TRANSFORMATION OPTICS TECHNIQUES
FOR DESIGNING GRADIENT INDEX DEVICES

A Thesis in
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by
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ABSTRACT

Transformation optics provides a powerful design tool aiding in the development of gradient index systems across a vast range of applications. Here, such techniques are applied towards two diverse objectives. In the first case, quasi-conformal transformation optics is used to design integrated photonic components and other gradient index devices on graphene. Such devices provide an alternative to conventional mediums, which are severely limited in system performance and design flexibility. The marriage of transformation optics design techniques with the emerging field of graphene offers the possibility for developing novel systems in the mid-infrared regime. It also has the potential for extending the functionality of existing systems in a compact geometry. In the second case, anti-reflective coating designs are developed for gradient index lens systems. Conventional design techniques are leveraged in conjunction with transformation optics to develop these coatings. First, narrowband solutions are presented; then the design techniques are extended to develop multi-layer broadband solutions. The result of this work demonstrates the great utility that transformation optics holds in the design of novel electromagnetic systems.
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CHAPTER 1: Fundamentals of Transformation Optics

1.1 Transformation Optics

Transformation optics (TO) is a computational design technique that facilitates the manipulation of electromagnetic waves [1]. The origins of this technique derive from the 2006 Science articles by Leonhardt [2] and Pendry [3], in which they discussed using coordinate distortions to create exotic lenses and electromagnetic cloaks. One of the great advantages of TO is the ability to realize complex systems with a minimum amount of computational difficulty. This is due to its reliance on coordinate transformations as its primary design tool. These coordinate distortions provide an intuitive and visual approach to controlling electromagnetic waves that seemingly effortlessly enables the development of complex and novel systems [4]–[6].

TO design is accomplished in two primary steps. First, a coordinate transformation is performed that realizes a desired conceptual distortion of space. In this process, a virtual geometry is mapped into a physical domain. The electromagnetic fields residing in these domains are correspondingly distorted. The coordinate mappings are not limited to simple Cartesian, cylindrical, and spherical grids, but can be distorted into complex systems, such as the transformation depicted in Figure 1.

![Coordinate Distortion](image)

**Figure 1.** A virtual coordinate grid is transformed into a desired physical coordinate grid. This transformation accomplishes a desired distortion of space and a corresponding distortion of the electromagnetic fields residing in the domain.
Next, the form invariance of Maxwell’s equations is exploited to determine a theoretical system with a material distribution that performs the same operation as the coordinate distortion. Assuming no sources, in a Cartesian grid, the electric field $E(x,y,z)$ and the magnetic field $H(x,y,z)$, with time dependence $e^{j\omega t}$, must satisfy Maxwell’s equations as denoted in (1), where the $\epsilon$ and $\mu$ are permittivity and permeability tensors. A coordinate transformation can be applied to obtain a new system $(x',y',z')$. Maxwell’s equations preserve the same form in this new system, as denoted in (2). The relationship between the tensors of the original (unprimed) and transformed (primed) is represented in (3). In this expression, $A$ is a Jacobian matrix, denoted in (4), that establishes the relationship between the systems. This matrix is a measure of the geometric variation, including expansion, compression, and rotation, described by partial spatial derivatives, represented as subscripts in (4).

$$\nabla \times E = -j\omega \mu H \quad \nabla \times H = j\omega \epsilon E$$  \hspace{1cm} (1)

$$\nabla' \times E' = -j\omega \mu' \quad \nabla' \times H' = j\omega \epsilon' E'$$  \hspace{1cm} (2)

$$\epsilon' = \frac{A\epsilon A^T}{|A|} \quad \mu' = \frac{A\mu A^T}{|A|}$$  \hspace{1cm} (3)

$$A = \begin{bmatrix} x'_x & x'_y & x'_z \\ y'_x & y'_y & y'_z \\ z'_x & z'_y & z'_z \end{bmatrix}$$  \hspace{1cm} (4)

TO design offers a few major benefits over conventional electromagnetic system design techniques. The TO approach significantly simplifies the electromagnetic design process. In general, a virtual coordinate system is chosen such that wave propagation can be described in simple forms. When the system is then transformed into a more complex physical domain, the equivalent properties are immediately realized from (1) and (2). This enables the design of highly complex systems in a fast and intuitive way without the
requirement for a computationally intensive process. The TO design process also provides insight allowing for the development of novel devices that would be too complex to efficiently realize otherwise.

However, the generic TO approach features drawbacks that limit its utility in electromagnetic design. The first major disadvantage is that the material distributions produced by the technique are generally complex, featuring both inhomogeneity and anisotropy in both $\epsilon$ and $\mu$ tensors. These characteristics are extraordinarily difficult to realize limiting the ability to fabricate practical systems. Furthermore, the TO process also do not provide any insight as to how to realize complex systems. Although designers are armed with the required material tensors, it is unclear how to achieve bulk regions of inhomogeneous and/or anisotropic properties. These limitations can be alleviated with concepts outlined in the following sections.

1.2 Conformal and Quasi-Conformal Mappings

Conformal mappings are a subset of TO in which the transformation preserves the angles between intersecting curves, as well as the aspect ratio of the resulting grid. For example, when considering a locally orthogonal system, such as the Cartesian grid, the coordinate lines form a rectangular gridding. A conformal mapping preserves this local orthogonality such that the angles formed by any two intersecting curves in one system are conserved in the transformed system. These restrictions on the transformation result in constitutive parameters that are much easier to physically implement than general TO designs. The parameters are locally isotropic and can be implemented with dielectric only media [1]. Eliminating the requirement for a magnetic response is particularly beneficial for devices in the terahertz (THz) and optical regimes where magnetic materials are essentially nonexistent. Furthermore, it is advantageous to develop designs that require only one degree of inhomogeneity to significantly reduce system complexity.

The conditions for a conformal mapping are highly restrictive, but they offer designs with highly desirable constituent parameters. Quasi-conformal mappings relax the
severe restrictions on conformal mappings and open up the possibility of designing more systems. These mappings allow for transformations between domains with dissimilar conformal modules. To compensate for the mismatched conformal modules, the virtual domain is mapped to an intermediate domain with the same conformal module as the physical domain. This is most simply realized using a uniform dilation of the form $y' = My$. This intermediate domain can then be conformally mapped to the physical domain. The combination of the two stages, the dilation and the conformal mapping, produce material tensors described by (5). It is apparent in (5) that the in-plane material tensors are now not equal to each other, thus the design features a degree of anisotropy. However, these perturbations to the conformal module are generally small and thus the resulting anisotropy can typically be ignored [7]. In general, this technique does not have a closed form solution and rather must be solved numerically using iterative methods.

$$\epsilon'' = \mu'' = \text{Diag}[M^{-1}, M, (M|\mathcal{A}_c|)^{-1}] \tag{5}$$

Furthermore, to implement this transformation without the use of magnetic materials, we may represent the permittivity tensors as (6). The resulting parameters are now approximated as an isotropic in-plane dilation. Now rather than the intermediate transformation accomplished with an anisotropic dilation, it is simply an isotropic scaling.

$$\epsilon'' = \text{Diag}[1, 1, (M|\mathcal{A}_c|)^{-1}] \tag{6}$$

These quasi-conformal mappings still do face several limitations. Even when implementing the quasi-conformal mapping with the anisotropic material parameters, the Neumann boundary conditions allow coordinate lines to slip along the boundary of the transformed domain resulting in aberrations. These are often minute errors and can be mitigated by increasing the overall size of the transformation domain [1]. Furthermore, the introduction of the isotropic approximations results in a deviation
from the perfect TO transformation. This can manifest in undesirable wave propagation properties and aberrations. Larger spatial deformations will result in correspondingly larger variation to the conformal module, exacerbating the aberrations. Some of these aberrations may be mitigated by exploiting additional degrees of freedom in the transformation. Despite these limitations, quasi-conformal mappings hold great utility in bringing TO-inspired designs closer to realization.

1.3 Metamaterials

In general, such complex TO designs cannot be realized with naturally occurring materials. These systems often demand both inhomogeneous and anisotropic material properties, whose gradients cannot be achieved with natural materials. Furthermore, although TO provides the required constituent parameters to achieve a desired functionality, the implementation is often nontrivial. The development of metamaterial technology allows access to previously unavailable electric and magnetic material properties. Metamaterials realize these unique material characteristics with periodically arranged sub-wavelength structures whose resonant or non-resonant electric and magnetic responses can be tailored [8]–[10]. Such structures are able to realize negative [11], [12], near-zero [13], [14], and high permittivity and permeability values [15], [16], as well as limited anisotropic mediums [17], [18]. These materials offer the unique characteristics to enable the realization of TO devices. However, in general, metamaterials are highly sensitive to frequency shifts and absorption losses, making TO-inspired systems difficult to implement and only plausible for narrowband applications. Implementing tunable elements into metamaterial structures significantly extends their capability and mitigates some of these drawbacks [19]. Additionally, to reduce absorptive losses, dielectric blocks can be drilled with holes for both microwave [20] and optical [21] TO devices. Despite their drawbacks, metamaterials offer a practical method for implementing TO-inspired devices.
1.4 Transformation-Optics Inspired Devices

Since the pioneering works in the field [2], [3], transformation optics has experienced dramatic growth, leading to a broad spectrum of novel designs with unique functionalities. Some of the more exciting and exotic works have targeted developing invisibility cloaks and other illusion devices. The invisibility cloak is a material coating that distorts the incident radiation around a desired geometry or object, such that the object cannot be observed from outside the cloak [22]. This idea is expanded for more general illusion devices that manipulate the scattering of an object so it resembles another target object [23]–[25]. Aside from applications in cloaking and illusion devices, TO has been used to realize a plethora of other functionalities in novel devices. Some of these include field concentrators [25]–[27], waveguide bends [28]–[30], wave collimators [30]–[32], beam splitters [33], [34], flattened Luneburg lenses [20], [35], [36], and multi-beam antenna lenses [37]–[40] among many others. The following chapters present the utility of TO to realize new functionalities with novel gradient index (GRIN) devices.
CHAPTER 2: Transformation Optical Design for Gradient Index Devices on Graphene

2.1 Introduction

The utility of TO and quasi-conformal mappings extend to both bulk and planar devices and functionalities. For planar mappings, TO manipulates surface-plasmon polariton (SPP) waves, which are conventionally hosted on noble metals, such as gold and silver. At optical wavelengths, these materials suffer from significant losses that degrade the plasmon resonance and reduce the propagation lengths of the SPP waves [41]. Furthermore, it is very difficult to control the material characteristics of these mediums, limiting their potential for realizing GRIN devices.

Recently, graphene has emerged as a suitable host for novel TO-inspired devices, such as GRIN lenses [42]–[44]. The authors theorized the possibility for embedding non-uniform conductivity patterns on graphene slabs by altering characteristics of the local substrate [41]. Later efforts also experimentally demonstrated the incorporation of non-uniform electronic transport properties on graphene sheets [45]. These patterns are realized by controlling the graphene’s substrate thickness, permittivity, or electric field distribution. Graphene is a particularly attractive alternative to conventional host mediums due to its single-atomic thickness, limited footprint, and highly tunable electronic transport properties. Such properties enable graphene to be used for the design of miniaturized, high functionality integrated components that can be fabricated on a single graphene sheet.

Here we leverage TO design techniques in conjunction with patterning non-uniform conductivity patterns on graphene sheets to design GRIN components for the near-IR frequency regime. First, we present the relevant design equations required to link the TO material tensors to the electronic properties of graphene. We then outline the development process and observe the functionality of the resulting designs followed by a brief discussion on fabrication considerations for realizing such devices.
2.2 Quasi-Conformal Transformation Optics Design Techniques

The utility of TO to design GRIN components on graphene hosts relies on the ability to embed inhomogeneous patterns. This is accomplished by considering the unique electronic transport properties of graphene. The electronic transport properties of graphene are highly tunable, making it a particularly attractive host for TO-inspired device design. The manipulation of these properties can be directly observed in the sheet’s complex conductivity, $\sigma_g$, denoted in (7), which is governed by the angular frequency $\omega$, particle scattering rate $\Gamma$, temperature $T$, and chemical potential $\mu_c$. This relationship is shown in the intra-band (8) and inter-band (9) conductivity expressions [46]. In these expressions, $k_B$ is Boltzmann’s constant and $\hbar$ is Planck’s constant. The inter-band conductivity term is approximated assuming that $k_B T \ll |\mu_c|, \hbar \omega$. As is clear from these terms, the conductivity of the graphene sheet can be varied by altering the chemical potential. This dependence is depicted in Figure 2. Here, we consider $T = 300 \, \text{K}$ and $\Gamma = 0.43 \, \text{meV}$. It is clear from the figure that the tunability of a graphene sheet manifests itself in the imaginary component of the complex conductivity, while the real component remains constant. Therefore, it is necessary to establish a relationship between the imaginary component of the complex conductivity and the material tensors from the quasi-conformal mapping.

$$\sigma_g = \sigma_{\text{intra}} + \sigma_{\text{inter}} = \sigma_{g,r} + j\sigma_{g,i}$$  \hspace{1cm} (7)

$$\sigma_{\text{intra}} = -j \frac{e^2 k_B T}{\pi \hbar^2 (\omega - j 2 \Gamma)} \left( \frac{\mu_c}{k_B T} + 2 \ln \left[ e^{-\mu_c/k_B T} + 1 \right] \right)$$  \hspace{1cm} (8)

$$\sigma_{\text{inter}} = -j \frac{e^2}{4 \pi \hbar} \ln \left( \frac{2|\mu_c|-(\omega-j2\Gamma)\hbar}{2|\mu_c|+(\omega-j2\Gamma)\hbar} \right)$$  \hspace{1cm} (9)
Figure 2. Graphene complex conductivity variance against frequency and chemical potential. Assumes $T = 300 \text{ K}$ and $\Gamma = 0.43 \text{ meV}$. (a) Real component of complex conductivity. (b) Imaginary component of complex conductivity.
The essential link connecting the TO design process to the graphene electronic transport properties is the dispersion relation of SPP waves denoted in (10), where $\beta$ and $k_0$ are the wave numbers of the guided mode and of free space respectively, $\eta_0$ is the intrinsic impedance of free space, and $n_{SPP}$ is the effective index of refraction of the SPP. It directly relates the material tensors resulting from the TO distortion and the complex conductivity of graphene. From (10), we can approximate (11), which indicates the inverse proportionality of the effective refractive index and the imaginary component of the complex conductivity [41]. This relationship is depicted in Figure 3. From the approximated expression, we can also observe that when $\sigma_{g,i} > 0$, the graphene sheet supports transverse magnetic SPP waves. However, when $\sigma_{g,i} < 0$, the same modes are no longer supported. Instead, in this case the medium supports a transverse electric SPP wave. This dependence indicates that using quasi-conformal TO design techniques, the resulting near-isotropic, dielectric-only distributions will feature corresponding conductivity values that support transverse magnetic SPP waves. Additionally, when considering a graphene host, the background indices must be chosen such that the resulting TO material tensors correspond to conductivity values that are realizable. This constraint does not significantly inhibit the design flexibility since the TO process is highly scalable. Furthermore, these effective material parameters do not have physical meaning, but are rather only representative of the behavior of the surface plasmons when supported on graphene sheets. Now that the link has been established between the TO process and the conductivity of graphene, it is necessary to specify how to control this surface conductivity.

$$\beta = k_0^2 \left[ 1 - \left( \frac{2}{\eta_0 \sigma_g} \right)^2 \right]$$  \hspace{1cm} (10)

$$\sigma_{g,i} \sim \frac{2}{\eta_0 n_{SPP}}$$  \hspace{1cm} (11)
Figure 3. Graphene dispersion relation depicts the inverse relationship between the effective permittivity of a SPP wave and the imaginary conductivity component of the graphene sheet.

As Figure 2 indicates, the imaginary component of the graphene sheet’s conductivity relies on the chemical potential, which is governed by the carrier density, denoted in (12) and (13) [47]. In these expressions, \( v_f \) is the electron velocity. Expression (12) indicates that the chemical potential may be manipulated by the substrate permittivity \( \epsilon_d \), the gate voltage \( V_b \), or the thickness of the substrate \( d \). Controlling these parameters in turn enables the realization of TO-inspired gradient index devices. The tunability of the chemical potential is demonstrated in Figure 4, where it is plotted against an applied electric field. It is clear that graphene is able to achieve a wide degree of tunability over a rather small variation in field bias.

\[
\begin{align*}
n_s &= \frac{2}{\pi \hbar^2 v_f} \int_0^\infty \epsilon [f_d(\epsilon) - f_d(\epsilon + 2\mu_c)] d\epsilon = \frac{2\epsilon_d V_b}{ed} \\
f_d(\epsilon) &= \left[ e^{(\epsilon - \mu_c)/k_B T} + 1 \right]^{-1}
\end{align*}
\]
Figure 4. Demonstration of the tunability of graphene electronic transport properties accomplished by varying the applied electric field bias.

The design equations outlined above provide the foundations needed to develop TO-inspired GRIN devices. The electronic transport properties of graphene in the infrared frequency regime make it particularly attractive for developing integrated photonic components.

2.3 Integrated Photonic Component Design

Integrated photonic components have become increasingly crucial to the development of novel fields, such as optical communications, computing, and sensing. Recent trends in these fields feature an increasing demand for reductions in component cost, weight, and size. It is highly desirable to develop systems that can integrate high functionality components in a single compact geometry [48]. Previous works demonstrated that TO techniques can be applied towards the design of practical optical components, such as collimators, adapters, crossers, couplers, and more [48].
2.3.1 Quad-Beam Collimator

The power and flexibility of the quasi-conformal TO design technique is evident when considering the design of novel devices. Here we demonstrate this technique by outlining the design of a planar quad-beam collimator based on a graphene host. The quad-beam collimator efficiently couples wave fronts from an isotropic source at its origin, into collimated wave fronts at each of its interfaces. Typically, the wave fronts radiated from an isotropic source can be considered planar at an infinitely large distance from the source. Here, the collimator performs the same function within only a few wavelengths of the source. This functionality is realized from the coordinate transformation depicted in Figure 5. The four arcs of the circle are mapped onto the corresponding interfaces of the square, which in turn distorts the source grid.

![Figure 5](image.png)

**Figure 5.** (a) Virtual space geometry for quad-beam collimator in arbitrary units. (b) Physical space geometry of quad-beam collimator in arbitrary units.
The coordinate distortion yields a nearly-isotropic material distribution that can be implemented with dielectric materials. The resulting distribution is shown in Figure 6(a). The background permittivity when extracting the material tensors is appropriately selected to result in a desired range of graphene conductivity values. From the effective permittivity, (11) is used to calculate the corresponding imaginary conductivity values, shown in Figure 6(b) using the fact that \( n_{SPP} = \sqrt{\varepsilon_{eq}} \). The chemical potential, depicted in Figure 6(c), can then be computed using (7) – (9) and assuming that \( \Gamma = 0.43 \text{ meV} \), \( T = 300K \), and \( f = 30 \text{ THz} \). These required distributions can be realized by controlling the graphene’s substrate permittivity, thickness, or applied bias voltage. This example solves for the required substrate permittivity, presented in Figure 6(d), assuming \( d = 50 \text{ nm} \) and \( V_g = 25 \text{ V} \). The substrate permittivity distribution is numerically calculated from the chemical potential values using (12) and (13).

The functionality of this device was evaluated through simulation. COMSOL Multiphysics was used to perform the full-wave simulations and demonstrate the performance of the collimator. Since the software package cannot implicitly model the electronic transport properties of graphene, the domain was populated with the effective permittivity distribution shown in Figure 6(a). This effective material distribution is representative of the electronic properties on the graphene sheet, such that the simulation replicates the propagation of SPP waves. The system is excited with a 30 THz line source perpendicular to the plane and the simulation domain is then enclosed by absorbing boundary conditions. Figure 7 depicts the normalized electric field distribution of the system. It is clear that the GRIN profile efficiently collimates the circular wave front into planar wave fronts at each of the interfaces. Furthermore, this functionality is achieved within only a few wavelengths of the epicenter. Each of these interfaces can theoretically be terminated with waveguides that lead to other components in an integrated circuit. It would enable four separate signals to be excited from a singular source on the circuit board.
Figure 6. Quad-beam collimator material properties. (a) Effective permittivity obtained from QCTO techniques. (b) Corresponding imaginary conductivity using (11) (c) Graphene chemical potential obtained from (7)-(9) (d) Required substrate permittivity to achieve desired device functionality acquired from (12) and (13).
2.3.2 Waveguide Adapter

Following the same design procedure, it is possible to develop other novel planar photonic devices. Another example is a waveguide adapter, which efficiently couples the energy between waveguides of differing size. Traditional waveguide couplers require long tapering sections to reduce losses, whereas the TO approach is capable of achieving the same functionality in a compact geometry.

The design process is similar to the collimator. Here, a larger curved aperture at the input (left) and a smaller curved aperture at the output (right) are mapped into planar interfaces. Using the same constants presented above, we obtain the material distributions shown in Figure 8. Here, the required graphene substrate permittivity is clipped at $\epsilon_d = 4$, limiting the permittivty range such that $SiO_2$ can be considered as a potential substrate for the graphene system. Clipping the upper range of the distribution does not significantly alter the performance of the device since those regions are small and manifest only from the corners of the virtual domain.
Figure 8. Waveguide adapter material parameters. (a) Effective permittivity obtained from QCTO techniques. (b) Corresponding imaginary conductivity using (11) (c) Graphene chemical potential obtained from (7)-(9) (d) Required substrate permittivity to achieve desired device functionality acquired from (12) and (13).
This device was also simulated using COMSOL Multiphysics. Figure 9 shows the resulting electric field distribution. It is clear that energy is efficiently coupled from the larger input waveguide into the smaller output waveguide without the use of a long tapered section. Since this device was derived using TO techniques, it is highly scalable and can be tailored for any waveguide dimension.

![Electric field distribution](image)

**Figure 9.** Normalized electric field distribution of GRIN waveguide adapter from full-wave simulation.

### 2.3.3 Waveguide Crosser

Another essential integrated photonic component is the waveguide crosser, which enables optical signals to be channeled and integrated within a limited geometry. This functionality is essential to miniaturizing photonic circuits. For the development of the waveguide crosser, quasi-conformal TO techniques were again used and the required graphene electronic transport properties were synthesized. However, here the initial virtual domain has an inhomogeneous Luneburg distribution.
This inhomogeneous region is then transformed. The results of this process are depicted in Figure 10. The required substrate permittivity is again clipped to the same range as the quad-beam collimator and waveguide adapter.

Figure 10. Waveguide crosser material parameters. (a) Effective permittivity obtained from QCTO techniques. (b) Corresponding imaginary conductivity using (11) (c) Graphene chemical potential obtained from (7)-(9) (d) Required substrate permittivity to achieve desired device functionality acquired from (12) and (13).
2.3 Additional Transformation Optics-Inspired Devices

The simulated results are presented in Figure 11 where only the top-left waveguide is radiating. It is clear from the electric field distribution that energy from the input waveguide crosses to the bottom-right output waveguide with minimal loss. This functionality is particularly useful when routing and interconnecting lines in an extremely limited geometry. This is an invaluable feature that would help enable the continued miniaturization of integrated photonic circuit technology.

Figure 11. Normalized electric field distribution of GRIN waveguide crosser from full-wave simulation.

2.4 Additional Transformation Optics-Inspired Devices

The utility of TO techniques in conjunction with graphene hosts also extends beyond integrated photonic components to a plethora of other GRIN devices. As an example, we present the design of a flattened convex interface that focuses incident radiation. Figure
Figure 12 shows the material parameters and the performance of the lens. It is clear that the incident radiation focuses to a desired focal point. Although this is a simple system, it demonstrates the great potential of these techniques to develop a wide range of other mid-IR devices.
2.5 Fabrication Considerations

Although this work is primarily based on the theoretical design of TO-inspired devices, it is important to consider the potential for realizing these systems with existing fabrication techniques and technologies. These graphene-based integrated photonic systems can be supported on $SiO_2/Si$ wafers. Previous works demonstrated the excitation of SPP waves on such substrates using a scattering-type scanning near-field optical microscope [45]. Illuminating the sharp tip of an atomic force microscope with an infrared beam produces the desired SPP wave fronts that resemble the isotropic source considered in the quad-beam collimator device. In the system presented above, tunability relied on the ability to embed a gradient index distribution on the host medium’s substrate. Here, the substrate is a layer of $SiO_2$. One possible method for embedding a gradient on the substrate is to use a slurry-based three-dimensional printing process [49]. Here the silica wafer is doped with alumina to achieve gradient indices. Another approach is to pattern sub-wavelength hole lattices of varying density in the $SiO_2$ [21]. This technique has been used to develop optical cloaks made purely out of dielectrics. Although these techniques have not been used to fabricate systems on graphene, they point to possible methods for realizing these complex novel devices.

2.6 Conclusions

We presented the potential of quasi-conformal TO design techniques to develop GRIN devices on graphene hosts. These devices hold great potential for integrated photonic component technology and other optical devices in the mid-IR frequency regime. The material tensors from TO techniques are used to calculate the required constituent parameters of graphene hosts to design non-uniform patterns on graphene sheets. This work provides a significant contribution to the development of optical and photonic technologies.
CHAPTER 3: TRANSFORMATION OPTICS INSPIRED ANTI-REFLECTIVE COATINGS FOR GRADIENT INDEX LENSES

3.1 Introduction

Quasi-conformal transformation optics has piqued interest in designing GRIN systems [50], [51], relaxing the material requirements predicted by the more general TO approach [1], [3]–[5]. GRIN systems hold great potential for reducing the size, weight, and power requirements of sensitive optical devices by adding additional design flexibility. However, despite the plethora of literature on GRIN lenses, previous works have not yet investigated the application of anti-reflective coating (ARC) technologies to these systems, leaving a crucial gap in the adaptation of the devices, since high performance optical systems necessitate ARCs to mitigate return losses. ARC design is well-established in literature for homogeneous lens systems [52], [53]. This includes a wide variety of methodologies extending beyond the narrowband quarter-wave transformer, to include multi-layer, GRIN, and nature-inspired coatings that are capable of achieving wideband solutions [54]–[57]. Furthermore, ARC design leverages the rich field of filter theory as well, offering a seemingly endless array of techniques. It is unclear how these coatings translate to inhomogeneous systems. Here, we demonstrate ARC designs to reduce return losses from GRIN lenses. Conventional ARC design techniques are leveraged in conjunction with QCTO to develop these coatings for both narrowband and broadband systems.
3.2 Anti-Reflective Coatings for Dielectrics and Conventional Lens Systems

Reflections originate from discontinuities along the propagation trajectory of electromagnetic radiation. These reflections occur when radiation propagates between regions of differing impedances. Often these reflections are undesirable and must be mitigated for optimal system performance. For normal incident radiation on bulk dielectric regions of differing indices, such as the scenario depicted in Figure 13, energy is partially transmitted, and partially reflected. This phenomenon is described by (14) and (15) where $\Gamma$ is the reflection coefficient and $T$ is the transmission coefficient from the interface.

$$ \Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} $$

(14)

$$ T^b = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma^b $$

(15)

**Figure 13.** Illustrative example of regions of differing refractive index separated by a planar interface. (a) Normal incident radiation is partially reflected due to the impedance mismatch between the regions. (b) A quarter wave transformer is implemented between the regions to mitigate reflections from the system back to the source. This transformer relies on destructive interference to reduce reflections at a single frequency.
These reflections can be mitigated by introducing a layer between the two regions such that there are no reflections from the interface of the layer back towards the direction of the source. The simplest method for accomplishing perfect anti-reflection at a single frequency is the quarter-wave transformer. The quarter-wave transformer leverages destructive interference to achieve the desired functionality. The index of this layer is chosen to be the geometric mean of the two regions, as denoted in (16). The thickness of this layer takes after its namesake, a quarter-wavelength long corresponding to the index of the ARC. This is because the reflected wave then travels a total of half of a wavelength to reach back to the input interface, thus observing the effect of perfect destructive interference. This is demonstrated in Figure 14, where in (a) the system does not have an ARC and therefore, an interference pattern is formed in the input region. In (b) the ARC mitigates the reflections perfectly at the design frequency and thus no interference pattern observed. However, this perfect anti-reflection property only persists at a single frequency, since it relies on the electric length of the ARC layer. There are methods to increase the operating bandwidth of ARCs, which will be discussed in later sections.

\[ \eta_{ARC} = \sqrt{\eta_1 \eta_0} \]  

(a) \hspace{1cm} (b)

Figure 14. Magnitude of the electric field of scenario depicted in Figure 13 when radiated with a normal incidence plane wave from the left region. (a) Scenario with an ARC features an interference pattern due to reflection. (b) Scenario with an ARC does not feature an interference pattern since reflections are mitigated.
Furthermore, the perfect anti-reflection condition only persists for normal incidence when the ARC is implemented in this manner. The relationships between reflection coefficient, transmission coefficient, and impedance denoted in (14) and (15) are altered to (16) and (17) when considering radiation at oblique incidence. To demonstrate this, the same system with the ARC layer was simulated at varied angles, shown in Figure 15. It is clear that at normal incident, the system achieves perfect anti-reflective properties. However, as the incidence angle diverges from normal, the reflection coefficient increases, thus more energy is reflected from the interfaces. For the purposes of this work, we do not consider the effects of oblique incidence, thus minimal system reflections will remain despite the implementation of ARCs. Previous works have considered the effect of oblique incidence on ARC design [53]. This work can later leverage existing research on this behavior to further characterize the following designs.

\[
\Gamma^b = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}
\]  \hspace{1cm} (16)

\[
T^b = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}
\]  \hspace{1cm} (17)

**Figure 15.** Reflection and transmission coefficients of waves through bulk dielectric regions of differing indices, separated with a quarter-wave transformer. Reflections increase as the incident radiation moves away from normal incidence to oblique incidence.
This study considers a flattened converging lens based on a biconvex-to-flat mapping [51]. The lens system provides a suitable case study, where the behavior is well-documented and the design is simplistic. Here the lens is assumed to be silicon, which at the design wavelength of $\lambda_0 = 3.5\mu m$ corresponds to a refractive index of 3.43. Furthermore, the lens is optically small, with an aperture that is $15\lambda_0$, a thickness of $2.14\lambda_0$, and a radius of curvature corresponding to a focal length that is the same as the aperture, $15\lambda_0$. This lens is designed using the lensmaker’s equation in (18), where $f$ is the focal length, $R_1$ and $R_2$ are the radii of curvature, $n$ is the refractive index of the lens, and $d$ is the thickness of the lens.

$$\frac{1}{f} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$  

(18)

The performance of this lens was evaluated using a 2D full-wave simulation in COMSOL Multiphysics. The bi-convex lens is populated with its homogeneous index and is radiated with a $3.5\mu m$ plane wave at normal incidence. The boundaries of the simulation domain are terminated with perfectly matched layers. The resulting normalized electric field distribution at the design wavelength is depicted in Figure 16(a). The mismatch between the lens material and free space results in a significant reflectance of 30% at the design wavelength. These reflections can be mitigated by implementing ARCs to the input and output interfaces of the lens. The quarter-wave transformers described above are implemented to the lens interfaces and the performance of the resulting system is shown in Figure 16(b). The coatings effectively reduce the reflectance at the design wavelength to approximately 0%. Insignificant reflections remain in the simulation domain due to diffraction from the edge of the lens and oblique incidence originating from the lens curvature. This single-layer coating provides effective mitigation of reflections over a narrow operating bandwidth. Figure 17 shows that the lens system achieves a bandwidth of 29% corresponding to a reflectance of less than 0.02, or 2%. The performance of this bi-convex lens provides the model for analyzing the performance of the GRIN system with ARCs.
Figure 16. Normalized electric field of a conventional bi-convex lens (a) without an ARC and (b) with an ARC.
3.3 Single-Layer Anti-Reflective Coatings for Gradient Index Lenses

The conventional bi-convex lens is mapped into a GRIN flat lens using QCTO techniques. A virtual coordinate grid, here the conventional homogeneous bi-convex lens, is distorted into a physical domain, here the flattened GRIN lens. The required material parameters to realize the equivalent distortion are then extracted, a process illustrated in Figure 18. QCTO significantly relaxes the material requirements produced using traditional TO mappings, enabling designers to realize nearly-isotropic, dielectric-only devices as shown by the refractive index distribution in Figure 18(b). The mapping essentially compresses the convex interfaces into planar interfaces, thus the resulting indices are higher than that of the original lens index. If the bi-convex lens were to feature a larger radius of curvature, the interface would not require as much compression to realize a planar interface. Therefore, the gradient would feature a smaller range.

Figure 17. Reflectance of a conventional bi-convex lens with single-layer quarter-wave transformer ARCs. The system achieves a bandwidth of 29% corresponding to a reflectance below 2%.
Figure 18. Bi-convex to flat QCTO transformation. (a) A virtual coordinate grid is mapped into a physical geometry. (b) the required material parameters needed to realize the distortion are then extracted. QCTO techniques enable the development of dielectric-only distributions as shown by the refractive index distribution here.

The GRIN flat lens again features a mismatch between the lens materials at the input and output of the system. Figure 19(a) shows that this GRIN lens has the same functionality as the bi-convex lens in Figure 16(a). The GRIN lens focuses the incident plane wave to the desired focal plane. Furthermore, it also suffers from significant reflections. Here, the flattened GRIN lens features a reflectance of 50%. This reflectance is higher than that of the bi-convex lens because the higher indices here result in a larger mismatch between the lens region and free space. The reflections from the GRIN lens can also be mitigated by incorporating ARC layers to the input and output interfaces. There are several possible approaches to accomplish this functionality.
Figure 19. Normalized electric field of a GRIN flattened bi-convex lens (a) without an ARC and (b) with a homogeneous ARC.
The first intuitive ARC design is to implement a homogeneous quarter-wave transformer. Since it is unclear which index to reference in the GRIN profile, a parametric study was performed to determine the optimal ARC index and corresponding quarter-wave thickness to achieve peak performance. Figure 20 shows that a homogeneous index of 2.06, referencing the lens index of 4.24, effectively mitigates the most reflections. To characterize this behavior in a more quantitative manner, consider the GRIN profile along the interface of the flat lens in the top subplot of Figure 20. This profile can be viewed as a random variable, since the QCTO mapping extracts the material tensors based on a geometrical distortion, and cannot initially be identified. The optimal ARC index corresponds to the mode of the discrete probability density function of this set of values, as shown in the bottom subplot of Figure 20. The performance of the resulting system is depicted in Figure 19(b). The homogeneous ARC effectively reduces the total system reflectance to approximately 0% at the design wavelength. The coating again provides a narrow operating bandwidth, here of 20% with reference to a reflectance below 2%, as shown in Figure 21.

![Graph showing reflectance as a function of ARC coating index](image)

**Figure 20.** (Large) Parametric study of homogeneous ARC index and corresponding thickness to obtain minimum system reflectance. (Top Subset) GRIN refractive index distribution of lens along the input and output interfaces. (Bottom Subset) The number of instances of each index value of the set shown above.
The homogeneous lens system does not perform as well as the homogeneous ARC for the conventional lens. There is still room for additional performance enhancement. Another possible design method considers implementing a radial GRIN distribution to the ARC layer. This GRIN corresponds to the geometric mean of the input index and the GRIN index along the input interface of the lens. The resulting coating applies the quarter-wave transformer as a radial GRIN, but remains homogeneous in the axial direction. This system is also capable of achieving approximately 0% reflectance at the design wavelength, but does not feature significant performance improvement over the homogeneous ARC, as evident from Figure 22 and Figure 25. The performance is nearly identical and the system realizes the same bandwidth as the homogeneous case. Given the added complexity in the ARC region for this design, it appears the homogeneous case is still better option. Therefore, it is necessary to looks towards another ARC design to realize better performance.

Figure 21. Reflectance of a GRIN lens with a homogeneous ARC on the input and output interface. The system achieves a bandwidth of 20% corresponding to a reflectance below 2%.
Figure 22. Reflectance of a GRIN lens with a radial GRIN ARC on the input and output interface. The system achieves a bandwidth of 20% corresponding to a reflectance below 2%.

Alternatively, the ARC design can be executed simultaneously with the QCTO mapping. This method produces a single flattened GRIN region that performs the desired operation whilst mitigating reflections. As shown in Figure 23(a), a homogeneous lens with homogeneous ARCs is concurrently flattened into a single inhomogeneous lens system with a GRIN lens profile and an embedded GRIN ARC. The resulting refractive index profiles are depicted in Figure 23(b), where the GRIN distributions corresponding to the lens and the ARC layers are exploded to clearly discern the gradients. This system again is capable of achieving a reflectance of roughly 0% at the design wavelength. The primary performance distinction between this design and the previous implementation is its bandwidth. Here, the system is capable of realizing a 23% bandwidth corresponding to a reflectance below 2%, as shown in Figure 24. This extra bandwidth is acquired at the expense of additional complexity in the ARC profile. However, this methodology enables designers to directly leverage the rich field of filter theory and ARC design in the QCTO process. The system can be designed entirely using traditional methods and can later be incorporated as a single entity into the QCTO mapping. These single-layer coatings mitigate reflections over a narrow bandwidth. However, the same design concepts are easily extended to multi-layer coatings for broadband operation.
Figure 23. (a) QCTO mapping of a bi-convex lens system including ARC coatings on the input and output interfaces. The computation converts a homogeneous lens with homogeneous ARCs to a system with a GRIN lens with GRIN ARCs. (b) The resulting refractive index distribution of the lens system with embedded ARC layers.

Figure 24. Reflectance of a GRIN lens with an embedded GRIN ARC on the input and output interface. The system achieves a bandwidth of 23% corresponding to a reflectance below 2%.
Figure 25. Reflectance of a flattened GRIN lens with a homogeneous ARC, radial GRIN, ARC, and an embedded GRIN ARC.

3.4 Multi-Layer Anti-Reflective Coatings for Gradient Index Lenses

The same conventional bi-convex lens is considered for broadband coatings. Although there are a plethora of techniques to realize broadband matching, here we consider a 3-layer binomial ARC design [58]. This design methodology relies on the binomial approximation given by (19) - (21), where $\Gamma$ is the reflection coefficient and $N$ is the number of layers.

$$
\Gamma_n(f) \cong \sum_{n=0}^{N} \Gamma_n e^{-j2n\theta} \cong 2^{-N} \frac{\eta_L-\eta_0}{\eta_L+\eta_0} \sum_{n=0}^{N} C_n^N e^{-j2n\theta}
$$

(19)

$$
C_n^N = \frac{N!}{(N-n)!n!}
$$

(20)

$$
\theta = \beta_n d_n = \frac{\pi f}{2 f_0}
$$

(21)
For a lens with an index of 3.43, the binomial approximation provides the corresponding indices for each corresponding layer of 2.64, 1.74, and 1.15, as shown in Figure 26. The resulting design again performs the desired functionality as depicted in Figure 28(a); the normally incident plane wave is focused down to the specified focal plane with minimal reflections from the system. Furthermore, this multi-layer ARC design significantly enhances the bandwidth of the lens system. Figure 27 depicts that the system achieves a bandwidth of 115% with a reflectance below 2%.

![Index distribution of a conventional bi-convex lens with a 3-layer binomial ARC system applied to the input and output interfaces.](image)

**Figure 26.** Index distribution of a conventional bi-convex lens with a 3-layer binomial ARC system applied to the input and output interfaces.

![Reflectance of a conventional bi-convex lens with a 3-layer binomial ARC applied to the input and output interfaces. This system achieves a bandwidth of 115% corresponding to a reflectance below 2%.](image)

**Figure 27.** Reflectance of a conventional bi-convex lens with a 3-layer binomial ARC applied to the input and output interfaces. This system achieves a bandwidth of 115% corresponding to a reflectance below 2%.
Figure 28. Normalized electric field of (a) a conventional bi-convex lens system with 3-layer binomial ARCs and (b) a flattened GRIN lens system with 3-layer homogeneous binomial ARCs.
Using the same process described for the single layer case, consider first a homogeneous ARC for a broadband GRIN system. Referencing the same index as in the single layer case, 4.24, the binomial approximation is applied to the GRIN lens. The homogeneous ARC again significantly reduces reflections from the lens and greatly increases the operating bandwidth compared to the single-layer narrowband design. Here, the system achieves a bandwidth of 106% as shown in Figure 29. However, as with the narrowband design, the GRIN lens system with the homogeneous ARC does not perform as well as the bi-convex lens. Another ARC design method may produce better results.

![Figure 29](image)

**Figure 29.** Reflectance of a flattened GRIN lens with a 3-layer binomial ARC applied to the input and output interfaces. This system achieves a bandwidth of 106% corresponding to a reflectance below 2%.

The ARC design can again be incorporated with the QCTO mapping in the same manner as the single layer case. This mapping starts as a homogeneous bi-convex lens with a 3-layer homogeneous ARC. The indices of the ARC layers are as presented above for the broadband conventional bi-convex lens. The system is then flattened resulting in a single GRIN region depicted in Figure 30. Here, the ARC layers feature radial and axial
GRIN distributions. Each ARC layer is displayed in a separate color scheme to emphasize the gradient distribution. This system also significantly enhances the bandwidth compared to the single layer embedded GRIN ARC implementation. Figure 31 shows that this system achieves a bandwidth of 131% corresponding to a reflectance below 2%. As with the single layer case, this implementation features a larger bandwidth than the homogeneous ARC design. The embedded GRIN ARCs achieve a 25% larger bandwidth than the homogeneous ARCs as shown in Figure 32. However, this additional bandwidth is realized at the expense of additional ARC complexity.

![Figure 30](image_url)

Figure 30. Refractive index distribution of a flattened GRIN lens with a 3-layer binomial embedded GRIN ARC. This system was produced using a QCTO mapping on a homogeneous lens with homogeneous 3-layer binomial ARCs.
Figure 31. Reflectance of a flattened GRIN lens with an embedded GRIN 3-layer binomial ARC on the input and output interface. This system achieves a bandwidth of 131% corresponding to a reflectance below 2%.

Figure 32. Reflectance performance comparison between a flattened GRIN lens with a 3-layer homogeneous binomial ARC and an embedded 3-layer GRIN binomial ARC.
3.5 Conclusions

This work has demonstrated that it is possible to develop ARCs for GRIN systems. We presented several ARC designs for a flattened bi-convex lens. Homogeneous ARCs can still effectively mitigate surface reflections. However, embedded GRIN ARCs perform the same functionality over a broader bandwidth. The choice of which solution to implement is largely a tradeoff between performance criteria and design / fabrication complexity. These design methodologies for narrowband solutions are also easily extended to multi-layer broadband solutions. This indicates that the rich field of ARC design and filter theory can be leveraged in conjunction with QCTO to realize GRIN lens systems with mitigated reflections.

There is still additional research required moving forward with the project. This study only considered the performance assuming normal incidence. However, in many applications, the system must be able to perform under oblique incidence as well. The system performance under oblique incidence must be characterized. Furthermore, this study also considered the performance of a flattened bi-convex lens. Additional lenses should also be considered, such as diverging or multi-lens systems. Also, it is essential to characterize the ARC’s effects on system aberrations, such as spot size. Once these properties are characterized, fabrication considerations must be developed to realize these systems. The difference between the indices in the ARC and the lens regions could present material challenges that may need to be addressed.
CHAPTER 4: CONCLUDING REMARKS

This work demonstrated the utility of applying TO techniques across diverse applications. In the first case, QCTO was used to design GRIN devices on graphene host mediums. This primarily focused on developing integrated photonic components for miniaturized optical applications, but also encompassed more general GRIN devices for a wide range of applications in the mid-infrared frequency regime. These TO-inspired devices offered a higher degree of tunability, functionality, and design flexibility. This work also investigated the design of ARCs for GRIN systems. Conventional ARC design techniques are leveraged in conjunction with QCTO to develop the coatings. First, narrowband solutions are presented and then these design methodologies are extended to multi-layer broadband solutions. These applications together provide representative examples of the great utility that TO holds in the design of novel electromagnetic systems.
REFERENCES


