STATISTICALLY-BASED AIR BLAST LOAD FACTORS BASED ON IMPRECISE PARAMETER STATISTICS FOR REINFORCED CONCRETE WALL

A Dissertation in

Civil Engineering

by

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Submitted in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

May 2015
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ABSTRACT

The determination of acceptable air blast load factors for Load and Resistance Factor Design (LRFD) is complicated due to highly variable loads and to non-linear and rate dependent material and structural response that can result. This has resulted in the use of blast load factors that are set equal to unity since load uncertainty, which is typically used as a probabilistic basis for developing LRFD load factors, is not considered. A precise distribution of random variables also has been required for load and resistance factor determination in LRFD. However, in the case where their distributions are uncertain due to insufficient data, such as for blast events, assumptions will be made that overlook uncertainties that should be accounted in the derivation. In this study, a combination of Response Surface Metamodels (RSM), Monte Carlo Simulations (MCS), and Probability Box (P-Box) that incorporate nonlinear finite element models were used to derive statistically-based blast load factors for LRFD. Load factor development centered on a case study involving a reinforced concrete (RC) cantilevered wall subjected to free air blasts. The resulting load factor was found to be 1.41 when precise parameter statistics were assumed. This blast load factor was then used to design a new RC cantilevered wall and this new wall was found to have its reliability close to the target value. However, when parameter uncertainty was considered, the resulting load factors based on P-Box representation were found to be a range between 1.16 and 1.74, indicating that there was a possibility that the load factor of 1.41 obtained from precise parameter statistic assumptions could be unreliable.
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ACKNOWLEDGEMENTS

This dissertation would not have been possible without support from many people. To express this appreciation, I would like to thank my advisor, Dr. Daniel Linzell, for his patience, encouragement, and guidance that he had continually provided since I began this Ph.D. program. I would also like to thank my advisor and committee chair, Dr. Peggy Johnson, for her constant support and feedback until I could finish the research for this dissertation. I would also like to thank members of my dissertation committee: Dr. Andrew Scanlon; Dr. Swagata Banerjee; and Dr. Renata Engel for their productive advice and Dr. William McGill who had provided very useful advice when I started working on this research.

In addition, I would like to thank the Thai Government for their financial support to pursue this Ph.D. degree. I would also like to thank my friends at Penn State for their friendship; especially to Suppawong Tuarob who has constantly offered very useful suggestions to my research. I would also like to thank Dr. Sathian Charoenrien and Surachai Pornpattrakul who had supported and encouraged me to pursue this Ph.D. degree.

Lastly, I would like to thank my family for their unconditional support, either mental or financial, and encouragement throughout my quest for this Ph.D. at Penn State.
Chapter 1

Introduction

1.1 Overview

Load and Resistance Factor Design (LRFD) is a statistically based structural design methodology that is used for many structures, components, and material types. The concept considers uncertainties in structural loads \( S \) and member capacities or resistance \( R \) and applies load factors \( \phi \) and resistance factors \( \gamma \) to \( S \) and \( R \) to ensure that the structure’s resistance will be greater than the anticipated load effects during its lifetime. This LRFD definition can be explained using Equation 1-1,

\[
\phi R_n \geq \sum \gamma_i S_{ni}
\]  

(Eq. 1-1)

in which \( R_n \) and \( S_n \) are the nominal strengths and loads, respectively. The load factors are influenced by the degree of accuracy of load effect calculations and their variation during the structure’s lifetime. On the other hand, the resistance factors are influenced by the probability of structural members being under-strength due to variations in material properties and component dimensions, inaccuracies in design equations, degrees of ductility, and their importance in the design (ACI, 2008). The application of LRFD can be found in many design codes, such as the American Concrete Institute’s (ACIs) Building Code Requirements for Structural Concrete (ACI 318-08) (ACI, 2008), the American Institute of Steel Construction’s (AISCs) Manual of Steel Construction (AISC, 2006), or the American Association of State Highway and Transportation Officials’ (AASHTOs) LRFD Bridge Design Specifications (AASHTO, 2004).

The development of \( \phi \) and \( \gamma \) found in these codes for low probability, extreme events such as a seismic event has been traditionally based on limiting structural responses so that structural elements can retain most of their original load-carrying capacity. However, blast resistant design often allows for structural elements to experience large plastic deformations
where the structural elements could be failed and alternate load-carrying mechanisms could be developed to resist progressive collapse. Therefore, many assumptions that form the basis for the development of conventional design load and resistance factors might not be valid for blast loads, and factors that have been developed for conventional design might not be applicable to blast design (Dusenberry, 2010). However, many blast resistant design guidelines suggest using a conservative resistance factor of 1.0 because: 1) material strength is shown to increase due to strain-rate effects during a blast event and 2) the air blast pressure time history is idealized and likely conservative (Department of Defense, 2008). Those guidelines, on the hand, suggest a unity blast load factor based on a conservative assumption for low-probability, extreme events like blast, not based on statistical information of blast load parameters that contradict the underpinnings of LRFD. The literature states that difficulties associated with statistically-based blast load factor development arise from complexities related to predicting structural response under blast and determining joint probability density functions of random variables (Cormie et al., 2005; Campidelli et al., 2013) that are required when performing reliability analyses.

Therefore, the current study developed a methodology to overcome these two shortcomings by: 1) applying Response Surface Metamodeling (RSM) techniques centered on fully-nonlinear finite element models to develop mathematical functions for predicting structural response under blast and 2) applying Monte Carlo Simulation (MCS) techniques in conjunction with the RSM functions and a thorough survey of available data related to reinforced concrete structure blast reliability to develop an empirical response probability distribution that was further used to determine subsequent blast load factors. The proposed method was demonstrated by the case of a blast-resistant reinforced concrete cantilevered wall that was originally designed using a traditional, single-degree of freedom (SDOF) approach.

Not only limiting nonlinearity of the response, early development of load and resistance factors also simplified incorporated parameter statistics so that the traditional reliability method
(e.g. First Order Second Moment (FOSM) method, Advance First Order Second Moment (AFOSM or ASM) method) could be performed (MacGregor, 1976; Winter, 1979). For example, precise normal or lognormal distributions, incorporating statistical independency were assumed when determining load and resistance factors for a RC beam under typical dead load and live load (MacGregor, 1976). This simplification is valid for these typical loadings when their extensive statistical information is available. Parameter statistics for blast loads, on the other hand, have been found to be highly uncertain because statistical studies for blast are limited and their results are normally unavailable due to security concern (Low and Hao, 2002; Bogosian et al., 2002; and Hao et al., 2010). When parameter distributions are uncertain, it has been reported that assuming different distributions can provide a significant difference of the subsequent reliability results (Zhou et al., 1999, Jimenez et al., 2009; Haldar and Mahadevan, 2000). Therefore, for circumstances where parameter distributions are imprecise such as in blast, traditional reliability analysis methods (e.g. FOSM, AFOSM, Monte Carlo Simulation (MCS)) that depend on precise parameter distributions may give inaccurate results and can, ultimately, affect the subsequent load and resistance factors. Therefore, the current study developed a methodology that implemented Probability Box (P-Box), to account parameter statistic uncertainty, in conjunction with RSM method and the iteration method (Allen et al., 2005) to determine subsequent blast load factors when parameter statistic uncertainty is included. The proposed P-Box method was demonstrated using the same case study used to demonstrate the RSM and MCS method discussed previously so that the blast load factors resulted from precise and uncertain parameter statistics can be compared.
1.2 Problem Statement

There is an extensive series of load and resistance factors available for many loading types, including blast loads, for LRFD. The development of these factors is typically based on extensive statistical information of parameters related to resistance and loads. However, for blast resistant design, there is no evidence of statistically-based blast load factor while most design guidelines provide only its simple, conservative version as a unity applied for a low probability, extreme event. This shortcoming comes from complex responses of structures under blast load and, as a result, a lack of explicit nonlinear response functions that are required when performing reliability analyses and subsequently determining probabilistic blast load factors. Moreover, parameter statistics related to blast loads have been found to be highly uncertain due to limited statistical information for blast load parameters and this contradicts the requirement of precise distribution when using traditional reliability methods.

1.3 Objectives

The primary objective of this study is to develop a methodology that can systematically determine statistically-based blast load factors used for blast resistant reinforced concrete design when such design factors have been conservatively assumed as a unity based on low-probability, high-consequence characteristic of blast event. The secondary objective is to develop another methodology used in conjunction the previous methodology to provide blast load factors that are resulted not only from blast load parameter statistics but also from the statistic uncertainty found in the literature.
1.4 Scope

The scope of this study focused on development of a modified reliability method to determine blast load factors for isolated reinforced concrete retaining wall under an ultimate strength limit state when variations of material strength, geometry imperfection, and loading statistical information obtained from the literature. The ultimate strength limit state for flexure was considered because it is the typical design criteria for structures under extreme events and the retaining wall was chosen because this type of structures is commonly used as a blast resistant structure. Single Degree of Freedom (SDOF), a traditional design approach for blast resistant design, was used to design the wall and the reliability from using this method, considered as the current state of blast resistant design, could be investigated. The development was also based on experimental programs done by Finite Element Models (FEMs) which were created and validated against theoretical calculation and published blast test results. The validated FEMs were then used in conjunction with Response Surface Metamodels (RSM) and Central Composite Design (CCD) to simulate response data points that were further used to develop response predicting functions of the wall. The developed response functions were then used in conjunction with MCS and P-Box to determine the subsequent blast load factors.

1.5 Tasks

To accomplish the objectives, the research will be organized as follows:

- The completion of a literature search that presents background on: the development of load and resistance factors for reinforced concrete; traditionally implemented tools for the reliability based load and resistance factors derivation; the application of PBA to reliability analyses when random variable distributions are not well-established; the availability of statistical information for random variables related to this study; the development of load and resistance factors for structures subjected to blast loads; the development of nonlinear response functions
using RSMs; and the modeling techniques for computational study of reinforced concrete members subjected to blast loads using LS-DYNA.

- The development of RSMs that are implemented to predict the nonlinear response of reinforced concrete members under blast loads and to determine subsequent blast load factors when precise parameter statistics are assumed.
- The development, application and validation of P-Boxes and a modified reliability method to determine blast load factors that account parameter statistic uncertainty.

This section summarizes the tasks that are needed to be done to complete the present study. Details for each task will be discussed in the subsequent chapters.
Chapter 2

History and Backgrounds for Load and Resistance Factor Development

In this chapter, current reliability approaches used to determine load and resistance factors and history behind the development of current load and resistance factors for reinforced concrete structures are discussed to provide some understanding the concepts and applications of load and resistance factor and possible deficiencies associated with their development.

2.1 Reliability Approaches to Determine Load and Resistance Factors

Load and resistance factors design (LRFD) is based on the requirement that the nominal resistance, $R_N$, reduced by a resistance factor, $\phi$, that account uncertainties in the actual strength of the elements and the consequences of failure, shall be greater than the linear combination of nominal load effects, $Q_{Ni}$, magnified by load factors, $\gamma_i$, that account for uncertainties in the actual magnitudes and occurrences of the loadings (Avrithi and Ayyub, 2010). This relationship is shown in Equation 2-1.

$$\phi R_N \geq \sum_{i=1}^{n} \gamma_i Q_{N_i}$$  \hspace{1cm} (Eq.2-1)

The factors are determined probabilistically using a reliability analysis approach and correspond to a predefined level of safety represented by a target reliability index, $\beta_T$.

To begin a reliability analysis that is used to determine load and resistance factors, a performance function that includes relevant parameters for the limit states of interest and describes a relationship between resistance and load effects must be developed. The parameters would usually be characterized as random variables and relationships between them might be
simple or complex depending on theoretical backgrounds of the resistance and load effects. As stated in Sections 1.1 and 1.2, in typical reliability analyses, known distributions are usually assumed for random variables based on the availability of data and a suitable reliability method would then be implemented to determine load and resistance factors.

Irrespective of the random variables characteristics and the complexity of the resistance, \( R \), and load effects, \( Q \), the generalized format for a limit state function can be presented as \( R - Q = 0 \) and can contain many interrelated random variables. Once distribution functions for \( R \) and \( Q \) are obtained, the probability of failure, \( P_f \), can be determined from the overlapping area between \( R \) and \( Q \) as shown in Figure 2-1.

![Figure 2-1. Probability of failure for a structural system](image)

Typically integration, as shown in Equation 2-2, can be used to determine \( P_f \) as long as explicit forms of \( Q \) and \( R \) are available.

\[
P_f = \int_{0}^{\infty} F_R(q)f_Q(q)\,dq
\]

(Eq.2-2)
where

- $P_f$ = probability of failure,
- $F_R$ = cumulative distribution function (CDF) of resistance $R$,
- $f_Q$ = probability distribution function (PDF) of load effect $Q$, and
- $q$ = load effect value.

In general, direct integration is difficult because many parameters are interrelated in a performance function and a joint PDF for those parameters is practically impossible to obtain. Therefore, analytical approximations of the integral are necessary (Haldar and Mahadevan, 2000). Currently, there are some approximate methods that can be used to simplify this integral, such as the first-order reliability method (FORM) or the second-order reliability method (SORM), and these methods have been applied to determine load and resistance factors for structural design. FORM or SORM solutions are given as safety or reliability indices, denoted as $\beta$, and they refer to the probability of failure for a particular performance function. Basically, FORM and SORM complete reliability analyses based on an underlying assumption that the random variable distributions are normal (Lu et al., 1994). For random variables whose distributions are not normal, transformation to an equivalent normal distribution occurs before following FORM or SORM procedures (Haldar and Mahadevan, 2000).

Haldar and Mahadevan (2000) demonstrated the derivation of load and resistance factors for a simply-supported beam using FORM. Resulting expressions for the derived load and resistance factors are shown in Equation 2-3 and 2-4, respectively:

$$\gamma_i = \frac{1 + \varepsilon_{mn} \beta \delta_{Q_i}}{1 + k_{Q_i} \delta_{Q_i}}$$

(Eq.2-3)
\[
\phi = \frac{1 - \varepsilon \beta \delta_R}{1 - k_R \delta_R}
\]  
(Eq.2-4)

where

- \( \delta_R \) = resistance variance,
- \( \delta_Q \) = load effects variance,
- \( \beta \) = target safety or reliability index,
- \( k_R \) = distance from mean resistance to nominal resistance \( (R_N/\mu_R) \),
- \( k_Q \) = distance from mean load effects to nominal load effects \( (S_N/\mu_S) \),
- \( \varepsilon = \frac{\sigma_R^2 + \sigma_Q^2}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \), and
- \( \varepsilon_{\text{nn}} = \sqrt{\sigma_{Q_1}^2 + \sigma_{Q_2}^2 + \ldots + \sigma_{Q_n}^2} \).

Two assumptions were made during the derivation of Equations 2-3 and 2-4. The first was that statistical independency was assumed for \( R \) and \( Q \) and the second was that \( R \) and \( Q \) were assigned normal distributions. However, results from the sensitivity study of reliability indices and distribution types indicated that assuming all random variable had normal distributions reduced \( \beta \) by approximately 1.3 from cases assuming lognormal distributions that provided a \( \beta \) of 5.26. This reduction indicates an increase in the probability of failure by approximately 18,400% and it may lead to a reduction in structural reliability if the structures are designed with load and resistance factors developed using normal distribution assumptions.

Complication associated with the calculation of load and resistance factors for a structural member arises when random variable distributions are neither normal nor lognormal. This is because some statistical parameters used to characterize the distributions for \( R \) and \( Q \) (e.g. the
standard deviation or covariance) cannot be calculated in conjunction with applying the previously discussed simplified statistical methods.

One method used to determine probability of failure when random variables are neither normal nor lognormal distribution is known as the Advance First-Order Second Moment method (AFOSM or ASM). It was initially developed by Hasofer and Lind (1974) for normal variables and adopted for non-normal variables by Rackwitz and Fiessler (1978). A flow chart of the ASM procedure is shown in Figure 2-2. This flow chart explains procedures to determine load and resistance factors using ASM for non-normal random variables. Firstly, the non-normal means and standard deviations are transformed to equivalent normal means and standard deviations whose distributions having the same CDF values as the corresponding non-normal CDFs. Secondly, design points for all random variables and the reliability index, $\beta$, are computed using iterative procedures. These iterative procedures are performed in a reduced coordinate system, a coordinate system of normal variables that has a mean of 0.0 and a standard deviation of 1.0, until the target $\beta$ is obtained. Lastly, load and resistance factors are calculated from the means and design points obtained for each random variable. Haldar and Mahadevan (2000) stated that the ASM approach has a drawback whereby approximations of non-normal distributions become more and more inaccurate as the original distributions are increasingly skewed, which means that the ASM approach is still dependent to the original random variable distributions. The sensitivity of the ASM method to distribution types (normal vs. lognormal) to determine $\beta$ was carried out by Haldar and Mahadevan (2000) using the same case study completed for the simplified approach. Results showed significant reduction in the difference for $\beta$ when assuming normal and lognormal from 1.3 for the simplified method to 1.0 for the ASM method.
Figure 2-2. ASM flow chart

Obtain data: mean, STD, COV, and type,
Assume 1st design point ($x_i^*$)

$\Phi^1(F_{X_i}(x_i^*))$, $\phi(\Phi^1(F_{X_i}(x_i^*))$)

$\sigma_{x_i}^N = \frac{\phi(\Phi^{-1}(F_{X_i}(x_i^*)))}{f_{x_i}(x_i^*)}$
$\mu_{x_i}^N = X^* - \Phi^{-1}(F_{X_i}(x_i^*))\sigma_{x_i}^N$

$x_i' = \frac{x_i^* - \mu_{x_i}^N}{\sigma_{x_i}^N}$

New $x_i'^N = \frac{1}{\sum(\frac{\partial g}{\partial X_i'})^2} \sum(\frac{\partial g}{\partial X_i'}) x_i' - g() \{\left(\frac{\partial g}{\partial X_i'}\right)^N\}$

New $x_i'^N = \mu_{x_i}^N + \sigma_{x_i}^N x_i'^N$

New $\beta = \sqrt{\sum(x_i'^N)^2}$

Check New $\beta - \beta \leq$ acceptable level

Check $g(new x_i'^N) - g(x_i'^N) \leq$ acceptable level

$\varphi = \frac{R(Y^*)}{R(Y_n)}, Y_i = \frac{L(Z_i^*)}{L(Z_{n,i})}$
Another approach to determine load and resistance factors is the Rackwitz-Fiessler procedure (Rackwitz and Fiessler, 1978). This approach is a graphical method that can obtain reliability indices directly from CDFs of $R$ and $Q$. However, this approach is applicable only if the performance function is linear (in this case $R - Q = 0$ is linear) (Allen et al, 2005). To perform the Rackwitz-Fiessler procedure, the cumulative probability distributions are plotted from both the load and resistance data. A trial design point ($R^*$ and $Q^*$) is selected at $R = Q$ (for limit state function, $g_1 = R - Q = 0$). Using tangents to the distribution curves at the selected $R^*$ and $Q^*$ points, load and resistance data means, $\bar{R}$ and $\bar{Q}$, and standard deviations, $\sigma_R$ and $\sigma_Q$, can be determined directly from the curves. Their mean values are represented by points where tangent lines to their distribution curves intersect the horizontal axis where probability is zero ($z = 0$) and their standard deviations are the inverse of the curve slopes for each tangent. By obtaining these four parameters, the reliability index can be estimated using Equation 2-5.

$$\beta = \frac{\bar{R} - \bar{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$  \hspace{1cm} (Eq.2-5)

Once $\beta$ is obtained for the first iteration, additional design points are selected and the process is repeated to obtain a new $\beta$. Iterations stop when $\beta$ converges to within a specific tolerance. However, for calibrating load and resistance factors, a desirable $\beta$ has already been identified. Therefore, for each iteration different trial load and resistance factors are applied to the CDFs for $R$ and $Q$ and then the process is repeated until load and resistance factors giving the desired $\beta$ are obtained. Application of the Rackwitz-Fiessler procedure to the determination of load and resistance factors for the pullout of steel grid reinforcement in a reinforced soil wall was demonstrated by Allen et al (2005). The study demonstrated that this method had the capability to
determine load and resistance factors even when CDFs for \( R \) and \( Q \) fit either normal or lognormal distributions.

Monte Carlo simulation was also used to calibrate load and resistance factors for an interior steel bridge girder by Allen et al. (2005). Random variables were generated based on known statistical characteristics (i.e. mean, standard deviation, distribution type). Trial load and resistance factors were assigned using an explicitly established flexure limit state function (\( g \)). A target reliability index (\( \beta_T \)) of 3.5 was selected as a desired reliability index for typical buildings based on existing literature. Each set of generated random variables were placed into the limit state function and the probability of failure (\( P_f \)) was calculated (\( g < 0.0 \)). The reliability indices (\( \beta \)) were obtained from inverse normal functions of the probability of failure as shown in Equation 2.6.

\[
\beta = \Phi^{-1}(-P_f) \quad \text{(Eq.2-6)}
\]

Iteration of load and resistance factors occurred until the calculated \( \beta \) was equal to \( \beta_T \). Allen et al. (2005) also stated that Monte Carlo simulation could be applicable for unknown distribution random variables when they were approximated by a normal or lognormal distribution and the reliability index could be calculated using Equations 2-5 and 2-7, respectively:

\[
\beta = \frac{\ln\left[\frac{R/Q}{\sqrt{\left(1+\text{COV}_Q^2\right)/\left(1+\text{COV}_R^2\right)}}\right]}{\sqrt{\ln\left(\left(1+\text{COV}_Q^2\right)/\left(1+\text{COV}_R^2\right)\right)}} \quad \text{(Eq.2-7)}
\]

where

- \( \ln(\cdot) = \text{natural logarithm} \)
• \( \bar{R} \) = the mean of resistance \( R \),
• \( \bar{Q} \) = the mean of load effects \( Q \),
• \( COV_R \) = the coefficient of variation for resistance,
• \( COV_Q \) = the coefficient of variation for load effects, and
• \( \beta \) = the reliability index.

2.2 Development of Reinforced Concrete Load and Resistance Factors

The load and resistance factors for structural concrete in the American Concrete Institute’s (ACIs) Building Code Requirements for Structural Concrete (ACI 318) were initially proposed in the 1962 revision (ACI 318-62) to the 1956 code (ACI 318-56), which was based on the ultimate strength design (USD) concept. The basis of the development of load and resistance factors was the implied assumption for a probability of failure of \( 10^{-5} \) that was derived from a \( 10^{-3} \) probability of member understrength coupled with a \( 10^{-2} \) probability of member overload. This assumed probability of failure was stated to be not quite correct statistically because the variation of strength and load was not considered but it was deemed as an adequate starting point (MacGregor, 1976). All random variable distributions were assumed normal. The probability of understrength was estimated from the probability that design strengths for concrete \( (f^*_{c}) \) and steel \( (f^*_{y}) \) fell below \( 0.67 f'_{c} \) and \( 0.8 f_{y} \), respectively, and the probability of understrength for a cross section calculated using \( f^*_{c} \) and \( f^*_{y} \) would be \( 10^{-3} \) or smaller corresponding to these assumptions. The probability of overload was estimated assuming that dead load had a coefficient of variation of 8 to 10% and live load a coefficient of variation of 33.5 to 50% and the corresponding dead load and live load factors providing the probability of overload to be \( 10^{-2} \) were 1.3.

The revisions of load and resistance factors in ACI 318-56 were adopted in ACI 318-63. Major changes were to increase of the probability of overload to \( 10^{-3} \) and reduce the probability of understrength to \( 10^{-2} \). These modifications increased the dead load and live load factors by
15%. Reduction in the understrength probability changed the design strengths for concrete and steel to $0.85 f'_c$ and $0.9 f_y$ respectively. ACI 318-71 reduced load factors by approximately 6% due to additional research and experience that produced more accurate load modeling methods and improved concrete and steel production quality control.

However, the load and resistance factors presented in ACI 318-71 were developed based on a statistical error associated with the procedures used to estimate the probability of failure from the probability of understrength and overload. MacGregor (1976) illustrated this problem using Figure 2-3. This figure shows the distribution of load, $U$, and strength, $R$. The $45^\circ$ line represents $U = R$. The area above this line was described as the failure region and the area below this line was described as the safe region. Combinations of $U$ and $R$ above the $45^\circ$ line resulted in failure such as, for example, load $U_1$ acting on structure $R_1$ as shown in Figure 2-3(a). On the other hand, combinations of $U$ and $R$ below the $45^\circ$ line, for example, load $U_2$ acting on structure $R_2$ in Figure 2-3(a), represented a safe combination. The shaded areas in Figure 2-3(b) represent the probability of overload and understrength as defined in the derivation of the load and resistance factors. As presented in Figure 2-3(b), the failure region was divided into three regions (A, B, and C) and region B represented the probability of failure at $10^{-5}$ as expected in ACI 318-71. This definition of the probability of failure ignores the probability in regions A and C and thus underestimation in the actual probability of failure could occur. Therefore, it was expected that structures designed using load and resistance factors from the 318-71 could have a lower reliability than what was anticipated.
To achieve a correct estimate of the probability of failure, MacGregor (1976) suggested using Equation 2-8 or 2-9 if the distribution of $R$ and $U$ are normal or lognormal to determine probability of failure:

$$P_f = P \left[ Y = (R - U) < 0 \right]$$  \hspace{1cm} (Eq.2-8)

$$P_f = P \left[ Y = \ln \left( \frac{R}{U} \right) < 0 \right]$$  \hspace{1cm} (Eq.2-9)

Figure 2-3. (a) Definition of failure (b) ACI definition of failure probability (MacGregor, 1976)
Equation 2-8 and 2-9 can be presented graphically with the plot of the distribution of $Y$ as shown in Figure 2-4. The probability of failure is represented by the shaded area and defined by the number of standard deviations, $\beta \sigma_Y$, where the mean value of $Y, \mu_Y$, is above zero. Here, $\beta$ denotes the reliability index. This definition of probability of failure becomes a fundamental concept in the derivation of load and resistance factors in ACI 318-99 and can be expressed as:

$$\mu_Y = \beta \sigma_Y$$

(Eq.2-10)

Figure 2-4. Definition of probability of failure and reliability index $\beta$ (MacGregor, 1976)

However, determination of the mean and standard deviation for $Y, \mu_Y$ and $\sigma_Y$, is complicated and dependent on the distribution of $R$ and $U$. Correct distributions for $R$ and $U$ cannot be determined theoretically, especially for the extreme tails of those distributions where failure would occur (Allen, 1975). Therefore, given limitations associated with reliability methods that were available at the time, assumptions for the distribution of $R$ and $U$ and approximations of the distribution parameters (e.g. mean, standard deviation, covariation) were needed to determine load and resistance factors. MacGregor (1976) demonstrated the derivation of load and resistance factors by assuming that the distributions for $R$ and $U$ are lognormal.
because they tended to be skewed. Using this assumption, the resistance, $\phi$, and load factors, $\lambda$, could be approximated using Equation 2-11 and 2-12:

$$\phi = \gamma_R e^{-\beta \alpha R}$$

(Eq.2-11)

$$\lambda = \gamma_U e^{\beta \alpha U}$$

(Eq.2-12)

where $\gamma$ is the ratio between the actual mean and its nominal value (i.e. a bias function).

If statistical information for $R$ and $U$ were available and a target for $\beta$ was chosen, load and resistance factors could be calculated directly from Equation 2-11 and 2-12. However, distributions for $R$ and $U$ had to be lognormal to be consistent with the equation derivations. MacGregor (1976) demonstrated the derivation of load and resistance factors for a reinforced concrete beam and column with given statistical information for their load and resistance factors using Equation 2-11 and 2-12 and resulting load and resistance factors were as follows:

- $\phi$ for flexure in the reinforced concrete beam = 0.802;
- $\phi$ for shear in the reinforced concrete beam = 0.667;
- $\phi$ for axially loaded reinforced concrete column = 0.606;
- $\lambda$ for dead load for a ductile member ($\beta = 3.5$) = 1.23;
- $\lambda$ for live load for a ductile member ($\beta = 3.5$) = 1.42;
- $\lambda$ for dead load for a brittle member ($\beta = 4.0$) = 1.27; and
- $\lambda$ for live load for a brittle member ($\beta = 4.0$) = 1.576.

He compared his $\phi$ factors against those from ACI 318-71 and the results showed that Macgregor’s were lower than ACI 318-71’s by 5% for the flexural limit state and 19% for the shear limit state. This indicated that, when considering all failure regions above the 45° line in Figure 2-3(b), the load and resistance factors in ACI 318-71 underestimated the probability of
failure and, as a result, provided a lower reliability for structures designed using those factors. His proposed load and resistance factors were slightly modified and implemented in ACI 318-95.

In ACI 318-02, resistance factors were adjusted to be compatible with load combinations from the ASCE’s *Minimum Design Loads for Buildings and Other Structures* (SEI/ASCE -7) (ASCE, 2010). These factors were essentially the same as those in ACI 318-95 except the flexure (tension controlled) resistance factor was increased from 0.80 to 0.90 based on a study by Nowak and Szerszen (2001) and the opinion of the committee overseeing the ACI 318 that performance of concrete structures in the past supported using a $\phi$ of 0.90 (ACI 318-08). In Nowak and Szerszen (2001), new $\phi$ factors were determined based on an assumption that material property data used to develop $\phi$ factors in ACI 318-95 was inadequate because material quality had improved over the years. To calibrate the $\phi$ factors, a target reliability index, $\beta_T$, was selected from reliability indices calculated from material test data in the 1970’s and early 1980’s and new test data available at that time. When completing their reliability analyses, Nowak and Szerszen (2001) assumed material properties distributions to be normal. Statistical parameters (i.e. mean, standard deviation, coefficient of variation) for the resistance were determined using Monte Carlo simulation and the reliability indices were calculated from Equation 2-5. Limit state functions were formulated using load combinations from SEI/ASCE -7 and ACI 318-99. Using the updated material data and their reliability analysis simulation technique, recommended resistance factors for cast-in-place RC members from Nowak and Szerszen (2001) are as follows:

- $\phi$ for RC beam in flexure = 0.90 with $\beta_T = 3.5$;
- $\phi$ for RC beam in shear = 0.85 with $\beta_T = 3.5$; and
- $\phi$ for axial loaded RC column = 0.75 with $\beta_T = 3.5$.

However, only the resistance factor for RC beams in flexure was implemented in ACI 318-02.
As discussed in this section, load and resistance factors in ACI have been repeatedly revised. Each revision used different reliability approaches and different assumptions for the random variables distributions, such as simplified method with no distribution assumption in ACI 318-63, the first-order second-moment (FOSM) method using lognormal distribution for resistance and load effects in ACI 318-95 by MacGregor (1976), or Monte Carlo simulation associated with the FOSM method and normal distributions for the material properties in ACI 318-02 by Nowak and Szerszen (2001). Additionally, updated material statistical data was taken into account during each update and, as a result, each revision provided different load and resistance factors.

2.3 Conclusion

This chapter presented information related to reliability approaches used to determined load and resistance factors and ACI 318 load and resistance factor development. The reviews demonstrated that the current load and resistance factors were developed using assumptions regarding random variable distributions so that conventional reliability analysis approaches (i.e. FOSM, AFOSM, and SORM) could be applied. However, load and resistance statistical information (i.e. mean, COV, distribution type) could, in many instances, be uncertain and conventional reliability methods could not take these uncertainties into account.
Chapter 3
Statistically-based Air Blast Load Factor Development

3.1. Introduction and Backgrounds

The development of $\phi$ and $\gamma$ found in many design codes such as the American Concrete Institute’s (ACIs) *Building Code Requirements for Structural Concrete* (ACI 318-08) (ACI, 2008), the American Institute of Steel Construction’s (AISCs) *Manual of Steel Construction* (AISC, 2006), or the American Association of State Highway and Transportation Officials’ (AASHTOs) *LRFD Bridge Design Specifications* (AASHTO, 2004) has been traditionally based on studies of structural responses that corresponds to frequent, in-service, demands and conditions. Less frequent and more extreme demands, such as those that occur during a seismic event, require limiting anticipated response so that structural elements can retain most of their original load-carrying capacity. Blast resistant design, on the other hand, often allows for structural elements to experience large deformations and to be loaded well into their inelastic range, where the structural elements could be failed and alternate load-carrying mechanisms (e.g., catenary action) could be developed to resist progressive collapse. Therefore, many assumptions that form the basis for the development of conventional design load and resistance factors might not be valid for blast loads, and factors that have been developed for conventional design might not be applicable to blast design (Dusenberry, 2010).

According to the Unified Facilities Criteria (UFC) Report UFC-3-340-02, *Structures to Resist the Effects of Accidental Explosions* (Department of Defense, 2008), blast-resistant design often uses assumed values of 1.0 for $\phi$ and $\gamma$. The resistance factor is selected assuming that: (a) the air blast pressure time history is idealized and likely conservative; (b) equivalent single degree of freedom (SDOF) models are used for load effect calculations; and (c) strain-rate effects are conservatively estimated. The load factor is selected assuming an unpredicted design threat.
probability, one that cannot be accurately determined, exists. These factors have no statistical basis and, as a result, confidence related to predicting structural response with acceptable precision is reduced (Dusenberry, 2010). For the case of reinforced concrete structural elements, which are the focus of the study summarized herein, UFC-3-340-02 recommends using a resistance factor of 1.0 even when it is understood that, during a blast event, material strength is shown to increase due to strain-rate effects until it reaches its failure point. Historically, when rate dependent changes to component response have been addressed for blast design, they have been based on mechanistic, not statistical, approaches that contradict the underpinnings of LRFD.

The literature states that difficulties associated with probabilistic air blast load factor development arise from complexities related to predicting structural response under blast and determining joint probability density functions of random variables (Cormie et al., 2005; Campidelli et al., 2013) that are required when performing reliability analyses. In the current study, the first problem was addressed by applying Response Surface Metamodelling (RSM) techniques centered on fully-nonlinear finite element models to develop mathematical functions for predicting structural response under blast. The second issue was addressed by applying Monte Carlo Simulation (MCS) techniques in conjunction with the RSM functions and a thorough survey of available data related to reinforced concrete structure blast reliability to develop an empirical response probability distribution that considered both material and load variations. A simulation technique, capable of determining load and resistance factors when input distributions are either unknown or not normally distributed (Allen et al., 2005), was then used to determine probabilistically based air blast load factors. The proposed technique was developed and demonstrated for the case of a blast-resistant reinforced concrete cantilevered wall that was originally designed using a traditional, single-degree of freedom (SDOF) approach.
3.2. Parameter Statistics

3.2.1 Blast Load

The magnitude of an air blast pressure wave \( P \) is mainly dependent on two parameters: the type and amount of explosive charge \( W \); and the relative location between the center of the charge and the target, otherwise known as the stand-off distance \( R \). The blast pressure is often defined as roughly being proportional to these parameters as follows (Department of Army, 1990):

\[
P \propto \frac{W}{R^2}. \tag{Eq.3-1}
\]

Therefore, the variation of \( P \) mainly results from the variation of \( W \) and \( R \). However, blast load variations found in published structural reliability studies are generally represented by variations of peak reflected pressures, \( Pr_{\text{max}} \), and positive phase durations, \( t_0 \), instead because \( W \) and \( R \) statistical information is not publically available due to security concerns. Variations assumed for \( Pr_{\text{max}} \) and \( t_0 \) can be found in a number of publications (Eamon, 2007; Low and Hao, 2002; Low, 2000; Bogosian et al., 2002; Hao et al., 2010).

Eamon (2007) investigated existing statistical information for \( R \), \( W \), and explosive material properties and found that three blast load levels, as defined by impulse delivered to the structures and the resulting structural behavior and classified as low, moderate, and high, have Coefficients of Variation (COVs) of 0.17, 0.09, and 0.08, respectively. Their distributions were also investigated, and it was found that low and high blast load data best fit a lognormal distribution, while moderate blast load data best fit a normal distribution. For the distribution upper tail, the low, moderate, and high blast load data best fit lognormal, normal, and extreme Type I distributions, respectively.
Low and Hao (2002) investigated blast load variation by comparing $Pr_{\text{max}}$ and $t_o$ estimated from 8 different prediction models and found that the average COV of $Pr_{\text{max}}$ and $t_o$ for scaled distance, a constant proportional of $R/W^{1/3}$ (referred to as called $Z$), ranging between 0.24 to 40 m/kg$^{1/3}$ were 0.3227 and 0.130, respectively. The mean value of the peak reflected pressure was found to be a function of $Z$, and could be estimated in kilopascals (kPa) using Equation 3-2 (Low, 2000):

$$Pr_{\text{max}} = \frac{154.67}{Z} + \frac{617.19}{Z^2} + \frac{3069.3}{Z^3} - 1.2024$$ (Eq. 3-2)

where the blast positive duration mean was obtained from a design chart in UFC-3-340-02.

Bogosian et al. (2002) also investigated the variation of $Pr_{\text{max}}$ and $t_o$ when $Pr_{\text{max}}$ and $t_o$ were estimated using three blast load prediction software packages, CONWEP (Hyde, 1992), SHOCK (Wager, 2005) and BlastX (Britt and Lumsden, 1994). Computed values were compared against measured data from 300 blast tests with $Z$ ranging between 1.19 and 23.79 m/kg$^{1/3}$. The study found that the average COV for $Pr_{\text{max}}$ and $t_o$ were 0.24 and 0.18, respectively.

Hao et al. (2010) developed equations as a function of $Z$ to estimate means, standard deviations, and COVs of $Pr_{\text{max}}$ and normalized $t_o$. The equations were determined using empirical relations, design charts, and analyses completed using CONWEP. $Pr_{\text{max}}$ and $t_o$ for a given $Z$ were also assumed to be normally distributed, having their means, standard deviations, and COVs calculated using their equations. However, these equations were found to be accurate for $Z$ ranging between 1.0 to 10 m/kg$^{1/3}$.

These findings (Low and Hao, 2002; Low, 2000; Bogosian et al., 2002; Hao et al., 2010) indicate significant uncertainty associated with derived blast load parameters as summarized in Table 3-1. As a result, this uncertainty should be carefully taken into consideration in the current study to appropriately assess structural response variations.
Table 3-1. Summary of blast load parameter variations

<table>
<thead>
<tr>
<th>Reference</th>
<th>Parameters</th>
<th>Conditions</th>
<th>Distribution Parameters</th>
<th>Distribution Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low and Hao (2002)</td>
<td>$Pr_{max}$</td>
<td>$Z \sim 0.24$ to $40$ m/kg$^{1/3}$</td>
<td>COV = 0.3227 Mean = 5,805 kPa*</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>$t_0$</td>
<td></td>
<td>COV = 0.130 Mean = 4.83 ms*</td>
<td></td>
</tr>
<tr>
<td>Bogosian et al.</td>
<td>$Pr_{max}$</td>
<td>$Z \sim 1.19$ to $23.79$ m/kg$^{1/3}$</td>
<td>COV = 0.24 Mean = 7,024 kPa*</td>
<td>Normal</td>
</tr>
<tr>
<td>(2002)</td>
<td>$t_0$</td>
<td></td>
<td>COV = 0.18</td>
<td></td>
</tr>
<tr>
<td>Hao et al. (2010)</td>
<td>$Pr_{max}$</td>
<td>$Z \sim 1.0$ to $10$ m/kg$^{1/3}$</td>
<td>COV = 0.260* Mean = 3.83 ms*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_0$</td>
<td></td>
<td>COV = 0.146*</td>
<td></td>
</tr>
</tbody>
</table>

*These values were obtained based on $W = 100$ kg and $R = 4$ m that were used in the case study.

3.2.2 RC Structures

Extensive statistical information on reinforced concrete structure material and geometric attributes are available in a number of reliability studies (Lu et al., 1994; Low and Hao, 2002; Real et al., 2003). The current study obtained material and geometric property information from past investigations of reinforced concrete structures that focused on blast reliability. As a result, random variables that should be considered were included: concrete compressive strength ($f'_c$); steel bar yield strength ($f_y$); concrete modulus of elasticity ($E_c$); steel modulus of elasticity ($E_s$); reinforcement ratio ($\rho$); and section geometry. Compiled statistical information and sources are summarized in Table 3-2, with STD referring to the variable's standard deviation.
Table 3-2. Summary of RC structure random variables and their statistical information

<table>
<thead>
<tr>
<th>Variables</th>
<th>Reference</th>
<th>Mean</th>
<th>COV</th>
<th>STD</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_{c}$ (MPa)</td>
<td>Low and Hao (2002)</td>
<td>$(f'<em>{c})<em>m = (f'</em>{c})<em>k + 1.64\sigma</em>{f'</em>{c}}$</td>
<td>0.12</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td>$E_{c}$ (GPa)</td>
<td>Low and Hao (2002)</td>
<td>$0.043W_{c}^{1.3}\sqrt{(f'_{c})_m}$</td>
<td>0.10</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td>$f_{y}$ (MPa)</td>
<td>Low and Hao (2002)</td>
<td>$(f_{y})<em>m = (f</em>{y})<em>k + 1.64\sigma</em>{f_{y}}$</td>
<td>0.08</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td>Geometry</td>
<td>Low and Hao (2002)</td>
<td>design</td>
<td>0.03</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Lu et al. (1994)</td>
<td>design</td>
<td>0.01</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Real et al. (2003)</td>
<td>design</td>
<td>5 - 10</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td>$\rho$ (%)</td>
<td>Low and Hao (2002)</td>
<td>design</td>
<td>0.03</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Lu et al. (1994)</td>
<td>design</td>
<td>0.09</td>
<td>-</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Hao et al. (2010)</td>
<td>design</td>
<td>0.10</td>
<td>-</td>
<td>normal</td>
</tr>
</tbody>
</table>

*W, normal weight concrete density

In Table 3-2, the subscripts $m$ and $k$ refers parameter mean and characteristic value, respectively, and $\sigma$ represents parameter standard deviation. These findings indicate that the section geometry and reinforcement ratio COVs are highly uncertain. This uncertainty was considered in the current study.
3.3. Finite Element Modeling

Finite element models (FEM) were used in association with MCS for RSM development to provide an accurate estimation of nonlinear response of RC elements subjected to blast. LS-DYNA was selected as the FEM package for the current study because CONWEP is integrated into the package, and because extensive material models are available to help accurately predict reinforced concrete response under varying strain rates. A summary of model construction techniques used for the current study is follows.

3.3.1 Concrete

Concrete is a nonlinear, brittle, heterogeneous material, and its failure characteristics need to be carefully represented in FEM to accurately describe behavior well into the inelastic and plastic ranges. In LS-DYNA, the Karagozian and Case concrete model (KCC) and the Continuous Surface Cap Model (CSCM) have been shown to be appropriate for modeling reinforced concrete members subjected to blast (Coughlin, 2008; Magallanes, 2008; Murray et al., 2007; O’Hare, 2011).

The KCC model, or Material 72 Release 3 in LS-DYNA, is the most recent version of Material 72 developed by Karagozian and Case. This model derives the failure surface by interpolating between two of three independent surfaces, including: a yield surface \( F_y \) indicating where damage begins; a maximum strength surface \( F_m \); and a residual strength surface \( F_r \). Each of these three independent surfaces, denoted as \( F_i \), has a parabolic form defined using experimentally calibrated constant, \( a_{0i}, a_{1i}, \) and \( a_{2i} \) as described by Equation 3-3:

\[
F_i(p) = a_{0i} + \frac{p}{a_{1i} + a_{2i}p} \tag{Eq.3-3}
\]
where $p$ is pressure (Magallanes, 2008). The relationship between these three surfaces occurs when the principal stress reaches the yield surface, at which point damage occurs, then the stress increases to the maximum strength surface and decreases to the residual strength surface. From this phenomenon, the failure surface functions, $F$, for this material model can be defined by Equation 3-4 or 3-5 depending on the assigned plastic strain value, $\lambda$:

$$F = \left[ \eta(\lambda) \left( F_m - F_y \right) + F_y \right] \quad \text{for } 0.0 \leq \lambda \leq 1.0 \quad (\text{Eq.3-4})$$

$$F = \left[ \eta(\lambda) \left( F_m - F_r \right) + F_r \right] \quad \text{for } 1.0 > \lambda \leq 2.0 \quad (\text{Eq.3-5})$$

where $\eta(\lambda)$ is a damage function, which is a function of $\lambda$ and can be defined from experimental data. This function initially increases to unity and then softens to zero as damage is accumulated. Strain rate effects are incorporated by a radial rate enhancement factor, taken from a user-defined curve that includes tensile and compressive dynamic increase factors (Coughlin, 2008). However, for this version of Material 72, most of the failure surface parameters are auto-generated as a function of the unconfined concrete compressive strength, $f'_c$ (LSTC, 2007).

The CSCM model, or Material 159 in LS-DYNA, is a cap model with smooth intersection between the shear yield surface and hardening cap, as shown in Figure 3-1. The initial damage surface coincides with the yield surface. Rate effects are modeled via viscoplasticity (LSTC, 2006). This model uses three invariant yield surfaces to formulate the damage function and a radial return algorithm to represent plastic strain after damage occurs. The function is established as a function of fracture energy, $G_f$, instead of stress, $\sigma$, to eliminate element size dependency (LSTC, 2006).
Both models also include rate effects that are required for blast design, but the CSCM was selected for the current study because it was shown to better predict cracking of a reinforced concrete column subjected to blast loads than the KCC model (Magalleanes, 2008). The model was also deemed suitable for the current study because it required $f'_c$ and the maximum aggregate size as inputs. As a result, concrete properties were easily modified when $f'_c$, one of the selected random variables in the study, changed. The CSCM model requires three-dimensional solid elements (LSTC, 2007). 8-noded hexahedron elements were selected because they were shown to be reliable for modeling concrete behavior (Wissmann, 1984-1986).

3.3.2 Steel Reinforcement

In LS-DYNA, steel reinforcement has been modeled as beam element having an elastoplastic material model that incorporates yielding, hardening, rate effects and plastic strain-based failure parameters (Murray et al., 2007). Two material models, Material 3 and Material 24, were investigated for RC beams under impact loadings and the results for both showed good agreement with experimental data (Murray et al., 2007).
Material 3 is a proper model for isotropic and kinematic hardening plasticity materials under quasi-static and cyclic loadings and it has an option of including rate effects. This model uses a bilinear elastic-plastic profile, as shown in Figure 3-2, to characterize the material nonlinear behavior (LSTC, 2007).

\[
\text{Figure 3-2 Elastic-plastic behavior of steel under uniaxial tension (LSTC, 2007)}
\]

Here \(l_0\) and \(l\) are undeformed and deformed lengths of the specimen, respectively. The modulus of elasticity, \(E\), and tangent modulus, \(E_t\), are used to define the elastic and plastic stages of the material. The strain rate effect is taken into account by using the Cowper and Symonds model to scale the yield stress with the factor of \(1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/P}\), where \(\dot{\varepsilon}\) is strain rate and \(C\) and \(P\) are Cowper and Symonds strain rate parameters. These parameters are set to zero if strain rate effect is not considered. Isotropic and kinematic hardening are specified by setting the hardening parameter, \(\beta\), to 1.0 and 0.0, respectively (LSTC, 2007).

Material 24 is an elasto-plastic material model with an arbitrary stress-strain curve. This model can simulate the behavior of isotropic materials in the plastic region and it includes strain
rate effects in the formulation of its yield function. To implement this material model, the
deviatoric stress, a condition in which the stress components at a point in a body are not the same
in every direction, is updated elastically and checked against the yield function. If it is satisfied,
the deviatoric stresses are accepted (LSTC, 2007). In this model, there is an option to define the
strain rate effects using Cowper and Symonds strain rate model or a user-defined strain rate
effects curve.

For this study, *Material 3* was selected because it provided an adequate representation of
isotropic and kinematic hardening under quasi-static and cyclic loadings, and it has the option of
including rate effects (LSTC, 2007). Beam elements were selected for the reinforcement (Murray
et al., 2007).

In addition to the recommended material model and element types for this study, element
aspect ratios of 1:1:1 and element sizes between 10-15 mm (0.4-0.6 in) were found to be
appropriate for structural solid element modeling. The selected aspect ratio and element size were
uniformly assigned whenever possible to reduce erroneous results (Mohan, 2003). The wall FEM
used in this study is presented in Figure 3-3.
3.3.3 Constraints and Boundary Conditions

A perfect bond between concrete and steel reinforcement is commonly assumed for FEM even though, in reality, some slippage of the reinforcement occurs during failure (Coughlin, 2008; Murray et al., 2007; O’Hare, 2011). A perfect bond can be represented using direct coupling between common concrete and reinforcement element nodes, but this method can lead to meshing complications (O’Hare, 2011). Coupling can also be represented in LS-DYNA without the need of coincident nodes by slaving reinforcement element nodes to their embedding concrete elements, so that they respond with the same velocity and acceleration (Murray et al., 2007; O’Hare, 2011). This technique was used because it was found to be effective for finite element modeling of reinforced concrete structures under blast loads (O’Hare, 2011).

As stated earlier, a cantilevered, RC wall was selected as the structure for the current study. This is a common and basic reinforced concrete structural element, and facilitated easy
implementation of appropriate parameter variations for blast load factor development. A unit width strip of the wall with appropriate boundary conditions was selected in the current study to further simplify FEM that could represent the entire wall while requiring less computational time for parametric runs. Boundary conditions at the wall base consisted of complete restraint of all nodal deformations and continuity boundary conditions, consisting of horizontal translation restraints within the wall plane, were established along both vertical edges of the strip. These boundary conditions are depicted in Figure 3-4.

![Figure 3-4. Wall strip boundary conditions](image)

3.4. Reliability Analysis of Reinforced Concrete Members Nonlinear Responses under Blast

When completing a structural design, one of the most common historical assumptions has been that the structures perform in linearly elastic fashion under given circumstances (Soares et al., 2002). This assumption is not true for blast resistant design. As discussed in Section 3.1, blast resistant design must address inelastic and highly nonlinear structural responses. However, obtaining nonlinear responses of large structural systems is complex when material and geometric nonlinearities are taken into account and, as a result, exact solutions for those responses may be
not available. Therefore, approximate analytical solutions, such as using the Single-Degree of Freedom (SDOF) method, are employed.

The SDOF in conjunction with Monte Carlo simulation have shown to be an acceptable solution for reinforced concrete member reliability analyses under blast loads. Acito et al. (2011) performed reliability analyses of a reinforced concrete beam subjected to explosive loadings using SDOF and Monte Carlo simulation. Maximum displacement was considered as the reliability measure when the randomness of blast loads, material properties, and dimensional imperfection were taken into account. Acito et al. (2011) found that the reliability of the beam as a function of its displacement limit depended on the slenderness and the span length of the beam. The beam was less reliable if its slenderness increased or its span length decreased.

Though the SDOF method is capable of estimating nonlinear dynamic response, it is generally suitable for simple, symmetric structural elements in bending and can determine only maximum deformation from the first response peak (Acito et al., 2011). Instead of using SDOF, nonlinear response can be obtained using computational tools such as finite element analyses (FEA) that can consider material and geometric nonlinearities of reinforced concrete members including: stiffness reductions due to cracking; softening effects after reaching the reinforced concrete member’s ultimate strength; and tension stiffening of the reinforcement (Val et al., 1997). These nonlinearities can be modeled using commercial finite element modeling (FEM) programs such as ABAQUS, ANSYS, and LS-DYNA, and results from these packages have demonstrated a good agreement with the experimental results for blast analyses (Rong and Li, 2007).

When performing reliability analyses for structures that respond in a nonlinear fashion, many studies have implemented FEM software in conjunction with common reliability approaches (i.e. FORM or SORM) to determine structure failure probabilities (Soares et al., 2002). FEM has been utilized to determine the nonlinear resistance \( R \) and load effects \( Q \) for a
particular combination of variables that were randomly sampled from appropriate distributions and multiple FEM simulations were used to build a resistance or load effect distribution (Dar et al., 2002). The sampling methods are largely based on Monte Carlo simulation technique, where the occurrence probability for each random variable is randomly generated and transformed to its corresponding random variable value based on statistical information (i.e. mean, COV, distribution type). However, the accuracy of Monte Carlo simulation is dependent on the sample size or the number of numbers generated for each random variable (Haldar and Mahadevan, 2000) and using Monte Carlo simulation leads to huge computational costs and effort due to large sample sizes being needed for accurate probabilistic results (Soares et al., 2002).

Instead of performing a large number of simulations, a less complex and more efficient analytical model, as known as metamodelling, has been implemented in typical reliability analyses (e.g. FORM, SORM) to represent a nonlinear structural response function in terms of associated random variables using a response surface approach. The response surface metamodels (RSM) have been used in conjunction with FEA and Monte Carlo simulation in nonlinear probabilistic analysis situations (Soares et al., 2002; Rong and Li, 2007; Kappos et al., 1999). RSM provides an estimated closed-form function for nonlinear responses in terms of inputs that are formulated using linear, quadratic, or polynomial functions. The response surface is basically derived from FEA results but uses fewer simulations to construct when compared against using more traditional Monte Carlo simulation in conjunction with FEA.

Rong and Li (2007) investigated the probabilistic response of simply supported, reinforced concrete flexural members subjected to blast loadings. 2,000 random variable samples were simulated to ensure accuracy of the statistical analysis. However, only 500 design cases were selected for deriving the response surface function that represented an explicit form for the nonlinear displacement of a beam subjected to blast loads. The resulting nonlinear displacements calculated from the response functions showed good agreement when compared to experiments.
Kappos et al. (1999) used RSM combined with Monte Carlo simulation to investigate the strength and ductility uncertainty of reinforced concrete beams and columns when subjected to blast loads. RSM was implemented because of the absence of suitable closed-form solutions for reinforced concrete member nonlinear behavior. Three random variables were considered, including concrete compressive strength, \( f'c \), steel yield strength, \( f_y \), and the ultimate strain, \( \varepsilon_{su} \). A total of 27 different combinations of the extreme values for these variables were selected to derive appropriate response functions and 1,000 simulations were performed to determine strength and ductility variations. Experimental and analytical results were compared and results showed that the bias ratio varied from 0.99 to 1.02. These results indicated good accuracy for the developed, simplified, response function.

3.5. The RSM and MCS Method

Difficulties associated with development of reliable air blast load factors, as stated earlier, arise from complexities associated with predicting structural response under blast coupled with a lack of known probability density functions that accurately depict response variability. To overcome these issues, the current study proposed an approach to determine air blast load factors using a combination of RSM and MCS. RSM utilizes a combination of FEM and Design of Experiments (DOE) to develop an appropriate set of varied finite element models whose responses were gathered and used to: 1) determine significant parameters to the response, called parameter screening; and 2) establish explicit prediction functions of inelastic structural response for the RC, cantilevered wall under blast. MCS was then implemented with RSM to simulate empirical distributions of wall response and to determine load factors when this response was known to be non-normally distributed. Backgrounds behind both RSM development and MCS implementation will be briefly discussed in the following sections.
3.5.1 Response Surface Metamodel (RSM)

RSM has been recommended for nonlinear reliability analyses of structural systems when their explicit failure formulas are not available (Soares et al., 2002). This technique has shown its robustness when compared to other methods because it can significantly reduce computational costs, while still providing accurate results (Soares et al., 2002; Kappos et al., 1999; Seo, 2009). RSM is generally represented by first- or second-order polynomial equations. A first order equation, as shown in Equation 3-6, is recommended for investigating parameter relationships or to assist with parameter screening. A second-order polynomial equation, as shown in Equation 3-7, is recommended for predicting nonlinear responses from structures subjected to blast (Seo, 2009; Bradley, 2007; Towashiraporn, 2004):

\[
y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \epsilon \quad \text{(Eq. 3-6)}
\]

\[
y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^{k} \beta_{ij} x_i x_j + \epsilon \quad \text{(Eq. 3-7)}
\]

where: \( y \) is the dependent response surface (i.e. nonlinear displacement in this study); \((x_i, x_j)\) are independent random variables (i.e. strength, load); \((\beta_0, \beta_i, \beta_{ii}, \beta_{ij})\) are coefficients to be estimated; \( k \) is the number of random variables; and \( \epsilon \) is the total error.

The relationship between random variables, \( x \), and response, \( y \), can be determined using a regression model. If there are more than two random variables multiple-regression model is used and is identical to Equations 3-6 and 3-7 for first order and second-order response surfaces, respectively. The general matrix form of the multiple-regression model is shown in Equation 3-8.

\[
[Y] = [X][\beta] + [\epsilon] \quad \text{(Eq. 3-8)}
\]
where

- $Y$ = a vector of $n$ observations,
- $X$ = a matrix of levels of independent variables,
- $\beta$ = a vector of regression coefficients, and
- $\epsilon$ = a vector of random errors (Montgomery, 2005).

For an example, the matrix $Y$, $X$, $\beta$, and $\epsilon$ for a first-order response model with $k$ random variables and $n$ observations can be represented by Equation 3-9 to 3-12, respectively.

\[
Y = \begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{bmatrix} \quad \text{(Eq. 3-9)}
\]

\[
X = \begin{bmatrix}
1 & x_{11} & x_{12} & \cdots & x_{1k} \\
1 & x_{21} & x_{22} & \cdots & x_{2k} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_{n1} & x_{n2} & \cdots & x_{nk}
\end{bmatrix} \quad \text{(Eq. 3-10)}
\]

\[
\beta = \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_n
\end{bmatrix} \quad \text{(Eq. 3-11)}
\]

\[
\epsilon = \begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\vdots \\
\epsilon_n
\end{bmatrix} \quad \text{(Eq. 3-12)}
\]

If the number of observations equals the number of random variables ($n = k$), then Equation 3-8 has an estimated least square solution, $\hat{\beta}$, as shown in Equation 3-13 (Bradley, 2007). This solution can be used to estimate $\beta$ and, in turn, estimate the response surface function.
\[ \hat{\beta} = (X'X)^{-1}X'Y \]  
(Eq. 3-13)

Once the estimated \( \hat{\beta} \) matrix is obtained, the response surface function can be determined using Equations 3-6 or 3-7 based on the complexity of the anticipated failure surface and this estimated response function is compared to the observation results to evaluate its accuracy. There are many statistical methods can be used to evaluate adequacy but the simplest measure when using deterministic computer experiments is via the coefficient of determination \( (R^2) \) as shown in Equation 3-14 (Seo, 2009).

\[ R^2 = \frac{SSR}{SST} \]  
(Eq. 3-14)

where

- \( SSR = \hat{\beta}'X'Y - \frac{(\sum_{i=1}^{n}y_i)^2}{n} \) is the error sum of squares, and
- \( SST = Y'Y - \frac{(\sum_{i=1}^{n}y_i)^2}{n} \) is the total sum of squares.

\( R^2 \) lies in the interval \([0,1]\) and a better estimation of a response surface function is indicated when \( R^2 \) is closer to 1. However, taking one more variable into account always increases \( R^2 \), regardless of whether or not that variable is statistically significant (Bradley, 2007).

3.5.2. Parameter Screening

The RSM method began with gathering all variable parameters of interest and their statistical information, as discussed in Sections 3.2.1 and 3.2.2. Significant parameters were then determined using screening procedures, which were a combination of a ranked-ordered output yield method that identified a Pareto optimal solution (Montgomery, 2005) and best parameter subset selection adopted for multiple linear regression (Kutner et al., 2004). The first method
provided parameters whose variation significantly influenced the response variation and the latter
provided parameters whose variation significantly influenced model accuracy. In the Pareto
analysis, relationships between the parameters and the response item of interest which, for the
current study was the maximum wall displacement, were represented through a first-order
regression equation, Equation 3-3. This equation was developed from experimental results
obtained using Plackett-Burman Design (PBD), a method commonly used for first-order response
function development that requires fewer experimental runs than Full Factorial Design (FFD) to
study main effects of parameters (Wu and Hamada, 2000). PBD always has a run number, \( n \), that
is a multiple of four (e.g. 12, 16, 20, …) with the minimum \( n \) being 12. The proper \( n \) is based on
the number of parameters, \( k \), which must be less than \( n \). For example, a design with \( n = 12 \) runs
can be used to estimate the main effects for \( k \) up to 11 parameters.

Using PBD, the matrix \( X \) in Equations 3-8 and 3-10 which has a size of \( n \times k \) was
constructed by specifically assigning the values of -1 and +1, where -1 and +1 represent
parameter lowest and highest values, respectively (Nair and Pregibon, 1988). For an example
of \( n = 12 \) and \( k = 8 \) which is a case study in this study, the first column of the matrix \( X \) was
assigned with a specific set of -1 and +1 given by Nair and Pregibon (1988) as shown in
Figure 3-5 (a) and then these -1 and +1 values were replicated to the next column but their
positions were changed by sliding each cell down one step while the last cell was moved to the
first cell as shown in Figure 3-5 (b). This process was repeated for the other columns to get an \((n
- 1) \times k \) matrix as shown in Figure 3-5 (c) and then an additional row containing values equal to -
1 was added at the bottom of the matrix to complete the experimental design as shown in Figure
3-5 (d).
Figure 3-5. Matrix $X$ construction for PBD when $n = 12$ and $k = 8$
The PBD $X$ matrix that contained a set of parameter combinations for finite element models, and the subsequent response matrix, $Y$, were then regressed to determine appropriate parameter coefficients in the $\beta$ matrix.

Once the $\beta$ matrix was obtained, $\beta_k$ of each parameter were multiplied by the parameter’s standard deviations to obtain scaled estimates used to represent their comparative contribution to the response. These scaled estimates were then normalized to the sum of all scaled estimates and plotted in a Pareto chart. Significant parameters were selected from a parameter set that provided a cumulative scaled estimate of at least 80% (Seo, 2009; Towashiraporn, 2004). Next, using the best subset method, candidate parameter subsets according to specific criteria, such as the maximum coefficient of determination ($R^2$), the minimum total mean squared error ($C_p$), or the minimum standard deviation of error ($S$ or $SE$), were obtained from all possible subsets formed from the pool of all parameters. The best parameter subsets, which could be greater than one, were chosen from those that were mostly consistent with the criteria. Finally, the finalized significant parameter set was chosen from the best parameter subset, which was consistent with the parameters obtained from the Pareto method.
3.5.3 Regression Models for Nonlinear Responses

A nonlinear response prediction equation, again focusing on the maximum wall displacement, was then developed using the significant parameters. Central Composite Design (CCD) was employed to develop an experimental design for gathering FEM response data that corresponds to the CCD inputs (Seo, 2009; Towashiraporn, 2004) and the resulting RSM was obtained by regressing inputs and outputs. CCD is a popular of second-order experimental design method. It consists of a complete \(2^k\) factorial design with additional design points. CCD can be depicted graphically by creating \(k\) axes that represent each random value space and then plotting all \(2^k\) design points. Additional design points are located at a distance of \(\pm \alpha\) from the design center point \((n_0)\), a point containing all random variable medians and coded as 0, on each variable axis. The last design point is located at the center points. Thus, the total number of points in a CCD is \(n = 2^k + 2k + n_0\). The value of \(n_0\) provides the orthogonality property, where \(X'X\) is diagonal, and has uniform precision. The value of \(\alpha\) provides rotatability of the CCD that ensures the variance of the estimated response is constant and small at a fixed distance from the center point (Seo, 2009). The value of \(\alpha\) is recommended to equal to \((2^k)^{1/4}\) to cause CCD to be rotatable (Khuri and Mukhopadhyay, 2010; Seo, 2009). However, \(\alpha = 1\) can be used if random variables are independent of each other with good prediction (Wu and Hamada, 2000; Towashiraporn, 2004). Figure 3-6 demonstrates CCD design points for \(k = 2\) and \(\alpha = 1\).
Figure 3-6. Design points of the CCD for $k = 2$ and $\alpha = 1$

The CCD parameter combinations and their subsequent FEM responses were then regressed to determine an appropriate RSM. Statistical methods have been used to evaluate RSM accuracy when experimental error is anticipated. However, in the case of RSMs developed in conjunction with finite element models, which are deterministic and have no experimental error, these methods are not recommended. In such case, RSM accuracy can be evaluated using Average Absolute Error ($%\text{AvgErr}$), the Maximum Absolute Error ($%\text{MaxErr}$), and the Root Mean Square Error ($%\text{RMSE}$) (Bradley, 2007) as expressed in Equation 3-15 to 3-17.

\[
%\text{AvgErr} = 100 \cdot \frac{\frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|}{\frac{1}{n} \sum_{i=1}^{n} y_i} 
\]  
(Eq. 3-15)

\[
%\text{MaxErr} = \text{Max} \left[ 100 \cdot \frac{1}{\frac{1}{n} \sum_{i=1}^{n} y_i} \left| y_i - \hat{y}_i \right| \right] 
\]  
(Eq. 3-16)

\[
%\text{RMSE} = 100 \cdot \frac{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\frac{1}{n} \sum_{i=1}^{n} y_i} 
\]  
(Eq. 3-17)

where $y_i$ and $\hat{y}_i$ are the observed and predicted responses, respectively, from each experimental run and $n$ is the number of experimental runs.
3.5.4. Air Blast Load Factor Development

MCS is a fundamental approach that can be used to determine the failure probability \( P_f \) of a structural system when derivation of analytical joint probability function of its performance function, \( g = R - S \), is impossible (Allen et al., 2005), where RSM represented \( S \) in this study. Failure is defined where \( R < S \) or \( g < 0 \), therefore the failure probability can be determined by counting events in MCS that \( g < 0 \), called \( N_f \), and by determining \( P_f \) as \( N_f/N \), where \( N \) is total simulated data of each parameter. The accuracy of the estimated \( P_f \) depends on \( N \), with estimates approaching a true \( P_f \) as \( N \) approaches infinity. Failure probability accuracy can be evaluated using COV of the estimated \( P_f \) (Ayyub and Haldar, 1985). The required \( N \) is defined by a point at which \( P_f \) begins to stabilize and its COV is small (less than 0.01 is recommended) (Johnson and Ayyub, 1992).

In the context of structural reliability, \( P_f \) is generally converted to a reliability index called \( \beta \), where \(-\beta\) equals to a standard normal value, \( z \), that provides a probability equal to \( P_f \), as shown in Equation 3-18.

\[-\beta = z = \Phi^{-1}(P_f) \quad \text{(Eq. 3-18)}\]

where \( \Phi^{-1} \) is the inverse normal function. Trial \( \gamma \) was applied to each simulated data \( S \) if \( \beta \) was not equal to the target \( \beta (\beta_T) \), and the new \( \beta \) was re-calculated. The trial process was repeated until the finalized \( \gamma \) resulted \( \beta_T \).

The proposed method discussed in this section is summarized in a flowchart as shown Figure 3-7:
3.6. Case Study: Blast-Resistant, Cantilevered RC Wall

A 3-m high cantilevered, RC wall having a design $f'_c$ of 28 MPa and a design $f_y$ of 415 MPa was chosen as the case for the current study. It was selected to demonstrate proposed method implementation for determining a probabilistic blast load factor because it has regular material and geometry attributes. The wall was subjected to blast pressure from an explosive charge weight equivalent to 100 kg of TNT at 4 m standoff distance giving a $Z$ of 0.861 m/kg$^{1/3}$, which is in the range that most empirically based computer packages can accurately estimate blast loads (0.50 – 10 m/kg$^{1/3}$) (Hao, et al., 2010).
3.6.1. Blast Resistant Design and Finite Element Modeling

For this case study, design of the wall was accomplished using an initial analysis involving a SDOF, elastic model having an equivalent mass and stiffness (Cormie et al., 2005) with an equation of motion applied that represented the total structural response of the wall. Dynamic Increase Factors (DIFs) associated with applicable failure modes which, for the current study, consisted solely of flexural failure at the wall base, were applied to material strengths to account for strain rate effects.

The blast load and standoff distance were modeled as a triangular pressure-time history applied to the SDOF model with a peak over pressure, $P_{\text{so}}$, of 1,931 kPa and positive phase duration, $t_d$, of 4.83 ms that produced an impulse of 5,030 kPa-ms. These values were obtained from empirical curves in UFC 3-340-02 (Department of Defense, 2008) for a $Z$ of 0.861 m/kg$^{1/3}$.

Given that plastic deformation is expected, support rotation, $\theta$, and member ductility, $\mu$, are recommended as governing design criteria (Dusenberry, 2010) to control damage levels established using support rotation limits presented in Table 3-3 (Department of Defense, 2008; Cormie et al., 2005).

Table 3-3. Reinforced concrete support rotation limits for different blast protection levels (Department of Defense, 2008; Cormie et al., 2005)

<table>
<thead>
<tr>
<th>Protection levels</th>
<th>Support rotations limits, $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protection of personnel and equipment</td>
<td>$2^\circ$</td>
</tr>
<tr>
<td>Protection of structural elements from collapse</td>
<td>$4^\circ$</td>
</tr>
</tbody>
</table>
For $\theta$ less than 2°, concrete surfaces are assumed to remain intact and the structural element can still provide additional moment capacity. For $\theta$ between 2° and 4°, concrete is assumed to have been crushed, and any additional compression forces are resisted by the reinforcement. Sufficient compression reinforcement is required for this rotation limit to fully develop full tension steel capacity. For $\theta$ greater than 4°, reinforced concrete elements experience large plastic deformation and lose their stability (Cormie et al., 2005). In this study, the maximum support rotation of 4° was chosen as the design criteria because the wall is a non-residential structure, which can be permitted to sustain plastic deformations without collapsing.

The aforementioned analysis and design criteria were used to design a blast resistant wall section, with the final section shown in Figure 3-8. Similar amounts of longitudinal reinforcement were used for both faces of the wall to guarantee that there was no compression failure when $\theta > 2°$. Adequate shear reinforcement was added to the section to prevent shear failure. Minimum reinforcement (ACI, 2008) was added in the horizontal direction of the wall to prevent rupture due to out-of-plane bending. More details about the wall design are presented in Appendix B-1.

Figure 3-8. (a) Wall geometry (b) Wall cross-section
3.6.2 FEM Validation

Finite element models are commonly validated via comparisons to applicable experimental results, if possible. However, due to the lack of blast tests of a wall similar to that shown in Figure 3-8, theoretical response comparisons were used to validate the FEM. This method has been shown to be an acceptable alternative for validation of FEM when experimental data is not available (O’Hare, 2011). Moment-curvature and moment-deflection relationships were chosen as response comparisons used to ascertain model accuracy. These quantities were selected to assist with validation because they permit examination of member ductility well into the plastic range (Wight and MacGregor, 2009). Validation data was supplied from FEM’s sectional curvature at the model base and the maximum displacement at the wall top under quasi-static loadings. However, it should be stated that, under quasi-static loads, rate-dependent responses resulting from blast could not be validated. Therefore, to investigate rate effects on model accuracy, techniques used to create the wall finite element model were used to model a panel that was tested under blast and had results published. Experimental data from this study was compared to FEM predictions.

Theoretical moment-curvature ($M$-$\Phi$) relationships for the RC, cantilevered wall shown in Figure 3-8 were calculated using an elastic, perfectly-plastic steel material model shown in Figure 3-9 and a parabolic concrete material model having a linear softening function (Hognestad’s model) (Hognestad, 1952) shown in Figure 3-10.
Basic assumptions in flexure theory for reinforced concrete design were applied to the $M$-$\Phi$ calculations. Two important points in $M$-$\Phi$ plots, the initiation of cracking and yielding, and resulting member behavior at those points were observed. Theoretically, a RC member will exhibit linearly elastic behavior up to the yielding point with $M$-$\Phi$ slope changing at the initiation of cracking where member stiffness changes. Cracking in a flexural concrete member occurs
when stress in the extreme tensile fiber reaches the modulus of rupture for the concrete, $f_r$, which equal to $0.6\sqrt{f_c}$. The moment that causes cracking, $M_{cr}$, can be determined using elastic theory as expressed in Equation 3-19.

$$ M_{cr} = \frac{f_r A_g}{y_t} \tag{Eq. 3-19} $$

where $I_g$ is the moment of inertia for the gross section and $y_t$ is the distance from the section centroid to the extreme tension fiber.

To calculate the yielding moment ($M_y$), section equilibrium is used instead of an elastic method to overcome sectional stiffness changes that occur due to cracking. First, the strain in the tension steel is set equal to the yield strain ($\varepsilon_y$), which is determined using Equation 3-20.

$$ \varepsilon_y = \frac{f_y}{E_s} \tag{Eq. 3-20} $$

where $f_y$ is the yielding stress of steel reinforcement and $E_s$ is the steel modulus of elasticity. The section’s neutral axis is adjusted until equilibrium is established with concrete’s contribution to the tensile resistance being excluded. After section equilibrium is established, $M_y$ is calculated as the sum of the moment of the internal forces about the neutral axis (Wight and MacGregor, 2009).

The section will then exhibit plastic deformation after yielding, until the maximum concrete compressive strain of 0.003 (Wight and MacGregor, 2009) is reached. For the plastic region, the internal moment and section curvature are determined using a section equilibrium method that incorporates the steel and concrete material models shown in Figure 3-8 and 3-9.
In the current study, the $M\cdot\Phi$ relationships were obtained from the curvatures at the base of the wall FEM models under three quasi-static lateral loadings, 133, 152, and 171 kN, that were applied at the top (free end) of the wall. A maximum load of 152 kN was determined from the wall moment capacity of $456 \times 10^6$ N-mm while the other two loadings were arbitrarily selected to be 12.5% below and above 152 kN to examine wall behavior in the elastic and plastic ranges. Resulting FEM $M\cdot\Phi$ relationships are compared to those from the theoretical model and shown in Figure 3-11.

![Figure 3-11. $M\cdot\Phi$ relationships from theoretical calculations and FEMs](image)

It can be observed from Figure 3-11 that FEM $M\cdot\Phi$ relationships are identical to the theoretical values in the uncracked and cracked elastic regions. As expected, the wall with a lateral load of 133 kN behaved elastically because its maximum moment is less than the anticipated yielding moment. The walls subjected lateral loads of 152 and 171 kN behaved...
elastically up to their yielding points and then behaved plastically thereafter. The FEMs for these load cases demonstrated slightly higher moment capacity in the plastic region when compared to theory. The increased moment capacity was presumed to result from additional stiffness provided by compression reinforcement in the FEM, which was not included in the theoretical calculations, and rate-stiffening effects in FEM constitutive models that strengthened the wall section and were not considered in the theoretical model.

The moment-deflection \((M-\Delta)\) relationships for the RC, cantilevered wall were also investigated under the same loadings and results shown in Figure 3-12 to 3-14. The comparisons consist of three regions: uncracked-elastic; cracked-elastic; and plastic. Similarly to the \(M-\Phi\) relationships, two important points in \(M-\Delta\) plots, cracking and yielding, and resulting member’s behavior were observed. The cracking and yielding moments were calculated as discussed earlier. Theoretical deflections in the uncracked-elastic region were calculated using elastic theory using a constant gross, cross sectional moment of inertia, \(I_g\).

Theoretical \(M-\Delta\) relationships transitioned to a cracked-elastic region when the concrete reached its cracking limit. Deflections were calculated using an effective moment of inertia, \(I_e\), that accounts for changing stiffness between the uncracked and cracked sections. Branson (1971) and Bischoff (2005, 2007) published equations to calculate \(I_e\), expressed in Equations 3-21 and 3-22, respectively, and they both were used to determine deflections. Theoretical \(M-\Delta\) relationships for this region were still linear with a reduced stiffness until reinforcement started yielding, and thereafter plastic deformation was assumed.

\[
I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[ 1 - \left(\frac{M_{cr}}{M_a}\right)^3 \right] I_{cr} \leq I_g \quad \text{(Eq. 3-21)}
\]
\[ I_e = \frac{I_{cr}}{1 - \eta (\frac{M_{cr}}{M_a})^2} \leq I_g \]  

(Eq. 3-22)

where

\( I_g \) = gross uncracked moment of inertia,

\( I_{cr} \) = cracked transformed moment of inertia,

\( M_{cr} \) = cracking moment,

\( M_a \) = applied load moment, and

\( \eta = 1 - I_{cr} / I_g \).

For a cantilevered wall used in this study, the theoretical maximum deflections (\( \Delta_{\text{max}} \)) in uncracked-elastic and cracked-elastic region were then calculated using an elastic deflection equation as shown Equation 3-23. However, it should be noted that the application of \( I_e \) either from Branson (1971) or Bischoff (2005, 2007) is limited to the elastic range and, as a result, there was no theoretical deflection could be obtained beyond the yielding points in all three cases.

\[ \Delta_{\text{max}} = \frac{1}{3} \cdot \frac{M_{neg} h^2}{E_c l} \]  

(Eq. 3-23)

where: \( M_{neg} \) = moment at the wall base;

\( h \) = the wall height;

\( E_c \) = concrete modulus of elasticity = 4,500\( \sqrt{f'_c} \) (MPa); and

\( I = I_e \) for uncracked-elastic region or \( I_e \) for cracked-elastic region.
Figure 3-12. $M$-$\Delta$ relationships from theoretical calculations and FEMs at $P=133$ kN

Figure 3-13. $M$-$\Delta$ relationships from theoretical calculations and FEMs at $P=152$ kN
Figure 3-14. $M$-$\Delta$ relationships from theoretical calculations and FEMs at $P=177$ kN

Again, FEM $M$-$\Delta$ relationships from the three load levels show that their uncracked, elastic regions are identical to theory while their cracked, elastic regions show slightly more ductility than the theoretical predictions. However, this difference alludes to the fact that the FEM $M$-$\Delta$ relationships for this region could be close to the realistic wall response because the deflections calculated using $I_e$ from Equations 3-21 and 3-22 were found to be slightly under-predicted (Bischoff and Gross, 2011). In similar fashion to the $M$-$\Phi$ plots, it also can be observed that the FEM with an applied 133 kN load performs elastically while FEMs with applied 152 and 171 loads behaved plastically after they reached yield.

To further validate FEM effectiveness when modeling structures subjected to air blasts, data from an experimental study of blast resistant RC panels by El-Dakhakhni et al. (2010) was examined and compared to FEM predictions. A 1,000 x 1,000 x 70 mm RC panel reinforced having concrete with a compressive strength of 42 MPa and reinforced with a welded steel mesh
with individual wires having cross-sectional areas of 25.8 mm$^2$ and a yield strength of 480 MPa was modeled in LS-DYNA following techniques discussed in Section 3.3. The panel was subjected to an explosive equivalent to 29.1 kg TNT located 3.0 m from the panel center. The resulting maximum displacement and damage from the FEM were observed and compared with those from El-Dakhakhni et al. (2010). The FEM maximum displacement was 13.1 mm while the reported maximum experimental displacement was 12.1 mm. Figure 3-15 shows a comparison between the final damage state for the panel from the blast test and that obtained from the FEM and good qualitative agreement is shown.

![Figure 3-15. Test-observed damage (El-Dakhakhni et al., 2010) and FEM predicted damage of the RC panel](image)

3.6.3. Parameter Screening

Significant parameters were determined using the Pareto method as discussed in Section 3.5.2. Table 3-4 summarizes statistical information and chosen minimum (-1) and maximum (+1) values of parameters discussed in Section 3.2 that were used to develop the PBD for this screening procedure. Variations for $W$ and $R$ were converted from $Pr_{\text{max}}$ and $t_o$ means, standard
deviations, and COVs obtained from Low and Hao (2002), Bogosian et al. (2002), and Hao et al. (2010) and their distributions were assumed to be uniform due to a lack of their statistical information. For normal distributed parameters shown in Table 3-2, -1 and +1 were approximated at two standard deviations above and below parameter means to cover standard 95% probability of the parameter variation.

Table 3-4. PBD Statistical Information

<table>
<thead>
<tr>
<th>Variables</th>
<th>Nominal</th>
<th>Mean</th>
<th>COV</th>
<th>STD</th>
<th>-1</th>
<th>0</th>
<th>+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_c$ (MPa)</td>
<td>28</td>
<td>34.86</td>
<td>0.12</td>
<td>4.18</td>
<td>26.66</td>
<td>34.86</td>
<td>43.06</td>
</tr>
<tr>
<td>$f_y$ (MPa)</td>
<td>415</td>
<td>477.67</td>
<td>0.08</td>
<td>38.21</td>
<td>402.77</td>
<td>477.67</td>
<td>552.57</td>
</tr>
<tr>
<td>$E_s$ (MPa)</td>
<td>210</td>
<td>210</td>
<td>0.025</td>
<td>5.25</td>
<td>199.71</td>
<td>210</td>
<td>220.29</td>
</tr>
<tr>
<td>$d$ (mm)</td>
<td>500</td>
<td>500</td>
<td>0.03</td>
<td>15</td>
<td>470.60</td>
<td>500</td>
<td>529.40</td>
</tr>
<tr>
<td>$H$ (mm)</td>
<td>3,000</td>
<td>3,000</td>
<td>-</td>
<td>90</td>
<td>2,823.60</td>
<td>3,000</td>
<td>3,176.40</td>
</tr>
<tr>
<td>$\rho$ (%)</td>
<td>0.50</td>
<td>0.50</td>
<td>-</td>
<td>0.050</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>$W$ (kg)</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>38.99</td>
<td>33.55</td>
<td>111.69</td>
<td>189.83</td>
</tr>
<tr>
<td>$R$ (m)</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>0.69</td>
<td>3.23</td>
<td>4.50</td>
<td>5.76</td>
</tr>
</tbody>
</table>

Parameter combinations and resulting maximum displacements from the analyses, summarized in Table A-1, were then used with linear regression to produce the resulting equation, Equation 3-24, that was used to determine each parameter significance.

$$\Delta_{max} = -318 + 1.61f'_c - 0.199f_y - 2.65E_s + 0.125d + 0.184H + 96.204\rho + 1.27W - 42.7R$$  
(Eq. 3-24)
It should be noted that the $R^2$ of this model was 88.6%, which has been shown to be acceptable for explaining the influence of multiple parameters on a calculated value (Towashiraporn, 2004). Each parameter contribution to maximum deflection was then determined from its scaled response as discussed in Section 3.5.2. All scaled responses and their normalized values are plotted in the Pareto chart shown in Figure 3-16.

![Pareto chart of scaled responses](image)

**Figure 3-16.** Pareto chart of scaled responses

Significant parameters are those that contributed at least 80% to the response (Seo, 2009; Towashiraporn, 2004). Figure 3-16 indicates that $W$, $R$, and $H$ are needed to obtain at least an 80% influence level on the maximum displacements (represented by the horizontal, dashed line).

Additional parameters required for an acceptable RSM accuracy level were then determined using the best subset method as discussed in Section 3.5.2. $E_s$ and $\rho$ were found to be the parameters that should be included in the regression model to have the maximum $R^2$ of 86.0%.
and the minimum $SE$ of 66.62 mm. Therefore, parameters $W, R, H, E$, and $\rho$ were then used for a nonlinear RSM construction for blast load factor development.

### 3.6.4 Blast Resistant Wall RSMs

The wall RSM in this section was regressed from the CCD parameter combinations and the subsequent FEM results of each combination that are summarized in Table A-2. A polynomial regression model as shown in Equation 3-7 was chosen for this section because the prediction function required more accuracy than Equation 3-24, an equation which could provide only the main effects of parameters on the response. Minimum (-1), maximum (+1), and middle values (0) for each parameter in Table 3-4 resulted in 43 CCD runs, which composed of $2^5 = 32$ factorial design points, $2 \times 5 = 10$ axial points, and one center point. LS-DYNA FEMs for these 43 cases were then developed and analyzed, and maximum displacement and model inputs were used to develop the polynomial regression function shown in Equation 3-25 for predicting the wall maximum displacement under blast:

$$
A_{\text{max}} = -46 + 2.750 W - 99.4 R + 1.2 E_s + 119 h - 324 \rho + 0.00286 W^2 + 8.35 R^2 - 0.0020 E_s^2 - 5.0 h^2 + 279 \rho^2 - 0.5229 R*W - 0.00052 E_s*R + 0.039 E_s^*W + 0.503 h^*W - 3.59 h^*R - 0.174 h^*E_s - 2.071 \rho^*W + 68.20 \rho^*R - 74.7 \rho^*h
$$

(Eq. 3-25)

$R^2$ for this regression function was 99.9% and it was further validated via comparisons to an additional 43 finite element analyses that contained randomly generated parameter sets. The corresponding maximum displacements from each model were compared with the predicted maximum using Equation 3-25. The parameter combinations, the corresponding LS-DYNA
maximum displacement, and the predicted maximum displacement are summarized in Table A-3. Model accuracy measures were then determined and are summarized in Table 3-5, with small percentage errors being shown. Therefore, Equation 3-25 is deemed accurate for FEM maximum displacement prediction and it was then used in association with MCS to determine a probabilistic air blast load factor. That process is summarized in the following section.

Table 3-5. Summary of RSM Accuracy Measures

<table>
<thead>
<tr>
<th>Measures</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>%AvgErr</td>
<td>1.64%</td>
</tr>
<tr>
<td>%MaxErr</td>
<td>6.58%</td>
</tr>
<tr>
<td>%RMSE</td>
<td>2.38%</td>
</tr>
</tbody>
</table>

3.6.5 Blast Load Factor Determination using RSM and MCS

To determine blast load factor, reliability analyses were performed using the performance function $g = R - S$. In this study, $R$ was defined as deterministic and set equal to a maximum displacement of 210 mm, based on a maximum of $4^\circ$ of rotation at the wall base as discussed in Section 3.6.1 and $S$ was the maximum displacement obtained from MCS using Equation 3-25 with $W$, $R$, $E_s$, $H$, and $\rho$ being randomly generated based on their underlying distributions shown in Table 3-1 and 3-2. Required simulation cycles, as discussed in Section 3.5.4 were determined first. MCSs involving $10^3$, $10^4$, $2\times10^4$, $5\times10^4$, and $10^5$ simulation cycles were completed and $P_f$ was determined for each simulation set. Resulting failure probabilities are summarized in Table 3-6.
Table 3-6. MCS Failure Probabilities

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$P_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1K</td>
<td>0.1930</td>
</tr>
<tr>
<td>10K</td>
<td>0.1692</td>
</tr>
<tr>
<td>20K</td>
<td>0.1677</td>
</tr>
<tr>
<td>50K</td>
<td>0.1679</td>
</tr>
<tr>
<td>100K</td>
<td>0.1680</td>
</tr>
</tbody>
</table>

The COV for each $P_f$ in Table 3-6 was calculated and plotted in Figure 3-17. It can be observed that all simulation sets provide good accuracy, with COVs less than 0.1. MCS using $5 \times 10^4$ simulations were selected for future work related to blast load factor determination, because its COV equaled 0.01. CDFs for $g = R - S$ obtained from $5 \times 10^4$ MCSs were calculated and are shown in Figure 3-18. They indicate that the probability of the wall maximum displacement greater than the 210 mm criteria or $P_f$ is 0.1679.

![Figure 3-17. MCS failure probability COVs](image-url)
The reliability index, $\beta$, was determined using Equation 3-18 for a $P_f$ of 0.1679 obtained from the $5 \times 10^4$ MCSs. $\beta$ was found to be 0.962, much less than the recommended $\beta$ of 3.5 for flexural design (ACI, 2008; Allen et al., 2005). As expected, a blast load factor, $\gamma_B$, was needed to obtain a higher reliability level for the wall that was studied. Trial blast load factors were applied to the $5 \times 10^4$ MCSs and $P_f$ and $\beta$ were recalculated for each trial factor, as described with the flowchart shown in Figure 3-6. This process was repeated until $\beta$ equaled 3.5 and the resulting $\gamma_B$ was found to equal 1.41.

3.6.6. Blast Load Factor Validation

To examine the validity of the derived load factor, the RC wall was redesigned by using the same input parameters except applying the load factor of 1.41 to an impulsive energy obtained from $W$ equal to 100 kg of TNT and $R$ equal to 4 m. This factored impulse was then used to redesign the wall using the SDOF method. Redesign calculations are presented in Appendix B-2.
The new wall section was found to have a thickness of 600 mm (initially 500 mm) with main reinforcement consisting of No.20 bars at 125 mm (initially No.20 bars at 150 mm) and shear reinforcement being No.15 bars at 125 mm (initially No.15 bars at 150 mm). The redesigned wall cross section is shown in Figure 3-19.

RSM and MCS methods used to perform initial reliability analyses were repeated for the new section to determine a new $P_f$ and $\beta$. The CCD parameter combinations and the subsequent FEM results of each combination for this new wall section are summarized in Table A-4 and the resulting RSM used to predict the new wall maximum displacement is expressed in Equation 3-26.

$$\Delta_{\text{max,new}} = -10 + 1.22W - 46.9R - 0.12E_s + 105h - 202\rho + 0.00139W^2 + 5.19R^2 - 0.0001E_s^2 - 14.6h^2 + 99\rho^2 - 0.276R*W + 0.000049E_s*R + 0.0010E_s*W + 0.294h*R - 3.44h*W + 0.022h*E_s - 1.05h*W + 33.6h*R + 0.15\rho*E_s - 16.1\rho*W$$

(Eq. 3-26)
The resulting $P_f$ was $1.6 \times 10^{-4}$ and $\beta$ was 3.60. CDFs of the initial wall design ($\gamma_B$ of 1.00) and the revised design ($\gamma_B$ of 1.41) are plotted and shown in Figure 3-20. It can be observed that, applying $\gamma_B$ of 1.41 to the impulsive loads improves wall reliability significantly.

Figure 3-20. Comparison of the CDFs of the wall designed with $\gamma_B$ of 1.00 and 1.41

### 3.7. Summary and Conclusions

The study summarized herein presented a novel method to determine an accurate, probability-based, air blast load factor for RC structures when a development for this probability-based factor has not been found. The complication of this development arose from complex, rate-dependent behaviors of RC structures under blast and limitations of available tools for reliability analyses when parameters were non-normal distributed or highly uncertain. The proposed method involved utilizing a combination of RSM and MCS that relied on FEM modeling to simulate actual blast experiments and its demonstration was centered on a blast resistant RC, cantilevered wall that was designed using SDOF method in order to minimize modeling and computing effort. FEMs were constructed in LS-DYNA and validated against prevailing theory and available experimental data. RSM and FEMs were used to develop nonlinear response functions that could
predict maximum wall displacement when significant parameters, which included \( f' \), \( f_y \), \( E_s \), \( h \), and \( \rho \), were altered. For the selected case study, RSM was found acceptable for maximum deflection prediction of the studied cantilevered, RC wall. Given the performance function \( g = R - S \) where \( R \) represented maximum allowance displacement of 210 mm and \( S \) was represented with RSM, the developed and validated RSMs were then used in association with MCS to produce 50,000 empirical response data points which were impossible to be collected from actual blast experiments. These data points were subsequently used to determine response distribution, consequent failure probabilities, and a resulting blast load factor for this study based on the given performance function. The resulting probability-based air blast load factor for flexure was found to be 1.41 and the wall section designed using this load factor was found to have its \( \beta \) slightly different from the target \( \beta \) of 3.50, signifying that the typically-assumed unity blast load factor provided insufficient reliability when it was used to design a blast resistant RC, cantilevered wall and the blast load factor of 1.41 was recommended to reach the target reliability for such case.

Based on results from the study, it can be concluded that:

1) While there is no trace of statistically-based blast load factor development, the current unity blast load factor is not suitable for blast resistant structural design because it does not account parameter uncertainty and, as a result, provides insufficient structural reliability;

2) Although nonlinear behaviors of RC structures under blast and their analytical functions are complex and can result huge computational cost for reliability studies, the maximum plastic deformation of RC, cantilevered wall subjected to blast in the current study can be accurately predicted with RSMs which are more simple and consume less computational effort when applied for reliability analyses; and

3) With validated RSM, the proposed method which consists of RSM and MCS method can utilize parameter statistics found in the literature to develop a probability-based air blast load
factor for the blast resistant RC, cantilevered wall and the resulting load factor, which was found to be 1.41 for flexure in the current study, can result reliable structures.

Several assumptions were made to be able to employ the proposed method and they should be addressed. The distribution parameters including minima, maxima, means and COVs, were selected from their extreme values or the underlying distribution was assumed to be uniform, as their true distributions are unknown. Therefore, a further study is needed to develop a methodology that accounts all uncertainties of the distribution parameters to appropriately assess the blast load factor when parameter distributions are uncertain. It also should be noted that the blast load factor developed herein is only for one design blast load level (or scaled distance, Z).
Chapter 4

Uncertain Distributed Parameter Applications for Statistically-based Air Blast Load Factor Determination

4.1. Introduction and Backgrounds

Precise statistics, such as specific distributions with known parameters, of random variables have played an important role in load and resistance factor development since the beginning of Load and Resistance Factor Design (LRFD) era because they were required by reliability analysis approaches available at the time, such as First Order Second Moment (FOSM) or Advance First Order Second Moment (AFOSM or ASM) methods (MacGregor, 1976; Winter, 1979). Normal and lognormal distributions with specific means and standard deviations incorporating statistical independency among random variables were most commonly used in load and resistance factor determination to simplify computation of resulting performance functions used to describe the state of failure (MacGregor, 1976). However, the random variable distributions are uncertain and assigning different distributions will affect the reliability analysis results (Zhou et al., 1999, Jimenez et al., 2009; Haldar and Mahadevan, 2000). Therefore, in such circumstances, traditional reliability analysis approaches (e.g. FOSM, AFOSM) which depend on precise statistics may give inaccurate results and can, ultimately, affect the determination of load and resistance factors. Thus, a reliability approach that can be applied in such situations would greatly benefit in more accurately determining load and resistance factors.

The focus of the current study was on developing a modified reliability approach to determine load factors for reinforced concrete members in blast events where precise distributions are unknown. Probability Boxes (P-Boxes), a statistical structure that can account for distribution uncertainty of random variables, were used to represent parameter variation. P-Boxes associated with a Response Surface Metamodel (RSM) and Monte Carlo Simulation (MCS) was proposed in
the study for probabilistic air-blast load factor determination. RSM was used to develop nonlinear response prediction function and MCS was used to determine probabilistic load factor when distributions are non-normal. Demonstration of the proposed P-Box method was centered on a case study used in Section 3.5 for air blast load factor derivation using RSM and MCS but, in this chapter, distribution parameter uncertainty was accounted into the problem instead of assuming a deterministic value. However, due to huge calculations required to comprehend all uncertainty, MATLAB was utilized to perform such amount of calculations to determine blast load factor from P-Boxes.

4.2. Parameter Statistic Uncertainty in Blast Resistant Design

Parameters in relation to blast resistant design for RC structures were gathered and summarized in Table 3-1 and 3-2. Blast load variations were found to be dependent to the variations of the type and amount of explosive charge (W) equivalent to a weight of trinitrotoluene (TNT) and the relative location between the charge and the target or the stand-off distance (R). However, blast load variations found in published structural reliability studies were generally represented by variations of peak reflected pressures ($Pr_{\text{max}}$) and positive phase durations ($t_0$) (Low and Hao, 2002; Bogosian et al., 2002; and Hao et al., 2010) because W and R statistical information was not publically available due to national security concerns. Variations assumed for $Pr_{\text{max}}$ and $t_0$ found in the literature were summarized in Table 3-1. It can be observed from Table 3-1 that COVs of $Pr_{\text{max}}$ and $t_0$ are uncertain with a range between 0.24-0.3227 and 0.13-0.18, respectively. Hao et al. (2010) assumed normal distribution, and Low and Hao (2002) and Bogosian et al. (2002) did not provide this information. These findings indicate inconclusive distributions for these blast load parameters and, as a result, they should be considered to be uncertain.
Random variables related to RC structures used for blast reliability studies (Low and Hao, 2002; Real et al., 2003; Lu et al., 1994; Hao et al., 2010), were also summarized in Section 3.2. The resulting random variables included concrete compressive strength \( f'c \), steel bar yield strength \( f_y \), concrete modulus of elasticity \( E_c \), steel modulus of elasticity \( E_s \), reinforcement ratio \( \rho \), and section geometry. Their statistical information was compiled and summarized in Table 3-2. In contrast to \( f'c, f_y, E_c, \) and \( E_s \) whose distributions are well established, the geometry and \( \rho \) distributions were found to be uncertain due to the COV variations. From Table 3-2, it can be observed that the geometry distribution is uncertain due to the variation of its COVs, ranging from 1% to 3%, or its standard deviations, ranging from 5 to 10 mm. The reinforcement ratio distribution is also uncertain due the variation of its COVs, ranging from 3% to 10%. These uncertain statistics for all significant parameters used to develop RSM in Chapter 3 are summarized in Table 4-1. Again, it should be noted that the \( W \) and \( R \) variations shown in Table 4-1 were converted from the \( Pr_{max} \) and \( t_o \) statistics presented in Table 3-1 and their distributions were assumed to be uniform due to a lack of their statistical information.

Table 4-1. Uncertain parameter statistics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>COV</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W ) (kg)</td>
<td>33.55-56.92</td>
<td>150.92-189.83</td>
<td></td>
<td></td>
<td>uniform</td>
</tr>
<tr>
<td>( R ) (m)</td>
<td>3.23-3.49</td>
<td>4.83-5.76</td>
<td></td>
<td></td>
<td>uniform</td>
</tr>
<tr>
<td>( E_s ) (MPa)</td>
<td></td>
<td></td>
<td>210</td>
<td>2.5%</td>
<td>normal</td>
</tr>
<tr>
<td>( H ) (m)</td>
<td></td>
<td></td>
<td>3</td>
<td>0.17-3%</td>
<td>normal</td>
</tr>
<tr>
<td>( \rho ) (%)</td>
<td></td>
<td></td>
<td>0.5</td>
<td>3-10%</td>
<td>normal</td>
</tr>
</tbody>
</table>

Parameter statistics shown in Table 4-1 were provided based on the statistics required for their underlying distribution when performing MCS. For example, uniform distribution requires
minimum and maximum values when simulating data points for MCS. It also can be observed in Table 4-1 that $W$ and $R$ have uncertain minimum and maximum value and $H$ and $\rho$ have uncertain COV while only $E_s$ has precise distribution.

4.3. P-Box Application for Parameters with Imprecise Distribution

Reliability analysis approaches discussed in Section 2.1 and background behind the derivation of load and resistance factors in the ACI codes provided in Section 2.2 were found to be dependent on available statistical information, especially the distribution of random variables, and how closely actual distributions match those that are assumed. It has been shown that, as a minimum, normal or lognormal distributions could be used to reasonably represent unknown distributions (Allen et al., 2005). It should also be emphasized that, as discussed Section 4.2, there was uncertainty in random variable distributions used in the literature stemming from assumptions made regarding underlying distribution types and available statistical data at the time that research was completed. To properly treat distribution uncertainty, there are some probabilistic modeling methods that can be used for the present research and are known to be utilized in fields where all uncertainties should be considered, such as risk assessment. The methods include interval probabilities, Dempster-Shafer structures, and probability boxes (P-Boxes) (Ferson et al., 2004). Interval probabilities and Dempster-Shafer structures will be briefly discussed in this section. P-Boxes will be discussed in more detail because they are found to be the most appropriate statistical approach for the problem being studied herein and, subsequently, will be applied to this study.

The interval probabilities method has been used to characterize the probability of an event when it was difficult to specify precisely but its upper and lower bounds were available. The types of calculations involving interval probabilities for risk and reliability assessment were reviewed by Hailperin (1986) and it was found that basic arithmetic operations (e.g. add, subtract,
multiply, divide) could be used for some calculations with this method (Moore, 1966; Alefeld and Herzberger, 1983; Neumaier, 1990). However, this method was only applied to fault tree analyses, when underlying distributions were not required in the analyses, because it could not account for distributions of the events (Ferson et al., 2004).

Dempster-Shafer theory is a variant probability theory where the elements in the sample space, \( X \), are not single points but, instead, sets of real values that represent ranges of possible evidence. Sets associated with nonzero probability mass are called focal elements. One example of a Dempster-Shafer structure is a discrete probability mass function giving the probability for each random variable value, \( x \), with an interval rather than a point value (Ferson et al., 2004). Unlike a discrete probability distribution, the focal elements of a Dempster-Shafer structure may overlap one another and this is the fundamental difference that distinguishes Dempster-Shafer theory from traditional probability theory. Dempster-Shafer theory has been widely studied in computer science and artificial intelligence, but has never been accepted by probability researchers and traditional statisticians, even though it can be rigorously interpreted as a classical probability theory (Ferson et al., 2003).

A Probability Box, or P-Box, is a statistical structure that is used to represent imprecise random variable distributions when analysts could not specify: (1) precise parameter values for input distributions of point estimates in the risk model (e.g., minimum, maximum, mean, median, mode, etc.); (2) precise probability distribution for some or all of the variables; (3) the precise nature of dependency among the variables; and (4) true structures of the model or the model uncertainty. P-Boxes are somewhat similar to Dempster-Shafer structures by using intervals to represent distribution uncertainty except P-Boxes are constructed from cumulative distribution functions (CDFs) where Dempster-Shafer structures are constructed from probability mass functions (PMFs). Due to their close connection, Yager (1986) found that P-Boxes could be converted to Demspster-Shafer structures and Dempster-Shafer structures could be converted to
P-Boxes. However, P-Boxes have been shown to more easily describe uncertainty than Dempster-Shafer structures (Ferson et al., 2004). P-Box applications have been used in many areas related to risk or safety assessment (Tucker and Ferson, 2003; Ferson and Donald, 1998; Ferson and Tucker, 2006; Aughenbaugh et al., 2006; and Ferson et al., 2002) and they can be traced back to the eighteenth century when analysts wished to construct a distribution for variables when only their mean or variance or both were available. P-Boxes have since been applied broadly in circumstances where analysts do not have enough statistical information to understand the random variables. P-Boxes are currently applied to risk and safety assessment in some areas of engineering, especially environmental engineering, when highly uncertainties exist.

P-Boxes consist of lower and upper bounds encasing the actual cumulative distribution function \( (F) \) and can be expressed by Equation 4-1.

\[
d(p) \geq F^{-1}(p) \geq u(p)
\]

(Eq. 4-1)

where \( p \) is probability level and \( d(p) \), \( u(p) \), and \( F^{-1}(p) \) are the lower, upper, and actual value of a random variable at each \( p \), respectively. P-Box can be constructed from uncertain parameter statistics such as parameter mean \( (\mu) \), standard deviation \( (\sigma) \), and coefficient of variation \( (\text{COV}) \) and its lower and lower bound can be defined as the parameter maximum and minimum, respectively, value at each probability when all possible scenarios for those statistics are considered. These lower and upper bound definition can be expressed as shown in Equations 4-2 and 4-3, respectively.

\[
d(p) = \text{Max}_{\alpha} N_{\alpha}^{-1}(p)
\]

(Eq. 4-2)

\[
u(p) = \text{Min}_{\alpha} N_{\alpha}^{-1}(p)
\]

(Eq. 4-3)
where $N^{-1}$ represents inverse normal probability and $\alpha$ refers all possible scenarios (Tucker and Ferson, 2003). An example of this P-Box was demonstrated by Tucker and Ferson (2003) when a normal distributed parameter $X$ contained uncertain $\mu$ and $\sigma$ in a range between 0.5 to 0.6 and 0.05 to 0.1, respectively. All possible scenarios of $\mu$ and $\sigma$ combinations were considered and their CDFs were plotted as shown in Figure 4-1 (a). The lower and upper bounds of $X$ at each probability level were determined using their definition from Equations 4-2 and 4-3 and they were plotted as shown in Figure 4-1 (b).

![Figure 4-1. (a) all possible CDFs and (b) resulting lower and upper bounds](image)

(Tucker and Ferson, 2003)
The P-Box shown in Figure 4-1 (b) indicates that the actual CDF of $X$ can be anywhere in between these lower and upper bounds.

A P-Box can be derived even when the distribution is not available if some constraints for the data set, such as its maximum value, minimum value, mean, or median can be obtained. A P-Box also can be constructed from a set of empirical data with a specific confidence level using the Kolmogorov-Smirnov ($K$-$S$) confidence interval method (Sokal and Rohlf, 1994). This method has been used to construct P-Boxes because it can account for sampling uncertainty and it does not require assumptions be made regarding random variable distributions (Ferson et al., 2002). The $K$-$S$ method also provides an opportunity to decrease or increase the size of the P-Box depending on the level of confidence associated with the data.

Similarly to typical random variables, P-Boxes can be used when performing reliability analyses. Williamson et al. (1990) provided an explicit numerical method for P-Box basic calculations including addition, subtraction, multiplication, and division but this method was found to be inaccurate when performance function was complex, such as nonlinear RSM. For more sophisticated calculation, two-phase MCS (Frantzich, 1998; Hofer et al., 2002; and Stephen, 1996) was found to be an alternative method to obtain output P-Box when input parameters contained uncertain statistics (Karanki et al., 2009). The method consists of two MCS loops called inner and outer loop. The outer loop simulates parameter statistics (e.g. $\mu$ and $\sigma$) for $N_1$ values based on their uncertainty and supplies each parameter statistic set to the inner loop to use to simulate $N_2$ data points for each parameter and to perform typical MCS. Karanki et al. (2009) summarized these procedures in a flowchart as presented in Figure 4-2 where the terms “Epistemic” and “Aleatory” refers to parameter statistical uncertainty, which can be improved when more data is obtained, and parameter variation, which comes from parameter randomness in nature, respectively.
where the number of simulations required for the outer and the inner loop were defined as $N_1$ and $N_2$, respectively.

The ability of the two-phase MCS when compared to the numerical method for P-Box calculation was demonstrated by Karanki et al. (2009) for a product of two lognormal variables, $A$ and $B$, whose $\mu$ and $\sigma$ were uncertain. Parameter $A$ contained $\mu$ and $\sigma$ in a range from 10.89 to 24.97 and 4.8 to 18.72 and parameter $B$ contained $\mu$ and $\sigma$ in a range from 43.32 to 148.85 and 19.09 to 83.36, respectively. First, their P-Boxes and the product P-Box ($AxB$ P-Box) were determined using the numerical method provided by Williamson et al. (1990). The resulting P-Box was plotted and represented by two red solid lines as shown in Figure 4-3. On the other hand,
for the two-phase MCS method, the outer loop simulated a number of \( \mu \) and \( \sigma \) for \( A \) and \( B \) within the given range. Each set of \( \mu \) and \( \sigma \) was then used to simulate 10,000 data points \( (N_2 = 10,000) \) for \( A \) and \( B \) and they were used toward \( AxB \) calculation. CDFs for \( AxB \) from each \( \mu \) and \( \sigma \) set were then plotted and represented by black solid lines as shown in Figure 4-3. However, two iteration sets for the outer loop including 100 and 1,000 iterations \( (N_1 = 100 \text{ and } 1,000) \) were chosen to investigate computational effort for the two-phase MCS in order to obtain the same uncertainty of \( AxB \) as provided by its P-Box and the resulting CDFs from those iteration sets were plotted as shown in Figure 4-3 (a) and (b), respectively.

![Figure 4-3. A x B MCS with (a) \( 10^2 \times 10^4 \) iterations (b) \( 10^3 \times 10^4 \) iterations](image)

Karanki et al. (2009) found that: 1) CDFs from both iteration sets were enclosed by the P-Box; 2) if the number of the iterations increased, the two-phase MCS results converged to the results from the numerical method by filling up the P-Box as presented in Figure 4-3 (b); and 3) the two-phase MCS consumed much more computational time than the numerical method, 130.3 seconds for 100 iterations and 1,295 seconds for 1,000 iterations when compared to 1.5 seconds for the numerical method. However, the two-phase MCS can be used to perform complex
calculations for P-Boxes when comparing to the numerical method (Williamson et al., 1990) which is only suitable for basic calculations. In the current study, such tremendous amount of calculations was completed by an assistance of computer coding software MATLAB.

4.4. Air Blast Load Factor Development by Accounting Parameter Statistic Uncertainty

In Chapter 3, air blast load factor for a RC, cantilevered wall designed using SDOF method was determined by using the proposed RSM and MCS method developed in that chapter. The summary of the RSM and MCS method was depicted using a flowchart shown in Figure 3-6. The method was used in conjunction with precise parameter statistic assumption to obtain the resulting air blast load factor of 1.41. However, parameter statistics were found to be uncertain rather than to be deterministic as discussed in Section 4.2. To account this uncertainty when determine air blast load factor, the current study proposes a methodology that implements P-Box to represent parameter distributions and two-phase MCS to adapt P-Box to the RSM and MCS method used to determine air blast load factor in Chapter 3. Therefore, the P-Box and two-phase MCS method was demonstrated using the same case study from Chapter 3 so that the resulting air blast load factors could be compared. The main changes of P-Box and two-phase MCS from the RSM and MCS method were that the a response CDF in Figure 3-6 was replaced by a response P-Box that was constructed using all parameter statistic combinations, RSM, and MCS as demonstrated by Tucker and Ferson (2003). After the response P-Box was completed, simulated responses CDFs were produced using two-phase MCS and RSM as demonstrated by Karanki et al. (2009) and subsequent air blast load factors for each simulated CDF were determined by using the RSM and MCS method again. CDFs were simulated until they could populate all area in the P-Box meaning all uncertainty was considered (Karanki et al., 2009). The P-Box and two-phase MCS method was then completed when all simulated response CDF were used to determine their associate blast load factors. This P-Box and two-phase MCS method was summarized in a
flowchart as shown in Figure 4-4. It should be noted there was no change for RSM development section used in the P-Box and two-phase MCS method because the parameter space was not changed from such used in Chapter 3. The application of the P-Box and two-phase MCS method and Figure 4-4 will be demonstrated as follows.

To construct the response P-Box for the same RC, cantilevered wall in Chapter 3, 64 factorial combinations obtained from all possible combinations of the statistic bounds shown in Table 4-1 were developed. For each statistic set, each statistic was implemented to simulate 50,000 simulated data points for each parameter. This simulation set was required to provide reliable results as found in Chapter 3. Each data point set that contained a simulated data point for all parameters was then applied to RSM (Equation 3-25) to determine the resulting maximum displacement for that data point set. 50,000 subsequent maximum displacements were then determined for all 50,000 data point set and they were ranked. The ranked maximum displacements were then plotted and the plots represented the response CDF for a statistic set. The CDF construction was repeated for all 64 statistic combinations. The 64 CDFs are plotted as shown in Figure 4-5 (a) and a maximum displacement P-Box was constructed following its lower and upper bounds determined using Equations 4-2 and 4-3, respectively. The response P-Box is shown in Figure 4-5 (b).
Figure 4-4. The Proposed P-Box and Two-Phase MCS Method
Figure 4-5. P-Box construction

The resulting P-Box shown in Figure 4-5 (b) indicates that maximum wall displacement CDF is uncertain and its actual CDF can fall anywhere within this P-Box. Therefore, to account this uncertainty when determining air blast load factors, all CDFs possibly contained in the P-Box were generated by an assistance of the two-phase MCS and those CDFs were applied back to the RSM and MCS method to determine the subsequent blast load factors as discussed earlier.

Similar to the two-phase MCS demonstration by Karanki et al. (2009), the outer loop randomly
selected parameter statistics within their bounds as shown in Table 4-1 and they were used to simulate data points that were then applied to Equation 3-25 to determine the resulting response CDFs.

The number of simulations required for inner loop \((N_2)\) was already found in Chapter 3 to be 50,000 while \(N_I\) was investigated for two set of simulations, 1,000 and 10,000 (Karanki et al. (2009) found \(N_I = 1,000\) to be able to populate P-Box area). The ability to adequately populate the P-Box for each \(N_I\) was determined by visually investigating the CDF set to populate area in the P-Box. The resulting CDFs from both parameter statistic set were plotted as shown in Figure 4-6 (a) and (b), respectively, and it can be observed that both simulation sets can generate CDFs that provide the same coverage in the P-Box area but the CDFs from the 1,000 simulation set show gaps between CDFs when they are close to the lower and upper bounds. In contrast, the CDFs from 10,000 simulation set completely fill their coverage. However, it also can be observed that both cases cannot populate most of the P-Box area, especially small area next to the lower and upper bounds. This small incomplete area can be observed in Karanki et al. (2009) as same as shown in Figure 4-3. However, since this area is very small when compared to the coverage area, it is acceptable to conclude that the 10,000 simulation set can provide a set of CDFs large enough to account all uncertainty in the P-Box.
Based on these 10,000 CDFs, $P_f$, $\beta$, and subsequent air blast load factors were determined using the RSM and MCS method. After all 10,000 CDFs were considered, the resulting blast load factors were found to be a range between 1.16 and 1.74. This range contains the blast load factor
of 1.41 found in Chapter 3 when precise distributions were assumed. The resulting range indicates that there is a possibility that the air blast load factor of 1.41 can result unreliable blast resistant structures because in some circumstances the recommended air blast load factor can be up to 1.74.

4.5. Summary and Conclusion

In the current study, random variable statistical uncertainty (i.e. maximum, minimum, mean, and COV values) was addressed during the determination of probabilistic air blast load factors. P-Boxes and two-phase MCS were applied to the RSM and MCS method developed in Chapter 3, a method that utilized deterministic parameter statistics, to incorporate uncertainty with the new method being called the “P-Box and two-phase MCS method.” Development and demonstration of the proposed method centered on the same blast resistant RC, cantilevered wall used in Chapter 3 so that the resulting blast load factors could be compared. Following the flowchart shown in Figure 4-4, the response P-Box was constructed using compiled parameter statistics from the literature and the RSM for maximum wall displacement developed in Chapter 3. Reliability analyses were performed for the performance function $g = R - S$, where $S$ was represented by the response P-Box and $R$ was assigned as the deterministic maximum permissible displacement of 210 mm, a maximum values based on limiting support rotation to $4^\circ$. Using two-phase MCS allowed for response CDFs to be randomly constructed until they populated all area in the response P-Box and resulting $P_f$, the subsequent $\beta$, and the corresponding blast load factor were determined for each CDF. 10,000 CDFs were found to be appropriate to account all response uncertainty. Blast load factors corresponding to these 10,000 CDFs were determined and found to be a range between 1.16 to 1.74, values which contained the blast load factor of 1.41 found in Chapter 3 when precise parameter statistics were assumed. The resulting range suggested that the air blast load factor of 1.41 could unreliable in certain circumstances.
It can be concluded that:

1. Parameter statistics related to blast resistant design are highly uncertain due to a lack of well-established statistical data and limited experimental data. The utilization of P-Boxes to address these uncertainties could be a preferred statistical structure to represent parameter variation for the studied application because they can not only represent parameter randomness but also parameter statistic uncertainty.

2. P-Box in conjunction with two-phase MCS can be used to determine air blast load factors. Resulting load factors indicate that parameter statistic uncertainty has significant influence on the reliability derived load factors, possibly rendering load factors that are obtained without considering these uncertainties unreliable.
Chapter 5

Conclusion, Impact, and Future Research

This study developed a methodology utilizing Response Surface Metamodeling (RSM), Monte Carlo Simulation (MCS), and Probability-Box (P-Box) to determine statistically-based air blast load factor that not only corresponds to parameter variations but also parameter statistic uncertainty. This chapter summarizes important findings learned from the study, contributions from the study, and future research required to advance this topic.

5.1 Summary and Conclusion

The objective of this study is to determine statistically-based air blast load factors when the current blast load factors available in blast resistant design guidelines have been assumed to be a unity based on the low-probability, high-consequence characteristic of blast events. The complications for statistically-based blast load factor determination arises from complex, highly nonlinear interactions between structural components and blast loads that cannot be derived explicitly. Limited statistical information due to national security concern is also another reason that results high uncertainty of blast parameters and leads to the complication for blast load factor determination when all uncertainty cannot be accounted.

In the study, the first issue was resolved by developing Response Surface Metamodels (RSMs) that are acceptably accurate for predicting nonlinear response of reinforced concrete members under blast loads. The RSM development was carried out in Chapter 3. The development was centered on a blast resistant reinforced concrete, cantilevered wall that was designed using Single Degree of Freedom (SDOF) method. Finite Element Models (FEMs) were used in conjunction with RSM method to develop the response predicting function for the wall. FEMs were created in LS-DYNA package using the material models available for reinforced
concrete members and the blast pressure predicting model embedded in the package. The FEM
static responses under quasi-static loadings were validated against their theoretical calculation
and its rate-dependent response was validated with published blast test results. Parameters related
to blast reliability study for reinforced concrete members were collected from available literature
and the potential key parameters affecting RSM accuracy were determined. The screening
process was carried out in conjunction with experimental design and FEMs and the five key
parameters were obtained including:

- Charge weight equivalent to TNT
- Distance between the explosion and the structure
- Elastic modulus of steel reinforcement
- Wall height
- Reinforcement ratio

These key parameters were then used along with experimental design for nonlinear RSM
to generate experimental data that was regressed for the wall nonlinear response predicting
function. The developed function was then used in conjunction with MCS and deterministic
parameter statistics to determine the subsequent statistically-based air blast load factor.

The uncertain parameter distribution was treated in Chapter 4 by developing a
methodology in conjunction with the RSM and MCS method to account this uncertainty into the
blast load development. Instead of using a response CDF to determine the subsequent blast load
factor, a response P-Box was developed from uncertain parameter statistics found in the literature
and it was use in conjunct with the two-phase MCS and the RSM and MCS method to determine
blast load factors.

Notable findings obtained from the development of statistically-based air blast load
factors included:
• While there is no published evidence of statistically-based blast load factor development, blast resistant reinforced concrete cantilevered wall designed with the conventional blast load factor of 1.0, as recommended in many design guidelines, has reliability lower than the target level;

• Although the nonlinear behavior of RC structures under blast is complex and resulting analytical techniques to predict response can involve large computational cost, the maximum plastic deformation of a RC, cantilevered wall subjected to blast in the current study was accurately predicted using RSMs, an approach that requires considerably less computational effort than other techniques. The proposed method that utilized the validated RSMs with MCS was able to effectively incorporate parameter statistics from the literature to develop a statistically-based air blast load factor for the RC, cantilevered wall; and

• Parameter statistics related to blast resistant design are highly uncertain due to a lack of well-established statistical data. The use of P-Boxes was shown to be an appropriate statistical structure to represent parameter variation in such a situation because it can not only represent parameter randomness but also parameter statistic uncertainty. The proposed method involving P-Boxes in conjunction with two-phase MCS was shown to be viable to determine air blast load factors. Resulting load factors, found to range between 1.16 and 1.74, demonstrated that parameter statistic uncertainty had significant influence on the reliability of derived load factors. It was also shown that assuming deterministic statistical information to obtain a similar load factor can produce a factor, found to be 1.41, that could be unreliable.

5.2 Impact

The key impact of this research is the capability to determine statistically-based air blast load factor used for SDOF blast design using RSM, MCS and P-Box. The study illustrated the need of sophisticated, statistically-based blast load factors for blast resistant design when the
current recommended blast load factors are not statistically-based and found to result unreliable blast resistant structures. This framework also results in a number of benefits and contributions included:

- RSM development which is practical, systemically organized, and acceptably accurate for nonlinear response predicting function and probably for the application of RSM in further blast reliability study.
- Application of P-Box to determine load and resistance factor when parameter distribution is uncertain.

5.3 Area of Future Research

Work from the present study could be extended through additional research in the following areas:

- The development of air blast load factors was centered only on a RC, cantilevered wall in order to pilot the application of RSM and P-Box for this field. However, it would be greatly benefit if other types of blast resistant structures are investigated for the application of this method.
- The development of air blast load factors was also based on a single load and protection level. Additional studies for other load and/or protection level would be greatly benefits for the blast resistant design community.
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Appendix A

Design of Experiment

Table A-1. Plackett-Burman Design table for 8 input parameters and resulting maximum displacement

Table A-2. Central Composite Design table for 5 input parameters and resulting maximum displacement for wall section designed using $\gamma_b = 1.00$

Table A-3. Additional 43 LS-DYNA runs and the corresponding and the predicted maximum displacement

Table A-4. Central Composite Design table for 5 input parameters and resulting maximum displacement for wall section designed using $\gamma_b = 1.41$
Table A-1. Plackett-Burman Design table for 8 input parameters and resulting maximum displacement

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Table A-2. Central Composite Design table for 5 input parameters and resulting maximum displacement for wall section designed using $\gamma_B = 1.00$

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Table A-3 (Cont’d). Additional 43 LS-DYNA runs and the corresponding and the predicted maximum displacement

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Table A-3 (Cont’d). Additional 43 LS-DYNA runs and the corresponding and the predicted maximum displacement

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Table A-4. Central Composite Design table for 5 input parameters and resulting maximum displacement for wall section designed using $\gamma_b = 1.41$

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Table A-4 (Cont’d). Central Composite Design table for 5 input parameters and resulting maximum displacement for wall section designed using $\gamma_B = 1.41$

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Appendix B

SDOF Blast Resistant Design Sheet

B-1. SDOF Design Sheet for Blast Resistant RC, Cantilevered Wall with $\gamma_B = 1.00$

B-2. SDOF Design Sheet for Blast Resistant RC, Cantilevered Wall with $\gamma_B = 1.41$
B-1. SDOF Design Sheet for Blast Resistant RC, Cantilevered Wall with $\gamma_B = 1.00$

**Design for Flexure**

Given: $W = 100$ kg (220 lbs)

- $R = 4.0$ m (13.12 ft)
- $H = 3.0$ m
- $\rho = 0.5\%$
- Concrete density ($w_c$) = 2,500 kg/m$^3$
- $f_y = 415$ MPa
- $f_{c'} = 28$ MPa

**Step 1:** Determine equivalent pressure-time history

Scaled distance ($Z$) = $\frac{R \cdot W^{1/3}}{100^{1/3}} = 0.862$ m/kg

From Figure 2-15 in UFC 3-340-02, for $Z = 0.862$ m/kg

- $\frac{i_r}{W^{1/3}} = 1.085$ kPa-ms/kg$^{1/3}$

  $\therefore \; i_r = 1.085 \times 100^{1/3} = 5.030$ kPa-ms

- $\frac{t_0}{W^{1/3}} = 1.04$ ms/kg$^{1/3}$

  $\therefore \; t_0 = 1.04 \times 100^{1/3} = 4.83$ ms

- $P_{so} = 1,790$ kPa
Figure B1-1. Equivalent pressure-time history for $W = 100$ kg and $R = 4$ m

**Step 2**: Determine resistance-deflection relationship

(i) Determine maximum resistance ($R_m$)

- From Table 3-1 in UFC 3-340-02, for one-way cantilever element

$$R_m = \frac{2M_N}{L}$$

- For type II section, maximum support rotation ($\theta$) = 4°

- Thus, with $\theta = 4°$ concrete in compression is disintegrated as shown in Figure 4-13 in UFC 3-340-02.

![Type II reinforced concrete section](image)

Figure B1-2. Type II reinforced concrete section

$$M_N = A_s f_{dy} d_c \text{ kN-m per unit width}$$
\[ R_m = \frac{2Af_{dy}d_c}{L} \text{kN per unit width or} \quad R_m = \frac{2}{L} \rho f_{dy}d_c^2 \text{kN when } A_s = \rho d_c \text{ mm}^2 \]

per unit width

- From Table 4-1 in UFC 3-340-02

  Reinforcing steel: Bending, DIF = 1.23
  Direct shear, DIF = 1.10

  Concrete: Compression, DIF = 1.25
  Direct shear, DIF = 1.10

  Hence \( f_{dy} = 1.23 \times 415 = 510 \text{ MPa} \)
  \( f'_{dc} = 1.25 \times 28 = 35 \text{ MPa} \)

  Thus \( R_m = \frac{2}{3.0} (0.005 \times 510 \times d_c^2) = 1.7 \times 10^3 d_c^2 \text{ kN per unit width} \)

(ii) Determine maximum deflection \((X_m)\)

For \( \theta = 4^\circ \), \( X_m = L \tan(4^\circ) = 210 \text{ mm} \)

(iii) Determine elastic deflection \((X_e)\)

- From Table 3-8 in UFC 3-340-02, for uniformly distributed loading cantilevered member

  Equivalent elastic stiffness \((K_e) = \frac{8EI}{L^3}\)

- Moment of inertia of cracked section \((I_c) = Fbd^3 \cong Fbd_c^3\), \(F\) is obtained from Figure 4-12 UFC 3-340-02

\[ E_s = 2.1 \times 10^5 \text{ MPa} \]

\[ E_c = 0.043\sqrt{F_{c}^{1.5}f_{c}^{1.5}} = 0.043 \times 2,500^{1.5}\sqrt{28} = 28,442 \text{ MPa} \]

\[ n = \frac{E_s}{E_c} = \frac{2.1 \times 10^5}{28,442} \approx 7.4 \]
• From Figure 4-12 in UFC 3-340-02, for $\rho = 0.005$ and $n = 7.4$, $F = 0.028$

\[ I_c = 0.028d_c^3 \]

\[ K_E = \frac{8 \times 28,442,000 \times 0.028}{3.0^3} d_c^3 = 232.3 \times 10^3 \, d_c^3 \, \text{kN/m} \]

\[ X_E = \frac{R_m}{K_E} = \frac{1.7 \times 10^3 d_c^2}{232.3 \times 10^3 d_c^2} = \frac{7.32 \times 10^{-3}}{d_c} \, \text{m} \]

\[ \text{Figure B1-3. Resistance-deflection relationships} \]

**Step 3:** Determine wall thickness and reinforcement

• From impulse equation (Eq. 3-94 in UFC 3-340-02) for elements subjected to impulsive loading

\[ \frac{i^2 A^2}{2m_a} = \frac{R_m X_E}{2} + \frac{m_a}{m_a} R_m (X_m - X_E) \]

Where:

- $A$ is loading area $= 3.0 \times 1.0 = 3.0 \, \text{m}^2$;
- $m_a$ is effective plastic mass $= K_{LM} \times $ actual mass;
- $m_a$ is an average of effective elastic and plastic mass $= m_a$
- $K_{LM} = 0.66$ (from Table 3-12 in UFC 3-340-02)
- Actual mass $= w_c d_c H = 2,500 \times 3.0 \, d_c = 7,500d_c \, \text{kg per unit width}$
For \( m_a \geq m_u \)

\[
\frac{i^2A^2}{2m_u} = \frac{R_m X_E}{2} + R_m (X_m - X_E) = R_m \left( X - \frac{X_E}{2} \right)
\]

\[
\therefore \frac{(5,030^2)(3^2)}{2(0.66)(7,500d_c)} = (1.7 \times 10^3 d_c^2) \left( 0.210 - \frac{(7.32 \times 10^{-3})}{2d_c} \right)
\]

\[
227,708,100 = 3,534,300d_c^3 - 61,598d_c^2
\]

\[
\therefore d_c = 0.401 \text{ m, says 400 mm}
\]

\[
\therefore A_s = 0.005 \times 1,000 \times 400 = 2,000 \text{ mm}^2 \text{ per unit width}
\]

\[
\therefore \text{Use No. 20 bars} \ (A_b = 300 \text{ mm}^2) \text{ for 7 bars, total } A_s = 2,100 \text{ mm}^2 \text{ (150 mm spacing) per unit width}
\]

- Total thickness = 40 + 10 + 400 + 10 + 40 = 500 mm
- Transverse reinforcement = minimum required reinforcement (ACI 318-08)

\[
A_{s,\text{min}} = \frac{0.25\sqrt{f'_c}}{f_y} b d_c = \frac{0.25\sqrt{28}}{415} (1,000)(400) = 1,275 \text{ mm}^2
\]

- Check maximum stirrup spacing, for Type II section

\[
S_{\text{max}} = \frac{d_c}{2} = \frac{400}{2} = 200 \text{ mm}
\]

\[
\therefore \text{Use No. 15 bars} \ (A_b = 200 \text{ mm}^2) \text{ at 150 mm center-to-center, total } A_s = 1,400 \text{ mm}^2
\]

**Step 4:** Check if impulsive loading design is valid

- From Table 3-1 in UFC 3-340-02, for uniformly distributed loading cantilevered member

\[
r_u = \frac{2M_N}{L^2} = \frac{R_m}{L} = \frac{(1.7\times10^3)(0.40^2)}{3.0} = 90.7 \text{ kN/m per unit width}
\]

- From Eq. (3-96) in UFC 3-340-02
\[ t_m \approx \frac{r}{r_u} = \frac{5.030}{90.7} = 55.5 \text{ ms} \]

\[ \therefore \frac{r_m}{t_d} = \frac{55.5}{4.83} = 11.5 \geq 3 \]

\[ \therefore \text{Impulsive loading design is valid.} \]

**Design for Shear** (shear reinforcement is required for \( \theta > 2^\circ \))

**Step 1:** Determine ultimate shear force \((V_u)\) at critical section (at \(d_c = d_e\) from the support face)

- From Table 4-6 in UFC 3-340-02, for uniformly distributed loading cantilevered member

\[ v_u = \frac{r_u(l-d_e)}{d_e} = \frac{90.7(3-0.4)}{0.4} = 590 \text{ kN/m}^2 \]

\[ \therefore V_u = v_u b d_c = 590 \times 1.0 \times 0.4 = 236 \text{ kN} \]

**Step 2:** Determine shear capacity \((V_c)\) of concrete

- For \(\theta > 2^\circ\), shear capacity of concrete is taken as zero and shear force is carried by shear reinforcement

**Step 3:** Determine shear reinforcement

- From Eq. 4-26 in UFC 3-340-02

\[ A_v = \frac{(V_u - V_c)}{s} = \frac{236 \times 10^3}{0.75} \frac{1.1 \times 415 \times 10^6}{(1.1 \times 415 \times 10^6) \times 0.4} = 1.72 \text{ mm}^2/\text{mm} \]

Assume \(s = 150 \text{ mm}\) (use same spacing as transverse reinforcement)

\[ \therefore A_v = 1.72 \times 150 = 258 \text{ mm}^2 \]

- Check minimum shear reinforcement

\[ A_{v,min} = 0.0015 b s = 0.0015 \times 1,000 \times 150 = 225 \text{ mm}^2 \]
\[ \therefore \textbf{Use No. 10 bars (} A_b = 100 \text{ mm}^2 \text{)} \text{ for 7 legs, total } A_v = 700 \text{ mm}^2 \text{ with 150 mm spacing} \]

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure}
\caption{Figure B1-4. Details of the cross-section for } \gamma_b = 1.00 \end{figure}
**B-2. SDOF Design Sheet for Blast Resistant RC, Cantilevered Wall with \( \gamma_b = 1.41 \)**

**Design for Flexure**

**Step 1:** Apply \( \gamma_b = 1.41 \) to the impulsive loading \( (i_r) \) obtained in Appendix B-1

\[
i_r = (1.41)(5,030) = 7,092.3 \text{ kPa-ms}
\]

**Step 2:** Determine new wall thickness and reinforcement

- Solve impulse equation with new impulsive loading

\[
\frac{(7,092.3^2)(3^2)}{2(0.66)(7,500d_c)} = (1.7 \times 10^3 \ d_c^2) \left(0.210 - \frac{(7.32 \times 10^{-3})}{2d_c}\right)
\]

\[
452,706,473.6 = 4,158,000d_c^3 - 87,021d_c^2
\]

\[
\therefore \ d_c = 0.480 \text{ m, says 500 mm}
\]

\[
\therefore \ A_s = 0.005 \times 1,000 \times 500 = 2,500 \text{ mm}^2 \text{ per unit width}
\]

\[
\therefore \ \text{Use No. 20 bars (} A_b = 300 \text{ mm}^2\text{) for 9 bars, total } A_s = 2,700 \text{ mm}^2 \text{ (125 mm spacing) per unit width}
\]

- Total thickness = 40 + 10 + 500 + 10 + 40 = 600 mm

- Transverse reinforcement = minimum required reinforcement (ACI 318-08)

\[
A_{s,min} = \frac{0.25 \sqrt{f'_c}}{f_y}bd_c = \frac{0.25 \sqrt{28}}{415} (1,000)(500) = 1,594 \text{ mm}^2
\]

- Check maximum stirrup spacing, for Type II section

\[
S_{max} = \frac{d_c}{2} = \frac{500}{2} = 250 \text{ mm}
\]

\[
\therefore \ \text{Use No. 15 bars (} A_b = 200 \text{ mm}^2\text{) at 125 mm center-to-center, total } A_s = 1,600 \text{ mm}^2
\]
**Step 3:** Check if impulsive loading design is valid

- From Table 3-1 in UFC 3-340-02, for uniformly distributed loading cantilevered member
  \[ r_u = \frac{2M_N}{L^2} = \frac{R_m}{L} = \frac{(1.7 \times 10^3)(0.50^2)}{3.0} = 141.7 \text{ kN/m per unit width} \]

- From Eq. (3-96) in UFC 3-340-02
  \[ t_m \cong \frac{i_r}{r_u} = \frac{7.092.3}{141.7} = 50.05 \text{ ms} \]
  \[ \therefore \frac{t_m}{t_d} = \frac{50.05}{4.83} = 10.4 \geq 3 \]
  \[ \therefore \text{Impulsive loading design is valid.} \]

**Design for Shear** (shear reinforcement is required for \( \theta > 2^\circ \))

**Step 1:** Determine ultimate shear force \( (V_u) \) at critical section (at \( d_e = d_c \) from the support face)

- From Table 4-6 in UFC 3-340-02, for uniformly distributed loading cantilevered member
  \[ v_u = \frac{r_u(L-d_e)}{d_e} = \frac{141.7(3-0.5)}{0.5} = 708.5 \text{ kN/m}^2 \]
  \[ \therefore V_u = v_u b d_c = 708.5 \times 1.0 \times 0.5 = 354.2 \text{ kN} \]

**Step 2:** Determine shear capacity \( (V_c) \) of concrete

- For \( \theta > 2^\circ \), shear capacity of concrete is taken as zero and shear force is carried by shear reinforcement
Step 3: Determine shear reinforcement

- From Eq. 4-26 in UFC 3-340-02

\[
\frac{A_v}{s} = \frac{(V_u - V_c)}{f_{ds} d_c} = \frac{354.2 \times 10^3}{0.75} \times \frac{0.75}{(1.1 \times 415 \times 10^6) \times 0.5} = 2.07 \text{ mm}^2/\text{mm}
\]

Assume \(s = 125\) mm (use same spacing as transverse reinforcement)

\[\therefore A_v = 2.07 \times 125 = 259 \text{ mm}^2\]

- Check minimum shear reinforcement

\[A_{v, \text{min}} = 0.0015bs = 0.0015 \times 1,000 \times 125 = 187.5 \text{ mm}^2\]

\[\therefore \text{Use No. 15 bars (} A_b = 200 \text{ mm}^2\) for 9 legs, total } A_v = 1,800 \text{ mm}^2 \text{ with } 125 \text{ mm spacing}\]

Figure B2-1. Details of the cross-section for \(\gamma_b = 1.41\)
VITA

Tanit Jaisa-ard

Tanit Jaisa-ard was born on February 8, 1977 in Phitsanuloke, Thailand. He graduated with a Bachelor of Engineering in Civil Engineering in the Summer of 1998 from Kasetsart University. He then entered Kasetsart University again in the Fall of 2001 to pursue a Master of Engineering in Civil Engineering with a focus on Structural Engineering. After receiving his Master of Engineering degree in Civil Engineering in the Summer of 2004, Tanit started working on his Ph.D. research with Dr. Daniel Linzell in the fall of 2008.