DEVELOPMENT OF AN IMPROVED ALGORITHM FOR

WORKPIECE LOCALIZATION OF RAW MATERIAL

A Thesis in

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by

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ABSTRACT

One of the problems that manufacturing industries are faced with is the proper localization of workpieces within the raw blanks while maintaining sufficient machining allows. This is especially important in case where the raw material is of near net shape. The solution of this problem is defined as workpiece localization. The corresponding decision-making is especially critical when the size and shape of the blank are closely specified to the designed model to ensure sufficient material.

In this paper, we present an improved workpiece localization algorithm for machining, which is achieved by two-step point cloud localization. Firstly, the two point clouds are created (one for the part and one for workpiece) and localized roughly by the Principal component analysis algorithm. Secondly, a more precise algorithm, i.e., least square-based algorithm, is used to search for the best translation and rotation of the workpiece within the blank. The algorithm allows an optimal setup of the part to ensure that no shortage of material occurs during machining. Through transformation, the algorithm determines whether or not the designed model is totally enclosed in the actual raw material to be machined. The two-step localization algorithm can reduce the computational time. The input to the new algorithm is simplified for a 2D workpiece localization process by using point clouds. A 2D example of plasma cutting of the blank and subsequent machining is used to test the algorithm. The results show the processing time is faster than other localization methods using the simplified inputs.
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Chapter 1

Introduction

1.1 Background

In the manufacturing industry, one critical problem of machining is the proper localization of the designed part relative to the raw material. To obtain the expected part successfully, the designed part must be enclosed in the raw material with sufficient machining allowance. The machining allowance is a planned deviation between an actual dimension and a theoretical dimension, or between an intermediate-stage dimension and an intended final dimension. The machining allowance is contrasted with a tolerance, which accounts for expected but unplanned deviations. The inspection for the machining allowance of raw material, which is also defined as workpiece localization, is significantly important, since it ensures there is enough material available for machining.

Workpiece localization is defined as the following: given a rigid raw material arbitrarily placed in a coordinate system, the position and orientation of the corresponding final machined part within the raw material needs to be determined relative to the raw material, with sufficient machining allowance. This ensures that there is no material shortage in the raw material. Consider a raw material that is randomly placed on the coordinate system as shown in Fig 1.1(a). We wish to transform the reference model’s position and orientation relative to the raw material to ensure that no
shortage of material occurs during machining. Fig 1.1(c) shows the machining allowance of the part, which is set before machining.

Figure 1.1. Workpiece localization process
In the workpiece localization process, the raw material must have sufficient machining allowance based on the applied machining requirement. In the machining process of complicated parts from casting or forging, the machining allowance has an important effect on the workpiece’s machining efficiency, since larger machining allowance requires longer processing time. However, material shortage may happen if the designed machining allowance for the workpiece is too small. Hence, workpiece localization is extremely critical for precise machining when the raw material size and shape are closely specified to the design model to reduce the non-conformance.

The traditional localization methods of checking the machining allowance are by manual measurement. According to the designed model, the nominal dimensions of the raw material are measured and then the location of the machining allowance is assigned and scribed empirically on the raw material. Giving an accurate evaluation is significantly difficult and time-consuming, even though accurate tools and gauges can be used.

In recent years, the competition in the manufacturing industry has become increasingly intense, causing a drop in gross profits of manufacturing companies. In order to survive in the fierce competitive environment, companies are trying to develop and adopt automatic production technology to replace, as much as possible, human intervention. In turn, this leads to improved efficiency and product’s accuracy. In order to make the workpiece localization process more efficient, the automatic workpiece localization methods have been developed, which can help avoid the human intervention.
1.2 Automatic Workpiece Localization

The development of technology, such as computer aided design and numerical control, makes the automatic workpiece localization possible to replace the manual measurement. In the automatic localization processes, the reference modeling provides the reference for the localization, while the data of the raw material can be obtained by some measuring instruments.

Before machining, a workpiece localization process should be conducted to ensure that the reference model is totally enclosed in the raw material with enough machining allowance. Even small deviations produced by the raw material manufacturing process can create material shortage and result in rework. When the reference model cannot be entirely enclosed in the raw material, or the machining allowance is insufficient for machining in certain direction, it means the raw material is not acceptable for the designed model. Therefore, the evaluation of the machining allowance is a very important procedure before actual machining.

To process the workpiece localization, data point clouds of the raw material and the reference model are indispensable in transforming the location of the reference model to the raw material. The development of CMM technology and 3D optical metrology makes automatic workpiece localization possible. The point cloud of the raw material can be measured by a CMM machine or a 3D optical scanner. The process can be roughly divided in two main steps. First, a raw material arbitrarily fixed on a machine table is measured by sampling a number of points on the surfaces of the raw material using a touch trigger probe or a laser scanner. Second, the position and orientation, which allows
designed model enclosed in the raw material, is optimized based on the data supplied by the scanned data and the corresponding reference model. Meanwhile, the data point cloud of the reference model can be generated by some CAD softwares, such as Solidworks, Imageware etc.

In recent years, several automatic methods have been developed to evaluate workpiece localization based on the data of point clouds, with the goal of increasing efficiency and accuracy of localization. Through the workpiece localization algorithm, the point cloud of the reference model transforms to the point cloud of the raw material with sufficient machining allowance in each point. The main goal of the workpiece localization process is to determine whether the reference model is totally enclosed in the raw material with the required machining allowance, which leads to no material shortage of the raw material.

1.3 Problem Statement

This paper presents an automatic workpiece localization algorithm for machining of raw material. The new algorithm focuses on the 2D process such as cutting with simpler inputs and shorter processing time compared to other methods which are used for the 3D process.

The simplified inputs contain:

1) The measured point cloud dataset of raw material, which is obtained from an optical scanner;
2) The point cloud dataset of corresponding reference model, which is generated from the designed model by software.

The new method removes the input of the relationship between dataset of raw material and the reference data, which reduces most of the workload of acquiring data.

The goal of this paper is creating an efficient algorithm with

1) Reliable result for workpiece localization;

2) Simplified inputs;

3) Less human intervention;

4) Shorter processing time.

The paper is organized as follows: Section 2 discusses several existing methods of workpiece localization. Section 3 describes the improved workpiece localization algorithm for 2D problem with simplified input and shorter processing time. Section 4 shows the result of the example, which is used to test the algorithm, and concludes the paper.
Chapter 2

Literature Review

2.1 Iterative Closest Point Algorithm

Iterative closest point (ICP) is an algorithm which defines an objective function and constraints to minimize the difference between two point clouds. ICP is often used to reconstruct 2D or 3D surfaces from different sources, in order to evaluate the potential location and material shortage issues.

In the algorithm, one point cloud (usually from the design model) is kept fixed as the reference or target, while the other point cloud, as the source, (usually captured by CMM or 3D optical scanner) is transformed from its original place to best match the reference iteratively. The transformation combines translation and rotation, which minimize the distance from each point of the source database to the reference point cloud step by step.

The ICP algorithms are found in a number of previous researches. Some papers focused on rigid transformation \cite{2,3,4,5,15,16,17,18,19}, while the others were looking for solution of non-rigid situation \cite{20,21,22,23,24,25}. For the workpiece localization problem, both the size and the shape of the raw material should not be changed during transformation. Therefore, the rigid ICP-based algorithm should be considered in the workpiece localization.
The inputs of the ICP algorithm include:

1) Measurement point cloud;
2) Reference point cloud;
3) Initial estimation of the transformation to align the source to the reference;
4) Criteria for stopping the iterations.

The basic steps of the algorithm are:

1) Finding the closest point from the reference point cloud dataset for each point in the measurement point cloud in order to minimize the difference between two point clouds;
2) Calculating the transformation matrix which including translation and rotation matrix, using a mean squared error cost function which will best align each measured point to its match found in the previous steps;
3) Transforming the measured points using the transformation matrix obtained in step 2;
4) Iterating the previous steps until the result fits the criteria.

Chen [16] proposed a localization algorithm for constructing a complete surface from different views of an object. When locating multiple overlapping views of an object, an accurate transformation was needed for surface localization in order to combine them. In his research, a localization algorithm was presented for range image localization which works on the images directly. This localization algorithm was based on minimizing the
distance measurement function which was derived from the definition of 3D surface localization. This function does not require a point to point match, which achieves the localization between different views.

Paul [15] described an ICP-based algorithm for the accurate and efficient localization of 3D shapes including free-form curves and surfaces. The rate of convergence of this algorithm was quicker compared to the generic nonlinear optimization algorithm. The quaternion-based algorithm was replaced the singular value decomposition (SVD) method, which was usually used in other ICP algorithm, for searching the transformation of the reference model.

Yan [17] presented a fast and robust ICP algorithm for workpiece localization problem by exploiting the biometrics application context. This research introduced the “Pre-computed Voxel Closest Neighbors” strategy to improve the speed of the original ICP-based algorithm. In the ICP algorithm, the most time consuming process is linking each point in the measured point cloud in order to find the closest point on the reference point cloud. In their research, the distance from any point in the 3D space to the reference surface was precomputed, and when the distance was needed, it could be directly utilized, which reduced the search time.

Jost [18] proposed an accelerated ICP algorithm for fast shape localization. The algorithm accelerated the process by finding multiple solutions in which each solution at the lower level could be successively improved at a higher level of representation. A K-D tree search and a neighbor search method were used for multiple solutions which had been theoretically and experimentally compared in a 3D shape matching test. Using
either the K-D tree search or the neighbor search, multiple solutions speeded up the localization process, which improved the convergence speed and matching quality.

The ICP-based workpiece localization algorithms are widely used with different 2D and 3D geometric data. However, since the basic ICP algorithm uses a mean square function as the objective function, the algorithm cannot push all points in the measurement point cloud out of the reference point cloud, which can cause a shortage of material in certain places. Additionally, the processing time of the ICP algorithm is slower comparing to other algorithms.

2.2 Geometric Algorithms for Workpiece Localization

Chu [26][27][28] defined a hybrid localization algorithm solved by nonlinear programming for partially finished workpieces. The algorithm first aligned the reference model with the raw material on its finished surfaces. Next, the geometric problem was converted to a nonlinear programming problem with a convex objective function. The transformation of the raw material was aligned with all the unfinished surfaces out of the model to guarantee the allowances for the future machining. For an arbitrarily fixed raw material, the algorithm computed an appropriate solution with high robustness and computational efficiency.

Li et al [1][7][8][9] developed an alternating variational approach to localize the parts for both general and symmetric workpieces. Firstly, a least square method was used to formulate the localization problem for a general 3D workpiece. The objective function of
the algorithm gave conditions for optimal reference surface points. Then, an iterative approach was developed for solving the workpiece localization problem. While the reference point cloud was fixed, the measurement point cloud transformed to the corresponding reference of the transformation matrix, which in turn provided translation and rotation information. The transformation matrix was obtained from the singular value decomposition method. In each iteration, the algorithm calculated a new transformation matrix and directed the raw material point cloud to move closer to the reference until the localization fits the objective function.

Xu et al. [29] introduced another geometric algorithm for symmetric workpiece localization. The algorithm used a geometric function for the optimal Euclidean transformation which localized the measured point cloud to the corresponding reference surface point cloud. The reference surface point cloud was given by two nonlinear equations. In the research, formulas for different symmetrical features were described for workpiece localization respectively. Experimental results showed this algorithm was more computationally efficient than the variational algorithm. However, the algorithm could not be applied to discreet symmetric workpiece problems.

Compared to the ICP-based algorithm, the geometric algorithm has shorter processing time and stronger reliability. However, the input of the algorithm includes the relationship linking each point in the raw material point cloud to the reference surface respectively, which involves manual work.
2.3 Feature Based Workpiece Localization Algorithm

A two-step workpiece localization algorithm was introduced by Xudong, Li et al.\cite{30,31} to solve this dilemma. The Principle Component Analysis (PCA) based algorithm was used to roughly localize the measured point cloud closer to the reference point cloud, which reduced the computation time for the future precise localization. Then, a feature-based localization algorithm extracted the proper surface from both point clouds and the measured point cloud to the reference point cloud with sufficient machining allowance. In the paper, an improved cube-dividing-based approach was recommended to extract planes from point cloud. The point cloud was divided into several cubes with corresponding points in them. Next, the algorithm merged these cubes into different feature planes by calculating the relationship of each cube with its neighbor cubes. A constrained localization approach was described to link the feature planes from each point cloud together to achieve the final result with sufficient machining allowance.

The feature based workpiece localization algorithm can potentially solve the material shortage-checking problem. However, the proposed approach is not fully automatic, which required specifying some critical parameters, such as the thresholds for the algorithm. Additionally, it is not useful in the situation where there is no dominating feature plane in the point cloud.

All the algorithms mentioned above required mapping the measured point cloud dataset to the nominal surfaces of the reference model. This input cannot be obtained directly, which means additional human work is required. In this paper, we introduce an
algorithm, which ignores this input in a 2D workpiece localization process. The efficiency of the algorithm is also higher than previous algorithms in a 2D case.
Chapter 3

Development of the Improved Algorithm for Workpiece Localization

3.1 Algorithm Introduction

To evaluate the workpiece localization process successfully, one question must be clarified: Can the designed reference model of a given workpiece be entirely enclosed in the corresponding measured part with enough allowance? If the answer is yes, then the raw material is proved to satisfy the machining process with sufficient stock allowance, and the algorithm will calculate the best location of the part. If the answer is no, it means one or more places of the raw material will be missing material. When machining, without the proper localization, the workpiece will either be scrapped or reworked after being machined correctly. In this case, the algorithm will preferentially push the designed model out of the raw material to satisfy the machining requirements fully and minimize the related rework cost.

To solve this problem, the machining allowance evaluation is accomplished by a two-step localization of the CAD-discretized point cloud and the measured point cloud, as shown in Fig. 3.1. The constraints are needed to push every measurement point outside the reference model in order to ensure sufficient stock allowance when possible.
A point cloud of the raw material could be captured by a laser scanner or Coordinate Measuring Machine (CMM). The point cloud of the raw material is arranged in counter-clockwise sequence, which is used for calculating the normal vector of point cloud later. Then, two point clouds are roughly localized based on Principle Component Analysis (PCA) algorithm. Through calculating normal vectors of lines created by the point with its former point and with latter point, an estimated normal vector of each point in the point cloud is calculated by averaging these two vectors. Next, an iterative least
squares algorithm is introduced for the precise localization. When the minimum distance between the two point clouds is more than the required machining allowance, the iteration stops and a feasible solution is created.

3.2 Preprocessing of Point Cloud Data

In this paper, a filtering process is necessary for the preprocessing of the 3D point cloud which can remove noise points and simplify the point number to increase the computational speed and accuracy \[^{32}[33]\]. Meanwhile, the reference model is discretized to obtain point cloud by using 3D modeling software (Imageware).

Fig. 3.2 shows the part’s original scanned data (a), its after-processed point cloud (b), and the discretized reference model (c).

Since the sequence of point stored in the data file is unknown, for the future calculation, the point cloud of the raw material data is arranged in counter-clockwise sequence.

3.3 Rough Localization of Measured Point Cloud

Commonly, the reference point cloud and the scanned point cloud are separately captured, for they are obtained from different points of view, as shown in Fig. 3.2(a). To localize the reference point cloud and the scanned point cloud roughly, a PCA \[^{34}[35][36]\] based method is used to accomplish the rough localization.
Fig 3.2. Original scanned data (a); After-processed data (b); discretized reference model (c)
Principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components.

The approach of PCA is defined as follow:

Step 1: Input the whole point cloud \( \mathcal{X} \);

Step 2: Calculate the \( n \)-dimensional mean vector of each point cloud;

Step 3: Compute the covariance matrix of the standardized dataset.

Standardization of features will have an effect on the outcome of a PCA (assuming that the variables are originally not standardized).

The equation for standardization of a variable is written as

\[
Z = \frac{X_i - \bar{X}}{s_x}
\]  
(3-1)

The original covariance matrix is:

\[
\sigma_{xy} = \frac{1}{n-1} \sum_{i} (X_i - \bar{X})(Y_i - \bar{Y})
\]  
(3-2)

While, \( X'_i = \frac{X_i - \bar{X}}{s_x} \) and \( Y'_i = \frac{Y_i - \bar{Y}}{s_y} \), after standardizing, the covariance matrix is:
\[
\sigma_{xy}' = \frac{1}{n-1} \sum_{i=1}^{n} (x_i' - \bar{x})(y_i' - \bar{y})
\]

(3-3)

\[
\sigma_{xy}' = \frac{xy'}{sxy}
\]

Step 4: Compute the eigenvectors and eigenvalues of the covariance matrix

Compute the matrix \( V \) of eigenvectors which diagonalizes the covariance matrix \( \sigma_{xy}' \):

\[
V^{-1} \sigma_{xy}' V = D'
\]

(3-4)

where \( D' \) is the diagonal matrix of eigenvalues of \( \sigma_{xy}' \). This step will typically involve the use of a computer-based algorithm for computing eigenvectors and eigenvalues. These algorithms are readily available as sub-components of most matrix algebra systems, such as R, Matlab;

Step 5: Rearrange the eigenvectors and eigenvalues.

Sort the columns of the eigenvector matrix \( V \) and eigenvalue matrix \( D' \) in order of decreasing eigenvalue. Make sure to maintain the correct pairings between the columns in each matrix.

Step 6: Choose \( k \) eigenvectors that correspond to the \( k \) largest eigenvalues where \( k \) is the number of dimensions of the new feature subspace

In this case, suppose the scanned point cloud is \( P_s \), and the designed reference point cloud is \( P_c \). \( \bar{P}_s \) and \( \bar{P}_c \) are the center of two point clouds respectively. \( \bar{P}_s - \bar{P}_c \) is the vector which translates the scanned point cloud to reference point cloud.
By decomposing its covariance matrix, the point cloud eigenvalues \([\lambda_1, \lambda_2]\) and its corresponding eigenvectors \([\alpha_1, \alpha_2]\) could be calculated. Rotation matrixes of two point clouds are \(R_S = [\alpha_{s1}, \alpha_{s2}]\) and \(R_C = [\alpha_{c1}, \alpha_{c2}]\). The rotation matrix which transforms \(P_c\) to \(P_S\) is \(R_C R_S^{-1}\).

In the linear algebra a rotation matrix is a matrix that is used to perform a rotation in Euclidean space. Eq. (3-5) shows a counter-clockwise rotation matrix which rotates an angle \(\theta\) from the original coordinate system.

\[
R = \begin{pmatrix}
\cos \theta & \sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]  

(3-5)

To perform the rotation, the position of each point from point cloud must be represented by a column vector, containing the coordinates of the point. Rotation matrices also provide a means of numerically representing an arbitrary rotation of the axes about the origin, without appealing to angular specification. These coordinate rotations are a natural way to express the orientation of a camera, or the attitude of a spacecraft, relative to a reference axes-set.

3.4 Precise Localization of Measured Point Cloud

As mentioned before, when evaluating precision machining allowance, the scanned point cloud and the reference point cloud are not exactly the same because of the existence of the machining allowance. Hence, a precise localization optimization, which involves a set of scanned points \(P_S = \{P_{Si} \mid i = 1, 2, \ldots, n\}\) and a set of reference
points \( P_c = \{P_{ci} \mid i = 1, 2, \ldots, n\} \) is necessary for the workpiece localization problem. The least-square principle is introduced to transform the reference point cloud to coincide with the scanned data as close as possible. An objective function \( F \) is shown as follows:

\[
F = \sum_{i=1}^{n} \left\| d_i \right\|^2
\]

(3-6)

where \( \left\| \cdot \right\| \) is the Euclidean norm distance; \( d_i \) is the distance between the scanned point cloud and its corresponding point on the reference point cloud can be calculated with the following function:

\[
d_i = \left\| R P_{si} + T - Q_{ci} \right\|
\]

(3-7)

where the point \( Q_{ci} \) is the nearest point on reference point cloud to scanned data point cloud, \( T \) is the translation vector that includes \( t_x \) and \( t_y \), where \( t_i \) is the translation along the ith axis, and \( R \) is the rotation matrix, where \( \theta \) represents the angle of rotation about the axis. The translation vector \( T \) and the rotation matrix \( R \) describe the rigid transformation of the scanned data related to the reference data respectively.

Let \( X = [\theta, t_x, t_y] \) be the transformation variables, then Eq. (3-7) can be written as a function of \( X \) with the following expressions:

\[
d_i(X) = \left\| R(X) P_{si} + T(X) - Q_{ci} \right\|
\]

(3-8)

Replacing Eq. (3-7) into Eq. (3-5):

\[
F(X) = \sum_{i=1}^{n} \left\| d_i(X) \right\|^2
\]

(3-9)
The objective function \( F \) is calculated as minimization of the summation of squared distances of scanned points in \( P_s \) from nearest reference points \( Q_c \) with respect to the three rigid transformation parameters included in \( X \). The optimal solution for minimizing the objective function \( F(X) \) is the calculated vector \( X \).

For workpiece localization, the vital objective is to push each scanned point out of the reference model in order to make sure that there is sufficient machining allowance for certain workpiece to be machined. Therefore, it is necessary to set effective constraints of oriented Euclidean distances \( d_i^0 \) from the scanned points to the reference model.

\[
d_i^0
\]

where \( \varepsilon \) is the minimum machining allowance of the given workpiece, and the oriented Euclidean distance \( d_i^0 \) can be defined as

\[
d_i^0(X) = (R_k(X)P_{Si}k + T_k(X)Q_{Cik}) \times n_{ik}^q
\]

where \( n_{ik}^q \) is the unit outward normal vector of the designed reference model at the point \( Q_{Cik} \). With this definition, scanned points with negative value between the points and the reference data means insufficient material at these positions and scanned points with positive value represent enough material.

Normally, the workpiece localization problem is focused on minimizing the objective function \( F(X) \) with \( n \) constraints of oriented Euclidean distance \( d_i^0(X) \geq \delta \).

The algorithm will converge the scanned data point cloud to a feasible solution if the reference model is entirely within the raw material. Otherwise, it will remind that the raw
material is infeasible in this case. The mathematical model of localization optimization can be described as follows:

\[
\min_{\mathbf{X}} F(\mathbf{X}) = \sum_{i=1}^{n} \left\| \mathbf{d}_i(\mathbf{X}) \right\|^2 \\
\text{subject to } d_i^0(\mathbf{X}) \leq e \left(3 - 1^2\right) \tag{3-12}
\]

Applying the transformation, the scanned data point cloud would have the following coordinates:

\[
\begin{pmatrix}
P^X_{si} \\
P^Y_{si}
\end{pmatrix} = \begin{pmatrix}
\cos & \sin \\
\sin & \cos
\end{pmatrix} \begin{pmatrix}
P^X_{si} \\
P^Y_{si}
\end{pmatrix} + \begin{pmatrix}
dx \\
dy
\end{pmatrix} \tag{3-13}
\]

Substituting Eq. (3-13) into Eq. (3-11):

\[
d_i^0(\mathbf{q}, dx, dy) = \begin{pmatrix}
\cos & \sin \\
\sin & \cos
\end{pmatrix} \begin{pmatrix}
P^X_{si} \\
P^Y_{si}
\end{pmatrix} + \begin{pmatrix}
dx \\
dy
\end{pmatrix} \tag{3-14}
\]

\[
\begin{pmatrix}
Q^X_{ci} \\
Q^Y_{ci}
\end{pmatrix} \cdot \begin{pmatrix}
nX_i^q \\
nY_i^q
\end{pmatrix}
\]

Therefore, the Eq. (11) can be expressed as follows:

\[
\min_{\mathbf{X}} F(\mathbf{q}, dx, dy) = d_i^0(\mathbf{q}, dx, dy) \tag{3-15}
\]

\[
\text{subject to } d_i^0(\mathbf{q}, dx, dy) \leq e \left(3 - 1^2\right)
\]

If \(\theta\) is very small (less than 0.03°), in order to simplify the formula and increase the calculating speed, sine and cosine values of \(\theta\) can be approximated by \(\sin \approx q\) and \(\cos \approx 1\)\(^{[37]}\). Through using these approximations in Eq. (3-14), an equivalent function can be expressed as follows:
A quadratic programming algorithm is used to optimize Eq. (3-11). Quadratic programming (QP) is a special type of mathematical optimization problem. It is the problem of optimizing (minimizing or maximizing) a quadratic function of several variables subject to linear constraints on these variables. The standard function of the algorithm is defined as:

\[
\text{minimize} \quad \frac{1}{2} X^T H X + f^T X
\]

subject to \( d_i^0(\ , \ dx, \ dy) \)

\[
\begin{pmatrix}
QX_{ci} \\
QY_{ci}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial^2 f}{\partial q^2} \\
\frac{\partial^2 f}{\partial q \partial x} \\
\frac{\partial^2 f}{\partial q \partial y}
\end{pmatrix}
+ \begin{pmatrix}
dx \\
dy
\end{pmatrix}
\]

where \( f \) is an n-dimensional real vector of first order parameters, \( H \) is the Hessian Matrix:

\[
H(\ , \ x, \ y) =
\begin{bmatrix}
\frac{\partial^2 f}{\partial x \partial x} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial y} \\
\frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y \partial y}
\end{bmatrix}
\]

The calculation of minimum distances between scanned points and the reference model are an essential and critical step in the phase of optimization. It has an important influence on the computing efficiency of the optimization since large numbers of distance
calculations are required in minimizing objective function. Fig.3.3 shows the flow chart of the workpiece localization algorithm.

Fig 3.3. Flow chart of the workpiece localization algorithm
Step 0: Initialize the input data $P_{S/0}$, $P_{C/0}$ and $\varepsilon$;

Step 1: Search the nearest point from $P_{C/0}$ relating to each point in $P_{S/k}$, and save
in data set $Q_{C/k}$;

Step 2: Calculate the Hessian Matrix based on Eq. (3-14);

Step 3: Apply QP algorithm to optimize the objective function Eq. (3-11), and get
$R_k(X), T_k(X)$;

Step 4: Evaluate the solution. If it is feasible, go to Step 7. Otherwise, go to Step 5;

Step 5: Let absolute value of rotation angle $\theta_k$ equal to $0.03^\circ$. Calculate
$F_k(0.03^\circ)$ and $F_k(-0.03^\circ)$. Then select $\theta_k$ with smaller $F_k$ value.

Step 6: Rotate $P_{S/k}$ with angle chosen $\theta_k$ to get $P_{S/(k+1)}$, and then go to Step 1
for next iteration.

Step 7: If the solution is feasible, transform $P_{S/k}$ with $R_k(X)$ and $T_k(X)$. Stop
iteration.
Chapter 4

Result and Conclusion

In this chapter, a workpiece localization example of the improved algorithm is shown. The objective of the example is to demonstrate the algorithm. A comparison between the new method and other existing methods is discussed later.

4.1 Example of Workpiece Localization Algorithm

Consider the bottle opener example where the raw material has been cut with plasma arc. After scanning the raw material, the point cloud obtained is shown in figure. The CAD file of the final part is processed through Imageware and point cloud obtained is shown in figure. Since the two point clouds are in different coordinate systems, the superimposition of them in a single co-ordinate frame is shown in Fig. 4.1(a). A machining allowance of 2.5 millimeter is required to be maintained.

A rough localization is used to move the two point clouds closer, in order to reduce the processing time of precise localization. The transformation contains a translation matrix $\vec{P}_s - \vec{P}_c$ and a rotation matrix $R_c R_s^{-1}$. The result of the transformation is shown in Fig. 4.1.

After rough localization, two point clouds are closed. However, there are still some reference points outside the precision casting point cloud, as shown in Fig. 4.1(c), which implies the precise registration is necessary.
Fig 4.1. Original location (a); center-superposed point clouds (b); rough localization result (c).
Since the structure and the appearance between the reference point cloud and measured point cloud usually vary significantly, a precise localization has to find the relationship between each point in the measured point cloud and the reference point cloud. In the previous research, this relationship should be set before processing the algorithm. In this algorithm, this input can be replaced by estimating the normal vector of reference point cloud after organizing the sequence of the reference point cloud. The normal vector of each point in the reference point cloud could be estimated by calculating the normal vectors with points before and after this point.

The inputs of the bottle opener example only include point clouds of reference model and measured data from equipment, no additional information is required for the algorithm. The result shows in Fig. 4.2.

![Fig 4.2. Precise localization result](image)
Assuming the machining allowance of 3.0 millimeter is required for this part, which leads to material shortage, the algorithm stops in the last iteration with an infeasible solution, which shows in Fig. 4.3.

![Fig 4.3. Result with insufficient machining allowance](image)

**4.2 Comparison of the new algorithm and previous algorithm**

For the new workpiece localization algorithm, the inputs are much simpler than the previous algorithm since the relationship between two point clouds and the normal vector of each point in the reference point cloud are not required. Additionally, the new algorithm uses rough localization before precise localization and an approximated Trigonometric function, which increase the calculating speed.

Table 4.1 shows the comparison results between different algorithms. From Table 4.1, the improved least-square method exhibits the significant advantages in computing speed while the accuracy of localization is as same as other algorithms.
Table 4.1 Comparisons between different algorithms.

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Fig. 4.4 exhibits the comparison of convergence between different methods. Since the improved LSM method uses an approximated Trigonometric function which leads to limited changes in each iteration, the algorithm need more iteration than other methods and the convergence speed is slower.
Fig. 4.4. Convergence of different methods
4.3 Conclusion

A new two-step localization algorithm is developed in this paper for localization of workpiece in raw material. It enables avoiding the material shortage of the workpiece. The optimization problem firstly narrows the difference between measured point cloud of the raw material and the point cloud of the reference model by a rough localization in order to reduce the calculation time. Subsequently, a least-square based optimization method is proposed to solve the precise localization. The process converges to a feasible solution under certain constraints, if the machining allowance of the workpiece is sufficient for the designed model. The proposed approach is faster and more automatic than the other algorithms in 2D case. It can improve the efficiency and the possibility of successful workpiece localization, and provides a way to evaluate the machining allowance automatically. The example has shown that the proposed algorithm can be effectively used in workpiece localization in raw material checking. The processing time of the improved algorithm is much faster than the previous algorithms in 2D workpiece localization process. The simplified inputs reduce most human intervention during workpiece localization. The future efforts will be placed on extending the algorithm from 2D to 3D cases by arranging the points in a well-organized sequence. Additionally, the tolerance of the parts should be considered in the future work to avoid rejecting feasible workpiece.
References


## Appendix A

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<td>55.1575</td>
<td>56.3693</td>
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<td>55.6783</td>
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<tr>
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<td>27.3041</td>
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<td>54.5828</td>
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<tr>
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<td>220</td>
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<tr>
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<tr>
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<tr>
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<td>-12.8407</td>
<td>22.5932</td>
<td>227</td>
<td>71.1826</td>
<td>56.5903</td>
</tr>
</tbody>
</table>
function pointset = scan_anticlockwise(pointset)
    for i = 1:size(pointset,1)
        for j = i+1:size(pointset,1)
            if (atan2(pointset(i,2)-0.5,pointset(i,1)-0.5) >
                atan2(pointset(j,2)-0.5,pointset(j,1)-0.5));
                temp = pointset(i,:);
                pointset(i,:) = pointset(j,:);
                pointset(j,:) = temp;
            end
        end
    end
end

Error=2.5;
contour1=xlsread('LIONdata','CAD');
CAD=contour1';
CAD(1,:)=CAD(1,:);
CAD(2,:)=CAD(2,:);
CAD=flipud(CAD);
contour2=xlsread('LIONdata','Blank');
contour3=contour2;
contour2=contour2';
R=[cos(0.3),-sin(0.3);
    sin(0.3),cos(0.3)];
contour2=R*contour2;
Scan=contour2;
CA=cov(CAD');
CB=cov(Scan');
Scanx=mean(Scan(1,:));
Scany=mean(Scan(2,:));
Scan(1,:)=Scan(1,:)-Scanx;
Scan(2,:)=Scan(2,:)-Scany;
contour2=Scan;
[A1,D1]=eig(CA);
[B1,D2]=eig(CB);
B2=B1^(-1);
R=A1*B2;
CAD2=CAD'*R;
CAD2=CAD2';
CADx=mean(CAD2(1,:));
CADy=mean(CAD2(2,:));
CCAD(1,:)=CAD2(1,:)-CADx;
CCAD(2,:)=CAD2(2,:)-CADy;
QNumber=size(CCAD,2);
SNumber=size(Scan,2);
CR1=contour2(1,:);
CR2=contour2(2,:);  
CR3=CR1(end: -1:1);  
CR4=CR2(end: -1:1);  
Anticlockwise_Scan(1,:)=CR3;  
Anticlockwise_Scan(2,:)=CR4;  
for NNumber=1:SNumber  
    if NNumber==SNumber  
        n_vector(1,NNumber)=(Anticlockwise_Scan(2,1)-  
            Anticlockwise_Scan(2,NNumber))/norm(Anticlockwise_Scan(:,NNumber)-  
            Anticlockwise_Scan(:,1));  
        n_vector(2,NNumber)=-(Anticlockwise_Scan(1,1)-  
            Anticlockwise_Scan(1,NNumber))/norm(Anticlockwise_Scan(:,NNumber)-  
            Anticlockwise_Scan(:,1));  
    else  
        n_vector(1,NNumber)=(Anticlockwise_Scan(2,NNumber+1)-  
            Anticlockwise_Scan(2,NNumber))/norm(Anticlockwise_Scan(:,NNumber+1)-  
            Anticlockwise_Scan(:,1));  
        n_vector(2,NNumber)=-(Anticlockwise_Scan(1,NNumber+1)-  
            Anticlockwise_Scan(1,NNumber))/norm(Anticlockwise_Scan(:,NNumber+1)-  
            Anticlockwise_Scan(:,1));  
    end  
end  
for VNumber=1:SNumber  
    if VNumber==1  
        n_vector2(1,VNumber)=(n_vector(1,1)+n_vector(1,SNumber));  
        n_vector2(2,VNumber)=(n_vector(2,1)+n_vector(2,SNumber));  
        Nnorm=norm(n_vector2(:,VNumber));  
        n_vector2(1,VNumber)=n_vector2(1,VNumber)/Nnorm;  
        n_vector2(2,VNumber)=n_vector2(2,VNumber)/Nnorm;  
    else  
        n_vector2(1,VNumber)=(n_vector(1,VNumber)+n_vector(1,VNumber-  
            1));  
        n_vector2(2,VNumber)=(n_vector(2,VNumber)+n_vector(2,VNumber-  
            1));  
        Nnorm=norm(n_vector2(:,VNumber));  
        n_vector2(1,VNumber)=n_vector2(1,VNumber)/Nnorm;  
        n_vector2(2,VNumber)=n_vector2(2,VNumber)/Nnorm;  
    end  
end  

tempN=n_vector;  
n_vector=n_vector2;  
for i=1:QNumber  
    PCAD=CCAD(:,i);  
    Cor=[PCAD,Anticlockwise_Scan]';  
    d=pdist(Cor);  
    D=squareform(d);  
    LineQ=D(1,2:size(D,1));  
    Dmin=min(LineQ);  
    [m,lm]=find(LineQ==Dmin);  
    Q(:,i)=Anticlockwise_Scan(:,lm);  
    n(:,i)=n_vector(:,lm);  
end  
for k=1:10  
    H=[sum((CCAD(2,:).^2+CCAD(1,:).^2)),sum((  
        CCAD(2,:))),sum(CCAD(1,:))];
sum((-CCAD(2,:))), size(CCAD,2), 0;
sum(CCAD(1,:)), 0, size(CCAD,2));
f=[2*sum(((Q(1,:)).*CCAD(2,:))+((-Q(2,:)).*CCAD(1,:)));
2*sum((CCAD(1,:) - Q(1,:)));
2*sum((CCAD(2,:) - Q(2,:)))];
A=[n(1,:).*CCAD(2,:) - n(2,:).*CCAD(1,:); -n(1,:); -n(2,:)];
A=A';
b=[Q(1,:).*n(1,:) - n(1,:).*CCAD(1,:) + Q(2,:).*n(2,:) - 
n(2,:).*CCAD(2,:) - Error];
b=b';
[Solution, fval, exitflag] = quadprog(H, f, A, b);
Output(1,k) = fval;
if exitflag == 1
  if Solution(1,1) <= 0.1 && Solution(1,1) >= -0.1
    R2=[cos(Solution(1,1)), sin(Solution(1,1)); -sin(Solution(1,1)), cos(Solution(1,1))];
    CCAD=R2*CCAD;
    Temp1=CCAD;
    CCAD(1,:) = CCAD(1,:) - Solution(2,1);
    CCAD(2,:) = CCAD(2,:) - Solution(3,1);
  else
    R2=[1-0.01^2, 0.01; -0.01, 1-0.01^2];
    CCAD=R2*CCAD;
    Temp1=CCAD;
    CCAD(1,:) = CCAD(1,:) - Solution(2,1);
    CCAD(2,:) = CCAD(2,:) - Solution(3,1);
  end
  for i=1:QNumber
    PCAD=CCAD(:,i);
    Cor=[PCAD, Anticlockwise_Scan]';
    d=pdist(Cor);
    D=squareform(d);
    LineQ=D(1,2:size(D,1));
    Dmin=min(LineQ);
    [m, lm]=find(LineQ==Dmin);
    Temp2=Q;
    Q(:,i) = Anticlockwise_Scan(:, lm);
    n(:,i) = n_vector(:, lm);
  end
  for j=1:size(CCAD,2)
    if sum((Q(:,j)-CCAD(:,j)).*n(:,j))<Error
      flag=1;
      CCAD=Temp1;
      Q=Temp2;
      break
    else
      flag=2;
    end
  end
  if flag==2
    break
  end
else
  R3=[cos(pi/180), sin(pi/180); -sin(pi/180), cos(pi/180)];
  CCAD=R3*CCAD;
CCAD(1,:) = CCAD(1,:) - Solution(2,1);
CCAD(2,:) = CCAD(2,:) - Solution(3,1);
for i = 1:QNumber
    PCAD = CCAD(:,i);
    Cor = [PCAD, Anticlockwise_Scan]';
    d = pdist(Cor);
    D = squareform(d);
    LineQ = D(1,2:size(D,1));
    Dmin = min(LineQ);
    [m, lm] = find(LineQ == Dmin);
    Q(:,i) = Anticlockwise_Scan(:,lm);
    n(:,i) = n_vector(:,lm);
end
end
if exitflag == 2
    msgbox('There is enough machining allowance for the part')
else
    msgbox('No enough machining allowance for the part')
end
plot(Anticlockwise_Scan(1,:), Anticlockwise_Scan(2,:), '+k')
axis equal
hold on
plot(CCAD(1,:), CCAD(2,:), '+k')
hold off
axis([-80 80 -50 50]);