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ABSTRACT

The first chapter of my thesis “Reputation for Two Audiences: Rating Agencies, Auditors, and Issuer-Pays Markets” considers a dynamic model of reputation formation with two audiences. The motivation for studying this model comes from the issuer-pays feature of many certification intermediary markets such as auditors and credit rating agencies. In financial markets, certification intermediaries like auditors and credit rating agencies acquire information about the financial health of a firm. The information these intermediaries provide is used by investors in order to mitigate information risks. This paper investigates whether in a litigation free world reputation concerns can lead to a socially efficient outcome, that is, whether reputation concerns can provide incentives for auditors to expend effort in order to produce high quality auditing. This paper also investigates whether competition among the auditors improves reputational incentives. Reputation of an auditor is modeled as the market belief about his informativeness, which is exogenously given. There are two types of auditors, “informative” and “uninformative”. In a monopoly set up, under imperfect monitoring, the informative auditor is diligent only for a low range of reputation. Gains from reputation shrink as the market becomes almost convinced about the auditor's type which leads to a continuum of threshold equilibria. The desired "high effort" equilibrium, which is also socially efficient, occurs only under the restrictive assumption of perfect monitoring. Comparing a duopoly and a monopoly model, I show that the range of reputation for which diligence can be sustained (when cost of diligence is small) is larger under monopoly. Reputation incentives are further weakened in a duopoly set up when firms have private information about the quality of their projects.

The second chapter “Competition: Boon or Bane for Reputation Building Behavior” is a joint work with Yu Awaya. This paper investigates whether competition aids or hinders reputation building behavior in experience goods markets. We examine a market where long lived firms face a short term incentive to put low effort. There are two types of firms, “good” firms to whom high effort is costless and “opportunistic” firms who have to pay a small cost for high effort. We characterize the equilibrium strategies of a monopolist and a duopolist for a two period model. Contrary to the prevalent idea that competition improves reputation-building behavior we find that competition may hinder reputation building behavior by shrinking expected future payoffs. Horner (2002) talks about a perfectly competitive market and emphasizes the importance of outside options generated through competition. Our model on the other hand compares a duopoly model with a monopoly model in an environment of price competition. This paper focuses on how competition shrinks expected future payoff and reduces reputation incentives. We also examine the case where a planner can observe the hired firm's type and can dictate the chosen firm's actions. We show under such circumstances the duopolist's choice of effort always coincides with or falls below the effort level the planner prescribes.
The third chapter “Informal Insurance and Group Size Under Individual Liability Loans” is a joint work with Souvik Dutta. There has been a recent shift from joint liability to group loans with individual liability by the Grameen Bank and some other prominent micro lending institutions across the world. Under the joint liability lending mechanism a group of individuals were given a loan and individuals in a group were jointly liable for the loan given. Under the new lending regime a group of individuals are given their individual shares of a group loan. Although they have to be in a group in order to have access to the loan, individuals are not liable for the loan of other members in the group. An individual is only liable for her share of the loan. Some recent field experiments observed no change in repayment rates with this regime change. This paper investigates the role of informal insurance among group members to explain the success of group lending with individual liability. We consider a model with finite number of players (villagers) who interact repeatedly. Each villager can invest in a project that can be a success or a failure. Villagers simultaneously obtain a loan from the microcredit organization (bank) at a fixed interest rate. The bank specifies a punishment function which is increasing and convex in the amount of loan not repaid. Individuals have exogenous bilateral arrangements specifying the amount a successful individual transfers to an unsuccessful individual. The realization of output is private information and villagers pay back their loans in a public meeting. An individual’s true outcome is revealed with a positive probability in the meeting. We consider a Perfect Bayesian Equilibrium where players repay the entire amount to the bank and keep their promises if successful. We show that with informal insurance individual liability lending can lead to the same repayment rates as joint liability. However individuals’ welfare is strictly lower under individual liability lending. In addition, this paper also sheds light on the optimal group size that villagers should maintain under the new lending mechanism.
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DEDICATION

To my parents

Swapna and Swapan Kumar Basu
Chapter 1

Reputation for Two Audiences: Rating Agencies, Auditors and Issuer-Pays Markets

1.1 Introduction

Certification intermediaries are information intermediaries who play an important role in mitigating information risk for a buyer by acquiring information about a seller. The most interesting feature of many of these institutions is the two audience feature resulting from the issuer-pays payment structure. In an issuer pays payment structure the subject of opinion is the client itself. Examples of such certification intermediaries are: laboratories in markets for industrial products (a famous example is Underwriters Laboratory), schools rating the ability of students, investment banks and underwriters evaluating the quality of firms that want to raise capital etc. Bond-rating agencies and auditors are the most significant ones in the context of financial markets.

This paper presents a model of reputation in the context of issuer-pays markets. The model is particularly interesting because of its relevance in the domain of certification intermediaries in financial markets. Both the rating agencies and the big audit firms came under severe public scrutiny through lawsuits, government investigations, and media attention after the Enron scandal in 2001 and the financial crisis of 2007-2008. The Credit rating firms are partly blamed in the major corporate failures of the last decade for their lack of diligence in identifying credit problems. The auditors on the other hand are often alleged to be involved in major accounting scandals, Enron(2001) being the most prominent example, leading to the demise of its auditor Arthur Anderson.

The issuer-pays payment structure has been vehemently criticized by the regulators because of its inherent conflict of interest. The proponents of issuer-pays model however argue that the conflict of interest can be mitigated through reputation concerns. The underlying idea is that if buyers determine that a certification is of low quality, they will stop crediting the certification intermediary, and its business will lose value. Adherents of the reputational capital model believe
that market discipline, in the form of fear of loss of reputation, does or at least can provide the right incentives for high-quality certifications. They believe that poor performance is deterred by the prospect of loss of reputation, and accordingly do not believe that liability or other ex post legal remedies are an appropriate adjunct to the reputational mechanism. The standard positive theory of intermediary behavior yields the proposition that regulatory behavior in these markets are superfluous at best and distortionary at worst in terms of incentives to act diligently.

Another criticism of these markets is the high degree of market concentration. The rating agency market comprises of three big firms, Moody’s, S&P, and Fitch while the audit market is dominated by the "Big Four": PWC, KPMG, Ernst & Young, and Deloitte. The prevalent argument against market concentration is that competition stimulates reputation building behavior as dissatisfied clients have the option of switching auditors. Better reputation is typically associated with higher market shares and higher profits and competition is typically perceived to be a devise to sustain high effort.

For concreteness I discuss the problem in an audit market setting. High quality auditing is a central component of corporate governance and what leads to high quality auditing is an open question. The two forms of incentives that has been discussed in the Accounting literature is litigation incentives and reputation incentives. The litigation incentives work in a straightforward way by making the auditor legally liable for the opinion he issues. Reputation incentives on the other hand takes a rather longer and indirect way of reaching the auditor and the question of interest is whether auditors’ incentives to maintain reputation sufficiently powerful, absent litigation?

This paper seeks answers to the following questions. First, does even a well-functioning reputation mechanism provide enough incentives for the auditors to produce high quality auditing and lead to a socially efficient outcome? Second, does competition among the auditors indeed improve reputational incentives? In order to answer these questions I use a stylized model that solely focuses on the role of reputation as a devise to sustain high effort.

This model of reputation incorporates both unobservable types, i.e. whether the auditor is informative, and unobservable actions, that is, whether the informative auditor exerts due diligence to acquire more relevant information. In the model, risk neutral entrepreneurs or firms are endowed with projects and investors who are also risk neutral are endowed with cash. Investors
face a new firm each period as an investment prospect. The outcome of the project can be "good" or "bad" and the investor has no information about the quality of the project. There is an auditor who can acquire information about the quality of the project and is hired by the firm for a report or a rating. The auditor if hired receives his payment upfront. The quality of information depends on how much effort the auditor expends. The auditor has short term incentives for not being diligent as effort is costly and his revenue comes from the firm. However in order to have value to the firm, his reports must also appear credible to the investors.

The auditor can be “informative” or “uninformative” and the auditor’s type is private information. The uninformative auditor can not improve informativeness by being diligent. An informative auditor’s evaluation on the other hand is inherently more informative than that of an uninformative auditor and he can choose to improve precision through diligence. Reputation of the auditor has been modeled in the usual way, where reputation is both the investor and the firm’s posterior expectation that the auditor is “informative”.

Firms and investors only observe noisy signals about how much information the auditor acquires, and beliefs about the auditor’s reputation is revised using Bayes’ rule. If the auditor’s reputation is determined once and for all then the informative auditor has no incentive to exert diligence.

The main results of this paper are captured in a simple two period model. In a monopoly set up, under imperfect monitoring, the informative auditor is diligent only for a low range of reputation. The auditor has low incentives to expend effort when there is no immediate credible threat of termination and the point beyond which users prefer a non diligent “informative” auditor to an uninformative auditor. There can still be reputation gains from diligence through increased expected future payoff. However, gains from reputation shrink as the market becomes almost convinced about the auditor’s type which leads to a continuum of threshold equilibria. In a threshold equilibrium, the monopolist stops exerting effort beyond a threshold reputation. This result invalidates the claims of the reputational capital model, which predicts that the auditor will be diligent whenever he is hired. The desired "high effort" equilibrium, which is also socially efficient, occurs only under the restrictive assumption of perfect monitoring. This theory thus predicts that intermediary failures may occur even in the most successful certification markets.

In this paper I also analyze a duopoly model to investigate how competition affects reputation building behavior. The duopoly model has two identical auditors whose types are not known.
Each period only one auditor is hired by the firm. The hired auditor’s actions determine the probability that he is hired next period and the payoffs he receives if hired. Competition drives down the prices these auditors can charge in a Bertrand competition and hence reduces expected future payoff of the auditors. This in turn reduces their incentives to expend effort which leads to low quality of certification. The situation worsens when firms have private information about the quality of the project they own.

Preview: The rest of the paper is organized as follows. Section 1.2 discusses related research. Section 1.3 presents the dynamic model along with strategies, beliefs and equilibrium concept. Section 1.4 presents the results of the two period monopoly model and Section 1.5 analyses the two period duopoly model. Section 1.6 provides brief discussions on report contingent payments, investor pays vs. issuer pays model and also contains a discussion on the infinite horizon version of the model. Section 1.7 concludes.

1.2 Related Literature

This paper contributes to three strands of literature: the literature concerning auditors’ behavior, the literature concerning rating agency behavior and the reputation literature. The key modeling features of this paper are the two audience model, the reputation dynamics and competition among auditors.

My paper shares some modeling similarities with Narayanan (1994), Dye et al. (1990) and Dye (1993). Dye et al. (1990) look at a model of audit market and examine how auditors and their clients respond to report contingent audit contracts. Narayanan (1994) compares audit quality under “joint and several” liability rules and “proportionate liability” rules and suggests that switching from joint and several liability to proportionate liability may actually increase audit quality. Dye (1993) also builds a model of audit market relating auditor liability and auditing standards. He shows how auditing fees depend on the informational value of the audit and the option value of the claim financial statement’s users have on auditors wealth in the event the audit is determined to have been substandard.

Reputation dynamics for auditors has always been a question of interest to the Accounting researchers. In a recent paper Douglas and Skinner (2012) investigate the importance of auditors’
reputation for delivering quality using data from the litigation free country Japan, while my paper theoretically explores the reputation dynamics in a litigation free world. The reputation models in the auditors’ behavior literature are fairly new and largely unexplored. Corona, Randhawa (2010) studies a two period game of repeated interactions between a manager and an auditor. They investigate reasons and circumstances under which reputational concerns induce an audit firm to misreport. With long lived managers in their paper they also incorporate the flavor of relationship building in their paper. Unlike existing papers in the auditors’ behavior literature my paper also addresses the issue of competition in audit market with a dynamic model of reputation formation. The paper incorporates the following institutional features discussed in Sunder (2003). First, the paper builds on the assumption that outside independent auditors help inform investors about the integrity of financial statements. Second, the results of an audit service is both imperfectly correlated with audit quality and rarely observed by the managers and investors. The imperfect correlation aspect has been captured by a model of two sided imperfect monitoring in my paper. However my model does not explicitly assume that the outcome of an audit service is rarely observed. This feature can be easily incorporated by assuming that the probability with which a true outcome is revealed at the end of any period is strictly less than one. In this event the results will still go through and the additional assumption will only have a scale effect. In addition this exercise will give us insights on how probability of the truth being discovered affect reputation incentives. The results will provide a formal representation of Sunder’s (2003) argument against competition as a reputation building device. Third, the model also captures the switch to an analytical review intensive production function by introducing a choice of effort for an auditor who has private information. This paper builds on the features of private information and uncertainty (Akerlof, "Market for Lemons") by adding moral hazard to the model to formally analyze the reputation dynamics.

Papers that deal with rating agencies and need to be mentioned in reference to this paper are Skreta and Veldkamp (2009) and Sangiorgi and Spatt (2012). Skreta and Veldkamp show that increase in asset complexity leads to increase in credit shopping. Competition worsens the problem in their framework too. Sangiorgi and Spatt develop a rational expectations model where a debt issuer purchases credit ratings(s) to provide useful information to investors and attract demand. The issuer purchases rating(s) sequentially and decides which to disclose. Their analysis emphasizes the importance of opacity about the contracts between the issuer and rating
agencies, leading to potential asymmetric information by investors about which ratings have been obtained. My paper in contrast, rules out certification shopping and focuses on isolating the effect of competition with reputation being the sole motivator.

Two papers that are closest to my paper in terms of modeling similarities are by Mailath and Samuelson (2001) and Mathis, McAndrews and Rochet (2008). Mailath and Samuelson (2001) builds a model of reputation in the context of experience goods market which is a one-audience market. They examine a market in which firms face a moral hazard problem: they have a short-term incentive to exert low effort, but could earn higher profits if it were possible to commit to high effort. They view reputation as a commitment device that allows firms to solve the moral hazard problem. There are two types of firms. The “competent” firms have a choice between exerting high effort and low effort while the “inept” firms can only exert low effort. There are occasional exits and firms that replace the existing firm may be competent or inept. These replacements are crucial in sustaining a high effort equilibrium. My model assumes away the occasional exits and shows that a high effort equilibrium exists only under restrictive assumptions. This result is in similar spirit to the result obtained by Mailath and Samuelson in the context of one-audience markets. I also assume that the “informative” auditor is inherently better than the “uninformative” auditor even if he does not exert diligence. This feature of the model helps to avoid the no effort equilibrium observed in Mailath and Samuelson’s paper. In their paper, they assume that the “competent” firm produces the same quality of output as the “inept” firm produces, if he does not expend effort. This assumption is crucial for the “no effort” equilibrium they observe. Moreover they do not discuss issues related to competition in their paper.

Mathis, McAndrews and Rochet (2008) deal with reputation building behavior by a Credit Rating Agency in a dynamic set up. They build on two important papers by Benabou and Laroque (1992) and Diamond (1989). While the paper by Benabou and Laroque examines the behavior of a Guru who can influence the behavior of uninformed investors, the paper by Diamond studies why new firms with insufficient reputation have to borrow from banks before they can issue direct debt. In the paper by Mathis, McAndrews and Rochet each period a new cashless firm wants to issue security for financing a complex investment project. The project quality is a priori unknown, including to the issuer himself. A Credit Rating Agency can observe the quality of the project and communicate it to the market through a rating. They show in a
monopoly set up that reputation concern fails to prevent rating agencies from inflating ratings when rating complex products becomes a major source of income for the rating agency. My paper in some sense investigates a more fundamental issue by looking at the role of reputational incentives to find out the truth by exerting effort while Mathis, McAndrews and Rochet primarily investigate incentives to reveal the truth. By assuming upfront payment method instead of report-contingent payment I show that upfront payment may improve reputational incentives but does not provide complete cure to the problem. Moreover my analysis includes how competition among certification intermediaries affects reputational incentives to acquire information. I also formally model the case where firms have private information about the quality of their projects.

The vast literature on reputation that this paper relates to mostly deals with one-audience models. Holmstrom (1999) analyses a managerial incentive problem where the manager works to induce erroneous market beliefs that its ability type is higher than in reality. Initially the manager’s talent is incompletely known to both the manager and the market. There are no explicit output contingent contracts but wages depend on expected output which essentially depends on the market’s assessment of the manager’s talent. The manager takes unobservable actions which gives rise to an incentive problem. In Board and Meyer-ter-Vehn (2009), reputation is modeled directly as the market belief about quality which is persistent and depends stochastically on the firm’s past investments. They build their model in a continuous time framework. However quality improvements are made at discrete points of time. Quality at a point in time is determined by the firm’s investment at the most recent technology shock, between shocks quality is constant. My model follows the existing literature in terms of how reputation is modeled. But the model differs from existing models of reputation through the two audience feature of the issuer pays markets. The two audience feature generates a self absorbing state at the lower range of reputation and can potentially lead a negatively sloped return function. Reputation incentives depend on the probability of reaching the self absorbing along with how much the firm values an informative auditor.

Horner (2002) shows how competition generates reputation building behavior in the context of “experience goods” markets. While Horner talks about a perfectly competitive market and emphasizes on the importance of outside options generated through competition, my model compares a duopoly model with a monopoly setting in an environment of price competition. My paper focuses on how competition shrinks expected future payoff and reduces reputation
incentives for certification intermediaries.

While the reputation models largely focus on one audience markets, the two audience market models in the economic literature are predominantly static models. Bhattacharya and Ritter (1983) and Gertner, Gibbons and Scharfstein (1988) have this feature of two audiences in their models, however in a different context. They analyze situations where an asymmetrically informed agent is motivated to communicate its privately known attribute but can do so only through channels or signals which convey directly useful information to competing agents. This revelation to the competition serves to reduce the value of the private information held by the first agent. Bhattacharya and Ritter (1983) develop their model in the context of a set of firms engaged in research and development rivalry. Pursuing R&D activity requires firms to raise external financing in the capital market. The model is structured so that the only way the informed firm can communicate its prospects to the capital market is through the disclosure of technological information of direct usefulness to competitors. The informed firm therefore faces a tradeoff between (i) reducing the value of its informational advantage, and (ii) raising financing at better terms. In Gertner, Gibbons and Scharfstein (1988) when a firm reveals information to the capital market, it often does so by a publicly observable action (such as a dividend) that reveals information to otherwise uninformed agents in other markets (such as product-market rivals). These agents then condition their behavior on this information, and this affects the profit (gross of financing costs) of the informed firm. Both these models are essentially two audience signaling models built in a static framework. This paper in contrast talks about an auditor who manipulates reputation for two audiences in a dynamic framework.

1.3 Environment

Players, actions and payoffs: This is a discrete time two period model where time is indexed \( t = 1, 2 \). There are three types of players; firms, investors and auditors. Firms and investors are short-lived while auditors are long-lived. Each period a cashless firm wants to raise capital for financing a project it owns. The investor, who faces the firm as an investment prospect, is endowed with cash of amount \( w \). The auditor is equipped with an information acquiring technology which can acquire information about the quality of the project the firm owns.

Production takes place only if the entire \( w \) is invested in the project. The outcome of the project
can be good(G) or bad(B) and the project quality is a priori unknown to the investor and the firm itself. A good outcome occurs with probability p and returns to the investor $r_1$ is strictly positive. With probability $(1 - p)$ the outcome is bad in which case the project yields a negative return $r_2 \in [-1, 0)$, to the investor. $r_1$ and $r_2$ is exogenously given.

Firms’ gains as a function of investment is given by $f(.)$ where,

$$f(x) = \begin{cases} 1, & x = w \\ 0, & otherwise \end{cases}$$

Investors are risk neutral and they are faced with the following problem

$$\max_{a \in [0, 1]} w [\pi (1 + ar_1) + (1 - \pi)(1 + ar_2)]$$

where $\pi$ is the probability that the outcome is good and the maximization gives

$$a^* = \begin{cases} 1, & \pi r_1 + (1 - \pi)r_2 \geq 0 \\ 0, & \pi r_1 + (1 - \pi)r_2 < 0 \end{cases}$$

Define, $\overline{p}$ such that $\pi r_1 + (1 - \pi)r_2 = 0$. $\overline{p}$ is the posterior probability (that the outcome of the project is good) that makes the investor indifferent between investing all her wealth and not investing. Clearly, if $p \geq \overline{p}$ the firm is capable of getting investment in the auditor’s absence. However, for priors lower than $\overline{p}$ firms fail to obtain investment without certification. This paper focuses on the case where $p < \overline{p}$.

**Informativeness of the auditor:** The auditor’s information acquisition technology generates publicly observable signals $s \in \{g, b\}$, where $g$ signifies a good outcome and $b$, a bad one. Auditors are of two types “informative” and “uninformative”. The “uninformative” auditor’s signals carry no additional information as he receives signals $s \in \{g, b\}$ independent of the firm’s true type. The “uninformative” auditor is also the non-strategic type who generates $g$ with probability $p$ and $b$ with probability $1 - p$.

The “informative” auditor’s informativeness is captured by the parameters $\alpha$ and $\epsilon$, where $\alpha$ is
the probability that a bad project is given the signal $g$ (false positive) and $\epsilon$ is the probability that a good project is given $b$ (false negative). Assume that $\epsilon < (1 - \epsilon)\alpha$, i.e. ex-ante the informative auditor is more likely to commit an error when the project quality is bad. 

Notice that $\alpha$ and $\epsilon$ are the parameters that depict the extent to which information is distorted. Higher are the parameters lower is the informativeness of the auditor. Consequently, the higher the parameters the lower is the credibility of the signal $s$. Suppose that the informative auditor is informative enough so that $Pr(G|g) > \bar{p}$ which in turn implies $Pr(G|b) < \bar{p}$.

The informative auditor can improve precision by being diligent for which he has to bear a small cost $c > 0$. By being diligent the auditor passes a bad project as good with a lower probability $\alpha' \in (0, \alpha)$ and passes a good project as good with probability 1. Notice that the informative auditor’s signals are useful to the investor even if he is not diligent. In case of perfect monitoring $\alpha' = 0$, that is, the informative auditor can perfectly distinguish between a good project and a bad project.

At any period $t$, reputation of an auditor, denoted $\theta_t$ gives the probability that the auditor is “informative”.

Suppose $c > 0$ is small enough so that the cost of diligence is lower than the information gain, that is, the informative auditor exerting diligence is efficient for the society.

**Timeline:** The sequence of events is as follows. At the beginning of period 1 the auditor decides whether to be diligent by paying the cost $c$ and acquires information about the project quality of the firm. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor’s type using Bayes’ rule. The auditor posts

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1. This assumption ensures that the return function for the auditor is increasing in reputation and the auditor has incentives to improve reputation to begin with. However this assumption is not crucial as far as the results of the paper are concerned. Whether the return function is increasing or decreasing in reputation also depends on the probability with which the uninformative auditor generates the signal $g$. The above assumptions capture conditions conducive to reputation building behavior and they are in place only for the ease of exposition.
2. This is a simplifying assumption. The results do not change if the informative auditor commits an error even if he is diligent.
a price $P$ and the firm decides whether to hire the auditor\(^3\). If the auditor is hired the payment is made upfront. If the auditor is hired he decides whether to be diligent by paying cost $c$ and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest.

**Strategies and beliefs:** A stationary strategy for the auditor is a pair $(P, \gamma)$, i.e., a fee and a choice of effort. Formally, $\gamma : [0, 1] \rightarrow [0, 1]$, where $\gamma(\theta)$ is the probability that the auditor is diligent in period 1 and $P : [0, 1] \rightarrow \mathbb{R}$ gives the fee the auditor posts in period 2.

A firm’s strategy is a hiring function $h$ from $\mathbb{R} \times [0, 1] \rightarrow \{0, 1\}$, where $h = 0$ implies the firm does not hire the auditor and $h = 1$ implies the auditor is hired.

The investor’s investment strategy $a^* : [0, 1] \times \{g, b, \phi\} \rightarrow [0, 1]$ depicts how much the investor invests from a wealth $w$ as a function of reputation and observed signal $s$.

The belief function $\pi : [0, 1] \times \{g, b, \phi\} \rightarrow [0, 1]$ gives the probability that the project is good given signal $s$ and $\pi(\theta, s)$ is calculated using Bayes’ rule.

At the beginning of each period the investor and the firm observes a signal $s \in \{g, b, \phi\}$ and at the end of that period one of the following three outcomes is observed. A good $(G)$ outcome is observed if a good project is financed, a bad $(B)$ outcome is observed if a bad project is financed and no $(N)$ outcome is observed if no investment takes place. We denote by $\varphi(\theta|s, i)$ or $\varphi_i^G$ the posterior probability that the auditor is “informative” given signal $s \in \{g, b\}$, outcome $i \in \{G, B, N\}$ and prior probability $\theta$. If the “informative” auditor is diligent with probability $\gamma(\theta)$, then the posterior beliefs are

\[
\varphi^G_g(\theta) = \frac{\theta[\gamma(\theta) + (1 - \gamma(\theta))(1 - \epsilon)]}{\theta[\gamma(\theta) + (1 - \gamma(\theta))(1 - \epsilon)] + (1 - \theta)p}
\]

\[
\varphi^B_g(\theta) = \frac{\theta[\gamma(\theta)\alpha' + (1 - \gamma(\theta))\alpha]}{\theta[\gamma(\theta)\alpha' + (1 - \gamma(\theta))\alpha] + (1 - \theta)p}
\]

\(^3\)The focus of this paper is to analyze the impact of reputation incentives and reputation should be the sole motivator in my model. Therefore, to set aside the issue of price signaling I assume that the auditor does not post a fee in the first period.
\[
\varphi_h^\gamma(\theta) = \frac{\theta [p(1 - \gamma(\theta))e + (1 - p)\{\gamma(\theta)(1 - \alpha') + (1 - \gamma(\theta))(1 - \alpha)\}]}{\theta[p(1 - \gamma(\theta))e + (1 - p)\{\gamma(\theta)(1 - \alpha') + (1 - \gamma(\theta))(1 - \alpha)\}] + (1 - \theta)(1 - p)}
\]  

(3)

The maximum willingness to pay by the firm for an auditor’s certification is:

\[
v(\theta) = \theta [p\{\hat{\gamma}(\theta) + (1 - \hat{\gamma}(\theta))(1 - \epsilon)\} + (1 - p)\{\hat{\gamma}(\theta)\alpha' + (1 - \hat{\gamma}(\theta))\alpha\}] + (1 - \theta)p
\]

where, \( \theta \) is the auditor’s reputation and \( \hat{\gamma}(\theta) \) is the belief about the auditor’s choice of effort.

The equilibrium concept is Perfect Bayes. The equilibrium consists of a hiring strategy by the firm, a choice of effort and a fee by the auditor, an investment strategy by the investor, a posterior function and an updating rule such that, players maximize their payoffs and beliefs coincide with actions.

**Definition:** Equilibrium consists of a hiring strategy \( h \) by the firm, a choice of effort \( \gamma \) and a fee \( P \) by the auditor, an investment strategy \( a^* \) by the investor, a posterior function \( \pi \) and an updating rule \( \varphi \) such that,

1. \( h \) is optimal for the firm.
2. \( \gamma \) maximizes expected lifetime payoff for the auditor.
3. \( P \) maximizes period 2 payoff.
4. \( a^* \) is optimal for the investor.
5. \( \pi \) is obtained using Bayes’ rule.
6. \( \varphi \) satisfies (1)-(3).

**1.4 Analysis**

This section analyzes the monopoly model and characterizes equilibria under the assumption of imperfect monitoring and perfect monitoring.
Notice that the investor invests only if she observes $g$. The investor’s decision depends on reputation($\theta$) in the following way. There exists a $0 < \theta < 1$, such that the investor is indifferent between investing and not investing if the auditor is diligent and the observed signal is $g$. That is, $\pi(\theta|\gamma = 1) = \bar{p}$. Thus for all $\theta \in [0, \theta)$ the auditor is not hired even if he puts effort. This is a self absorbing state and the auditor can never make it to the market once he falls into this region. Thus, the auditor has no incentive to be diligent if his reputation falls in the region $\theta \in [0, \theta)$. There exists another important threshold reputation given by $\bar{\theta} \in (\theta, 1)$, such that, if the observed signal is $g$, the investor is indifferent between investing and not investing even if the auditor does not exert diligence. That is, $\pi(\theta|\gamma = 0) = \bar{p}$. Therefore, in equilibrium the auditor must exert diligence for the range $\theta \in [\bar{\theta}, \theta)$ in order to get hired. Now, whether the auditor puts effort beyond $\bar{\theta}$, is the question of interest. The desired “high effort” equilibrium that supports the reputation concern arguments is one where the auditor exerts diligence whenever he is hired by the firm.

The following three Lemmas characterize how reputation is revised following signals and outcomes.

**Lemma 1.1:** $\varphi^G_g(\theta) > \theta$ and $\varphi^N_g(\theta), \varphi^B_g(\theta) < \theta$.

Proof:

$$
\varphi^G_g(\theta) - \theta = \frac{\theta(1 - \theta)[\gamma(\theta) + (1 - \gamma(\theta))(1 - e) - p]}{\theta[\gamma \theta + (1 - \gamma(\theta)(1 - e)] + (1 - \theta)p} > 0
$$

$$
\varphi^B_g(\theta) - \theta = \frac{\theta(1 - \theta)[\gamma(\theta)\alpha' + (1 - \gamma(\theta)\alpha - p]}{\theta[\gamma \theta + (1 - \gamma(\theta)(1 - e)] + (1 - \theta)p} < 0
$$

$$
\varphi^N_g(\theta) - \theta = \frac{\theta(1 - \theta)[p(1 - \gamma(\theta)e + (1 - p)\{(\gamma(\theta)(1 - \alpha') + (1 - \gamma(\theta))(1 - \alpha))\} - (1 - p)]}{\theta[p(1 - \gamma(\theta)e + (1 - p)\{(\gamma(\theta)(1 - \alpha') + (1 - \gamma(\theta))(1 - \alpha))\} + (1 - \theta)(1 - p)} < 0
$$

Lemma 1.1 shows that reputation is revised upwards when a good signal is followed by a good outcome. On the other hand a bad signal and a good signal followed by a bad outcome leads to downwards revision of reputation.
Lemma 1.2: \( \varphi_g^B(\theta) - \varphi_b^N(\theta) < 0 \) when \( \gamma = 1 \) and \( \gamma = 0 \).

Proof:

With \( \gamma = 1 \),

\[
\varphi_g^B(\theta) = \frac{\theta \alpha'}{\theta \alpha + (1-\theta)p} \quad \text{and} \quad \varphi_b^N(\theta) = \frac{\theta(1-\alpha')}{\theta(1-\alpha') + (1-\theta)}.
\]

\[
\varphi_g^B(\theta) - \varphi_b^N(\theta) = \frac{\theta(1-\theta)[\alpha'(1-p) - p]}{[\theta \alpha' + (1-\theta)p][\theta(1-\alpha') + (1-\theta)]} < 0
\]

\[\Rightarrow \varphi_g^B(\theta) < \varphi_b^N(\theta)\]

With \( \gamma = 0 \),

\[
\varphi_g^B(\theta) = \frac{\theta \alpha}{\theta \alpha + (1-\theta)p} \quad \text{and} \quad \varphi_b^N(\theta) = \frac{\theta p + (1-p)(1-\alpha)}{\theta p + (1-p)(1-\alpha)(1-\theta)(1-\theta)p}
\]

\[
\varphi_g^B(\theta) - \varphi_b^N(\theta) = \frac{\theta(1-\theta)[\alpha - \alpha p^2 - \epsilon p^2 - p + p^2]}{[\theta p + (1-p)(1-\alpha)] + (1-\theta)(1-p)[\theta \alpha + (1-\theta)p]} < 0
\]

\[\Rightarrow \varphi_g^B(\theta) < \varphi_b^N(\theta)\]

Lemma 1.2 shows that reputation falls more when a good signal is followed by a bad outcome in comparison to the fall in reputation followed by a bad signal.

Lemma 1.3: Given \( \theta \),

\[
pe(\varphi_g^G(\theta) - \varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta) - \varphi_g^B(\theta))\bigg|_{\hat{\gamma} = 1} > pe(\varphi_g^G(\theta) - \varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta) - \varphi_g^B(\theta))\bigg|_{\hat{\gamma} = 0}
\]

Proof:

Notice that \( \varphi_g^G(\theta) \) is increasing in \( \gamma \) and \( \varphi_g^B(\theta) \) is decreasing in \( \gamma \), that is revisions are milder when the believed choice of effort is zero. For notational convenience let us denote the believed choice of effort as \( \gamma \) (instead of \( \hat{\gamma} \)) for the rest of the proof.
Let, $\varphi_b^N(\theta)|_{\gamma=0} - \varphi_b^N(\theta)|_{\gamma=1} = \Delta$.

Now if $pc > (1-p)(\alpha - \alpha')$, $\Delta > 0$.

Also, the difference between $\varphi_b^N(\theta)|_{\gamma=0} - \varphi_b^B(\theta)|_{\gamma=0}$ and $\varphi_b^N(\theta)|_{\gamma=1} - \varphi_b^B(\theta)|_{\gamma=0}$ equals $\Delta$ which is same as the difference between $\varphi_g^G(\theta)|_{\gamma=0} - \varphi_b^N(\theta)|_{\gamma=1}$ and $\varphi_g^G(\theta)|_{\gamma=0} - \varphi_b^N(\theta)|_{\gamma=0}$.

Since, $pc > (1-p)(\alpha - \alpha')$

$$pc(\varphi_g^G(\theta)-\varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta)-\varphi_b^B(\theta))|_{\gamma=1} > pc(\varphi_g^G(\theta)-\varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta)-\varphi_b^B(\theta))|_{\gamma=0}$$

Similarly, when $pc < (1-p)(\alpha - \alpha')$, $\Delta < 0$.

Also, the difference between $\varphi_b^N(\theta)|_{\gamma=0} - \varphi_b^B(\theta)|_{\gamma=0}$ and $\varphi_b^N(\theta)|_{\gamma=1} - \varphi_b^B(\theta)|_{\gamma=0}$ equals $\Delta$ which is same as the difference between $\varphi_g^G(\theta)|_{\gamma=0} - \varphi_b^N(\theta)|_{\gamma=1}$ and $\varphi_g^G(\theta)|_{\gamma=0} - \varphi_b^N(\theta)|_{\gamma=0}$.

Since, $pc < (1-p)(\alpha - \alpha')$

$$pc(\varphi_g^G(\theta)-\varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta)-\varphi_b^B(\theta))|_{\gamma=1} > pc(\varphi_g^G(\theta)-\varphi_b^N(\theta)) + (1-p)(\alpha - \alpha')(\varphi_b^N(\theta)-\varphi_b^B(\theta))|_{\gamma=0}$$

Suppose the auditor’s reputation is such that the auditor is hired in the second period with probability 1. Lemma 1.3 implies that at any such reputation $\theta$, the auditors expected payoff from exerting effort with $\hat{\gamma} = 1$, is strictly greater than his expected payoff from putting effort with $\hat{\gamma} = 0$.

**Proposition 1.1:** There exists $\overline{c} > 0$ such that for all $c \leq \overline{c}$ there exist $\theta^1 \in [0, \overline{\theta}]$ and $\theta^2 \in [\overline{\theta}, 1)$ so that the following pure strategy profile constitutes an equilibrium. At $t=2$, the auditor is never diligent. The auditor posts the fee $P(\theta) = \theta[p(1-c) + (1-p)\alpha] + (1-\theta)p$ and is hired only if $\theta \geq \overline{\theta}$. The investor invests only if $\theta \geq \overline{\theta}$ and $s = g$. At $t=1$, the auditor is not diligent, i.e. $\gamma(\theta) = 0$ for $\theta \in [0, \theta^1)$. $\gamma(\theta) = 1$ for $\theta \in [\theta^1, \theta^2)$ and $\gamma(\theta) = 0$ for $\theta \in [\theta^2, 1]$. The investor invests only if $\theta \geq \theta^1$ and $s = g$. 

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Proof:

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. His decision in that period does not affect his payoff in period 2. Thus, not putting effort in period 2 is a dominant strategy for the auditor. Now $P(\theta) = \theta[p(1-\epsilon) + (1-p)\alpha] + (1-\theta)p = v(\theta)$ at $t = 2$, that is the auditor extracts all the rents as he gets to post the price.

At $t = 1$, suppose the auditor exerts due diligence. With probability $p$ the informative auditor meets a good firm and moves to a higher level of reputation, say to $\theta_G = \varphi_{G}^{G}(\theta)$. With probability $(1 - p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha')$. No investment takes place and reputation is revised to $\theta_N = \varphi_{b}^{N}(\theta)$. With probability $\alpha'$ the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B = \varphi_{b}^{B}(\theta)$.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pv(\theta_G) + (1-p)[\alpha'v(\theta_B) + (1 - \alpha')v(\theta_N)] - c$.

Now if the auditor does not put effort in period 1, he moves to a higher level of reputation $\theta_G$ with probability $p(1 - \epsilon)$. With probability $\epsilon$ he makes a mistake and receives $s = b$ which is followed by no investment and reputation falls to $\theta_N$. With probability $(1 - p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha)$. No investment takes place and reputation is revised to $\theta_N$. With probability $\alpha$ the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B$.

Therefore, the expected payoff by not exerting effort in period 1 is given by, $p[(1 - \epsilon)v(\theta_G) + \epsilon v(\theta_N)] + (1 - p)[\alpha v(\theta_B) + (1 - \alpha)v(\theta_N)]$.

The auditor will put effort in period 1 if,

$$c \leq pe[v(\theta_G) - v(\theta_N)] + (1 - p)(\alpha - \alpha')[v(\theta_N) - v(\theta_B)]$$

(4)

The right hand side equals

$$[(1 - p)\alpha - pe][p(\theta_G - \theta_N) + (1 - p)(\alpha - \alpha'(\theta_N - \theta_B))] \text{ if } \theta_B \geq \bar{\theta}$$
Now from Lemma 1.1, \( \varphi_G^G(\theta) > \theta \) and \( \varphi_N^N(\theta), \varphi_B^B(\theta) < \theta \).

Consider the range \( \theta \in [0, \overline{\theta}] \). \( \varphi_N^N(\theta), \varphi_B^B(\theta) < \overline{\theta} \) for this range. Thus \( v(\theta_N) = v(\theta_B) = 0 \).

Also, there exists a \( \theta' < \overline{\theta} \) such that \( \varphi_G^G(\theta') = \overline{\theta} \).

Notice that, there does not exist \( c > 0 \) such that the auditor puts effort for \( \theta \in [0, \theta') \).

Now, as \( \theta \to 1 \), \( \varphi_G^G(\theta), \varphi_N^N(\theta), \varphi_B^B(\theta) \to 1 \) which implies that the right hand side of the above inequality goes to zero. Thus there exists a \( \theta^* < 1 \) such that for all \( \theta > \theta^* \) right hand side of inequality (4) is decreasing in \( \theta \).

Define, \( \overline{\theta} = \min_{\theta \in [\overline{\theta}, \theta']} [p_\epsilon [v(\varphi_G^G(\theta)) - v(\varphi_N^N(\theta))] + (1 - p)(\alpha - \alpha') (v(\varphi_N^N(\theta)) - v(\varphi_B^B(\theta))] \]

Fix \( c \leq \overline{\theta} \).

There exists \( \theta'' \in [\theta', \overline{\theta}] \) such that \( \gamma = 1 \) is optimal for the auditor for \( \theta \in [\theta'', \overline{\theta}] \). This is because \( v(\theta) \) is increasing.

Define, \( \theta^1 = \max\{\theta, \theta''\} \).

Now, with \( \gamma = 1 \), there exists a threshold \( \hat{\theta} > \overline{\theta} \), beyond which putting effort is not optimal for the auditor. Beyond this threshold \( c \) is strictly greater than \( p_\epsilon [v(\theta_G) - v(\theta_N)] + (1 - p)(\alpha - \alpha') (v(\theta_N) - v(\theta_B)] \).

Now if the investor believes that the informative auditor does not put effort then there exists another threshold \( \tilde{\theta} \) such that the auditor has no incentive to be diligent if \( \theta > \tilde{\theta} \).

From Lemma 1.3 we know that, given \( \theta \), \( p_\epsilon (\varphi_G^G(\theta) - \varphi_N^N(\theta)) + (1 - p)(\alpha - \alpha') (\varphi_N^N(\theta) - \varphi_B^B(\theta)) \) is increasing in \( \gamma \). Thus \( \tilde{\theta} < \hat{\theta} \).

\( \theta^2 \) can take any value between \( \tilde{\theta} \) and \( \hat{\theta} \) and the threshold equilibrium holds.

Therefore there exists a continuum of thresholds \( \theta^2 \in [\tilde{\theta}, \hat{\theta}] \) such that the strategy described in Proposition 1.1 is an equilibrium. \( \blacksquare \)

Proposition 1.1 crucially depends on the fact that there exists a range of \( \theta \) where the posterior exceeds \( \overline{\theta} \) even if the informative auditor does not exert due diligence. The informative auditor
can shirk in that range of reputation only if three conditions are satisfied. First, the investor invests even when she believes that the auditor is not diligent. Second, following a mistake, reputation does not fall below \( \bar{\theta} \), which is the threshold below which the auditor is not hired in period 2. If reputation does not fall below \( \bar{\theta} \) the auditor continues to operate in the market even after making a mistake. Third, effort is costly, i.e. the difference between future payoff generated by high effort and low effort is smaller than the cost of putting effort.

The auditor meets a firm with a good project with probability \( p \). By expending effort the auditor improves his chances of moving to a higher level reputation \( \varphi_g^G(\theta) \) instead of moving to a lower level of reputation \( \varphi_b^N(\theta) \). The gains from effort are captured by the difference between the expected capital raised at \( \varphi_g^G(\theta) \) and \( \varphi_b^N(\theta) \). The auditor meets a firm with a bad project with probability \( 1 - p \). By expending effort the auditor reduces his chances of moving to a even lower level of reputation \( \varphi_g^B(\theta) \). Gains from reputation is captured by the difference between the fee the auditor can charge by moving to the level of reputation \( \varphi_b^N(\theta) \) and the fee he can charge if he moves to a lower level of reputation \( \varphi_g^B(\theta) \) by passing a bad firm as good. The auditor is hired in the second period, only if his reputation is above \( \bar{\theta} \). Thus for any reputation \( \theta \) below \( \bar{\theta} \), from where \( \bar{\theta} \) can not be reached, the auditor is not diligent.

In equilibrium, for the auditor to be diligent, the difference between the value of choosing high effort and the value of choosing low effort must exceed the cost of choosing high effort. Suppose cost of expending effort is small. If the auditor is believed to be non-diligent, the value functions corresponding to high effort and low effort approach each other as \( \theta \to 1 \). This is because the values diverge only through the effect of current outcome on future posteriors. Current outcomes have very little effect on future posteriors if the market is quite sure of the auditor’s type. Thus for posteriors close to unity, if the market believes that the auditor is not diligent, no matter how small the cost is, the gains from being diligent is surpassed by the cost of putting effort which gives rise to the threshold equilibria.

Proposition 1.1 formally shows that even when cost of effort is small, reputation concerns fail to solve the moral hazard problem under imperfect monitoring when reputation of the auditor is high. However, a high effort equilibrium, in which the auditor is diligent whenever he is hired, can be sustained with small costs under a more restrictive and non-generic assumption of perfect monitoring.
**Perfect monitoring:** With the error structure being perfect monitoring, the informative auditor can perfectly distinguish between a good project and a bad project when he chooses to be diligent. The “informative” auditor’s informativeness is still captured by the parameters $\alpha$ and $\epsilon$, where $\alpha$ is the probability that a bad project is given the signal $g$ and $\epsilon$ is the probability that a good project is given $b$. The informative auditor can improve precision by being diligent for which he has to bear a small cost $c > 0$. By being diligent the auditor passes a good project as good and a bad project as bad with probability 1.

**Proposition 1.2:** There exists $\bar{c} > 0$ such that for all $c \leq \bar{c}$ there exist $\theta^1 \in [0, \bar{\theta})$ so that the following pure strategy profile constitutes an equilibrium. At $t=2$, the auditor is never diligent. The auditor posts the fee $P(\theta) = \theta[p(1-\epsilon) + (1-p)\alpha] + (1-\theta)p$ and is hired only if $\theta \geq \bar{\theta}$. The investor invests only if $\theta \geq \bar{\theta}$ and $s = g$. At $t=1$, the auditor is not diligent, i.e. $\gamma(\theta) = 0$ for $\theta \in [0,\theta^1)$. $\gamma(\theta) = 1$ for $\theta \in [\theta^1, 1)$ and $\gamma(\theta) = 0$ for $\theta = 1$. The investor invests only if $\theta \geq \theta^1$ and $s = g$.

**Proof:**

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. His decision in that period does not affect his payoff in period 2. Thus, not putting effort in period 2 is a dominant strategy for the auditor. Now at $t = 2$, $P(\theta) = v(\theta)$, that is, the auditor extracts all the rents.

At $t = 1$, suppose the auditor is diligent. With probability $p$ the informative auditor meets a good firm and moves to a higher level of reputation, say to $\theta_G = \varphi_g^G(\theta)$. With probability $(1-p)$ he meets a bad firm and receives signal $s = b$ with probability 1. No investment takes place and reputation is revised to $\theta_N = \varphi_b^N(\theta)$.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pv(\theta_G) + (1-p)v(\theta_N) - c$.

Now if the auditor does not put effort in period 1, he moves to a higher level of reputation $\theta_G$ with probability $p(1-\epsilon)$. With probability $\epsilon$ he makes a mistake and receives $s = b$ which is followed by no investment and reputation falls to $\theta_N$. With probability $(1 - p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha)$. No investment takes place and reputation is revised to $\theta_N$. With probability $\alpha$ the auditor makes a mistake and receives $s = g$.
The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B = \varphi^B_g(\theta)$.

Therefore, the expected payoff by not exerting effort in period 1 is given by, $p[(1 - \epsilon)v(\theta_G) + \epsilon v(\theta_N)] + (1 - p)[(1 - \alpha)v(\theta_N) + \alpha v(\theta_B)]$.

The auditor will put effort in period 1 if,

$$c \leq pe[v(\theta_G) - v(\theta_N)] + (1 - p)\alpha[v(\theta_N) - v(\theta_B)]$$

$$= [(1 - p)\alpha - pe][\alpha v(\theta_G - \theta_N) + (1 - p)\alpha(\theta_N - \theta_B)] \text{ if } \theta_N \geq \bar{\theta}$$

Consider the range $\theta \in [0, \bar{\theta}]$. $\varphi^N_g(\theta), \varphi^B_g(\theta) < \bar{\theta}$ for this range. Thus $v(\theta_N) = v(\theta_B) = 0$.

Also, there exists a $\theta' < \bar{\theta}$ such that $\varphi^G_g(\theta') = \bar{\theta}$.

Notice that, there does not exist $c > 0$ such that the auditor puts effort for $\theta \in [0, \theta')$.

Also notice that with $\hat{\gamma} = 1$ and $\alpha' = 0$, $\varphi^B_g(\theta) = \theta_B = 0$.

Now, as $\theta \to 1$, right hand side of the above inequality goes to $(1 - p)\alpha[(1 - p)\alpha - pe] > 0$.

Also, $pe[v(\theta_G) - v(\theta_N)] + (1 - p)\alpha[v(\theta_N) - v(\theta_B)]$ is increasing in $\theta$ for $\theta \in [\theta', 1)$

Define $\bar{c} = pev(\varphi^G_g(\bar{\theta}))$.

Fix $c \leq \bar{c}$.

There exists $\theta'' \in [\theta', \bar{\theta}]$ such that $\gamma = 1$ is optimal for the auditor for $\theta \in [\theta'', \bar{\theta}]$. This is because $v(\theta)$ is increasing.

Define, $\theta^1 = \max\{\theta, \theta''\}$.

Therefore for $c \leq \bar{c}$, the auditor has incentive to be diligent whenever hired and the “high effort” equilibrium holds.

The auditor meets a firm with a good project with probability $p$. In this event, gains from reputation approaches zero as $\theta \to 1$. Now the auditor meets a firm with a bad project with
probability \(1 - p\) in which event, reputation gains may not diminish as \(\theta\) approaches 1. This is sustained by the market belief that the informative auditor never makes a mistake. If he chooses to be non diligent, there is always a positive probability of making a mistake and moving to a lower reputation \(\varphi^B_g(\theta)\). If the firm and the investor believes that the informed auditor never makes a mistake then \(\varphi^B_g(\theta) = 0\). Therefore the difference between the expected capital raised at \(\varphi^G_g(\theta)\) and \(\varphi^N_b(\theta)\) is positive for values of \(\theta\) for which the expected capital raised at \(\varphi^N_b(\theta)\) is positive. As long as the expected cost of this mistake is higher than the cost of being diligent, the informative auditor will choose to be diligent. The mere fear of losing the market does not allow him to be lazy. At \(\theta = 1\), learning stops and the optimal action for the auditor is not to exert diligence at that level of reputation. However, \(\theta = 1\) is never reached if reputation in the first period is strictly less than 1.

The pure strategy equilibrium of Proposition 1.2 is the high effort equilibrium one might be interested in and this equilibrium arises only under the restrictive assumption of perfect monitoring coupled with a particular way of belief revision. However it is not the only pure strategy equilibrium in the perfect monitoring framework. There exist a continuum of threshold equilibria as described in Proposition 1.1. Equilibria with higher values of \(\theta^2\) are more efficient in the sense that they support high effort over a larger range of reputation.

1.5 Competition Among Auditors

Our analysis so far assumed monopoly of the auditor and no competition from other auditors in this market was assumed in the model. We now shift our attention to a duopoly market with two competing auditors, one investor, and one firm. The new model keeps the key aspects of the monopoly framework presented in section 1.3 and gives rise to similar threshold equilibria as described in Proposition 1.1. Equilibria with higher values of \(\theta^2\) are more efficient in the sense that they support high effort over a larger range of reputation.

Consider two identical auditors, auditor 1 and auditor 2 with the same reputation \(\theta\). Each of these auditors can be informative or uninformative. In each period, the firm must hire only one auditor. The analysis in this section does not focus on report(rating) shopping aspect of the issuer pays markets. The focus of the analysis on the other hand is to capture the effect of competition on reputation incentives when auditors compete for clients.
Timeline: The sequence of events is as follows. At the beginning of period 1, the auditor who has been hired by the firm decides whether to be diligent by paying the cost \( c \) and acquires information about the project quality. He receives signal \( s \in \{g, b\} \) depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor’s type using Bayes’ rule.

At the beginning of period 2, the investor and the firm believes that the auditors’ are informative with probability \( \theta \) and \( \theta' \) respectively. The auditors post prices \( P \) and \( P' \) respectively and the firm decides which auditor to hire. The hired auditor receives his fee upfront. The hired auditor decides whether to be diligent by paying the cost \( c \) and acquires information about the project quality. He receives signal \( s \in \{g, b\} \) depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest.

Strategies and beliefs: A stationary strategy for the auditor is a pair \((P, \gamma)\), i.e., a fee and a choice of effort. Formally, \( \gamma : [0, 1] \times [0, 1] \to [0, 1] \), where \( \gamma(\theta, \theta') \) is the probability that the hired auditor is diligent in period 1 which is a function of his own reputation and his rival’s reputation. Similarly, \( P : [0, 1] \times [0, 1] \to \mathbb{R} \) gives the fee an auditor posts in period 2.

A firm’s strategy is a hiring function \( h \) from \( \mathbb{R} \times \mathbb{R} \times [0, 1] \times [0, 1] \to \{0, 1, 2\} \), where \( h = 0 \) implies the firm does not hire any auditor, \( h = 1 \) implies that auditor 1 is hired and \( h = 2 \) implies that auditor 2 is hired.

The investor’s investment strategy \( a^* : [0, 1] \times \{g, b, \phi\} \to [0, 1] \) depicts how much the investor invests from a wealth \( w \) as a function of reputation of the hired auditor and the observed signal \( s \).

The belief function \( \pi : [0, 1] \times \{g, b, \phi\} \to [0, 1] \) gives the probability that the project is good given signal \( s \) and \( \pi(\theta, s) \) is calculated using Bayes’ rule.

If the “informative” auditor is diligent with probability \( \gamma(\theta) \), then the posterior beliefs are updated using (1)-(3)
The expected capital raised by the firm for an auditor’s certification is:

\[ v(\theta) = \theta[p\{\hat{\gamma}(\theta) + (1 - \hat{\gamma}(\theta))(1 - \epsilon)\} + (1 - p)\{\hat{\gamma}(\theta)\alpha' + (1 - \hat{\gamma}(\theta))\alpha\}] + (1 - \theta)p \]

where, \( \theta \) is the auditor’s reputation and \( \hat{\gamma}(\theta) \) is the belief about the auditor’s choice of effort.

**Definition:** Equilibrium consists of a hiring strategy \( h \) by the firm, a choice of effort \( \gamma \) and a fee \( P \) by an auditor, an investment strategy \( a^* \) by the investor, a posterior function \( \pi \) and an updating rule \( \varphi \) such that,

1. \( h \) is optimal for the firm.
2. \( \gamma \) maximizes expected lifetime payoff for the auditor.
3. \( P \) maximizes period 2 payoff.
4. \( a^* \) is optimal for the investor.
5. \( \pi \) is obtained using Bayes’ rule.
6. \( \varphi \) satisfies (1)-(3).

**Proposition 1.3:** There exists \( \bar{c} > 0 \) such that for all \( c \leq \bar{c} \) there exist \( \theta^1 \in [0, \bar{\theta}] \) and \( \theta^2 \in [\bar{\theta}, 1) \) so that the following pure strategy profile constitutes an equilibrium.

At \( t=2 \), an auditor is never diligent. An auditor whose reputation is \( \theta \) and whose rival’s reputation is \( \theta^0 \) posts a fee

\[ P(\theta, \theta^0) = \begin{cases} \max\{0, (\theta - \theta^0)(1 - p)\alpha - pc\} & \text{if } \theta^0 \geq \bar{\theta} \\ v(\theta) & \text{otherwise} \end{cases} \]

An auditor is hired only if \( \theta \geq \bar{\theta} \) and expected capital raised by hiring that auditor is higher than that of his rival. The investor invests only if \( \theta \geq \bar{\theta} \) and \( s = g \).

At \( t=1 \), the auditor is not diligent, i.e. \( \gamma(\theta) = 0 \) for \( \theta \in [0, \theta^1) \). \( \gamma(\theta) = 1 \) for \( \theta \in [\theta^1, \theta^2) \) and \( \gamma(\theta) = 0 \) for \( \theta \in [\theta^2, 1] \). The investor invests only if \( \theta \geq \theta^1 \) and \( s = g \).
Proof:

At $t = 2$, if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. His decision in that period does not affect his payoff in period 2. Thus, not putting effort in period 2 is a dominant strategy for the auditor. Now $P(\theta) = \max\{0, v(\theta) - v(\theta')\}$ at $t = 2$, that is, the hired auditor can post a positive fee only if his reputation is higher than his rival’s reputation. If the auditor with reputation $\theta$ posts any price above $P(\theta)$, his rival can cut price and get hired by the firm.

Now, at $t = 1$, suppose the auditor exerts due diligence. With probability $p$ the informative auditor meets a good firm and moves to a higher level of reputation, say to $\theta_G = \varphi^G_g(\theta)$. With probability $(1 - p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha')$. No investment takes place and reputation is revised to $\theta_N = \varphi^N_b(\theta)$. With probability $\alpha'$ the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B = \varphi^B_g(\theta)$.

Therefore, the expected payoff by exerting effort in period 1 is given by, $pP(\theta_G) + (1 - p)[\alpha'P(\theta_B) + (1 - \alpha')P(\theta_N)] - c$.

If the auditor does not put effort in period 1, he moves to a higher level of reputation $\theta_G$ with probability $p(1 - \epsilon)$. With probability $p\epsilon$ he makes a mistake and receives $s = b$ which is followed by no investment and reputation falls to $\theta_N$. With probability $(1 - p)$ he meets a bad firm and receives signal $s = b$ with probability $(1 - \alpha)$. No investment takes place and reputation is revised to $\theta_N$. With probability $\alpha$ the auditor makes a mistake and receives $s = g$. The investor invests and the true quality of the project is revealed. This pushes reputation down to $\theta_B$.

Therefore, the expected payoff by not exerting effort in period 1 is given by, $p[(1 - \epsilon)P(\theta_G) + \epsilon P(\theta_N)] + (1 - p)[\alpha P(\theta_B) + (1 - \alpha)P(\theta_N)]$.

The auditor will put effort in period 1 if,

$$c \leq p\epsilon[P(\theta_G) - P(\theta_N)] + (1 - p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)]$$

Since, from Lemma 1.1 we know that, $\varphi^G_g(\theta) > \theta$ and $\varphi^N_b(\theta), \varphi^B_g(\theta) < \theta$, $P(\theta_N) = P(\theta_B) = 0$ and

$$p\epsilon[P(\theta_G) - P(\theta_N)] + (1 - p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)] = p\epsilon[v(\theta_G) - v(\theta)]$$
Now,
\[ p[c(v(\theta G) - v(\theta))] = p(c(\theta G - \theta))(1 - p)\alpha - pc \quad \text{if } \theta \geq \bar{\theta}. \]

Consider the range \( \theta \in [0, \bar{\theta}] \). \( \varphi^N_b(\theta), \varphi^B_g(\theta) < \bar{\theta} \) for this range.

There exists a \( \hat{\theta} < \bar{\theta} \) such that \( \varphi^G_g(\hat{\theta}) = \bar{\theta} \).

Notice that, there does not exist \( c > 0 \) such that the auditor puts effort for \( \theta \in [0, \hat{\theta}) \).

Also, as \( \theta \to 1 \), \( \varphi^G_g(\theta) \to 1 \) which implies that the right hand side of the above inequality goes to zero. Thus there exists a \( \theta^{**} < 1 \) such that for all \( \theta > \theta^{**} \) right hand side of inequality (6) is decreasing in \( \theta \).

Define, \( \bar{c} = \min_{\theta \in [\bar{\theta}, \theta^{**}]} [pc(v(\varphi^G_g(\theta)) - v(\theta))] \).

Fix \( c \leq \bar{c} \).

There exists \( \theta'' \in [\tilde{\theta}, \bar{\theta}] \) such that \( \gamma = 1 \) is optimal for the auditor for \( \theta \in [\theta'', \bar{\theta}] \). This is because \( v(\theta) \) is increasing.

Define, \( \theta^1 = \max\{\tilde{\theta}, \theta''\} \).

Now, there exists a threshold \( \hat{\theta} > \bar{\theta} \), above which putting effort is not optimal for the auditor when \( \hat{\gamma} = 1 \).

Also if the investor believes that the informative auditor does not put effort then there exists another threshold \( \tilde{\theta} \) such that the auditor has no incentive to be diligent if \( \theta > \tilde{\theta} \).

From Lemma 1.3 we know that, given \( \theta \), \( \varphi^G_g(\theta) \) is increasing in \( \gamma \). Thus \( \tilde{\theta} < \hat{\theta} \).

\( \theta^2 \) can take any value between \( \tilde{\theta} \) and \( \hat{\theta} \) and the threshold equilibrium holds.

Therefore there exists a continuum of thresholds \( \theta^2 \in [\tilde{\theta}, \hat{\theta}] \) such that the strategy described in Proposition 1.3 is an equilibrium.

Proposition 1.3 characterizes the equilibria in a duopoly set up. Notice that the high effort equilibrium does not exist in the duopoly set up. No matter how small cost of effort is competition
leads to threshold equilibria as described in Proposition 1.3. The next proposition addresses our second research question: does competition aid or inhibit reputation building behavior?

Consider the range of $c$ for which the “threshold” equilibria as described in Proposition 1.1 and Proposition 1.3 holds for both monopoly and duopoly. The following proposition shows for any such $c$ the range of reputation for which high effort can be sustained under monopoly is strictly larger than the range for which high effort can be sustained under duopoly. Define, 

$$c^M = \min_{\theta \in [\bar{\theta}, \theta^*]} [pe[v(\varphi_g^G(\theta)) - v(\varphi_b^N(\theta))] + (1 - p)(\alpha - \alpha')[v(\varphi_b^N(\theta)) - v(\varphi_b^B(\theta))]$$

and

$$c^D = \min_{\theta \in [\bar{\theta}, \theta^{**}]} [pe[v(\varphi_g^G(\theta)) - v(\theta)]]$$

where $\theta^*$ and $\theta^{**}$ are the way they have been defined in Proposition 1.1 and 1.3 respectively. Also define, $\overline{c} = \min\{c^M, c^D\}$.

**Proposition 1.4:** For $c \leq \overline{c}$, the range of reputation for which $\gamma = 1$ can be sustained in equilibrium is larger under monopoly.

**Proof:**

Define, $\theta'$ such that $\varphi_g^G(\theta') = \bar{\theta}$.

Under monopoly, the auditor is diligent at $\theta'$ if $c \leq pev(\bar{\theta})$.

Under duopoly, the auditor is diligent at $\theta'$ if $c \leq peP(\bar{\theta})$.

Now, $P(\bar{\theta}) = v(\bar{\theta})$.

Define, $\theta^1 = \max\{\bar{\theta}, \theta'\}$ as before.

Thus if $c \leq pev(\bar{\theta})$ the auditor will be diligent for $[\theta^1, \bar{\theta}]$.

Now, consider the range $\theta \in [\bar{\theta}, 1]$ and suppose that the auditor is diligent.

Define, $\theta^N$ such that $\varphi_b^N(\theta^N) = \bar{\theta}$

Fix $\theta$. From Proposition 1.3 we know that,

$$pe[P(\theta_G) - P(\theta_N)] + (1 - p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)] = pe(\theta_G - \theta)(1 - p)(\alpha - \epsilon)$$

From Proposition 1.1, we know
where, \( \theta_G = \varphi^G_g(\theta) \), \( \theta_N = \varphi^N_b(\theta) \) and \( \theta_B = \varphi^B_g(\theta) \).

Now, \( \varphi^N_b(\theta) < \theta \) and with \( \gamma = 1 \),

\[
\varphi^B_g(\theta) = \frac{\theta \alpha'}{\theta \alpha' + (1-\theta)p} \quad \text{and} \quad \varphi^N_b(\theta) = \frac{\theta(1-\alpha')}{\theta(1-\alpha') + (1-\theta)}.
\]

From Lemma 1.2, we know that \( \varphi^B_g(\theta) < \varphi^N_b(\theta) \).

Thus, the right hand side of equation (7) is equal to the right hand side of equation (8) for the range \( \theta \in [\bar{\theta}, \theta^N] \) and is strictly greater for the range \( [\theta^N, 1) \).

Therefore, the maximum range for which \( \gamma = 1 \) can be sustained in equilibrium is larger under monopoly. ■

Notice that, no matter what \( c \) is, reputation incentives for a monopolist is captured by \( pe[v(\theta_G) - v(\theta_N)] + (1-p)(\alpha - \alpha')[v(\theta_N) - v(\theta_B)] \) while the reputation incentives for a duopolist is captured by \( pe[v(\theta_G) - v(\theta_B)] \). Since, \( \theta_B < \theta \) and \( v(\theta) \) is increasing, gains from building reputation is always greater under monopoly. The above result is counter intuitive in some sense as reputation incentives are lowered under competition. The prevalent argument in favor of competition is, competition stimulates reputation building behavior as dissatisfied clients have the option of switching auditors. The very fear of losing customers to rivals is what drives reputation building behavior. Better reputation is typically associated with higher market shares and higher profits and competition is typically perceived to be a devise to sustain high effort.

The argument in support of the above result is as follows. In these kinds of markets competition among certification intermediaries is often manifested in the form of price competition and not quality competition. In order to attract clients, the fee the duopolists charge in equilibrium are strictly lower than the fee they would charge under monopoly. Thus, competition may reduce expected future profit of an auditor and hence reduce incentives to put effort. In a setting as described in my model, an auditor can best reap the benefits generated by building reputation when he is not faced with price competition.
In equilibrium, for the auditor to be diligent, the difference between the value of choosing high effort and the value of choosing low effort must exceed the cost of choosing high effort. With probability $p$ the auditor meets a firm with a good project. By expending effort the auditor improves his chances of moving to a higher level reputation $\varphi_g^G(\theta)$. By moving to a higher level of reputation the auditor can charge a positive fee which equals the difference between the expected capital the firm can raise by hiring the auditor and the expected capital the firm can raise by hiring his rival (whose reputation is $\theta$). Under the monopoly set up the fee that the auditor can charge at $\varphi_g^G(\theta)$ equals the expected capital raised at that level of reputation. Gains from reputation under monopoly is captured by the difference between the fee the auditor can charge by moving to a higher level of reputation $\varphi_g^G(\theta)$ and the fee he can charge if he moves to a lower level of reputation $\varphi_b^N(\theta)$ by committing a mistake. The fee that the duopolist can charge if he moves to a lower level of reputation $\varphi_b^N(\theta)$ is zero, as his rival (who has a higher reputation $\theta$) can cut price and still charge a positive fee. Thus the gains from building reputation is captured by the difference between the expected capital raised at $\varphi_g^G(\theta)$ and $\varphi_b^N(\theta)$ in a monopoly set up, while the gains from building reputation under duopoly is captured by the difference between the expected capital raised at $\varphi_g^G(\theta)$ and $\theta$.

With probability $(1 - p)$ the auditor meets a firm with a bad project. By expending effort the auditor reduces his chances of moving to a even lower level of reputation $\varphi_g^B(\theta)$. Gains from reputation under monopoly is captured by the difference between the fee the auditor can charge by moving to the level of reputation $\varphi_b^N(\theta)$ and the fee he can charge if he moves to a lower level of reputation $\varphi_g^B(\theta)$ by passing a bad firm as good. The fee that the duopolist can charge if he moves to any of these levels of reputations, $\varphi_b^N(\theta)$ or $\varphi_g^B(\theta)$ is zero, as his rival (who has a higher reputation $\theta$) can cut price and still charge a positive fee. The gains from building reputation under duopoly disappears fully while the gains from building reputation under monopoly is still positive and is captured by the difference between the expected capital raised at $\varphi_b^N(\theta)$ and $\varphi_g^B(\theta)$. The presence of a rival leads to a reduction in fees that the auditors can charge and this in turn reduces their incentive to expend effort.

1.5.1 Firms Have Private Information About Their Own Type

Firms and investors may be asymmetrically informed about the quality of the project owned by the firm. This section deals with a situation when firms have private information about the
project quality. Auditors do not possess prior information about the project quality and can acquire information only if hired by the firm. Thus auditors can not price discriminate and have to post a single fee for the firms. The fee can be a pooling fee or a separating fee.

Suppose the firms are of two types, the high type or H firms and the low type or L firms. These types can be perceived as firms’ virtual types, as there is still some uncertainty about the firms’ true project quality. The true project quality can either be good or bad. A high type firm produces a good outcome with probability \( \rho \in (p, 1) \) and a bad outcome with probability \( 1 - \rho \). A low type firm on the other hand produces a good outcome with probability \( \rho' < p \) and a bad outcome with probability \( 1 - \rho' \). The firm has private information about its own type. However the firm does not know the true quality of the project it owns. Whether the project is good or bad is revealed after the investor invests in it.

In period 2, the H firm’s maximum willingness to pay for an auditor with reputation \( \theta \) is

\[
P_H(\theta) = \theta[\rho(1 - \epsilon) + (1 - \rho)\alpha] + (1 - \theta)p
\]

The L firm’s maximum willingness to pay for an auditor with reputation \( \theta \) on the other hand is

\[
P_L(\theta) = \theta[\rho'(1 - \epsilon) + (1 - \rho')\alpha] + (1 - \theta)p
\]

Suppose that \( \rho'(1 - \epsilon) + (1 - \rho')\alpha < p < \rho(1 - \epsilon) + (1 - \rho)\alpha \), i.e. a H firm receives the signal \( g \) from an informative auditor with a higher probability than it does from an uninformative auditor. The assumption also ensures that the L firm receives the signal \( g \) from an informative auditor with a lower probability than it does from an uninformative auditor.\(^4\) Notice that \( P_H(\theta) > P_L(\theta) \) for \( \theta \geq \theta_d \). Therefore, \( P_L \) is the pooling price at which both the H firms and the L firms hire an auditor. An auditor can charge the separating price \( P_H \) in which event he is only hired by the H firms. The H firms hire an auditor as there is no other way they can signal their type and investors invest only if the signal is \( g \). This is because there is still some uncertainty about the firms’ type and the signal \( s \) is the only source of information to the investor. Now, at a particular \( \theta \), which fee gives higher revenue to an auditor depends on the proportion of high type firms in the economy. Let \( x \) be the probability of meeting a high type firm every period.

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\(^4\)This assumption is both plausible and innocuous. It is in place for the ease of exposition.
Thus, \( x\rho + (1 - x)\rho' = \rho \implies x = \frac{\rho - \rho'}{\rho - \rho'}. \) Now, revenue of an auditor from posting the separating price is \( x\theta[p(1 - \epsilon) + (1 - \rho)\alpha] + x(1 - \theta)p, \) which is increasing in \( \theta. \) Revenue from posting the pooling price on the other hand is \( \theta[\rho'(1 - \epsilon) + (1 - \rho')\alpha] + (1 - \theta)p \) which is decreasing in \( \theta. \)

For \( \theta \geq \theta', \) define \( R(\theta) \) such that,

\[
R(\theta) = \max\{x\theta[p(1 - \epsilon) + (1 - \rho)\alpha] + x(1 - \theta)p, \theta[\rho'(1 - \epsilon) + (1 - \rho')\alpha] + (1 - \theta)p\}
\]

Consider two identical auditors, auditor 1 and auditor 2 with the same reputation \( \theta. \) Each of these auditors can be informative or uninformative. Because of the presence of a rival the only fee an auditor can post in equilibrium is the pooling fee.

**Timeline:** The sequence of events is as follows. At the beginning of period 1 an auditor, who has been hired by the firm in period 1, decides whether to be diligent by paying the cost \( c \) and acquires information about the project quality. He receives signal \( s \in \{g, b\} \) depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period 2 the firm and the investor updates their belief about the auditor’s type using Bayes’ rule.

At the beginning of period 2, the investor and the firm believes that the auditors are informative with probability \( \theta \) and \( \theta' \) respectively. The firm privately observes its own type. The auditors post prices \( P \) and \( P' \) respectively and the firm decides which auditor to hire. The hired auditor receives his fee upfront. The hired auditor decides whether to be diligent by paying the cost \( c \) and acquires information about the project quality. He receives signal \( s \in \{g, b\} \) depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest.

**Strategies and beliefs:** A stationary strategy for the auditor is a pair \((P, \gamma)\), i.e., a fee and a choice of effort. Formally, \( \gamma : [0, 1] \times [0, 1] \to [0, 1], \) where \( \gamma(\theta, \theta') \) is the probability that the hired auditor is diligent in period 1 as a function of his own reputation and his rival’s reputation. Similarly, \( P : [0, 1] \times [0, 1] \to \mathbb{R} \) gives the fee an auditor posts in period 2.
A firm’s strategy is a hiring function $h$ from $\mathbb{R} \times \mathbb{R} \times [0, 1] \times [0, 1] \times \{H, L\} \rightarrow \{0, 1, 2\}$, where $h = 0$ implies the firm does not hire any auditor, $h = 1$ implies that auditor 1 is hired and $h = 2$ implies that auditor 2 is hired.

The investor’s investment strategy $a^* : [0, 1] \times \{g, b, \phi\} \rightarrow [0, 1]$ depicts how much the investor invests from a wealth $w$ as a function of reputation of the hired auditor and the observed signal $s$.

The belief function $\pi : [0, 1] \times \{g, b, \phi\} \rightarrow [0, 1]$ gives the probability that the project is good given signal $s$ and $\pi(\theta, s)$ is calculated using Bayes’ rule.

If the “informative” auditor is diligent with probability $\gamma(\theta)$, then the posterior beliefs are updated using (1)-(3)

where, $\theta$ is the auditor’s reputation and $\gamma(\theta)$ is the belief about the auditor’s choice of effort.

*Definition:* Equilibrium consists of a hiring strategy $h$ by the firm, a choice of effort $\gamma$ and a fee $P$ by an auditor, an investment strategy $a^*$ by the investor, a posterior function $\pi$ and an updating rule $\varphi$ such that,

1. $h$ is optimal for the firm.
2. $\gamma$ maximizes expected lifetime payoff for the auditor.
3. $P$ maximizes period 2 payoff.
4. $a^*$ is optimal for the investor.
5. $\pi$ is obtained using Bayes’ rule.
6. $\varphi$ satisfies (1)-(3).

**Proposition 1.5:** There exists $\bar{\varepsilon} > 0$ such that for all $c \leq \bar{\varepsilon}$ there exist $\theta^1 \in [0, \bar{\theta}]$ and $\theta^2 \in [\bar{\theta}, 1)$ so that the following pure strategy profile constitutes an equilibrium.

At $t=2$, an auditor is never diligent. An auditor whose reputation is $\theta$ and whose rival’s reputation is $\theta'$ posts a fee

$$P(\theta, \theta') = \begin{cases} \max\{0, (\theta' - \theta)[\rho'(1 - \epsilon) + (1 - \rho')(1 - p)]\} & \text{if } \theta' \geq \bar{\theta} \\ R(\theta) & \text{otherwise} \end{cases}$$
An auditor is hired only if \( \theta \geq \bar{\theta} \) and expected capital raised by hiring that auditor is higher than that of his rival. The investor invests only if \( \theta \geq \bar{\theta} \) and \( s = g \).

At \( t=1 \), the auditor is not diligent, i.e. \( \gamma(\theta) = 0 \) for \( \gamma(\theta) = 0 \) for \( \theta \in [0, \theta^1) \). \( \gamma(\theta) = 1 \) for \( \theta \in [\theta^1, \theta^2) \) and \( \gamma(\theta) = 0 \) for \( \theta \in [\theta^2, 1] \). The investor invests only if \( \theta \geq \theta^1 \) and \( s = g \).

Proof:

At \( t = 2 \), if the auditor is hired the firm pays him first and then the auditor decides whether to be diligent. Thus, not putting effort in period 2 is a dominant strategy for the auditor.

Now consider the range \( \theta \in [\bar{\theta}, 1] \)

If \( \theta' < \bar{\theta} \) the auditor posts a monopoly price \( R(\theta) \).

If \( \theta' \geq \bar{\theta} \), the auditor faces his rival and \( P(\theta) = \max \{0, P_L(\theta') - P_L(\theta)\} = \max \{0, (\theta' - \theta)[\rho'(1 - \epsilon) + (1 - \rho')\alpha - p]\} \). The hired auditor can post a positive fee only if his reputation is lower than his rival’s reputation.

Now, at \( t = 1 \), suppose the auditor exerts due diligence.

Therefore, the expected payoff by exerting effort in period 1 is given by, \( pP(\theta_G) + (1 - p)[\alpha' P(\theta_B) + (1 - \alpha')P(\theta_N)] - c \) where, \( \theta_G = \varphi^G_g(\theta) \), \( \theta_N = \varphi^N_b(\theta) \) and \( \theta_B = \varphi^B_g(\theta) \).

Now if the auditor does not put effort in period 1, his expected payoff is given by, \( p(1 - \epsilon)P(\theta_G) + \epsilon P(\theta_N)] + (1 - p)[\alpha P(\theta_B) + (1 - \alpha)P(\theta_N)] \).

The auditor will put effort in period 1 if,

\[
c \leq p[\epsilon[P(\theta_G) - P(\theta_N)] + (1 - p)(\alpha - \alpha')[P(\theta_N) - P(\theta_B)]\]

Now define, \( \hat{\theta} \) such that \( \varphi^B_g(\hat{\theta}) = \bar{\theta} \) and consider \( \theta \geq \hat{\theta} \).

Since, \( \varphi^G_g(\theta) > \theta \) and \( \varphi^B_g(\theta) < \varphi^N_b(\theta) < \theta \), \( P(\theta_G) - P(\theta_N) < 0 \) and \( P(\theta_N) - P(\theta_B) < 0 \)

Therefore, for any \( c > 0 \) \( \gamma = 1 \) can not be sustained in equilibrium for \( \theta \geq \hat{\theta} \).

Consider the range \( \theta \in [0, \bar{\theta}] \). \( \varphi^N_b(\theta), \varphi^B_g(\theta) < \bar{\theta} \) for this range.
Define $\bar{\theta} < \bar{\theta}$ such that $\varphi^C_g(\bar{\theta}) = \bar{\theta}$.

Notice that, there does not exist $c > 0$ such that the auditor puts effort for $\theta \in [0, \bar{\theta})$.

Let $g(\theta)$ be the equilibrium expected gain of an auditor by being diligent.

Let $\bar{P} = \min_{\theta \in [\bar{\theta}, \bar{\theta}]} g(\theta)$.

Fix $c \leq \bar{P}$.

There exists $\theta'' \in [\bar{\theta}, \bar{\theta}]$ such that $\gamma = 1$ is optimal for the auditor for $\theta \in [\theta'', \bar{\theta}]$.

Define, $\theta^1 = \max\{\bar{\theta}, \theta''\}$.

Set $\theta^2 = \bar{\theta}$ and this concludes the proof. ■

The presence of a rival makes it impossible for a duopolist to post any fee above the pooling fee. The return function that the duopolist takes into account is guided by the pooling fee which is decreasing in reputation. For a lower range of reputation close to $\theta^1$ the gains from reputation is same as the gains from reputation of a monopolist. This is because the rival may drop out in the second period because of low reputation. Now let us focus on the range of reputation where an auditor faces his rival in period 2 with probability 1, that is, no auditor’s reputation falls below $\bar{\theta}$. We are particularly interested in this range of reputation as we are interested in finding out the impact of competition on reputation building behavior. For this range the relevant return function is guided by the pooling fee which is a decreasing function of reputation.

The auditor meets a firm with a good project with probability $p$. By expending effort the auditor improves his chances of moving to a higher level reputation $\varphi^C_g(\theta)$. By moving to a higher level of reputation the auditor does not gain anything as the fee he can charge is zero. His rival (whose reputation is $\theta$) on the other hand can charge a positive fee which equals the difference between the expected capital a L type firm can raise by hiring the rival and the expected capital a L type firm can raise by hiring the auditor. The fee that the duopolist can charge if he moves to a lower level of reputation $\varphi^N_h(\theta)$ is positive and equals the difference between the expected capital a L type firm can raise by hiring the auditor and the expected capital a L type firm can raise by hiring his rival (who has a higher reputation $\theta$).
The auditor meets a firm with a bad project with probability $1 - p$. By expending effort the auditor reduces his chances of moving to a even lower level of reputation $\varphi^B_g(\theta)$. If he moves to a lower level of reputation $\varphi^B_g(\theta)$ by passing a bad firm as good the fee he can post equals the difference between the expected capital a L type firm can raise by hiring the auditor and the expected capital a L type firm can raise by hiring his rival (who has a higher reputation $\theta$). This is higher than the fee he can charge if he moves to $\varphi^N_g(\theta)$ by not committing a mistake. The fee the duopolist can charge at $\varphi^N_g(\theta)$ is difference between the expected capital a L type firm can raise by hiring the auditor at $\varphi^N_g(\theta)$ and the expected capital a L type firm can raise by hiring his rival (who has a higher reputation $\theta$).

Thus for the range of reputation where both auditors remain in the market in period 2 with probability 1, the hired auditor in period 1, has no incentive to put effort. The intuitive explanation for the result is as follows. The lower range of reputation where the rival drops out in period 2 the auditor acts like a monopolist and may find it profitable to post a separating fee in the next period. The range of reputation where the auditor faces his rival in period 2 for sure if he is in the market, but he himself has a higher probability of dropping out next period if he is not diligent, then he will have incentive to put effort. But when he faces his rival for sure and is also sure that he himself will not have to drop out for not being diligent, the auditor stops putting effort. When firms have private information about the project quality they own the auditors compete to attract clients and can never charge a fee above the pooling fee and end up attracting the bad clients along with the good ones. The H firms pay much less than their maximum willingness to pay and pricing decision is solely driven by the maximum willingness to pay by the L type clients. This hampers reputation incentives by making the return function decreasing in reputation. When firms and investors are symmetrically informed, as cost of being diligent goes to zero the threshold beyond which effort can not be sustained in equilibrium approaches unity. However when firms have private information about the project quality, the duopolist stops putting effort beyond the threshold as described in Proposition 1.5. No matter how small the cost of being diligent is sustaining high effort beyond that threshold is not possible in equilibrium.
1.6 Discussion

This section provides discussions on issues that are closely related to these two audience markets. Also these are issues that are frequently discussed when it comes to the understanding of the certification intermediary markets. Subsection 6.1 and 6.2 talks about report contingent payments and issuer-pays vs. the investor pays model. The discussions help us understand these issues as well as the functioning of these markets in light of the model presented in section 1.3. In subsection 1.6.3 I present a model with infinite horizon and discuss why the results of my model are robust.

1.6.1 Report Contingent Payments

Bolton et al. (2008) suggested regulatory intervention requiring upfront payments for rating services (before CRAs propose a rating to the issuer) combined with mandatory disclosure of any rating produced by CRAs to substantially mitigate the conflicts of interest of both CRAs and issuers. In the audit market up front payment is a regular practice. This section discusses how upfront payment can mitigate conflicts of interest when it comes to the choice of effort by an auditor in an issuer pays market. Though upfront payment can substantially weaken adverse effects of conflict of interest it can not eliminate the moral hazard problem embedded in these two sided markets.

Consider the following report contingent payment structure. The firm pays the auditor only if the signal is $g$ and no payment is made if the signal is $b$.

**Timeline:** The sequence of events is as follows. At the beginning of period 1 the auditor posts a price $P$ and the firm decides whether to hire the auditor. If the auditor is hired he decides whether to be diligent by paying cost $c$ and acquires information about the project quality. He receives signal $s \in \{g, b\}$ depending on his choice of effort and the signal is observed by the investor and the firm. The investor decides how much to invest. The firm pays $P$ to the auditor if and only if $s = g$.

If the investor invests her entire wealth, then the true project quality is revealed at the end of period 1. The investor does not observe project quality if no investment takes place. In period
the firm and the investor updates their belief about the auditor’s type using Bayes’ rule. The auditor posts a price \( P \) and the firm decides whether to hire the auditor. If the auditor is hired, he decides whether to be diligent by paying the cost \( c \) and acquires information about the project quality. He receives signal \( s \in \{g, b\} \) depending on his choice of effort and the signal is observed by the investor. The investor then decides how much to invest. The firm pays \( P \) to the auditor if and only if \( s = g \) and the game ends.

**Analysis:** In each period, an auditor who is a monopolist posts a fee \( P = 1 \), that is, the auditor extracts all the rent when \( s = g \). Notice that, now the choice of effort not only affects expected future payoff, but also affects current payoff of the auditor. In the one shot game the informative auditor puts effort if \( c \leq (1 - p)(\alpha' - \alpha) + p\epsilon \). Suppose, \((1 - p)(\alpha - \alpha') - p\epsilon > 0\), that is, the auditor has short term incentive to shirk. Therefore, in the second period, as there is no reputation concern the auditor will not put effort. In the first period, the auditor has to face a short term loss in order to have reputation gain in the second period. Now in the second period, the expected payoff of an informative auditor who knows his own type is independent of reputation as long as the auditor is hired by the firm. Therefore if the auditor is hired in the second period, \( EP(\theta) = p(1 - \epsilon) + (1 - p)\alpha \) for \( \theta \geq \overline{\theta} \). Also, there exists \( \hat{\theta} \in (\overline{\theta}, 1) \) such that \( \varphi_2^B(\hat{\theta}) = \overline{\theta} \). Hence, the auditor never exerts diligence in the first period for \( \hat{\theta} \) no matter how small \( c \) is. On the other hand as \( c \to 0 \), the range of reputation for which high effort can be sustained in the first period approaches unity with a upfront payment structure.

In a duopoly set up with two auditors, one firm and one investor, the auditor with a higher net expected payoff is hired. In case of a tie the auditor who receives \( s = g \) with a higher probability is hired by the firm. Now notice that in period 2, if the auditor with a higher reputation charges \( P = 1 \), the auditor with lower reputation can cut price and get hired by the firm. Thus, the price the higher reputation auditor must charge in period 2 must be such that the expected net gain from hiring the auditor with higher reputation is same as that of hiring the auditor with lower reputation. Hence the expected second period payoff of the auditor with a higher reputation is the difference between the probability of him generating \( g \) and the probability that the lower reputation auditor generates \( g \). Thus report contingent payments generate the same reputation incentives as upfront payments in terms of expected payoff in period 2. However the upfront payment structure performs better than the report contingent payment structure by not
allowing the choice of effort affect the current payoff of the auditor.

1.6.2 Issuer Pays vs Investor Pays

First, it is important to explain what investor pays means in these kinds of two sided markets where issuer pays is the rule. Investors are the ones who gain the most from the information value of the opinion issued by the auditors. In a litigation free world, investors bear all the risks of investment. It is often difficult to identify investors as one entity for publicly held companies. However, if the investor is a single entity there are several ways in which the investor can enter into a contract with the auditor.

In an “investor pays” model the investor hires the auditor who acquires information about the firm the investor wants to invest in. The auditor can be of two types “informative” and “uninformative”. The “informative” auditor’s informativeness is captured by the parameters $\alpha$ and $\epsilon$, where $\alpha$ is the probability that a bad project is given the signal $g$ and $\epsilon$ is the probability that a good project is given $b$. Assume that $\epsilon < (1 - p)\alpha$, i.e. ex-ante the informative auditor is more likely to commit an error when the project quality is bad. Without loss of generality, suppose that $r_2 = -1$, that is the investor loses all her money if the project is bad. The investor’s gain from the auditor being diligent is bounded above by $w[(1 - p)(\alpha - \alpha') + p\epsilon r_1]$. Thus, whenever $c > w[(1 - p)(\alpha - \alpha') + p\epsilon r_1]$ it is not socially optimal for the auditor to exert diligence. Consequently, the auditor is hired only at a high level of reputation that is for $\theta \geq \overline{\theta}$.

The one shot game: Suppose the auditor is “informative” and the type is public information. In an issuer pays model with upfront payment the auditor is never diligent. In the “issuer pays” model the firm hires the auditor as long as $\alpha$ and $\epsilon$ is such that $Pr(G|g) > \overline{\theta}$ and $Pr(G|b) < \overline{\theta}$. The investor invests her entire wealth whenever $s = g$.

However under the “investor pays” model there exists $\overline{\epsilon} > 0$ such that for $c \leq \overline{\epsilon}$ the investor can design a contract contingent on outcomes, that can provide incentives for the auditor to exert diligence. Let $y \in \{-1, 0, 1\}$ denote the outcome in one period. $y = 1$, if $s = g$ and the outcome is good. $y = 0$, if $s = b$ and no investment is made. $y = -1$, if $s = g$ and the outcome is bad. Let $T(y)$ be the payment to the auditor when outcome is $y$. Now consider the following
contract, $T(1) = x'$, $T(0) = x$ and $T(-1) = 0$. For the auditor to exert diligence, the following condition must hold

$$px' + (1-p)(1-\alpha')x \geq p(1-\epsilon)x' + (1-p)(1-\alpha)x + p\epsilon x$$

That is,

$$p\epsilon (x' - x) + (1-p)(\alpha - \alpha')x \geq c$$

Any combination of $x$ and $x'$ that satisfies the above inequality will make the auditor expend effort even if his type is known to be informative.

**Dynamic contract:** Now consider the situation where the type of the auditor is not known. The prior probability that the auditor is informative is given by $\theta_0$. At $\theta$, the investor is indifferent between hiring and not hiring the auditor if the auditor exerts diligence.

Since, at $t = 2$ there exists a contract that can induce effort the state contingent contract at $t = 1$ can be of the following form. At $t = 1$, a state contingent contract is a map from $[0,1] \times \{-1,0,1\}$ to $\mathbb{R} \times \Psi$ which specifies a transfer from the investor and a contract for period 2. Let $\Psi$ be the set of incentive compatible and individually rational contracts at $t = 2$. To look for an efficient contract is not the purpose of this exercise. The objective of this discussion is to argue for the existence of contracts that are incentive compatible and individually rational.

Consider the following contract. At $t=1$, the investor pays according to the transfers specified in the one shot game. Any outcome that pushes reputation $\theta$ below $\theta_0$ is followed by the auditor not being hired at $t = 2$. Any outcome that keeps reputation $\theta$ above $\theta_0$ is followed by the same state contingent transfers as the one shot game at $t = 2$. By offering the contract described above the investor can make sure that the auditor exerts effort in both periods for the range of reputation $\theta \in [\theta_0, 1]$. Thus under investor pays model with state contingent contracts the range of reputation for which effort can be sustained in equilibrium is larger.

Now suppose there are frictions that prohibit the investor from entering into state contingent contract with the auditor. However the investor can pay the auditor upfront for the information
the auditor acquires about the project owned by the firm. The auditor makes a take it or leave it
offer by posting a fee. In this kind of set up where the return function of the auditor is increasing
in reputation, reputation concern may fail to incentivize effort after a threshold reputation and
lead to qualitatively similar outcomes as the issuer pays model.

1.6.3 Infinite Horizon:

This section aims to provide some intuition for why the results of the two period model have
similar implications as one would get under an infinite horizon model. This is an interesting
extension of the existing two period model. However this model does not provide us with any
new insights. Hence the following discussion abstains from providing a rigorous analysis. In this
model, time proceeds in discrete steps, indexed by \( t = 1, 2, \ldots \) and has an infinite horizon. Each
period, emerges a new firm that can yield a good outcome or a bad outcome. The auditor is
long lived and has a discount factor \( \delta \in (0, 1) \).

Timeline: The sequence of events is as follows. At the beginning of period \( t \), the investor and
the firm believes that the auditor is informative with probability \( \theta_t \). The firm which does not
know its own type decides whether to hire the auditor and if the firm decides to hire the auditor
it makes its payments in advance and the payment equals its maximum willingness to pay for
the auditor. The auditor then decides whether to be diligent by paying the cost \( c \) and acquires
information about the project quality. He receives signal \( s \in \{ g, b \} \) depending on his choice of
effort and the signal is observed by the investor. The investor then decides how much to invest.
If the investor invests her entire wealth, then the true quality of the project is revealed at the
end of period \( t \). Reputation is revised using Bayes’ rule.

Strategies and beliefs: A Markov strategy for an informative auditor at any \( t \) and \( \theta_t \)
is a map \( \gamma_t : [0, 1] \rightarrow [0, 1] \). Where, \( \gamma_t \) is the probability of exerting due diligence. A firm’s
strategy is a hiring function \( h \) from \( [0, 1] \rightarrow \{0, 1\} \), where \( h = 0 \) implies the firm does not
hire the auditor and \( h = 1 \) implies the auditor is hired. The investor’s investment strategy
\( a^* : [0, 1] \times \{ g, b, \phi \} \rightarrow [0, 1] \) depicts how much the investor invests from a wealth \( w \) as a function
of reputation and observed signal \( s \). The Markov belief function \( \pi : [0, 1] \times \{ g, b, \phi \} \rightarrow [0, 1] \)
gives the probability that the project is good given signal \( s \) and \( \pi(\theta, s) \) is calculated using Bayes’ rule.

At the beginning of each period the investor and the firm observes a signal \( s \in \{g, b, \phi\} \) and at the end of that period one of the following three outcomes is observed. A good(\( G \)) outcome is observed if a good project is financed, a bad(\( B \)) outcome is observed if a bad project is financed and no(\( N \)) outcome is observed if no investment takes place. At any \( t \), the history of outcomes until period \( t \), is denoted \( y^t = (y_1, y_2, ..., y_t) \) and reputation of the auditor \( \theta_t \) is common knowledge. Reputation at period \( t+1 \), \( \theta_{t+1} = \theta_{t+1}(\theta_t, \^y_t, y^t) \) is a function of reputation at period \( t \), the firm and the investor’s common belief about the competent auditors’ strategies and the history of outcomes until period \( t \). \( \theta_{t+1} \) is updated using Bayes’ rule. \( \varphi(\theta|s, i) \) or \( \varphi^i_s \) is the posterior probability that the auditor is “informative” given signal \( s \in \{g, b\} \), outcome \( i \in \{G, B, N\} \) and prior probability \( \theta \). If the “informative” auditor is diligent with probability \( \gamma(\theta) \), then the posterior beliefs are calculated using (1)-(3).

A Markov perfect equilibrium for a given prior \( p \) consists of a hiring strategy \( h \) by the firm, a choice of effort \( \gamma \) by the auditor, an investment strategy \( \alpha^* \) by the investor, a posterior function \( \pi \) and an updating rule \( \varphi \) such that,

1. \( h \) is optimal for the firm.
2. \( \gamma \) maximizes expected lifetime payoff for the auditor.
3. \( \alpha^* \) is optimal for the investor.
4. \( \pi \) is obtained using Bayes’ rule.
5. \( \varphi \) satisfies (1)-(3).

Let \( V(\theta) \) be the continuation payoff of the auditor at reputation \( \theta \). An auditor with reputation \( \theta \) will put effort if,

\[
c \leq \delta p e[V(\theta_G) - V(\theta_N)] + \delta (1 - p)(\alpha - \alpha') [V(\theta_N) - V(\theta_B)]
\]

where \( \theta_G = \varphi^G(\theta) \), \( \theta_N = \varphi^N(\theta) \) and \( \theta_B = \varphi^B(\theta) \).
Notice that the continuation payoff $V(\theta)$ is bounded as $\delta < 1$. Clearly, under imperfect monitoring, as $\theta \to 1$ the revisions become small and $\theta_G$, $\theta_N$, $\theta_B \to 1$. Therefore, the right hand side of the above inequality approaches zero. Thus there exists a threshold reputation beyond which effort can not be sustained in equilibrium. However with perfect monitoring high effort can be sustained in equilibrium for the range of $\theta \in [\bar{\theta}, 1)$ for $c$ small enough.

Now under the duopoly model, there are two infinitely lived auditors whose reputation at $t = 1$ is same. Notice that the per period return for the duopolists are lower than that of the monopolist. This is because of the presence of the rival. The duopolist whose rival is in his close vicinity can charge a lower price than the monopolist and reputation gains are also smaller. The lower per period payoffs leads to the flattening of the return function which in turn reduces reputation incentives.

1.7 Conclusion

This paper provides an analysis of the issuer pays markets with the help of a simple model of reputation. The analysis focuses on an environment with upfront payments and publicly observable signals and seeks answers to the following questions. Does even a well-functioning reputation mechanism provide enough incentives for the auditors to produce high quality certification? Does competition among auditors improve reputational incentives? These are questions concerning the fundamentals of audit market and other issuer pays markets.

I show that in a monopoly set up reputation may provide incentives to generate effort under certain circumstances, but not always. In a monopoly set up an auditor may have incentives to be non-diligent for very high levels of reputation even if the cost of diligence is small. The range of reputation where the market is quite sure about the auditor’s type the auditor may have incentives to shirk. The high effort equilibrium where the auditor exerts diligence whenever he is hired, is a fragile equilibrium and it holds only under the restrictive assumption of perfect monitoring. The threshold equilibria on the other hand do not depend on the error structure and they hold under both perfect monitoring and imperfect monitoring.

The answer to the second question to the readers’ surprise is a No. Competition among auditors does not necessarily improve reputation building behavior. Instead, competition may reduce the
incentive to put effort by shrinking long term expected profit. In the two period duopoly model I show that the range of reputation for which diligence can be sustained (when costs are small) is larger under monopoly. Reputation incentives are further weakened in a duopoly set up when firms have private information about the quality of their projects.
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Chapter 2

Competition: Boon or Bane for Reputation Building Behavior

2.1 Introduction

On one hand, competition enhances reputation-building behavior, because dissatisfied clients have the option of switching firms. The fear of losing customers to rivals forces firms to keep their reputations. This view is formulated by Klein and Leffler (1981), and later Horner (2002) and Vial (2010).

However, there is another effect that works in an opposite direction. Competition reduces expected future payoff which precludes a firm from putting high effort. Kranton (2003) points out the second effect. By means of numerical examples, Bar-Issac (2005) shows that competition has non-monotone effects to reputational commitments for quality.\(^5\)

In this paper, we provide an analytical framework and explicitly derive conditions for which competition aids reputation-building behavior and for which it hinders. This is a simple two-period model. The economy consists of long-lived seller(s) and short-lived buyers. In each period, every seller who is matched with a buyer produces an indivisible good, which can be either of high or of low quality. There are two types of sellers: good or opportunistic. A good seller always produces high-quality goods without any cost. An opportunistic seller incurs a positive cost to produce high-quality goods and may or may not mimic a good seller.

We compare two cases, namely, *monopoly* and *duopoly*. In case of monopoly, there is only one long-lived seller. In the first period, the seller has a buyer and he decides on the quality (if he is opportunistic). In the second period, the seller posts price. A potential buyer shows up with probability half. She observes the quality of the product in the first period as well as the price, and then decides whether to purchase from the seller.

In case of duopoly, there are two ex ante identical long-lived sellers who compete for the buyers. In the first period, a seller has a buyer while the other doesn’t. A seller who has a buyer decides...\(^5\)Bar-Issac and Tadelis (2008) provide a convenient survey.
on the quality of output. In the second period, both sellers post prices. In this case, sellers engage in Bertrand competition. Then a buyer shows up. Hence, the ratio of sellers and buyers are the same regardless of monopoly or duopoly. She observes the quality of the product that a seller made in the first period as well as the prices. Then she decides whether to purchase from a seller, and if so, from whom.

The main finding of the paper is as follows. First, if the ex ante probability of seller(s) being good is more than half, then monopoly outperforms duopoly in terms of effort level. Second, if the ex ante probability of seller(s) being good is less than half, then the effort level depends on the difference of utilities of consuming high and low quality. If the difference is sufficiently high, the monopoly effort level is higher than the duopoly effort level, while if it is low then the opposite is true.

In addition we also show that the effort choice of a duopolist either coincides or falls below the effort choice prescribed by a benevolent social planner with minimum control.

2.2 The Model

Environment: Consider a market in which firms and consumers repeatedly trade. Time is discrete and there are only two periods indexed \( t = 1, 2 \). Firms are long lived while a new consumer arrives in every period. The consumer may trade with only one firm in a given period. In this event the consumer pays upfront and enjoys a product whose quality depends on the unobserved effort level exerted by the chosen firm. The product can be of high (H) quality or of low (L) quality yielding utility levels \( v_h \) and \( v_l \) respectively. \( v_h > v_l \geq 0 \), where the utility from not consuming the good is normalized to be zero. Let \( q \in \{h, l\} \) denote the quality of output. Firms are of two types, good or opportunistic, which is private information. A firm is good with probability \( \alpha > 0 \) and firms’ types are drawn independently. Good firms produce high quality goods at no cost while opportunistic firms choose quality. For opportunistic firms cost of producing high quality output is given by \( c > 0 \) while the cost of producing low quality output is normalized to zero. We assume that \( \Delta v = v_h - v_l > c \), that is, it is economically efficient for the opportunistic firm to produce the high quality output.

Timeline: The sequence of events is as follows.
In period 1, the consumer decides from whom to buy and firms decide on the quality of the product.

In period 2, the new consumer observes the quality of the good produced in the last period and updates her belief about the firm’s type. Firms post prices (in case of duopoly there is Bertrand competition). The consumer then decides from whom to buy.

**Equilibrium concept:** In equilibrium,

1. Firms choose effort level in the first period and prices in the second period to maximize their payoff.
2. The consumer behaves optimally in the second period given her belief.
3. Beliefs are updated using Bayes rule.
4. Beliefs coincide with actions.

### 2.2.1 Monopoly

In the monopoly model there is one long lived firm and a new consumer in each period. In period 1, the consumer decides whether to buy from the monopolist and the monopolist decides on the quality of the product if the consumer is interested in trade. In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the firm’s type. The monopolist posts a price. The consumer then decides whether to buy from the monopolist.

**Strategies:** A stationary strategy for the firm is a pair \((P, x)\), i.e. a price and a choice of effort. Formally, \(x : [0, 1] \rightarrow [0, 1]\), where \(x(\alpha)\) is the probability that the opportunistic firm exerts effort in period 1 and \(P : [0, 1] \rightarrow \mathbb{R}\) gives the fee the firm posts in period 2.

A consumer’s strategy is whether to buy from a firm given her belief about the firm’s type in the second period and the price the firm posts in the second period.
Belief update: Let $\phi(\alpha, q)$ be the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior $\alpha$ and the quality of output $q \in \{h, l\}$.

Clearly, $\phi(\alpha, l) = 0$. This is because the good firm produces a high quality output at no cost and the good firm is assumed to be non-strategic, that is, the good firm only produces a high quality output. Therefore, whenever the consumer observes a low quality output she concludes that the producer of the good is of the opportunistic type. When $q = h$, the belief update function obtained using Bayes rule is as follows

$$
\phi(\alpha, H) = \frac{\alpha}{\alpha + (1-\alpha)x}
$$

where, $\alpha$ is the prior probability that the firm is good and $x$ is the probability that the opportunistic firm exerts effort.

Analysis: Let us first consider the price posting behavior of the firm in period 2. In equilibrium, the firm posts a price that makes the consumer indifferent between purchasing the good and not purchasing it. The firm can do better if the price is such that the consumer strictly prefers trade to autarky. Hence the price must equal the expected utility the consumer obtains from the transaction and the price is given by,

$$
P = \phi(\alpha, q)v_h + (1 - \phi(\alpha, q))v_l
$$

Given the the belief update rule and the price posting behavior described above, the firm’s gains from putting effort and producing a high quality good in the first period is given by

$$
\frac{\alpha}{\alpha + (1-\alpha)x}v_h + \left(1 - \frac{\alpha}{\alpha + (1-\alpha)x}\right)v_l - c.
$$

Instead if the the firm does not put effort in period 1 and produces a low quality output, the consumer becomes sure that the firm is an opportunistic type. In this event the only price the firm can post in period 2 is $v_l$. The firm will be indifferent between exerting effort and not exerting effort if the following condition holds

$$
\frac{\alpha}{\alpha + (1-\alpha)x}v_h + \left(1 - \frac{\alpha}{\alpha + (1-\alpha)x}\right)v_l - c = v_l
$$

(11)
Solving for $x$ from the above equation we get,

\[ x = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\Delta v - c}{c} \right) \]  

(12)

where $\Delta v = v_h - v_l$.

**Equilibrium conditions:** In equilibrium the monopolist posts a price in period 2 which depends on the prior probability that the firm is good, the equilibrium choice of effort by the monopolist in period 1 and the observed quality of the output in period 1. Thus the equilibrium pricing rule is given by

\[ P^M = \begin{cases} 
  v_l & \text{if } q = l \\
  \frac{\alpha}{\alpha + (1-\alpha)x} v_h + \left( 1 - \frac{\alpha}{\alpha + (1-\alpha)x} \right) v_l & \text{if } q = h
\end{cases} \]  

(13)

In equilibrium, the choice of effort of the opportunistic firm in period 1 is given by

\[ x^M = \begin{cases} 
  \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{\Delta v - c}{c} \right) & \text{if } \alpha \Delta v < c \\
  1 & \text{if } \alpha \Delta v \geq c
\end{cases} \]  

(14)

The equilibrium condition specified above implies that the monopolist exerts effort with positive probability for all possible parameter values. The monopolist exerts effort for sure beyond the threshold reputation $c/\Delta v$.

**Proposition 2.1:** $x^M$ is increasing in $\alpha$ and $\Delta v$ while decreasing in $c$.

**Proof:**

Partially differentiating $x^M$ with respect to $\alpha$, $\Delta v$ and $c$ we get,

\[ \frac{\partial x^M}{\partial \alpha} = \left( \frac{\Delta v - c}{c} \right) \frac{1}{(1 - \alpha)^2} > 0 \]
The monopolist’s choice of effort is increasing in reputation, which means, as the consumer becomes more convinced that the monopolist is of the good type the monopolist can reap the gains from reputation better. This is because a higher reputation is associated with a higher expected quality of the good and hence a higher payment for the monopolist. By not putting effort the opportunistic firm produces a bad quality output and that pushes reputation down to zero. In this event the only price the monopolist can charge in period 2 is $v_l$. Thus reputation incentives increases as reputation of the firm increases.

Reputation incentives are also increasing in $\Delta v$ that is the increment in quality resulting from high effort as opposed to low effort. For a better understanding of the result, let us fix $v_l$ and vary $v_h$. Note that in equation (13), for any given level of reputation $\alpha > 0$ the price the monopolist can post in period 2, is increasing in $v_h$. On the other hand the gains from not putting effort is fixed at $v_l$. Thus as the difference between the quality of the output increases reputation incentives become stronger.

The third result of Proposition 2.1 is straight forward. A higher cost of effort is associated with a lower choice of effort. Therefore, as $c$ increases the threshold beyond which the monopolist puts effort with probability 1 increases too. The range of reputation where the monopolist exerts effort with positive probability is now characterized by lower level of effort.

2.2.2 Duopoly

In this section we discuss a duopoly model with two firms and one consumer. This model captures reputation building behaviors of firms when they compete for clients through price posting. Since this model assumes that there is only one consumer in each period, the results we obtain in this section is also affected by the reduction in market size. In the monopoly model the market size for the firm was given by one consumer and the consumer-seller ratio
was 1. In the duopoly model the ratio reduces to 1/2. In this section we analyze the model with consumer-seller ratio 1/2. In the next section we analyze a model that isolates the effect of competition from the market size effect.

There are two long lived firms and a new consumer in each period. Consumers trade only with one firm in each period. In period 1, the consumer chooses a firm and the chosen firm decides on the quality of the product. In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the type of the firm who was hired in period 1. The firms post prices. the consumer then decides from whom to buy.

**Strategies:** A stationary strategy for the firm is a pair \((P,x)\), i.e. , a price and a choice of effort. Formally, \(x : [0,1] \to [0,1]\), where \(x(\alpha)\) is the probability that the opportunistic firm exerts effort in period 1 and \(P : [0,1] \to \mathbb{R}\) gives the price the firm posts in period 2.

A consumer’s strategy specifies from which firm to buy, given her beliefs about the firms’ types in the second period and the prices they post in the second period.

**Consumer’s belief and behavior:** The belief update rule is same as described in the monopoly model. \(\phi(\alpha,q)\) is the belief update function where, \(\phi(\alpha,q)\) gives the probability that the firm is good given the prior \(\alpha\) and the quality of output \(q \in \{h,l\}\).

Clearly, \(\phi(\alpha,l) = 0\). This is because a good firm produces a high quality output at no cost and a good firm is also non-strategic. Therefore, whenever the consumer observes a low quality output she concludes that the producer of the good is the opportunistic type. When \(q = h\), the belief update function obtained using Bays rule is as follows

\[
\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}
\]

where, \(\alpha\) is the prior probability that the firm is good and \(x\) is the probability that the opportunistic firm exerts effort.

Given the belief update rule, the consumer in period 2 purchases only from a firm that gives her higher net expected utility. Net expected utility of a consumer is defined as the difference between the expected utility of the consumer from a transaction and the price the consumer has
to pay for the good. In case of a tie, we assume that the consumer purchases from the firm whose product yields higher expected utility. This assumption implies that reputation is valuable to a firm. Even when a rival firm cuts price in order to match the expected net utility of a firm, the consumer trades only with the firm that has a higher reputation.

**Analysis:** Let us first consider the price posting behavior of the firms in period 2. In equilibrium, the firm posts a price that makes the consumer indifferent between purchasing from him and purchasing from his rival. Therefore, the price a firm posts depends on whether the consumer in period 1 purchased from him, the observed quality of the good produced in period 1 and the current reputation of the firm. Consider the following price posting behavior.

In the second period, the firm who did not have trade in the first period posts price $P' \text{ such that}$,

$$
P' = \begin{cases} 
0 & \text{if } q = h \\
\alpha v_h + (1 - \alpha)v_l - v_l & \text{if } q = l 
\end{cases}
$$

(15)

The firm who had trade in the first period posts price $P$ such that,

$$
P = \begin{cases} 
(\phi(\alpha, h) - \alpha)\Delta v & \text{if } q = h \\
0 & \text{if } q = l 
\end{cases}
$$

(16)

Suppose both firms have same reputation $\alpha$ in the first period. Also suppose one firm is chosen by the consumer in the first period. Our model does not assume price posting in the first period in order to set aside the issue of price signaling. The focus of our analysis is to capture the effect of competition on reputation building behavior. For this reason we assume that a firm is chosen in the first period by the consumer for reasons outside the arena of this model. However the firms are strategic when it comes to posting prices in the second period and the consumer in the second period chooses a firm whose product yields a higher net expected utility for the consumer. In case of a tie the consumer trades with the firm that yields a higher expected utility.

Given this selection behavior of the consumer, the above price posting behavior is optimal for the firms. A firm who did not have trade in the first period, must take into account the quality of the good his rival produced in the first period. If the quality of the good was observed to be
high in the first period, the rival firm who had trade in the first period moves to a higher level of reputation \( \phi(\alpha, h) \). A higher level of reputation is associated with a higher expected utility \( \phi(\alpha, h)v_h + (1 - \phi(\alpha, h))v_l \). For any \( P' \in (0, \alpha v_h + (1 - \alpha)v_l) \) the rival firm whose reputation is \( \phi(\alpha, h) \) can post \( P = \phi(\alpha, h)v_h + (1 - \phi(\alpha, h))v_l - \alpha v_h - (1 - \alpha)v_l + P' > 0 \) and have trade with the consumer in period 2. Similarly, for any \( P > (\phi(\alpha, h) - \alpha)\Delta v \) the firm who did not have trade in the first period can cut price such that \( \alpha v_h + (1 - \alpha)v_l - P' > \phi(\alpha, h)v_h + (1 - \phi(\alpha, h))v_l - P \). Thus the firm who did not have trade in the first period, can only post \( P' = 0 \).

When the observed quality of the output in the first period is low, the rival firm’s reputation drops to zero. The expected utility from purchasing the good from the rival firm is \( v_l \). The expected utility from purchasing the good from the firm who did not trade in the first period is \( \alpha v_h + (1 - \alpha)v_l \). Therefore in equilibrium, the only price the firm can charge is \( \alpha v_h + (1 - \alpha)v_l \).

Now consider the firm who traded with the consumer in the first period. Following similar arguments, if he produces a low quality output his reputation falls to zero. The expected utility from buying from his rival is higher and the only price he can charge in equilibrium is \( \alpha v_h + (1 - \alpha)v_l \). Therefore in equilibrium, the only price the firm can charge is \( \alpha v_h + (1 - \alpha)v_l - v_l \).

Given the belief update rule and price posting behavior we can now find the equilibrium effort level of the firm chosen by the consumer in the first period. If the opportunistic firm does not put effort in the first period, his reputation moves to zero and the only price he can charge in the second period is zero. However if he puts effort, he can charge the price \( P \) in the second period and the consumer in the second period has trade with him for sure. Thus the opportunistic firm is indifferent between putting effort and not putting effort in the first period if \( P = c \). The indifference condition can be rewritten as

\[
\left( \frac{\alpha}{\alpha + (1 - \alpha)x^D} - \alpha \right) \Delta v = c \tag{17}
\]

where \( x^D \) is the probability that the opportunistic firm exerts effort. Thus,

\[
x^D = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{(1 - \alpha)\Delta v - c}{c + \alpha \Delta v} \right) < 1 \tag{18}
\]
**Equilibrium conditions:** In equilibrium, the following conditions hold.

The firm who has trade in the first period charges $P$ and his rival charges $P'$ in the second period. The consumer in the second period purchases from the firm whose product yields higher net utility. In case of a tie the consumer purchases from the firm whose product yields higher expected utility.

In the first period, the effort choice of the opportunistic firm is given by,

$$x^D = \begin{cases} 
0 & \text{if } (1 - \alpha)\Delta v - c \leq 0 \\
\frac{\alpha}{1 - \alpha} \left(\frac{(1-\alpha)\Delta v - c}{c + \alpha \Delta v}\right) & \text{if } (1 - \alpha)\Delta v - c > 0
\end{cases} \quad (19)$$

**Proposition 2.2:** $x^D$ is increasing in $\alpha$ and $\Delta v$ while decreasing in $c$.

**Proof:**

$x^D$ can be re written as

$$x^D = \frac{\Delta v - \frac{c}{1-\alpha}}{\frac{\alpha}{\alpha} + \Delta v}$$

Partially differentiating $x^D$ with respect to $\alpha$, $\Delta v$ and $c$ we get,

$$\frac{\partial x^D}{\partial \alpha} = \left(\frac{\Delta v + \frac{c}{\alpha}}{(1-\alpha)^2}\right) + \left(\frac{\Delta c - \frac{c}{1-\alpha}}{\alpha}\right) \frac{c^2}{\alpha^2} > 0$$

$$\frac{\partial x^D}{\partial \Delta v} = \left(\frac{\alpha}{1 - \alpha}\right) \frac{c}{(c + \alpha \Delta v)^2} > 0$$

$$\frac{\partial x^D}{\partial c} = \left(\frac{\alpha}{1 - \alpha}\right) \frac{-\Delta v}{(c + \alpha \Delta v)^2} < 0 \quad \square$$

Qualitatively the duopolist’s choice of effort moves in the same direction as the monopolist’s.

Choice of effort is increasing in reputation below a threshold reputation $\alpha = \frac{\Delta v - c}{\Delta v}$. Above this threshold reputation the duopolist does not expend effort. As $\alpha \to 1$, $\phi(\alpha) \to 1$ and
Reputation revision becomes smaller and smaller as \( \alpha \) approaches 1. Because of the presence of a rival, the only price the duopolist can charge at this range of reputation equals the difference between the expected quality of its own product and the expected quality of its rival’s product. Thus gains from reputation shrink as the market becomes almost convinced about the firm’s type and hence the duopolist does not have incentives to put effort for this range of reputation.

Reputation incentives are increasing in \( \Delta v \) that is the increment in quality resulting from high effort as opposed to low effort. For a better understanding of the result, let us fix \( v_l \) and vary \( v_h \). Note that in equation (16), for any given level of reputation \( \alpha > 0 \) such that \((1 - \alpha)\Delta v - c > 0\), the price the duopolist can post in period 2, is increasing in \( v_h \). On the other hand the gains from not putting effort is fixed at 0. Thus as the difference between the quality of the output increases reputation incentives become stronger. Notice that, given \( \alpha \) and \( c \) the duopolist does not expend effort for a low range where \( \Delta v < c/(1 - \alpha) \). Above this range, as \( \Delta v \) increases the duopolist’s gains from building reputation increases as well.

The third result of Proposition 2.2 is straightforward. A higher cost of effort is associated with a lower choice of effort. Therefore, as \( c \) increases the threshold beyond which the monopolist puts effort with probability 1 increases too. The range of reputation where the monopolist exerts effort with positive probability is now characterized by lower level of effort.

### 2.3 Comparing Monopoly and Duopoly

In the previous section we analyzed the model for two cases namely, monopoly and duopoly. In this section we compare the results obtained in the previous section and see how reputation building behavior is affected by competition.

Define, \( D = x^M - x^D \). Notice that \( x^M > 0 \) for all \( \Delta v > 0 \). Also, \( x^M = 1 \) for higher values of \( \Delta v \). However, \( x^D < 1 \) for all parameter values and \( x^D = 0 \) for low range of \( \Delta v \).

**Theorem 2.1:** The equilibrium effort level is always strictly higher in monopoly, that is, \( x^M > x^D \).

**Proof:**
When \( \Delta v \leq c/(1 - \alpha) \), \( x^D = 0 \) and

\[
x^M = \begin{cases} 
\left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\Delta v - c}{c} \right) & \text{if } \alpha \Delta v < c \\
1 & \text{if } \alpha \Delta v \geq c 
\end{cases}
\]

(20)

Thus, \( x^M > x^D \).

Now suppose, \( c/(1 - \alpha) < \Delta v < c/\alpha \).

\[
x^M - x^D = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\Delta v - c}{c} \right) - \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{(1 - \alpha) \Delta v - c}{c + \alpha \Delta v} \right)
\]

\[
= \frac{\alpha^2 \Delta v^2}{(1 - \alpha)c(c + \alpha \Delta v)} > 0
\]

When \( \alpha \Delta v \geq c \), \( x^M = 1 \) and \( x^D < 1 \). Hence the proof. ■

The primary intuition for the above result is that price competition reduces future profit hence the value of putting effort. Thus gains from reputation are best reaped under monopoly. The presence of a rival who can cut price does not allow a firm to extract all the rents from building reputation. This makes reputation building less rewarding under duopoly as compared to monopoly. To see more formally, let \( \Delta \pi^M \) and \( \Delta \pi^D \) be the difference in profits generated by putting effort under monopoly and duopoly respectively. Now

\[
\Delta \pi^M = \left( \frac{\alpha}{\alpha + (1 - \alpha)x} \right) \Delta v
\]

and

\[
\Delta \pi^D = \left( \frac{\alpha}{\alpha + (1 - \alpha)x} - \alpha \right) \Delta v
\]

Thus \( \Delta \pi^M > \Delta \pi^D \) for a given choice of effort and reputation. Thus the monopolist’s benefits are higher than that of the duopolist from the same choice of effort. Hence in equilibrium, the monopolist has higher incentives to put effort in equilibrium than the duopolist.

The following observations summarizes how the effort choice of a monopolist differs from that of a duopolist. Notice that under monopoly, the firm always puts effort with strictly positive
probability and for some range the monopolist puts effort for sure. Under duopoly, for some range, the firm does not put effort at all and for no range of parameter values the firm puts effort for sure. Also for the range \( c/(1 - \alpha) < \Delta v < c/\alpha \), \( D = x^M - x^D \) is increasing in both \( \Delta v \) and \( \alpha \).

\[2.4 \text{ Consumer Arrives with Probability } 1/2\]

Since the model described in section 2.2 assumes that there is only one consumer in each period, the results we obtained in the previous section is also affected by the reduction in market size. In the monopoly model the market size for the firm was given by one consumer and the consumer-seller ratio was 1 while in the duopoly model the ratio reduces to 1/2. In this section we analyze a model that isolates the effect of competition from the market size effect. This section assumes that the monopolist meets a consumer with probability 1/2 in the second period. This assumption ensures that the consumer seller ratio is the same irrespective of the market structure.

\textbf{Monopoly:}

In the monopoly model there is one long lived firm and a new consumer in each period. In period 1, the consumer decides whether to buy from the monopolist and the monopolist decides on the quality of the product if the consumer is interested in trade. In period 2, a new consumer arrives with probability 1/2 who observes the quality of the good produced in the last period and updates her belief about the firm’s type. The monopolist posts a price. The consumer then decides whether to buy from the monopolist.

\textbf{Strategies:} A stationary strategy for the firm is a pair \((P, x)\), i.e. , a price and a choice of effort. Formally, \( x : [0, 1] \rightarrow [0, 1] \), where \( x(\alpha) \) is the probability that the opportunistic firm exerts effort in period 1 and \( P : [0, 1] \rightarrow \mathbb{R} \) gives the fee the firm posts in period 2.

A consumer’s strategy is whether to buy from a firm given her belief about the firm’s type in the second period and the price the firm posts in the second period.
Belief update: Belief update rule is same as the belief update rule described in the section that assumes that the monopolist meets as consumer for sure. \( \phi(\alpha, q) \) is the belief update function where, \( \phi(\alpha, q) \) gives the probability that the firm is good given the prior \( \alpha \) and the quality of output \( q \in \{h, l\} \).

Clearly, \( \phi(\alpha, l) = 0 \). This is because the good firm produces a high quality output at no cost and the good firm is assumed to be non-strategic. Therefore, whenever the consumer observes a low quality output she concludes that the producer of the good is the opportunistic type. When \( q = h \), the belief update function obtained using Bayes rule is as follows

\[
\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}
\]

where, \( \alpha \) is the prior probability that the firm is good and \( x \) is the probability that the opportunistic firm exerts effort.

Analysis: Let us first consider the price posting behavior of the firm in period 2. In equilibrium, the firm posts a price that makes the consumer indifferent between purchasing the good and not purchasing it. The firm can do better if the price is such that the consumer strictly prefers trade to autarky. Hence the price must equal the expected utility the consumer obtains from the transaction and

\[
P = \phi(\alpha, q)v_h + (1 - \phi(\alpha, q))v_l
\]

Given the the belief update rule and the price posting behavior described above, the firm’s gains from putting effort and producing a high quality good is given by

\[
\frac{1}{2} \left[ \frac{\alpha}{\alpha + (1 - \alpha)x} v_h + \left( 1 - \frac{\alpha}{\alpha + (1 - \alpha)x} \right) v_l \right] - c.
\]

Instead if the the firm does not put effort in period 1 and produces a low quality output, the consumer becomes sure that the firm is the opportunistic type. In this event the only price the firm can post in period 2 is \( v_l \). Now since the firm meets a consumer in the second period with probability 1/2 the gains from effort is now lower than the previous case. The firm will be indifferent between exerting effort and not exerting effort if the following condition holds

\[
\frac{1}{2} \frac{\alpha}{\alpha + (1 - \alpha)x} (v_h - v_l) = c
\]

(21)
Solving for $x$ from the above equation we get,

$$x = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\Delta v - 2c}{2c} \right)$$

(22)

**Equilibrium conditions:** In equilibrium the monopolist posts a price in period 2 which depends on the prior probability that the firm is good, the equilibrium choice of effort by the monopolist in period 1 and the observed quality of the output in period 1. Thus the equilibrium pricing rule is given by

$$P^M = \begin{cases} v_l & \text{if } q = l \\ \frac{\alpha}{\alpha + (1 - \alpha)x_M} v_h + \left( 1 - \frac{\alpha}{\alpha + (1 - \alpha)x_M} \right) v_l & \text{if } q = h \end{cases}$$

(23)

In equilibrium, the choice of effort of the opportunistic firm in period 1 is given by

$$x^M = \begin{cases} 0 & \text{if } \Delta v \leq 2c \\ \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\Delta v - 2c}{2c} \right) & \text{if } \Delta v \in (2c, 2c/\alpha) \\ 1 & \text{if } \Delta v > 2c/\alpha \end{cases}$$

(24)

The equilibrium condition specified above implies that the monopolist does not exert effort for low range of $\Delta v$, puts effort with strictly positive probability in the middle range and puts effort with probability 1 for high range of $\Delta v$. Also $x^M$ is increasing in $\Delta v$ for the range $\Delta v \in (2c, 2c/\alpha)$.

**Duopoly:**

In the duopoly model there are two long lived firms and a new consumer in each period. Consumers trade only with one firm in each period. In period 1, the consumer chooses a firm and the chosen firm decides on the quality of the product. In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the type of the firm who was hired in period 1. The firms post prices, the consumer then decides from whom to buy.
**Strategies:** A stationary strategy for the firm is a pair \((P, x)\), i.e., a price and a choice of effort. Formally, \(x : [0, 1] \rightarrow [0, 1]\), where \(x(\alpha)\) is the probability that the opportunistic firm exerts effort in period 1 and \(P : [0, 1] \rightarrow \mathbb{R}\) gives the price the firm posts in period 2.

A consumer’s strategy specifies from which firm to buy, given her beliefs about the the firms’ types in the second period and the prices they post in the second period.

**Consumer’s belief and behavior:** The belief update rule is same as described in the monopoly model. \(\phi(\alpha, q)\) is the belief update function where, \(\phi(\alpha, q)\) gives the probability that the firm is good given the prior \(\alpha\) and the quality of output \(q \in \{h, l\}\).

Clearly, \(\phi(\alpha, l) = 0\). When \(q = h\), the belief update function obtained using Bays rule is as follows

\[
\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}
\]

where, \(\alpha\) is the prior probability that the firm is good and \(x\) is the probability that the opportunistic firm exerts effort.

Given the belief update rule, the consumer in period 2 purchases only from a firm that gives her higher net expected utility. In case of a tie, we assume that the consumer purchases from the firm whose product yields higher expected utility.

The analysis and equilibrium condition is same as described in the earlier section. In the first period, the effort choice of the opportunistic firm is given by,

\[
x^D = \begin{cases} 
0 & \text{if } (1 - \alpha)\Delta v - c \leq 0 \\
\left(\frac{\alpha}{1 - \alpha}\right) \left(\frac{(1 - \alpha)\Delta v - c}{c + \alpha\Delta v}\right) & \text{if } (1 - \alpha)\Delta v - c > 0
\end{cases}
\]  (25)

**Comparing Monopoly and Duopoly**

First let us compare the monopoly results under the new set up with the monopoly results under the earlier set up. Notice that, in the new set up there exists a range of \(\Delta v\) where the monopolist also does not put effort. In the earlier set up the monopolist would always put effort with strictly positive probability in equilibrium. However the monopolist under the new set up
puts effort with positive probability for a high values of $\Delta v$ as he did in the earlier case and there exists a threshold beyond which he puts effort with probability 1. The monopolist puts effort with probability 1 for values of $\Delta v$ higher than $c/\alpha$ and $2c/\alpha$ under the old and new set up respectively. Thus the threshold beyond which the monopolist puts effort for sure is bigger when the consumer arrives with probability 1/2. Also in the range for which the monopolist puts effort with strictly positive probability, the choice of effort is strictly higher under the old set up. Therefore it’s evident that market size does play an important role in determining the choice of effort and larger market shares lead to higher reputation incentives. Consequently a firm with a larger market share exerts more effort than a firm with a smaller market share. Because market size has such a considerable impact on the choice of effort in experience goods markets, it’s worth isolating the impact of competition from the market size effect. Notice that the effort choice of the duopolist is same as earlier. The duopolist does not expend effort for a low range of $\Delta v$ and puts effort with strictly positive probability beyond a threshold. However the duopolist never puts effort with probability 1.

**Proposition 2.3:** If $\alpha \geq 1/2$, the duopolist’s choice of effort is no greater than the monopolist’s choice of effort.

If $\alpha < 1/2$ there exists a threshold $\bar{\Delta}v$ such that for $\Delta v < \bar{\Delta}v$ the monopolist’s choice of effort is higher than the duopolist’s choice of effort and, for $\Delta v > \bar{\Delta}v$ the opposite holds.

**Proof:**

From equation (24) and (25) we know that $x^M = 0$ for $\Delta v \leq 2c$ and $x^D = 0$ for $\Delta v \leq c/(1 - \alpha)$.

Therefore, when $\alpha = 1/2$, the monopolist and the duopolist does not put effort for the exact same range.

When $\alpha > 1/2$ the range for which the monopolist does not put effort is smaller than the range for which the duopolist does not put effort.

Similarly, with $\alpha < 1/2$ the range for which the monopolist does not put effort is larger than the range for which the duopolist does not put effort.

Now define, $D = x^M - x^D$ and consider $\alpha \geq 1/2$. 
For $\Delta v \in (c/(1-\alpha), 2c/\alpha]$, we know that both $x^M$ and $x^D$ are monotonically increasing in $\Delta v$.

At $\Delta v = 2c/\alpha$, $x^M = 1$ and $x^D < 1$. Therefore for $\Delta v \in (c/(1-\alpha), 2c/\alpha]$, $D = x^M - x^D > 0$. }

For $\Delta v > 2c/\alpha$, $x^M = 1$ and $x^D < 1$. Hence, $D = x^M - x^D > 0$.

Now consider $\alpha < 1/2$.

c/(1-\alpha) < 2c.

Thus, for $\Delta v \in (c/(1-\alpha), 2c]$, $x^M = 0$ and $x^D > 0$. Hence, $D = x^M - x^D < 0$.

Now at $\Delta v = c/\alpha$, $x^M = x^D = \frac{1}{(1-\alpha)} \frac{c-2c\alpha}{2c}$.

Since, $x^M$ and $x^D$ are monotonically increasing in $\Delta v$ for $\Delta v \in (2c, c/\alpha)$, $D = x^M - x^D < 0$.

Following similar logic as earlier, for $\Delta v > c/\alpha$, $D = x^M - x^D > 0$. ■

The following figures illustrate how the monopolist’s choice of effort differs from that of the duopolist.

![Figure 2.1: Choice of effort with α=1/2](image)
Figure 2.2: Choice of effort with $\alpha \in (1/2, 2/3)$

Figure 2.3: Choice of effort with $\alpha > 2/3$
For a better understanding of the result let us consider the indifference conditions of the monopolist and the duopolist respectively.

\[ \frac{1}{2} \left[ \phi(\alpha)v_h + (1 - \phi(\alpha)v_l - v_l) \right] = c \]  \hspace{1cm} (26)

\[ \phi(\alpha)v_h + (1 - \phi(\alpha))v_l - (\alpha v_h + (1 - \alpha)v_l) - 0 = c \]  \hspace{1cm} (27)

\( \phi(\alpha)v_h + (1 - \phi(\alpha))v_l \) in equation (26) depicts the returns of the monopolist from expending effort and \( v_l \) captures the return from not putting effort. In equation (27) the component that captures the duopolist’s returns from putting effort is \( \phi(\alpha)v_h + (1 - \phi(\alpha))v_l - (\alpha v_h + (1 - \alpha)v_l) \) and returns from not putting effort is 0. Notice that if \( v_l \) is high, punishment from not putting effort is low for the monopolist. Punishment for the duopolist on the other hand is always set to the maximum punishment, that is the duopolist loses his market in the second period if he does not put effort. Now let us fix \( v_h \) and vary \( v_l \) to see how the choice of effort moves in equilibrium as \( \Delta v \) changes.

As \( \Delta v \) increases, that is, as \( v_l \) decreases the monopolist’s punishment from not putting effort decreases which in turn reduces his incentives to put effort. For the duopolist, a reduction in \( v_l \) reduces his rewards from putting effort, his punishment from not putting effort stays put on...
the other hand. Thus, a decrease in $v_l$ leads to lower incentives to put effort for the duopolist as well. Now when $\alpha$ is high enough, the returns from putting effort is small for the duopolist. This is because, $\phi(\alpha)v_h + (1 - \phi(\alpha))v_l - (\alpha v_h + (1 - \alpha)v_l)$ is small for high values of $\alpha$. The returns for the monopolist on the other hand is still positive. Thus the monopolist’s incentive to put effort is higher than that of the duopolist for higher values of $\alpha$. When $\alpha$ is sufficiently small and $v_l$ decreases, that is, $\Delta v$ increases, the punishment for the monopolist is less severe. For the duopolist on the other hand punishment level remains the same. Thus the monopolist has weaker incentives to put effort as compared to the duopolist’s incentives to put effort.

2.5 The Planner’s Problem

In this section we discuss a planner’s problem when the planner has “some” information about the hired firm’s type. We are interested in a situation where the planner does not have “full” information about the firms’ types and hence the first best can not be implemented. Since the cost of effort $c$ is strictly less than the quality increment $\Delta v$, it is socially efficient for the opportunistic firm to expend effort whenever he has trade with the consumer. Now consider the following situation. The social planner has no control over the consumers’ decision and the planner can only dictate the chosen firm’s action in the first period. If the monopolist is opportunistic, the planner in the first period will dictate him to expend effort and produce a high quality output. In the second period the monopolist will choose to produce a low quality output.

Now under the duopoly set up suppose the planner only observes the chosen firm’s type and he dictates the chosen firm’s action. If the chosen firm is opportunistic and he produces high quality output by expending effort in the first period, the consumer fails to detect the opportunistic firm. Consequently the opportunistic firm is hired in the second period and produces low quality output. On the other hand if the opportunistic firm’s type is revealed in the first period and its rival is good, then the output quality in the second period is high and the cost of producing the high quality output is zero. Thus in a duopoly set up it may not be socially efficient for an opportunistic firm to put effort in the first period. In order to understand the welfare consequences and to compare the choice of effort prescribed by the social planner with the choice of effort of a duopolist we consider the following model. In our model we assume minimum control
of the planner.

**Players and actions:** There are two long lived firms, a social planner who lives only for the first period and a new consumer in each period. Consumers choose a firm and trades only with one firm in each period. In period 1, if the consumer chooses a firm, the planner observes the chosen firm’s type. The consumer in the first period and the social planner share the same beliefs about the firm that has not been hired by the firm. The planner dictates the choice of effort by the chosen firm. In period 2, a new consumer arrives who observes the quality of the good produced in the last period.

**Strategies:** Strategy for the firm is a price $P : [0, 1] \rightarrow \mathbb{R}$ that gives the price the firm posts in period 2.

The Planner’s strategy is a choice of action the planner chooses for the firm hired in the first period. The planners’s strategy $x^P : [0, 1] \times \{\text{good, opportunistic}\} \rightarrow [0, 1]$ is a function of the chosen firm’s type and the planner’s belief about its rival.

A consumer’s strategy specifies from which firm to buy, given her beliefs about the the firms’ types in the second period and the prices they post in the second period.

**Timeline:** In period 1, if the consumer chooses a firm, the planner observes the chosen firm’s type. The consumer in the first period and the social planner share the same beliefs about the firm that has not been hired by the firm. The planner dictates the choice of effort by the chosen firm.

In period 2, a new consumer arrives who observes the quality of the good produced in the last period and updates her belief about the type of the firm who was hired in period 1. The firms post prices. the consumer then decides from whom to buy.

**Consumer’s belief and behavior:** The belief update rule is same as earlier. $\phi(\alpha, q)$ is the belief update function where, $\phi(\alpha, q)$ gives the probability that the firm is good given the prior $\alpha$ and the quality of output $q \in \{h, l\}$. 
Clearly, $\phi(\alpha, l) = 0$. When $q = h$, the belief update function obtained using Bayes’ rule is as follows

$$\phi(\alpha, H) = \frac{\alpha}{\alpha + (1 - \alpha)x}$$

where, $\alpha$ is the prior probability that the firm is good and $x$ is the probability that the opportunistic firm exerts effort. Notice that if the planner prescribes the chosen (opportunistic) firm to expend effort with probability 1, the consumer can not update her belief about the firm.

Given the belief update rule, the consumer in period 2 purchases only from a firm that gives her higher net expected utility. In case of a tie, we assume that the consumer purchases from the firm whose product yields higher expected utility. Also if firms are such that they yield same expected utility as well as same expected net utility, a firm is chosen at random with probability $1/2$.

**Analysis:** Suppose the chosen firm in the first period is ‘opportunistic’. If the planner dictates the firm not to put effort, the firm produces low quality output in the first period and his type is revealed. The consumer in the second period has trade with the rival firm and the expected quality of output in the second period is $\alpha v_h + (1 - \alpha) v_l$. Thus the value the planner generates by dictating the firm in the first period to not put effort is $v_0 = v_l + \alpha v_h + (1 - \alpha) v_l$.

If the planner dictates the opportunistic firm to put effort in the first period, the consumer in the second period fails to update her belief about the firm’s type. Thus the opportunistic firm is hired in the second period with probability $1/2$ and he produces output $v_l$ in the second period.

With probability $1/2$ his rival is hired and the expected quality of output is $\alpha v_h + (1 - \alpha) v_l$. Thus the value the planner generates by dictating the firm in the first period to put effort is $v_1 = v_h - c + \frac{1}{2} [v_l + \alpha v_h + (1 - \alpha) v_l]$.

Now suppose the planner mixes with probability $\gamma = Pr(e = 1)$, that is the planner dictates the opportunistic firm to put effort with probability $\gamma$. The social gain in the first period is $\gamma(v_h - c) + (1 - \gamma) v_l$ and the social gain in the second period is $v_l$. Define $v_\gamma = \gamma (v_h - c) + (1 - \gamma) v_l + v_l$.

Now $v_\gamma$ is strictly increasing in $\gamma$. Define, $\overline{v}_\gamma = sup_{\gamma \in (0,1)} v_\gamma$.

**Lemma 2.1:** $v_1 > \overline{v}_\gamma$. 

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Proof:

\[ v_\gamma = v_h - c + v_l \]

\[ v_1 - v_\gamma = v_h - c + \frac{1}{2} [ v_l + \alpha v_h + (1 - \alpha) v_l ] - v_h + c - v_l = \frac{1}{2} [ v_l + \alpha v_h + (1 - \alpha) v_l ] - v_l > 0. \]

Therefore, whenever the planner dictates the opportunistic firm to put effort with positive probability he dictates the opportunistic firm to put effort with probability 1. Now our concern is under what circumstances the planner dictates the bad firm to put effort with probability 1. We need to compare \( v_0 \) and \( v_1 \) to obtain the effort level the planner prescribes.

**Lemma 2.2:** When \( (1 - \alpha) \Delta v > c \), \( v_0 < v_\gamma \).

Proof:

\[ v_0 - v_\gamma = v_l + \alpha v_h + (1 - \alpha) v_l - v_h + c - v_l = c - (1 - \alpha) \Delta v. \]

Thus if \( (1 - \alpha) \Delta v > c \), \( v_0 < v_\gamma \). ■

**Proposition 2.4:** The duopolist’s choice of effort either falls below or coincides with the choice of effort the planner prescribes.

Proof:

**Case 1:** \( (1 - \alpha) \Delta v > c \)

From Lemma 2.2, we know that \( v_0 < v_\gamma \) and from Lemma 2.1 we know that \( v_\gamma < v_1 \).

Thus \( v_0 < v_1 \). The planner dictates the duopolist to put effort with probability 1.

However, from equation (25) we know that, \( x^D = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{(1 - \alpha) \Delta v - c}{c + \alpha \Delta v} \right) < 1 \).

**Case 2:** \( (1 - \alpha) \Delta v \leq c \)

From Lemma 2.2, we know that \( v_0 \geq v_\gamma \).

Now, \( v_0 - v_1 = (1 - \frac{\alpha}{2}) v_l - (\frac{3}{2} - \alpha) v_h + c \) which is ambiguous.

When \( v_0 > v_1 \), the planner’s solution coincides with the market outcome and the duopolist does not put effort.

When \( v_0 < v_1 \) the firm does not put effort under duopoly even if the planner would dictate him to put effort with probability 1. ■
2.6 Conclusion

In this paper, we provide an analytical framework and explicitly derive conditions for which competition aids reputation-building behavior and for which it hiders. This is a simple two-period model. Contrary to the prevalent idea that competition is conducive to reputation building behavior, our analysis suggests that under certain circumstances competition can hinder reputation building behavior. Consequently, competition has a negative impact on the overall quality of production. The intuition supporting this kind of result is as follows. Competition may reduce future expected payoff and hence can reduce incentives to put effort in the current period. This effect is more severe when the difference in values is large. Our result is robust to the market size effect embedded in imperfect competition models.

We also compare the market outcome under duopoly with that of a planner’s solution. We show that the duopolist’s choice of effort either falls below or coincides with the planner’s choice of action. This analysis implies that there is inefficiency in the duopoly market whenever it is socially optimal for a duopolist to expend effort. On the other hand there is efficiency under duopoly when it is socially optimal for the duopolist to not expend effort.
References


Chapter 3

Informal Insurance and Group Size Under Individual Liability Loans

3.1 Introduction

It has been well established in economic literature that access to credit can empower the poor. Formal financial intermediaries, such as commercial banks, usually refuse to serve poor households because of the high cost of small transactions and lack of traditional collateral. As a consequence, an enormous pool of potential abilities and talents remain untapped by the society. Providing access to credit to the poor households helps not only to improve their economic condition, but also provides a way to maintain or improve their quality of life. This also encourages self-development of the poor households by helping them to integrate with the broader economic life. The economy benefits from a better utilization of human resources which in turn translates into an overall development.

The Grameen Bank is the world’s best known lender to the poor and reaches more poor people than most of the other micro-lending organizations. It reaches to more than 8.35 million borrowers with the total amount of loan disbursement being Tk 684.13 billion (US $ 11.35 billion) since inception. The Nobel Peace Prize 2006 was awarded jointly to Muhammad Yunus (founder of the Grameen Bank) and Grameen Bank “for their efforts to create economic and social development from below”. The unique feature of this bank had been the joint liability loans and similar banking models were subsequently adopted by hundreds of organizations around the world.

Under joint liability, a group of five borrowers were given individual loans, but held jointly liable for repayment. If any member defaulted, future loans to all the group members would be denied or delayed. The economic literature has focussed on this aspect of joint liability as the major factor behind the success of the Grameen Bank. It was believed that joint liability would encourage mutual insurance among group members and generate social pressure on borrowers to repay loans creating a sustainable model of lending.

http://www.grameen-info.org
In recent years, there has been a shift from joint liability to individual liability loans undertaken by some prominent micro lending institutions including the Grameen Bank. Providing useful insights regarding this regime shift, Giné and Karlan (2011) points out some of the drawbacks of group liability lending. First, borrowers disliked the tension caused by group liability as one had to face punishment even when one was not at default. This could also harm social capital among group members by giving rise to free riding problems. Second, clients may decide not to repay their loans believing that other clients will pay it for them while the bank is indifferent because it still gets its money back. Third, group liability is more costly for clients with less risky projects because they are often required to pay back the loans of other group members with riskier projects.

Due to the growing dissatisfaction with joint liability lending among its members, the Grameen Bank in 2002, replaced their model of group lending with Grameen II, which no longer involves joint liability. According to Dr. Yunus,

“Grameen Bank II has emerged. The transition is now complete. The last branch of Grameen Bank switched over to Grameen II on August 7, 2002, completing the process of transition. The new Grameen Bank II is now a real and functioning institution. This second-generation microcredit institution appears to be much better equipped than it was in its earlier version.”

Following the introduction of the new system, the total number of borrowers increased from 3 to 8 million. At present, all members are individually liable for their loans. Under the new system access to future credit by an individual borrower is not conditional on the performance of others in the group.

Much of the previous literature has focussed on the fact that under joint liability successful group members might help unsuccessful group members repay, hence risk is shared within the group. The theoretical models inspired by Grameen I suggest that joint liability clauses are key to efficient lending. Giné and Karlan (2011) conducted a field experiment with the Green Bank in Philippines, in which they compared randomly selected branches with joint liability to those

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without and found no significant change in repayment rates. Their experiment suggests that joint liability may not be the key feature of successful micro lending.

The natural question that arises at this point is, what leads to the smooth functioning of a micro lending system with individual liability loans? Before moving on to the answer to this question, we must shed light on the institutional features of Grameen II. The new system abandoned one of the most celebrated features of the old format of Grameen lending, “joint liability”. It also introduced a much more flexible punishment scheme as opposed to Grameen I which unleashed stricter punishments for defaulters. According to Yunus,

"There is no reason for a credit institution dedicated to provide financial services to the poor to get uptight because a borrower could not pay back the entire amount of a loan .... Many things can go wrong for a poor person during the loan period..... Since she is paying additional interest for the extra time, where is the problem?"

Despite the shift from “joint liability” to “individual liability” it is still mandatory for individuals to be in a group in order to obtain a loan from the bank. As before individuals should have their groups formally recognized by Grameen staff. There is a clear distinction between “group liability” and “group lending”. As Giné and Karlan (2011) pointed out, “group liability” refers to the terms of the actual contract, whereby individuals are both borrowers and simultaneously guarantors of other clients’ loans. “Group lending” means there is some group aspect to the process or program, perhaps only logistical, like the sharing of a common meeting time and place to make payments.

Another important feature of Grameen I that has been retained by Grameen II is public repayment meetings. Repayments are made in public meetings where all the borrowers are present. These meetings allow the borrowers to learn about each other which helps in alleviating frictions. As observed by Armendariz and Morduch (2005), when repayments are made in public, “the villagers know who among them is moving forward and who may be running into difficulties”.

The shift to individual liability was not merely done by the Grameen Bank and a few other large, well-known lenders, but many lenders around the world are adopting individual liability. This paper is an attempt to explain the smooth functioning of the micro lending system with
individual liability under a theoretical framework. We argue that there are still possibilities of informal insurance and risk sharing under individual liability through a different mechanism. The heart of this paper lies in explaining that though liability is individualized, borrowers can informally insure each other in “groups” leading to repayment rates similar to those as under joint liability. Our paper provides a stepping stone in explaining why the shift from group liability to individual liability loans does not lead to a change in repayment rates as observed by Giné and Karlan. In the process, this paper also suggests that benefits of informal insurance are best reaped when individuals are in an optimal “group” size. The bank can increase social welfare by stipulating a “group” size its clients should maintain to obtain a loan.

In this paper agents are connected through a social network. They borrow from a micro-credit organization under individual liability. Though individuals are not jointly responsible for a loan or default, they still enter into informal arrangements with their neighbors in the social network to avoid the cost of default. The public repayment here plays an important role as observed by Armendariz and Morduch (2005).

In our model we have $N$ risk-neutral villagers who interact in a social network. A bilateral link is given: it comes from two individuals getting to know each other for reasons exogenous to the model. Such links may be destroyed, but no new links can be formed. Each villager can invest in a project that can be a success with probability $p$ or a failure with probability $1 - p$. Villagers can obtain a loan from a microcredit organization to fund this project. Each borrower must pay back the loan along with the interest charged by the bank. A borrower who defaults is punished by the bank and this non-pecuniary cost of punishment is increasing in the amount of default. Our analysis focuses on continuous cost functions that are convex and this assumption is motivated by the fact that Grameen II has switched to a more flexible punishment scheme than Grameen I.

Since default is costly, individuals can enter into informal arrangements with people whom they have direct links with. They cannot get into arrangements with individuals with whom they do not have direct links. A contract of value $x$ specifies the amount the successful individual transfers to her unsuccessful counterpart. The links are undirected in nature. When both are successful or unsuccessful, no transfers are made. At every period $t$, individuals observe their own output levels, but they do not observe others’ output levels. Unsuccessful individuals approach
their neighbors with whom they have an arrangement in order to repay their loans. Individuals with positive probability learn about their neighbors’ outcomes at the public meeting. Once individuals learn about their neighbors’ outcomes, they decide on whether to sever links with the neighbors who did not keep their promises.

In this paper we consider regular networks where each individual has \( n \) direct links and each link is characterized by a promise value \( x \). The analysis focuses on the role of network size on welfare for different cost functions. We show that when punishment equals the amount of default, welfare does not depend on the number of links an individual has. However individuals gain from links whenever cost of default is larger than the amount of default. We show for convex costs, individuals’ expected utility under risk sharing is non decreasing in the number of neighbors. With a small cost of maintaining each link, there exists an optimal network size for which individual’s gains from informal insurance are maximized. Though this result holds true for any general regular network, it has an important policy implication in the context of group loans with individual liability. The bank can stipulate village specific group sizes that individuals need to maintain in order to get a loan.

**Literature:** Much of the earlier literature discusses several institutional features of joint liability which has been summarized by Ghatak and Guinnane (1999). The literature on individual liability lending is still at a nascent stage. In this paper we provide a theoretical framework that emphasizes on the role of informal insurance under a social network in the context of individual liability loans. Our analysis largely relies on the assumption that individuals face internal frictions and cannot enter into arrangements that maximize joint utility.

The work of Townsend (1994) and Udry (1994) stimulated interest in the internal contractual arrangements of poor villagers which can potentially insure them against idiosyncratic shocks. They look at how good or how bad these informal risk sharing institutions are for the villages in southern India. They point out that the informal arrangements are imperfect, because they typically suffer from various kinds of informational and enforcement problems.

However, enforcement problems may be less severe in informal arrangements which are enforced by social sanctions i.e. which rely on social capital instead of traditional collateral. Besley and Coate (1995) discuss the role of social capital in the context of Grameen I. They provide a
game-theoretic analysis of repayment decisions under group lending. The two incentive effects they emphasized on are: first, there is always a possibility that a successful borrower may repay the loan of a partner who obtains a bad return on her project. Second, group lending may be able to harness social collateral. Under an individual lending contract, all that the borrower has to fear, if she defaults, are the penalties that the bank can impose on her. Under group lending, she may also incur the wrath of other group members. Our paper retains both these features in the context of individual liability loans with the help of a social network. A successful individual may still be interested in helping her unsuccessful neighbor with the expectation that her neighbor will help her in bad times. Links are valuable to an individual because they can insure them against idiosyncratic shocks.

Rai and Sjöström (2004) design a lending mechanism that efficiently induces mutual insurance under joint liability lending. The role of bilateral insurance schemes across networks of individuals has been studied by Bloch et al. (2008). They investigate the structure of a self enforcing insurance network where transfers are based on social norms and are publicly observable. Our paper preserves their feature of bilateral insurance schemes across networks of individuals, however in a different context. Our paper also assumes away the observability of transfers across individuals.

Rai and Sjöström (2010), with whom we share some basic modeling similarities, show that in a Coasean world without frictions the village functions as a “composite agent” who minimize the joint expected cost of default. In such a world, the design of the lending contract is rather unimportant and joint liability loans are no better than individual liability loans. However the cost structure they assume is discrete. Punishment is constant for any positive default amount. This cost structure resembles the cost structure of Grameen I which has been too rigid in enforcing repayments. This strict adherence of rigid rules is not desirable, particularly if the credit institution is dedicated to provide financial services to the poor. Many things can go wrong and if these poor people are forced to default they may not find their way back to the credit market. Grameen II allows for more flexible repayments which are structured more in line with the borrower’s cash-flows. Our model differs from Rai and Sjöström (2010) by assuming punishment cost to be a continuous and increasing function of default. This indeed is a better reflection of the highly flexible punishment scheme that has been adopted by Grameen II. Also in contrast to their two agent framework we introduce a general $N$ agent model.
Findings of our paper are strongly supported by two recent field experiments. Giné and Karlan (2009) in their field experiment with Green Bank, a Grameen replica in the Philippines, compares randomly selected branches with joint liability to those without and find no change in repayment rates. They also find that those with weaker social networks prior to the conversion are more likely to experience default problems after conversion to individual liability, relative to those who remain under group liability. This phenomenon is captured in one of our results that shows too few neighbors is not welfare maximizing. A neighbor in our model acts as an insurance possibility rather than a monitoring devise. In other words, lack of neighbors means lack of insurance.

Feigenberg et al. (2010) provide an experimental evidence on the economic returns to social interaction in the context of micro finance. Their results also provide a rationale for the current trend among MFIs of maintaining repayment in group meetings despite the transition from group to individual liability contracts. They emphasize on the role of frequent social meetings to facilitate cooperative behavior.

In response to Feigenberg et al. (2011), Quidt de et al. (2012) derive conditions under which more frequent meetings, modeled as an increase in the amount of time borrowers and loan officers must spend in loan repayment meetings, increases borrowers’ incentive to invest in social capital. They also show that individual lending with or without groups may be welfare improving as long as borrowers have sufficient social capital to sustain mutual insurance. In their paper each link is characterized by pair-specific social capital which is conceptualized as the net present value of lifetime payoffs in a repeated “social game” played alongside the borrowing relationship. In our paper welfare improvement is driven by pair-specific informal insurance arrangements in regular networks. The value of a link comes from the fact that a link functions as an insurance possibility.

Section 3.2 describes the model and section 3.3 carries out the analysis under regular networks. Section 3.4 argues that individuals would not enter into informal arrangements or make promises to her neighbors that they may not be able to keep. In section 3.5 we discuss the case where individuals can enter into punishment sharing arrangements. Section 3.6 briefly discusses about the moral hazard and adverse selection aspects of the model and Section 3.7 concludes.
3.2 The Model

Suppose there are $N$ villagers, $i \in \{1, 2, \ldots, N\}$. Each villager has an investment opportunity which requires an investment of one dollar. The project can be a “success” or a “failure”. Agent $i$’s output is denoted by $y_i = \{0, h\}$. A successful project yields $h > 0$ amount of output. In case of a failure an individual gets 0. The project can be successful with probability $p$ and fail with probability $(1 - p)$. Hence the state of the world is a $N$-tuple $(y_1, y_2, \ldots, y_N) \in Y = \{0, h\}^N$. The random variables $y_1, y_2, \ldots, y_N$ are independent. The villagers are risk-neutral. They have no assets, so neither self-financing nor borrowing from commercial banks is possible as banks require collateral.

Now suppose there is a benevolent not-for-profit microcredit organization who unlike commercial banks provides credit without collateral. It is assumed that $h$ is high enough so that the projects are viable i.e. it is efficient to fund the investment opportunities.

$$h[Np^N + \left(\frac{N}{N-1}\right)p^{N-1}(1-p)(N-1) + \ldots + \left(\frac{N}{1}\right)p(N-p)^{N-1}] > N$$

The bank cannot observe the state of the world. So it cannot observe whether a project succeeds or fails. Each villager needs to repay $(1 + r)$. The interest rate charged by the bank, which is exogenously given, is the opportunity cost borne by the bank.

Default is not costless. A borrower who defaults is punished by the bank. It is often argued that villagers may face hard times because of some exogenous shocks and hence may find it difficult to pay back. However for the lender it can be difficult to verify such shocks. If default is costless, then the borrower has a strategic incentive to default, claiming that for certain exogenous reasons she failed which may be difficult to verify. To prevent such strategic default we introduce that the bank imposes punishment depending by how much an individual defaulted. This non-pecuniary punishment can be interpreted as loans may be given at a higher interest rate.

Let $C(d)$ be the punishment an individual faces when the amount of default is $d$ and $C' > 0$. Punishment is increasing in the amount of default with $C(0) = 0$. Much of the previous literature assumes that a borrower is punished whenever she defaults irrespective of the default amount. The continuous and increasing cost function is a better reflection of the new punishment scheme.
introduced in Grameen II which is more flexible as opposed to the stricter punishment scheme in the older system. We analyze the model with both linear and convex cost functions later in Section 3.3.

A villager who invests in a project and takes a loan of one dollar from the bank has an expected return

\[ p[h - (1 + r)] - (1 - p)C(1 + r) \]

We assume this to be positive so that villagers have an incentive to invest in a project. If \( h \) is high enough this is easy to satisfy and hence incentive compatible for a villager to invest in a project.

Agents interact in a social network. Formally, a network \( g \) consists of the \( N \) villagers as the set of nodes and a graph—a collection of pairs of agents— with the interpretation that the pair \( ij \) belongs to \( g \) if they are directly linked. In this paper, a bilateral link is given: it comes from two individuals getting to know each other for reasons exogenous to the model. While such links may be destroyed (for instance, due to an unkept promise) no new links can be created.

An individual can enter into informal arrangements with people whom they have direct links with. We refer to them as “neighbors” of the individual. An arrangement of value \( x \) specifies the amount a successful individual transfers to her unsuccessful counterpart. When both are successful or unsuccessful, no transfers are made. A contract of value \( x = 0 \) means that the linked individuals have no informal arrangement. Hence each link in this network is characterized by a value \( x \). Since default is costly and project returns are independent, villagers can benefit from mutual insurance. If an individual fails while her neighbors are successful, then the successful neighbors can help the individual repay her loan reducing the cost of default. In short these arrangements can act as informal insurance.

3.2.1. Timeline

We consider an infinite horizon framework where agents are infinitely lived. Time is discrete and every period individuals face the same investment opportunity as described earlier. The villagers need to take a loan every period because the amount they save after paying back
the interest to the bank can only suffice their sustenance. Given that they are very poor and their savings can only sustain their livelihood, they cannot avoid borrowing from the bank. At every period $t$, there are four stages. At stage 1, individuals observe only their own output levels, but they do not observe others’ output levels. A public meeting is convened by the bank where individuals declare in public whether they have been successful or not. At stage 2, an unsuccessful individual approaches her neighbors with whom she has an agreement, in order to repay her loan. The neighbors respond by keeping or not keeping her promise. An individual will be interested in helping out her neighbor with the expectation that her neighbor will help her in her bad times. At stage 3, individuals repay their loans to the bank official in the public meeting. These meetings are held in each center (kendra) or branch of the bank. We make a simple assumption that the $N$ villagers that we consider belong to the same branch. We rule out the possibility that individuals may have neighbors who belong to other branches of the bank. The public repayment meetings facilitate the borrowers to learn about each other. We assume that an individual’s true outcome is revealed with probability $\beta > 0$. At stage 4, individuals decide on which links to sever.

3.2.2. Constraints, Payoffs and Equilibrium

For an individual $i$ with $k$ links, we assume the following is satisfied

$$\sum_{j=1}^{k} x_{ij} \leq h - (1 + r)$$

(28)

where $x_{ij} = x_{ji}$ is the promise value that characterizes the bilateral link between $i$ and $j$. This implies that when individual $i$ is successful the total amount promised to all her links should not exceed the resource available to her after repayment of her loan. In other words the budget constraint is satisfied. This assumption is relaxed later in Section 3.4 where we argue that individuals do not promise any $x$ that does not satisfy the budget constraint.

We make an additional assumption on the parameters of the model,

$$1 + r \geq \frac{h}{2}$$

(29)
This assumption in the context of two person joint liability implies that a successful individual alone cannot repay the entire amount they jointly owe to the bank. In the context of our model this is the “no leftover” condition which ensures that even when all neighbors of an unsuccessful individual are successful, the maximum amount of help does not exceed the amount to be repaid.\footnote{The "no-leftover" condition is a simplifying assumption. If this assumption is violated then situations may arise when an unsuccessful individual can repay her loan in full and still enjoy some surplus. However this does not alter the main results of the paper as discussed in Section 3.3.}

Given a network $g$ and a state of the world $y$, the per-period utility of an individual $i$ is given by

$$u_i(g, y) = \begin{cases} h - (1 + r) - \sum x & \text{if successful} \\ -C(d) & \text{if unsuccessful} \end{cases}$$

where $\sum x$ is the total amount the successful individual transfers to her unsuccessful neighbors. The sum is over the number of unsuccessful links.

**Equilibrium:** An equilibrium consists of a network $(g, N)$ and a set of arrangements $[x^1, x^2, ..., x^N]$ where $x^i \in \mathbb{R}^{n_i}$ and $n_i$ is the number of links an individual has, satisfying

1. Each individuals’ expected welfare is maximized.
2. Promises are kept whenever possible.

### 3.3 Analysis

In this paper we consider regular networks, that is each individual has $n$ direct links. Each link is characterized by a specific promise value $x_{ij} = x, \forall i, \forall j$. The expected utility or welfare of an individual with $n$ neighbors is

$$W = p \left[ (h - (1 + r)) - \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} (n - k) x \right] - (1 - p) \left[ \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} C(1 + r - kx) \right]$$

(30)
Individuals choose $x$ to maximize $W$ subject to the budget constraint

$$nx \leq A, \text{ where } A = h - (1 + r)$$  \hspace{1cm} (31)$$

This constraint implies that $x \in [0, \frac{A}{n}]$ i.e. the maximum possible insurance level is bounded above by $A/n$.

We analyze the role of network on an individual’s welfare for linear and convex cost functions which are increasing and has the property $C(0) = 0$. This property ensures that an individual who pays back in full is not punished by the bank. We find that network plays no role when punishment equals the default amount as is explained in Proposition 3.1. When punishment is greater than the default amount but linear in nature, individuals with informal arrangements are better off than in autarky. However this welfare increment is network size invariant as discussed in Proposition 3.2. Proposition 3.3 considers strictly convex cost functions which drive the main result of our paper. With strictly convex cost functions, individual welfare increases with network size.

**Proposition 3.1:** If cost of punishment is linear i.e. $C(d) = d$ then network plays no role in increasing welfare from the autarkic level.

**Proof :** By replacing the cost function in equation (28) we obtain

$$W = p \left[ (h - (1 + r)) - (1 - p)^n nx - \binom{n}{1} p(1 - p)^{n-1}(n - 1)x - ... - p^n 0 \right] +$$

$$(1 - p) \left[ -(1 - p)^n(1 + r) - \binom{n}{1} p(1 - p)^{n-1}(1 + r - x) - ... - p^n(1 + r - nx) \right]$$

Note that, all terms containing $x$ cancels out. This is true because,

$$\binom{n}{r - 1} (n - (r - 1)) = \binom{n}{r} r$$

Canceling out terms we get,

$$W = p[h - (1 + r)] - (1 - p)(1 + r)$$

$$= ph - (1 + r)$$
which is the same as the autarkic utility.

This implies that with linear costs the expected benefit is the same as the expected cost of maintaining a link and hence the welfare of an individual is unchanged at the autarkic value. Ex-ante, the amount a successful individual transfers to her unsuccessful neighbors is exactly the amount she gets back from her neighbors when she is unsuccessful. Since cost of default equals the default amount, the gain from links is same as the cost of maintaining links.

Now assume that the cost of default is greater than the default level, $C(d) > d$. This ensures that no individual has an incentive to strategically default. Under this cost structure we would be interested in finding the optimal amount of insurance $x^*$, which maximizes the expected utility of an individual. We look for the optimal insurance when the budget constraint (31) is satisfied.

** Lemma 3.1:** If $C' > 1$ the optimal insurance is the maximum insurance possible i.e. $x^* = A/n$.

**Proof:** For a fixed number of links $n$, we want to find the optimal insurance $x$ that maximizes the expected utility $W$ as in equation (30) subject to the budget constraint (31).

The individual solves the following optimization problem

$$
\max_x W \quad \text{st} : \quad nx \leq A
$$

With fixed $n$, we take the partial derivative of $W$ with respect to $x$ and obtain

$$
\frac{\partial W}{\partial x} = -p(1-p)^n + \left( \begin{array}{c} n \\ 1 \end{array} \right) p^2(1-p)^{n-1}(n-1) + \cdots + \left( \begin{array}{c} n \\ n-1 \end{array} \right) p^n(1-p)
$$

$$
+(1-p) \left[ \left( \begin{array}{c} n \\ 1 \end{array} \right) p(1-p)^{n-1}C'(1+r-x) + 2 \left( \begin{array}{c} n \\ 2 \end{array} \right) p^2(1-p)^{n-2}C'(1+r-2x) \cdots + np^nC'(1+r-nx) \right]
$$

(32)
Now, \((\frac{n}{k-1})(n-k+1) = h(\frac{n}{k})\).

We can simplify equation (32) and write

\[
\frac{\partial W}{\partial x} = p \left[ (1-p)^n n (C'(1 + r - x) - 1) + \binom{n}{1} p(1-p)^{n-1}(n-1) (C'(1 + r - 2x) - 1) + \ldots \right.
\]

\[
+ \binom{n}{n-1} p^n (1-p) (C'(1 + r - nx) - 1) \right]
\]

(33)

Under the assumption \(C' > 1\), \(\frac{\partial W}{\partial x}\) is increasing in \(x\). Hence the constraint holds with equality implying that the optimal insurance is \(x^* = \frac{A}{n}\) ■

This result applies to linear cost functions of the form \(C(d) = \alpha d\), \(\alpha > 1\) and to cost functions which are strictly convex. The above lemma implies that when cost is convex and is greater than the default amount, individuals always gain from insurance. In fact, under such circumstances individuals opt for the maximum possible insurance. Ex-ante the amount a successful individual transfers to her unsuccessful neighbors is less than her welfare gain when she is unsuccessful.

We now analyze the effect of network-size on welfare with linear costs where cost is greater than the default amount.

**Proposition 3.2**: If \(C(d) = \alpha d\), \(\alpha > 1\), individuals gain from informal insurance and welfare is higher than autarky. However network size does not play a role, that is, welfare is network-size invariant.

**Proof**: See Appendix A.

The intuitive explanation is similar to Proposition 3.1. Like the previous case, ex-ante, the amount a successful individual transfers to her unsuccessful neighbors is exactly the amount she gets back from her neighbors when she is unsuccessful. However the gains from transfers is higher due to the structure of the cost function. Although welfare is higher than the autarkic level, it is independent of the number of links an individual has.

Now we assume that the costs of default is not only strictly increasing but also strictly convex, \(C'' > 0\). Under this assumption we obtain that welfare is increasing in network size. In other words individuals are better off with more neighbors.
**Proposition 3.3**: If \( C(d) > d \) and \( C'' > 0 \) then welfare is increasing in \( n \).

**Proof**: See Appendix A.

The assumption of convex costs implicitly implies that individuals are risk averse. From Lemma 3.1, we know that the optimal insurance is the maximum insurance possible. We obtain that the expected utility is increasing in \( n \) evaluated at the optimal level of insurance. Each neighbor can be perceived as an insurance possibility and more neighbors can be associated with higher risk diversification. Since individuals are risk averse they gain from bigger neighborhoods through higher risk diversification and better insurance possibility.

### 3.3.1 Example

In light of the above discussion, for a better understanding of the reader we present a simple example with a particular form of the cost function given by \( C(d) = e^{x} - 1 \). This cost function satisfies all the properties assumed in our model.

Figure 3.1 depicts the welfare function for the cost function \( C(d) = e^{x} - 1 \) with parameter values \( h = 3, p = 0.9, r = 0.6 \).

*Figure 3.1: Shape of the example welfare function*
This specific example motivates us to find the shape of $W$ for any cost function which is increasing and strictly convex with $C(0) = 0$. We know from Proposition 3.3, $\sigma^2 = \frac{A^2}{n} p(1 - p)$. Thus as $n \to \infty$, $\sigma^2 \to 0$ and hence $\Gamma(n) \to C(1 + r) + \mu C'(1 + r)$, which is constant. This in turn implies that for large values of $n$ the welfare function is increasing and converges asymptotically. However we do not know the exact shape of the welfare function. Figure 3.2 depicts a particular form that the welfare function can possibly assume.

Now suppose that there is a small cost of maintaining each link. Let $\theta$ be the cost of maintaining a link where $\theta > 0$. Hence an individual with $n$ neighbors has to spend $n\theta$ amount of resources to maintain her links. This cost may or may not be non pecuniary in nature. Pecuniary costs may involve exchanging gifts among neighbors while non-pecuniary costs may come from socializing with them (opportunity cost of labor or compromise on leisure). This motivates us to the next proposition.

**Proposition 3.4:** If $C(d) > d$ and $C'' > 0$ and there is a small cost $\theta$ of maintaining each link, then there is an optimal number of links, $n^*$.

**Proof:** We have already established that $W$ increases with $n$ and converges to a constant as $n \to \infty$. Since there is a cost $\theta$ for maintaining each link an individual will maintain the number

![Figure 3.2: Shape of the welfare function](image)
of links where the marginal gain from an additional link is no less than the marginal cost. The 
optimal number of links is obtained at the point after which the marginal gain of an additional 
link is less than $\theta$.

Since, $\Gamma(n) \rightarrow C(1+r)+\mu C'(1+r)$, there exists a $n^*$ such that for all $n > n^*$, $\Gamma(n+1)-\Gamma(n) < \theta$. 
In order to obtain the optimal number of links we pick the minimum of such $n^*$s. ■

![Figure 3.3: Optimal number of links](image)

Abusing the technical intricacies, figure 3.3 roughly depicts the optimal number of links $n^*$ 
beyond which the marginal benefit is less than the cost of maintaining an additional link.

Proposition 3.3 along with Proposition 3.4 conveys the main results of the paper. The results 
remain qualitatively unaltered if we relax the “no leftover” condition. Under the assumption 
$1 + r < h/2$, we redefine the utility function of an individual as

$$u_i(g, y) = \begin{cases} 
    h - (1 + r) - \sum x & \text{if successful} \\
    -C(d) + surplus & \text{if unsuccessful} 
\end{cases}$$
An utility maximizing individual enjoys a surplus of \(kx - (1 + r)\) when she is unsuccessful, \(k\) of her neighbors are successful and \(kx > 1 + r\). For punishment costs satisfying \(C(d) > d\) and \(C' > 1\), the optimal insurance level remains at \(x^* = A/n\) as obtained in Lemma 3.1. The welfare function \(W\) is now a concave function with a kink and is no longer strictly concave. It can be easily shown that the lottery \(z_{n+1}\) as defined in the proof of Proposition 3.3, second order stochastically dominates \(z_n\). Hence, individual welfare in equilibrium is non decreasing in \(n\). Moreover individual welfare is bounded above implying that there exists an optimal number of links \(n^*\) when there is a small cost of maintaining each link. Though this result holds true for any general regular network, it has an important policy implication in the context of group loans with individual liability. This potentially provides a guidance for an optimal group size that individuals need to maintain in order to get a loan.

### 3.4 Relaxing the Budget Constraint

Under assumption 1, the choice of \(x\) is restricted to satisfy the budget constraint of an individual. This implies that if an individual is successful and all her neighbors are unsuccessful then the individual is able to keep her promises. Now suppose an individual enters into an arrangement such that her total promise exceeds the budget when some or all of her neighbors are unsuccessful. Individuals may want to promise higher values of \(x\) as they can be thought of as better insurance possibilities. However in equilibrium promises must be kept as links are severed as punishment.

Suppose players have a common discount factor \(\delta \in (0, 1)\). We argue that if players are patient enough then the budget constraint is satisfied endogenously. This leads us to our next proposition.

**Proposition 3.5:** Suppose \(n \leq n^*\). Consider the following strategy. An individual pays \(x = A/n\) to her unsuccessful neighbors whenever she is successful. She severs link with her neighbor whenever she finds out that her neighbor has not kept her promise. As \(\delta \to 1\), the above strategy constitutes a Nash Equilibrium.

**Proof:** See Appendix B.

The argument that supports the above proposition is as follows. Suppose individuals’ promises are such that there exist events where promises can not be kept and these events occur with
positive probabilities. An individual who can not keep her promise is revealed to be successful with probability $\beta$. Given the strategy specified in the above proposition, the neighbors who do not receive the promised value sever links with the individuals who did not keep their promises. Since, links are valuable insurance possibilities, long term gains from a link exceeds short term gains from deviation.

The natural question that arises here is whether the above strategy constitutes a Subgame Perfect Equilibrium (SPE). Once a neighbor does not keep her promise, it is necessary to check whether the link will indeed be severed. Now consider the following strategy. A successful individual keeps her promise, i.e., she pays $x$ to her unsuccessful neighbors whenever she is successful. Whenever an individual finds out that her neighbor did not keep her promise the link in question is severed. An individual severs her link with a neighbor whenever her neighbor is found out to deviate from the actions prescribed above.

For the above strategy to be SPE we need some degree of observability of actions. Links in our model are bilateral in nature and the transfers are private to the links. To implement the punishment strategy we need transactions to be verifiable. If there is a positive probability of information leakage from which one’s neighbors can infer if she has deviated from the equilibrium strategy, then the above strategy can be sustained as SPE. However, as discussed in Proposition 3.5 for Nash Equilibrium to hold we do not require any form of information leakage particular to bilateral transactions.

Consider an individual $i$ whose neighbor $j$ has not kept her promise, i.e. $j$ did not transfer the promised value $x$ even when she was successful and $i$ was not. Now, $i$’s optimal response is to sever the link with $j$ if she finds out that $j$ did not keep her promise. If $i$ instead decides not to sever the link with $j$, and her neighbors find this out, then all $i$’s neighbors sever their links with $i$. Losing all the links is costlier than losing a single link. From the point of view of $i$’s neighbors, they will not deviate from the prescribed action as deviation will make them lose all their links. Repeating the argument in Proposition 3.5, one can show that the strategy prescribed above is indeed a subgame perfect equilibrium.
3.5 Collusion

In a world with no frictions, if there is a benevolent non-profit microcredit organization providing credit, then resources would be efficiently used so that individuals maximize their joint welfare. However frictions cannot be ignored in the real world. In our model, frictions impede individuals from maximizing joint welfare through punishment sharing. In this section we abstract away from such frictions and look for the first best solution where individuals can collude to share punishment.

Since cost of default is assumed to be a continuous convex function, punishment sharing can be welfare improving. Suppose there are two individuals and we allow for collusion among individuals in the form of punishment sharing. In our earlier setting individuals pay back their entire loan whenever they are successful and the successful individual helps her unsuccessful neighbor with \( x^* = [h-(1+r)] \). Consequently, the successful individual does not face punishment from the bank while her unsuccessful neighbor faces the punishment cost \( C(1 + r - x^*) \). However the successful individual can help out her neighbor with some additional \( \varepsilon > 0 \) for which she faces a punishment \( C(\varepsilon) \) and her neighbor’s punishment reduces to \( C(1 + r - x^* - \varepsilon) \). Since cost is convex, for \( \varepsilon \) small enough, the reduction in the unsuccessful individual’s punishment is larger than the punishment cost faced by the successful individual. Thus individuals can gain by entering into arrangements that allow for punishment sharing.

Suppose the arrangement between two individuals is given by \( x \) which specifies the amount a successful individual transfers to her unsuccessful neighbor. Now the welfare function of an individual is given by

\[
W = p [h - (1-p)C(1 + r - h + x)] - (1-p) [(1-p)C(1 + r) + pC(1 + r - x)]
\]

Welfare maximization leads to the optimal arrangement \( x^{**} = h/2 \). Punishment is shared up to the point where both individuals face the same level of punishment. Notice that when only one individual is successful, the bank collects \( h \) in the form of repayments. This is the same amount the bank collects under individual liability loans with no punishment sharing.

The same argument holds when an individual has \( n \) neighbors with whom she enters into pun-
ishment sharing arrangements. The welfare of an individual with \( n \) neighbors is then given by

\[
W = p[h - \sum_{k=0}^{n} p^k(1 - p)^{n-k}C(1 + r - h + (n - k)x)] \\
-(1 - p)[\sum_{k=0}^{n} p^k(1 - p)^{n-k}C(1 + r - kx)]
\]

and the optimal arrangement is given by \( x^{**} = h/(n + 1) \). When the number of successful individuals is \( m \), the bank collects \( m \times h \) which is same as the amount the bank collects under individual liability without punishment sharing.

The optimal punishment sharing arrangement is conceptually equivalent to joint welfare maximization of a group of individuals. In other words, individuals behave as a “composite agent” who minimizes joint expected punishment. Joint welfare maximization is the first best solution that joint liability aims to enforce. Joint liability is sometimes justified as a way to encourage the group members to help each other in bad times by “formalizing” the idea of mutual insurance. However internal frictions often impede this kind of collusive behavior. Our analysis in Section 3.3 provides an alternative model of risk mitigation where internal frictions preclude individuals from punishment sharing. The ex post collection of the bank in equilibrium is same as the amount it collects under the first best solution.

3.6 Discussion

In this section we briefly discuss important issues like moral hazard and adverse selection in the context of our model. The following discussion suggests that stricter punishments are essential to remove moral hazard and sustain the all effort equilibrium where every member of the group exerts high effort. When faced with the adverse selection problem, stricter punishments might help in removing social segregation.

3.6.1 Moral Hazard

When output depends on the choice of effort, an individual’s expected payoff not only depends on her own action but also depends on the action of her neighbors. Since neighbors act as insur-
ance possibilities, neighbors’ choice of effort can affect an individual’s payoff in two ways. The expected amount a successful individual transfers to her unsuccessful neighbor decreases with the neighbor’s effort choice while the expected transfer an unsuccessful individual receives from her successful neighbor increases with the choice of effort. Thus an individual has incentives to take remedial action against a neighbor who does not put effort. However when peer monitoring is prohibitively costly strict punishment schemes may alleviate the moral hazard problem as discussed below.

Suppose there are \( n + 1 \) individuals. Each individual puts effort \( e \in \{0, 1\} \). When an individual puts effort \( e = 1 \), the probability of success is given by \( p \) whereas when \( e = 0 \), the probability of success is given by \( q \), \( p > q \). Let \( c > 0 \) be the cost of putting effort.

Let \( \gamma \) be the probability of putting effort and \( \gamma^* \in [0, 1] \) be the symmetric equilibrium effort choice. Let \( p' = \gamma p + (1 - \gamma)q \) be the probability of success of an individual putting effort with probability \( \gamma \).

Given other players play the mixed strategy \( \gamma \), an individual’s gross payoff from putting effort is given by

\[
\pi_1(p') = p (h - (1 + r)) - p * T_{x^*}(p') - (1 - p) Z_{x^*}(p')
\]

The payoff of the individual from not putting effort is given by

\[
\pi_0(p') = q (h - (1 + r)) - q * T_{x^*}(p') - (1 - q) Z_{x^*}(p')
\]

where, \( T_{x^*}(\cdot) \) is the total expected transfer an individual makes to her unsuccessful neighbors when she is successful given an arrangement \( x^* \). \( Z_{x^*}(\cdot) \) is the expected punishment an individual faces when she is unsuccessful under the same arrangement \( x^* \).

Suppose \( \gamma^* \in [0, 1] \) be the symmetric equilibrium strategy (assuming existence). Now \( p^* = \gamma^* p + (1 - \gamma^*) q \). Notice that the previous analysis (about optimal insurance and optimal network size) carries over with \( p = p^* \) and \( x^* = \frac{h - (1 + r)}{n} \).

Since we know \( p^* \) and \( x^* \), we can compute \( T_{x^*}(p^*) \) and \( Z_{x^*}(p^*) \). Now
\[
\pi_1(p^*) - \pi_0(p^*) = (p - q)(h - (1 + r)) - (p - q)T_{x^*}(p^*) + (p - q)Z_{x^*}(p^*)
\]

Since \( h - (1 + r) - T_{x^*}(p^*) \geq 0 \), \( \pi_1(p^*) - \pi_0(p^*) > 0 \).

For an individual to be indifferent between \( e = 1 \) and \( e = 0 \), we must have

\[
\pi_1(p^*) - \pi_0(p^*) = c
\]  
(34)

Observe that \( T_{x^*}(\cdot) \) and \( Z_{x^*}(\cdot) \) are decreasing functions of \( \gamma \). This is because as neighbors of an individual put more effort, they are unsuccessful with a lower probability and hence the expected transfer is lower. Similarly, a higher \( \gamma \) works as a better insurance possibility. Neighbors putting more effort are successful with higher probability and hence reduces expected punishment of an individual who is unsuccessful. Also \( \pi_1(\cdot) - \pi_0(\cdot) \) is decreasing in \( \gamma \) as \( Z_{x^*}(\cdot) \) decreases at a higher rate than \( T_{x^*}(\cdot) \). This is because transfers are linear and cost of punishment is convex.

\textbf{Equilibrium Analysis:} For \( \gamma^* \in [0, 1] \) to be a symmetric equilibrium, (34) must be satisfied. It is evident from equation (34) that a symmetric mixed strategy equilibrium will exist only for certain parameter values. For example, if \( p \) and \( q \) are not significantly different, then an interior solution will exist only for small values of \( c \).

An all effort equilibrium exists if \( (p - q)(h - (1 + r)) - (p - q)T_{x^*}(p) + (p - q)Z_{x^*}(p) \geq c \). This condition implies that it is optimal for an individual to put effort with probability one when all her neighbors do the same. All individuals put effort in this symmetric pure strategy Nash equilibrium.

Let \( \gamma^* \in (0, 1) \) be a symmetric mixed strategy equilibrium, i.e. \( \pi_1(p') - \pi_0(p') = c \). Now suppose the cost of default \( C(d) \) becomes steeper for all default levels maintaining \( C(0) = 0 \). At \( \gamma^* \), \( Z_{x^*} \) increases making \( \pi_1 - \pi_0 > c \). Since, \( \pi_1 - \pi_0 \) is decreasing in \( \gamma \) the new equilibrium (assuming existence) \( \gamma^{**} > \gamma^* \). This implies that steeper punishments lead to higher equilibrium effort.

When peer monitoring is not prohibitively costly and individuals are sufficiently patient a high effort equilibrium can be sustained through a different mechanism. An individual can punish by
severing the link with her neighbor who shirks. Suppose the cost of monitoring each neighbor is \( \epsilon \) which reveals the neighbor’s actions with probability \( \epsilon \). Using similar argument as in the proof of Proposition 3.5, high effort equilibrium can be sustained as \( \delta \to 1 \).

### 3.6.2 Segregation

So far our analysis suggests when the economy consists of homogeneous individuals, a larger network corresponds to insurance possibilities no worse than a smaller network. However in reality, the economy may not consist of homogeneous agents and this may lead to different implications in the context of our model. We capture heterogeneity among individuals by introducing differences in productivity. A high productive individual has a higher probability of success than that of a low productive individual.

Our previous analysis relies on an individual’s willingness to engage herself in informal insurance arrangements with her neighbor. In a homogeneous population with linear or convex costs of default, individuals are never worse off with more links. In a heterogenous economy, a low productive individual fails to act as an insurance possibility as good as a high productive individual. Since a low productive individual fails more frequently, the high productive individual has to help her low productive neighbor more often than being helped by her neighbor. This may preclude the high productive individuals from entering into arrangements with low productive neighbors leading to social segregation. In a segregated society, individuals end up keeping links only with individuals of similar ability. However as already discussed in Section 3.3, larger number of links imply better insurance possibility, severe punishment schemes adopted by the bank may trigger the need for insurance. This in turn encourages high productive individuals to keep links with low productive neighbors and hence eliminate social segregation.

Suppose a high(low) productive individual is successful with probability \( p(q) \), where \( p > q \). Also suppose there is one high productive individual and one low productive individual in the economy.

The expected welfare of the high productive individual under autarky is

\[
W_A = p(h - (1 + r)) - (1 - p)C(1 + r)
\]
Expected welfare of the high productive individual when she has a link with the low productive individual

\[ W_L = p \left[ h - (1 + r) - (1 - q)x \right] - (1 - p) \left[ (1 - q)C(1 + r) + qC(1 + r - x) \right] \]

Now

\[ W_A - W_L = (1 - p)qC(1 + r - x) - C(1 + r) \]  
\[ \left\{ \begin{array}{cl} \text{(-ve)} & \text{if } 1 - p < p(1 - q) \text{ and is convex} \\ \text{(+ve)} & \text{if } 1 - p > p(1 - q) \text{ and is convex} \end{array} \right. \]

Given \( p > q \), we have \( (1 - p)q < p(1 - q) \). Therefore, \( |C(1 + r - x) - C(1 + r)| \) has to be sufficiently higher than \( x \) for \( W_L \) to be larger than \( W_A \). The strictness of the punishment function is captured by the degree of convexity of \( C(\cdot) \). More convex the punishment function higher is the welfare gain from having a link. With linear punishment functions of the form \( C(d) = \alpha d, \alpha > 1 \), social segregation is eliminated for values of \( \alpha \) higher than \( p(1 - q)/q(1 - p) \).

### 3.7 Conclusion

This study is primarily motivated by the success of Grameen II, individual liability loans in particular. Although joint liability facilitated mutual insurance by formalizing it to a great extent, it is reasonable to think that informal insurance plays a significant role under individual liability lending as well. Individuals help each other in bad times and informal insurance is facilitated by repeated interactions among individuals.

We investigate the role of networks when individuals are not able to enter into contracts which maximize joint welfare for reasons exogenous to the model. We show that informal arrangements play an important role in protecting individuals from idiosyncratic shocks. When the punishment cost is greater than the default amount and is convex, individuals opt for maximum possible insurance. This in equilibrium leads to the same repayment rates as would be observed if individuals could share punishments and maximize joint welfare.

This paper not only provides an alternative explanation for the success of individual liability loans, it also takes up an important policy question that has not been addressed by the existing
literature. In an attempt to provide an alternative explanation of Grameen II we emphasize on
the importance of bilateral arrangements in a given regular network. This set up in the context
of micro-lending is novel in the sense that most of the papers in this literature only deal with
groups containing two individuals.

It has been observed in Grameen II and its various replicas that individuals are encouraged to
maintain an implicit group structure among themselves even under individual liability. A natural
question to ask is if there is an optimal group size that maximizes individual’s welfare. Since
under convex costs insurance possibility increases with the number of links an individual has,
should the entire village act a single group? Does it really take a village to maximize individual’s
welfare? The answer is “no”. Marginal welfare declines as the number of links increases, and
if there is a small cost of maintaining each link, only a finite number of neighbors maximize
welfare.

The analysis in this paper thus potentially provides a guidance for the optimal group size that
needs to be implicitly maintained by the villagers. In accordance with the punishment scheme
adopted by the bank a social planner can adopt policies to encourage villagers to maintain the
optimum group size. Our results however remains true for any regular network and finding the
optimal group size is one mere application of a more general result.
Appendix A

Proof of Proposition 3.2: Given this cost structure we can write the welfare of an individual as

\[ W = -p \left[ -(h - (1 + r)) + (1 - p)n_0n_1 + \left( \frac{n_1}{n} \right) p(1 - p)^{n-1}(n - 1)x + \ldots + p^n0 \right] \\
- (1 - p) \left[ (1 - p)^nC(1 + r) + \left( \frac{n}{1} \right) p(1 - p)^{n-1}C(1 + r - x) + \ldots + p^nC(1 + r - nx) \right] \\
= -p \left[ -(h - (1 + r)) + (1 - p)n_0n_1 + \left( \frac{n_1}{n} \right) p(1 - p)^{n-1}x + \ldots + p^n0 \right] \\
- (1 - p)\alpha(1 + r) + (1 - p) \left[ \left( \frac{n}{1} \right) p(1 - p)^{n-1}\alpha x + \ldots + p^n\alpha nx \right] \\
\]

From Lemma 3.1 we know that the optimal level of insurance will be \( x^\star = \frac{A}{n} \). We now evaluate the welfare of an individual at this level of insurance.

Let \( \alpha = 1 + \gamma \) where \( \gamma > 0 \).

Thus,

\[ W = p[h - (1 + r)] - (1 - p)\alpha(1 + r) + (1 - p)\gamma \left[ \left( \frac{n}{1} \right) p(1 - p)^{n-1} + \ldots + p^n0 \right] \frac{A}{n} \\
= p[h - (1 + r)] - (1 - p)\alpha(1 + r) + (1 - p)\gamma \left[ \sum_{r=0}^{n} \left( \frac{n}{r} \right) p^r(1 - p)^{n-r}r \right] \frac{A}{n} \\
= p[h - (1 + r)] - (1 - p)\alpha(1 + r) + (1 - p)\gamma n_0A \frac{A}{n} \\
= p[h - (1 + r)] - (1 - p)\alpha(1 + r) + (1 - p)\gamma pA \]

So the welfare is greater than autarky by the amount \( (1 - p)\gamma pA \) which is positive. Moreover the welfare level is independent of \( n \) i.e. the size of the network. \( \blacksquare \)

Proof of Proposition 3.3: The welfare function is given by

\[ W = -p \left[ -(h - (1 + r)) + (1 - p)n_0n_1 + \left( \frac{n_1}{n} \right) p(1 - p)^{n-1}(n - 1)x + \ldots + p^n0 \right] \\
- (1 - p) \left[ (1 - p)^nC(1 + r) + \left( \frac{n}{1} \right) p(1 - p)^{n-1}C(1 + r - x) + \ldots + p^nC(1 + r - nx) \right] \\
= p[h - (1 + r)] - (1 - p) \left[ \sum_{k=0}^{n} \left( \frac{n}{k} \right) p^k(1 - p)^{n-k} (C(1 + r - kx) + kx) \right] \\
\]
Substituting the optimal value of $x$ in the welfare function from Lemma 3.1 we get,

$$W = p[h - (1 + r)] - (1 - p) \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} \left( C(1 + r - k \frac{A}{n}) + k \frac{A}{n} \right)$$

$$= p[h - (1 + r)] - (1 - p) \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} C(1 + r - k \frac{A}{n}) - (1 - p) \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} k \frac{A}{n}$$

$$W = p[h - (1 + r)] - (1 - p)pA - (1 - p) \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} C(1 + r - k \frac{A}{n}) \tag{A1}$$

Now we want to show that in equation (A1) the last term is decreasing in $n$.

Let

$$\Gamma(n) = \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} C(1 + r - k \frac{A}{n}). \tag{*}$$

Define $z_n$ to be a lottery with outcomes $\{0, \frac{A}{n}, \frac{2A}{n}, ..., A\}$ following binomial distribution with parameters $n$ and $p$. Given an individual has $n$ neighbors, the $(k+1)^{th}$ outcome of $z_n$ which equals $\frac{kA}{n}$, represents the amount an unsuccessful individual receives when $k$ of her neighbors are successful.

Hence, we can rewrite (*) as $\Gamma(n) = EC((1 + r) + z_n)$.

Taking second order Taylor series expansion of $C$ around $(1 + r)$ we get,

$$\Gamma(n) = EC((1 + r) + z_n)$$

$$\approx E \left[ C(1 + r) + C'(1 + r) z_n + \frac{1}{2} C''(1 + r) z_n^2 \right]$$

$$= E[C(1 + r)] + C'(1 + r) E(z_n) + \frac{1}{2} C''(1 + r) E(z_n^2)$$

$$= C(1 + r) + \mu C'(1 + r) + \frac{1}{2} (\mu^2 + \sigma^2) C''(1 + r)$$

where, $\mu = Ap$ and $\sigma^2 = \frac{A^2}{n} p(1 - p)$.

As $n$ increases the mean of the lotteries remain unchanged while the variance decreases. Since $C'' > 0$, the expected cost $\Gamma(n)$ decreases. Hence welfare increases as $n$ increases. ■
Appendix B

Proof of Proposition 3.5: We know that if the budget constraint is satisfied then the optimal insurance is $A/n$. Now we want to see if two individuals have incentives to enter into an arrangement $(A/n + \varepsilon)$ where $\varepsilon > 0$. This means under certain events which occur with positive probability at least one link is severed.

Let $E_{\varepsilon}$ denote the set of events where no link is severed under the arrangement $(A/n + \varepsilon)$. Let $E'_{\varepsilon}$ denote the set of events where a link is severed i.e. an individual fails to keep her promise when she is successful and her true type is revealed. $E'_{\varepsilon}$ may correspond to situations where all neighbors of an individual is not successful and the individual is successful. In such a situation the successful individual fails to keep her promise to at least one of her neighbors and her true state is revealed with probability $\beta$. It also includes situations where the individual is unsuccessful, her neighbor is successful and all her neighbor’s neighbors are unsuccessful.

Let $\gamma$ be the probability that $E'_{\varepsilon}$ occurs and $(1 - \gamma)$ be the probability that $E_{\varepsilon}$ occurs.

Let $V_{\varepsilon}$ be the normalized expected discounted payoff under the arrangement $(A/n + \varepsilon)$ and $\overline{V}$ be the normalized expected discounted payoff under the arrangement $A/n$.

Let $V_0$ be the normalized expected discounted payoff when a link is severed. Also let $\overline{\sigma}$ and $\overline{\pi}$ be the one period payoffs under $(A/n + \varepsilon)$ and $A/n$ respectively.

Suppose $\overline{\sigma} > \overline{\pi}$, i.e. one period deviation is profitable. Notice that, $\overline{V} = \overline{\pi} > V_0$. This follows from the assumption $n \leq n^*$.

Now,

$$V_{\varepsilon} = (1 - \delta)\overline{\sigma} + \delta \gamma V_0 + \delta (1 - \gamma) V_{\varepsilon}$$

$$V_{\varepsilon} = \frac{1 - \delta}{1 - \delta (1 - \gamma)} \overline{\sigma} + \frac{\delta \gamma}{1 - \delta (1 - \gamma)} V_0$$

Subtracting $\overline{V}$ from $V_{\varepsilon}$,

$$V_{\varepsilon} - \overline{V} = \frac{1 - \delta}{1 - \delta (1 - \gamma)} (\overline{\sigma} - \overline{\pi}) + \frac{\delta \gamma}{1 - \delta (1 - \gamma)} (V_0 - \overline{\pi})$$
As $\delta \to 1$, $1 - \delta(1 - \gamma) \to \gamma$ implying $\frac{1 - \delta}{1 - \delta(1 - \gamma)} \to 0$ and $\frac{\delta \gamma}{1 - \delta(1 - \gamma)} \to 1$. Therefore, in the limit $V_e - \nabla < 0$.

This proves our proposition.
References


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