ESSAY ON NETWORK EFFECTS, CONSUMER DEMAND, AND FIRMS’ DYNAMIC PRICING

A Dissertation in Economics by Rong Luo

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Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

August 2015
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Abstract

This dissertation includes three chapters on estimating structural economics models. My research focuses on empirically study consumers’ utility from different products, the impact of network effects on consumers’ demand for products, and multi-network firms’ dynamic pricing strategies. The three chapters share the same feature of estimating a discrete choice demand model, but differ in the static versus dynamic setting and the underlying economics question and strategic behaviors that I’m interested.

Chapter 1

The Operating System Network Effect and Telecom Carriers’ Dynamic Pricing of Smartphones.

The utility a consumer realizes from owning a smartphone increases with its operating system (OS) network size. Due to this OS network effect, multi-network telecom carriers have a different pricing strategy for smartphones than the single-network manufacturers in a dynamic environment. While manufacturers choose higher prices for larger networks, carriers, who can internalize competition across OSs, have incentives to choose lower prices for larger networks. The carriers’ pricing strategy contributes to the increasing smartphone users and OS concentration. In this paper, I first analyze a theoretical model to compare the pricing strategies of the carriers and manufacturers. Then I design a structural model of consumers’ demand and the carriers’ dynamic pricing game for smartphones, and empirically study the impact of the OS network effect and carriers’ two-year contract policy on the smartphone market penetration and OS concentration. I estimate the model using product level data from August 2011 to July 2013 in the US. I deal with the empirical challenges of dynamic prices for multi-product carriers, high dimension continuous state variables, and asymmetric oligopolistic firms in the estimation. The results show that the OS network size has a positive and significant impact on consumer utility. I then study two counterfactual cases in which I eliminate the OS
network effect and the carriers’ pricing strategy, respectively. I find that, without the OS network effect, the smartphone penetration rate would decrease by 54.7% and the largest OS share difference decrease by 31.7% by May 2013. Without the carriers’ pricing strategy, the penetration rate would decrease by 29.1% and the OS market share difference decrease by 11.2%

Chapter 2

The Operating System Network Effect and Consumers’ Dynamic Demand of Smartphones with Two-Year Contracts.

This paper studies consumers’ dynamic demand of smartphones on two-year wireless contracts. Individuals’ demand decisions are affected by the improving quality and changing prices of smartphones, and the OS network effect, and their current smartphone contract status. Consumers need to pay high early termination fees if they end active contracts. The dynamic demand model in this paper incorporates the evolving choice set, prices, endogenous OS network sizes, and the termination policies in the smartphone industry. The preliminary results find that the OS network effect is large and significant. In addition, compared with dynamic model results, a static demand model tends to underestimate the OS network effect and overestimate price coefficient.

Chapter 3

Store Brands and Retail Grocery Competition in Breakfast Cereals.

This paper empirically analyzes the impacts of store brands on grocery retailers and consumers in the market for breakfast cereals. On the supply side, store brands help a retailer to avoid direct competition with other retailers and change the set of retailer’s products. On the demand side, introducing store brands changes the national brands prices and consumers’ choice set. We analyze the effects via demand estimation for a single grocery store chain Dominick’s at Chicago in 1997 and counterfactual exercises. The estimation results show that consumers’ unobserved utility of buying at a competing retailer is higher for consumers that value national brands, and is lower for ones that value Dominick’s store brands. This is consistent with the claim that store brands help a retailer to avoid competition. The counterfactual calculations show that the profit loss from removing store
brands is higher if the retailer has more competitors, with a median loss of 4.33% of profits from cereals. Existence of store brands increases national brands prices and consumer welfare increases slightly when store brands are removed.
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Acknowledgments

I am deeply indebted to my adviser Mark Roberts, who always expected the best research out of me, selflessly shared his knowledge, spent countless afternoons on meetings with me, patiently guided me to the best solution on many problems, and offered many financial support. I was extremely lucky to have him as an adviser. He is the role model for my academic career.

I would also like to express my appreciation to my other adviser Paul Grieco. It is his thorough comments and high standards on what are not reasonable or acceptable that pushed me to explore my better potential and made my dissertation more reasonable and more acceptable. His sharp economics mind makes my every single meeting with him a learning experience.

My research has greatly benefited from discussions with Peter Newberry, Joris Pinkse, Charles Murry, Martin Hackmann, Kala Krishna, Ed Green, Marc Henry, and other Penn State faculty.

The journey of graduate school at Penn State wouldn’t be possible or such a pleasant time without the help from Professor John Riew, who brought me to the program, believed in me, and cared about me.

My empirical research became less of a burden thanks to the help from my friend Huihui Li. I thank all my bright and supportive classmates. I especially thank the best staff Krista Winkelblech, whose help and work was very important.

Lastly, I am grateful to my boyfriend Yue, for the discussions that improved my understanding of economics, for all the encouragement I needed, and for being a wonderful companion.
Chapter 1  
The Operating System Network Effect and Carriers’ Dynamic Pricing of Smartphones.

1.1 Introduction

In markets with network effects, such as computers and smartphones, consumers value the size of the installed consumer base because it can lead to higher utility. Due to the network effect, the current price of a product affects its network size and thus future demand. This leads the suppliers of the product to make dynamic pricing decisions, no matter whether the products are sold by single-network manufacturers or by multi-network retailers. However, in this paper, I show that the multi-network retailers have a quite different pricing strategy than the manufacturers. The retailers choose lower prices for products with initially larger networks while the manufacturers do the opposite.

In this paper, I study the pricing patterns and incentives of multi-network retailers selling smartphones. These products are subject to a network effect that arises through the operating systems for two reasons. First, application stores themselves generate an indirect network effect. Application developers choose to develop more apps for large OSs. In return, more consumers adopt large OSs because of the greater number and better quality of applications. Second, there is a direct OS network effect. Friends and family members prefer adopting the same OS. The benefits of doing so include convenient communication (e.g., FaceTime, iMessage), as well as ease of sharing files and purchases, and lower learning costs.

Smartphones are sold by telecom carriers who act like multi-network retailers. During the 2011-2013 period, each carrier sold products of all OSs. For example, Verizon sold smartphones with iOS, Android, Blackberry, and Windows Phone.

\[\text{\textsuperscript{1}}\text{The only exception is that T-Mobile only started to sell iPhones in April 2013.}\]
The carriers give discounts on smartphone purchases when sold with a two-year wireless contract. Consumers pay the discounted prices instead of the manufacturer retail prices for smartphones. As will be shown in Section 6, the carrier discounts rise in line with past OS shares.

The goal of this paper is to study, the impact of the OS network effect on the multi-network retailers’ dynamic prices. To build intuition, I solve a two-period, two-OS theoretical model to compare the price for the large OS with that for the small OS in two different settings. In the first setting, a multi-network carrier sells products with both a large and small OS. In the second setting, single-network manufacturers sell their own products. I find that the carrier chooses a lower price for the large OS than the small OS in the first period, while the manufacturers do the opposite.

The intuition of the pricing strategies of the single-network manufacturers and the multi-network carrier is as follows. In a dynamic environment with network effects, both the manufacturers and the carrier would choose low prices to grow the network to attract future customers. The difference between them lies in their competitive environments. A manufacturer faces competition from other manufacturers and its initial network size affects its market power. In equilibrium, manufacturers with larger initial networks choose higher prices. In contrast, the multi-network carrier sells products of all networks. This implies that the carrier is able to internalize the competition across networks. It prefers a very large network to several small networks, so that future consumers will be more likely to buy the product as opposed to the outside option. Therefore, the carrier chooses relatively lower prices for initially larger networks, which will grow fast and gain customers.

I then I develop a structural model of consumers’ demand and carriers’ dynamic pricing game for smartphones to empirically quantify the OS network effect and measure its impact on the carriers’ two-year contract discounts. I estimate the model using product level data from the period August 2011-October 2013. In the estimation, I deal with several empirical challenges including dynamic prices of multiple products for each carrier, high dimension continuous state variables, and asymmetric oligopolistic firms.

The estimation results show that the OS network effect is positive and significant. This implies that consumers’ utility from a smartphone is affected by the number of its OS users, and the carriers’ prices are affected by the OS network sizes. I study
two counterfactual scenarios to measure the impact of the OS network effect and the carriers’ discounts on the growth of smartphone users and OS concentration. In the first counterfactual case, I eliminate the OS network effect. I find that, without the OS network effect, the smartphone penetration rate would decrease by 54.69% and the largest OS share difference would decrease by 31.66% when the carriers make static pricing decisions. In the second counterfactual case, I eliminate the carrier discounts on two-year contracts. Without the carriers discounts, the smartphone penetration rate would decrease by 29.06% and the largest OS share difference would decrease by 11.18%. The results show that both the OS network effect and carrier discounts are very important to smartphone penetration and OS concentration, but the OS network effect is more important than the carrier discounts.

The paper proceeds as follows. In Section 2, I discuss the related literature and contributions of this paper. Section 3 provides the background to the US smartphone industry. In Section 4, I study a simple two-period, two-OS model to compare the large OS price with that of the small OS in two different settings. Section 5 sets up consumer demand and the carriers’ supply model for smartphones with two-year contracts. The data used in this paper is described in Section 6. Section 7 discusses identification and estimation details. The estimated results are presented in Section 8. Section 9 studies the two counterfactual scenarios. Section 10 concludes the paper.

1.2 Literature Review

The main contribution of this paper is to empirically study the impacts of a network effect on the prices charged by multi-network retailers. Neither the theoretical literature nor the empirical literature has studied this topic. Theoretical research on the network effect has focused on the competition between single-network manufacturers, but not the prices of multi-network retailers. Katz and Shapiro (1985), Farrell and Saloner (1986), Katz and Shapiro (1992), Katz and Shapiro (1994), Shapiro and Varian (1999), Rochet and Tirole (2003), Armstrong (2006), Zhu and Iansiti (2007), Rysman (2009), and Weyl (2010) all study the impact of the network effect on the prices of either monopoly or oligopoly manufacturers. In this paper, I focus on the network effect’s impact on multi-network sellers’ prices,
which differs from the impact on single-network sellers’ prices.

The empirical literature also focuses on markets in which manufacturers choose prices. This includes papers that study the network effect in the video game industry (Zhu and Iansiti (2007), Lee (2013), Dubé, Hitsch, and Chintagunta (2010)), the DVD player industry (Gowrisankaran, Park, and Rysman (2010)), the VCR industry (Park (2004)), the yellow page industry (Rysman (2004)), the ACH banking industry (Ackerberg and Gowrisankaran (2006)), videocalling technology (Ryan and Tucker (2012)), and the PDA industry (Nair, Chintagunta, and Dubé (2004)). In all these markets, manufacturers sell products directly to consumers. In this paper, I focus on the impact of the network effect upon carrier prices across different OSs.

This paper makes a further contribution by estimating a structural model of dynamic pricing with asymmetric multi-network carriers. I deal with several empirical challenges including dynamic prices of multiple products, high-dimension continuous state variables, and asymmetric oligopolistic firms.

There are literatures on estimating discrete choice dynamic games and continuous choice games with single-product firms. However, none of those methods is ideal to be applied to this paper, where the dynamic game features both continuous choices and multi-product firms.

Aguirregabiria and Mira (2007), Pakes, Ostrovsky, and Berry (2007), Kasahara and Shimotsu (2012) have proposed estimation methods for dynamic discrete choice games. Bajari, Benkard, and Levin (2007) propose a two-step method that can estimate dynamic games with continuous choices. Their first step estimates the policy functions and the second step estimates the model’s parameters using simulated minimum distance estimator. Their two-step method requires monotonicity of the policy functions on shocks and linearity of the value functions in the model’s parameters. These two requirements make the method not applicable to this paper, because it is hard to prove monotonicity of policy functions and the value function is nonlinear in this paper. In addition, first step noise could arise without a sufficiently large number of observations and a flexible parametric assumptions on policy functions. The noise could bias the second step estimates.

Several papers have studied the dynamic pricing problems of single-product firms. Liu (2010) and Dubé, Hitsch, and Chintagunta (2010) analyze the dynamic decisions of two oligopolistic video game console manufacturers. They assume parametric

With a small number of single-product firms, the dynamic problem could be solved with value function iteration or policy function iteration. However, when there are many firms and each firm has many products, previous methods cannot be used due to the high dimension choice space (prices of many products).

Instead, I solve the multi-network carriers’ dynamic pricing game within the estimation. In particular, I approximate the carriers’ value functions with basis functions and develop an efficient iterative method to solve the dynamic pricing game. I estimate the model using Generalized Methods of Moments (GMM) with MPEC, which was introduced by Su and Judd (2012). The moment conditions are based on the orthogonality between unobserved shocks and instrumental variables. The carriers’ Bellman equations are used as constraints. The value function approximation method is motivated by the sieve estimation literature. Ai and Chen (2003) propose a minimum distance estimator with sieve approximation and show its efficiency. Barwick and Pathak (2011) also use sieve approximations to solve a dynamic maximization problem.


This paper focuses on the consumer demand for smartphones with contracts and the impact of the OS network effect and the carrier discounts on those contracts. The carriers’ two-year contract policy is an inseparable part of the carriers’ pricing. Consumers receive discounts on the manufacturers’ smartphone prices if they sign
two-year wireless service contracts. Only with a two-year contract to guarantee a revenue flow from wireless service consumption can the carriers offer discounts on smartphone purchases. The impact of the OS network effect upon carrier prices explained above would no longer exist if the carriers don’t make pricing decisions.

1.3 Background of the U.S. Smartphone Industry

The major manufacturers in the U.S. smartphone market are Apple Inc., Research in Motion Limited (produces Blackberry models), HTC Corporation, Motorola, Inc., Samsung Electronics Co. Ltd., LG Corp., and Nokia Corporation. The top four operating systems in the U.S. smartphone industry are Android, iOS, Blackberry, and Windows Phone. The combined market share of the four increased from 94% to 99% during 2011 to 2014.

Every smartphone operating system has an online store where applications can be purchased and downloaded to extend the functionality of the smartphone. The proprietary OSs have application stores that are exclusive to the operating system. Licensable or open source OSs have application stores that work with any device that runs the OS, no matter the manufacturer. The application stores for the four OSs are: Google Play(for Android), App Store (for Apple iOS), BlackBerry App World (for BlackBerry), and Windows Phone Marketplace (for Windows Phone).

There are indirect and direct network effects at the operating system level. The indirect network effect arises through app development. Both the OS developers and third-party firms develop apps for the operating systems. The apps on different operating systems are exclusive. Consumers find value in apps because of the additional functionality of their smartphones. The firms maximize profits by developing apps for the large operating systems due to the large number of users. In this way, the network effect exists at the OS level. There is also a direct OS network effect. Family members and friends prefer adopting the same OS because they can enjoy convenient communication methods (FaceTime, iMessage) and easy ways to

\footnote{The two-year contract policy existed long before the smartphone industry developed. The question of why contracts are two-years long is also interesting, but it is not the focus of this paper. This paper takes the two-year length as given.}

\footnote{Cromar (2010) has a thorough description of the US smartphone industry.}

\footnote{I don’t separately model the carriers’ wireless network effect in this paper, because the carrier-OS fixed effects I add in consumers’ utility function can incorporate the carrier specific effects, including the wireless network network.}
share files; they can share their apps purchases; and there is a lower learning cost to use an OS.

A smartphone device is useful when connected to a wireless service provider’s network that allows the consumer to make calls and access data such as email and the internet. Thus, a consumer has to choose both the smartphone model and the service provider when buying a smartphone. The top four service providers (carriers) in the U.S. are Verizon Wireless, AT&T Mobility, Sprint Corporation, and T-Mobile US. Together, they account for 90% of the total market for smartphone sales. They have varying degrees of network coverage and different pricing plans for wireless service. According to Kantar World Panel data, the average combined share of smartphone sales for the big four carriers (service providers) during Oct. 2011 to Nov. 2013 is 88.72%. The average sales share of Verizon is 33.26%, 28.68% for AT&T, 15.58% for Sprint, and 11.10% for T-Mobile.

The wireless carriers offer discounts on smartphones if consumers purchase with long-term contracts, usually two years. For example, Apple’s retail price of the iPhone 5 was $649 in Oct 2012. Consumers could pay $199 for an iPhone 5 if they sign a two-year wireless service contract with the carriers. If consumers need to terminate the contract within two-years, they have to pay early termination fees. Depending on how many months left in an unfinished contract, the termination fee is between $150 to $350. According to the Statista.com, the average monthly churn rate for the four wireless carriers is 2%.

1.4 A Two-Period, Two-OS Model of Pricing

In this section, I study two two-period, two-OS theoretical models to show that multi-network sellers (carriers) choose lower smartphone prices for large OSs than for small OSs, while the single-network sellers (manufacturers) do the opposite. In the first model, a monopoly seller sells two smartphone models with different operating systems in the two periods. In the second model, two single-network firms choose prices and play a dynamic pricing game. The key difference between the two models is that the multi-network seller can internalize the competition, while single-network sellers can not.

\footnote{Churn rate is defined as the proportion of contractual customers or subscribers who leave a supplier during a given time period. Data source is from this link: \url{http://www.statista.com/statistics/}}
1.4.1 Demand Model Setup

There are two smartphone models, A and B, in a two-period economy, period 1 and 2. The two models have different operating systems, which are also denoted by \{A, B\}. Let \( \delta_j, j \in \{A, B\} \), be the consumer utility from the characteristics of model \( j \). Let the initial market shares of the two OSs be \((n_{At}, n_{Bt})\) in period \( t \).

**Assumption 1.** Without loss of generality, assume that network A has a higher market share than B initially: \( n_{A1} > n_{B1} \) and that the OS network effect exists: \( \gamma > 0 \).\(^6\)

**Assumption 2.** The two models provide the same characteristics: \( \delta_A = \delta_B \). The two models have the same unit cost, \( c = 0 \).\(^7\)

Assumption 1 and 2 will be assumed for the rest of this section. Assumption 2 implies that the only difference between A and B is their network size difference, so that this model focuses on the network size effect. Let the total mass of consumers be 1. At the beginning of period \( t \), only consumers who do not own smartphones enter the market, so the market size in period \( t \) is

\[
M_t = 1 - n_{At} - n_{Bt}.
\]

Consumer \( i \)'s utility of buying smartphone \( j \in \{A, B\} \) in period \( t \) is:

\[
u_{ijt} = \delta_j + \gamma n_{jt} - \alpha p_{jt} + \epsilon_{ijt},\]

in which \( n_{jt} \) measures the network size of OS \( j \) at the beginning of period \( t \), \( p_{jt} \) is the carrier’s smartphone price in period \( t \), \( \epsilon_{ijt} \) is the idiosyncratic utility shock of model \( j \) in period \( t \), and \( \gamma \) and \( \alpha \) are marginal utility of OS network size and price, respectively.

An outside option exists in each period and it means not to buy a smartphone. Let the mean utility of the outside option be zero. Consumer \( i \)'s utility of the outside option is \( u_{i0t} = \epsilon_{i0t} \). Assume that the utility shock \( \epsilon_{ijt} \) follows a Type-I

\(^6\)There are several reasons that one OS has a higher share than the other initially. Different OSs may enter the market at different years. Their companies may have different shocks. Different operating systems may have different openness towards smartphone manufacturers.

\(^7\)By normalizing costs to be zero, the prices in this section can be interpreted as markups that the carrier earns. When the costs are not zero, choosing prices is equivalent to choosing markups.
extreme value distribution and is i.i.d. across consumers, models, and periods. Then the sales market share of model $j$ in period $t$ is

$$s_{jt} = \frac{e^{(\delta_j+\gamma n_{jt}-\alpha p_{jt})}}{1+\sum_{k=A,B} e^{(\delta_k+\gamma n_{kt}-\alpha p_{kt})}}. \tag{1.1}$$

An OS network grows due to new sales of smartphones. At the beginning of the second period, the market share of OS $j \in \{A, B\}$ is

$$n_{j2} = n_{j1} + M_1 s_{j1}. \tag{1.2}$$

Therefore, the market size in the second period is:

$$M_2 = 1 - n_{A2} - n_{B2}$$
$$= 1 - n_{A1} - n_{B1} - M_1 (s_{A1} + s_{B1})$$
$$= M_1 (1 - s_{A1} - s_{B1})$$
$$= M_1 s_{01}, \tag{1.3}$$

where $s_{01}$ is the market share of the outside option in the first period. The market size in the second period is the measure of consumers who do not buy smartphones in the first period.

### 1.4.2 Model One: A Multi-Network Seller

Suppose that a seller sells both models and chooses prices to maximize the sum of discounted profits. Next, I solve the problem using backwards induction to analyze the multi-network seller’s pricing decisions.

#### 1.4.2.1 Optimal Prices in the Second Period

At the beginning of the second period, the multi-network seller observes the OS network sizes $(n_{A2}, n_{B2})$ and chooses the prices $(p_{A2}, p_{B2})$ for A and B. The seller faces a static problem after entering the second period, because it is the last period.
For second period prices \((p_{A2}, p_{B2})\), the seller’s profit in the second period is:

\[
\pi_2(p_{A2}, p_{B2}) = M_2(s_{A2}p_{A2} + s_{B2}p_{B2}) = M_2 \left[ e^{(\delta_A + \gamma n_{A2} - \alpha p_{A2})} p_{A2} + e^{(\delta_B + \gamma n_{B2} - \alpha p_{B2})} p_{B2} \right].
\]

where \(M_2 s_{j2}\) is the market demand for phone \(j\) in the second period. The first order conditions with respect (FOC) to price \(p_{A2}\) is:

\[
M_2 \left[ s_{A2} - \alpha p_{A2} s_{A2}(1 - s_{A2}) + \frac{\alpha p_{B2}^* s_{A2} s_{B2}}{\text{marginal profit from B}} \right] = 0. \tag{1.4}
\]

The price of A affects the seller’s profit not only though consumers’ demand for phone A but also though that of phone B, because it sells both A and B. Raising the price of A increases the seller’s profit from phone B, and vice versa. This implies that the multi-network seller can internalize the competition between A and B. Dividing the FOC by \(M_2 s_{A2}\), we have:

\[
1 - \alpha (p_{A2}^*)(1 - s_{A2}) + \alpha (p_{B2}^*) s_{B2} = 0. \tag{1.5}
\]

Rewrite equation 1.5:

\[
1 - \alpha p_{A2}^* + \alpha p_{A2}^* s_{A2} + \alpha p_{B2}^* s_{B2} = 0. \tag{1.6}
\]

The FOC for \(p_{B2}\) can be derived in a similar way:

\[
1 - \alpha p_{B2}^* + \alpha p_{B2}^* s_{B2} + \alpha p_{A2}^* s_{A2} = 0. \tag{1.7}
\]

By comparing the FOCs for A and B, 1.6 and 1.7, one immediately gets: \(p_{A2}^* = p_{B2}^*\). That is, in order to have the marginal profits from A and B both be zero, the carrier chooses the same price for A and B.

Denote the second period price for A and B by \(p_{n2}\). According to the sales share equation 1.1, \(s_{A2}\) and \(s_{B2}\) are functions of \((p_{n2}, n_{A2}, n_{B2})\). The FOC of price in the
second period becomes:

\[
0 = 1 - \alpha p^*_2 + \alpha p^*_2 \left[ s_{A2}(p^*_2, n_{A2}, n_{B2}) + s_{B2}(p^*_2, n_{A2}, n_{B2}) \right] \\
= 1 - \alpha p^*_2 \left[ 1 - s_{A2}(p^*_2, n_{A2}, n_{B2}) - s_{B2}(p^*_2, n_{A2}, n_{B2}) \right],
\]

which implies that \( p^*_2 \) is a function of the OS network sizes \((n_{A2}, n_{B2})\): \( p^*_2(n_{A2}, n_{B2}) \).

The FOC also implies that \( s_{A2} + s_{B2} = 1 - \frac{1}{\alpha p^*_2} \). The second period profit is:

\[
\pi_2 = p^*_2(s_{A2} + s_{B2})M_2 \\
= p^*_2(1 - \frac{1}{\alpha p^*_2})M_2 \\
= (p^*_2(n_{A2}, n_{B2}) - \frac{1}{\alpha})M_1s_{01}.
\]

where \( M_2 = M_1s_{01} \) as in equation (1.3). This implies that the profit in the second period is strictly increasing with price \( p^*_2 \) and market size \( M_2 \).

Before moving to the first period, it is helpful to first look at how the OS network sizes \((n_{A2}, n_{B2})\) affect the optimal price \( p^*_2 \), since the first period prices will make an impact on \( \pi_2 \) through \((n_{A2}, n_{B2})\) and \( p^*_2 \). Using the FOC (1.8) and Implicit Function Theorem, the first order derivative of price \( p^*_2 \) with respect to \( n_{j2}, j \in \{A,B\} \) can be derived:

\[
\frac{\partial p^*_2(n_{A2}, n_{B2})}{\partial n_{j2}} = \frac{\gamma}{\alpha s_{j2}} > 0.
\]

There are two implications of equation (1.10). First, the price \( p^*_2 \) increases with the OS network size \( n_{j2} \). Second, the larger OS network A has a stronger positive impact on \( p^*_2 \) than OS B, since the sales share of A is higher than B, \( s_{A2} > s_{B2} \). This is because A and B have the same price and A has a larger network than B \( n_{A2} > n_{B2} \) (See Appendix A.1 for the proof of \( n_{A2} > n_{B2} \)), so demand for A is higher than B: \( s_{A2} > s_{B2} \). Therefore, \( n_{A2} \) has a stronger impact on \( p^*_2 \) than \( n_{B2} \):

\[
\frac{\partial p^*_2(n_{A2}, n_{B2})}{\partial n_{A2}} > \frac{\partial p^*_2(n_{A2}, n_{B2})}{\partial n_{B2}} > 0.
\]

From equation (1.10), the second order derivatives can also be derived to check
the convexity of price $p^*_2$ as a function of $n_{j2}$, $j \in \{A,B\}$:

$$\frac{\partial^2 p^*_2(n_{A2}, n_{B2})}{\partial n_{j2}^2} = \frac{\gamma}{\alpha} s_{j2}(1 - s_{j2}) > 0.$$  \hspace{1cm} (1.12)

$$\frac{\partial^2 p^*_2(n_{A2}, n_{B2})}{\partial n_{A2}\partial n_{B2}} = -\frac{\gamma}{\alpha} s_{A2}s_{B2} < 0.$$  \hspace{1cm} (1.13)

Equation (1.12) and (1.13) imply that the price $p^*_2$ is a convex function in the OS network sizes $(n_{A2}, n_{B2})$. Due to the convexity, $p^*_2$ increases as more consumers join the same OS network, either OS A or B. For example, consider two possible states in the second period, $(n^g_{A2}, n^g_{B2}) = (0.4, 0.1)$ and $(n^h_{A2}, n^h_{B2}) = (0.25, 0.25)$. The two states have the same total measure of existing users, 0.5. However, the price $p^*_2$ is higher in state $g$ than $h$ because of the convexity. Notice that, a third state $(n'^g_{A2}, n'^g_{B2}) = (0.1, 0.4)$ will lead to the same $p^*_2$ as state $g$, because A and B have the same characteristics. Therefore, $p^*_2$ increases with $|n_{A2} - n_{B2}|$, given the total existing users. More importantly, since $\pi_2$ increases in $p^*_2$, the convexity implies that the seller gets the maximum $\pi_2$ when all consumers use the same OS at the beginning of the second period, given the market size $M_2$.

Whether the seller chooses OS A or B to be the larger network in the second period depends on their initial OS network sizes. Intuitively, the seller will choose the initially larger network A, which is less costly for the seller than to reach the same level of OS concentration by choosing the smaller network B. In Appendix A.1, I prove that, if $n_{A1} > n_{B1}$, then the profit maximizing prices must lead to $n_{A2} > n_{B2}$.

The second period state $(n_{A2}, n_{B2})$ has two more effects on $\pi_2$, in addition to that $\pi_2$ increases with their difference $|n_{A2} - n_{B2}|$. One is that, as $n_{A2}$ and $n_{B2}$ increase, consumers’ demand in the second period increases because larger OS network sizes raise consumer utility. The other one is that, as $n_{A2}$ and $n_{B2}$ increase, the market size $M_2$ decreases, so the demand decreases. Depending on which of the two effects dominates the other, the seller may increase or decrease the prices for both A and B simultaneously in the first period. However, the two effects do not change the seller’s preference in the OS concentration in the second period to increase price.

Next, I solve the multi-network seller’s problem in the first period. Equation (1.11) will be the key to compare the prices of A and B in the first period.
1.4.2.2 Optimal Prices in the First Period

In the first period, the seller observes the initial OS market sizes \((n_{A1}, n_{B1})\) and chooses prices \((p_{A1}, p_{B1})\) which will determine profits in both periods. The seller’s profit maximization problem in the first period is:

\[
\max_{p_{A1}, p_{B1}} \left\{ \pi_1(p_{A1}, p_{B1}) + \beta \pi_2(n_{A2}, n_{B2}|p_{A1}, p_{B1}) \right\} = \max_{p_{A1}, p_{B1}} \left\{ M_1(s_{A1}p_{A1} + s_{B1}p_{B1}) + \beta \left[ p^*_2(n_{A2}, n_{B2}|p_{A1}, p_{B1}) - \frac{1}{\alpha} \right] M_1 s_{01} \right\}
\]

(1.14)

where \(\beta\) is the discount factor across periods. The equality in (1.14) is from the profit function (1.9). The OS network sizes \((n_{A2}, n_{B2})\) depend on the first period prices \((p_{A1}, p_{B1})\), as in equations (1.1) and (1.2).

The first order condition w.r.t. \(p_{A1}\) is:

\[
M_1 \left\{ \begin{array}{l}
1 - \alpha p^*_A s_{A1} (1 - s_{A1}) + \frac{\alpha p^*_B s_{A1} s_{B1}}{1 - s_{A1}} + \\
\beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{A1} s_{01} + \beta M_1 s_{01} \left[ -\alpha \frac{\partial p^*_2}{\partial n_{A2}} s_{A1} (1 - s_{A1}) + \frac{\alpha}{\alpha} \frac{\partial p^*_2}{\partial n_{B2}} s_{A1} s_{B1} \right] \end{array} \right\} = 0.
\]

(1.15)

The FOC implies that the price \(p_{A1}\) makes an impact on the first period profit from A and B, the market size in the second period, and the price in the second period. Divide the FOC by \(M_1 s_{A1}\):

\[
1 - \alpha p^*_A + \alpha p^*_A s_{A1} + \alpha p^*_B s_{B1} + \\
\beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{01} + \beta M_1 s_{01} \left[ -\alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{A1} s_{B1} \right] = 0.
\]

(1.16)

The FOC for \(p_{B1}\) can be derived similarly.

\[
1 - \alpha p^*_B + \alpha p^*_B s_{B1} + \alpha p^*_A s_{A1} + \\
\beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{01} + \beta M_1 s_{01} \left[ -\alpha \frac{\partial p^*_2}{\partial n_{B2}} (1 - s_{B1}) + \alpha \frac{\partial p^*_2}{\partial n_{A2}} s_{A1} \right] = 0.
\]

(1.17)

Comparing the FOCs in (1.16) and (1.17), then:

\[
\alpha p^*_A + \beta M_1 s_{01} \alpha \frac{\partial p^*_2}{\partial n_{A2}} = \alpha p^*_B + \beta M_1 s_{01} \alpha \frac{\partial p^*_2}{\partial n_{B2}}.
\]

(1.18)
According to equation (1.11), \( \frac{\partial p^*_A}{\partial m_{A2}} > \frac{\partial p^*_B}{\partial m_{B2}} > 0 \), which means that the large OS A has a stronger positive impact on \( p_2 \) than B. Hence, the price of A in the first period should be lower than B, \( p_{A1} < p_{B1} \), so that their marginal profits will both be equal to zero.

To be more specific, plug equation (1.10) in equation (1.18), then

\[
\alpha p^*_A + \beta M_1 s_0 \gamma s_{A2} = \alpha p^*_B + \beta M_1 s_0 \gamma s_{B2}.
\]

Divide both sides by \( \alpha \), the relationship between the prices is:

\[
p^*_A - p^*_B = \frac{\beta \gamma}{\alpha} M_1 s_0 (s_{B2} - s_{A2}).
\]  

(1.19)

Since phone A has a higher sales share than B in the beginning of the second period, \( s_{A2} > s_{B2} \), the price of A is lower than B in the first period, \( p^*_A < p^*_B \).

The first period prices \((p_{A1}, p_{B1})\) have two effects on the discounted total profit. First, as explained above, the seller may increase or decrease the two prices to balance the tradeoff between market sizes across periods, depending on the discount factor \( \beta \), price elasticity \( \alpha \), and OS network effect \( \gamma \). Second and more importantly, the seller chooses a lower price for the initially larger network A than B in the first period, so that A has as many users as possible so that the second period profit is maximized.

1.4.2.3 The Dynamic Effect and the OS Network Effect

The dynamic model makes an impact on the first period prices. Without the dynamic effect, \( \beta = 0 \), equation (1.19) implies that the two products have the same price in the first period as well, \( p_{A1} = p_{B1} \), no matter which product has a larger network. However, when the model is dynamic (\( \beta > 0 \)), then the large network has a lower price than the small network in the first period, \( p_{A1} < p_{B1} \). The prices of A and B in the second period are the same, \( p_{A2} = p_{B2} \), no matter whether the problem is dynamic or not.

The OS network effect makes a similar effect on \((p_{A1}, p_{B1})\) like the dynamic effect. Equation (1.19) implies that, when there is no network effect, \( \gamma = 0 \), A and B will have a same price in the first period. In this case, the seller’s problem is still dynamic, but the OS sizes do not affect pricing. However, when the network
effect exists, $\gamma > 0$, the larger network A has lower price than B in the first period. Because the seller’s second period profit increases with network concentration.

1.4.2.4 Model One Summary

The following proposition summarizes the properties of the multi-network seller’s optimal prices $(p_{A1}^{*c}, p_{B1}^{*c}, p_{A2}^{*c}, p_{B2}^{*c})$.

**Proposition 1.** The following statements hold for the carrier’s optimal prices.

1. The price of A is lower than that of B in the first period: $p_{A1}^{*c} < p_{B1}^{*c}$.
2. The price difference between the two models $|p_{A1}^{*c} - p_{B1}^{*c}|$ increases as the OS network effect $\gamma$ increases.
3. The OS market share difference at the beginning of the second period $(n_{A2} - n_{B2})$ increases in the OS network effect $\gamma$.

**Proof.** See Appendix A.1.

The first statement of Proposition 1 says that, when the OS network effect exists, the multi-network seller chooses a lower price for the initially larger network A than for network B in the first period. As explained above, the intuition is that, to increase consumer demand, the seller prefers for most consumers to use the initially larger network A in the first period, so that the seller can charge higher prices in the second period to increase profit.\(^8\)

The second statement means that price difference between A and B increases with the OS network effect parameter $\gamma$. This is because that the seller’s incentive to choose lower price for A increases with the OS network effect.

The third statement says that the market becomes more concentrated in the large OS as the network effect increases.\(^9\) This result says that the multi-network seller’s prices contribute to the OS concentration of smartphones.

The results in Proposition 1 are derived using the logit demand model. The logit model is an ideal choice to compare carrier prices in a static and dynamic

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\(8\)The reason why the carriers sell products with different OSs is that consumers are heterogeneous. Some consumers might prefer B no matter how large network A is. So the carrier still sells B, which can increase its profits.

\(9\)I do not model the vertical relationship between the carrier and the manufacturers. It is probably not optimal for a carrier to sell only one manufacturer’s model, because in this case, the manufacture has the monopoly power and can charge carriers a high wholesale price. Accordingly, the carrier’s profit could decrease. The question of why the carrier is selling the both operating systems is also interesting, but is beyond the focus of this paper.
model. That is because the static logit model provides a clear benchmark to study the carrier’s incentive in a dynamic model. The static logit demand model predicts that the carrier will choose the same price for both OSs, no matter which one has the initial network advantage. When moving from a static model to a dynamic one, the changes in the seller’s pricing strategy reflect the impact of the dynamic factors, OS network sizes and market size.

More importantly, the multi-network seller’s incentive to have the large OS dominate the small OS exists no matter which demand model is used in the two-period model. With other demand functions, the seller would also choose a lower price for the large OS than for the small OS, as long as the network effect is strong and the discount factor is sufficiently large.

Though I consider a monopoly carrier case, the carrier’s pricing strategy in a dynamic environment carries over to oligopoly carriers. The competition among the carriers doesn’t change their incentives to take advantage of the initially larger network in equilibrium.

1.4.3 Model Two: Two Single-Network Sellers

To compare the multi-network seller’s pricing strategy with single-network sellers’ prices, I now consider two single-network sellers, with each selling one of the two smartphones and play a dynamic pricing game in the two periods. The demand model setup is the same as before. Denote the two sellers as \{A, B\}. Seller \(j \in \{A, B\}\) produces and sells smartphone \(j\) that uses OS \(j\) in the two periods. In this section, I use superscript \(m\) to denote the sellers. For seller \(j\), the profit in each period \(t\) is

\[
\pi_{jt}^m(p_{At}^m, p_{Bt}^m) = p_{jt}^m s_{jt}(p_{At}^m, p_{Bt}^m) M_t.
\]

This reflects the fact that the cross-derivatives of prices \(\alpha_{At} s_{Bt}\) are symmetric for both OSs, so the carrier’s two first-order conditions are symmetric, as shown in Appendix A.

It is difficult to theoretically prove this result. However, by solving numerical examples, I find that for demand functions which also reflect the increasing share in model’s mean utility, there exist critical values of \((\gamma, \beta, \alpha)\) such that at these values, the multi-network seller chooses same price for both models in the first period. And if either \(\gamma\) or \(\beta\) increases, carriers choose a lower price for the large OS than for the small OS in the first period, and vice versa.

I show this in Appendix A.3. In addition, I used numerical examples with oligopolistic carriers to check the equilibrium prices. The results find that the carriers still choose lower prices for the large network than the small network in the first period.
The dynamic problem of seller $j$ is

$$
\max_{p_{j1}} \{ \pi_{j1}^m(p_{A1}, p_{B1}) + \beta \max_{p_{j2}} \{ \pi_{j2}^m(p_{A2}, p_{B2}|p_{A1}, p_{B1}) \} \},
$$

in which $\beta$ is the discount factor across periods. The Subgame Perfect Nash Equilibrium of the finite period dynamic game can be solved backwards. The following proposition summarizes the properties of the sellers’ equilibrium prices $(p_{A1}^*, p_{B1}^*, p_{A2}^*, p_{B2}^*)$.

**Proposition 2.** The following statements hold for the single-network sellers’ equilibrium prices.

1. The optimal price of A is higher than that of B in both periods: $p_{A1}^* > p_{B1}^*$, for $t = 1, 2$.

2. The larger OS A keeps its network advantage in the second period, though model A is more expensive in the first period.

**Proof.** See Appendix A.2.

The first statement of Proposition 2 says that when the single-network sellers choose prices, seller A chooses higher prices than B in both periods, because the initial OS network advantage gives seller A market power. Suppose both models have the same price in the first period in equilibrium, then B’s marginal profit is zero at the price. However, since A’s initial network size is larger, A’s marginal profit at the price is positive, so seller A could increase profit by raising price. As a result, A’s price is higher than B’s price in the first period. The second statement says that A keeps its network advantage over B in the second period, though A is more expensive in the first period.

### 1.4.4 Comparing the Two Models

By comparing Proposition 1 and Proposition 2, it is clear that in the first period, the multi-network seller chooses a lower price for A than B, while single-network sellers do the opposite. These price differences in the two models are due to the fact that the multi-network seller can internalize the competition effect of the two OSs. In the single-network seller model, the initially larger OS maintains the network advantage and has higher prices than the small OS in each period. In the
multi-network model, the initially larger OS has a lower price than the small OS in the first period, thus it grows faster than the small OS. Therefore, the multi-network seller’s pricing strategy accelerates the concentration of the OS market.

To clearly show a multi-network firm’s dynamic pricing strategy, individual heterogeneity, carrier competition, and infinite periods are ignored in this section. Next, I set up an empirical model of the consumers’ demand and the carriers’ pricing of smartphones, which is more general than the two-period, two-OS setting.

### 1.5 An Empirical Model of Demand and Supply of Smartphones

In this section, I design a structural model of consumers’ demand and carriers’ dynamic pricing of smartphones with two-year wireless service contracts. There are four leading wireless carriers, four operating systems, and hundreds of smartphone models. I assume consumers make static purchase decisions of smartphones with a random coefficient model. The carriers sell different sets of smartphone models and they play an infinite horizon dynamic pricing game.\(^{13}\)

The US Wireless Industry Overview 2011 reported that more than 78% of users are on two-year contracts. This number includes both the feature phone and smartphone subscribers. The percentage is expected to be even higher for the smartphone market because smartphones are much more expensive if bought without a contract. The data used in this paper is from Aug. 2011 to Oct 2013, during which most smartphone users were on two-year contracts. Because of this, I focus on the consumers’ demand for smartphones with two-year contracts.

One challenge in modeling the consumers’ demand for smartphones is that there is no available data on smartphone model level market shares. Smartphone characteristics and prices are at the model level, while the market shares are observed at carrier-OS level. To deal with this challenge and use data at both

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\(^{13}\)In this paper, I focus on the price decisions of the carriers. I do not model the manufacturers’ prices in a Stackelberg leader-and-follower framework. In other words, I analyze the “follower” part of the full game given the manufacturers’ wholesale prices fixed. This simplification is not problematic in terms of model specification and estimation. In a Stackelberg leader-and-follower model, the manufacturers choose prices first and then the carriers choose prices accordingly. So the carriers are only affected by the manufacturers’ prices. Since I do model the carriers’ wholesale costs paid to the manufacturers, the simplification of focusing on the“follower” part is not problematic.
levels, I introduce a carrier-OS specific unobserved quality shock, which will be explained in detail in this section.

1.5.1 Consumer Demand

Each period, consumers who don’t own any smartphone or have ended previous two-year contracts enter the market. Each consumer chooses one option from the available choice set to maximize utility. The choice set in period $t$ is $\Omega_t = \{(j, s, c, t)\}_{j, s, c} \cup \{(0, t)\}$, where $j$ is a smartphone model, $s$ is an operating system, and $c$ is a carrier. $(0, t)$ is the outside option, which means not buying any smartphone. If a consumer purchases model $(j, s, c, t)$, s/he signs a two-year wireless service contract with carrier $c$. Let $J_t$ be the total number of models in $\Omega_t$.

Assume consumer $i$’s utility of buying model $(j, s, c)$ in period $t$ to be:

$$u_{ijst} = x'_{jsc} \beta_i - \alpha_i (p_{jst} + f_{ct}) + \gamma N_{st} + \psi_{sc} + \xi_{sct} + \epsilon_{ijst}.$$  \hspace{1cm} (1.20)

$x_{jsc}$ is a $K$ by 1 vector of observed smartphone characteristics. $p_{jst}$ is the price of the smartphone if purchased with a two-year contract. $f_{ct}$ is the carrier $c$’s plan price for the two years in the contract. $N_{st}$ is the number of users of OS $s$ at the beginning of period $t$. $\psi_{sc}$ is the carrier-OS $(s, c)$ dummy, which captures the fixed effect of the carrier and operating system. It captures the carrier-OS quality that is constant across periods. $\xi_{sct}$ is the carrier-OS level unobserved quality shock in period $t$. It represents the shocks both in carrier service quality and the operating system quality. $\epsilon_{ijst}$ is the consumer idiosyncratic utility shock.

The unobserved quality shocks $\xi_{sct}$ is assumed to be carrier-OS specific because the market shares are observed at the carrier-OS level. The implication of the assumption is that the variation in market shares across different models in the same carrier-OS group is determined by their observed characteristics and prices, not by their individual unobserved quality shocks. The normal distribution assumption will used in the carriers’ pricing model in the next section.

I assume that the consumer pays the wireless plan price for two years when signing the contract. This assumption is not really restrictive since the monthly plan price is the same across the different models sold by the same carrier. So this assumption shouldn’t affect consumers’ decisions within each carrier’s models.

Consumers may also care about future OS network, but they do not observe the future OS network sizes. In my paper, ‘Consumers’ Dynamic Demand for Smartphones with Two-Year Contracts,’ 2014, I find that static model tends to underestimate the OS network effect.
The parameters $\theta_i = (\beta_i, \alpha_i)$ are consumer specific, as the random coefficient demand model in Berry, Levinsohn, and Pakes (1995). Let $\theta = (\beta, \alpha)$, where $\beta$ is the mean of $\beta_i$ over all consumers and $\alpha$ is the constant part of $\alpha_i$ that is the same for all consumers. Consumers are heterogeneous in income levels and tastes for smartphone characteristics. Consumer $i$ is described by his/her characteristics $v_i = (y_i, v_{i1}, ..., v_{iK})$, in which $y_i$ is income and $K$ is the dimension of the smartphone characteristics. The elements in $v_i$ are assumed to follow independent standard normal distributions $N(0, I_K)$, where $I_K$ is identity matrix with dimension $K$. Consumer characteristics $v_i$ is assumed to be independent of the unobserved quality shock $\xi_{sc}$. Consumer $i$’s coefficient vector $\theta_i$ is assumed to be:

$$\theta_i = \theta + \Phi v_i,$$

in which $\Phi$ is a diagonal matrix that determines the impact of consumer characteristics on the parameters. Rewrite the utility function (1.20) as:

$$u_{ijsc} = \delta_{jsc} + \mu_{ijsc} + \epsilon_{ijsc},$$

in which

$$\delta_{jsc} = x_{jsc}' \beta - \alpha (p_{jsc}^c + f_{ct}) + \gamma N_{st} + \psi_{sc} + \xi_{sc},$$

$$\mu_{ijsc} = [x_{jsc}' (p_{jsc}^c + f_{ct})] * \Phi v_i.$$

The outside option means not to buy a new smartphone. The mean utility of the outside option is normalized to zero. Consumer $i$’s utility of the outside option is:

$$x u_{i0t} = \epsilon_{i0t}.$$

Assume that the $\epsilon_{ijsc}$ follows a Type-I extreme value distribution and is i.i.d. across $(i, j, s, c, t)$. Then consumer $i$’s probability of choosing product $j$ in period $t$ is:

$$s_{ijsc}(v_i) = \frac{e^{(\delta_{jsc} + \mu_{ijsc})}}{1 + \sum_{j',s',c' \in \Omega_t} e^{(\delta_{j's'c't} + \mu_{ij's'c't})}}.$$

Let $A_{jsc}$ be the set of consumer characteristics such that $j$ has the highest utility for these consumers. That is, $A_{jsc} = \{v_i | u_{ijsc}(v_i) \geq u_{ij's'c't}(v_i), \text{ for all } (j's'c't) \in \Omega_t\}$. 

20
Then the market share of product $j$ in period $t$ is:

$$s_{j_{sct}} = \int_{A_{j_{sct}}} s_{i_{j_{sct}}}(v_i) dF(v_i).$$

(1.21)

I aggregate the smartphone model shares to carrier-OS levels, observe smartphone model level shares in the data, but the carrier-OS group level shares. Let $\Omega_{sct}$ be the set of all models with OS $s$ by carrier $c$ in period $t$. The market share of the carrier-OS group $(s, c)$ in period $t$ is the sum of shares over models in $\Omega_{sct}$:

$$s_{sct} = \sum_{j \in \Omega_{sct}} s_{j_{sct}}.$$  

(1.22)

### 1.5.2 The Carriers’ Dynamic Pricing Model

The four carriers play a dynamic pricing game because of the evolving OS network sizes and market size. In this section, I model the carriers’ costs of a smartphone, the aggregate market size, and the carriers’ dynamic pricing game of smartphones. Each period, the carriers observe the market shares of all OSs and choose the prices of smartphones when sold with two-year wireless service contracts. For example, the carriers’ two-year contract price for an iPhone is $199, while its manufacturer retail price is $649.

#### 1.5.2.1 The Carriers’ Unit Costs

A carrier pays a wholesale cost, a service cost, and an unobserved cost shock on each smartphone model. Carrier $c$’s wholesale cost of model $(j, s, c, t)$ is the product of a manufacturer wholesale price rate $\omega_j$ and the manufacturer retail price $p_{m_j_{sct}}$. $\omega_j$ is assumed to be manufacturer specific and the same across carriers. It captures

---

16 The manufacturers may also affect the two-year contract prices of smartphones by signing price contracts with carriers. However, a carrier will only sign a price contract with a manufacturer if the two-year contract prices can maximize its long run profit, together with other manufacturers’ products. Therefore, the carriers are indirectly choosing the two-year contract prices even when they sign price contracts with manufacturers.

17 Another option is to assume the wholesale discounts be a function of operating system shares. Intuitively, the manufacturers’ wholesale prices could be affected by the OS network sizes. For example, the wholesale costs could be higher for smartphone models with larger OS network sizes. However, this effect could already be reflected in the manufacturer retail prices. In this case, modeling discount rates as depending on OS shares would double count the OS share’s effect. Instead, I use manufacturer specific wholesale discounts to measure the manufacturers’ bargaining
the discount the carriers receive from the manufacturers if there is any.

In addition to the wholesale cost, carrier $c$ also pays a monthly service cost $\kappa_{sc}$, which is carrier-OS specific. The service cost includes the costs of selling a phone, maintaining wireless coverage, and providing customer services. Different OSs may have different service costs. For example, users of different OSs consume different amounts of data and request different types of customer service. There is also an unobserved cost shock, $\lambda_{jsct}$. Hence, carrier $c$’s unit cost of selling model $(j, s, c)$ with a two-year contract in period $t$ is:

$$c_{jsct} = \omega_j p^m_{jsct} + 24\kappa_{sc} + \lambda_{jsct},$$

where $p^m_{jsct}$ is the manufacturer’s retailer price.

### 1.5.2.2 The Market Size

At the beginning of period $t$, the state variables for the carriers are the cumulative market shares of different OSs: $n_t = (n_{1t}, ..., n_{St})$, where $S$ is the number of OSs. The OS shares are defined relative to the number of potential smartphone users $M$: $n_{st} = N_{st}/M$.

Two types of consumers enter the market each period, those who do not own any smartphone yet and those who have finished their previous contracts. The proportion of consumers without any smartphone in period $t$ is $1 - \sum_{s=1}^{S} (n_{st})$. I assume that each existing user has the same probability to end his/her current contract each period.\(^{18}\) This probability is fixed to be $1/8$ because a contract is two years long and one period is three months in this paper.\(^{19}\) Let $M_t$ be the market size at the beginning of period $t$. Then given state variable $n_t$, the market size is:

$$M_t = \{1 - \sum_s n_{st}\}M + \frac{1}{8} \sum_s n_{st}M = \{1 - \frac{7}{8} \sum_s n_{st}\}M.$$  \hspace{1cm} (1.24)

---

\(^{18}\)In reality, the share of population that have ended their two-year contract in period $t$ should be endogenous. But tracking the distribution of the smartphone owners’ contract status would require this distribution being taken as a state variable for the carriers. This complicates the state space of the dynamic problem a lot.

\(^{19}\)Since one period is three months and the contract is 24 months long. The number of periods left in a contract at the end of each period is in $\{0, 1, 2, ..., 7\}$. The assumption that a user has $1/8$ probability to end his/her contract is equivalent to that $1/8$ of the users will have 0 period left at the end of the period.
The market size is decreasing in the sum of OS shares. By assumption, the market size will not become zero, because each period one eighth of previous users end their contracts and re-enter the market again.

1.5.2.3 Timing of the Pricing Game and the Carriers’ Bellman Equations

The timing of the dynamic pricing game is as follows. At the beginning of period \( t \), all carriers observe the state variables \( n_t \). Then the demand and cost shocks \((\xi_t, \lambda_t)\) are realized. Each carrier observes all carriers’ shocks and costs. The carriers choose prices simultaneously and then the consumers make choices accordingly. At the end of period \( t \), the state variables update to \( n_{t+1} \) and the market enters the next period.

At the end of period \( t \), each operating system loses \( 1/8 \) of its existing users, who end their current contracts and enter the market at period \( t + 1 \). This implies that, \( 7/8 \) of the existing users of an OS keep using the OS. Meanwhile, the OS also gets new users from the sales in period \( t \). Let \( \Omega_{st} \) be the set of smartphones with OS \( s \) in period \( t \). The transition of the cumulative market share of operating system \( s \) is:

\[
 n_{st+1}(n_t, p^c_t) = \frac{7}{8} n_{st} + \left(1 - \frac{7}{8}\right) \sum_{s'=1}^{S} n_{s't} \sum_{(j,c) \in \Omega_{st}} s_{j;ct}(p^c_t(\xi_t, \lambda_t), \xi_t). \tag{1.25}
\]

Denote the set of carrier \( c \)'s smartphones in period \( t \) by \( \Omega_{ct} \). Let \( \lambda_t \) be the vector of all cost shocks in period \( t \). Then the profit of carrier \( c \) in period \( t \), given price \( p^c_t \), is:

\[
 \pi_{ct}(p^c_t, \xi_t, \lambda_t) = \sum_{(j,s) \in \Omega_{ct}} (p^c_{j;ct} + f_{ct} - c_{j;ct}) s_{j;ct}(p^c_t, \xi_t) M_t, \tag{1.26}
\]

where \( c_{j;ct} \) is the unit cost as in equation \( 1.23 \) and \( f_{ct} \) is carrier \( c \)'s price of two years’ wireless service.

Carrier \( c \)'s Bellman equation is:

\[
 V_c(n_t) = E_{\xi, \lambda} \left[ \max_{p^c_{j;ct}(\xi_t, \lambda_t), (j,s) \in \Omega_{ct}} \left\{ \pi_{ct}(p^c_t, \xi_t, \lambda_t) + \beta^d V_c(n_{t+1}(n_t, p^c_t(\xi_t, \lambda_t))) \right\} \right], \tag{1.27}
\]

subject to equation \( 1.25 \). \( \beta^d \) is a discount factor across periods. The expectation
is over the unobserved quality shock vector $\xi$ and the cost shock vector $\lambda$. To avoid including the shocks as state variables, I define the value functions before the carriers observe the shocks. In addition to the i.i.d. assumption of $\lambda_{jst}$, I make the following assumption to calculate the carriers’ value functions.

**Assumption 3.** (1) The random cost shock $\lambda_{jst}$ follows normal distribution $N(0, \sigma^{2}_{\lambda})$.
(2) The unobserved quality shock $\xi_{sct}$ follows the normal distribution $N(0, \sigma^{2}_{\xi})$ and is i.i.d. across carrier-OS groups and periods.
(3) Cost shock $\lambda_{jst}$ and quality shock $\xi_{sct}$ are independent with each other.

The value functions are carrier specific because the carriers are different in their smartphone sets, wireless service costs, and wireless plan prices. In an ideal model, the evolving sets of smartphone models and their characteristics should also be state variables. However, the number of smartphone models by each carrier is relatively stable and the characteristics of all operating systems are improving in the data, as will be shown in Section 6. To keep the problem tractable, I don’t add them as state variables.

For any $(\xi_{t}, \lambda_{t})$, carrier $c$’s first-order conditions with respect to price can be derived. Given the prices of other models in period $t$, the equilibrium prices $p^{c}_{jst}(\xi_{t}, \lambda_{t})$ must satisfy the following first-order condition:

$$M_{t}s_{jst}(p^{c}_{t}, \xi_{t}) + M_{t} \sum_{(j',s') \in \Omega_{ct}} m_{j's'ct} \frac{\partial s_{j's'ct}}{\partial p^{c}_{jst}} + \beta \frac{\partial V_{c}(nt+1(n_{t}, p^{c}_{t}))}{\partial p^{c}_{jst}} = 0,$$

where $m_{jst}$ denotes the carrier markup:

$$m_{jst} = p^{c}_{jst} + f_{ct} - c_{jst}.$$

The price $p^{c}_{jst}$ affects both the current sales of all models of carrier $c$ and future state variables. The FOC implies that the marginal long run profit should be zero at the equilibrium prices.

**1.5.3 Equilibrium**

The equilibrium concept used in the carriers’ dynamic pricing game is Markov perfect Nash equilibrium (MPNE). In this paper, a MPNE is a subgame perfect
equilibrium where the strategies depend only on the past through the OS market shares updated from last period. For any shock vector \((\xi, \lambda)\) in a period, one equilibrium is a price vector \(p^c(n, \xi, \lambda)\) and value functions \(\{V^c(n)\}\) such that (1) given \(p^c(n, \xi, \lambda)\) and \(\{V^c(n)\}\)'s are the expected discounted long run profits and (2) given \(\{V^c(n)\}\), the price vector \(p^c(n, \xi, \lambda)\) maximizes the long run profit for each carrier, given the rivals' strategy following the price.

Theoretically, the dynamic game may have multiple equilibria for a set of model parameters. However, as often argued in the empirical literature, the multiple equilibria possibility is not an issue for the identification, as long as all the observed actions are outcomes from the same equilibrium. The assumption implies that the observed prices are the carriers' best responses to their opponents' strategies under the true parameters.

1.5.4 Discussion of the Demand and Supply Model

Ideally, the consumers' demand for smartphones with contracts should be modeled as dynamic decisions. The main concern with using a static demand model is that it ignores the consumers' expectation of future prices, new launched models, and OS network effect. However, the static demand model still gives the carriers the incentive to choose prices dynamically. As long as consumers value the OS network sizes, no matter in a static or dynamic way, the carriers will always endogenously make an impact on the future OS network sizes through current prices. The static model might underestimate the importance of the OS network effect, because forward-looking consumers may postpone their adoption of a large OS which can be even larger in the future. A static model would treat this low current demand for a large OS as if that consumers do not valuing the OS network effect enough. Therefore, the results with the static demand model could be interpreted as lower bounds of the impact of OS network effect on the industry.

Besides, it is quite challenging to nest a dynamic demand model into the dynamic pricing problem of asymmetric players with multiple products. There are two reasons. First, it's very difficult to have explicit forms of the derivatives of the market shares in a dynamic demand model because consumers' value functions are unknown. These derivatives will be required to analyze the first-order conditions of the dynamic pricing problem of the carriers.
Second and more importantly, the carriers’ state variables have to include the distribution of smartphone ownership, whose dimension is very high (more than a thousand) in this paper. The consumers’ state variables are their current smartphone ownership status (current model and number of periods left in the current contract) in a dynamic demand model. It is extremely hard to solve a dynamic problem with such high dimension state variables.

In this paper, I don’t endogenously model the carriers’ service prices $f_{ct}$. There are several reasons for this. First, each carrier’s plan price rarely changed during the sample periods. Second, the problem of choosing the optimal service price is not closely related to the OS network effect because the service price is carrier specific, not OS or model specific. Third, the plan prices are not adjusted periodically in the data and the very few adjustments of the carriers are not made simultaneously. This makes it hard to clearly define a period. Therefore, I take the carriers’ service prices as given in this paper.

1.6 Data

The data used in this paper comes from several sources. The sample period is from Aug 2011 to Oct 2013. The comScore.com reports the U.S. cumulative smartphone subscriber market shares every month. Sales market shares for the past three months are published every month by the Kantar World Panel. Carrier prices and manufacturer retail prices are collected via the web archive website. The smartphone characteristics data are collected from phonearena.com.

This paper focuses on the leading four carriers and four operating systems. The four carriers are Verizon, AT&T, Sprint, and T-Mobile. Their combined sales market shares during the sample period is around 90%. The four operating systems are iOS, Android, Blackberry, and Windows Phone. Their combined sales market share is more than 99%. During the data period, Verizon carries around 25 models of smartphones a month on average. AT&T, Sprint, and T-Mobile have 28, 19, and 17 models, respectively.

I exclude the population younger than 12 years old and older than 70 years old as smartphone consumers. This assumption makes the potential market size of smartphone consumption to be 75% of the population, according to the 2010 US population distribution by age. The website comScore.com reports the total
number of smartphone subscribers and the cumulative market share of each OS every month. I calculate the market size $M_t$ and state variables $n_t$ using these data.

The Kantar World Panel publishes the sales market share of each carrier for the previous three months each month. For example, in Feb 2012, it published the sales market shares for the three months ending in Jan 2012. In addition, the website also reports the OS sales shares of AT&T and Verizon. Combining the sales market shares by carrier and the OS sales within each carrier, I get the sales market share of each carrier-OS group. One missing piece of the sales data is the conditional OS sales market shares for Sprint or T-Mobile. Since only the combined OS sales market share for the two are observed, I construct the OS sales market shares for them as proportional to the number of models they have on different OSs. In the end, there are sales market shares for 16 carrier-OS groups for 26 months.

The web archive website has been archiving the carriers’ webpages several times every month from 2008.\(^{20}\) The carriers’ two-year contract price and the listed manufacturer retail price of each model can be collected by month. In the sample period, the data has 2283 model-month observations. The highest two-year contract price is $399 for the 64 GB iPhones from multiple carriers.

I also collect the monthly wireless plan prices from the web archive website. Each carrier offers multiple plans each month. I use the single line price for medium amount of data and minutes\(^{21}\). Verizon’s wireless plan price was the highest at $70 and T-Mobile’s was the lowest at $50. The average across carriers is $60 per month during 2011 to 2013. This matches the $61 average reported by New Street Research company for 2013.

To match the sales shares data, a period is three months in the structural model, while smartphone models and their prices are observed every month. Thus, to use as much information as possible, I construct consumers’ choice set every period in this way: if a smartphone model is observed in multiple months in a period, I treat them as different choice options in that period.

The smartphone characteristics include camera pixels, built-in storage, 4G dummy, weight, screen size, resolution, processor speed, system memory, and battery capacity. All the characteristics are scaled so that their values are in similar

\(^{20}\) The web archive website link is: [http://archive.org/web/](http://archive.org/web/)

\(^{21}\) I use the prices for the following minutes and data bundles for the 4 carriers: Verizon (unlimited minutes, 2GB), AT&T (450 minutes, 300MB), Sprint (unlimited minutes, 1GB), T-Mobile (unlimited minutes and data).
<table>
<thead>
<tr>
<th>Carrier-OS Group</th>
<th>No. of Carrier Models</th>
<th>Carrier Price</th>
<th>Manuf. Price</th>
<th>Battery mAh</th>
<th>Screen Pixel/100 inches</th>
<th>Processor GHz</th>
<th>Sales Share%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verizon-iOS</td>
<td>4.69</td>
<td>2.26</td>
<td>6.74</td>
<td>1.43</td>
<td>7.14</td>
<td>3.65</td>
<td>1.72</td>
</tr>
<tr>
<td>Verizon-And</td>
<td>16.15</td>
<td>1.22</td>
<td>5.04</td>
<td>1.39</td>
<td>5.03</td>
<td>3.01</td>
<td>0.47</td>
</tr>
<tr>
<td>Verizon-Bla</td>
<td>3.08</td>
<td>1.45</td>
<td>4.66</td>
<td>1.33</td>
<td>4.32</td>
<td>1.31</td>
<td>0.08</td>
</tr>
<tr>
<td>Verizon-Win</td>
<td>4.07</td>
<td>1.03</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>2.60</td>
</tr>
<tr>
<td>AT&amp;T-iOS</td>
<td>5.07</td>
<td>1.98</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>2.60</td>
</tr>
<tr>
<td>AT&amp;T-And</td>
<td>13.88</td>
<td>0.98</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>2.60</td>
</tr>
<tr>
<td>AT&amp;T-Bla</td>
<td>3.81</td>
<td>0.86</td>
<td>4.86</td>
<td>1.25</td>
<td>4.32</td>
<td>1.31</td>
<td>0.08</td>
</tr>
<tr>
<td>AT&amp;T-Win</td>
<td>4.81</td>
<td>4.36</td>
<td>6.71</td>
<td>1.43</td>
<td>6.75</td>
<td>3.66</td>
<td>2.60</td>
</tr>
<tr>
<td>Sprint-iOS</td>
<td>4.91</td>
<td>0.86</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>2.60</td>
</tr>
<tr>
<td>Sprint-And</td>
<td>11.38</td>
<td>0.86</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>2.60</td>
</tr>
<tr>
<td>Sprint-Bla</td>
<td>3.69</td>
<td>2.04</td>
<td>1.30</td>
<td>1.37</td>
<td>1.35</td>
<td>1.39</td>
<td>0.08</td>
</tr>
<tr>
<td>Sprint-Win</td>
<td>1.00</td>
<td>1.30</td>
<td>1.30</td>
<td>1.37</td>
<td>1.35</td>
<td>1.39</td>
<td>0.08</td>
</tr>
<tr>
<td>T-Mobile-iOS</td>
<td>4.00</td>
<td>1.58</td>
<td>6.96</td>
<td>1.44</td>
<td>6.05</td>
<td>4.05</td>
<td>2.01</td>
</tr>
<tr>
<td>T-Mobile-And</td>
<td>10.62</td>
<td>1.30</td>
<td>4.46</td>
<td>1.58</td>
<td>4.05</td>
<td>4.05</td>
<td>2.01</td>
</tr>
<tr>
<td>T-Mobile-Bla</td>
<td>3.09</td>
<td>1.30</td>
<td>4.46</td>
<td>1.58</td>
<td>4.05</td>
<td>4.05</td>
<td>2.01</td>
</tr>
<tr>
<td>T-Mobile-Win</td>
<td>1.31</td>
<td>1.46</td>
<td>5.19</td>
<td>1.39</td>
<td>4.46</td>
<td>4.46</td>
<td>2.48</td>
</tr>
</tbody>
</table>

Notes: The first three letters are used to denote the operating system. For example, “And” is for Android. T-Mobile eliminated the 2-contract policy since it started to sell iOS models. The sales shares are conditional shares among the listed groups and are recorded for the sales in the past three months every month.
range to compare their coefficients in the utility function.

Table 2.1 shows the summary statistics of the number of models, characteristics, average smartphone price, and manufacturer retail price of all carrier-OS groups by month. Each carrier has more than 10 Android models each month on average. Windows Phone has the fewest number of models, with monthly average lower than 2. iOS models have the highest carrier contract prices and manufacturer retail prices on average. Windows Phone models have the lowest carrier prices and manufacturer prices.

The pattern of hardware characteristics across OSs is mixed. The iOS models outperform other models in camera pixels and screen pixels per square inch. Android models have the best battery capacities, screen sizes, and processor speeds, and they dominate the sales. Most new iPhone users signed contracts with Verizon and AT&T. T-Mobile only started to sell iPhones from April 2013.

Figure 1.1: Increase of Smartphone Users by Month

Figure 1.1 shows the monthly increases in the number of smartphone subscribers. There are spikes in the new smartphone users during holiday seasons (December and January). To address this issue, the data are seasonally adjusted when estimating consumer demand model. The green curve shows the adjusted monthly increases of smartphone subscribers. The monthly increases are adjusted so that (1) the geometric mean of adjusted sales is the same across months of the year and (2) the total increase of smartphone users from Sep 2009 to Jan 2014 is equal to the
Table 1.2: Smartphone Prices ($100)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Carrier Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS share (lag 1)</td>
<td>-1.04***</td>
</tr>
<tr>
<td>Manufactal price</td>
<td>0.64***</td>
</tr>
<tr>
<td>Dummy iOS</td>
<td>2.92***</td>
</tr>
<tr>
<td>Dummy Android</td>
<td>1.87***</td>
</tr>
<tr>
<td>Dummy Blackberry</td>
<td>2.11***</td>
</tr>
<tr>
<td>Dummy Windows Phone</td>
<td>1.94***</td>
</tr>
</tbody>
</table>

Table 1.2 shows the reduced form estimation results from regressing carrier two-year contract price of smartphones on manufacturer prices, OS shares in the end of the past month, characteristics, OS dummies, and month dummies. Column 2 shows that the carriers’ two year contract prices decrease with OS shares in the last month. The coefficient $-1.04$ means that, if iOS’s share is 10% higher than Blackberry in the period, then an iPhone’s two-year contract price is $10.4 lower than a Blackberry model. This result is consistent with the Proposition 1 in the two-period-two-OS example.

Figure 2.1 shows the cumulative market shares of the four operating systems during the August 2011-October 2013 period. The market shares of iOS and Android have been increasing, while Blackberry have been decreasing. The market shares of iOS and Android both increased from below 6% to above 25%. Blackberry’s market share decreased from 10% to less than 5%. The Windows Phone market share is stable and small, at around 3%.

\[^{22}\text{Following Gowrisankaran and Rysman (2009), I first regress the monthly log increases on the month dummies. Then divide each monthly increase by the exponentiated dummy for its month of the year. The adjusted increase is constructed by multiplying the divided increases by a constant such that the total increase during Sep 2009 to Jan 2014 is the same with that in the original data.}\]
1.7 Identification and Estimation

The structural model parameters are the demand parameters \( \theta_d = (\alpha; \beta; \gamma; \psi; \Phi; \sigma_\xi) \) in the carrier-OS share equation (1.22) and the supply parameters \( \theta_s = (\omega; \kappa; \sigma_\lambda) \) in the carrier first-order condition equation (1.28). I use GMM with MPEC to estimate the parameters. The moment conditions are based on the carrier-OS level unobserved quality shock \( \xi_{sct} \) in equation (1.22) and the model level cost shock \( \lambda_{jsc} \) in equation (1.28).

To fit the data into the structural model, I first calculate model level shares in equation (2.14) using the smartphone model level data on characteristics and prices and data on the number of OS users. The model level shares are then used to calculate \( \xi \) and \( \lambda \). To calculate \( \xi_{sct} \), I aggregate the model level shares to carrier-OS level shares, so that they match the observed carrier-OS shares in the data. To calculate \( \lambda_{jsc} \), I solve the carriers’ FOCs, which also uses the model level shares and their derivatives w.r.t. prices, so that the observed prices are the equilibrium prices in the carriers’ dynamic pricing game model.

Next, I discuss how the data provides identification for the structural model.
parameters and explain how I calculate $\xi_{sct}$ and $\lambda_{jsct}$ in details.

1.7.1 Identification

The identification of the demand side parameters comes from the variation in sales market shares across carrier-OS groups and across periods. The smartphone prices and the plan prices identify the parameters in the supply model.

The identification of the price coefficient $\alpha$ requires instruments, since the carriers’ price $p_{j_{sct}}$ is correlated with the unobserved carrier-OS group quality $\xi_{sct}$. The candidate instruments for model price $p_{j_{sct}}$ include the average characteristics of models in the carrier-OS group $(s, c, t)$, that of other OSs by the same carrier $(-s, c, t)$, that of other carriers $(-c, t)$, and that of all models in period $t$. The characteristics are correlated with prices via the wholesale costs and the competition among the carriers. I assume the characteristics are not correlated with the unobserved quality shock $\xi_{sct}$. This implies that the average characteristics are valid instruments. Individual model characteristics could also be instruments, but I don’t use them because there are many models in one group. Instead, I use the averages by carrier-OS group. Once the endogeneity issue is dealt with, $\alpha$ is identified by the variation in market shares of carrier-OS groups that differ in prices of their models.

The OS network effect coefficient $\gamma$ is identified both by the demand side variation of sales shares across OS networks, and by the carriers’ pricing strategy across different OS networks on the supply side. On the demand side, when everything else is controlled, if a group with a larger OS network size has a higher market share, then the OS network effect is positive and the magnitude can be identified by the market share difference across OSs. On the supply side, as the network effect increases, carriers have stronger incentives to price differentiate across different OS networks.

Controlling for prices, the OS network effect, and the carrier-OS dummies, the characteristics parameter $\beta$ is identified by carrier-OS groups that differ in the characteristics of their models and their sales market shares.

Elements in the diagonal matrix $\Phi$ are the coefficients on the interaction between consumer characteristics and the prices and model characteristics. It is identified by the market shares in periods with varying distribution of the
consumer characteristics. I use the CPS (Current Population Survey) household income distribution each year during 2011 to 2013. The average household income increased from $69,677 to $72,641 and the standard deviation increased from $368 to $499. To see why the income distribution variation identifies $\Phi$, suppose that the average consumer income increases from one period to another, then the difference between the two periods’ product shares identifies the income’s impact on the price coefficient, everything else equal. If the coefficient on the interaction of income and price is zero, then the two periods will have the same market shares. But if the coefficient is positive (less price elastic), then the period with higher level of income will have higher market shares of more expensive smartphones.

The carrier service cost $\kappa_{sc}$ is identified by the average prices in the carrier-OS groups. The structural model predicts the markup and thus the unit cost $c_{jsc}$ for each smartphone model. The unit cost equation (1.23) then provides the identification of $\kappa_{sc}$ and the wholesale price parameters in $\omega$. With the control of the wholesale costs, the cost $\kappa_{sc}$ is identified by the average difference between the unit costs and the wholesale costs over all models in the same carrier-OS group. Similarly, by controlling the service costs, the wholesale price parameters in $\omega$ are identified by the average difference between unit costs and the service costs over models by the same manufacturer.

The standard variances of the unobserved demand and cost shocks, $\sigma_\xi$ and $\sigma_\lambda$, are not directly estimated. Instead, I calculate them after backing out the demand and cost shocks for any given set of the rest parameters. Following Goettler and Gordon (2011), I use $0.975$ as the discount rate $\beta^d$ for a three-month period model.

### 1.7.2 Estimation

#### 1.7.2.1 Moment Conditions and Objective Functions

I construct moment conditions based on the orthogonality between instrumental variables and the unobserved carrier-OS demand shock $\xi_{sct}$, and the unobserved cost shock $\lambda_{jsc}$. The details of calculating $\xi(\theta_d)$ and $\lambda(\theta_d, \theta_s)$ are described in the next subsection.

An endogeneity issue exists in the demand model between the carrier-OS-period specific demand shock $\xi_{sct}$ and the carrier prices, so proper instrument variables

---

23See Appendix D.1 for the details of simulation and normalization of individual income levels.
are required to get consistent estimates. By assumption, there is no endogeneity issue in the supply model because the cost shocks $\lambda_{jst}$ are not correlated with the carriers’ service costs and wholesale costs.

The moment conditions for the unobserved shocks are:

$$E[\xi_{sct}(\theta_d)|Z_{1sct}] = 0,$$

(1.30)

$$E[\lambda_{jst}(\theta_s, \theta_d)|Z_{2jst}] = 0,$$

(1.31)

where $Z_{1sct}$ is a vector of variables that are correlated with prices $p_{jst}^c$ but not correlated with the shock $\xi_{sct}$. It includes the average smartphone characteristics as listed in the identification part. There are 44 moment conditions in (1.30). $Z_{2jst}$ is a vector of variables that are orthogonal to the cost shocks $\lambda_{jst}$. It includes the variables in the unit cost (manufacturer dummies, carrier-OS dummies), the characteristics of $(j, s, c, t)$, the average characteristics over models by carrier $c$ in period $t$, the average over models with the same operating system $s$ in period $t$, and the average over models in the same period $t$. There are 67 moment conditions in (1.31). The number of structural parameters is 58 in the demand and supply model together.

In the estimation, I apply the moment conditions on $\xi_{sct}$ to each model in the carrier-OS group $\Omega_{sct}$, which implies that there is a vector of moment conditions $[Z_{1sct}\xi_{sct}; Z_{2jst}\lambda_{jst}]$ for shocks of model $(j, s, c, t)$. I use two-stage GMM to get efficient estimates.

In the first stage, the weight matrix for the demand side moment conditions is $W_{d1} = (Z_1'Z_1)^{-1}$ and that for the supply side moment conditions is $W_{s1} = (Z_2'Z_2)^{-1}$. Let the first stage estimates be $(\hat{\theta}_d, \hat{\theta}_s)$. The second stage uses the optimal weight matrix estimate $\hat{W}_2$, estimated using the first stage results. The second stage objective function is:

$$Q_2(\theta_d, \theta_s) = \left( \frac{1}{N}\sum_{jst} [Z_{1sct}\xi_{sct}(\theta_d); Z_{2jst}\lambda_{jst}(\theta_s; \theta_d)] \right)'$$

$$\hat{W}_2 \left( \frac{1}{N}\sum_{jst} [Z_{1sct}\xi_{sct}(\theta_d); Z_{2jst}\lambda_{jst}(\theta_s; \theta_d)] \right).$$

(1.32)
1.7.2.2 Calculating the Unobserved Shocks $\xi$ and $\lambda$.

In this subsection, I describe the algorithm of calculating the unobserved carrier-OS specific quality shock $\xi_{sct}$ and $\lambda_{j_{sct}}$. I calculate $\xi_{sct}$ by matching the model predicted carrier-OS market shares to the observed shares. The cost shock $\lambda_{j_{sct}}$ can be backed out from the carriers’ FOCs.

The model market share in equation (2.14) has the integration of individual probabilities over the distribution of consumer characteristics $v_i$. Since $v_i$ is a high dimension vector, I use numerical approximation to calculate the integration. I simulate $N_s = 300$ consumers with different characteristics each period and use the averages of their individual choice probabilities to approximate the sales market share of each model.

$$\tilde{s}_{j_{sct}}(\theta_d) = \frac{1}{N_s} \sum_{i=1}^{N_s} s_{i_{sct}}(\theta_d).$$ (1.33)

Denote the set of models with operating system $s$ and carrier $c$ in period $t$ by $\Omega_{sct}$. The predicted carrier-OS sales share is the sum over all models in the group.

$$\tilde{s}_{sct}(\theta_d) = \sum_{j \in \Omega_{sct}} \tilde{s}_{j_{sct}}(\theta_d).$$

Following [Berry (1994)], it can be shown that the quality shocks $\xi_s$ are uniquely determined by the observed carrier-OS shares for a given $\theta_d$. See Appendix C for the proof of the inversion from the carrier-OS sales shares to $\xi_{sct}(\theta_d)$.

As often used in the random coefficient demand model literature, an iterative procedure is applied to solve for the unobserved shocks $\xi(\theta_d)$. Let the observed carrier-OS shares be $s_{sct}^0$. Given an initial guess of the unobserved demand shocks, $\xi^0 = \{\xi^0_{sct}\}$, calculate the predicted market shares $\tilde{s}_{sct}(\theta_d, \xi^0)$. Then compare the predicted shares with the observed shares. The updating rule is to increase $\xi_{sct}$ if the predicted share is less than the observed market share for the group $(s, c, t)$ and decrease it otherwise. Repeat this updating process until the vector $\xi^k$ converges. The updating process is summarized in the following equation:

$$\xi_{sct}^{k+1}(\theta_d) = \xi_{sct}^k(\theta_d) + \chi (s_{sct}^0 - \tilde{s}_{sct}(\theta_d, \xi^k)),$$

where $\chi$ is a constant, set to be 0.9. The proof of unique fixed point in Appendix C guarantees that the iteration converges to the solution of $\xi(\theta_d)$. 

35
Next, I describe the algorithm for calculating the cost shock $\lambda_{j\text{sc}t}$ for a given value of $(\theta_d, \theta_s)$. I first solve for the smartphone model level markup $m_{j\text{sc}t}$ using data and FOCs, then the calculation of $\lambda_{j\text{sc}t}$ is straightforward, using the definition of markup. As in equation (1.28), the FOC w.r.t. price $p_{j\text{sc}t}$ is:

$$M_t s_{j\text{sc}t}(p^\epsilon_t, \xi_t; \theta_d) + M_t \sum_{(j',s')\in\Omega_{ct}} m_{j's'ct} \frac{\partial s_{j's'ct}}{\partial p^\epsilon_{j'sct}} + \beta_d \frac{\partial V_c(n_{t+1}(n_t, p^\epsilon_t(\xi_t)))}{\partial p^\epsilon_{j't}} = 0.$$  

(1.34)

An important feature of the FOC’s is that the the left hand sides are linear in the model markups $m_{j\text{sc}t}$’s, given prices. The first two terms in equation (1.34) can be calculated using the demand side market share functions. To calculate the last term in equation (1.34), I approximate each carrier’s value function with a linear combination of basis functions, so that that there is explicit functional form of the derivative $\frac{\partial V_c(n_{t+1})}{\partial p^\epsilon_{j't}}$.

Each carrier’s value function is a multivariate function of the four operating systems’ market shares $n_t$. I use the second-order complete polynomials as basis functions to approximate each carrier’s value function.\footnote{The Multivariate Adaptive Regression Spline (MARS) is another method used in empirical literature to approximate multivariate functions. Friedman (1991) proposed this algorithm. There is a Matlab package, ARESLab by Jekabsons (2011), to implement the MARS method to find basis functions. Similar method has been used by Jia and Pathak (2010). The problem of using this method is that the approximated functions are not differentiable in a small number of points where the splines connect. But in my paper, the first-order conditions require the value functions to be differentiable on its domain. So the MARS method is not used in this paper.} Hence, there are 15 basis functions for the four state variables. Denote the basis functions by $Bf(n) = (1, bf_1(n), \ldots, bf_{14}(n))$. Let $\theta^c_v$ be the basis function coefficient vector for carrier $c$. The coefficient vector $\theta^c_v$ is carrier specific.

$$V_c(n_t) = Bf(n_t) \ast \theta^c_v.$$  

Let $\theta^v$ be the vector of approximation coefficients for all carriers’ value functions. With the approximated value functions, model level markups can be solved using the FOCs. Appendix B derives the function form for markups with the approximated value functions. Let $m_{j\text{sc}t}(\theta_d, \theta_s)$ be the solution for the given $(\theta_d, \theta_s)$. The unobserved cost shock is:

$$\lambda_{j\text{sc}t}(\theta_d, \theta_s) = p_{j\text{sc}t}^\epsilon + f_{ct} - m_{j\text{sc}t}(\theta_d, \theta_s) - \omega_{j\text{sc}t}m^m_{j\text{sc}t} - 24\kappa_{sc}.$$  

(1.35)
Therefore, given an alternative of the parameters \((\theta_d, \theta_s)\), the shocks \(\xi_{set}(\theta_d)\) and \(\lambda_{jset}(\theta_d)\) can be solved using observed data and approximation of the value functions \(25\). Next subsection describes the restrictions on the approximation of value functions, which corresponds to the MPEC constraints in the estimation.

1.7.2.3 Equilibrium Constraints

In order to have well approximated value functions, the Bellman equations are imposed as constraints on the approximation parameters \(\theta^v\). The right hand side of the Bellman equations are expectations of the carriers’ discounted profits over \((\xi, \lambda)\), which requires solving for the optimal prices for any possible \((\xi, \lambda)\). Instead of integrating over the distributions of \((\xi, \lambda)\), I simulate \(R\) vectors of the demand and costs shocks \(26\). For each simulated \((\xi^r, \lambda^r)\), I solve for the equilibrium prices \(p_{jset}^{c,r}(\xi^r, \lambda^r)\) using a Newton-Raphson iteration method and take the advantage of the linearity of markups in the price FOCs \(27\). Then I use the average long run profit across simulations to approximate the right hand side of the Bellman equations:

\[
R\text{HS}_{ct}(\theta_d, \theta_s, \theta^v) = \frac{1}{R} \sum_{r=1}^{R} \left[ \sum_{(j,s)} (p_{jset}^{c,r} - \omega_j p_{jset}^m - 24\kappa_{sc} - \lambda_{jset}^r) M_{t}s_{jset}(p_{t}^{c,r}, \xi^r, \theta_d) \right.
\]

\[+ \beta^d V_c(n_{t+1}(n_t, p_{t}^{c,r}), \theta^v) \]

In the estimation, I use inequality constraints which allow some error between the two sides of the Bellman equations \(28\). Barwick and Pathak \(2011\) also imposed

\[25\] When calculating the GMM objective functions, I exclude observations with carrier prices at $0 and T-Mobile models in May 2013-July 2013. For the $0 carrier prices, since $0 is the lower bound of the carrier prices, so they are corner solutions in the optimal equilibrium prices. The corresponding cost shocks, calculated as if $0’s are interior solutions, could be far from the actual shocks. For the T-Mobile models after May 2013, they were sold at manufacturer prices without any contract. There are no carrier discounts for those models. So the cost shocks calculated in equation (1.35) for these observations have a different meaning. Therefore, I exclude the cost shocks of these observations in the GMM objective function.

\[26\] Due to computation time issue, I use \(R = 50\) in the estimation. After I get the parameter estimates, I check the impact of the number of Monte Carlo simulation on the value functions. See Appendix D.2 for details.

\[27\] See Appendix D.3 for the algorithm for solving the equilibrium prices in period \(t\) for a simulated shock pair \((\xi^r, \lambda^r)\).

\[28\] If the value function approximation is ideal, for instance, with infinite basis functions, then the Bellman equations should hold exactly. However, when a finite sequence of basis functions are used, there is inevitably some error between the two sides of each Bellman equation.
Bellman equations as equilibrium constraints when using approximations of the value functions. Specifically, for each carrier $c$ in each period $t$, I impose the following inequality on $\theta^v$.

$$|V_c(n_t; \theta_d, \theta_s, \theta^v) - \tilde{RHS}_{ct}(\theta_d, \theta_s, \theta^v)|/|V_c(n_t; \theta^v)| < \tau.$$  \hspace{1cm} (1.36)

The constraint (1.36) implies that the relative difference between the two sides of each carrier’s Bellman equation at any state $n_t$ should be smaller than the tolerance $\tau$. The smaller is $\tau$, the better the value functions are approximated. In the estimation, the value of $\tau$ is set to be 5% and the constraint is satisfied for all carriers in all periods.\(^{29}\)

I conclude this section by summarizing the GMM estimation objective with MPEC.

$$\min_{\theta_d, \theta_s, \theta^v} \{Q_2(\theta_d, \theta_s, \theta^v)\},$$

subject to (1.36).

where $Q_2$ is the GMM objective function as in equation (1.32).

### 1.8 Estimation Results

Table 3 shows the estimation results for the demand model parameters. The OS network effect coefficient estimate $\hat{\gamma}$ is positive and significant, which means that a consumer’s utility of buying a smartphone increases with the number of existing OS users. The price coefficient $\hat{\alpha}$ is also significant, meaning that consumer utility decreases with the carrier price of smartphones. The interaction between price and income has positive coefficient $\phi_1$, which means that consumers’ disutility of price decreases with income levels.

The coefficients for characteristics imply that consumers’ utility increases with storage, camera pixels, and 4G feature. The coefficients for battery, screen size, and pixels are negative. There are several reasons for the coefficients being negative. First, the best selling models do not have the most advanced characteristics. For example, iPhone 5’s battery is 1540mAh, while most other top models have capacities more than 2000mAh. Similarly, the average screen size for carrier-iOS

\(^{29}\)The estimation results actually imply a better approximation, with 2% maximum difference between the two sides of the Bellman equations for all periods and all carriers.
groups is 3.66 inches, while that for carrier-Android groups is 4.24 inches and that for carrier-Windows Phone groups is 3.94 inches.

Table 1.3: Demand Model Parameter Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Fixed Effects</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS Subscribers (Million), $\gamma$</td>
<td>0.0418***</td>
<td>Verizon-iOS, $\psi_{iv}$</td>
<td>4.1023***</td>
</tr>
<tr>
<td>(0.0007)</td>
<td></td>
<td>(0.8155)</td>
<td></td>
</tr>
<tr>
<td>Carrier Price ($100), $-\alpha$</td>
<td>-0.2731***</td>
<td>Verizon-Android, $\psi_{av}$</td>
<td>2.5591***</td>
</tr>
<tr>
<td>(0.0120)</td>
<td></td>
<td>(0.7282)</td>
<td></td>
</tr>
<tr>
<td>Price*Income, $-\phi_1$</td>
<td>0.01420***</td>
<td>Verizon-Blackberry, $\psi_{bv}$</td>
<td>1.1071*</td>
</tr>
<tr>
<td>(0.0086)</td>
<td></td>
<td>(0.6047)</td>
<td></td>
</tr>
<tr>
<td>Storage (GB), $\beta_1$</td>
<td>0.0310***</td>
<td>Verizon-Windows, $\psi_{wv}$</td>
<td>3.6198***</td>
</tr>
<tr>
<td>(0.0055)</td>
<td></td>
<td>(0.7527)</td>
<td></td>
</tr>
<tr>
<td>Battery (1000mAh), $\beta_2$</td>
<td>-0.3379***</td>
<td>AT&amp;T-iOS, $\psi_{ia}$</td>
<td>4.6184***</td>
</tr>
<tr>
<td>(0.0704)</td>
<td></td>
<td>(0.8458)</td>
<td></td>
</tr>
<tr>
<td>Camera (100MP), $\beta_3$</td>
<td>0.1248***</td>
<td>AT&amp;T-Android, $\psi_{aa}$</td>
<td>0.9682</td>
</tr>
<tr>
<td>(0.0410)</td>
<td></td>
<td>(0.6446)</td>
<td></td>
</tr>
<tr>
<td>Screen Size (inch), $\beta_4$</td>
<td>-1.0008***</td>
<td>AT&amp;T-Blackberry, $\psi_{ba}$</td>
<td>1.5645***</td>
</tr>
<tr>
<td>(0.3951)</td>
<td></td>
<td>(0.5366)</td>
<td></td>
</tr>
<tr>
<td>Dummy 4G, $\beta_5$</td>
<td>0.3522***</td>
<td>AT&amp;T-Windows, $\psi_{wa}$</td>
<td>2.5305***</td>
</tr>
<tr>
<td>(0.1210)</td>
<td></td>
<td>(0.6444)</td>
<td></td>
</tr>
<tr>
<td>Pixels (100/inch$^2$), $\beta_6$</td>
<td>-0.7273***</td>
<td>Sprint-iOS, $\psi_{is}$</td>
<td>1.0753</td>
</tr>
<tr>
<td>(0.0856)</td>
<td></td>
<td>(0.8169)</td>
<td></td>
</tr>
<tr>
<td>RAM (GB), $\beta_7$</td>
<td>0.1821</td>
<td>Sprint-Android, $\psi_{as}$</td>
<td>2.0045***</td>
</tr>
<tr>
<td>(0.1515)</td>
<td></td>
<td>(0.7506)</td>
<td></td>
</tr>
<tr>
<td>CPU (Ghz), $\beta_8$</td>
<td>0.1908</td>
<td>Sprint-Blackberry, $\psi_{bs}$</td>
<td>0.9243</td>
</tr>
<tr>
<td>(0.3116)</td>
<td></td>
<td>(0.6315)</td>
<td></td>
</tr>
<tr>
<td>Pixels (100/inch$^2$), $\beta_6$</td>
<td>-0.7273***</td>
<td>Sprint-iOS, $\psi_{is}$</td>
<td>1.0753</td>
</tr>
<tr>
<td>(0.0856)</td>
<td></td>
<td>(0.8169)</td>
<td></td>
</tr>
</tbody>
</table>

Second, there may be model level unobserved quality across smartphone models, which is not included in the demand model due to data limitation. For example, consider two smartphone models which differ in their pixels, but the model with a
low pixel density has a high unobserved quality, and it has a higher sales market share due to unobserved quality, though consumers do like high pixel density. In this case, the demand model would interpret the data as if consumers do not like high pixel density.\footnote{30}

The carrier-OS fixed effects, $\psi_{sc}$, measure the unobserved characteristics of these groups that are constant over time. The results show that iOS has the highest fixed effect among all OSs. The Windows Phone’s fixed effect is higher than Android. This implies that the high sales market shares of Android is not because Android has high fixed effects. Instead, the reason of Android having high sales shares is because of its number of models and the frontier characteristics of its models.

Given the approximated demand market share equation (1.33), the own and cross product demand elasticities are:

$$\frac{\partial s_{j's'c't}}{\partial p_{j'sc't}} \frac{p_{j'sc't}}{s_{j's'c't}} = \begin{cases} -\frac{p_{j'sc't} + f_{j'sc't}}{s_{j'sc't}} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijsc't} (1 - s_{ijsc't}) & \text{if } (j',s',c') = (j,s,c), \\ -\frac{p_{j'sc't} + f_{j'sc't}}{s_{j's'c't}} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijsc't} s_{ij's'c't} & \text{if } (j',s',c') \neq (j,s,c). \end{cases}$$

Table 1.4 shows the demand elasticities across carrier-OS groups in May 2013. The $(g,g')$th element in the table is the sum of model elasticities in carrier-OS group $g$ when prices of all products in group $g'$ increase by 1%. The own elasticities are stronger than cross elasticities. The own elasticities of Android are the highest, meaning that consumers’ demand of Android models is the most elastic. The own elasticities of iOS are relatively small. The cross elasticities show that Android and iOS are better substitutes than other OSs.

\footnote{30}However, ignoring the model level unobserved quality does not affect the identification of the model characteristics. Because the moment conditions on the carrier-OS unobserved quality are valid as long as the characteristics are exogenous. The carrier-OS unobserved quality includes the model level unobserved quality. Although there may be correlation among the carrier-OS unobserved qualities because that the different carriers may sell the same model, the moment conditions are based on the orthogonality between the carrier-OS unobserved qualities and the observed characteristics.
Table 1.4: Demand Elasticities w.r.t. Prices by Group

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V-iOS</td>
<td>-27.68</td>
<td>2.16</td>
<td>0.00</td>
<td>0.10</td>
<td>2.29</td>
<td>0.97</td>
<td>0.04</td>
<td>0.15</td>
</tr>
<tr>
<td>V-Android</td>
<td>6.80</td>
<td>-75.24</td>
<td>0.01</td>
<td>0.28</td>
<td>6.17</td>
<td>2.66</td>
<td>0.10</td>
<td>0.43</td>
</tr>
<tr>
<td>V-Blackberry</td>
<td>1.20</td>
<td>1.10</td>
<td>-15.45</td>
<td>0.05</td>
<td>1.08</td>
<td>0.47</td>
<td>0.02</td>
<td>0.077</td>
</tr>
<tr>
<td>V-Windows</td>
<td>1.71</td>
<td>1.54</td>
<td>0.00</td>
<td>-20.33</td>
<td>1.55</td>
<td>0.67</td>
<td>0.03</td>
<td>0.11</td>
</tr>
<tr>
<td>A-iOS</td>
<td>2.52</td>
<td>2.16</td>
<td>0.00</td>
<td>0.10</td>
<td>-27.91</td>
<td>0.97</td>
<td>0.06</td>
<td>0.15</td>
</tr>
<tr>
<td>A-Android</td>
<td>7.80</td>
<td>6.82</td>
<td>0.01</td>
<td>0.30</td>
<td>7.08</td>
<td>-87.35</td>
<td>0.11</td>
<td>0.47</td>
</tr>
<tr>
<td>A-Blackberry</td>
<td>2.03</td>
<td>1.90</td>
<td>0.00</td>
<td>0.09</td>
<td>1.84</td>
<td>0.79</td>
<td>-26.22</td>
<td>0.13</td>
</tr>
<tr>
<td>A-Windows</td>
<td>1.56</td>
<td>1.41</td>
<td>0.00</td>
<td>0.06</td>
<td>1.42</td>
<td>0.61</td>
<td>0.02</td>
<td>-18.02</td>
</tr>
<tr>
<td>Outside</td>
<td>0.12</td>
<td>0.12</td>
<td>0.00</td>
<td>0.01</td>
<td>0.11</td>
<td>0.05</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>


The last row in table 1.4 are elasticities of the outside option. The elasticity of the outside option is relatively small compared with other groups. This implies that when prices of models in a carrier-OS group increase, most consumers switch to other groups but not to the outside option.

Table 5 shows the estimates of the supply side parameters. The wholesale cost coefficients $\omega$ are significant, which are defined as the carrier’s wholesale cost divided by the manufacturer retail price. The average wholesale cost ratio is 82.83%. Apple’s wholesale price ratio is 85.01%. It implies that the carriers pay $552 to Apple for a $649 iPhone, which matches anecdotal evidences. Blackberry’s wholesale price ratio is the second highest among all manufacturers at 84.21%. Samsung’s wholesale price ratio is the lowest, at 80.84%. It implies that the carriers pay $484 for a Samsung S3 with manufacturer retail price $599.
Table 1.5: Supply Model Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale Price</td>
<td></td>
<td>Wholesale Price</td>
<td></td>
</tr>
<tr>
<td>Apple, ( b_a )</td>
<td>0.8501***</td>
<td>AT&amp;T-iOS, ( \hat{\kappa}_{ai} )</td>
<td>0.2410***</td>
</tr>
<tr>
<td>(0.0074)</td>
<td></td>
<td>(0.0375)</td>
<td></td>
</tr>
<tr>
<td>Samsung, ( b_s )</td>
<td>0.8084***</td>
<td>AT&amp;T-Android, ( \hat{\kappa}_{aa} )</td>
<td>0.2566***</td>
</tr>
<tr>
<td>(0.0027)</td>
<td></td>
<td>(0.0389)</td>
<td></td>
</tr>
<tr>
<td>Motorola, ( b_m )</td>
<td>0.8151***</td>
<td>AT&amp;T-Blackberry, ( \hat{\kappa}_{ab} )</td>
<td>0.2418</td>
</tr>
<tr>
<td>(0.0028)</td>
<td></td>
<td>(0.3795)</td>
<td></td>
</tr>
<tr>
<td>LG, ( b_l )</td>
<td>0.8305***</td>
<td>AT&amp;T-Windows, ( \hat{\kappa}_{aw} )</td>
<td>0.2407</td>
</tr>
<tr>
<td>(0.0029)</td>
<td></td>
<td>(0.3811)</td>
<td></td>
</tr>
<tr>
<td>HTC, ( b_h )</td>
<td>0.8164***</td>
<td>Sprint-iOS, ( \hat{\kappa}_{si} )</td>
<td>0.2185***</td>
</tr>
<tr>
<td>(0.0026)</td>
<td></td>
<td>(0.0436)</td>
<td></td>
</tr>
<tr>
<td>Blackberry, ( b_b )</td>
<td>0.8421***</td>
<td>Sprint-Android, ( \hat{\kappa}_{sa} )</td>
<td>0.2268***</td>
</tr>
<tr>
<td>(0.0043)</td>
<td></td>
<td>(0.0372)</td>
<td></td>
</tr>
<tr>
<td>Nokia, ( b_n )</td>
<td>0.8354***</td>
<td>Sprint-Blackberry, ( \hat{\kappa}_{sb} )</td>
<td>0.2305</td>
</tr>
<tr>
<td>(0.035)</td>
<td></td>
<td>(0.5933)</td>
<td></td>
</tr>
<tr>
<td>Monthly Service</td>
<td></td>
<td>Monthly Service</td>
<td></td>
</tr>
<tr>
<td>Verizon-iOS, ( \hat{\kappa}_{vi} )</td>
<td>0.2462***</td>
<td>Sprint-Windows, ( \hat{\kappa}_{sw} )</td>
<td>0.2217***</td>
</tr>
<tr>
<td>(0.0207)</td>
<td></td>
<td>(0.2257)</td>
<td></td>
</tr>
<tr>
<td>Verizon-Android, ( \hat{\kappa}_{vi} )</td>
<td>0.2360***</td>
<td>T-Mobile-iOS, ( \hat{\kappa}_{ti} )</td>
<td>0.2121</td>
</tr>
<tr>
<td>(0.0538)</td>
<td></td>
<td>(0.0100)</td>
<td></td>
</tr>
<tr>
<td>Verizon-Blackberry, ( \hat{\kappa}_{vb} )</td>
<td>0.2411</td>
<td>T-Mobile-Android, ( \hat{\kappa}_{ta} )</td>
<td>0.2281***</td>
</tr>
<tr>
<td>(0.2898)</td>
<td></td>
<td>(0.0679)</td>
<td></td>
</tr>
<tr>
<td>Verizon-Windows, ( \hat{\kappa}_{vw} )</td>
<td>0.2381**</td>
<td>T-Mobile-Blackberry, ( \hat{\kappa}_{tb} )</td>
<td>0.2251</td>
</tr>
<tr>
<td>(0.1253)</td>
<td></td>
<td>(0.6919)</td>
<td></td>
</tr>
<tr>
<td>T-Mobile-Windows, ( \hat{\kappa}_{tw} )</td>
<td>0.2262</td>
<td>T-Mobile-Windows, ( \hat{\kappa}_{tw} )</td>
<td>0.2262</td>
</tr>
<tr>
<td>(0.2060)</td>
<td></td>
<td>(0.2060)</td>
<td></td>
</tr>
</tbody>
</table>

The monthly service cost estimates \( \hat{\kappa} \) are at the carrier-OS levels. The average monthly service cost for all carriers is $23.32. Overall, Verizon and AT&T have higher service costs with an average of $24.27. Their monthly markups on wireless services are $39.47 and $35.50, respectively. The average monthly service cost for Sprint and T-Mobile is $22.36. Their monthly markups on wireless services are $37.56 and $27.71 respectively. Sprint has higher markups than AT&T because it has lower service costs.

Given the estimates above, the carriers’ profits from each two-year contract customer can be calculated. Take AT&T and iPhone 5 in December 2012 for example. On one hand, AT&T sells the iPhone 5 at $199 while paying $552 to the manufacturers, which implies a $353 net cost on the phone. On the other hand,
AT&T earns a net margin of $35.50 each month from the customer’s wireless service, which makes $852’s total net margin in the two years’ contract. Therefore, AT&T earns a net profit of $499 from a two-year contract from an iPhone 5 customer over the two years before counting the shocks. Similarly, I can calculate the carriers’ two-year contract markups on all models. Table 1.6 shows the carriers’ markups on two-year contract smartphones by carrier-OS.

Table 1.6: Carriers’ Markups on Two-Year Contract Smartphones by Carrier-OS ($100)

<table>
<thead>
<tr>
<th></th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>iOS</td>
<td>5.89</td>
<td>5.13</td>
<td>5.59</td>
<td>—</td>
</tr>
<tr>
<td>Android</td>
<td>6.72</td>
<td>5.50</td>
<td>6.26</td>
<td>4.01</td>
</tr>
<tr>
<td>Blackberry</td>
<td>7.39</td>
<td>5.65</td>
<td>6.52</td>
<td>4.19</td>
</tr>
<tr>
<td>Windows</td>
<td>6.81</td>
<td>5.95</td>
<td>5.98</td>
<td>4.26</td>
</tr>
</tbody>
</table>

Across operating systems, the carriers’ two-year contract markups on Blackberry and Windows Phone models are higher than iOS and Android models in general. This is because that 1) the carriers give lower discounts on smaller operating systems and 2) the carriers pay lower wholesale prices to manufacturers. Among the carriers, Verizon has the highest markups among all carriers. The reason is that Verizon has the highest margins on wireless service. Sprint’s average markups on contracts are higher than AT&T because Sprint has lower service cost estimates. T-Mobile’s markups are the lowest among all carriers due to their low margins on services.

The carriers’ value functions at any state can be calculated with the polynomial approximation. The value functions are plotted in Appendix E. Verizon’s value function has the highest value among all carriers. AT&T has the second highest value function. T-Mobile’s value function has the lowest value.

1.9 Counterfactuals

In this section, I study two counterfactual scenarios to measure the impact of the OS network effect on consumers’ demand for smartphones and the OS market concentration. I eliminate the OS network effect and the carrier discounts on two-year contracts in the two scenarios, respectively.
1.9.1 The Impact of the OS Network Effect

To find out the impact of the OS network effect on the carriers’ prices and the market concentration of operating systems, the OS network effect is eliminated in this counterfactual. This has two effects on the carriers’ prices. First, without the OS network effect, the carriers do not have the incentive to give higher discounts to larger OSs. Second, consumers’ utility of a smartphone decreases when there is no OS network effect.

Without the OS network effect, consumer $i$’s utility is:

$$u_{ijsc} = x'_{jsc} \beta_i - \alpha_i(p_{jt} + f_{ct}) + \psi_{sc} + \hat{\xi}_{sct} + \epsilon_{ijsc}$$

in which $\hat{\xi}_{sct}$ is the unobserved carrier-OS specific utility calculated using the estimates. For any given price vector $p_t^c$, the market share of each model $s_{jsc}^t(p^c_t)$ can be derived similarly as in the demand model.

The carriers play a static pricing game without the OS network effect. Carrier $c$’s profit in period $t$ is:

$$\bar{\pi}_{ct}(p_t^c; \hat{\xi}_t, \hat{\lambda}_t) = \sum_{(j,s) \in \Omega_{ct}} (p_{jsc}^c + f_{ct} - \omega_j p_{jsc}^m - 24 \kappa_{sc} - \hat{\lambda}_{jsc}) s_{jsc}^t(p^c_t; \hat{\xi}_t) \bar{M}_t,$$

where $\bar{M}_t$ is the new market size at the beginning of period $t$: $\bar{M}_t = (1 - \frac{7}{8} n_t) M$.

<table>
<thead>
<tr>
<th>Carrier-OS Group</th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>iOS</td>
<td>2.13</td>
<td>2.98</td>
<td>2.44</td>
<td>—</td>
</tr>
<tr>
<td>Android</td>
<td>1.34</td>
<td>2.62</td>
<td>1.76</td>
<td>4.06</td>
</tr>
<tr>
<td>Blackberry</td>
<td>0.79</td>
<td>2.48</td>
<td>1.60</td>
<td>3.89</td>
</tr>
<tr>
<td>Windows</td>
<td>1.30</td>
<td>2.19</td>
<td>2.10</td>
<td>3.77</td>
</tr>
</tbody>
</table>

I solve for the new equilibrium prices using the carriers’ first-order conditions. Table 1.7 shows the average price decreases by carrier-OS group. The carriers’ prices increase in the static pricing game compared with the data. This is because, the carriers don’t have incentives to use low prices to build up networks when there is no network effect. The prices of iOS and Android models increase more
than Blackberry and Windows prices. Without the OS network effect, the carriers would not give high discounts for large OSs because OS network sizes do not affect consumer demand.

Figure 1.3 compares the cumulative OS market shares with data. The bold curves show the OS shares without OS network effect and the dashed curves represent the OS shares in the data. Without the OS network effect, the smartphone penetration rate decreases for two reasons. First, consumers’ utility of smartphone decrease in general because the OS network effect no long exists. Second, the carriers increase their prices in static models. Both effects reduce consumers’ demand for smartphones. The aggregate smartphone penetration in the counterfactual case decreases from 45.06\% in Aug 2011 to 23.67\% Aug 2013. While in the data, the smartphone penetration rate increased from 45.06\% in Aug 2011 to 78.36\% in Aug 2013.

![Figure 1.3: OS Growth without OS Network Effect](image)

The OS market is less concentrated without the OS network effect. The market shares of iOS and Android decrease from more than 30\% to below 10\%. The joint market share of Blackberry RIM and the Windows Phone is 6.75\%, compared with
6.39% in the data. The OS market becomes less concentrated for two reasons. First, the large OSs do not have the “initial installed base advantage” anymore. This implies that, given the characteristics and prices, consumers are equally likely to buy products with large or small OS networks. Second, the carriers do not have the incentive to choose lower prices to the large networks anymore. Thus, the OS concentration slows down.

Table 1.8 shows the profit comparison by carrier. Without the network effect, the carriers’ total profit decreases by $53.84 billion (79.13%). This huge profit loss is caused by the decrease in consumer demand. The profit losses for Verizon, AT&T, Sprint, and T-Mobile are 76.11%, 78.28%, 84.04%, and 85.97% respectively.

<table>
<thead>
<tr>
<th></th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated</td>
<td>28.61</td>
<td>20.95</td>
<td>11.72</td>
<td>6.77</td>
</tr>
<tr>
<td>No OS network effect</td>
<td>6.83</td>
<td>4.55</td>
<td>1.87</td>
<td>0.95</td>
</tr>
</tbody>
</table>

### 1.9.2 The Impact of the Carrier Discounts

This counterfactual case studies the importance of the two-year contracts. I consider a scenario in which there is no two-year contract, consumers pay the manufacturer retail prices for smartphones, but carriers’ monthly service prices decrease by $15 each month. To make this counterfactual results comparable with the full model, consumers are assumed to use smartphones for two years. For example, a consumer can buy the iPhone 5 at the price of $649 and pay $360 less on wireless service for the two-year usage.

Suppose that the new plan price is $p_{ct}$ for carrier $c$. By assumption, $p_{ct} = f_{ct} - 3.6$ (in $100$ units). Since the consumers are paying the manufacturer retail price $p_{m_{j = c} t}$, the utility function is:

$$u_{ijset} = x_{j = c, t}^{i} - \alpha(p_{m_{j = c} t} + p_{ct}) + \psi_{sc} + \hat{\xi}_{sct} + \epsilon_{ijset},$$

where $\hat{\xi}_{sct}$ is the unobserved carrier-OS specific utility calculated using estimation results. The market shares can be derived from this utility function. Let the

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31In fact, carriers started to sell smartphones at the manufacturer prices and decrease monthly service prices by at least $15 from late 2013.
corresponding sales market shares be $\hat{s}_{j_{\text{set}}}$.

Though the service prices decrease, consumers actually may pay more on most smartphones in the counterfactual than in the data. Take the AT&T 16GB iPhone5 in December 2012 for example. A consumer gets a $450 discount if sign the two-year contract, while s/he only saves $360 on the service price. In this case, the consumer pays $90 more without the discount on contract.

Figure 4 compares the OS growth paths in this counterfactual case with the data. The overall smartphone penetration rate without the two-year contract is 49.30% (86.52 million), compared with 78.36% (137.51 million) in the data by the end of May 2013. The reason is that consumers now pay higher total prices on the most popular smartphone models. Without the two-year contract, the decrease in service price is only $360, which is less than the two-year contract discount on those models. Therefore, the carriers' discounts on two-year contracts contributed to the smartphone penetration.

Figure 1.4: OS Growth without two-year Contract

iOS and Android would still dominate the market for several reasons. First, they have initial OS network advantages. Second, iOS has high fixed effects across
carriers, so consumers’ demand for iPhones is relatively high compared with other OSs. However, Blackberry RIM and the Windows Phone together share 9.17% of the market, instead of 6.39% in the data. The gap between Android and Windows Phone market shares decreases from 28.57% in the data to 11.18%. It implies that the carrier discount also contributed to the concentration of the OS market shares.

The profit for carrier $c$ in period $t$ is:

$$
\pi_{ct}(p_t^c, \hat{\xi}_t, \hat{\lambda}_t) = \sum_{(j,s) \in \Omega_{ct}} \left( p_{jsc}^m - \omega_j p_{jsc}^m + f_{ct} - 24\kappa_{sc} - \hat{\lambda}_{jset} \right) s_{jset} M_t,
$$

where $(\hat{\xi}_{set}, \hat{\lambda}_t)$ are the shocks in period $t$.

Table 1.9 compares the profits in the counterfactual scenario with the profits estimated from the original model. The overall profit decreases by $23.52$ billion (34.91%) for the 4 carriers combined from Aug 2011 to Aug 2013.

<table>
<thead>
<tr>
<th></th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated</td>
<td>28.61</td>
<td>20.95</td>
<td>11.72</td>
<td>6.77</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>18.43</td>
<td>13.26</td>
<td>8.14</td>
<td>4.46</td>
</tr>
</tbody>
</table>

The two counterfactual cases imply that both the OS network effect and the carriers’ discounts on the two-year contracts have contributed significantly to the growth of the smartphone industry in the US and the concentration of the operating system networks. The OS network effect plays a relatively more important role than the two-year contract discounts.

1.10 Conclusion

The literature on the network effect has focused on the pricing problems of single-network firms, but not the prices of multi-network firms. In this paper, I analyze the impact of a network effect on multi-network retailers’ dynamic prices empirically. I first use a two-period, two-OS model to compare the price for the large OS with that for the small OS. I show that, when network effects exist, single-network firms and multi-network firms have quite different dynamic pricing strategies. While larger networks have higher prices in the single-network firm case, a multi-network firm
chooses lower prices for the products with larger networks. The difference in their pricing strategies is a result of the difference in their competition environments.

The above pricing strategy of retailers is present in the smartphone industry, where the smartphone operating system network effect exists and telecom carriers act like multi-network firms. I estimate a structural model of consumers’ demand and carriers’ dynamic pricing of smartphones, to quantify the OS network effect and measure its impact on carrier prices. I use smartphone model level data during 2011 to 2013 to estimate the structural model. The results show that there is positive and significant OS network effect. This implies that consumers’ utility and thus carriers’ prices are affected by the OS network sizes. With the estimates, I study two counterfactual cases in which I eliminate the OS network effect and the carriers’ two-year contract policy, respectively. The results show that both the OS network effect and carriers’ two-year contract discounts significantly accelerated the smartphone industry’s growth and the concentration of operating systems. Furthermore, the OS network effect is relatively more important than the carrier discounts to both smartphone growth and OS concentration.

This paper also makes a contribution by estimating a structural model of dynamic pricing of multiple products with high-dimension continuous state variables and asymmetric oligopolistic firms. The existing estimation algorithms for discrete choice dynamic games or continuous choice games with single-product firms can not be used in this paper. I solve the carriers’ dynamic pricing game by approximating the carriers’ value functions with basis functions. I develop an iterative procedure to efficiently solve for the equilibrium prices. As a result, the oligopolistic multiple-product carriers’ asymmetric pricing game can be solved efficiently within the estimation algorithm.
Chapter 2  
Consumers’ Dynamic Demand for Smartphones with Two-Year Contracts and Operating System Network Effect.

2.1 Introduction

Network effects play an important role in the evolution of market structure when different networks are not compatible. If networks are compatible, then the pooled network only affect aggregate penetration, but not the market concentration, which always draws economists’ attention and is a very important measure in policy making. This paper studies the impact of network effect on market concentration with incompatible networks in a dynamic framework. In a dynamic model with network effects, the network sizes evolve endogenously via consumers’ adoption decisions, and consumers make dynamic adoption decisions while predicting the endogenous network sizes. The dynamic model is applied to the smartphone industry, in which different operating systems (OS) are incompatible.

In addition to smartphone operating system network effect, another important policy that affects consumers’ dynamic demand of smartphones is the two-year contract associated with telecom carriers. The two-year contracts lock in consumers for two years with the same smartphone model and same carrier. A consumer’s current contract status affects his/her utility of buying a new smartphone. To switch to another smartphone or another carrier, existing users have to end their current unfinished contracts, but the carriers charge high early termination fees if customers terminate current contracts within two years. If a consumer has already been in the current contract for no less than two years, he/she is free of charge to switch to another model or another carrier. Therefore, the current contract status
makes an important impact on a consumer’s demand of new smartphones.

In this paper, I empirically address the two issues above together: the network effect’s impact on network concentration with incompatible networks and the two-year contract’s impact on consumer dynamic demand of smartphones.

In the literature of estimating network effects, many papers have adopted static demand models and a few used dynamic demand models. The papers that study network effects in static models include Park (2004), Rysman (2004), Ackerberg and Gowrisankaran (2006), Ryan and Tucker (2012), Nair, Chintagunta, and Dubé (2004), and so on. As shown by Chen, Esteban, and Shum (2008) and Gowrisankaran and Rysman (2009), the static models generate biased estimates when the true consumption decisions were dynamic. Hence, a dynamic model is necessary to get consistent estimates of the demand model.

A few papers have studied network effects in dynamic demand frameworks. The most relevant paper is Gowrisankaran, Park, and Rysman (2010). They measure the indirect network effect of DVD players, but they couldn’t study the network effect’s impact on network concentration since the DVD players are all compatible in their paper. The dynamic motivation of consumer’s demand in their paper heavily depends on a convex disutility function of owning multiple DVD players. Besides, though they deal with the endogeneity issue of available DVD titles, the growth of the titles is not endogenously modeled. However, in the this paper, the OS networks are incompatible across different smartphone operating systems, and the growth of networks is endogenous modeled.

Dubé, Hitsch, and Chintagunta (2010) and Lee (2013) also model consumers’ dynamic adoption decisions of video game consoles in dynamic models. Dubé, Hitsch, and Chintagunta (2010) study the tipping and market concentration in the video game industry with two duopoly firms. This paper differs from theirs in modeling consumers’ beliefs and the estimation method. Lee (2013) analyzes the vertical relationship between video game titles and consoles, while this paper focuses on the market concentration with network effect and the two-year contract’s impact on consumers’ dynamic demand.

This paper also fills the gap of studying the effect of the two-year contract policy in the literature on the smartphone industry. Parker and Van Alstyne (2010) analyze the innovation and the platform openness control. Zhu, Liu, and Chintagunta (2011) and Sinkinson (2011) study the incentive and the impacts of

In addition, this paper contributes to the literature on the static model estimation bias for underlying dynamic data. The literature has mixed findings about the bias direction. Chen, Esteban, and Shum (2008) find that the static estimated of the elasticity of demand is an overestimate of the true elasticity in the car industry. Gowrisankaran and Rysman (2009) find that “by incorrectly using prices instead of the difference in price, a static estimation applied to a durable good purchase decision with falling prices will result in mismeasurement that may tend to bias the price coefficient towards zero.” This paper find that the static model underestimate the OS network effect and over estimate the price disutility.

Section 2 describes the background of the US smartphone industry. Section 3 introduces the data used in this paper. In section 4, I model consumers’ dynamic demand of smartphones with two-year contracts. Section 5 discusses the identification and estimation algorithm of the model parameters. The estimation results are presented in section 6.

2.2 Background of the U.S. Smartphone Industry

A smartphone is a mobile electronic device which runs an operating system that is open to installing new applications, is always connected to the internet, and which provides very diverse functionality to the consumer.

The major manufacturers in the U.S. smartphone market include Apple Inc. (a U.S. corporation), Research in Motion Limited (or RIM, a Canadian corporation), HTC Corporation (a Taiwanese corporation), Motorola, Inc. (a U.S. corporation), Samsung Electronics Co. Ltd. (a subsidiary of the Korean corporation Samsung Group), LG Corp. (a Korean corporation), and Nokia Corporation (a Finnish corporation).

The top four operating systems in the U.S. smartphone industry are Android, iOS, Blackberry, and Windows Phone. The combined market share of the four
increased from 94% to 99% during 2011 to 2014. Smartphone operating systems have three forms: (1) proprietary, (2) licensable, and (3) open source.

Proprietary OS’s are developed in-house by smartphone manufacturers that manage the device development. Apple and RIM take this approach with their iOS and Blackberry. The proprietary approach gives the manufacturer a potential competitive edge over rivals as it allows the manufacturer to differentiate from others. It also allows them to more tightly integrate the function of the OS and the hardware of the phone. This comes at a high cost, as the software development is time consuming and expensive.

Licensable OS’s allow any manufacturer to use the OS for a device they produce. Microsoft’s Windows Phone OS is in this form. Licensable OS’s are largely used “as-is” with a manufacturer’s hardware, although some customization is available. Smartphone manufacturers chose a licensable OS because of the high cost of developing a operating system. They can also take advantage of the existing ecosystem of OS users.

Open source OS’s give the smartphone manufacturer access to an existing operating system this is free and freely customizable. Android is the most popular open source OS. Android was released by Google under a license that allows anyone to use the OS and include adding enhancements, without contributing those enhancements back.

Software applications are a significant part of the smartphone market. Every smartphone operating system has an online store where apps can be purchased and downloaded to extend the functionality of the smartphone. The proprietary OS’s have app stores that are exclusive to the operating system. Licensable or open source OS’s have app stores that work with any device that runs the OS, no matter the manufacturer. The app stores for the four OS’s are: The Google Play(for Android), The App Store (for Apple iOS), BlackBerry App World (for BlackBerry), and Windows Phone Marketplace (for Windows Phone).

Consumers find great value in apps because of the additional functionality of their smartphones. While many apps are free from the app stores, many quality apps need to be purchased. OSs with more users are more attractive to the app developers to increase revenue and profit. The fact that there is a loop of how consumers and app developers value each other generates the indirect network effect at the operating system level.
A smartphone device only become useful when mated with a wireless service provider that allows the consumer to make calls and access data such as email and the internet. Thus, a consumer has to choose both the smartphone model and the service provider when buying a smartphone. The top four service providers in the U.S. are Verizon Wireless, AT&T Mobility, Sprint Corporation, and T-Mobile US. They have varying degrees of network coverage and different pricing plans for service plans.

The wireless service providers offer subsidized prices on smartphones if consumers purchase with long-term contracts, usually two-year long. The contracts require consumers to stay with their providers to avoid large early termination fees. According to the Statista.com, the average monthly churn rate for the four wireless carriers is 2%. This means that consumers rarely switch carriers.

The combined share of smartphone sales for the big four carriers (service providers) during Oct. 2011 to Nov. 2013 is 88.72%. The average sales share of Verizon is 33.26%, 28.68% for AT&T, 15.58% for Sprint, and 11.10% for T-Mobile.

According to the US Wireless Industry Overview 2011, more than 78% users are on two-year contracts. This number include both the feature phones and smartphone subscribers. The percentage is expected to be even higher for the smartphone sub-market because smartphones are much more expensive if bought without a contract. Hence, this paper focuses on the consumers’ demand of smartphones with two-year contract.

2.3 Data

The data comes from several sources. The website comScore.com reports the U.S. cumulative smartphone subscriber market shares every month. The sales market shares for the past three months are published every month by the Kantar World Panel. The carrier prices and manufacturer retail prices are collected via the web archive website. The smartphone characteristics data are collected from the phonearena.com. The data sample period is from Aug 2011 to Oct 2013.

The four leading telecom carriers are Verizon, AT&T, Sprint, and T-Mobile. Their combined sales market shares during the sample period is 90% on average. The four operating systems are iOS, Android, Blackberry, and Windows Phone. Their combined sales market share is more than 99%. During the data period, the
average number of smartphone models that the four carriers have are 25, 28, 19, and 17, respectively.

The comScore.com data reports the total number of smartphone subscribers and the cumulative market share of each operating system $s_{it}^o$ every month. The total number of smartphone subscribers and the cumulative operating system shares are used to calculate the market size $M_t$ every period.

The Kantar World Panel’s sales market shares are for the past three months. For example, at the beginning of Feb 2012, it published the sales market shares for the 3 months ending in Jan 2012. It reports the sales market share of each operating system by carrier. In addition, it also reports the OS sales shares of AT&T and Verizon. Combining the sales market shares by carrier and the OS sales within each carrier, I get the sales market share of each carrier-OS group. One missing piece of the sales data is the OS sales market shares within Sprint or T-Mobile. Since only the combined OS sales market share for the two carriers are observed, I construct the sales market shares for them to be proportional to the number of models they have on different operating systems. In the end, there are sales market shares for 16 carrier-OS groups for 26 months, including 8 periods of 3-month.

The web archive website has been archiving the carriers’ webpages several times every month since 2008. The carriers’ price of each smartphone on contract and the listed manufacturer retail price of it can be collected by month. In the sample period, the data have 2283 model-month observations. The highest two-year contract price is $399 for the 64 GB iPhones.

Since the sales market shares are observed for the past three months, a period is three months in the demand model to match the sales shares. However, smartphone models and their prices are observed every month. To use as much information in the prices as possible, I construct the choice set every period in the following way. If a smartphone model is observed in multiple months in a period (three month), this paper treats them as different choice options in that period. Dealing with data this way, the consumers are allowed to behave “dynamically” within the three months of a period and have perfect information.

The smartphone characteristics include camera pixels, built-in storage, 4G dummy, weight, screen size, resolution, processor speed, system memory, and battery capacity. All the characteristics are scaled to be in similar range.
Figure 2.1: The Cumulative OS Market Shares (2009.09-2014.03)

Figure 2.1 shows the cumulative market shares of the four operating systems during the longer period from Q3 2009 to Q1 2014. The market shares of the Blackberry OS increased until early 2011 and then decreased continuously. The market shares of iOS and Android have been increasing since the beginning. And their market shares both increased from below 6% to above 25%. The Windows Phone market share is stable and small, at around 3%.

Table 2.1 lists the summary statistics of the number of models, characteristics, two-year contract price of smartphones, manufacturer retail prices, and conditional sales market shares by carrier-OS groups by month. All carriers have more than 10 Android models each month on average. Windows Phone has the least number of models. iOS models have the highest two-year contract prices and manufacturer retail prices on average. Windows Phone models have the lowest carrier prices and manufacturer prices.

The pattern of hardware characteristics comparison across operating systems is mixed. iOS models outperform other models in camera pixels and screen pixels. Android models have the best battery capacities, screen sizes, and processor speeds. iOS and Android models dominate the sales. Most new iPhone users signed contracts with Verizon and AT&T. Android models contributed most to the sales of each carrier.
Table 2.1: Descriptive Statistics by Carrier-OS group by Month

<table>
<thead>
<tr>
<th>Group</th>
<th>No. of Models</th>
<th>Contract Price 100$</th>
<th>Manuf. Price 100$</th>
<th>Battery 1000mAh</th>
<th>Camera megapixels</th>
<th>Screen inches</th>
<th>Pixel 100/inch²</th>
<th>Processor Ghz</th>
<th>Sales%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verizon-iOS</td>
<td>4.69</td>
<td>2.26</td>
<td>6.74</td>
<td>1.43</td>
<td>7.14</td>
<td>3.65</td>
<td>3.26</td>
<td>1.72</td>
<td>17.80</td>
</tr>
<tr>
<td>Verizon-And</td>
<td>16.15</td>
<td>1.22</td>
<td>5.04</td>
<td>1.92</td>
<td>6.86</td>
<td>4.24</td>
<td>2.68</td>
<td>2.60</td>
<td>18.94</td>
</tr>
<tr>
<td>Verizon-Bla</td>
<td>3.00</td>
<td>1.45</td>
<td>4.66</td>
<td>1.31</td>
<td>5.03</td>
<td>3.01</td>
<td>2.60</td>
<td>1.34</td>
<td>0.47</td>
</tr>
<tr>
<td>Verizon-Win</td>
<td>1.81</td>
<td>1.03</td>
<td>4.32</td>
<td>1.49</td>
<td>5.68</td>
<td>3.94</td>
<td>2.45</td>
<td>1.61</td>
<td>0.08</td>
</tr>
<tr>
<td>AT&amp;T-iOS</td>
<td>5.07</td>
<td>1.98</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>3.11</td>
<td>1.66</td>
<td>20.75</td>
</tr>
<tr>
<td>AT&amp;T-And</td>
<td>13.88</td>
<td>0.98</td>
<td>4.69</td>
<td>1.90</td>
<td>7.04</td>
<td>4.31</td>
<td>2.66</td>
<td>2.98</td>
<td>9.58</td>
</tr>
<tr>
<td>AT&amp;T-Bla</td>
<td>3.81</td>
<td>0.86</td>
<td>4.48</td>
<td>1.25</td>
<td>5.30</td>
<td>3.07</td>
<td>2.59</td>
<td>1.25</td>
<td>1.4</td>
</tr>
<tr>
<td>AT&amp;T-Win</td>
<td>4.81</td>
<td>0.75</td>
<td>4.36</td>
<td>1.67</td>
<td>7.82</td>
<td>4.26</td>
<td>2.47</td>
<td>1.81</td>
<td>1.36</td>
</tr>
<tr>
<td>Sprint-iOS</td>
<td>4.91</td>
<td>2.04</td>
<td>6.71</td>
<td>1.43</td>
<td>7.36</td>
<td>3.66</td>
<td>3.28</td>
<td>1.79</td>
<td>3.07</td>
</tr>
<tr>
<td>Sprint-And</td>
<td>11.38</td>
<td>0.90</td>
<td>4.41</td>
<td>1.80</td>
<td>5.77</td>
<td>3.97</td>
<td>2.45</td>
<td>2.03</td>
<td>11.98</td>
</tr>
<tr>
<td>Sprint-Bla</td>
<td>1.65</td>
<td>1.30</td>
<td>4.53</td>
<td>1.18</td>
<td>5.06</td>
<td>2.68</td>
<td>2.76</td>
<td>1.11</td>
<td>0.38</td>
</tr>
<tr>
<td>Sprint-Win</td>
<td>1.13</td>
<td>0.79</td>
<td>4.42</td>
<td>1.57</td>
<td>5.60</td>
<td>3.77</td>
<td>2.57</td>
<td>1.36</td>
<td>0.45</td>
</tr>
<tr>
<td>TMobile-iOS</td>
<td>4.00</td>
<td>—</td>
<td>6.96</td>
<td>1.44</td>
<td>7.94</td>
<td>3.98</td>
<td>3.26</td>
<td>2.55</td>
<td>0.36</td>
</tr>
<tr>
<td>TMobile-And</td>
<td>10.62</td>
<td>1.58</td>
<td>4.05</td>
<td>1.77</td>
<td>6.10</td>
<td>5.19</td>
<td>2.43</td>
<td>2.62</td>
<td>11.44</td>
</tr>
<tr>
<td>TMobile-Bla</td>
<td>3.69</td>
<td>2.01</td>
<td>4.46</td>
<td>1.39</td>
<td>5.19</td>
<td>2.94</td>
<td>2.61</td>
<td>1.19</td>
<td>0.83</td>
</tr>
<tr>
<td>TMobile-Win</td>
<td>1.31</td>
<td>1.15</td>
<td>3.49</td>
<td>1.46</td>
<td>5.80</td>
<td>3.91</td>
<td>2.48</td>
<td>1.89</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Notes: The first three letters are used to denote the operating system. For example, “And” is for Android. T-Mobile eliminated the two-year contract policy since it started to sell iOS models. The sales shares are conditional shares among the listed groups and are recorded for the sales in the past three months every month.
2.4 Dynamic Demand Model

2.4.1 Consumers’ Dynamic Problem

In this section, I model consumers’ dynamic demand of smartphones with two-year contracts. In each period, a consumer has an initial state of his/her smartphone status, and chooses whether to buy a new phone with a new two-year contract or not.

At the beginning of each period $t$, a consumer’s state variables are the flow utility of his/her choice from last period $f_{0t}$, the number of periods to go if still in a two-year contract, $l_t$, the operating system of the current phone $o_t$, the market share of the current operating system $s_{t-1}$, and the information set $H_t$ about the smartphone models, prices, characteristics, and network sizes of all operating systems.

A consumer’s initial status could be having a smartphone or not having any. If a consumer doesn’t have a smartphone, then there is no contract, which implies $l_t = 0$, and the initial operating system is $o_t = 0$. If a consumer has a smartphone on contract at the beginning of period $t$, then the number of periods to go in a contract is $l_t \in \{0, 1, ..., L\}$, in which $L$ is the maximum periods left in a contract. The operating system status is $o_t \in \{1, 2, ..., S\}$. $S$ is the number of operating systems.

The market size of smartphone consumers is assumed to be 65% of the population, percentage of the population between 15 years old (starting of high school) and 65 years. Denote the market size by $M$. All consumers enter the market and choose from the choice set in each period. The choice set in period $t$ includes all available smartphone models and the outside option: $\Omega_t = \{(j, s, c, t)\}_{j, s, c} \cup \{(0, t)\}$, where $j$ is a smartphone model, $s$ is the operating system, $c$ is the carrier, and $t$ is the period. $(0, t)$ is the outside option, which means do not buy any new smartphone.

A consumer’s current smartphone contract status makes an importance impact on his/her demand in each period. If a consumer has been in the current contract for no less than two years or doesn’t own a smartphone yet, which means that $l_0 = 0$, then there is no termination fee if a consumer buys a new smartphone. However, if a consumer has only been being in the current contract for less than two years, which means that $l_t > 0$, then an early termination fee $T + a * l_t$ applies.
if buying a new smartphone, no matter from the same carrier or a different carrier. $T$ is a fixed fee and $a$ is a per month charge. They are assumed to be the same across carriers. Consumers get utility from the OS network size $s_{os_{t-1}} M$ due to network effect. This utility includes utility from both direct and indirect network effect at operating system level. On one hand, the direct network effect arises from the convenience of communicating among family members or friends who use the same operating system. For example, two iOS users can make free and high quality video calls via FaceTime and can easily share pictures through PhotoStream or AirDrop. On the other hand, smartphone operating systems serve as exclusive platforms for consumers and the app developers. Consumers value the variety and quality of apps, and the app developers value the user base of each operating system in return. Hence, an indirect network effect also exists.

A consumer pays the smartphone wireless service plan price $p^c_t$ each period, if he/she uses a smartphone in that period and the service is provided by carrier $c$. The service plan price is carrier specific and assumed to be constant over time. During the data period, 2011-2013, the carriers’ service prices change once a year maximum. In this paper, I take the service prices to be exogenous.

2.4.1.1 Choose the Outside Option

The outside option of a consumer is to not buy a new smartphone. Consider a consumer $i$ with state $(f_{0t}, l_t, o_t, s_{os_{t-1}})$ at the beginning of period $t$, where I subtract the individual subscript for notation simplification. If consumer $i$ doesn’t have a smartphone at the beginning of period $t$, $o_t = 0$, then the outside option means to use a feature phone in period $t$. If consumer $i$ has a smartphone, then $o_t \in \{1, 2, ..., S\}$, and the outside option means to use the same smartphone from last period, and the contract is fulfilled by one more period if there is any left.

Consumer $i$’s utility of choosing the outside option in period $t$ is:

$$u_{i0t} = f_{0t} + I(o_t > 0)\gamma s_{os_{t-1}} M + \epsilon_{i0t},$$

(2.1)

where $f_{0t}$ is the flow utility of the existing phone. If a consumer has a smartphone,
then there is utility from the OS network size $\gamma s_{ot-1}^s M$, where $\gamma$ measures the network effect strength. $\epsilon_{itot}$ is a random utility shock.

If a consumer has a smartphone at the beginning of period $t$, then the flow utility of the outside option in period $t$ is:

$$f_{ot} = x_0 \beta - \alpha p_0^c,$$

where $x_0$ is a vector of the smartphone characteristics and $p_0^c$ is the carrier’s wireless service price per period. If a consumer doesn’t have a smartphone at the beginning of period $t$, then the flow utility of the outside option is normalized to be zero.

$$f_{ot} = 0,$$

if doesn’t own a smartphone initially.

The state variables $s_{ot+1}^s$ and $H_t$ are determined by the choices of all consumers in period $t$. The state variable updating rules will be discussed later. For the state variables $(f_{ot}, l_t, o_t)$, then the updating rules if choosing the outside option are:

$$f_{ot+1} = f_{ot},$$

$$o_{t+1} = o_t,$$

$$l_{t+1} = \max\{l_t - 1, 0\}.$$  \hspace{1cm} (2.2)

The outside option flow utility and the operating system doesn’t change since the consumer didn’t buy any new smartphone. If the consumer still have multiple periods left in a contract, then the contract is one more period closer to end. If the consumer’s contract is already finished, then $l_{t+1}$ stays at zero.

2.4.1.2 Buy a New Smartphone

Consumer $i$’s utility of buying a new model $j$ with operating system $s$ from carrier $c$ in period $t$ is:

$$u_{jset} = f_{jkc} - \alpha (p_{jset}^c + (T + a_l)I(l_t > 0)) + \gamma s_{st-1}^s M + \xi_{set} + \epsilon_{ijset}.$$  \hspace{1cm} (2.3)

$f_{jkc}$ is the flow utility of using smartphone $(j, s, c)$, and is constant over time since the phone characteristics don’t change. $p_{jset}^c$ is the carrier’s price of the smartphone.

---

2 Consumers are allowed to buy the same model repeated in the model.
with a two-year contract. \((T + al_t)\) is the early termination fee if there is an existing active contract, \(l_t > 0\). Due to the OS network effect, the consumer gets utility from the OS network size \(s_{st-1}M\). \(\xi_{sc}t\) is a carrier-OS specific unobserved quality and is assumed to be i.i.d. across groups and periods. \(\epsilon_{ijsc}t\) is the random utility shock and i.i.d. across periods but not across groups.

Assume \(\epsilon_{ijsc}t\)s have a nested logit structure. \(\epsilon_{ijsc}t\)s are correlated for models by the same carrier with the same OS, and the correlation parameter is \(1 - \rho\). They are independent across carrier-OS groups. The random shocks become perfectly correlated as \(\rho\) goes to 0 and become independent as \(\rho\) goes to 1. For example, a consumer’s utility shocks for the models in the Verizon-Android group are correlated.

The flow utility of the model \((j, s, c)\) is:

\[
f_{jsc} = x_{jsc}\beta - \alpha p_c^p.
\]

If the consumer chooses to buy model \((j, s, c)\) in period \(t\), then the updating rules for the \((f_{0t}, l_t, o_t)\) are:

\[
\begin{align*}
f_{0t+1} &= f_{jsc}, \\
o_{t+1} &= s, \\
l_{t+1} &= L - 1.
\end{align*}
\] (2.4)

The first equation means that the new flow utility at the beginning of period \(t + 1\) is that from the new smartphone \(f_{jsc}\). The second equation means that the new operating system is \(s\) for the new initial state. Since the consumer signs a new contract in period \(t\), the updated contract periods left at the beginning of period \(t + 1\) is \(L - 1\).

### 2.4.1.3 Dynamic Problem

Conditional on the initial state \((f_{0t}, l_t, o_t, s_{ot-1}^o)\) and the information set \(H_t\) at the beginning of period \(t\), a consumer chooses an option from the choice set \(\Omega_t\) to maximize discounted long run profit. Assume consumers face an infinite period dynamic problem. A consumer’s value function is defined as the expectation over all possible values of the utility shock vector \(\epsilon_{it}\). Consumers make choice decisions after observing the realized utility shocks.
Given the updating rules above, consumer $i$’s value function in period $t$ is:

$$V(f_0, l_t, o_t, s_{ot-1}^o, H_t) = E_i[\max\{u_{i0t} + \beta E[V(f_0, \max\{0, l_t - 1\}, o_t, s_{st}^o, H_{t+1})|H_t],$$

$$\max_{j sc \in \Omega_t} [u_{ij st} + \beta E[V(f_{jsc}, l - 1, s, s_{st}^o, H_{t+1})|H_t]\},] $$

(2.5)

where $\beta$ is the discount rate across periods. The information set updates to $H_{t+1}$ in the next period. The operating system market share at the beginning of period $t+1$ is $s_{ot}^o$ if the consumer chooses operating system $s$ in period $t$.

Given the nested logit distribution of the $\epsilon_{ij st}$’s and equations (2.1) and (2.3), the value function can be rewritten as:

$$V(f_0, l_t, o_t, s_{ot-1}^o, H_t) = \ln[\exp(f_0 + \gamma s_{ot-1}^o M + \beta E[V(f_0, \max\{0, l_t - 1\}, o_t, s_{st}^o, H_{t+1})|H_t])$$

$$+ \exp(\delta_t - \alpha(T + al_t)I(l_t > 0))],$$

(2.6)

in which the termination fee $(T + al_t)I(l_t > 0)$ is not $(j, s, c)$ specific and should be paid as long as choosing a new smartphone. $\delta_t$ is the log inclusive value of buying a new smartphone before paying the termination fee:

$$\delta_t = \ln(\sum_{(s,c) \in \Omega_t} \exp(\rho \delta_{sc})),$$

(2.7)

where $\delta_{sc}$ is the log inclusive value of buying a model from the carrier-OS group $(s, c)$ in period $t$:

$$\delta_{sc} = \ln(\sum_{j \in \Omega_{sc}} \exp[(f_{j sc} + \xi_{sc} + \gamma s_{st-1}^o M - \alpha p_{j sc}^c + \beta E[V(f_{j sc}, l - 1, o_{t+1}, s_{ot}^o, H_{t+1})|H_t])]/\rho]).$$

(2.8)

In this dynamic model, the state variables in period $t + 1$ determine the continuation value in the next period, hence the consumer’s choice in period $t$. The consumer has perfect insight for the state variables determined by his/her individual choice, $(f_{ot+1}, l_{t+1}, o_{t+1})$. However, a consumer doesn’t have perfect information about the aggregate state variables $(s_{ot+1}^o, H_{t+1})$, which are determined by the aggregation of consumer choices in period $t$ and the carrier prices and new models in period $t + 1$. Hence, a consumer forms a belief about $(s_{ot+1}^o, H_{t+1})$ before making decisions in period $t$. To make the dynamic problem tractable, I make three simplifying assumptions. The first assumption provides the consumers a statistic
updating rule for the operating system market shares. The second and the third assumptions let the inclusive value $\delta_t$ represent the information in $H_t$ and provide a updating rule for $\delta_t$.

**Assumption 4.** The market shares of the operating systems follow an VAR(1) process. Let $s_t^{os}$ be the vector of OS market shares in period $t$.

$$s_{t+1}^{os} = \Gamma s_t^{os} + \nu_{t+1}^{os}$$  \hspace{1cm} (2.9)

where $\Gamma$ is the transition matrix and $\nu_{t+1}^{os}$ follows the standard normal distribution: $N(0, \Sigma)$, where $\Sigma$ is the variance-covariance matrix.

With Assumption 4, consumers form beliefs about operating system shares in the next period, with transition matrix $\Gamma$. Let $h_\delta(\delta(H_{t+1})|H_t)$ be the conditional distribution function of the inclusive value $\delta(H_{t+1})$ in period $t + 1$ given period $t$ information set $H_t$.

**Assumption 5. Inclusive Value Sufficiency:** For two information sets in period $t$: $H_{1t}$ and $H_{2t}$, if they imply the same inclusive value in period $t$: $\delta(H_{1t}) = \delta(H_{2t})$, then the conditional distribution functions of next period $\delta(H_{t+1})$ are the same for the two information sets: $h_\delta(\delta(H_{1t+1})|H_{1t}) = h_\delta(\delta(H_{2t+1})|H_{2t})$.

Assumption 5 implies that the inclusive value $\delta_t$ represent all information in $H_t$. $\delta_t$ represents all information about the smartphone prices, characteristics, the OS market shares, and transition information of $s_{ot}^{os}$. The next assumption specifies the transition of the inclusive value across periods. Similar assumptions have been imposed in [Gowrisankaran, Park, and Rysman] (2010), [Gowrisankaran and Rysman] (2009), and [Lee] (2013).

**Assumption 6.** The inclusive value $\delta_t$ follows an AR(1) process:

$$\delta_{t+1} = \gamma_1 + \gamma_2 \delta_t + \nu_{t+1}$$  \hspace{1cm} (2.10)

in which $\nu_{t+1}$ follows normal distribution $N(0, \sigma^2)$.

Given the assumptions above, the state variable $H_t$ can be replace by the inclusive value $\delta_t$, and the state variables $s_{ot}^{os}$ and $\delta_t$ both have parametric updating.
processes. Rewrite the consumer’s problem as:

\[
V(f_0, l_t, o_t, s^o_{t-1}, \delta_t) = \ln[\exp(f_0 + \gamma s^o_{t-1} M + \beta E[V(f_0, \max\{0, l_t - 1\}, o_t, s^o_{t+1}|s^o_{t-1}, \delta_t)]) + \exp(\delta_t - \alpha(T + a l_t) I(l_t > 0))]
\]

(2.11)

where \( s^o_{t} \) and \( \delta_{t+1} \) follow equations (2.9) and (2.10) respectively. Next, I calculate the market shares of all smartphone models in period \( t \).

### 2.4.2 Sales Market Shares

Consumers at different states can make different choice decisions. The sales of each model in a period is the aggregate over all consumers. Denote the distribution of consumer states in period \( t \) by \( \tilde{P}_{rt}(f_0, o_0, l_0) \). Notice that \( s^o_{t-1} \) and \( \delta_t \) are state variables at the aggregate level. They are not consumer specific. Consumers differ only in \((f_0, o_0, l_0)\).

Consider a consumer at state \((f_0, o_0, l_0, s^o_{t-1}, \delta_t)\) in period \( t \). Given the nested logit utility shock in the utility function and equation (2.11), the probability of choosing the outside option for the consumer is:

\[
s_{0t}(f_0, o_0, l_0, s^o_{t-1}, \delta_t) = \exp(f_0 + \gamma s^o_{t-1} M + \beta d E[V(f_0, o_0, \max\{0, l_0 - 1\}, s^o_{t+1}|s^o_{t-1}, \delta_t)]) + \exp(\delta_t - \alpha(T + a l_0) I(l_0 > 0))
\]

(2.12)

The probability of a consumer in state \((f_0, o_0, l_0, s^o_{t-1}, \delta_t)\) buying a new smartphone is:

\[
s_{buy}(f_0, o_0, l_0, s^o_{t-1}, \delta_t) = 1 - s_{0t}(f_0, o_0, l_0, s^o_{t-1}, \delta_t)
\]

Conditional on buying a new smartphone, the probability of choosing a model in the carrier-OS group \((s, c, t)\) is:

\[
s_{sct|buy} = \frac{\exp(\rho \delta_{sct})}{\sum_{s',c'\in\Omega_t} \exp(\rho \delta_{s'c't})}
\]

(2.13)

The conditional probability of the consumer buying model \( j \) in group \((s, c, t)\)
conditional on buying from the group is:

\[
    s_{j|\text{set}} = \frac{\exp[(f_{jsc} - \alpha p^c_{jsc} + \gamma s^o_{st-1}M + \xi_{set} + \beta E[V(f_{jsc}, L - 1, s, s^o_{ot}, \delta_{t+1})|\delta_t)]/\rho]}{\exp(\delta_{set})}
\]

(2.14)

The consumer’s initial state variables \((f_0, o_0, l_0)\) affect the decision of whether to buy a new smartphone or not. \(s_{0t}\) depends on \((f_0, o_0, l_0)\) in equation (2.12). The conditional probabilities in equations (2.13) and (2.14), \(s_{j|\text{set}}\) and \(s_{\text{set}|\text{buy}}\), do not change with consumer specific state variables \((f_0, o_0, l_0)\). That is, once decide to buy a new model, consumers’ initial states do not affect their choices of which model to buy.

Given the probabilities above, the unconditional probability of a consumer with state \((f_0, o_0, l_0, s^o_{ot-1}, \delta_t)\) to buy the model \((j, s, c)\) in period \(t\) is:

\[
    s_{j|\text{set}}(f_0, o_0, l_0, s^o_{ot-1}, \delta_t) = s_{j|\text{set}} * s_{\text{set}|\text{buy}} * (1 - s_{0t}(f_0, o_0, l_0, s^o_{ot-1}, \delta_t))
\]

(2.15)

The sales market share of a model is the aggregate over all consumer states. Given the distribution of consumers’ initial states \(\tilde{\mathcal{P}}_t(f_0, o_0, l_0)\), the sales market share of model \((j, s, c, t)\) is the aggregation over consumers in all states.

\[
    s_{j|\text{set}} = \int_{(f_0, o_0, l_0)} s_{j|\text{set}}(f_0, o_0, l_0, s^o_{ot-1}, \delta_t) \tilde{\mathcal{P}}_t(f_0, o_0, l_0)
\]

The integration in the market share is over \((f_0, o_0, l_0)\). \(o_0\) and \(l_0\) both have finite discrete values: \(o_0 \in \{0, 1, ..., S\}\) and \(l_0 \in \{0, 1, ..., L - 1\}\). \(f_0\) is a continuous variable and is bounded by definition. Let \([f, \bar{f}]\) be the range of the flow utility in all periods. Partition the range into \(N_f\) adjacent sub-intervals \([T_1, T_2, ..., T_{N_f}]\). Each sub-interval \(T_n\) is denoted by its middle point \(f_n\). Let the probability of consumer state falling in \((T_n, o_0, l_0)\) be \(\mathcal{P}_t(f_n, o_0, l_0)\), then the market share in equation can be rewritten as:

\[
    s_{j|\text{set}} = \sum_{n=1}^{N_f} \sum_{o_0=1}^{S} \sum_{l_0=0}^{L-1} s_{j|\text{set}}(f_n, o_0, l_0, s^o_{ot-1}, \delta_t) \mathcal{P}_t(f_n, o_0, l_0)
\]

(2.16)

Let \(\Omega_{\text{set}}\) be the set of smartphone models in the carrier-OS group \((s, c)\) in period
The sales market share of the carrier-OS group is:

\[
s_{sc} = \sum_{j \in \Omega_{sc}} s_{jsc}, \tag{2.17}
\]

### 2.4.3 Transition of the Consumer State Distribution

A consumer’s exact state is determined by his/her current smartphone model and the contract status, which implies that there is a distribution of consumer states:

\[
\Delta_t = (\tilde{P}_t(jsc, l)) \text{ at the beginning of period } t. \ (jsc) \text{ is the model that the consumer initially owns. } (jsc) = 0 \text{ if the consumer doesn’t own a smartphone. } l \text{ is the number of periods left in a contract, } l \in \{0, 1, ..., L - 1\}.
\]

The distribution of \(\tilde{P}_t(f_0, o_0, l_0)\) is introduced to reduce the number of possible states, since there are more \((j, s, c)\) combinations than \((f_0, o_0)\) in the data. This means that, the model needs to keep track of the two distributions each period and update them according to the consumers’ purchase decisions.

The distribution, \(\Delta_t\), can be transformed into the distribution of \((f_0, o_0, l_0)\) at the beginning of each period.

\[
\tilde{P}_t(f_n, o_0, l_0) = \sum_{(jsc)} \tilde{P}_t(jsc, l)I(f_{jsc} \in T_n)I(s = o_0)I(l = l_0).
\]

Let \(s_{jsc}(j's'c', l')\) be the probability of buying a new model \((j, s, c)\) for a consumer holding a model \((j', s', c')\) and with \(l'\) contract periods left at the beginning of period \(t\).

\[
s_{jsc}(j's'c', l') = \sum_{n=1}^{N_t} \sum_{o_0=1}^{S} \sum_{l_0=0}^{L-1} s_{jsc}(f_n, o_0, l_0, s_{of-1}^o, \delta_t)I(f_{j's'c'} \in T_n)I(s' = o_0)I(l' = l_0).
\]

The transition of the distribution \(\tilde{P}_t(jsc, l)\) can be derived using equation \(2.18\). For a smartphone model \((j, s, c)\) and \(l\) periods left at the beginning of period \(t + 1\) is:

\[
\tilde{P}_{t+1}(jsc, l) = \begin{cases} 
\tilde{P}_t(jsc, l + 1)s_0(jsc, l + 1) & : l = 0, 1, ..., L - 2 \\
\sum_{(j's'c', l')} \tilde{P}_t(j's'c', l')s_{jsc}(j's'c', l') & : l = L - 1
\end{cases}
\]

For states that imply already being in a contract for at least one period \(l =...
0, 1, ...L − 2, only consumers who choose the outside option can be in these states. For states with \( l = L - 1 \), only consumers who choose to buy a new smartphone can be in these states. Any consumer might buy a new smartphone with a new contract each period.

Non-smartphone owners \((j_{sc} = 0)\) in period \( t + 1 \) are those who didn’t own and didn’t buy a smartphone in period \( t \). For consumers with a smartphone contract in period \( t \), the outside option is to choose to stay in the contract. These consumers can’t be in a state with non-smartphone in period \( t + 1 \). For the non-smartphone state, flow utility is zero \((f_0 = 0)\), the OS state is zero \((o_t = 0)\), the contract periods left is always zero \((l_t = l_{t+1} = 0)\). Hence, the transition of the probability of non-smartphone state is:

\[
\tilde{P}_{r_{t+1}}(0, 0) = \tilde{P}_{r_{t}}(0, 0)s_{0t}(0, 0, 0, s_{ot-1}^{os}, \delta_t)
\]

### 2.5 Estimation

#### 2.5.1 Estimation Method

The structural parameters to be estimated are \((\beta; \alpha; \gamma; \rho; \Gamma; \Sigma; \gamma_1; \gamma_2; \sigma)\). The parameters \((\Gamma; \Sigma)\) in the operating system market share VAR(1) process are estimated using observed shares \(s_{st}^{os}\). Let the estimates be \((\hat{\Gamma}, \hat{\Sigma})\). These estimates are taken as data and passed into the estimation of the rest of the parameters in the model. In addition, the inclusive value defined in equations \((2.7)\) and \((2.8)\) imply that \((\gamma_1; \gamma_2; \sigma)\) are determined by the parameters in the utility function \(\theta_1 = (\beta; \alpha; \gamma; \rho)\). For a given \(\theta_1\), the AR(1) process of \(\delta_t\) can be approximated using the definition of \(\delta_t\), so \((\gamma_1; \gamma_2; \sigma)\) are determined by \(\theta_1\). This will be explained in detail soon. The remaining parameters to estimate are \(\theta_1\).

This paper uses Generalized Methods of Moments with moment conditions based on the carrier-OS specific unobserved quality \(\xi_{sct}\).

\[
E(\xi_{sct}(\theta_1; data)Z_{act}) = 0 \tag{2.19}
\]

where \(Z_{act}\) are instruments for the unobserved quality. As well recognized in the literature, endogeneity issue exists because prices are correlated with \(\xi_{sct}\)’s due to the carriers’ price decisions. Reasonable instrument variables for prices are the cost
shifters, for example, the characteristics. The objective function is:

\[ Q(\theta_1) = \xi(\theta_1)'Z(Z'Z)^{-1}Z'\xi(\theta_1). \]

2.5.2 Algorithm of Solving \( \xi(\theta_1) \)

The approach to solving for \( \xi_{sct} \) for a given \( \theta_1 \) is as follows. An iteration over the \( \xi_{sct} \)s is applied so that the model predicted market shares equal to the observed shares. To calculate the model predicted shares in equation (2.17), the value function is calculated using iteration and the state variable space is discretized based on \( \theta_1 \). The details of the algorithm to solve \( \{\xi_{sct}(\theta_1)\} \) is explained next.

First, given \( \theta_1 \), discretize the state spaces of \( (f_0, o_0, l_0, s_{ot}^{os}) \) and find the transition matrix of \( s_{ot}^{os} \). The transition of distribution of \( (f_0, o_0, l_0) \) is determined by the consumer’s choice, so there is a deterministic process on transition of \( (f_0, o_0, l_0) \). For the \( \theta_1 \), the flow utility of all models can be calculated to find the range \([f(\theta_1), \bar{f}(\theta_1)]\).

Partition the range and find the middle points \( f_n(\theta_1) \), for \( n \in \{1, 2, ..., N_f\} \). The spaces of \( (o_0, l_0) \) are discrete by definition. The space of \( s_{ot}^{os} \) is discretized according to the VAR(1) process with parameter \( (\hat{\Gamma}, \hat{\Sigma}) \) and the corresponding transition matrix \( Q_{sh} \) of \( s_{ot}^{os} \) can be calculated. The discretized spaces of \( (f_0, o_0, l_0, s_{ot}^{os})_{\theta_1} \) and \( Q_{sh} \) are fixed in the procedure to solve \( \{\xi_{sct}(\theta_1)\} \) for a specific \( \theta_1 \).

Let \( \theta_2 = (\gamma_1; \gamma_2; \sigma) \). The following algorithm shows the details of solving \( \{\xi(\theta_1)\} \). The \( \xi_s \) for different periods are solved simultaneously. It’s also straightforward to solve by period.

First, guess initial values for all \( (s, c, t), \xi^o \). Solve the implied parameter \( \theta_2(\theta_1, \xi^o) \) and \( \delta_t(\theta_1, \xi^o) \). I use the following steps to do so. (a), guess an initial \( \theta_2^0 \). Discretize the state space of \( \delta \) and calculate its transition matrix \( Q_{\delta}(\theta_2^0) \) according to the AR(1). Then use value function iteration to solve the bellman equation (2.11) and get \( V(\theta_1, \theta_2^0) \) as a vector. (b), solve \( \delta_t(\theta_1, \theta_2^0) \) for all periods using equations (2.7) and (2.8), and the value function \( V(\theta_1, \theta_2^0) \) from step (a). This can be done using an iteration over the vector of \( \delta_t \). (c), estimate the AR(1) process parameters using the \( \delta_t(\theta_1, \theta_2^0) \) solved in step (b). Denote the new estimate as \( \theta_2^1(\theta_1, \theta_2^0) \). (d), repeat steps (a)-(c) above until \( \theta_2^n(\theta_1, \theta_2^{n-1}) \) converges. Now I have solved \( \theta_2(\theta_1, \xi^o) \) and \( \delta_t(\theta_1, \xi^o) \) for all periods.

Second, calculate the model market shares, \( s_{j,sct}(\theta_1, \xi^o) \), for all \( (j, s, c, t) \), using \( \delta_t(\theta_1, \xi^o) \) in step 1. This step uses the probability function for all states and the
consumer state distribution transition rules in Section 4.

Third, Aggregate to group shares: \( s_{sct}(\theta_1, \xi^0) = \sum_{j \in \Omega_{sct}} s_{j,sct}(\theta_1, \xi^0) \). Update \( \xi \) using the observed market shares:

\[
\xi_{sct}^{b+1}(\theta_1) = \xi_{sct}^b + \psi(ln(s_{sct}^{data}) - ln(s_{sct}(\theta_1, \xi^b))),
\]

where \( \psi \) is a parameter that determines the iteration convergence speed and \( b \) is the count of finished iterations. \( b = 0 \) means the initial guess.

Lastly, repeat steps (1) to (3) until \( \xi_{sct}^{b+1}(\theta_1) \) converges. The converged value is the solution of \( \xi(\theta_1) \) that matches the model predicted shares and the observed carrier-OS shares.

This algorithm involves nested loops with 3 levels and 4 iterations. The first level is the iteration to solve \( \xi \). The second level is the iteration to solve \( \theta_2(\theta_1, \xi) \). The third level has two iterations, one to solve \( V(\theta_2, \theta_1, \xi) \) and one to solve \( \delta(\theta_2, \theta_1, \xi, V(\theta_2, \theta_1, \xi)) \).

Table (2.2) summarizes the steps of how to evaluate the step 1 objective function at a particular parameter vector \( \theta_1 \).

Table 2.2: GMM Objective Function

<table>
<thead>
<tr>
<th>Steps</th>
<th>Goal</th>
<th>How</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \delta(\xi^{old}, \theta_1) )</td>
<td>iteration on ( \delta ) and value function iteration</td>
</tr>
<tr>
<td>2</td>
<td>( s_{sct}(\delta(\xi^{old}, \theta_1)) )</td>
<td>market share equations and updates of state distribution.</td>
</tr>
<tr>
<td>3</td>
<td>( \xi^{new} )</td>
<td>( \xi^{old} + \psi(ln(s_{sct}(\delta(\xi^{old}, \theta_1))) - ln(s_{sct}^{data})) )</td>
</tr>
<tr>
<td>4</td>
<td>( \xi(\theta_1) )</td>
<td>repeat steps (1)-(3) until convergence</td>
</tr>
<tr>
<td>5</td>
<td>( Q(\theta_1) )</td>
<td>( \xi(\theta_1)'Z(Z'Z)^{-1}Z'\xi(\theta_1) )</td>
</tr>
</tbody>
</table>

2.5.3 Estimation Results

The following table shows the estimation results of the demand model. I compare the estimation results of a static nested Logit model, a static random coefficient model, and the dynamic model described above. Static models assume consumers
pay for the smartphone model at carriers’ discounted prices and the wireless service prices for the two years in a contract at the same time. In the nested Logit model, smartphones are divided into the same OS-carrier groups as in the dynamic model, and consumers have the same coefficients for product characteristics and price. The random coefficient demand model follows the static demand model of smartphones in Luo (2015), which allows more flexible substitution patterns across different smartphone models.

Table 2.3: Demand Model Estimation Results

<table>
<thead>
<tr>
<th>Variables</th>
<th>Nested Logit</th>
<th>Random Coeff.</th>
<th>Dynamic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS Subscribers (Million)</td>
<td>0.0427***</td>
<td>0.0341***</td>
<td>0.0437</td>
</tr>
<tr>
<td>Carrier Price ($100)</td>
<td>0.2965***</td>
<td>0.4166**</td>
<td>0.0778</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.6351**</td>
<td>—</td>
<td>0.7616</td>
</tr>
<tr>
<td>Verizon-iOS</td>
<td>5.0846***</td>
<td>4.6472***</td>
<td>2.9532</td>
</tr>
<tr>
<td>Verizon-Android</td>
<td>2.4625***</td>
<td>2.3854***</td>
<td>2.0353</td>
</tr>
<tr>
<td>Verizon-Blackberry</td>
<td>-0.4706</td>
<td>0.1634</td>
<td>0.6959</td>
</tr>
<tr>
<td>Verizon-Windows</td>
<td>3.0536***</td>
<td>3.2642***</td>
<td>2.4760</td>
</tr>
<tr>
<td>AT&amp;T-iOS</td>
<td>4.2989***</td>
<td>4.3210***</td>
<td>2.6733</td>
</tr>
<tr>
<td>AT&amp;T-Android</td>
<td>1.1204***</td>
<td>1.4186***</td>
<td>1.6101</td>
</tr>
<tr>
<td>AT&amp;T-Blackberry</td>
<td>0.7284</td>
<td>1.1499*</td>
<td>1.5575</td>
</tr>
<tr>
<td>AT&amp;T-Windows</td>
<td>2.8932***</td>
<td>1.7467***</td>
<td>2.6185</td>
</tr>
<tr>
<td>Sprint-iOS</td>
<td>1.6947***</td>
<td>0.9277</td>
<td>1.9311</td>
</tr>
<tr>
<td>Sprint-Android</td>
<td>1.5094***</td>
<td>1.3481***</td>
<td>1.7981</td>
</tr>
<tr>
<td>Sprint-Blackberry</td>
<td>-0.1599</td>
<td>0.7436</td>
<td>1.1610</td>
</tr>
<tr>
<td>Sprint-Windows</td>
<td>3.5382***</td>
<td>4.0886***</td>
<td>2.5838</td>
</tr>
<tr>
<td>Tmobile-iOS</td>
<td>2.3735***</td>
<td>1.9543***</td>
<td>2.0354</td>
</tr>
<tr>
<td>Tmobile-Android</td>
<td>1.0891***</td>
<td>1.0978***</td>
<td>1.6862</td>
</tr>
<tr>
<td>Tmobile-Blackberry</td>
<td>-0.7467</td>
<td>-0.3954</td>
<td>0.9740</td>
</tr>
<tr>
<td>Tmobile-Windows</td>
<td>2.6041***</td>
<td>2.8466***</td>
<td>2.3112</td>
</tr>
<tr>
<td>cpu(Ghz)</td>
<td>0.5000</td>
<td>0.2686</td>
<td></td>
</tr>
<tr>
<td>Dummy 4G</td>
<td>0.0747</td>
<td>0.7792</td>
<td>0.6126</td>
</tr>
<tr>
<td>Pixels (100/inch*2)</td>
<td>-0.5553</td>
<td>-0.8447</td>
<td>-0.9747</td>
</tr>
<tr>
<td>Camera(100MP)</td>
<td>-0.2128</td>
<td>-0.1665</td>
<td>-0.6491</td>
</tr>
<tr>
<td>Storage (GB)</td>
<td>-0.0369</td>
<td>-0.0136</td>
<td>0.0023</td>
</tr>
<tr>
<td>Screen Size (inch)</td>
<td>-1.4264***</td>
<td>-1.1176</td>
<td>-0.4250</td>
</tr>
</tbody>
</table>

The results show that static models underestimate the OS network effect and overestimate the price coefficient. The reason is as follows. If consumers do value
the OS network sizes and make dynamic decisions, then they would postpone buying a particular model if they predict that the OS network is going to grow. The static models will interpret the current market shares from a dynamic model as if consumers don’t value the network sizes as much. As a result, the static models underestimate the OS network effect coefficient. For similar reasons, static models overestimate the negative effect of prices, because the lower market shares caused by dynamic behaviors are interpreted as if consumers have high disutility on price.

Both the static and dynamic nested Logit model predict positive correlation between consumers’ random utility within OS-carrier groups. This implies that iPhone 5 and iPhone 5s by AT&T are better substitutes for each other than iPhone 5 and Samsung Galaxy S3 and iPhone 5s, and that iPhone 5 by AT&T is a better substitute for iPhone 5s by AT&T than iPhone 5 by Verizon.

All models predict similar rankings of the fixed effects for the four operating systems. iOS has the highest fixed effect, while Blackberry is the lowest. The dynamic model has different magnitudes of the fixed effects from the static models due to their difference in normalization. This is because that, in the static model, the mean utility of the outside option is normalized to be zero, and that outside option includes using one’s old smartphone. However, in the dynamic model, it is the utility of not owning any smartphone that is normalized to be zero. Hence, the overall utility level in the static model is higher than that in the dynamic model.

\section{Conclusion}

In this paper, I study consumers’ dynamic demand of smartphones on two-year contracts. The model takes into account the dynamic factors that play important roles in consumers’ demand of smartphones with two-year contracts, including endogenous individual smartphone contract status and exogenous aggregate smartphone market information. In particular, this paper considers the impact of the OS network effect and the two-year contract policy on consumers’ demand of smartphones in a dynamic framework.

With the estimation results, I can measure the lock-in effect of the two-year contract policy, on which consumers receive high discounts on handsets but have to stay in the contracts for two-years unless a high early termination fee is paid to end standing contracts.
This paper contributes to the literature of empirical analysis of dynamic demand models, which is an appropriate framework for modeling demand for durable goods. By comparing the estimation results to the static models, this paper also provides insights on how static demand models could lead to biased estimation results when the underlying demand decisions are made dynamic.
Chapter 3  
Store Brands and Retail Grocery Competition in Breakfast Cereals.

3.1 Introduction

This paper studies the impact of store brand cereals on grocery retailers’ profits as the number of local competing retailers varies. Grocery retailers sell both national brands and store brands of cereals. Since national brands are produced by national manufacturers, there is no product differentiation across retailers on national brand cereals. However, store brands are exclusive to the carrying retailers, so store brands create product differentiation for the retailers. In addition to differentiation, store brands are usually cheaper than national brands, and this makes them attractive to price sensitive consumers. In this paper, I emphasize the importance of store brands on both creating product differentiation when facing competition and retailers’ profits when consumers are heterogenous.

The product differentiation by selling both national brands and store brands leads to the following substitution pattern. For a retailer $A$ selling both national brands and store brands of cereal, entry of competing retail stores has different effects on the demand for national brands and store brands. Given that competing retailers sell the same set of national brands, $A$’s national brands’ consumers may switch to competing stores since they can buy exactly the same products there. However, $A$’s store brands’ consumers can’t find the exact same products in competing stores. That is to say, competing retailers sell products that are better substitutes for $A$’s national brands than $A$’s store brands. This substitution pattern justifies a retailer’s decision of introducing store brands. As more competitors enter to share the market, a retailer may use store brands to "lock-in" some consumers when its market share decreases with competition. As a result, store brands are...
expected to be more important in markets with more competing grocery stores. In this paper, I empirically analyze the relationship between a retailer’s profit gain from introducing store brands and its competition with other retailers with an application to the ready-to-eat cereals.

It’s important to analyze consumers’ demand of store brands products and the relationship between store brands profit and local retailers’ competition. First, the demand side welfare effect of store brands can’t be neglected. The market share of private label grocery products (mainly store brands) is about 20% in the United States in 2012, 35% in Europe and 50% in Britain in 2011. Store brands are a very important part to consumers’ grocery shopping. Second, studying the impact of store brands on profits is critical to retailers. Introducing store brands is a strategy for retailers to differentiate from competitors and increase profits. It is important to empirically measure the profit change by having store brands on shelves.

The main data used in this paper is the Dominick’s Dataset from Kilts Center at University of Chicago. Dominick’s is a Chicago-area grocery store chain, which had 87 different stores. The data includes detailed records of store level weekly prices and sales of all cereal brands in 1997. The Dominick’s stores have different numbers of local competitors. The data on the number of local grocery stores can be collected from the 1997 Economics Census dataset.

I estimate a structural model of consumers’ demand for ready-to-eat cereals. To allow flexible substitution patterns across products, I allow heterogenous consumers by using the random coefficient demand model, introduced by Nevo (2001). The main difference between the demand model in this paper and Nevo (2001) lies in the mean utility of outside option. Since only Dominick’s data is available, the outside option in this paper implies buying from a Dominick’s competitor or not buying cereals. In order to study consumers’ response to the varying grocery competition across different Dominick’s stores, I allow the mean utility of the outside to depend on number of local competitors. Intuitively, as the number of competitors increases, the mean utility of the outside option increases and Dominick’s market share.

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1 See Hong and Li (2013).
2 This importance is not limited to retail markets. For most multi-product firms, measuring the profit of introducing a new product that is a substitute for existing products has significant effect on market structure. For example, companies in industries like electronics and automobile frequently introduce new products, the effects of which can be studied by similar methods as developed in this paper.
3 Website: -
decreases. I estimate the demand model with Generalized Methods of Moments.

With the estimation results, I model Dominick’s profit maximization problem to study the impact of store brands cereals. I assume that each Dominick’s store chooses prices independently. \cite{Cohen and Cotterill (2011)} justifies this assumption by comparing different pricing models of milk products and finds that retailers maximize profits taken the wholesale prices as given. Given the demand estimates, each store knows the cross- and own-price elasticities of substitution of all products and solves its profit maximization problem. Given the observed prices are in equilibrium the cost of each product can be calculated from retailer’s first-order conditions. The costs will be used in counterfactual analysis.

The demand estimation results show that consumers’ unobserved utilities of national brands and the outside option are positively correlated, with correlation coefficient 0.99. The unobserved utilities of store brands and the outside option are negatively correlated, with correlation coefficient −.44. These correlations imply that the outside option is a better substitution of national brands than Dominick’s store brands.

With the estimation results, I study two counterfactual cases to measure the impact of store brands and local grocery competition on Dominick’s profit. In the counterfactual experiment, I evaluate the profit change of Dominick’s when store brands ready-to-eat cereals are removed, and check the relationship between the profit change and local grocery competition. The results show that the median profit loss across all stores is $108 per store every week, which is 4.33% of profits in the ready-to-eat cereal category. In addition, the percentage profit loss increases with local grocery competition, the profit loss will increase by 0.16% if the number of competing retailers increases by 1%. That is, if two Dominick’s stores $S_1$ and $S_2$ have 100 and 101 competitors respectively, then the percentage profit loss of $S_2$ is 0.16% higher than $S_1$. This counterfactual analysis implies that the profit from store brands is more important when a Dominick’s store faces more competitors.

With the demand model, I also evaluate consumer welfare change removing store brands cereals and find that consumer welfare loss increases with competition when store brands are removed.

In the second counterfactual case, I measure the impact of competition on Dominick’s profit. Specifically, I assume that the number of competing grocery stores increases by 5 for each Dominick’s store. I then calculate each store’s optimal
prices, profits, and consumer demand. The results show that the median profit loss is $502 per store every week, which is 20.21% of the ready-to-eat cereal profit. More importantly, Dominick’s reduces prices of all brands when competition increases. On the demand side, welfare increases when competition increases.

The rest of the paper proceeds as follows. Section 2 describes the related literature about store brands, demand estimation, and retailer-manufacturer relationships. In section 3, I introduce the data and some motivation facts on the substitution patterns across brands. The structural demand model and supply model are presented in section 4. Section 5 discusses the identification of parameters and estimation method of the demand model. The results of two counterfactual cases, removing store brands and increasing competitors, are presented in Section 6. Section 7 concludes the paper.

3.2 Literature

A few paper in the literature analyze the impact of introducing store brands on the manufacturer-retailer vertical relationship. However, no paper has studied the horizontal competition effect of introducing store brands, which is also very important. Chintagunta, Bonfrer, and Song (2002) analyze the demand side price elasticity changes and supply side retailer-manufacturer relationships. Their test can’t reject the hypothesis that there is no change in price elasticities after introducing store brands. They compare the markups before and after introducing store brands, and find that manufacturers taking a 'softer' stand after store brands are introduced. Steiner (2004) theoretically argues that store brands are competitive weapons and consumer welfare is maximized when store brands and national brands compete vigorously. Hong and Li (2013) analyzes the cost pass-through of national brands and private label products and finds that the pass-through rate of private labels is 40% higher than national brands. Cohen and Cotterill (2011) assesses the impact of retailer store brand products on national brands prices, profitability and consumer welfare. They calculate manufactural margins implied by different supply models between manufacturers and retailer and regress the implied costs of the models on input prices to choose the best fit supply model. They find that national brands prices are lower and consumer welfare is higher with store brands. In addition to analyzing prices and welfare effects of introducing store brands,
this paper also emphasizes the relationship between introducing store brands and competition among retailers, which hasn’t been studied in the literature.

Consumers make discrete choices among all cereal brands in the demand model. Following the discrete choice demand model literature (e.g., BLP(1995), Nevo (2001), Petrin (2001)), I allow heterogenous consumers by using a random coefficient model. The demand model in this paper has one important difference with the literature. That is, to incorporate the impact of local competition on demand, I assume the mean utility of outside option to depend on the number of local grocery stores. In contrast to the literature, which usually normalize mean utility of the outside option to be zero, the mean utility of the outside option in this paper depends on the number local grocery stores. Since only data on Dominick’s stores is available, this is a reasonable way to model the impact of the competition on consumers’ demand.

Analyzing effects of introducing store brands is also related to the vertical integration literature. Without store brands, retailers only sell national brands products produced by upstream manufacturers. Theoretically, store brands eliminate the double marginalization problem of national brands, lower the national brands prices by increasing competition, and raise consumer welfare. However, Hastings (2000) finds that vertical integration significantly increases prices in the gasoline market in South California. The reason behind is that the vertical integration in her paper reduces competition among gas stations instead of increasing competition because large refining companies gain stronger market power. In this paper, introducing low cost store brands makes the retailer to differentiate consumers, so the high quality national brands’ prices increase. Villas-Boas (2007) compares margins implied by different supply models between manufacturer and retailer to select the model that is most consistent with data. While comparing different supply models is important, I assume reasonable pricing problems of Dominick’s and focus on relationship between store brands and retailer competition.
3.3 Data and Facts

3.3.1 Data and Ready-to-eat Cereal Market

I analyze the ready-to-eat cereal products in Dominick’s chain stores in this paper. Dominick’s is a chain grocery store in Chicago area. The data used in this paper has three parts. The first part is store level weekly scanner data, which records product code, price, quantity sold, percentage markup, and product size in each Dominick’s store every week. This data also provides demographic characteristics for the market of each store. The second part is data on grocery retailer competition. I collect data on the number of supermarkets at zip code level from 1997 U.S. Economic Census report, and number of population at zip code level from Illinois state census. The last part is the product characteristics data, which can be found from Mrbreakfirst’s website.

I consider the most important 20 brands sold in every Dominick’s store. A brand here is a manufacturer-shape pair. There are four shape types: ring, flake, shredded and other. For example, all Cheerios by General Mills are combined into one brand. The reason to aggregate products in this way is to balance choice sets for consumers of different Dominick’s stores. The product characteristics contain information on the grain type dummy, fruit dummy and nut dummy of each brand. Since products are combined into brands, I use sales-weighted averages for brand characteristics.

A market in this paper is defined as a store in a week. For example, the same store at two different weeks are treated as two different markets. In 1997, Dominick’s had 86 stores, and almost all stores are located in areas with different zip codes. Eighteen stores are in the downtown Chicago area, where the number of competitors and population data are the same and large for all stores, so I drop observations of all stores in downtown Chicago area, which leaves 68 stores in the data. To match the data of competition and population, I only use Dominick’s data in 1997, including 28 weeks. To avoid constant prices in neighboring weeks, I only use data for the first week of each month during 1997, which leaves data for 288 markets. To further restrict the set of products to be the same for every market, I only use data for stores that carry all the 20 brands defined above. In the end, there are 285 markets.
The outside option is defined as not buying from a Dominick’s store. For each consumer in a market, the outside option includes two possibilities, buying from a retailer other than Dominick’s and not consuming ready-to-eat cereal. To calculate the market shares of the outside option, I assume each person’s potential demand for ready-to-eat cereal is 1 pound per week\(^4\). This leads the outside option’s market share belongs to [33%, 99%], with median 91%.

Dominick’s stores sell national brands and store brand cereals. The leading national brand manufacturers are Kelloggs, General Mills, Post, Quaker, and Ralston. The data shows that store brands cereals have lower costs than national brands. The average Dominick’s store brand cereal cost is $1.58 per pound, while the lowest national brands average cost is $2.35 per pound for Post and the highest is $3.35 for Ralston. Store brands prices are also relatively lower. The average price for store brands is $2.60 per pound, while the lowest national brands average price is $2.97 for Post and the highest is $4.23 for Ralston. While store brands have low prices, their market shares are quite low, 5.54% across stores on average. This may imply that Dominick’s store brands are not good substitutes for national brands. In the next section, I show some facts about the substitution pattern across brands and the relationship between store brands and local supermarket competition.

### 3.3.2 Facts about Substitution and Competition

In this section, I show the summary statistics of the data and reduced form regression results about the impact of retailer competition on national brands and store brands.

The ready-to-eat cereal market is highly concentrated. The five leading national manufacturers are: Kelloggs, General Mills, Post, Quaker and Ralston. According to the sales data of all Dominick’s stores in 1996, the total market share (in terms of quantity sold) of the five companies is 94.91%. The decomposed shares are: 36.92% for Kelloggs, 27.93% for General Mills, 15.85% for Post, 10.73% for Quaker and 3.20% for Ralston.

Table 3.1 shows the mean prices per pound, markups, and market shares of

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\(^4\)‘Granola and Breakfast Facts in Year 2006’ stated that ‘Americans consume 10 pounds per person per year’, which means that actual consumption per person was 0.2 pound per week and potential demand should be higher than that. Assuming potential demand lower than 1 lb/week causes unreasonably negative market shares of the outside option in some markets.
all brands across markets. Markup is defined as the ratio of profit per pound and price per pound. Market shares in the table are shares within each Dominick’s store. Dominick’s store brands have low prices (as low as $2.24 per pound), high markups and low market shares. Ralston brands have high prices (as high as $4.75 per pound) and low market shares. General Mill’s ring-shaped cereals and Kellogg’s flake cereals have the highest market shares, at 11.61% and 14.04% respectively.

Table 3.1: Product Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Retail Price($/lb)</th>
<th>Wholesale Cost</th>
<th>Markup</th>
<th>Within Store shares (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick’s_ring</td>
<td>2.58</td>
<td>1.55</td>
<td>39%</td>
<td>1.29</td>
</tr>
<tr>
<td>Dominick’s_flake</td>
<td>2.24</td>
<td>1.36</td>
<td>39%</td>
<td>2.93</td>
</tr>
<tr>
<td>Dominick’s_shred</td>
<td>2.94</td>
<td>1.90</td>
<td>39%</td>
<td>0.60</td>
</tr>
<tr>
<td>Dominick’s_puff</td>
<td>2.70</td>
<td>1.56</td>
<td>39%</td>
<td>0.54</td>
</tr>
<tr>
<td>General Mills_ring</td>
<td>3.36</td>
<td>2.68</td>
<td>23%</td>
<td>11.61</td>
</tr>
<tr>
<td>General Mills_flake</td>
<td>3.28</td>
<td>2.73</td>
<td>18%</td>
<td>9.48</td>
</tr>
<tr>
<td>General Mills_shred</td>
<td>3.50</td>
<td>2.90</td>
<td>19%</td>
<td>2.47</td>
</tr>
<tr>
<td>General Mills_puff</td>
<td>3.87</td>
<td>2.92</td>
<td>19%</td>
<td>4.37</td>
</tr>
<tr>
<td>Quaker_flake</td>
<td>3.16</td>
<td>2.45</td>
<td>21%</td>
<td>1.85</td>
</tr>
<tr>
<td>Quaker_shred</td>
<td>2.49</td>
<td>1.94</td>
<td>21%</td>
<td>4.11</td>
</tr>
<tr>
<td>Quaker_puff</td>
<td>2.91</td>
<td>2.13</td>
<td>27%</td>
<td>4.77</td>
</tr>
<tr>
<td>Kellogg’s_ring</td>
<td>3.44</td>
<td>2.60</td>
<td>21%</td>
<td>4.77</td>
</tr>
<tr>
<td>Kellogg’s_flake</td>
<td>2.93</td>
<td>2.37</td>
<td>19%</td>
<td>14.04</td>
</tr>
<tr>
<td>Kellogg’s_shred</td>
<td>2.75</td>
<td>2.07</td>
<td>22%</td>
<td>10.28</td>
</tr>
<tr>
<td>Kellogg’s_puff</td>
<td>3.55</td>
<td>2.92</td>
<td>14%</td>
<td>7.83</td>
</tr>
<tr>
<td>Post_flake</td>
<td>2.63</td>
<td>2.02</td>
<td>21%</td>
<td>8.07</td>
</tr>
<tr>
<td>Post_shred</td>
<td>2.90</td>
<td>2.28</td>
<td>21%</td>
<td>4.09</td>
</tr>
<tr>
<td>Post_puff</td>
<td>3.17</td>
<td>2.44</td>
<td>21%</td>
<td>3.69</td>
</tr>
<tr>
<td>Ralston_shred</td>
<td>3.84</td>
<td>2.70</td>
<td>27%</td>
<td>2.43</td>
</tr>
<tr>
<td>Ralston_puff</td>
<td>4.75</td>
<td>3.82</td>
<td>23%</td>
<td>0.77</td>
</tr>
</tbody>
</table>

The market demographics are population, number of grocery stores, logarithm of median income, age distribution, and percentage of people owning a car for each store. Table 3.2 summarizes the market characteristics. The mean number of food retailers within a zip code area is 17.56. The outside option is not buying from a Dominick’s store. The mean market share of the outside option is 86.40% across markets.
Table 3.2: Market Demographic Statistics

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Median Income</td>
<td>10.73</td>
<td>0.22</td>
</tr>
<tr>
<td>Population</td>
<td>36464</td>
<td>28419</td>
</tr>
<tr>
<td>Age&lt; 9 Percentage</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
<td>No car Percentage</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td># Competitors</td>
<td>17.56</td>
<td>11.18</td>
</tr>
<tr>
<td>Outside share</td>
<td>86.40%</td>
<td>12.18%</td>
</tr>
</tbody>
</table>

Next, I use simple regressions to show the impact of retailer competition on store brands and national brands. Let $j = 0, 1, ..., J$ denote different brands of cereal products, $i = 1, 2, ..., S$ denote consumers, and $t = 1, 2, ..., M$ denote markets. The first regression is:

$$m_{jt} = b_1 R_t + b_2 \text{Inc}_t + \epsilon_{1jt},$$

where $m_{jt}$ is the markup of product $j$ in market $t$, $R_t$ is the number of food retailers in market $t$, and $\text{Inc}_t$ is the logarithm of median income in market $t$. $m_{jt}$ is the ratio of gross profit per pound over price per pound, which is observed from data. For each product, gross profit is equal to revenue from sales of the product minus the wholesale cost of the product. For example, $m_{jt} = .3$ means that for every dollar revenue, $\$0.30$ is gross profit, and $\$ - .70$ is wholesale cost.

Table 3.3: Markups and Competition

<table>
<thead>
<tr>
<th></th>
<th>$\hat{b}_1$</th>
<th>$\hat{b}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick’s</td>
<td>0.0047</td>
<td>0.7021</td>
</tr>
<tr>
<td>GeneralMills</td>
<td>-0.0045</td>
<td>0.7503**</td>
</tr>
<tr>
<td>Quaker</td>
<td>-0.0050*</td>
<td>0.9814***</td>
</tr>
<tr>
<td>Kellogg’s</td>
<td>-0.0028</td>
<td>1.0157***</td>
</tr>
<tr>
<td>Post</td>
<td>-0.0090***</td>
<td>1.0373***</td>
</tr>
<tr>
<td>Ralston</td>
<td>-0.0102***</td>
<td>0.4394*</td>
</tr>
</tbody>
</table>

As Table 3.3 shows, markups of most national brands decrease significantly with grocery competition. However, store brands' markups don't decrease with
competition. This fact implies that competing retailers affect the profitability of national brands more than that of Dominick’s store brands. This is reasonable because all competitors sell the same national brands as Dominick’s, but don’t sell Dominick’s store brands.

Second, I regress the market share (quantity sold) of different brands within each store on number of competitors.

\[
\log \frac{q_{jt}}{q_t} = b_3 \log R_t + b_4 \log p_{jt} + \epsilon_{2jt}
\]

where \( q_{jt} \) is the quantity sold of brand \( j \) in market \( t \), \( q_t \) is total quantity sold in market \( t \), and \( p_{jt} \) is price of brand \( j \) in market \( t \).

Table 3.4: Brands Market Shares and Competition

<table>
<thead>
<tr>
<th></th>
<th>( \hat{b}_3 )</th>
<th>( \hat{b}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick’s</td>
<td>.0363***</td>
<td>−.2644***</td>
</tr>
<tr>
<td>GeneralMills</td>
<td>−.0194**</td>
<td>−2.3521**</td>
</tr>
<tr>
<td>Quaker</td>
<td>−.0210</td>
<td>−2.7443***</td>
</tr>
<tr>
<td>Kellogg’s</td>
<td>−.0032</td>
<td>−2.4333***</td>
</tr>
<tr>
<td>Post</td>
<td>.0103</td>
<td>−2.7393***</td>
</tr>
<tr>
<td>Ralston</td>
<td>−.0265</td>
<td>−2.8952**</td>
</tr>
</tbody>
</table>

Table 3.4 shows that the quantity shares of store brands increase significantly with number of competitors, while national brands’ shares decrease. This implies that when number of competitors increases, previous Dominick’s customers of national brands may switch to other stores, because these stores also sell national brands. However, Dominick’s store brands consumers do not switch to competing stores because store brands are unique to Dominick’s.

The next regression shows the impact of competition on the the gross profit shares of different brands. The gross profit shares are within store shares of profits by brand. Let \( \pi_{jt} \) be the gross profit from brand \( j \) in market \( t \), and \( \pi_t \) be the profit from market \( t \) of all brands. \( \pi_{jt}/\pi_t \) is the profit share of brand \( j \) in market \( t \).

\[
\log \frac{\pi_{jt}}{\pi_t} = b_5 \log R_t + \epsilon_{3jt}
\]
Table 3.5: Gross profit shares and Competition

<table>
<thead>
<tr>
<th>Gross Profit Share</th>
<th>$b_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick's</td>
<td>6.62% (3.45%)</td>
</tr>
<tr>
<td>General Mills</td>
<td>31.82% (22.24%)</td>
</tr>
<tr>
<td>Quaker</td>
<td>13.64% (16.07%)</td>
</tr>
<tr>
<td>Kellogg’s</td>
<td>28.58% (19.41%)</td>
</tr>
<tr>
<td>Post</td>
<td>11.49% (6.29%)</td>
</tr>
<tr>
<td>Ralston</td>
<td>7.85% (13.36%)</td>
</tr>
</tbody>
</table>

Table 3.5 shows that the gross profit share of store brands is relatively low compared to national brands. However, it increases significantly with the number of competitors across stores, while national brands’ profit shares, except for Post brands, decrease with competition. This regression implies that store brands are more important for stores that have more competitors.

In summary, the results shown above imply that local grocery competition has different impacts on the national brands and store brands cereals sold by Dominick’s. National brands’ markups and profit shares decrease with competition, while store brands’ profit shares and quantity shares increase with competition. To study the substitution pattern across brands and competitors and the importance of store brands cereals, I estimate a structural demand model and analyze Dominick’s stores’ profit maximization problems. In the next section, I describe the consumers’ demand model of cereals and the Dominick’s profit maximization model.

### 3.4 Model

In this section, I model consumers’ demand and Dominick’s stores’ pricing problem of cereals. In the demand model, consumers make discrete choices over different brands. I use a random coefficient model to allow for consumer heterogeneity, which follows Berry, Levinsohn, and Pakes (1995) and Nevo (2001). A consumer’s utility from cereals depends on the price, product characteristics, and consumer characteristics. The outside good is defined as not buying from the 20 brands by Dominick’s. One difference between the demand model in this paper and the
literature is that, a consumer’s utility of the outside option depends on the number of competing local grocery stores in this paper.

On the supply side, I assume that each store chooses prices for both national brands and store brands. The pricing decisions are independently across stores. Given the demand model, each store chooses the optimal prices for all brands in each period.

3.4.1 Demand Model

Let $j = 0, 1, ..., J$ be a brand of cereal, $i = 1, 2, ..., S$ be a an individual consumer, and $t = 1, 2, ..., M$ be a market. $j = 0$ denotes the outside option. Products are differentiated in their characteristics. Let $x_j$ be the vector of product characteristics of product $j$, which is same across markets and across periods. The characteristics used are dummies for wheat, oat, bran, corn/rice, fruit, and nut. $p_{jt}$ is the price of $j$ in market $t$.

A consumer $i$’s utility of product $j$ in market $t$ is:

$$u_{ijt} = x_j \beta + \alpha_i p_{jt} + \gamma_i d_j + \xi_{jt} + \epsilon_{ijt}, \text{ for } j = 1, 2, ..., J$$ (3.1)

where $\xi_{jt}$ is the unobserved demand shock of product $j$ in market $t$ and $\epsilon_{ijt}$ is unobserved individual demand shock of product $j$ in market $t$. $d_j$ is a vector group dummies. There are three product groups: store brands, national brands and outside option. For example, if $j$ is a national brand, then $d_j = [0, 1, 0]$, and if $j$ is outside option, then $d_j = [0, 0, 1]$. Since I allow consumers to have correlated preferences on different cereals, I will need to estimate a 20 by 20 covariance matrix if I use brand dummies instead. The implication of using the group dummies is that, if two national brands have the same characteristics, then any difference in their market shares will be explained by their unobserved demand shocks $\xi_{jt}s$, but not the brand level fixed effects.

The price coefficient, $\alpha_i$, is consumer specific. For example, consumers with different income levels have different disutility on price. $\gamma_i$ is a vector of group dummy coefficients, and depends on consumer characteristics. $\beta$ is a vector of

---

5These characteristics data are collected from this website: [link]. In principle, one could use the full nutrition facts data if only analyzing the national brands, since the nutrition facts data for national brands are available. However, there is nutrition facts data for Dominick’s store brands. Therefore, I use the dummies in this paper.
coefficients on product characteristics. I assume it to be the same across consumers. A more flexible model may allow heterogeneity in \( \beta \), since age may affect consumers’ preferences on product characteristics. However, if the shopping decisions are made at the household level, then the age effect can be averaged out. The main reason of using this assumption is to decrease the number of parameters to be estimated.

To allow local grocery competition to make an impact on consumers’ demand from Dominick’s, I assume that competition affects consumers’ mean utility of the outside good. Intuitively, consumers’ utility of consuming the outside good increases when there are more competitors. Let the utility of outside option in market \( t \) to be:

\[
-u_{i0t} = \eta_R R_t + \eta_I Inc_t + \epsilon_{i0t}, \text{for } j = 0
\]

where \( R_t \) is the number of local grocery stores in market \( t \) and \( Inc_t \) is the log median household income in market \( t \). \( \eta_R \) and \( \eta_I \) are coefficients that measure the impact of competition and local average income on the outside option. If \( \eta_R \) is positive, then as competition increases, the market share of Dominick’s decreases.

The price coefficient \( \alpha_i \) and group dummy coefficients \( \gamma_i \) depend on consumer \( i \)’s income \( Inc_i \), whether individual \( i \) is less than 9 years old (or the household has a child that is less than 9 years old), and whether the individual has a car. Income is an important characteristics that determines individual’s price sensitivity. Age may affect consumers’ preference on different brands. If national brands have higher quality than store brands, then kids may prefer national brands to store brands. Whether the individual has a car or not can affect his/her ability to shop around local stores, which can affect the utility from the outside option. The consumer specific coefficients are:

\[
\begin{pmatrix}
\alpha_i \\
\gamma_{is} \\
\gamma_{in} \\
\gamma_{i0}
\end{pmatrix} = \begin{pmatrix}
\alpha \\
\gamma_s \\
\gamma_n \\
0
\end{pmatrix} + \Pi \begin{pmatrix}
Inc_i \\
Inc_i^2 \\
D(age < 9) \\
D(car)
\end{pmatrix} + \begin{pmatrix}
\nu_{ip} \\
\nu_{is} \\
\nu_{in} \\
\nu_{i0}
\end{pmatrix}
\]

where \([\alpha; \gamma_s; \gamma_n; 0]\) are the common coefficients on group dummies for all individuals. \( \gamma_s \) is for store brands and \( \gamma_n \) for national brands. The mean of the random coefficient

\[\text{This assumption can be relaxed by making } \beta \text{ to be heterogenous across consumers. However, since this paper focuses on store brands effect and retailer competition, this assumption has a second-order importance in the model.}\]
of the outside dummy is normalized to be 0. The $4 \times 4$ matrix $\Pi$ has all coefficients on consumer characteristics. $D_i$ is a vector of individual characteristics, log income, square of log income, age dummy, and car dummy. Let $F_{D_i}(\cdot)$ be the distribution function of $D_i$ in market $t$.

The random part of the coefficients is a vector $\nu_i = [\nu_{ip}; \nu_{is}; \nu_{in}; \nu_{io}]$, each of which is assumed to follow the standard normal distribution. Denote the joint distribution function of vector $\nu_i$ by $F_\nu$. I assume the price coefficient shock $\nu_{ip}$ is independent with the $\nu_{is}$, $\nu_{in}$ and $\nu_{io}$. Hence, the covariance matrix is:

$$\Sigma = \begin{pmatrix} \sigma_p^2 & 0 & 0 & 0 \\ 0 & \sigma_s^2 & \sigma_{sn} & \sigma_{so} \\ 0 & \sigma_{sn} & \sigma_n^2 & \sigma_{no} \\ 0 & \sigma_{so} & \sigma_{no} & \sigma_o^2 \end{pmatrix}$$

$\Sigma$ is a symmetrical matrix. The diagonal terms are the variances of the random coefficients. The off-diagonal term $\sigma_{sn}$ is the covariance between the coefficient of store brands and that of national brands. Similarly, $\sigma_{so}$ and $\sigma_{no}$ are the covariances between different brand groups.

The random coefficient model allows flexible substitution patterns. Conditional on product characteristics and prices, brands that have high correlation in $\nu$ are better substitutes for each other. For example, if $\sigma_{no}$ is positive and high, then consumers who have high utility from national brands also have high utility from the outside option on average. This implies that, Dominick’s national brands consumers are likely to switch to outside option when the utility of the outside option increases. Similarly, if $\sigma_{sn} < 0$, then consumers’ coefficients on store brands dummy and national brands dummy are negatively correlated, meaning that consumers are less likely to substitute between store brands and national brands. Now the utility function can be rewritten as:

$$u_{ijt} = \delta_{jt} + \mu_{ijt} + \epsilon_{ijt}$$

(3.4)

where

$$\delta_{jt} = x_{jt} \beta + \alpha p_{jt} + \gamma d_j + \xi_{jt}$$

$$\mu_{ijt} = [p_{jt} \ d_j^t] (\Pi D_i + \nu_i)$$
δ_{jt} is the mean utility of brand \( j \) in market \( t \). \( \mu_{ijt} \) is individual deviation from mean utility. Following the literature, I assume that \( \epsilon_{ijt} \)'s follow Type-I extreme value distribution, and they are independent across brands and markets. Given the distribution of \( \epsilon_{ijt} \), I can calculate each individual’s probabilities of choosing all brands. Consumer \( i \) chooses product \( j \) if \( u_{ijt} \geq u_{ikt}, \forall k = 0, 1, ..., J \). The probability of \( i \) with individual characteristics \( \nu_i \) choosing brand \( j \) is:

\[
s_{ijt} = \frac{\exp(\delta_{jt} + \mu_{ijt})}{\exp(\eta_R R_t + \eta_I Inc_t) + \sum_{j'=1}^{J} \exp(\delta_{j't} + \mu_{ij't})}
\]  

(3.5)

\( s_{ijt} \) depends on the individual characteristics \((D_i, \nu_i)\). Let \( A_{jt} \) be the set of individual characteristics such that consumers with these characteristics choose brand \( j \) in market \( t \).

\[
A_{jt} = (D_i, \nu_i) | u_{ijt} > u_{ikt}, \forall k = 0, 1, ..., j
\]

Then the market share of brand \( j \) is

\[
s_{jt} = \int_{A_{jt}} s_{ijt}(D_i, \nu_i) dF_{Dt} dF_{\nu}
\]  

(3.6)

In the next section, I model Dominick’s profit maximization problem.

### 3.4.2 Supply Model

In this paper, I assume each Dominick’s store chooses optimal prices for all products it carries. I don’t estimate the supply side game played among different local grocery stores, because I don’t have data on the prices of other retailers in every market. Instead, I model the impact of competing stores by allowing the utility of the outside option to depend on the number of competitors.

For each brand \( j \), a Dominick’s store pays a wholesale cost \( c_{jt} \) to the manufacturer and a distribution cost \( w_{jt} \). Distribution cost includes the wage paid to store employees. Let \( M_t \) be the market size of market \( t \). Each store’s problem is to maximize total profit from all brands in a market \( t \).
\[
\max_{p_t} \sum_{j=1}^{J} (p_{jt} - c_{jt} - \omega_{jt}) s_{jt} (p_t, R_t, Inc_t) M_{jt}
\]  

(3.7)

where \( p_t \) is a vector of prices for all brands. The first order condition for price \( p_{jt} \) is:

\[
s_{jt} + \sum_{k=1}^{J} (p_{kt} - c_{kt} - \omega_{kt}) \frac{\partial s_{kt}(p_t)}{\partial p_{jt}} = 0, \forall j = 1, 2, ..., J
\]  

(3.8)

Let \( \Omega \) be the matrix of price elasticities, so the \((j,k)\)th element is \(\frac{\partial s_{jt}(p_t)}{\partial p_{kt}}\). From the FOCs, I can solve the vector of distribution costs. In matrix form, the distribution costs in market \( t \) are:

\[
\omega_t = p_t - c_t + \Omega^{-1} s_t
\]  

(3.9)

This supply side model is quite standard except for the distribution cost. The distribution cost here is important to rationalize the prices of each store. In Nevo(2000) and Villas-Boas(2007), because cost data is not available, price-cost-margins are calculated using demand side estimates and firm’s first order conditions. However, in this paper, both retail prices and wholesale prices are observed. If each store only pays a wholesale cost, but not the distribution cost, then the supply model can’t explain any difference between the wholesale costs calculated using demand model elasticities and the wholesale costs observed in data. In fact, it’s very likely that the wholesale cost do not reflect all costs associated with selling a unit of cereal. First, there may be measurement error in the cost data. Second, in addition to the wholesale costs, there are other costs to sell the products. Therefore, introducing the distribution costs justifies the observed prices given the observed wholesale costs.

In the next section, I discuss the estimation method of the demand model and the identification of the parameters.

### 3.5 Estimation

#### 3.5.1 Instruments and Estimation

Following the literature, I use Generalized Methods of Moments to estimate the structural demand model. The moment conditions are based on the orthogonality between unobserved product utility and instruments. In the random coefficient
demand model, the parameters to be estimated are \( \theta = (\beta, \alpha, \eta, \gamma, \Pi, \Sigma) \). \( \theta \) can
be rewritten into two pars: \( \theta = (\theta_1; \theta_2) \), \( \theta_1 = (\beta, \alpha, \eta, \gamma) \) and \( \theta_2 = (\Pi, \Sigma) \). The
parameters in \( \theta_1 \) enter the utility function linearly, while parameters in \( \theta_2 \) interacts
nonlinearly with distributions of observed and unobserved terms in utility function.

The moment conditions are:

\[
E(\xi_j z_{jt}) = 0.
\]

where \( z_{jt} \) is a vector of variables, including product characteristics and instrumental
variables for price. There is endogeneity issue because price \( p_{jt} \) is correlated with
the demand shock \( \xi_{jt} \), since the Dominick’s store choose prices after observing the
demand shocks. Hence, \( p_{jt} \) is not orthogonal to \( \xi_{jt} \). The instrumental variables for
prices used in this paper are the lagged price \( p_{jt-1} \) and lagged wholesale cost \( c_{jt-1} \).
The lagged price \( p_{jt-1} \) is correlated with current price because of the common cost,
but is uncorrelated with current demand shocks \( \xi_t \). Similarly, lagged costs \( c_{jt-1} \) is
also correlated with the current price \( p_{jt} \), but is independent with current demand
shocks.

The GMM objective function is:

\[
\min_{\theta} \{ \xi(\theta)'Z'WZ\xi(\theta) \}.
\]

where \( \xi(\theta) \) is the vector of unobserved demand shocks in all markets, \( Z \) is a matrix
of instrumental variables, and \( W \) is a weight matrix on different moment conditions.
By writing the objective function this way, all observations in all markets have equal
weights. The weight matrix \( W \) adjusts the weights of different moment conditions.
In the estimation, I use 2-step GMM method to find the optimal weight matrix.

To calculate the objective function for a given parameter \( \theta \), I solve for the
demand shocks \( \xi(\theta) \) by matching the model predicted market shares \( s_{jt}(\xi, \theta) \) with
observed shares. To calculate the model predicted shares, I first simulate \( N \) number
of consumers in each market \( t \) from the distributions of consumer characteristics
\( (D_i, \nu_i) \). The parameters \( \theta_1 \) and the unobserved shocks \( \xi \) only affect the mean utility
\( \delta \). Hence, the market shares are functions of \( (\theta_2, \delta) \). For a given \( \theta_2 \) and mean utility
\( \delta \), the model predicted market share \( s_{jt}(\theta_2, \delta) \) are the average over all consumers in
market $t$. 

\[ s_{jt}(\theta_2, \delta) = \frac{1}{N} \sum_{i=1}^{N} s_{ijt}(D_i, \nu_i; \theta_2, \delta). \]

By matching the model predicted shares with observed market shares, I can solve for the mean utility $\delta(\theta_2)$. I then regress $\delta(\theta_2)$ on the observed product characteristics.

\[ \delta_{jt}(\theta_2) = (x_j; p_{jt}; d_j)'\theta_1 + \xi_{jt}. \]

The residuals are the unobserved demand shocks $\xi$ for the given $\theta_2$. Now I can calculate the value of objective function for each given parameter $\theta_2$. The estimation will search over $\theta_2$ to find the value that minimizes the objective function.

### 3.5.2 Identification

The identification of the demand model is quite clear. $\beta$ is identified by the variation in market shares and variation in the product characteristics. Suppose that two products are identical except that one has fruit and the other one does not. The difference in their market shares imply the value of $\beta_{\text{fruit}}$. If $\beta_{\text{fruit}} = 0$, then they will have the same market share. As $\beta_{\text{fruit}}$ increases, the difference between the products increases. The identification of price coefficient $\alpha$ requires instrumental variables, due to the endogeneity pointed in the last subsection. The variation in prices and the variation in market shares identify the $\alpha \gamma_n$ and $\gamma_s$ are identified by the mean utility of store brands and national brands, controlling for the product characteristics, number of competitors and log of median income. Notice that the mean utility of the outside option dummy to be zero, $\gamma_o = 0$.

The nonlinear parameter is $\theta_2 = (\Pi, \Sigma)$. $\Pi$ is the matrix of coefficients on consumer demographics, and it is identified by the market shares of markets with different demographics. Suppose two markets have the same prices for all products, the same age distribution and same percentage of people having a car, then the difference between the two markets’ product shares identifies the impact of income on price coefficient in $\Pi$. If the coefficients on income are zeros, then the two markets will have the same market shares; if the coefficient on the interaction of income and price is positive (less elastic), then the market with higher level of income will have higher market shares of the more expensive products. Similarly, if the coefficients on the interaction of income and national brand group dummy is
positive, then a market with higher level of income will have higher market shares of the same national brands. The identification of all parameters in \( \Pi \) can be explained in similar way.

The diagonal terms of \( \Sigma \) are identified by the substitutability across different groups, controlling everything else. The larger the diagonal terms are, the higher utility consumers get from consuming these groups. For example, if \( \sigma_n \) is large, then consumers are very heterogenous in their evaluation of national brands and this will lead consumers to substitute across national brands when one national brand’s price increases. These consumers are not very likely to switch to store brands or the outside option when national brands’ prices increase. If \( \sigma_n \) is small, then consumers are less heterogeneous in their utility from the national brands’ group dummy. In this case, when the price of one national brand increases, the consumers are likely to switch to store brands or the outside option since they don’t get high utility from the national brand dummy. The extent to which national brands’ market shares change when national brands’ prices change identifies \( \sigma_n \).

The identification of \( \sigma_s \) and \( \sigma_o \) can be explained accordingly.

In BLP(1995), the variances are estimated while the covariance terms are assumed to be zero. However, the covariances are critical in this paper. This paper emphasizes the substitution patterns that are determined by correlation of peoples’ preferences across group dummies, which can’t be a common product characteristic for brands in different groups. The off-diagonal covariances determine the substitution pattern across groups. Suppose when \( \sigma_{no} = 0 \), market shares of national brands and store brands of Dominick’s decrease as the number of competing retailers increases. As \( \sigma_{no} \) increases to be positive, more national brands consumers will switch to the outside option and the market shares of national brands will decrease more than the case when \( \sigma_{no} = 0 \). Similarly, if \( \sigma_{so} < 0 \), then store brands consumers are not likely to prefer the outside option when the competition increases. Market shares of store brands will decrease less than the case when \( \sigma_{so} = 0 \).

### 3.6 Estimation Results

This section shows the estimation results of the demand model. To compare results, I also estimate the model using the Logit model with instrumental variables. In the random coefficient model, I fix some of the interaction parameters in the matrix.
Π to be zero, including the interaction terms between price and squared income, national brands group and squared income, outside option and squared income, price and age, and national brands group and age to be zero.

The covariance matrix $\Sigma$ can be rewritten as the product of an lower-triangular matrix and its transpose: $\Sigma = L * L'$, where

$$L = \begin{pmatrix} l_p & 0 & 0 & 0 \\ 0 & l_s & 0 & 0 \\ 0 & l_{ns} & l_n & 0 \\ 0 & l_{os} & l_{on} & l_o \end{pmatrix}$$

In the estimation, I use estimate the triangular matrix instead of the matrix $\Sigma$.

Table 3.6: Demand Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Logit-IV</th>
<th>Random Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{wheat}}$</td>
<td>-0.78 (0.08)</td>
<td>-0.75 (0.23)</td>
</tr>
<tr>
<td>$\beta_{\text{oat}}$</td>
<td>-0.48 (0.10)</td>
<td>-0.48 (0.19)</td>
</tr>
<tr>
<td>$\beta_{\text{bran}}$</td>
<td>1.20 (0.20)</td>
<td>1.95 (0.56)</td>
</tr>
<tr>
<td>$\beta_{\text{corn/rice}}$</td>
<td>0.70 (0.05)</td>
<td>0.71 (0.13)</td>
</tr>
<tr>
<td>$\beta_{\text{fruit}}$</td>
<td>-0.25 (0.06)</td>
<td>-0.25 (0.17)</td>
</tr>
<tr>
<td>$\beta_{\text{nut}}$</td>
<td>2.59 (0.15)</td>
<td>2.60 (0.30)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1.17 (0.09)</td>
<td>-2.19 (0.12)</td>
</tr>
<tr>
<td>$\gamma_{\text{store}}$</td>
<td>-6.11 (0.64)</td>
<td>-1.77 (0.56)</td>
</tr>
<tr>
<td>$\gamma_{\text{national}}$</td>
<td>-3.8 (0.68)</td>
<td>1.31 (0.48)</td>
</tr>
<tr>
<td>$\eta_R$</td>
<td>0.06 (0.00)</td>
<td>0.05 (0.00)</td>
</tr>
<tr>
<td>$\eta_{\text{Inc}}$</td>
<td>-0.26 (0.06)</td>
<td>-0.22 (0.00)</td>
</tr>
<tr>
<td>$l_p$</td>
<td>—</td>
<td>-0.01 (0.11)</td>
</tr>
<tr>
<td>$l_n$</td>
<td>—</td>
<td>2.66 (2.00)</td>
</tr>
<tr>
<td>$l_s$</td>
<td>—</td>
<td>-3.66 (1.06)</td>
</tr>
<tr>
<td>$l_o$</td>
<td>—</td>
<td>2.61 (1.58)</td>
</tr>
<tr>
<td>$l_{no}$</td>
<td>—</td>
<td>7.44 (0.80)</td>
</tr>
<tr>
<td>$l_{so}$</td>
<td>—</td>
<td>-0.06 (2.86)</td>
</tr>
<tr>
<td>$l_{ns}$</td>
<td>—</td>
<td>-0.21 (4.36)</td>
</tr>
</tbody>
</table>

The demand estimation results of the two models show that the price coefficient estimate $\hat{\alpha}$ is significant and negative. The Logit model underestimates the price
coefficient because consumers who are less price elastic are not differentiated with more elastic consumers. The Logit model doesn’t allow consumers’ characteristics to affect the price coefficient. Thus, high income consumers who have smaller price coefficients are mixed with lower income consumers who have larger price coefficients. As a result, price coefficient in Logit model is underestimated. The coefficient on the number of competitors in outside option’s mean utility is also significant and positive. This implies that, as the number of competitors increases, the mean utility of outside option increases. In both models, the mean utility of outside option increases as local median income rises.

The estimation results show the estimates of elements in the lower-triangular matrix \( L \). All elements but the last two are significant. The implied covariance matrix is:

\[
\hat{\Sigma} = \begin{pmatrix}
0.001 & 0 & 0 & 0 \\
0 & 68.72 & -9.72 & -9.54 \\
0 & -9.72 & 7.06 & 6.93 \\
0 & -9.54 & 6.93 & 6.80
\end{pmatrix}
\]

The variance of unobserved utility for store brands is 68.72. This high variance means that store brands consumers are very loyal to Dominick’s store brands. These consumers are not likely to switch to national brands or the outside option when store brands’ prices change. The variances for national brands and the outside option are lower, meaning that consumers of national brands and the outside option are relatively more likely to switch to other groups when prices change.

Let \( \rho \) denote the implied correlation between groups, and can be calculated using the estimated variances and covariances. In the random coefficient demand model, the estimates of the covariance matrix implies that correlation between random shocks on national brands and outside option is \( \rho_{no} = 0.99 \). This means that consumers with strong preferences for national brands will also have a strong preference for the outside option. The estimated correlation between the random coefficients of store brand dummy and outside option dummy is \( \rho_{so} = -0.44 \). If a consumer has a high value of store brands, he/she is likely to have low value of the outside option. Therefore, Dominick’s national brands and the outside option are close substitutes for each other. In addition, \( \rho_{ns} = -0.44 \), which means that consumers’ preferences for Dominick’s national brands and store brands are negatively correlated. Therefore, the local retailers’ competition will substantially
affect Dominick’s national brands’ demand, but not Dominick’s store brands.

The estimates of parameters in II are shown in the table 7. The positive coefficient of interaction term between price and income level is positive, which means that the higher income a consumer has, the less disutility he/she gets from paying for cereals.

Table 3.7: Demand Estimation Results

<table>
<thead>
<tr>
<th>Random Coefficient Model(std)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{price, inc}$</td>
</tr>
<tr>
<td>$\pi_{price, car \geq 1}$</td>
</tr>
<tr>
<td>$\pi_{national, inc}$</td>
</tr>
<tr>
<td>$\pi_{national, car \geq 1}$</td>
</tr>
<tr>
<td>$\pi_{store, inc}$</td>
</tr>
<tr>
<td>$\pi_{store, inc^2}$</td>
</tr>
<tr>
<td>$\pi_{store, age &lt; 9}$</td>
</tr>
<tr>
<td>$\pi_{store, car \geq 1}$</td>
</tr>
<tr>
<td>$\pi_{out, inc}$</td>
</tr>
<tr>
<td>$\pi_{out, age &lt; 9}$</td>
</tr>
<tr>
<td>$\pi_{out, car \geq 1}$</td>
</tr>
</tbody>
</table>

The own- and cross- elasticities of all cereal brands are shown in following table. The price elasticities are higher within the groups than across groups. The first column is the own-elasticities. On average, the own elasticities of the store brands are lower than national brands. This can be explained by the high variance $\hat{\sigma}_s^2 = 68.72$, which implies that store brands consumers highly value store brands. Hence, when the prices of store brands change, consumers do not switch to national brands or the outside option as much. The second column show the demand elasticities of all brands with respect to the price of Kellogg’s flake cereals. Again, when the prices of Kellogg’s flake cereals increase, consumers switch from Kellogg’s flake cereals to other national brands, but less to Dominick’s store brands. The last column shows the elasticities when the price of Dominick’s puff cereal increases. The numbers imply that consumers switch across store brands more than to national brands.
### Table 3.8: Demand Elasticities

<table>
<thead>
<tr>
<th></th>
<th>Own</th>
<th>KL’s_flake</th>
<th>Dominick’s_puff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick’s_ring</td>
<td>-3.03</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Dominick’s_flake</td>
<td>-2.64</td>
<td>0.01</td>
<td>0.22</td>
</tr>
<tr>
<td>Dominick’s_shred</td>
<td>-3.60</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>Dominick’s_puff</td>
<td>-3.30</td>
<td>0.00</td>
<td>-3.30</td>
</tr>
<tr>
<td>General Mills_ring</td>
<td>-4.13</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>General Mills_flake</td>
<td>-3.98</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>General Mills_shred</td>
<td>-4.31</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>General Mills_puff</td>
<td>-4.33</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Quaker_flake</td>
<td>-3.75</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Quaker_shred</td>
<td>-2.99</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Quaker_puff</td>
<td>-3.53</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Kellogg’s_ring</td>
<td>-3.98</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Kellogg’s_flake</td>
<td>-3.51</td>
<td>-3.51</td>
<td>0.05</td>
</tr>
<tr>
<td>Kellogg’s_shred</td>
<td>-3.18</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Kellogg’s_puff</td>
<td>-4.11</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Post_flake</td>
<td>-3.09</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Post_shred</td>
<td>-3.49</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Post_puff</td>
<td>-3.74</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Ralston_shred</td>
<td>-4.34</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Ralston_puff</td>
<td>-5.99</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### 3.7 Counterfactual Comparison

#### 3.7.1 Without Store Brands

One question that this paper is interested in is whether store brands are more important in markets with more grocery competitors. In order to answer this question, I consider the case in which there is no store brand. One important assumption made here is that wholesale costs stay the same when store brands are removed. Though this assumption might not be reasonable when analyzing the vertical relationships between retailer and manufacturers, it is not problematic to analyze the horizontal relationship between store brands and retailers’ competition. I compare the optimal profits with and without store brands, and analyze the relationship between the profit change and competition. In addition, I also measure the welfare effect of removing store brands.
Let $k = 1, 2, ..., K$ be the national brands. Now each store chooses optimal prices of all national brands to maximize profit, given the mean utility of the outside option. The profit maximization problem of any store is now:

$$\max_{p_t} \sum_{j=1}^{K} (p_{kt} - c_{kt} - \omega_{kt}) s_{kt} (p_{kt}) M_{kt}$$

The F.O.C.s with respect to prices of national brands in matrix form is:

$$p_{t}^* = c_{t} + \omega_{t} - \Omega^{-1}(p_{t}^*) \ast s(p_{t}^*)$$  \hspace{1cm} (3.10)

where the superscript $*$ denotes variables in counterfactual case. $\Omega$ is the matrix of partial derivatives, the $(j,k)$th element is $\frac{\partial s_{jt}(p_t)}{\partial p_{kt}}$ at any given price vector $p_t$. So the new optimal prices solve the vector equations. Let $\pi_t^*$ be the new profit for the Dominick’s store in market $t$. So

$$\pi_t^* = \sum_{j=1}^{K} (p_{kt}^* - c_{kt} - \omega_{kt}) s_{kt} (p_{kt}^*) M_{kt}$$

For stores in different markets, the profit loss from dropping the store brands will be different. Market sizes, consumer characteristic distributions, and competition degrees are all different for different markets. This paper is interested in the relationship between profit loss and local competition. The facts in Section 3 show that the within market profit share of store brands increases significantly with local competition. Hence, when store brands are removed, the profit loss in terms of profit percentage is also expected to increase with competition.

Table 9 shows the median counterfactual optimal prices of all brands when store brands are removed. The median prices of the 16 national brands decrease. For all national brands in all markets, prices of 65% of them decrease when store brands are removed, and 35% of national brands’ prices increase. The decrease in national brands prices implies that Dominick’s chooses higher prices for national brands after introducing store brands. Dominick’s increase national brands’ prices to differentiate national brands’ consumers from store brands’ consumers.

Table 10 shows the own elasticities of national brands with and without store
brands. We find that consumers’ demand of national brands are less elastic when store brands are dropped. This result is intuitive. First, consumers’ demand are more elastic when store brands are available because they have more alternatives in their choice set. Second, consumers are more elastic when prices are higher. The national brands’ prices are higher when store brands are sold, so the consumers’ elasticities are higher when store brands are available. This result show that though store brands and national brands are weak substitutes.

Next I compare Dominick’s profit changes when store brands are removed. The first two columns of Table 11 show the profit losses and market share changes of removing store brands. Dominick’s profits decrease in all 285 markets, and the median profit loss is $108 per week per store, which corresponds to 4.33% of profits before removing store brands. The total profit loss for all the 285 markets is $37853 every week. The $108 loss per week leads to a $5616 loss per store every year. Removing store brands increases market share of the outside option, with a median increase of 0.24%.

<table>
<thead>
<tr>
<th>Table 3.9: Median Prices of National Brands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>General Mills_ring</td>
</tr>
<tr>
<td>General Mills Flake</td>
</tr>
<tr>
<td>General Mills Shred</td>
</tr>
<tr>
<td>General Mills Puff</td>
</tr>
<tr>
<td>Quaker Flake</td>
</tr>
<tr>
<td>Quaker Shred</td>
</tr>
<tr>
<td>Quaker Puff</td>
</tr>
<tr>
<td>Kellogg’s Ring</td>
</tr>
<tr>
<td>Kellogg’s Flake</td>
</tr>
<tr>
<td>Kellogg’s Shred</td>
</tr>
<tr>
<td>Kellogg’s Puff</td>
</tr>
<tr>
<td>Post Flake</td>
</tr>
<tr>
<td>Post Shred</td>
</tr>
<tr>
<td>Post Puff</td>
</tr>
<tr>
<td>Ralston Shred</td>
</tr>
<tr>
<td>Ralston Puff</td>
</tr>
</tbody>
</table>
Table 3.10: Demand Elasticities

<table>
<thead>
<tr>
<th></th>
<th>Without Store Brands</th>
<th>Store Brands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick’s_ring</td>
<td>—</td>
<td>-3.03</td>
</tr>
<tr>
<td>Dominick’s_flake</td>
<td>—</td>
<td>-2.64</td>
</tr>
<tr>
<td>Dominick’s_shred</td>
<td>—</td>
<td>-3.60</td>
</tr>
<tr>
<td>Dominick’s_puff</td>
<td>—</td>
<td>-3.30</td>
</tr>
<tr>
<td>General Mills_ring</td>
<td>-4.06</td>
<td>-4.13</td>
</tr>
<tr>
<td>General Mills_flake</td>
<td>-3.93</td>
<td>-3.98</td>
</tr>
<tr>
<td>General Mills_shred</td>
<td>-4.24</td>
<td>-4.31</td>
</tr>
<tr>
<td>General Mills_puff</td>
<td>-4.25</td>
<td>-4.33</td>
</tr>
<tr>
<td>Quaker_flake</td>
<td>-3.71</td>
<td>-3.75</td>
</tr>
<tr>
<td>Quaker_shred</td>
<td>-2.98</td>
<td>-2.99</td>
</tr>
<tr>
<td>Quaker_puff</td>
<td>-3.50</td>
<td>-3.53</td>
</tr>
<tr>
<td>Kellogg’s_ring</td>
<td>-3.93</td>
<td>-3.98</td>
</tr>
<tr>
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<td>-3.47</td>
<td>-3.51</td>
</tr>
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<td>-3.16</td>
<td>-3.18</td>
</tr>
<tr>
<td>Kellogg’s_puff</td>
<td>-4.05</td>
<td>-4.11</td>
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<td>Post_flake</td>
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</tr>
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<td>Post_puff</td>
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<td>-3.74</td>
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<td>-4.25</td>
<td>-4.34</td>
</tr>
<tr>
<td>Ralston_puff</td>
<td>-5.80</td>
<td>-5.99</td>
</tr>
</tbody>
</table>

The last column of Table 11 shows the relationship between profit loss of removing store brands and retailers’ competition. To check this relationship, I regress log of percentage profit losses of each market on the corresponding log of number of competitors and log market median income. Let $\Delta \pi_{st}$ be profit loss in market $t$ after removing store brands.

$$
\log\left(\frac{\Delta \pi_{st}}{\pi_{st}}\right) = \lambda_1 \log(R(t)) + \lambda_2 \log(Inc_t)
$$

Table 3.11: Percentage of Profit Loss and Competition

<table>
<thead>
<tr>
<th></th>
<th>Profit Loss</th>
<th>Outside Share</th>
<th>Loss Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>4.33%</td>
<td>+0.24%</td>
<td>$\hat{\lambda}_1$ 0.16***</td>
</tr>
<tr>
<td>St.d</td>
<td>7.07%</td>
<td>0.73%</td>
<td>$\hat{\lambda}_2$ -0.21***</td>
</tr>
</tbody>
</table>
The results show that Dominick’s profit loss increases with competition significantly, with elasticity 0.16. That is to say, when the degree of competition increases by 1%, the profit loss increases by .16%. Suppose a Dominick’s store has 10 competitors, and its profit loss from removing store brands is $500 every week, then this loss will increase to $540 if it had 15 competitors. Profit loss decreases with the market level income, with elasticity −0.21. That is, when a Dominick’s store is in a market with higher income level, the profit loss is smaller when store brands are removed.

I then calculate consumers’ welfare change when store brands are removed. Given prices and characteristics of all brands, define consumer i’s expected utility as

$$E(u_{it}) = \sum_{k=0}^{20} s_{ikt} * (\delta_{kt} + \mu_{ikt}),$$

where 0 represents the outside option. I find that welfare increases when store brands are removed. The mean welfare increase is about $6662 per market every year. There are two effects of removing store brands that work in the opposite direction. On one hand, removing store brands reduces consumers’ welfare because there are less alternatives. On the other hand, removing store brands lowers the optimal prices of national brands, which increases consumers’ welfare. The increase in consumer welfare implies that the price effect dominates the choice set effect. These two effects lead to the relatively small increase of consumer welfare. Therefore, when store brands are removed, Dominick’s profits decrease while consumers’ welfare increases slightly.

3.7.2 Increasing Competition

In this subsection, I want to measure the impact of increasing competition on Dominick’s prices and profits. The counterfactual scenario is to increase the number of local retailers by five for each Dominick’s store.

The wholesale costs of national brands are assumed to be the same as in the data when there are more competitors. To check whether this assumption is reasonable, I check whether wholesale costs vary with number of retailers by using data across different markets. In particular, I regress the unit wholesale costs for all brands on the number of competitors and brand dummies, the results show that the coefficient in front of competition is 0, and not significant. This result implies that wholesale
prices don’t vary with number of retailers in a market. Thus, the assumption on wholesale cost is valid.

When the number of competitors increases, the mean utility of the outside option increases by $5\eta_R$. The demand model implies that Dominick’s will decrease prices of both national and store brands and lose profits. To measure the impacts, I re-solve each store’s profit maximization problem to find the new optimal prices and calculate the profit loss for each market.

The first two columns of Table 12 compare the median prices after and before increasing competition. It shows that prices of all brands decrease. The third column shows the median changes in markups of all brands. Markup for every brand is defined as the ratio of profit per unit over unit price. Markups of both national brands and store brands decrease. Though the median change of store brands are slightly higher than national brands, their standard deviations are much higher. This means that store brands’ markups do not decrease significantly while national brands’ markups decrease significantly when competition increases. The significant decreases of national brands markups can be explained by the high correlation (0.99) between consumers’ unobserved utility of national brands dummy and the outside option dummy. The percentage decreases of store brands’ markups are more volatile than national brands, because the correlation between consumers’ unobserved utility of store brands and the outside option is negative and relatively small (-0.44).

The median profit loss from having five more competitors is $502 per week per store, which is 20.21% of cereal profits before competition increases. The total profit loss for the 285 markets is $160,670 per week. I also calculate consumer welfare change when there are more competitors in each market. The results show that consumer welfare increases in all markets when competition increases. The welfare increase is around $7.91 per consumer every year, and the average welfare increase is $282,290 per zip code area every year.
Table 3.12: Median Prices of All Brands

<table>
<thead>
<tr>
<th></th>
<th>5 More Competitors/$(std)</th>
<th>Data/$(std)</th>
<th>△Markup$(std)/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominick’s_ring</td>
<td>2.58 (.31)</td>
<td>2.63 (.28)</td>
<td>-1.80 (3.15)</td>
</tr>
<tr>
<td>Dominick’s_flake</td>
<td>2.33 (.26)</td>
<td>2.37 (.22)</td>
<td>-1.62 (3.38)</td>
</tr>
<tr>
<td>Dominick’s_shred</td>
<td>2.97 (.25)</td>
<td>3.02 (.22)</td>
<td>-2.07 (3.46)</td>
</tr>
<tr>
<td>Dominick’s_puff</td>
<td>2.71 (.33)</td>
<td>2.76 (.31)</td>
<td>-1.90 (3.23)</td>
</tr>
<tr>
<td>General Mills_ring</td>
<td>3.49 (.27)</td>
<td>3.52 (.26)</td>
<td>-1.66 (1.42)</td>
</tr>
<tr>
<td>General Mills_flake</td>
<td>3.30 (.10)</td>
<td>3.32 (.09)</td>
<td>-1.67 (1.31)</td>
</tr>
<tr>
<td>General Mills_shred</td>
<td>3.55 (.13)</td>
<td>3.58 (.12)</td>
<td>-1.72 (1.38)</td>
</tr>
<tr>
<td>General Mills_puff</td>
<td>3.72 (.40)</td>
<td>3.74 (.41)</td>
<td>-1.73 (1.41)</td>
</tr>
<tr>
<td>Quaker_flake</td>
<td>3.10 (.14)</td>
<td>3.12 (.14)</td>
<td>-1.62 (1.24)</td>
</tr>
<tr>
<td>Quaker_shred</td>
<td>2.43 (.11)</td>
<td>2.46 (.11)</td>
<td>-1.41 (1.04)</td>
</tr>
<tr>
<td>Quaker_puff</td>
<td>2.92 (.28)</td>
<td>2.94 (.28)</td>
<td>-1.57 (1.20)</td>
</tr>
<tr>
<td>Kellogg’s_ring</td>
<td>3.32 (.24)</td>
<td>3.35 (.24)</td>
<td>-1.66 (1.32)</td>
</tr>
<tr>
<td>Kellogg’s_flake</td>
<td>3.05 (.34)</td>
<td>3.08 (.34)</td>
<td>-1.57 (1.20)</td>
</tr>
<tr>
<td>Kellogg’s_shred</td>
<td>2.65 (.20)</td>
<td>2.68 (.20)</td>
<td>-1.48 (1.12)</td>
</tr>
<tr>
<td>Kellogg’s_puff</td>
<td>3.60 (.46)</td>
<td>3.62 (.46)</td>
<td>-1.60 (1.36)</td>
</tr>
<tr>
<td>Post_flake</td>
<td>2.58 (.19)</td>
<td>2.61 (.19)</td>
<td>-1.45 (1.06)</td>
</tr>
<tr>
<td>Post_shred</td>
<td>2.85 (.11)</td>
<td>2.88 (.10)</td>
<td>-1.56 (1.18)</td>
</tr>
<tr>
<td>Post_puff</td>
<td>3.18 (.32)</td>
<td>3.21 (.32)</td>
<td>-1.61 (1.23)</td>
</tr>
<tr>
<td>Ralston_shred</td>
<td>3.95 (.79)</td>
<td>3.98 (.79)</td>
<td>-1.68 (1.40)</td>
</tr>
<tr>
<td>Ralston_puff</td>
<td>5.09 (.31)</td>
<td>5.12 (.31)</td>
<td>-1.90 (1.71)</td>
</tr>
</tbody>
</table>

3.8 Conclusion

The effects of store brands has been understudied in the literature. Existing literature on store brands emphasize the impact of store brands on national brands prices and consumer demand elasticities. There are several papers trying to find the impact of store brands on the vertical relationship between retailer and manufacturers. However, no study has paid attention to the impact of store brands’ on competition among local retailers. In this paper, I take a first step to fill this gap, and find that store brands are more important to the retailer Dominick’s as it faces more competitors in a geographic market.

In addition to the competition effect of store brands, this paper also finds interesting results on prices effects and welfare effects. I find that national brands’ prices are higher when Dominick’s also sells store brands. This result implies that the retailer discriminates consumers according to vertical product qualities.
when store brands are introduced, which increases national brands prices. As a result, in the counterfactual results, the national brands’ prices decrease. The decreased prices and high market shares of national brands lead to a small increase of consumer welfare when store brands are removed.

I estimate the whole covariance matrix of unobserved utility of all brand groups in the random coefficient model. Every element of the matrix plays an important role in determining the substitution patterns across groups. This is different from many papers using random coefficient models, which only estimate the diagonal entries of the covariance matrix. In this paper, the off-diagonal terms in the covariance matrix are estimated because they determine the substitution pattern across groups, after controlling for product characteristics. Since this paper is interested in the substitution pattern across groups, I also estimated several more restrictive covariance matrices by fixing the values of diagonal elements. All results agree that store brands are more important when a store faces more competitors.

There are several interesting potential extensions to the this paper. Store brands’ effect on the vertical relationships between retailer and manufacturer is quite interesting. Though several papers have theoretically analyzed the impact of introducing store brands on the vertical relationship, there is no empirical work that study this. In addition, the wide category range of store brands coverage makes the cross category management of store brands another important and interesting topic.
Chapter A  |  Proofs for the Two-OS, Two-Period Model

**Lemma 1.** Given the costs of the two models are the same (not necessarily 0), the multi-network seller chooses the same price for A and B in the second period: \( p_{A2}^* = p_{B2}^* \).

**Proof.** The seller’s pricing problem can be solved backwards. Let the unit cost of the two models be \( c \). The seller’s profit in the 2nd period is:

\[
\pi_2(p_{A1}, p_{B1}, p_{A2}, p_{B2}) = \sum_{j=A,B} (p_{j2} - c)M_2s_{j2}
\]

\[
= \sum_{j=A,B} (p_{j2} - c)M_2 \frac{e^{(\delta_j + \gamma p_{j2} - \alpha p_{j2})}}{1 + \sum_{k=A,B} e^{(\delta_k + \gamma p_{k2} - \alpha p_{k2})}}.
\]

The FOC with respect to \( p_{A2} \) is:

\[
s_{A2} - \alpha(p_{A2}^* - c)s_{A2}(1 - s_{A2}) + \alpha(p_{B2}^* - c)s_{A2}s_{B2} = 0,
\]

which is equivalent to:

\[
1 - \alpha(p_{A2}^* - c) + \alpha(p_{A2}^* - c)s_{A2} + \alpha(p_{B2}^* - c)s_{B2} = 0.
\]

Similarly for B, we have:

\[
1 - \alpha(p_{B2}^* - c) + \alpha(p_{A2}^* - c)s_{A2} + \alpha(p_{B2}^* - c)s_{B2} = 0.
\]

Compare equations (A.2) and (A.3), we immediately get \( p_{A2}^* = p_{B2}^* \).  \( \square \)
A.1 Proof of Proposition 1

Proof. Let the unit cost of the two models be 0. Let the price in the 2nd period of the two models be $p^*_2$. Then plug $p^*_2$ into equation (A.1). We get:

$$1 - \alpha p^*_2(1 - s_{A2} - s_{B2}) = 0.$$  

Plug in the sales market share equations and rearrange the terms. We get:

$$\alpha p^*_2 - 1 = e^{(\delta_A + \gamma_A s_{A2} - \alpha p^*_2)} + e^{(\delta_B + \gamma_B s_{B2} - \alpha p^*_2)}.$$  \hspace{1cm} (A.4)

Using total differentiation on (A.4), we get:

$$\alpha \frac{\partial p^*_2}{\partial n_{A2}} = e^{(\delta_A + \gamma_A n_{A2} - \alpha p^*_2)} (\gamma - \alpha \frac{\partial p^*_2}{\partial n_{A2}}) - \alpha e^{(\delta_B + \gamma_B n_{B2} - \alpha p^*_2)} \frac{\partial p^*_2}{\partial n_{A2}}.$$  

Then we can solve $\frac{\partial p^*_2}{\partial n_{A2}}$:

$$\frac{\partial p^*_2}{\partial n_{A2}} = \frac{\gamma}{\alpha} s_{A2}.$$  \hspace{1cm} (A.5)

Similarly for $\frac{\partial p^*_2}{\partial n_{B2}}$, we have:

$$\frac{\partial p^*_2}{\partial n_{B2}} = \frac{\gamma}{\alpha} s_{B2}.$$  \hspace{1cm} (A.6)

Then the profit in the 2nd period is:

$$\pi_2 = \sum_{j=A,B} p_j M_2 s_{j2}$$

$$= p^*_2 M_2 (s_{A2} + s_{B2})$$

$$= p^*_2 M_2 (1 - \frac{1}{1 + e^{(\delta_A + \gamma_A s_{A2} - \alpha p^*_2)} + e^{(\delta_B + \gamma_B s_{B2} - \alpha p^*_2)}})$$  \hspace{1cm} (A.7)

$$= p^*_2 M_2 (1 - \frac{1}{\alpha p^*_2})$$

$$= \frac{M_2}{\alpha} (\alpha p^*_2 - 1).$$
Then the maximization problem in the first period is:

$$
\max_{p_{A1}, p_{B1}} \pi_1(p_{A1}, p_{B1}) + \beta \pi_2(p_{A1}, p_{B1}, p^*_2(p_{A1}, p_{B1}))
$$

$$
= \max_{p_{A1}, p_{B1}} \sum_{j=A,B} p_j M_1 s_{j1} + \frac{\beta}{\alpha} (\alpha p^*_2(p^*_1, p^*_2) - 1) M_2
$$

$$
= \max_{p_{A1}, p_{B1}} \sum_{j=A,B} p_j M_1 s_{j1} + \beta (p^*_2(p_{A1}, p_{B1}) - \frac{1}{\alpha}) M_1 s_{01}.
$$

(A.8)

in which $s_{01} = 1 - s_{A1} - s_{B1}$ (0 means the outside option) and the market size in the second period is the proportion of the first period market size who didn’t buy any smartphone, $M_2 = M_1 s_{01}$. Then the FOC with respect to $p_{A1}$ is:

$$
0 = s_{A1} - \alpha p^*_A s_{A1}(1 - s_{A1}) + \alpha p^*_B s_{A1} s_{B1} + \alpha \beta (p^*_2(p_{A1}, p_{B1}) - \frac{1}{\alpha}) s_{A1} s_{01} + \beta \frac{\partial p^*_2}{\partial p_{A1}} s_{01}.
$$

(A.9)

in which the partial derivative of price $p^*_2$ with respect to 1st period price $p_{A1}$ is:

$$
\frac{\partial p^*_2}{\partial p_{A1}} = \frac{\partial p^*_2}{\partial n_{A2}} \frac{\partial n_{A2}}{\partial p_{A1}} + \frac{\partial p^*_2}{\partial n_{B2}} \frac{\partial n_{B2}}{\partial p_{A1}}
$$

$$
= \frac{\partial p^*_2}{\partial n_{A2}} M_1 (-\alpha) s_{A1}(1 - s_{A1}) + \frac{\partial p^*_2}{\partial n_{B2}} M_1 \alpha s_{A1} s_{B1}.
$$

Plug $\frac{\partial p^*_2}{\partial p_{A1}}$ into (A.9), we get:

$$
1 - \alpha p^*_A + \alpha p^*_A s_{A1} + \alpha p^*_B s_{B1} + \beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{01}
$$

$$
+ \beta M_1 s_{01} [-\alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{B1}] = 0.
$$

(A.10)

Similarly for OS B, the first-order condition of profit with respect to price $p_{B1}$ gives the following equation:

$$
1 - \alpha p^*_B + \alpha p^*_B s_{B1} + \alpha p^*_A s_{A1} + \beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{01}
$$

$$
+ \beta M_1 s_{01} [-\alpha - \frac{\partial p^*_2}{\partial n_{B2}} (1 - s_{B1}) + \alpha \frac{\partial p^*_2}{\partial n_{A2}} s_{A1}] = 0.
$$

(A.11)
Then taking the difference of equations (A.10) and (A.11), we get:

\[- \alpha p^*_1 + 1.0 + \beta M_1 s_{01}[- \alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{B1}] = - \alpha p^*_1 + 1.0 + \beta M_1 s_{01}[- \alpha \frac{\partial p^*_2}{\partial n_{B2}} (1 - s_{B1}) + \alpha \frac{\partial p^*_2}{\partial n_{A2}} s_{A1}] .\]

Plug in the partial derivatives in (A.5) and (A.6), we get:

\[ p^*_1 - p^*_B = \frac{\beta \gamma}{\alpha} M_1 s_{01} (s_{B2} - s_{A2}). \]  

(A.12)

Next we need to prove \( p^*_1 < p^*_b \) under the condition that \( n_{A1} > n_{B1} \). That is the 1st period price for the large OS is lower than that for the small OS. Notice that \( s_{B2} < s_{A2} \Leftrightarrow n_{B2} < n_{A2} \), given \( p^*_B = p^*_A \) and \( \delta_A = \delta_B = \delta \). Next I discuss three cases of relationships between \( p^*_1 \) and \( p^*_B \) to prove that the profit maximization solution satisfies \( p^*_1 < p^*_B \).

First, suppose \( p^*_1 = p^*_B \). Then \( n_{B2} < n_{A2} \). Because \( n_{j2} = n_{j1} + M_1 s_{j1} \), \( n_{A1} > n_{B1} \), and \( s_{A1} = s_{B1} \). But this means (A.12) is violated.

Second, suppose \( p^*_1 > p^*_B \). If this is the case, (A.12) implies that \( s_{B2} > s_{A2} \), which means that \( n_{B2} > n_{A2} \). That is, the initial OS network advantage of A is reversed in the 2nd period due to high price of A. Since \( n_{j2} = n_{j1} + M_1 s_{j1} \), \( n_{B2} > n_{A2} \) implies that \( s_{B1} > s_{A1} \). That is, the sales share of B is higher than that of A in the first period. Use the equation of sales market shares, we get:

\[ e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} > e^{(\delta + \gamma n_{A1} - \alpha p^*_A)}. \]  

(A.13)

Let \((p^*_1, p^*_B, p^*_2)\) bet the profit maximization prices in the two periods. Then the seller’s total profit is:

\[ \Pi^* = M_1 p^*_A e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + p^*_B e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} + (\alpha p^*_2 - 1) + e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} . \]

Now consider another price plan for the two periods \((p'_1, p'_B, p'_2)\), in which:

\[ \gamma n_{A1} - \alpha p'_A = \gamma n_{B1} - \alpha p'_B , \]
\[ \gamma n_{B1} - \alpha p'_B = \gamma n_{A1} - \alpha p'_A. \]
Then \( p'_{A1} = \frac{\gamma n_{A1} - \gamma n_{B1} + \alpha p^*_B}{\alpha} \) and \( p'_{B1} = \frac{\gamma n_{B1} - \gamma n_{A1} + \alpha p^*_A}{\alpha} \). The seller’s total profit with this price plan is:

\[
\Pi' = M_1 p'_{A1} e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + p'_{B1} e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} + \frac{\beta}{\alpha} (\alpha p^*_2 - 1)
\]

\[
= M_1 p'_{B1} e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + p'_{A1} e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} + \frac{\beta}{\alpha} (\alpha p^*_2 - 1)
\]

Take the difference of the two profits with the two price plans, we have:

\[
\Pi' - \Pi^* = M_1 \frac{\gamma/\alpha (n_{A1} - n_{B1})(e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} - e^{(\delta + \gamma n_{A1} - \alpha p^*_A)})}{1 + e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + e^{(\delta + \gamma n_{B1} - \alpha p^*_B)}}.
\]

The according to (A.13), we know that \( \Pi' > \Pi^* \). Hence, there exists another price plan that leads to higher profit than \( (p^*_A, p^*_B, p^*_2) \), when \( p^*_A > p^*_B \). Therefore \( p^*_A > p^*_B \) can not be the profit maximization solution.

Therefore, the seller’s profit maximization prices in the first period must satisfy \( p^*_A < p^*_B \). And this implies that \( n_{A2} > n_{B2} \) and so \( s_{A2} > s_{B2} \). And we have:

\[
p^*_A - p^*_B = \frac{\beta\gamma}{\alpha} M_1 s_{01} (s_{B2}^{os} - s_{A2}^{os}),
\]

\[
n_{A2} - n_{B2} = n_{A1} - n_{B1} + M_1 (s_{A1} - s_{B1}).
\]

So we have the following conclusions:

1. The optimal price of A is lower than that of B in the first period: \( p^*_A < p^*_B \).
2. The price gap \(|p^*_A - p^*_B|\) between the two models increase as the OS network effect becomes stronger (\( \gamma \) increases).
3. The OS market share difference \( (n_{A2} - n_{B2}) \) increases in the OS network effect \( \gamma \).

\( \square \)

**A.2 Proof of Proposition 2**

Proof. This proof has three steps. The first step shows that the price of the larger OS network is higher in the second period. The second step shows that the price of the larger OS network is higher in the first period. The third step shows that the operating system with initial advantage in the first period keeps its advantage in
the second period. Combine the above three steps, the proposition statements are proved.

- **Step 1.** This steps shows that, if \( n_{A2} > n_{B2} \) at the beginning of the second period, then \( p_{A2}^{m} > p_{B2}^{m} \). Manufacturer \( j \)'s problem in the second period is:

\[
\max_{p_{j2}} \left\{ \pi_{m}^{j}(p_{j2}, p_{j-2}) \right\} = p_{j2}^{m} s_{j2} M_{2} = p_{j2}^{m} \left( \frac{e^{(\delta_j + \gamma n_{j2} - \alpha p_{j2}^{m})}}{1 + \sum_{k=A,B} e^{(\delta_k + \gamma n_{k2} - \alpha p_{k2}^{m})}} \right) M_{2}.
\]

Then the FOC of the problem is:

\[
s_{j2} + p_{j2}^{*m} (-\alpha s_{j2} + \alpha s_{j2}^{2}) = 0,
\]

which is equivalent to the following equation since \( s_{j2} > 0 \):

\[
p_{j2}^{*m} = \frac{1}{\alpha (1 - s_{j2})}.
\]

By comparing the equations \((A.14)\) for model A and B, we have the following equation:

\[
\frac{p_{A2}^{*m}}{p_{B2}^{*m}} = \frac{1 - s_{B2}}{1 - s_{A2}}. \tag{A.15}
\]

From equation \((A.15)\) and the assumption that \( n_{A2} > n_{B2} \), the result \( p_{A2}^{*m} > p_{B2}^{*m} \) holds. The proof is as follows. Suppose that \( p_{A2}^{*m} \leq p_{B2}^{*m} \). This implies that model A not only has larger OS network size \( (n_{A2} > n_{B2}) \), but also lower prices in the second period. then \( s_{A2} > s_{B2} \). So the LHS (left hand side) of equation \((A.15)\) is less than 1 but the RHS (right hand side) is greater than 1. This contradiction shows that \( p_{A2}^{*m} > p_{B2}^{*m} \) when \( n_{A2} > n_{B2} \). Therefore, the model with larger OS network size at the beginning of the second period has higher price in the second period.

- **Step 2.** This steps shows that if \( n_{A1} > n_{B1} \) and \( n_{A2} > n_{B2} \), then \( p_{A1}^{*m} > p_{B1}^{*m} \). That is, the optimal price of the larger OS model is higher in the first period.
From the Step 1, the maximized profit in the second period for \( j \) is:

\[
\pi^*_{m_2} = p^*_{m_2} s_{j_2} M_2 \\
= \frac{s_{j_2}}{\alpha(1 - s_{j_2})} (1 - n_{A_2} - n_{B_2}) \\
= \frac{s_{j_2}}{\alpha(1 - s_{j_2})} M_1 s_{01},
\]

in which \( s_{01} \) is the market share of the outside option in the first period. Then manufacturer \( j \)'s profit maximization problem in the first period is:

\[
\max_{p^m_{j_1}} \left\{ \pi^m_{m_1}(p^m_{j_1}, p^m_{-j_1}) + \beta \pi^*_{m_2} \right\} \\
= p^m_{j_1} s_{j_1} M_1 + \beta \frac{s_{j_2}}{\alpha 1 - s_{j_2}} s_{01} M_1 \\
= p^m_{j_1} \frac{e^{(\delta_j + \gamma j_1 - \alpha p^m_{j_1})}}{1 + \sum_{k=A,B} e^{(\delta_k + \gamma k_1 - \alpha p^m_{k_1})}} M_1 + \beta \frac{s_{j_2}}{\alpha 1 - s_{j_2}} s_{01} M_1.
\]

Then the FOC w.r.t. \( p^m_{j_1} \) is:

\[
s_{j_1} - \alpha s_{j_1}(1 - s_{j_1}) + \beta s_{01} s_{j_1} \frac{s_{j_2}}{1 - s_{j_2}} \\
+ \frac{\beta}{\alpha} \frac{s_{01}}{(1 - s_{j_2})^2} \frac{\partial s_{j_2}}{\partial s_{j_1}} \frac{\partial s_{j_1}}{\partial p^m_{j_1}} = 0 \quad \text{(A.16)}
\]

Using the definition of \( s_{j_2} \) and \( s_{j_1} \), we get the following partial derivatives:

\[
\frac{\partial s_{j_2}}{\partial s_{j_1}} = \gamma s_{j_2}(1 - s_{j_2}) \\
\frac{\partial s_{j_1}}{\partial p^m_{j_1}} = -M_1 \alpha s_{j_1}(1 - s_{j_1}). \quad \text{(A.17)}
\]

Plug equations in (A.17) into equation (A.16) and rearrange the terms, we get the following equation:

\[
1 + \beta s_{01} \frac{s_{j_2}}{1 - s_{j_2}} = (1 - s_{j_1})(\alpha p^*_{j_1} - \beta M_1 s_{01} \frac{\gamma s_{j_2}}{1 - s_{j_2}}). \quad \text{(A.18)}
\]

Equation (A.18) can be applied to both model A and model B, then by
comparing the two sides for the two models, we get:

\[
1 + \beta \frac{s_{A2}}{1-s_{A2}} = 1 - \frac{s_{A1}}{1-s_{A1}} \frac{\alpha p_{A1}^{*m} - \beta M_1 s_{A1}}{\alpha p_{B1}^{*m} - \beta M_1 s_{B1}} \frac{s_{A2}}{1-s_{A2}}.
\] (A.19)

Given the assumption that \( n_{A2} > n_{B2} \), it is shown in Step 1 that \( s_{A2} > s_{B2} \). So for the equation (A.19), \( LHS > 1 \). Next I show that \( p_{A1}^{*m} > p_{B1}^{*m} \).

Suppose \( p_{A1}^{*m} \leq p_{B1}^{*m} \), then \( s_{A1} > s_{B1} \) because model A not only has the OS network advantage but also lower or equal price than model B. So on the RHS of equation (A.19), we have \( R1 < 1 \). In addition, since \( s_{A2} > s_{B2} \), then \( R2 < 1 \) in equation (A.19). Thus, the \( RHS < 1 \) for equation (A.19), which contradicts the result that \( LHS > 1 \).

Therefore, in the first period, the price of model A is higher than model B, \( p_{A1}^{*m} > p_{B1}^{*m} \), under the condition that \( n_{A2} > n_{B2} \) and the \( n_{A1} > n_{B1} \). In the next step, I show that \( n_{A2} > n_{B2} \) holds if \( n_{A1} > n_{B1} \).

- Step 3. This step shows that the manufacturer A keeps its OS network advantage to the second period: \( n_{A2} > n_{B2} \) given \( n_{A1} > n_{B1} \). Suppose on the contradictory that \( n_{A2} \leq n_{B2} \), then according to Step 1 result, \( s_{B2} > s_{A2} \). So \( LHS < 1 \) for equation (A.19). Also \( n_{A2} \leq n_{B2} \) implies that \( s_{A1} < s_{B1} \), so \( R1 > 1 \) for equation (A.19). In addition \( p_{A1}^{*m} > p_{B1}^{*m} \) and that \( s_{B2} > s_{A2} \) imply that \( R2 > 1 \) for equation (A.19). Hence, the \( RHS > 1 \), which contradicts that \( LHS < 1 \). Thus, \( n_{A2} \leq n_{B2} \) can’t hold, which means that \( n_{A2} > n_{B2} \) holds.

The three steps above shows that when the two manufacturers choose prices, the one with initial OS network advantage choose higher prices in both periods and keeps its advantage in the second period. So I have proved that: (1), \( p_{A1}^{*m} > p_{B1}^{*m} \), for \( t = 1, 2 \), and (2), \( n_{A2} > n_{B2} \).

\[ \square \]

A.3 Competition among Multi-Network Sellers

In this subsection, I use two steps to argue that even with multiple multi-network sellers in the two-period model, the equilibrium prices would still be that the
product with the large network has a lower price than the small network in the first period. Without loss of generality, suppose there are two symmetric sellers. Notice that, in the static game in the second period, the two sellers choose same prices for both A and B and have same profit. The following arguments is for the prices and profits in the first period.

First of all, there exists a symmetric equilibrium in which the two sellers choose the same prices for the two models. In such a symmetric equilibrium, the large platform will have a lower price than the small platform in the first period. Suppose instead, that both sellers choose a higher price for the large platform in the first period. Then given the rival’s prices, each seller has the incentive to deviate and chooses a lower price for the product with the large network. Because the deviating seller gain consumers from both the other seller and the outside option. This makes the deviation profitable. Hence, in a symmetric equilibrium, the price of the product with the large network is lower than that of the small network in the first period.

Second, there doesn’t exist an asymmetric equilibrium. Suppose instead, that seller 1’s price of the large network is lower than the small network, and seller 2’s price of the large platform is higher than the small network. Then seller 2 could be better off by choosing the same price as seller 1. Because to get the same profit as seller 1 in the first period, seller 2 has to choose a very low price for the small platform to compete for customers, when seller 1 can easily get consumers with a low price on the initially large platform.

Therefore, when there are multiple sellers, they would coordinate on choosing the same low price for the large network than the small network. Intuitively, the network effect is not seller specific, so the sellers can’t exclude others from benefiting from the growing OS networks. As a result, no seller would like to grow the small platform by sacrificing the first period profit.
Chapter B  
Solve for the Model Markups

The first-order conditions with respect to carrier prices are:

\[
M_t s_{jst} + \sum_{(k,s') \in \Omega_{ct}} (p_{jst}^c - \omega_j p_{jst}^M + p_{ct}^p - 24 \kappa_{sc} - \lambda_{jst}) \frac{\partial s_{ks'ct}}{\partial p_{jst}^c} + \beta_d \frac{\partial V_c(n_{t+1})}{\partial p_{jst}^c} = 0. 
\]  
\tag{B.1}

Next the partial derivatives in equation (B.1) will be explicitly derived. First, given the model market share in equation (2.14), the partial derivatives of shares with respect to prices can be derived. For \(\frac{\partial s_{j's'c't}}{\partial p_{jst}^c}\), the derivatives depend on whether the products are in the same group or not.

With the assumption that the coefficient matrix determining how consumer characteristics affect utility is diagonal, consumer \(i\)’s price coefficient can be written as:

\[
\alpha_i = \alpha + \phi_i y_i. 
\]

If \((j', s', c') = (j, s, c)\), then

\[
\frac{\partial s_{j's'c't}}{\partial p_{jst}^c} = -\frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst}(1 - s_{ijst}). \tag{B.2}
\]

If \((j', s', c') \neq (j, s, c)\), then the partial derivative is:

\[
\frac{\partial s_{j's'c't}}{\partial p_{jst}^c} = \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ij's'c't} s_{ijst}. \tag{B.3}
\]

where \(N_s\) is the number of simulated consumers.
Second, for $\frac{\partial V_c(n_{t+1})}{\partial p^c_{jst}}$, we have:

$$\frac{\partial V_c(n_{t+1})}{\partial p^c_{jst}} = \sum_{l=1}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{lt+1}} \frac{\partial n_{lt+1}}{\partial p^c_{jst}}.$$  \hspace{1cm} (B.4)

Given an approximate of the value function form $\hat{V}_c(n_t)$, then $\frac{\partial V_c(n_{t+1})}{\partial n_{st+1}}$ can be derived. And $\frac{\partial n_{st+1}}{\partial p^c_{jst}}$ can be derived from the network size transition rule. It also depends on whether $s = s'$ or not. The network size transition rule is:

$$n_{s't+1} = \frac{7}{8} n_{st} + \frac{M_t}{\text{pop}} \sum_{(j',c') \in \Omega_{st}} s_{j's'c'}(p^c_t).$$

If $s = s'$, then

$$\frac{\partial n_{s't+1}}{\partial p^c_{jst}} = \frac{M_t}{\text{pop}} \sum_{(j',c') \in \Omega_{st}} \frac{\partial s_{j's'c'}}{\partial p^c_{jst}} = \frac{M_t}{\text{pop}} \sum_{(j',c') \in \Omega_{st}} \left[- \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} + \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} s_{j's'c'} \right].$$

If $s \neq s'$, then the partial derivative of OS shares with respect to carrier price is:

$$\frac{\partial n_{s't+1}}{\partial p^c_{jst}} = \frac{M_t}{\text{pop}} \frac{1}{N_s} \sum_{(j',c') \in \Omega_{st}} \alpha_i s_{ijst} s_{ij's'c'}.$$

Then we have:

$$\frac{\partial V_c(n_{t+1})}{\partial p^c_{jst}} = \sum_{s' = 1}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \frac{\partial n_{s't+1}}{\partial p^c_{jst}}$$

$$= \frac{\partial V_c(n_{t+1})}{\partial n_{st+1}} \frac{M_t}{\text{pop}} \sum_{(j',c') \in \Omega_{st}} \left[- \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} + \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} s_{j's'c'} \right]$$

$$+ \sum_{l \neq s}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \frac{M_t}{\text{pop}} \frac{1}{N_s} \sum_{(j',c') \in \Omega_{st}} \alpha_i s_{ijst} s_{ij's'c'}$$

$$= \frac{M_t}{\text{pop}} \sum_{s' = 1}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \left( \sum_{j',c' \in \Omega_{st}} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} s_{ij's'c'} \right) - \frac{\partial V_c(n_{t+1})}{\partial n_{st+1}} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst}. \hspace{1cm} (B.5)$$

Then the equations (B.2)-(B.5) can be plugged back into equation (B.1). Define the markup as $m_{jst} = p^c_{jst} - \omega_j p^m_{jst} + p^p_t - \kappa_{sc} - \lambda_{jst}$. Then plug the derivatives
into the FOC, we have:

\[ s_{jstc} + \sum_{(j',s') \in \Omega_{stc}} m_{j's'tc} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijstc} d_{ij's'tc} - \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijstc} m_{jstc} + \frac{\beta^d}{M_t} \partial V_c(n_{t+1}) = 0. \]  

\text{(B.6)}

Since the individual market shares can be calculated from the demand model with estimated parameters \( \hat{\theta}_d \) and the carriers’ value functions are parametrically approximated, the only unknowns are the markups \( m_{jstc} \)'s in equation \text{(B.6)}, given the parameters. There are \( J_t \) equations and \( J_t \) unknowns. The equations are linear in the unknowns. So the markups \( m_{jstc} \)'s can be solved using matrix inversion. Then we can calculate the unobserved cost shock:

\[ \lambda_{jstc} = p_{jstc}^c - \omega_j p_{jstc}^m + f_{ct} - \kappa_{stc} - m_{jstc}. \]  

\text{(B.7)}
Chapter C
The Inversion of Carrier-OS Shares to Carrier-OS Unobserved Quality

The goal is to prove that the observed vector carrier-OS sales market shares $s_{sct}$ uniquely determine a vector of the carrier-OS unobserved quality $\xi_{sct}$. This proof is similar to the proof in Berry (1994). For notation simplicity, I use a logit model version of the demand model with carrier-OS unobserved quality. This proof goes through for the random coefficient demand model as well.

Consider the consumer utility function:

$$u_{ijst} = \bar{\delta}_{jst} + \xi_{sct} + \epsilon_{ijst},$$

where $\bar{\delta}_{jst}$ is the mean observed utility and $\xi_{sct}$ is carrier-OS specific unobserved quality. The utility of the outside option is assumed to be $u_{i0t} = \epsilon_{i0t}$. Then given the assumption that $\epsilon_{ijst}$’s follow Type-I extreme value distribution and are i.i.d. across consumers, models, and periods, the market share of carrier-OS group $(c, s)$ in period $t$ is:

$$s_{sct} = \mathcal{I}(\xi_t) = \sum_{j \in \Omega_{sct}} \frac{e^{(\bar{\delta}_{jst} + \xi_{sct})}}{1 + \sum_{(j', s', c') \in \Omega_t} e^{(\bar{\delta}_{j's't} + \xi_{s'c't})}}.$$  \hfill (C.1)

I shall prove that there is a unique $\xi_t = \mathcal{I}^{-1}(s_{data}^t) \in \mathbb{R}^K$ for any fixed finite $\bar{\delta}$ vector, where $s_{data}^t$ is the observed vector of carrier-OS market shares in period $t$. The equation \eqref{C.1} has the following properties. (1) $\partial \mathcal{I}_{sct}/\partial \xi_{sct} > 0$; (2) $\partial \mathcal{I}_{sct}/\partial \xi_{s'c't} < 0$, if $(s, c) \neq (s', c')$; and (3) $\mathcal{I}_{sct}$ approaches to zero as $\xi_{sct}$ goes to $-\infty$ and it approaches to 1 as $\xi_{sct}$ goes to $\infty$.

Define the element-by-element inverse, $r_{sc}(\xi_t, s_{data}^{ct})$. This function is defined as the unobserved quality value for carrier-OS $(s, c)$ such that the model predicted
carrier-OS share \( \mathcal{S}_{ct} \) equals the observed share \( s_{ct}^{data} \). That is:

\[
{s_{ct}^{data} = \mathcal{S}(r_{sc}(\xi_t, s_{ct}^{data}), \xi_{-ct}; \tilde{t})}.
\]

(C.2)

Since the market share function \( \mathcal{S} \) is continuously differentiable and satisfy the three properties above, the function \( r_{sc}(\xi_t, s_{ct}^{data}) \) is well defined and differentiable. In particular, \( r_{sc} \) is strictly increasing in \( \xi_{s', c'} \) for any \( (s', c') \neq (s, c) \) and doesn’t depend on \( \xi_{ct} \). So a vector \( \xi_t \) solves \( \mathcal{S}(\xi_t, s_{ct}^{data}) = s_{ct}^{data} \) if and only if it is a fixed point of the element-by-element inverse: \( \xi_t = r(\xi_t, d^{data}) \). Next, I first show existence of fixed point of \( r(\xi_t, d^{data}) \), then show the uniqueness of the fixed point.

First, \( r(\xi_t, d^{data}) \) has a lower bound \( \tilde{\xi} \). The lower bound for the \((s, c)\)th element is the value of \( r_{sc}(\xi_t', d^{data}) \), with \( \xi_{s', c'}' = -\infty \), for all \( (s', c') \neq (s, c) \). Define \( \tilde{\xi} \) as the smallest value across the products of these lower bounds. Note that there is no upper bound for \( r_{sc} \), but a slight variant of the element-by-element inverse has.

**Lemma 2.** There is a value \( \tilde{\xi} \), with the property that if one element of \( \xi_t \), say \( \xi_{ct} \) is greater than \( \tilde{\xi} \), then there is another carrier-OS pair \((s', c')\) such that \( r_{s', c'}(\xi_t, s_{ct}^{data}) < \xi_{s', c'} \).

*Proof.* To construct \( \tilde{\xi} \), set \( \xi_{s', c'} = -\infty \), for all \( (s', c') \neq (s, c) \). Then define \( \tilde{\xi}_{ct} \) as the value of \( \xi_{ct} \) that set the outside option market share \( \mathcal{S}(\tilde{\xi}_{ct}, \xi_{-ct}) = s_{ct}^{data} \). Define \( \tilde{\xi} \) as any value greater than the maximum of the \( \tilde{\xi}_{ct} \). Then, if for the vector \( \xi_t \), there is an element \( (s, c) \) such that \( \xi_{ct} > \tilde{\xi}_{ct} \), then \( \mathcal{S}(\tilde{\xi}_{ct}, \xi_{-ct}) < s_{ct}^{data} \), which implies that \( \sum_{s', c'} \mathcal{S}_{s', c'}(\xi_t; \tilde{\delta}_t) > \sum_{s', c'} s_{ct}^{data} \), so there is at least one carrier-OS pair \((s', c')\) such that \( \mathcal{S}_{s', c'}(\xi_t; \tilde{\delta}_t) > s_{ct}^{data} \). Then for this pair \((s', c')\), \( r_{s', c'}(\xi_t, s_{ct}^{data}) < \xi_{s', c'} \).

Now define a new function which is a truncated version of \( r_{sc} \): \( \tilde{r}_{sc}(\xi_t, s_{ct}^{data}) = \min\{r_{sc}(\xi_t, s_{ct}^{data}), \tilde{\xi}\} \). Then \( \tilde{r} \) is a continuous function which maps \( [\xi, \tilde{\xi}] \) into itself. Then by Brouwer’s fixed-point theorem, \( \tilde{r} \) has a fixed point \( \xi^* \). By the definition of \( \tilde{\xi} \) and \( \tilde{\xi} \), \( \xi^* \) can’t have a value at the upper bound, so \( \xi^* \) is in the interior of \( [\xi, \tilde{\xi}] \). This implies that \( \xi^* \) is also a fixed point of the function \( r(\xi_t, s_{ct}^{data}) \). So there exists a fixed point for the element-by-element inverse function.

Next I show the uniqueness of the fixed point. One sufficient condition for uniqueness is the diagonal dominance of the Jacobian matrix of the inverse functions. That is: \( \sum_{(s', c') \neq (s, c)} |\partial r_{sc} / \partial \xi_{s', c'}| < |\partial r_{sc} / \partial \xi_{ct}| \). By the implicit function theorem
on equation (C.2), we have:

\[
\frac{\partial r_{sc}}{\partial \xi_{s'c't}} = -\frac{\partial \mathcal{S}_{sct}/\partial \xi_{s'c't}}{\partial \mathcal{S}_{sct}/\partial \xi_{sct}},
\]

which implies that \( \frac{\partial r_{sc}}{\partial \xi_{sct}} = 1 \). Then the sum is:

\[
\sum_{(s',c') \neq (s,c)} \left| \frac{\partial r_{sc}}{\partial \xi_{s'c't}} \right| = \frac{1}{|\partial \mathcal{S}_{sct}/\partial \xi_{sct}|} \sum_{(s',c') \neq (s,c)} \left| \frac{\partial \mathcal{S}_{sct}}{\partial \xi_{s'c't}} \right|.
\]

(C.3)

Note that increasing all the unobserved quality levels (including the outside option \( \xi_{0t} \)) by the same amount wouldn’t change any market share. That is:

\[
\sum_{s'=0}^{K} \frac{\partial \mathcal{S}_{sct}}{\partial \xi_{s'c't}} = 0
\]

Then it implies that:

\[
\frac{\partial \mathcal{S}_{sct}}{\partial \xi_{sct}} = -\frac{\partial \mathcal{S}_{sct}}{\partial \xi_{0t}} \sum_{s' \neq (sc)} \frac{\partial \mathcal{S}_{sct}}{\partial \xi_{s'c't}}.
\]

Since all terms in the right hand side are strictly negative, so

\[
\left| \frac{\partial \mathcal{S}_{sct}}{\partial \xi_{sct}} \right| > \sum_{s' \neq (sc)} \left| \frac{\partial \mathcal{S}_{sct}}{\partial \xi_{s'c't}} \right|.
\]

Then the sum in equation (C.3) is:

\[
\sum_{(s',c') \neq (s,c)} \left| \frac{\partial r_{sc}}{\partial \xi_{s'c't}} \right| < 1 = \left| \frac{\partial r_{sc}}{\partial \xi_{sct}} \right|.
\]

Hence the sufficient condition for uniqueness is satisfied. Therefore, the element-by-element inverse function has unique fixed point \( \xi^* \), which is the solution of the market share inversion function.
Chapter D  
Computation Appendix

D.1 Simulating the Income Levels of Consumers

In the estimation, I simulate the income levels $Y_{it}$ of 300 individuals for each period $t$ based on household income distributions using yearly Current Population Survey (CPS) data. Each individual’s income level is assumed to independently follow a lognormal distribution, whose mean and standard deviation are from the CPS data.

I normalize the simulated individuals’ income levels by the log mean income in 2011, such that the mean of normalized log income levels is zero in 2011. That is, in the estimation, the log income level of consumer $i$ in year is $y_{it} = \log(Y_{it}) - \mu_{2011}$, where $\mu_{2011}$ is the log of mean income in 2011 and $Y_{it}$ is $i$’s income level in dollars. The income levels in 2012 and 2013 are also normalized by $\mu_{2011}$. In this way, the normalized log-income data keep the pattern of growing household income over time.

D.2 Number of Monte Carlo Draws to Approximate Integration over Shocks

In the estimation, the MPEC constraints are the carriers’ Bellman equations. These constraints are used so that the value functions are well approximated.

$$V_c(n_t) = E_{\xi,\lambda}[\max_{p_{jact}(\xi,\lambda_t), (j,s) \in \Omega_{ct}} \left\{ \pi_{ct}(p^*_t, \xi_t, \lambda_t) + \beta^d V_c(n_{t+1}(n_t, p^*_t(\xi_t, \lambda_t))) \right\}].$$

To calculate the expectation over $(\xi, \lambda)$ on the right hand side, I simulate $R$ vectors of quality shocks and cost shocks $(\xi^r, \lambda^r)$, $r = 1, ..., R$ to approximate the integration
of discounted profits over \((\xi, \lambda)\).

\[
\hat{V}_c^R(n_t) \approx \frac{1}{R} \sum_{r=1}^{R} \max_{p_{jsc}^r(\xi^r, \lambda^r), (j,s) \in \Omega_{ct}} \left\{ \pi_{ct}(p_t^r, \xi^r, \lambda^r) + \beta^d V_c(n_{t+1}(n_t, p_{ct}^r(\xi^r, \lambda_t))) \right\}.
\]

Given a set of parameter values, the algorithm solves the equilibrium prices of the carriers’ dynamic pricing game for each \((\xi^r, \lambda^r)\) and use the average over the draws to approximate the value functions.

In the estimation, I use 50 draws \((R=50)\) due to computation burden. But since the dimension of \((\xi, \lambda)\) is more than 200, I need to check whether \(R = 50\) is generating large error in the value function. To check this, I simulate more draws \((R = 100, 300, 1000)\) and use the estimation results to re-calculate the \(\hat{V}_c^R(n_t)\)'s for all carriers in all periods. I find that when \(R\) increases from 50 to 1000, \(\hat{V}_c^R\) changes by 1.35% on average with a standard deviation of 1.61%.

**D.3 Solving the Carriers’ FOC’s**

For each simulated shock \((\xi^r, \lambda^r)\), the algorithm for solving the equilibrium prices in period \(t\) follows four steps.

1. Guess an initial price vector for all models, \(p^{0r}\). With the price, I calculate the consumers’ choice probabilities \(s_{ijsc}^{0r}\) for all models and the next period state variable \(n_{t+1}(n_t, p^{0r})\) using \(p^{0r}\) and \(n_t\).

2. Calculate the markups \(m_{jsc}^{0r}\) using first-order conditions (B.6) for all models, which are linear in the markup \(m_{jsc}\)'s.

3. Update the price for all models simultaneously. The new price for model \((j, s, c, t)\) is:

\[
p_{jsc}^{1r} = m_{jsc}^{0r} + c_{jsc}(\theta_s, \lambda^r).
\]

4. Compare \(p^{1r}\) with \(p^{0r}\). If the distance between the two prices, in \(L^1\) norm, are larger than \(10^{-3}\), then repeat step 2-4. If the distance is smaller than \(a0^{-3}\), then the prices are solved.
D.4 Plots of the Carriers’ Value Functions

Each carrier’s value function is a function of the 4 state variables, the 4 OS market shares. Figure 5 and 6 are the plots of the value functions. The market shares of iOS and Android are varying in Figure 5. The market shares of Blackberry and Windows Phone are fixed at the July 2013 shares. In Figure 6, the market shares of Blackberry and Windows Phone are varying. The market shares of iOS and Android are fixed at the July 2013 shares.

Figure D.1: Value Functions (iOS and Android Market Shares)
Figure D.2: Value Functions (Blackberry and Windows Phone Market Shares)
Bibliography


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