \partial z} + \frac{\mu}{\rho_g} \left( \nabla^2 u_z \right) - \beta g (T - T_0) - \frac{\mu}{\rho_g k_a} u_z - \frac{C_f}{\sqrt{k_a}} |u| u_z
\]

(7-11b)

Using the continuity equation velocities in both radial and vertical directions can be calculated from a stream function $\psi$. Relations between stream function and velocities in a two dimensional domain can be written as:

\[
u_r = \frac{\partial \psi}{\partial z}
\]

(7-12a)

\[
u_z = -\frac{\partial \psi}{\partial r}
\]

(7-12b)

However, for an axisymmetric condition the relation between velocities in radial and vertical directions ($u_r$ and $u_z$) and stream function $\psi$ is different from the one expressed in Equation (7-12a). Continuity equation for an axisymmetric condition can be expressed as:

\[
u_r + \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} = 0
\]

(7-13)

Rearrangement of Equation (7-13) yields:
\[
\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \tag{7-14}
\]

Simplification of Equation (7-14) results in:

\[
\frac{\partial (ru_r)}{\partial r} + \frac{\partial (ru_z)}{\partial z} = 0 \tag{7-15}
\]

Using Equation (7-15) the radial and vertical components of velocity \( u \) in an axisymmetric condition can be calculated as:

\[
u_r = \frac{1}{r} \frac{\partial \psi}{\partial z} \tag{7-16a}
\]

\[
u_z = -\frac{1}{r} \frac{\partial \psi}{\partial r} \tag{7-16b}
\]

Taking the derivatives of Equations (7-11a) and (7-11b), respectively, over \( z \) and \( r \) directions, the derivative of Navier Stokes equation can be expressed as:

\[
\frac{1}{n} \left[ \frac{\partial^2 u_r}{\partial z \partial t} + \frac{1}{n} \left( \frac{\partial u_r}{\partial z} \frac{\partial u_r}{\partial r} + u_r \frac{\partial^2 u_r}{\partial z \partial r} + \frac{\partial u_r}{\partial z} \frac{\partial u_r}{\partial z} + u_r \frac{\partial^2 u_r}{\partial z^2} \right) \right] =

- \frac{\partial^2 p_a}{\rho_e \partial z \partial r} + \frac{\mu}{\rho_e n} \left[ \frac{\partial}{\partial z} \nabla^2 u_r \right]

- \frac{\mu}{\rho_e k_a} \frac{\partial u_r}{\partial z} - \frac{C_e}{\sqrt{k_a}} \left\{ \frac{\partial \left[ |u| u_r \right]}{\partial z} \right\} \tag{7-17a}
\]

\[
\frac{1}{n} \left[ \frac{\partial^2 u_z}{\partial r \partial t} + \frac{1}{n} \left( \frac{\partial u_z}{\partial z} \frac{\partial u_z}{\partial r} + u_z \frac{\partial^2 u_z}{\partial z \partial r} + \frac{\partial u_z}{\partial z} \frac{\partial u_z}{\partial z} + u_z \frac{\partial^2 u_z}{\partial z^2} \right) \right] =

- \frac{\partial^2 p_a}{\rho_e \partial z \partial r} + \frac{\mu}{\rho_e n} \left[ \frac{\partial}{\partial r} \nabla^2 u_z \right]

- \frac{\mu}{\rho_e k_a} \frac{\partial u_z}{\partial z} - \frac{C_e}{\sqrt{k_a}} \left\{ \frac{\partial \left[ |u| u_z \right]}{\partial r} \right\} \tag{7-17b}
\]

Subtracting Equation (7-17a) from Equation (7-17b), the pressure term can be eliminated.
\[
\frac{1}{n} \left[ \frac{\partial \omega}{\partial t} + \frac{1}{n} \left( \frac{\partial (\omega u_r)}{\partial r} + \frac{\partial (\omega u_z)}{\partial z} \right) \right] = \frac{\nu}{n} \left[ \nabla^2 \omega \right] - \frac{\nu}{k_s} \omega - \frac{C_f}{\sqrt{k_s}} \left[ \frac{\partial \left( |\mathbf{u}| u_z \right)}{\partial r} - \frac{\partial \left( |\mathbf{u}| u_r \right)}{\partial z} \right] + g \beta \frac{\partial T}{\partial r}
\]

(7-18)

where \( \nu \) is the kinematic viscosity and \( \omega \) is the vorticity function defined as:

\[
\omega = \frac{\partial u_r}{\partial r} - \frac{\partial u_z}{\partial z}
\]

(7-19)

For the axisymmetric condition Equation (7-18) should be expressed as:

\[
\frac{1}{n} \left[ \frac{\partial \omega}{\partial t} + \frac{1}{n} \left( \frac{\partial (\omega u_r)}{\partial r} \right) \right] = \frac{\nu}{n} \left[ \nabla^2 \omega \right] - \frac{\nu}{k_s} \omega - \frac{C_f}{\sqrt{k_s}} \left[ \frac{\partial \left( |\mathbf{u}| u_z \right)}{\partial r} - \frac{\partial \left( |\mathbf{u}| u_r \right)}{\partial z} \right] + g \beta \frac{\partial T}{\partial r} - \frac{\nu \omega}{n r^2 - 2 \frac{\partial^2 u_r}{\partial z \partial r} - \frac{\nu \partial \omega}{n r \partial r}}
\]

(7-20)

Considering a reference length \( H \) and undisturbed ground temperature \( T_{g0} \), Equation (7-18) can be expressed using non dimensional terms.

\[
\frac{1}{n} \left[ \frac{\partial \Omega}{\partial \tau} + \frac{1}{n} \left( \frac{\partial (U_r \Omega)}{\partial r'} + \frac{\partial (U_z \Omega)}{\partial z'} \right) \right] = \frac{\nu}{n} \left[ \nabla^2 \Omega \right] - (\gamma + \Gamma |\mathbf{U}|) \Omega

-\Gamma \left\{ U_r \frac{\partial \left( |\mathbf{U}| \right)}{\partial r'} - U_z \frac{\partial \left( |\mathbf{U}| \right)}{\partial z'} \right\} + \frac{Ra \partial \theta_g}{Pr \partial R}
\]

(7-21)

where \( \Omega \) is non-dimensional vorticity, \( U_r \) and \( U_z \) are the dimensionless velocity components, and \( \theta_g \) is the normalized temperature increments. All the parameters used in Equation (7-21) are defined as:

\[
r' = \frac{r}{H}
\]

(7-22a)

\[
z' = \frac{z}{H}
\]

(7-22b)
\[ \tau = \frac{\omega H}{\nu^2} \]  
(7-22c)

\[ U_r = \frac{H}{\nu} u_r \]  
(7-22d)

\[ U_z = \frac{H}{\nu} u_z \]  
(7-22c)

\[ \Psi = \frac{\psi}{\nu} \]  
(7-22f)

\[ \Omega = \frac{\omega H^2}{\nu} \]  
(7-22g)

\[ \theta = \frac{T - T_0}{T_{\text{max}} - T_0} \]  
(7-22h)

Also other than the aforementioned dimensionless parameters in Equation (7-21), Ra is Rayleigh number, Pr is Prandtl number, \( \gamma \) is dimensionless absolute permeability, and \( \Gamma \) is dimensionless Forchheimer drag term. All of these dimensionless input parameters can be calculated as:

\[ Ra = \frac{g \beta (T_{\text{max}} - T_0)}{\alpha \nu} \]  
(7-23a)

\[ \gamma = \frac{H^2}{k_a} \]  
(7-23b)

\[ \Gamma = \frac{C \nu H}{\sqrt{k_a}} \]  
(7-23c)

The stream function \( \psi \) and vorticity \( \omega \) are related through the Laplacian equation.

\[ \nabla^2 \psi = -\omega \]  
(7-24)
Substituting the dimensionless form of vorticity and stream function Equation (7-24) can be rewritten as:

\[ \nabla^2 \psi = -\Omega \]  

(7-25)

### 7.3 Solution Algorithm

In order to solve for the heat energy balance and momentum equations for an incompressible fluid, Equations (7-3), (7-21), and (7-25) should be solved simultaneously. A hybrid explicit-implicit approach is employed in the finite difference solution scheme to calculate the temperature increments within the porous medium and pore fluid velocities. Temperature increments are calculated explicitly using the velocities at the previous time step, and then the Navier-Stokes [Equation (7-21)] and the Laplacian [Equation (7-25)] equations are implicitly solved using the updated temperature gradient at the present time step. To solve the momentum equation, firstly an initial velocity is considered to calculate the vorticity for the next time step and then using the Laplacian equation, the stream function can be obtained using the Equation (7-25). Finally, the velocity vector will be updated using the calculated stream function.

For numerical solution of the Laplacian equation and implicit solution of the momentum equation alternate direct implicit (ADI) scheme together with successive over relaxation method is adopted in this study. To use the ADI scheme, Equation (7-21) can be split into two segmental time steps; derivative over \( r \) in \( t+1/2 \) time step and derivative over \( z \) in \( t+1 \) time step.

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\[
\frac{\Omega^{t+1/2} - \Omega^{t}}{\Delta \tau/2} = c_{1r} \Omega^{t+1/2}_r + c_{2r} \Omega^{t+1/2} + c_{1z} U_z^{t+1/2} \\
\frac{\Omega^{t+1} - \Omega^{t+1/2}}{\Delta \tau/2} = c_{1r}^{'} \Omega^{t+1/2}_r + c_{2z} \Omega^{t+1} + c_{1z}^{'} U_z^{t+1} 
\]

(7-26a)

(7-26b)
where the $c_{1r}, c_{2r}, c_{1z}, c'_{1r}, c'_{2z}$, and $c'_{1z}$ are the coefficients calculated based on the input parameters and velocity values.

After obtaining the dimensionless vorticity $\Omega$, the dimensionless stream function $\Psi$ can be calculated using the Equation (7-27).

\[
A[1:m,1:n] \times \Psi[1:n] = -\Omega[1:n]
\]

(7-27)

where $A$ is a coefficient matrix (of size $n \times n$) which depends on the coefficient of $\Psi_{i,j}$ and its neighborhood grids. The matrix $A$ can be expressed as:

\[
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 & 0 & \cdots & 0 \\
a_{21} & a_{22} & a_{23} & 0 & 0 & \cdots & 0 \\
0 & a_{32} & a_{33} & a_{34} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \cdots & 0 \\
0 & 0 & \cdots & a_{ij(i-1)} & a_{ij} & a_{ij(i+1)} & 0 \\
0 & 0 & \cdots & 0 & a_{m-1(n-2)} & a_{m-1(n-1)} & a_{m(n)} \\
0 & 0 & \cdots & 0 & a_{m(n-2)} & a_{m(n-1)} & a_{mn}
\end{bmatrix}
\]

(7-28)

Note that the determinant of $A$ is zero before applying the boundary conditions. The eigenvalues of the matrix are zero before applying boundary conditions. Note, this can be explained as the stiffness matrix is zero for any structural element before applying boundary conditions. The matrix is also banded and thus for reducing the computational cost only non-zero elements are saved in computer memory. Equation (7-29) is obtained by applying boundary conditions in Equation (7-27)
Successive over relaxation method with residual tolerance of $10^{-5}$ is used to solve the matrix equation in Equation (7-29). Using the over successive relaxation method each of the $\Psi_i$ term can be calculated as follows:

$$\Psi_i^{(k+1)} = \Psi_i^{(k)} - w_r \left[ \frac{\sum_{j=1}^{i-1} A_{ij} \Psi_j^{(i+1)} + \sum_{j=i}^{n} A_{ij} \Psi_j^{k} - B_i}{A_{ii}} \right]$$

(7-30)

where $w_r$ is the over relaxation parameter ($1 < w_r < 2$) and can be calculated as:

$$w_r = \frac{4}{2 + \sqrt{4 + c^2}}$$

(7-31)

The term $c$ in Equation (7-31) depends on the matrix dimension (the number of grid points) and can be expressed as:

$$c = \cos \left( \frac{\pi}{m} \right) + \cos \left( \frac{\pi}{n} \right)$$

(7-32)

Time step required to satisfy stability criteria depends on medium thermal diffusivity, grid spacing and velocity (Roache 1998).
\[
\Delta \tau \leq \min \left\{ \frac{\Delta r^2}{4D_c}, \frac{\Delta z^2}{4D_c}, \frac{1}{2D_c \left( \frac{2}{\Delta r^2} + \frac{|U_{i,j} + U_z|}{\Delta r} \right)} \right\}
\]

where \( D_c \) is the diffusion coefficient of the transport equation. Following the Equations (7-1) and (7-21), \( Pr \) can be considered as the coefficient of diffusion in the transport equation.

### 7.4 Finite Difference Formulations

The finite difference form of the (7-26) using the ADI method can be expressed as:

\[
\frac{1}{n} \frac{\Omega_{i,j}^{t+1/2} - \Omega_{i,j}^t}{\Delta \tau/2} + \frac{1}{n^2} \left( \frac{\partial(U_r \Omega)}{\partial r} + \frac{\partial(U_z \Omega)}{\partial z} \right) =
\]

\[
\frac{1}{n} \frac{\Delta r_{r+1} \times \Omega_{r+1,j}^{t+1/2} - (\Delta r_i + \Delta r_{r+1}) \Omega_{r,j+1}^{t+1/2} + \Delta r_i \Omega_{i,j+1}^{t+1/2}}{2} - \frac{1}{2} \left( \Delta r_i^2 \Delta r_{r+1} + \Delta r_{r+1}^2 \Delta r_i \right)
\]

\[
+ \frac{1}{n} \frac{\Delta z_{j+1} \times \Omega_{i,j+1}^{t+1/2} - (\Delta z_j + \Delta z_{j+1}) \times \Omega_{i,j}^{t+1/2} + \Delta z_j \times \Omega_{i,j+1}^{t+1/2}}{2} - \frac{1}{2} \left( \Delta z_j^2 \Delta z_{j+1} + \Delta z_{j+1}^2 \Delta z_j \right)
\]

\[
+ \left( \gamma + \Gamma \right) \frac{|U|^{t+1/2}}{\Omega_{r,j}^{t+1/2}} \Omega_{r,j}^{t+1/2}
\]

\[
- \Gamma \left( U_r \frac{|U|_{i+1,j}^{t+1/2} - |U|_{i,j}^{t+1/2}}{\Delta r_i + \Delta r_{i+1}} - U_z \frac{|U|_{i+1,j}^{t+1/2} - |U|_{i,j}^{t+1/2}}{\Delta z_j + \Delta z_{j+1}} \right) + \frac{Ra}{\Pr} \frac{\theta_{i+1,j}^{t+1/2} - \theta_{i,j}^{t+1/2}}{\Delta r_i + \Delta r_{i+1}}
\]
The convective terms for negative flows (in a direction opposite to the axis direction defined earlier in Chapters 3 and 4) can be approximated using Equations

(7-36a), and
(7-36b). For the negative flows, the upwind form of the finite difference scheme uses the east side points (E, and EE) for derivative over r, and two south points (S, SS) for definition of upwind difference scheme.

\[
\frac{\partial (U, \Omega)}{\partial r} = -\frac{\left( (\Delta r_{i+1} + \Delta r_{i+2})^2 - \Delta r_{i+1}^2 \right)}{(\Delta r_{i+1} + \Delta r_{i+2}) \Delta r_{i+1} \Delta r_{i+2}} \left[ U_{r_{i+1},j} \Omega_{i+1,j} + \left( (\Delta r_{i+1} + \Delta r_{i+2})^2 \right) U_{r_{i+1+2},j} \Omega_{r_{i+1+2},j} - \Delta r_{i+1}^2 U_{r_{i+1},j} \Omega_{r_{i+1},j} \right]
\]

(7-36a)

\[
\frac{\partial (U, \Omega)}{\partial z} = -\frac{\left( (\Delta z_{j+1} + \Delta z_{j+2})^2 - \Delta z_{j+1}^2 \right)}{(\Delta z_{j+1} + \Delta z_{j+2}) \Delta z_{j+1} \Delta z_{j+2}} \left[ U_{z_{i,j+1}} \Omega_{z_{i,j+1}} + \left( (\Delta z_{j+1} + \Delta z_{j+2})^2 \right) U_{z_{i,j+1+2}} \Omega_{z_{i,j+1+2}} - \Delta z_{j+1}^2 U_{z_{i,j+1}} \Omega_{z_{i,j+1}} \right]
\]

(7-36b)

The central difference scheme is also used to approximate the Equation (7-25).

\[
\begin{align*}
\Delta r_{i+1} & \times \Psi^{t+1}_{i+1,j} - (\Delta r_{i} + \Delta r_{i+1}) \Psi^{t+1}_{i,j} + \Delta r \Psi^{t+1}_{i+1,j} + \\
& \quad \frac{1}{2} (\Delta r^2 \Delta r_{i+1} + \Delta r^2 \Delta r_{i+2}) \\
\Delta z_{j+1} & \times \Psi^{t+1}_{i,j+1} - (\Delta z_{j} + \Delta z_{j+1}) \times \Psi^{t+1}_{i,j} + \Delta z \times \Psi^{t+1}_{i,j+1} + \\
& \quad \frac{1}{2} (\Delta z^2 \Delta z_{j+1} + \Delta z^2 \Delta z_{j}) = -\Omega^{t+1}_{i,j}
\end{align*}
\]

(7-37)

After obtaining the dimensionless stream function from Equation (7-37), the dimensionless velocities are updated using the following equations:

\[
U_{r_{i,j}} = \frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{(\Delta z_i + \Delta z_{i-1})}
\]

(7-38a)

\[
U_{z_{i,j}} = \frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{(\Delta r_i + \Delta r_{i-1})}
\]

(7-38b)

The heat energy equation is solved explicitly and the details of the explicit method to solve the conductive heat transport for different cases were discussed in detail in Chapters 3 and 4.
7.5 Solution of a Cavity Flow Problem

The solution algorithm described in the previous section and the refined FD code is validated through simulation of a cavity flow problem reported in literature (Hossain et al. 2013). Hossain et al. (2013) simulated heat conduction and induced pore fluid flow (considering the Forchheimer drag term) within a rectangular domain with constant temperature boundary. Figure 7-1 shows the problem geometry and boundary conditions, as described in Hossain et al. (2013).

Figure 7-1 Geometry and boundary conditions used in Hossain et al. (2013)
7.5.1 Verification of the Developed FD Model

Results obtained using the refined FD model, which can account for convective heat transport due to thermally induced pore fluid flow, are compared with two different cases using the drag term $\Gamma = 0$ and 25. The other input parameters adopted from Hossain et al. (2013) are: $\gamma = 1000$, $T_h = 1.5T_0$, $Pr = 0.7$, and $Ra = 2 \times 10^5$. Normalized temperature $\theta_g = \frac{T - T_{g0}}{T_{\max} - T_{g0}}$ obtained from the developed code for the first case in which the Frochheimer drag term $\Gamma$ is set equal to zero, is compared with the contour plots presented by Hossain et al. (2013). Figure 7-2 shows that the developed FD code can accurately predict the normalized temperature variation due to both heat conduction and convection through thermally induced pore fluid flow.
Figure 7-2 Comparison between isotherms for normalized temperature $\theta_g (\Gamma = 0)$: (a) obtained from the developed FD model and (b) result presented in Hossain et al. (2013)
The isotherms obtained from both conduction and convection through thermally induced pore fluid flow is also compared with results obtained using a common practice that considers only conductive form of heat transport even in saturated soil. Comparison of Figure 7-2 with Figure 7-3 reveals that pore fluid flow induced by temperature gradient may play an important role in quantification of long term temperature response of a two-phase porous media like saturated soil. Note all the figures for stream function and normalized temperature are obtained at the steady state condition \((t = 10\ \text{days})\).

![Figure 7-3 Isotherms for normalized temperature \(\theta_g\) obtained (using the developed FD code) without considering thermally induced pore fluid flow](image)

Dimensionless stream function \(\Psi\) contours obtained from the developed FD code is also compared with those presented by Hossain et al. (2013). Figure 7-4 demonstrates that two vortices form inside the analysis domain; one being stronger and with opposite direction compared to the other. Formation of such vortices results in variation of isotherms along the
direction of pore fluid flow. Stream function obtained from the FD model is in complete agreement with the results presented by Hossain et al. (2013). Note that the direction of vertical velocity component $u_z$ considered in this study is opposite to the sign convention (see Figure 7-1) followed in Hossain et al. (2013) and thus, the sign of stream functions is reversed in Figure 7-4(a) when compared with that in Figure 7-4(b).
Figure 7-4 (b)

Figure 7-4 Comparison between dimensionless stream function $\Psi$ contours (for $\Gamma=0$) (a) obtained from the developed FD model and (b) result presented in Hossain et al. (2013)

The normalized temperature $\theta_g$ and the dimensionless stream function $\Psi$ obtained using the developed FD model for the case when Forchheimer drag term $\Gamma$ is set to be equal to 25 are further compared with corresponding results presented in Hossain et al. (2013). Figure 7-5 and Figure 7-6 confirm that the developed FD model can accurately consider the Forchheimer drag term and both $\theta_g$ and $\Psi$ are in close agreement with results presented in Hossain et al. (2013).
Figure 7-5 Comparison between isotherms for normalized temperature $\theta_g$ ($\Gamma = 25$): (a) obtained from the developed FD model and (b) result presented in Hossain et al. (2013)
Figure 7-6 Comparison between dimensionless stream function Ψ contours (for Γ=0) (a) obtained from the developed FD model and (b) result presented in Hossain et al. (2013)
7.6 Simulation of Laboratory Thermal Performance Tests on Model Geothermal Pile Installed in Saturated Sand Bed

The thermal performance tests conducted on the model geothermal pile installed in saturated sand (described in Chapter 6) are numerically simulated using the refined (third generation) FD code that can capture the effect of thermally induced pore fluid flow on heat transport in saturated ground. The dimensionless input parameters obtained from the input parameters described in Chapter 6 are: $\gamma = 3.27 \times 10^9$, $Pr = 1.43$, $Ra = 5.59 \times 10^9$, $I' = 31441.4$, and $\Delta \tau = 1.003 \times 10^{-9}$. The thermal boundary conditions considered in this analysis are same as thermal boundary conditions discussed in Chapter 6. For solving Navier Stokes and continuity equations, no slip boundary ($u_r = u_z = 0$) are considered at all four sides of the tank as well as at the model pile surface and at the bottom of the model pile. The additional computational capabilities incorporated into the FD code described in Chapter 4 are described in sections 7.2, 7.3 and 7.4. Such advanced features significantly increases computational time as finer grid spacing is required (as compared to that used for simulations presented in Chapters 4, 5 and 6) to obtain accurate solution from the convective part of the problem. To reduce the computational cost without sacrificing the solution accuracy, two different grid spacings are used in the third generation FD model – (i) a coarse grid (which has previously been validated in Chapters 4, 5 and 6 to produce accurate solution) is used for solution of advection and convection within circulation fluid and heat energy balance equations in PVC tube, concrete and soil surrounding the geothermal pile and (ii) a finer grid with reduced vertical spacing is used to solve for Navier Stokes and flow continuity equations for thermally induced pore fluid (incompressible) flow (Figure 7-7). Such computational maneuverability is one of the major advantages of developing an independent code specific to the problem, as opposed to resorting to one of the commercially
available software. Since the heat transport surrounding a geothermal pile is mostly a radial phenomenon (Laloui et al. 2006, Kramer et al. 2014, Ozudogru et al. 2014) there is no need to refine the previously used grid spacing in the vertical direction (i.e., along z axis). However, thermally induced pore fluid flow is not just a radial phenomenon and both radial and vertical velocity components play important role in forming the thermally-induced vortices.

Figure 7-7 Numerical techniques to reduce computational cost (a) u-p formulation, (b) method used in this study

Power output calculated from results of numerical simulations of test TPHS3, with and without considering thermally induced pore fluid flow, is plotted in Figure 7-8. It is evident that the consideration of thermally induced pore fluid flow consistently predicts higher power output from pile-soil heat exchange. Note that the seemingly small difference between predictions with and without considering pore fluid flow may be attributed to laboratory-scale geometry and real time-scale of the simulations; for real piles this difference might be significant to affect design decisions. Figure 7-9 confirms the presence of thermally induced pore fluid flow (fluid vortex) within a zone surrounding the model geothermal pile; formation of such vortices eventually helps in increasing thermal efficiency of geothermal piles installed in saturated ground.
Figure 7-8 Effect of thermally induced pore fluid flow on predicted heat exchange efficiency (power output) of the model geothermal heat exchanger pile during test TPSH3

Figure 7-9 (a)
Ground temperature increments recorded during test TPSH3 is also compared with numerical predictions using the second and third generation FD models. Figure 7-10(a) shows that soil temperature increment $\Delta T_s$ is insensitive to the consideration of thermally induced pore fluid flow at a time shortly after ($t = \text{half a day}$) the start of the thermal operation of the model pile. This is because pore fluid velocity remains pretty small in the initial part of thermal operation and the convection through pore fluid flow barely changes soil temperature response. However, the effect of pore fluid flow on $\Delta T_s$ is evident at time $t = 4$ days (Figure 7-10b). Therefore, the related hypothesis formulated in Chapter 6 is tested to be positive.
Figure 7-10 Comparison of recorded temperature increment $\Delta T_s$ in saturated soil with FD predictions with and without considering thermally induced pore fluid flow (a) at $t = 12$ hours and (b) $t = 4$ days.
To investigate the effect of soil permeability on thermally induced pore fluid flow, two different dimensionless permeability values are used. As Figure 7-11 shows, increasing the soil permeability results in lower ground temperature increments near the geothermal pile. This is because there will be an enhanced pore fluid flow in a more permeable porous media.

Figure 7-11 Effect of soil permeability on temperature increment $\Delta T_s$ in saturated soil

### 7.7 Pore Pressure Measurements from Physical Model Test

The formation of temperature gradient induced pore fluid vortex within a zone surrounding the model geothermal pile installed in saturated sand bed is to be further explored using measurements from vibrating wire (VW) pressure transducers (Geokon model # 4580) embedded at eight different locations within saturated sand bed. The VW transducers are placed on the inlet side (hot side) of the geothermal model pile (Figure 7-12). These pressure transducers are connected to a 16 channel VW data logger (Geokon model LC-2x16). The data logger can
measure and record change in pressures at every 30 seconds. Calibration tests were performed on all eight VW piezometeres to check their accuracy and their dependency on temperature and pressure variations (Figure 7-13). Two calibration methods were used – linear and polynomial calibrations. Results from these calibration checks were compared with each other and verified for accuracy before embedding the VW within the sand bed. It is anticipated that meaningful data can be accrued during the series of tests that are underway. Variation of pore water pressure during the thermal test ($T_{\text{inlet}}=60^\circ \text{C}$) are shown at three different locations in Figure 7-14. This figure clearly shows that increasing the temperature of the ground surrounding the geothermal pile results in pore water pressure increment. However, the excess pore water pressure decreases immediately after stopping the hot fluid circulation within the circulation tube. As expected, thermally induced pore water pressure is maximum near the base of the geothermal pile ($z = 81.3$ cm and $r = 10.2$ cm) and the thermally induced excess pore water pressure decreases with increasing the radial distances. It is interesting to note that after the complete heating test (heating followed by cooling down to initial temperature) the excess pore water pressures are negative at all the locations. This is because that heating test results in moisture content reduction. More tests are underway to quantify the exact behavior of thermally induced pore fluid flow surrounding the geothermal pile.
Figure 7-12 Placement of VW pressure transducers inside the test bed; arrangement with respect to the model pile and side view exactly after the first half of the tank was filled

Figure 7-13 Calibration tests on VW pressure transducers
Figure 7-14 Variation of excess pore water pressure during the thermal test

7.8 Chapter Summary

The potential effects of thermally induced pore fluid flow on ground temperature increment and pile-soil heat exchange efficiency are investigated through finite difference analysis of heat transfer from a geothermal pile to the surrounding ground. Results show that heat injection from a geothermal pile to saturated ground creates a fluid vortex that further facilitates convective heat transport inside the saturated medium. Convection of pore water decreases ground temperature increment near the pile for most part of the pile length, except for a short length near the pile head. Thus thermally induced pore water flow within a saturated medium facilitates pile-soil heat exchange due to convective heat transfer in the medium. The positive effect of thermally induced pore water flow on heat exchange efficiency of geothermal piles is expected to increase with
increase in the values of volumetric coefficient of thermal expansion of the pore fluid and soil hydraulic conductivity.

7.9 Cited References


Chapter 8: Summary and Conclusions

8.1 Research Summary

Pile-soil heat exchange is analyzed through sequential development of three different numerical models that captures important features of heat flow within geothermal piles and in the soil surrounding it. Finite difference technique is used for simultaneous solution of partial differential equations that govern the physics of the problem. The developed models, with various levels of approximations and accuracy, can simulate thermal behavior of geothermal piles under under any combinations of climatic, geologic and project-specific operational conditions and can quantify energy efficiency of a pile-anchored geothermal energy harvesting system component (i.e., a single pile-soil system) for both short- and long-term operations. The developed models have been verified, under idealized conditions, using analytical solutions available in literature. Model predictions have also been compared extensively with results recorded during both field and large-scale laboratory tests performed under controlled conditions. Results from a series of numerical simulations are used to formulate a closed-form solution that can be readily used at the design stage to quickly predict power output from a geothermal pile with an embedded U-shaped circulation tube. A sensitivity analysis is performed to determine the hierarchy of different input variables based on their relative impacts on heat exchange efficiency of a geothermal pile.

8.2 Challenges and Significant Contributions

Major challenges involved in this research was in (i) modeling complex heat and fluid flow within circulation tubes, geothermal pile and surrounding ground with practical approximations
to reduce computational time without sacrificing accuracy of the solution (ii) eliminating the need for any further calibration of the developed model in order to predict thermal response of a pile-soil system, and (iii) establishing the models on a general framework so that the developed numerical model can predict response recorded from a wide variety of field and laboratory tests on geothermal piles and that for future thermal design of these piles.

The outcome of this research may facilitate modification of existing empirical and analytical rules commonly used for thermal design of geothermal heat exchanger piles (and boreholes). Significant contributions of this research are:

1. Potential effect of thermally induced pore fluid flow in saturated ground on thermal performance of geothermal piles (and boreholes) has been investigated through formation and systemic evaluation of a research hypothesis through numerical simulations of heat and thermally induced fluid flow in saturated ground during thermal operation of geothermal piles; comparison of model predictions with physical model test results further supports the hypothesis.

2. Results from numerical analyses are used to develop regression-based closed form expressions that can be used for a fast, and yet reasonably accurate, prediction of power output that can be expected from a geothermal pile. In this way, outcome of this research enables informed decision making during the design stage of geothermal piles. While extreme rigor has been the primary focus of this research, the practical application aspects have not been ignored.

3. Hierarchy of several input parameters in affecting heat exchange performance of a single geothermal pile has been established based on parameter sensitivity studies.
(4) Expected variance in heat exchange efficiency of geothermal piles due to individual 
and combined statistical variation of different design, operational and field 
parameters is established through Advanced First-order Second-moment (AFOSM) 
reliability analysis.

### 8.3 Specific Conclusions

Specific conclusions drawn from this research are:

1. Soil thermal conductivity, difference between fluid temperature at the inlet point and 
the initial ground temperature, and circulation tube radius are the three most 
important parameters in hierarchical order of their effects on power output efficiency 
of a geothermal pile.

2. Temperature loss within the circulation tube increases as the flow characteristics 
changes from turbulent to laminar; consequently, turbulent flow should be considered 
to maximize power output from pile-soil heat exchange system.

3. The effects of circulation velocity and circulation tube radius on heat exchange 
efficiency should be considered separately. Consideration of just the rate of 
circulation (i.e., flow rate), as often has been reported in literature, does not yield 
optimal design. Since an increase in circulation velocity increases geothermal power 
output, the optimum circulation flow rate depends on the electrical (input) power 
needed to create such flow.

4. Temperature gradient induced by heat injection or extraction using geothermal piles 
to or from the ground results in pore fluid flow in saturated ground. The induced fluid 
vortex in a zone surrounding the pile further facilitates heat transport in saturated soil 
and consequently, increases heat exchange efficiency of pile-soil system.
8.4 Recommendations for Future Research

Research in different related areas may promote and generate further knowledge on geothermal piles under thermal and mechanical loading and similar systems to harvest shallow geothermal energy. Some of these potential areas are listed herein:

- Thermo-hydro-mechanical (THM) modeling of axial pile-soil interaction for single geothermal pile using advanced soil constitutive models to know potential variations of shaft and base stiffness and resistance during thermo-mechanical loading.
- Investigate the effect of moisture migration in partially saturated zones surrounding geothermal piles.
- Study the climate region effect on thermal performance of geothermal piles and investigate the feasibility of using geothermal piles installed in permafrost regions like Alaska.
- Study the possibility of using similar systems (circulating fluid through PVC tube) in offshore structures or bridges foundations built over the oceans or sea. The flow of water can easily transport the heat energy. Heat transfer mechanism and power output should be investigated for different climate conditions.
- Simulation of thermal operation of geothermal piles while acting as part of a pile group. Such a study will facilitate optimizing the locations and distances between geothermal piles. Further investigation may also explore the effectiveness of embedding horizontal loops within the pile cap.
- Life cycle cost analysis to quantify the sustainability aspect of this promising alternative energy harvesting technology.
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RELATED PUBLICATIONS


J2. Ghasemi-Fare, O. and Basu, P. (2015). “Thermal performance of geothermal piles in different ground conditions – observations from laboratory, field and numerical tests”; will be submitted to Journal of Geotechnical and Geoenvironmental Engineering, ASCE.


G2. Ghasemi-Fare, O., Basu, P., (2015). “A finite difference model to predict thermal response of ground surrounding geothermal piles”, accepted in Geocongress, ASCE.